Lecture 3

Charge form factor

$(E, \vec{P})$ $= \dfrac{i}{\eta\left(E - \vec{P}^2/2M_{nc}\right) + \Delta} = i\, G_{o\,\text{dimer}}(E, \vec{P})$

Properly normalized state

$$G_{o\,\text{dimer}}(E, \vec{P}) = \frac{1}{E - \frac{\vec{P}^2}{2M_{nc}} + \Delta}$$
$(\eta = 1)$

$D_d(E, \vec{P}) =$  $= \dfrac{i}{\eta\left(E - \frac{\vec{P}^2}{2M_{nc}}\right) + \Delta - \Sigma_d(E, \vec{P})}$

$(E, \vec{P})$ $= -\dfrac{2\pi}{m_R} \dfrac{1}{-\frac{1}{a_0} + \frac{1}{2} r_0 k^2 - ik}$

$$g^2 D_d(E, \vec{P}) = -\frac{2\pi}{m_R} \frac{1}{-\frac{1}{a_0} + \frac{1}{2} r_0 k^2 - ik}$$

$$= \frac{Z}{E + B}$$

for $E \simeq -B$

$E = k^2/2m_R$

$\vec{P} = \vec{0}$

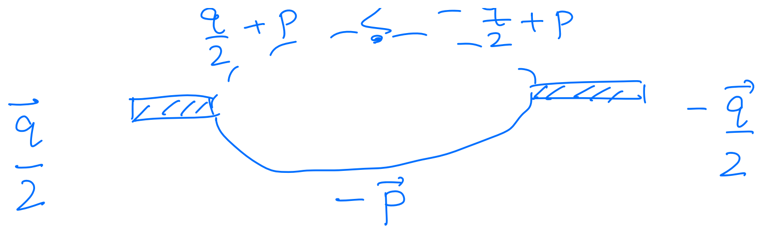
from Prof. Hammer's Exercise 10

with $Z = \dfrac{2\pi}{m_R^2} \dfrac{\gamma}{1 - \gamma r_0}$

$$\Rightarrow D_d(E, \vec{P}) = \frac{Z_d}{E + B}$$

$$Z_d = \frac{2\pi\gamma}{m_R^2 g^2} \frac{1}{1 - r_0\gamma} = \frac{\pi}{m_R^2 g^2} A_0^2$$

$\vec{q}$



$$-B+\frac{\vec{q}^{\,2}}{8M_{nc}}$$

$$\frac{i}{p_0-\vec{p}^{\,2}/2M_c}$$

$$\frac{i}{p_0-\vec{p}^{\,2}/2M_n}$$

$$-i|e|Z_c$$

$$i\partial_0 \longrightarrow iD_0 = i\partial_0 - |e|Z_c A_0$$

$$i\mathcal{M}_{(a)} = \cancel{-i|e|}\,\cancel{Z_c}\,Z_d\,(-ig)^2$$

$$\int\frac{d^4p}{(2\pi)^4}\;\frac{i}{-B-p_0-\frac{(\frac{\vec{q}}{2}+\vec{p})^2}{2M_c}+\frac{\vec{q}^{\,2}}{8M_{nc}}+i\varepsilon}$$

$$\frac{i}{-B-p_0-\frac{(\frac{\vec{q}}{2}-\vec{p})^2}{2M_c}+\frac{\vec{q}^{\,2}}{8M_{nc}}+i\varepsilon}\;\frac{i}{p_0-\frac{\vec{p}^{\,2}}{2M_n}+i\varepsilon}$$

$$\equiv \cancel{-i|e|}\,\cancel{Z_c}\,G_c^{(a)}(|\vec{q}|)$$

$$G_c^{(a)}(|\vec{q}|) = Z_d\, g^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{-B + \cancel{\frac{\vec{q}^{\,2}}{8M_{nc}}} - \cancel{\frac{\vec{q}^{\,2}}{8M_{nc}}} - \frac{(\vec{p} + f\vec{q}/2)^2}{2m_R}}$$

$$\frac{1}{-B - \frac{\left(\vec{p} - \frac{f\vec{q}}{2}\right)^2}{2m_R} + \frac{\vec{q}^{\,2}}{8M_{nc}} - \frac{\vec{q}^{\,2}}{8M_{nc}}}$$

$$f = \frac{m_n}{M_{nc}} = \frac{m_R}{M_c} \approx \frac{1}{A+1}$$

$$= 4 m_R^2\, g^2 Z_d \int \frac{d^3p}{(2\pi)^3} \frac{1}{2m_R B + (\vec{p} + f\vec{q}/2)^2} \frac{1}{2m_R B + \left(\vec{p} - \frac{f\vec{q}}{2}\right)^2}$$

$$= 4\pi A_0^2 \int \frac{d^3p}{(2\pi)^3} \int d^3r\, e^{i\left(\vec{p} + \frac{f\vec{q}}{2}\right)\cdot\vec{r}} \frac{e^{-\gamma r}}{4\pi r} \int d^3r'\, e^{-i\left(\vec{p} - \frac{f\vec{q}}{2}\right)\cdot\vec{r}\,'} \frac{e^{-\gamma r'}}{4\pi r'}$$

$\vec{p}$ integral $\displaystyle\int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot(\vec{r}-\vec{r}')} = \delta^{(3)}(r-r')$

$$= \cancel{4\pi} A_0^2 \int d^3r \, e^{if\vec{q}\cdot\vec{r}} \frac{e^{-\gamma r}}{\cancel{4\pi} r} \frac{e^{-\gamma r}}{4\pi r}$$

$$= \frac{A_0^2}{2} \int dr \, \cancel{r^2} \int_{-1}^{1} dx \, e^{ifqrx} \frac{e^{-2\gamma r}}{\cancel{r^2}}$$

$$= A_0^2 \int dr \, e^{-2\gamma r} j_0(fqr)$$

$$j_0(x) = \frac{\sin x}{x}$$

$$G_c^{(c)} = \frac{A_0^2}{fq} \arctan\left(\frac{fq}{2\gamma}\right)$$

$$= \frac{A_0^2}{fq} \frac{fq}{2\gamma} - \frac{A_0^2}{fq} \left(\frac{fq}{2\gamma}\right)^3 \frac{1}{3} + \cdots$$

$$= \frac{A_0^2}{2\gamma} - \frac{A_0^2 \rho^2 q^2}{24\gamma^3}$$

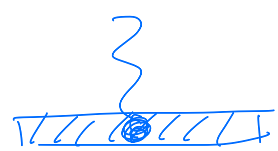
$$\frac{1}{1-\gamma_0 r_0} = 2\gamma$$

$$\langle r_E^2 \rangle = \frac{A_0^2 \rho^2}{4\gamma^3} = \frac{\rho^2}{2\gamma^2} \text{ at LO}$$

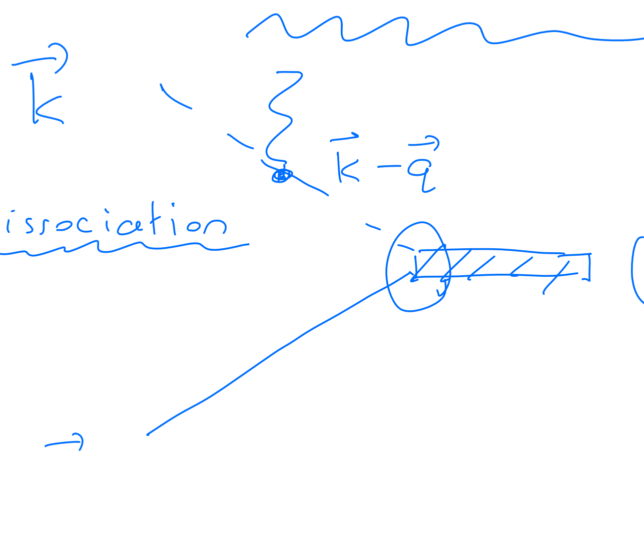
Charge $\neq 1$ at NLO!

Feynman rule associated with A_0 photon coupling directly to dimer $\Rightarrow$ for $\eta < 1$ additional negative contribution to probability density, all at $\vec{r} = \vec{0}$

$$d^\dagger \eta\, i\partial_0 d \longrightarrow d^\dagger i\partial_0 d - |e|\eta Z_c d^\dagger A_0 d$$



Photodissociation



$$\left(-B + \frac{\vec{q}^{\,2}}{2M_{nc}},\ -\vec{q}\right)$$

$-k$

$$-i\mathcal{M} = \sqrt{Z_d}\,(-ie|Z_c|)\,(-ig)\,\frac{i}{-B+\frac{\vec{q}^{\,2}}{2M_{nc}} - \frac{(\vec{k}-\vec{q})^2}{2M_c} - \frac{k^2}{2M_n}}$$

$$= \sqrt{Z_d}\,(-ie|Z_c)(ig)\,\frac{2m_R}{\gamma^2+(\vec{k}-f\vec{q})^2}$$

$$= \sqrt{4\pi}\,A_0\,|e|\,Z_c\,\frac{1}{\gamma^2+(\vec{k}-f\vec{q})^2}$$

$$\xrightarrow{LO} \sqrt{8\pi\gamma}\,|e|\,Z_c\,\frac{1}{\gamma^2+(\vec{k}-f\vec{q})^2}$$

3-body problem

$$E_{12} = E - \frac{\nabla_3^2}{2m_3}\,|\vec{q}_3\rangle$$

$$= E - \frac{q_3^2}{\;}$$

$2m_3$

$$\vec{P}_{12} = -\vec{q}$$

$$\therefore E_{rel} = E - \frac{q_3^2}{2m_3} - \frac{q_3^2}{2(m_1+m_2)}$$

$$\frac{1}{\nu_3} = \frac{1}{m_3} + \frac{1}{m_1+m_2} \equiv E - \frac{q_3^2}{2\nu_3}$$

$$\left[-\frac{\nabla_{12}^2}{2\mu_{12}} + V_{12}(\vec{r}_{12})\right]\chi_3(\vec{r}_{12})$$

$$= \left(E - \frac{q_3^2}{2\nu_3}\right)\chi_3(\vec{r}_{12})$$