Effective Field Theory for Halo Nuclei: Lecture 3

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The story so far

- Halo EFT seeks to describe halo nuclei for $k \sim 1/R_{halo} \ll 1/R_{core}$, i.e., $kr_0 \ll 1$.

- We are interested in shallow bound states (or virtual bound states), states for which $\gamma r_0 \ll 1 \Leftrightarrow r_0/a_0 \ll 1$

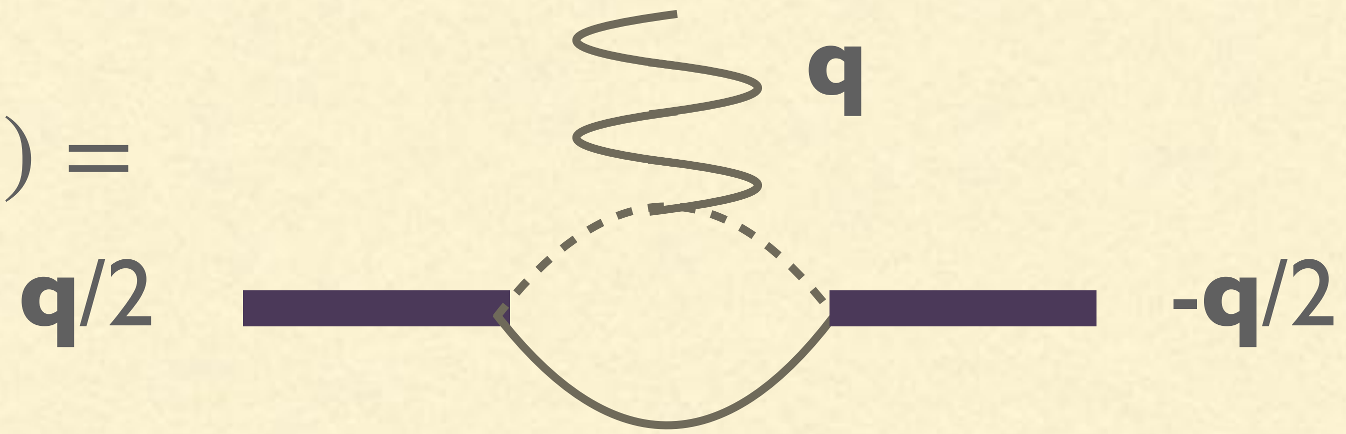
$$t_0^{NLO}(E) = -\frac{2\pi}{m_R}\frac{1}{-\frac{1}{a_0} + \frac{1}{2}r_0 k^2 - ik}$$

- Predicts a shallow bound state with co-ordinate space wave function $\psi(\mathbf{r}) = \frac{A}{\sqrt{4\pi}}\frac{e^{-\gamma r}}{r}$

- $A^2 = \frac{2\gamma}{1 - \gamma r}$

Electric form factor

$$-i|e|Z_c G_c^{NLO}(|\mathbf{q}|) =$$



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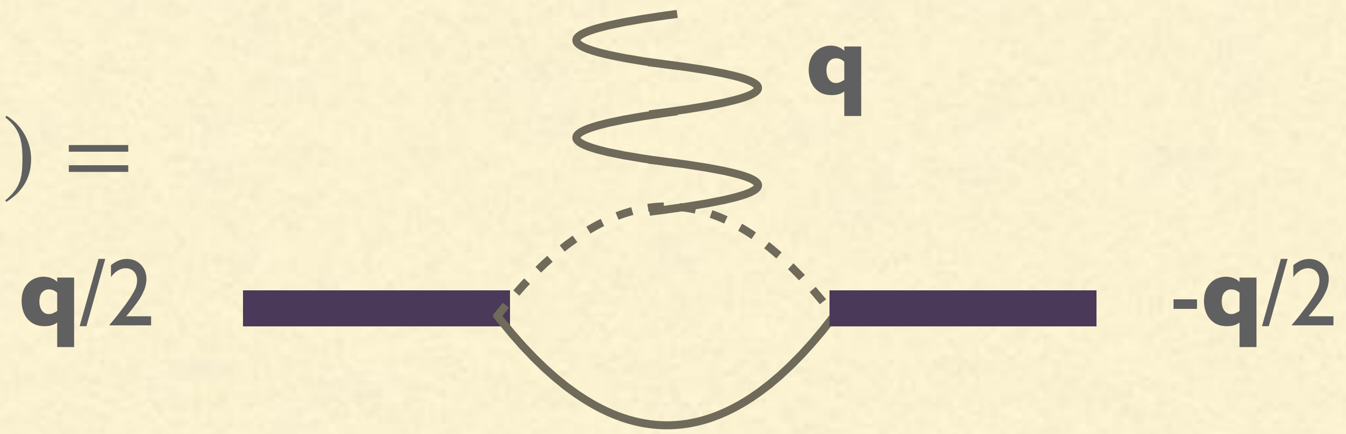

$$G_c(|\mathbf{q}|) = \frac{A_0^2}{f|\mathbf{q}|} \arctan\left(\frac{f|\mathbf{q}|}{2\gamma}\right)$$

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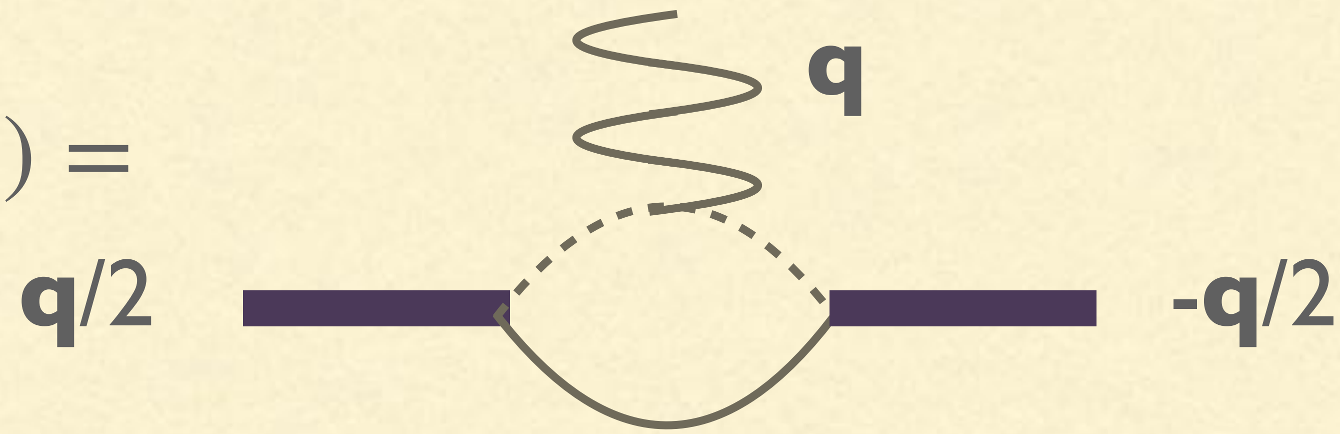

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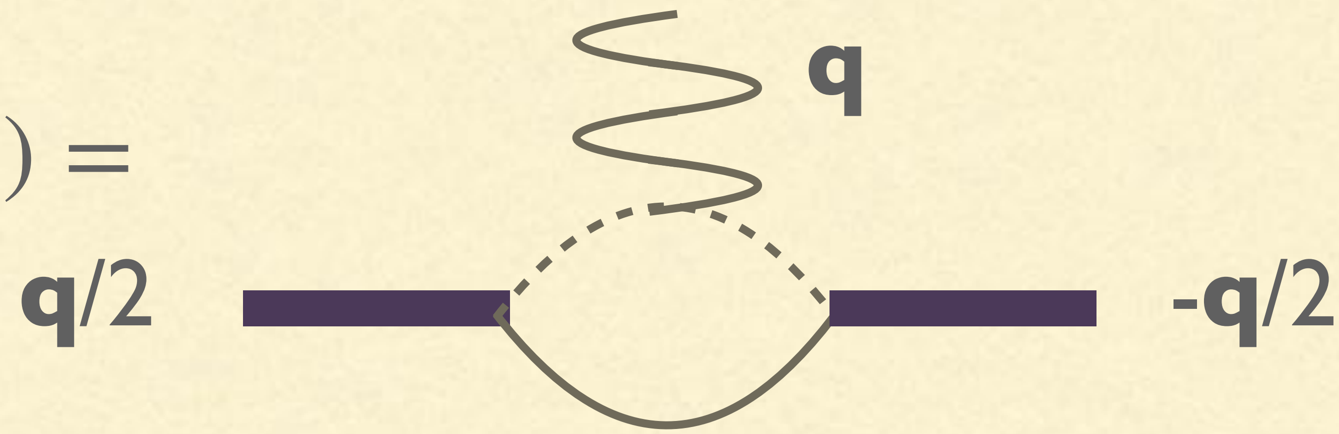

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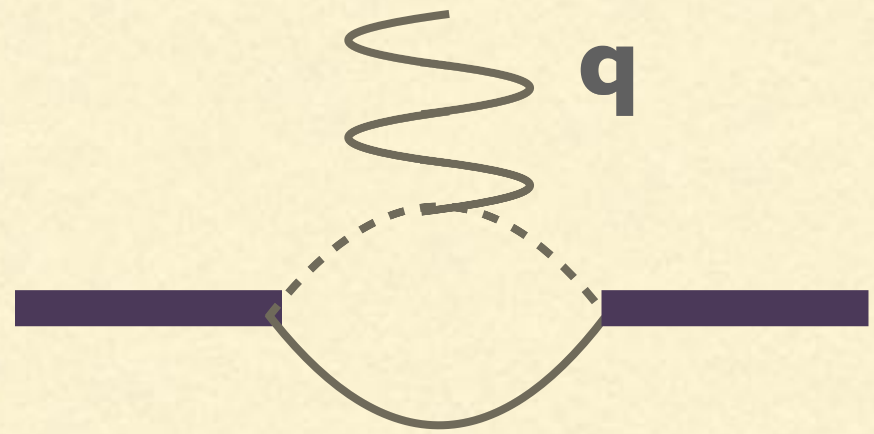

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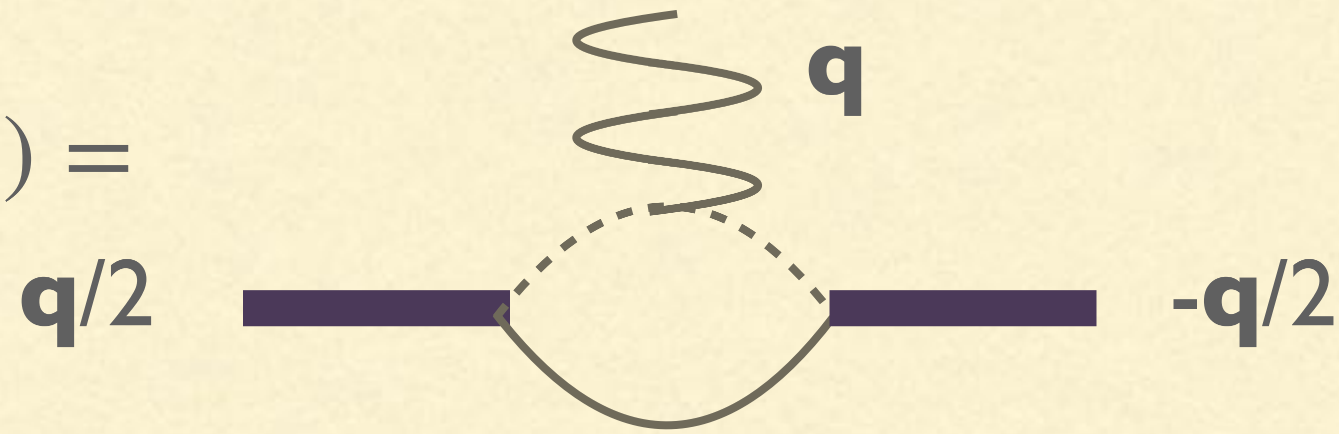

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1. Expand in powers of |**q**| to get: $\langle r_E{}^2 \rangle_{\text{halo}} = \dfrac{f^2 A_0^2}{4\gamma^3}$
2. How do we correct for finite size of core and neutron?

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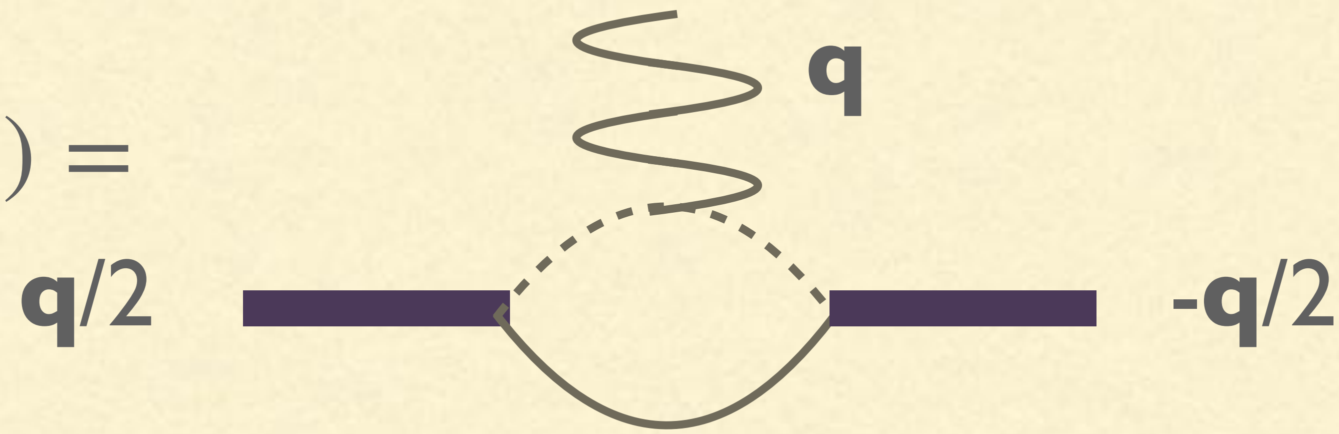

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$$\langle r_E{}^2\rangle_{\text{nucleus}} = \frac{f^2 A_0^2}{4\gamma^3} + \langle r_E^2\rangle_{\text{c}} + \frac{1}{Z_c}\langle r_n^2\rangle$$

Cf. Ryberg et al., EPJA 56, 7 (2020)

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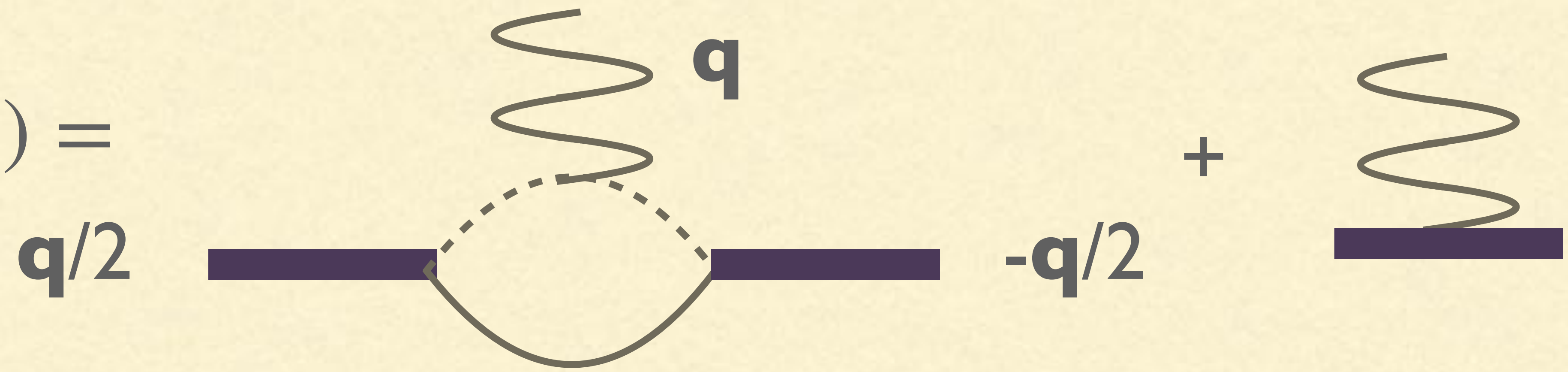

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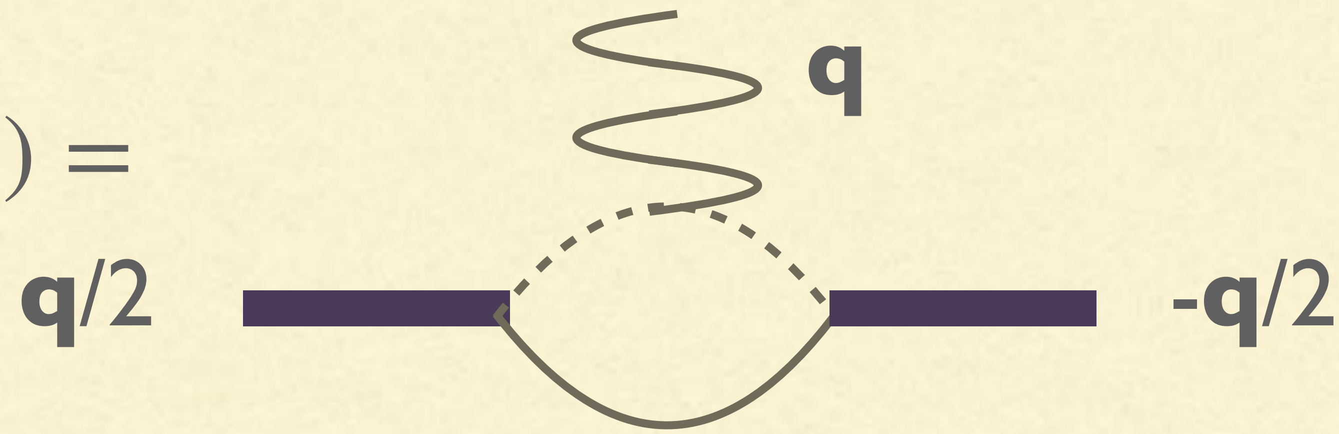

+

$$G_c(|\mathbf{q}|) = \frac{A_0^2}{f|\mathbf{q}|}\arctan\left(\frac{f|\mathbf{q}|}{2\gamma}\right) + 1 - \frac{A_0^2}{2\gamma}$$

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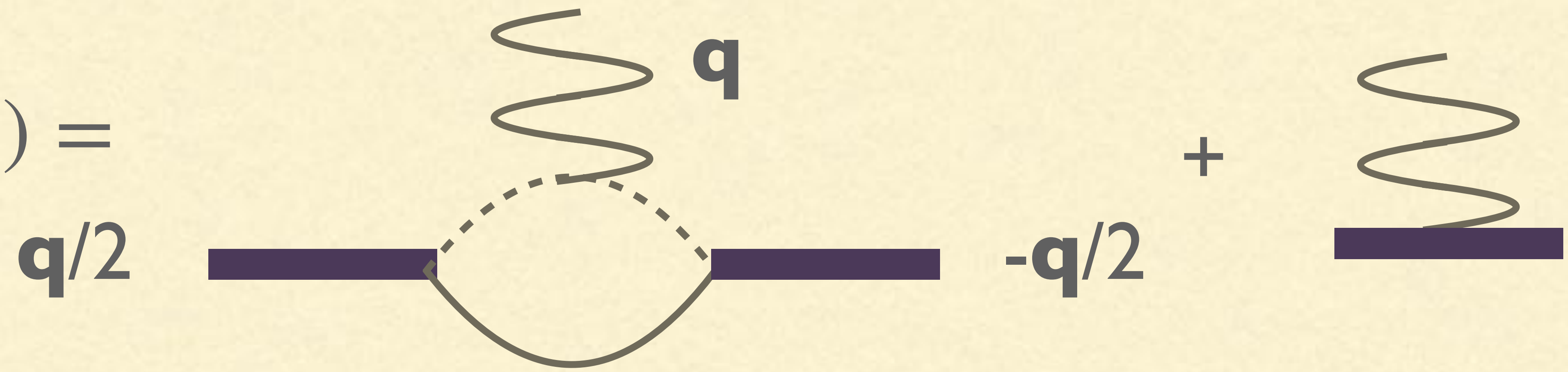

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LO:
$A_0 = \sqrt{2\gamma}$

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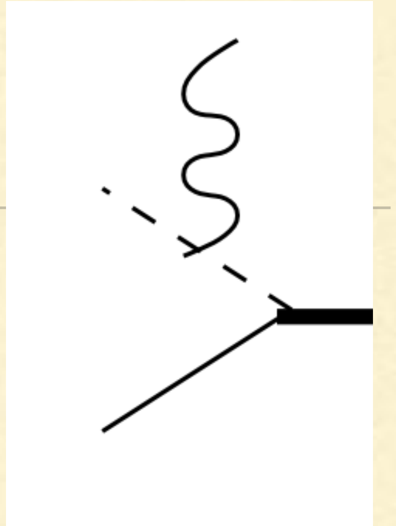
E1 photodissociation



Chen, Savage (1999)

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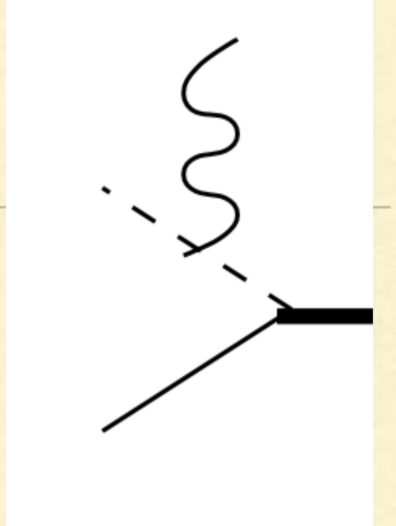

$$\mathscr{M}^{LO} = \sqrt{8\pi\gamma}\, e Z_c \frac{1}{\gamma^2 + (\mathbf{k} - f\mathbf{q})^2}$$

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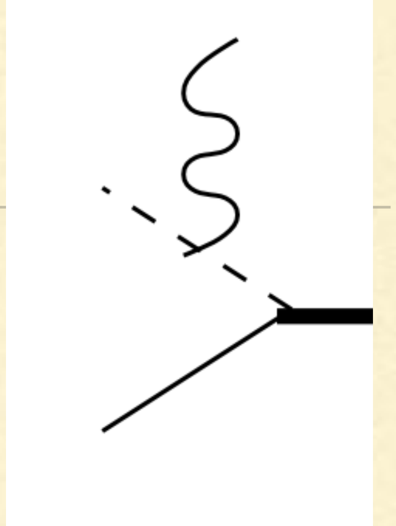

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1. Choose photon momentum to be aligned with z-axis
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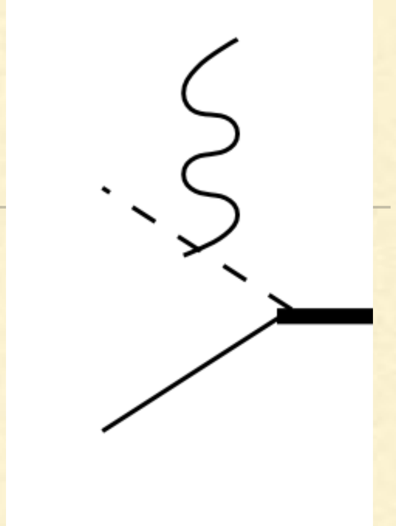

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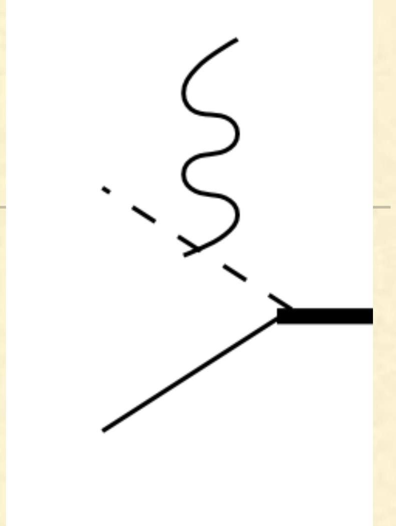

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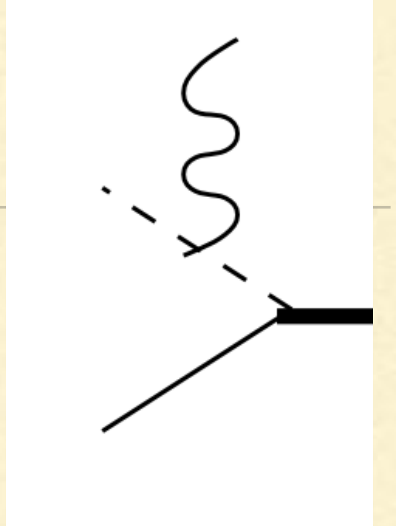

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Universal E1 strength formula

$$\frac{dB(E1)}{\alpha_{em} dE} = \frac{6 m_R}{\pi^2} \frac{Z_c^2}{(A_c+1)^2} A_0^2 \frac{k^3}{(\gamma^2 + k^2)^4}$$

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Bertulani, Typel, Baur

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- Universal E1 strength formula for S-wave halos
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- Short-distance piece of E1 m.e.: $L_{E1}\left[d^\dagger \mathbf{E} \cdot (n \overleftrightarrow{\nabla} c) + h.c.\right] \sim \left(\frac{R_{\rm core}}{R_{\rm halo}}\right)^4$

Universal E1 strength formula

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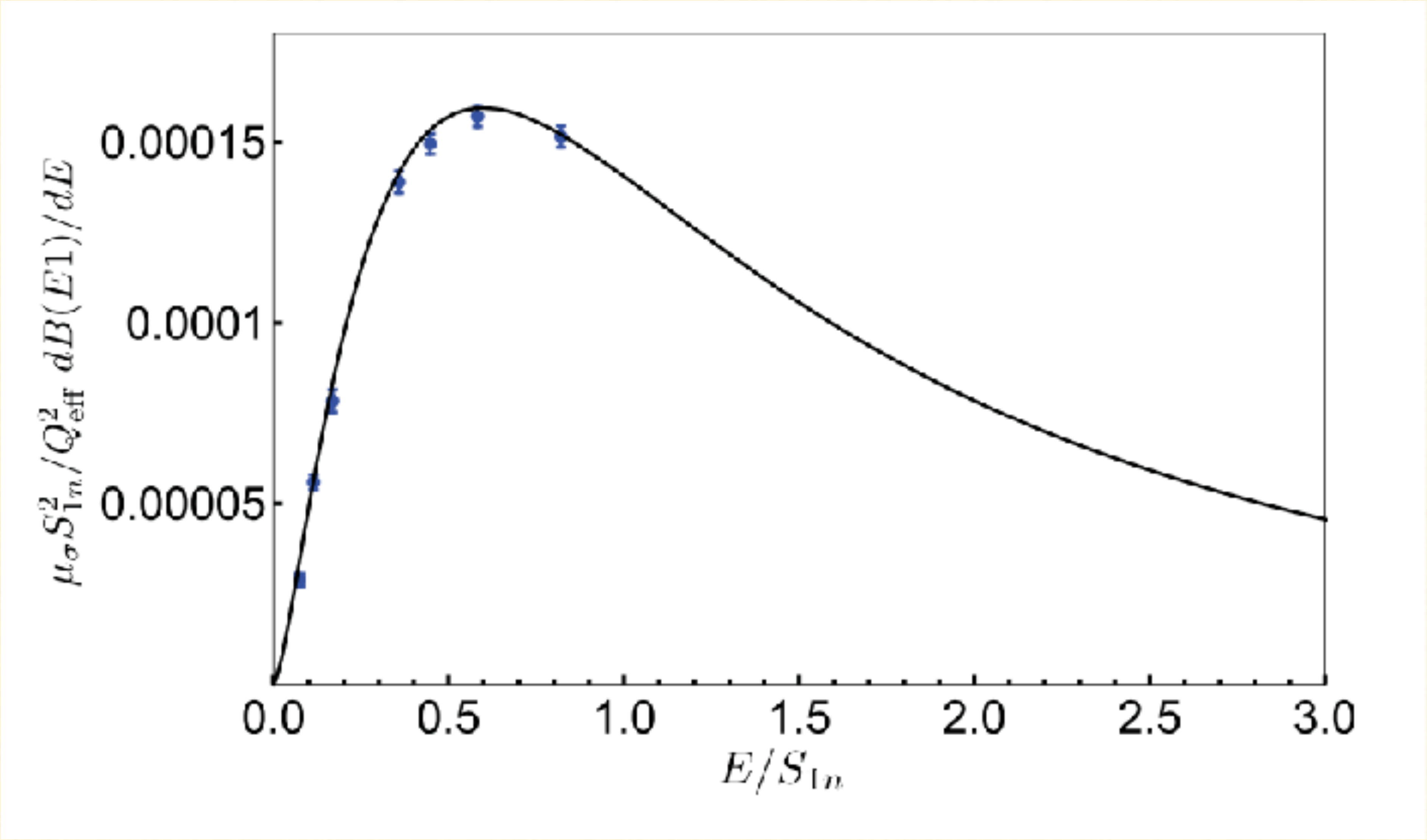
- Tells you how universal formula "breaks"

Results



Deuteron

Data: Tornow et al. (2003)
Analysis: Chen, Savage (1999); Hammer, Ji, DP (2017)

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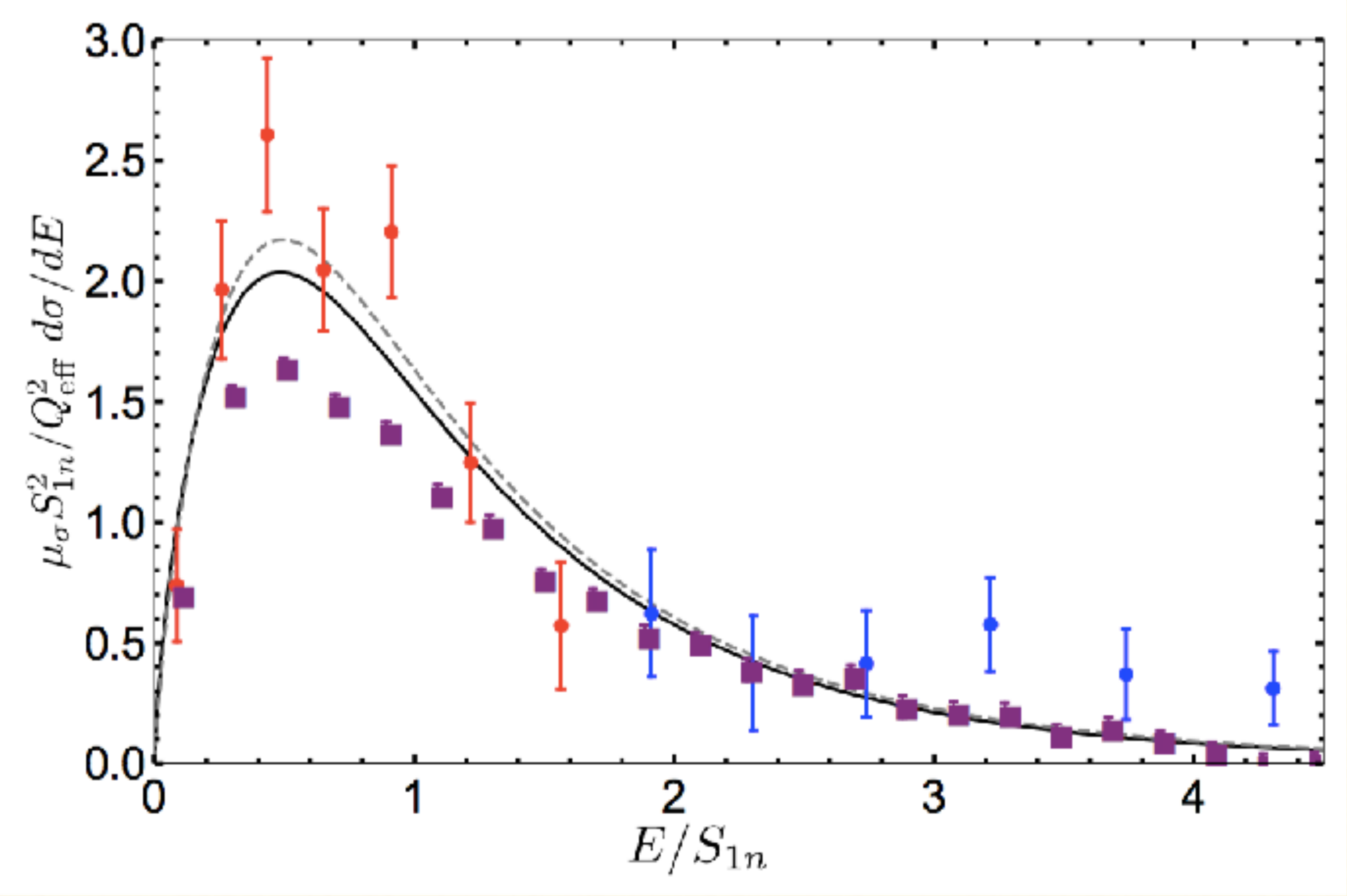

^{19}C

Data: Nakamura et al. (1999, 2003); Fukuda et al. (2004)
Analysis: Acharya, DP (2013); Hammer, Ji, DP (2017)

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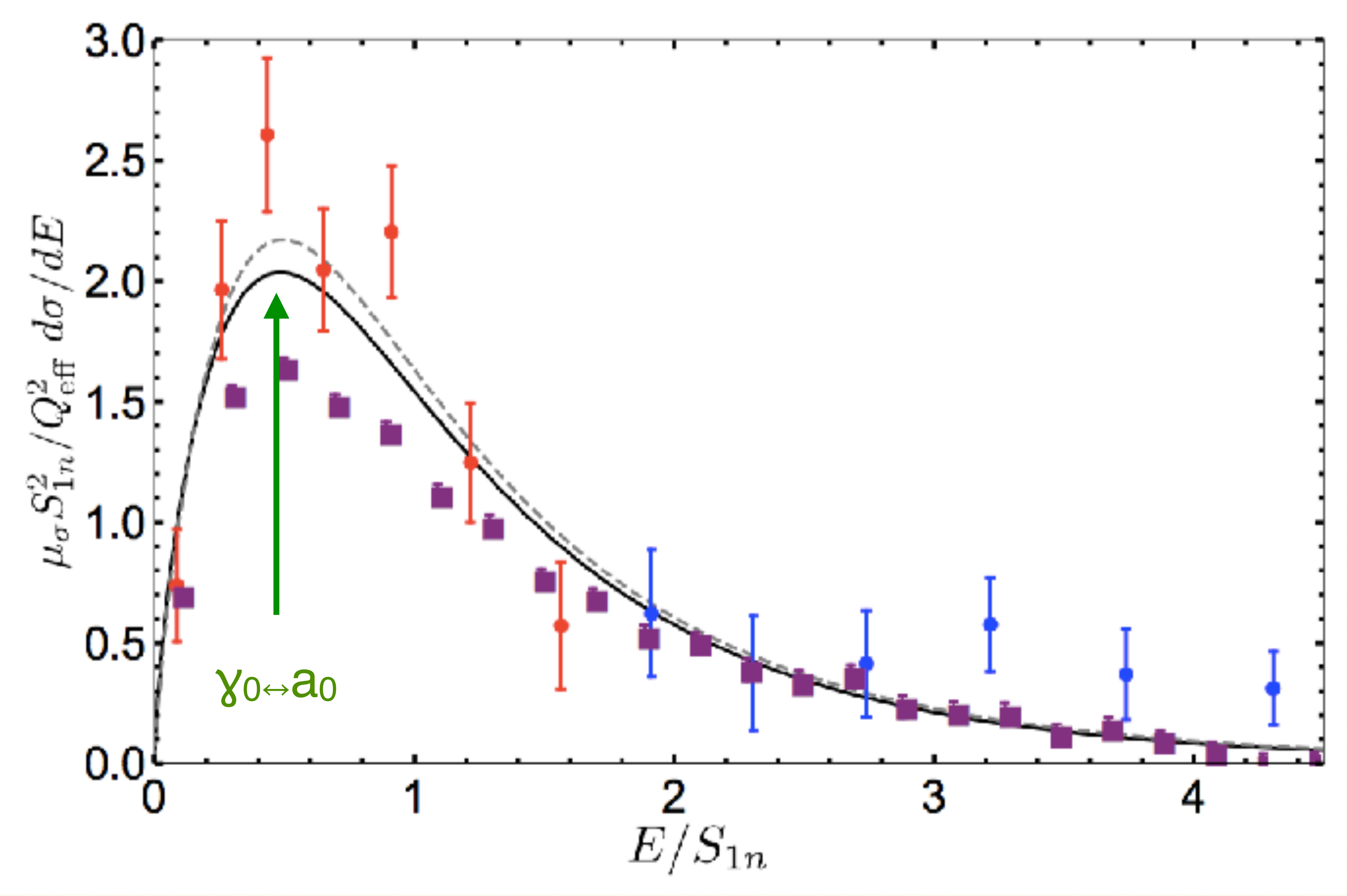

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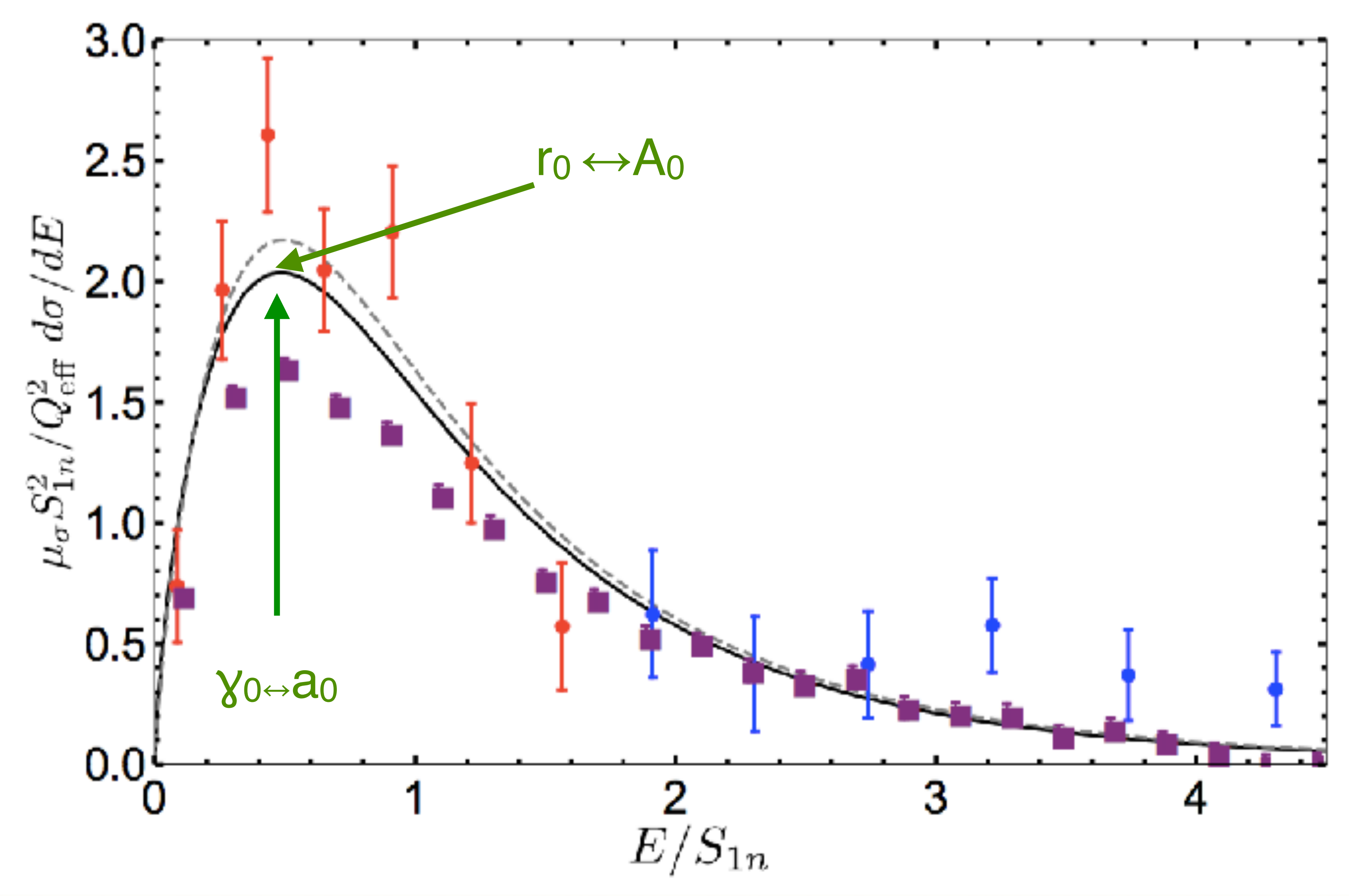

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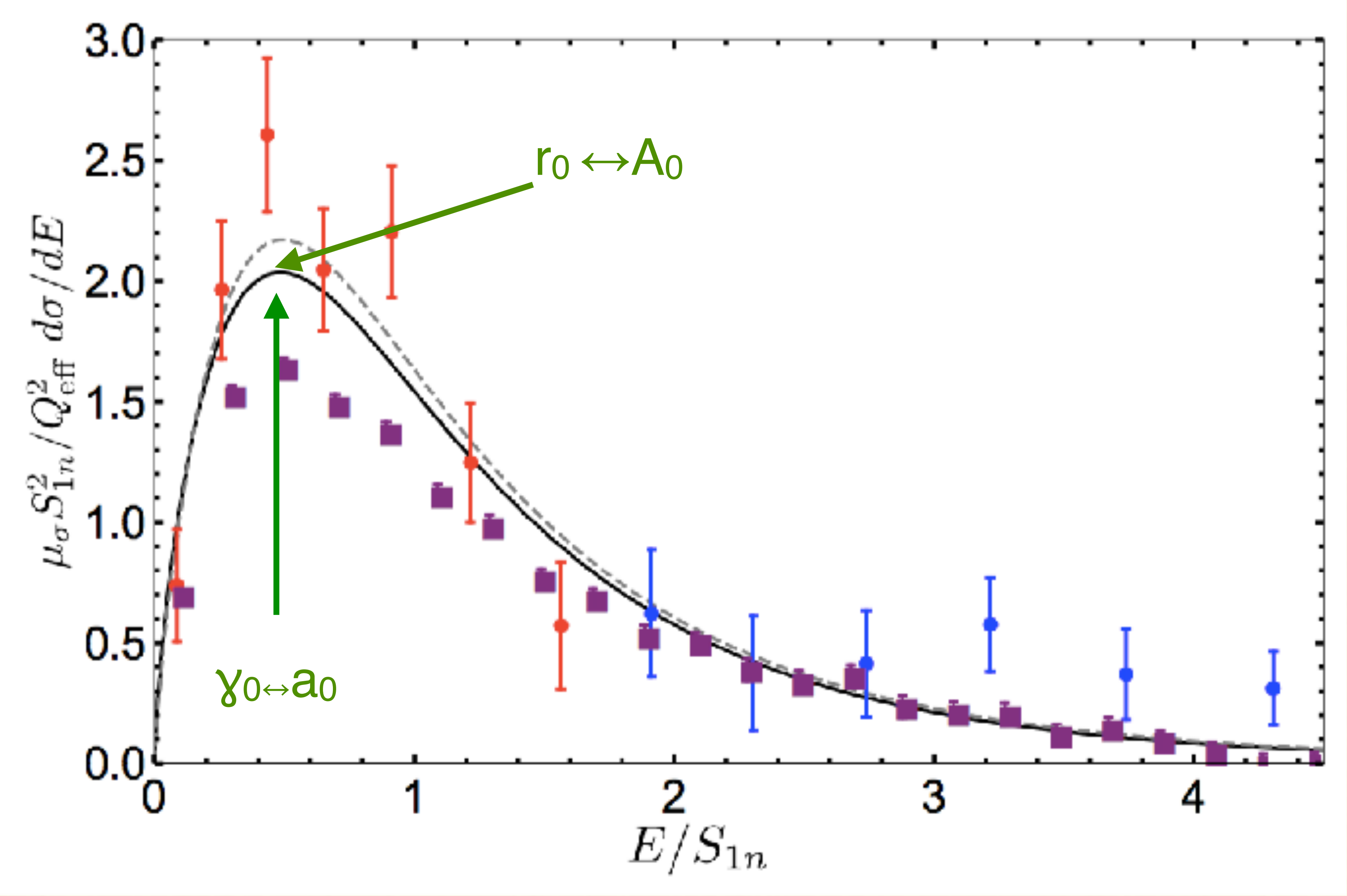

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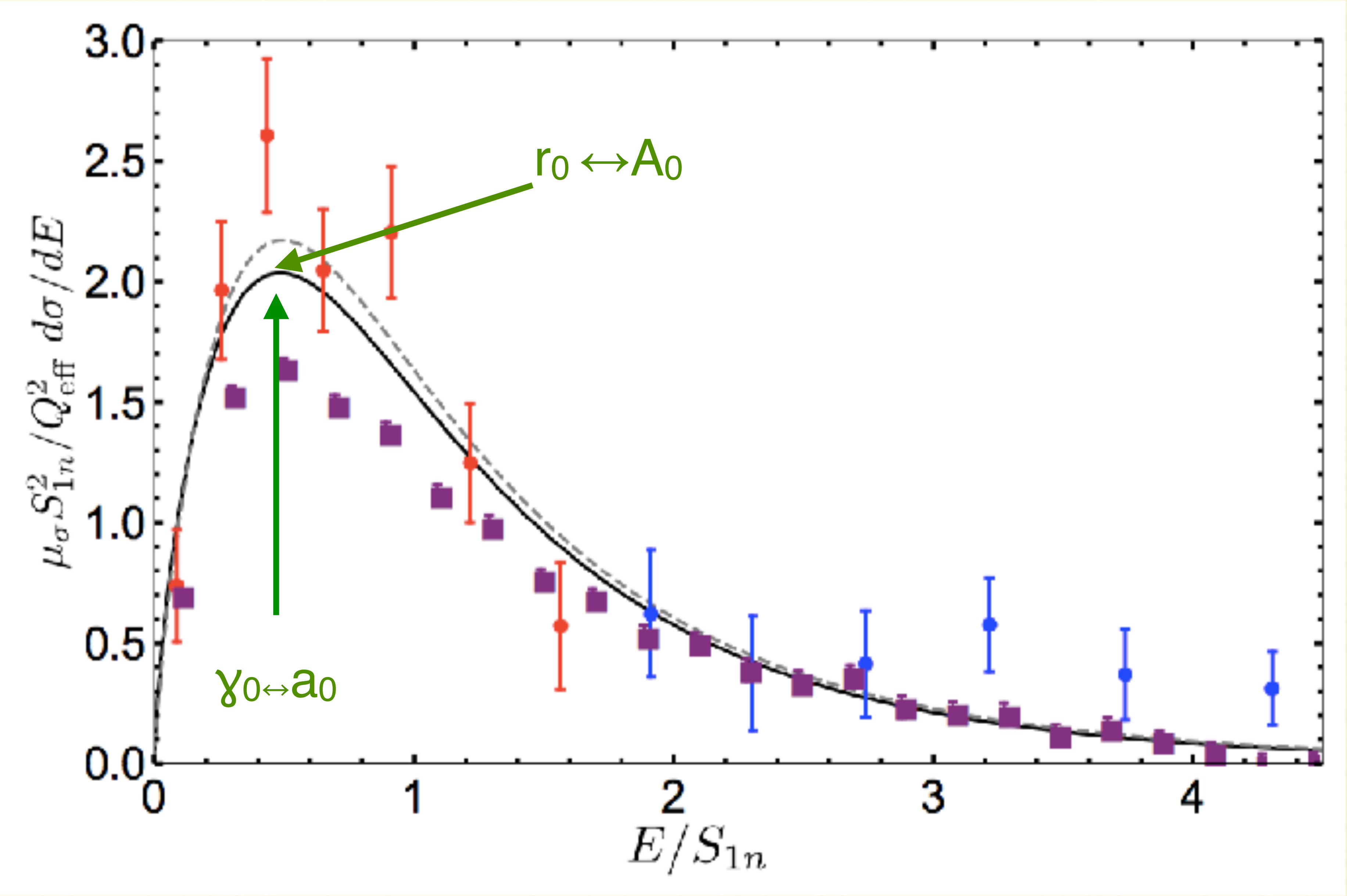
Determine S-wave ^{18}C-n scattering parameters⇔ ^{19}C ANC from dissociation data.

Results



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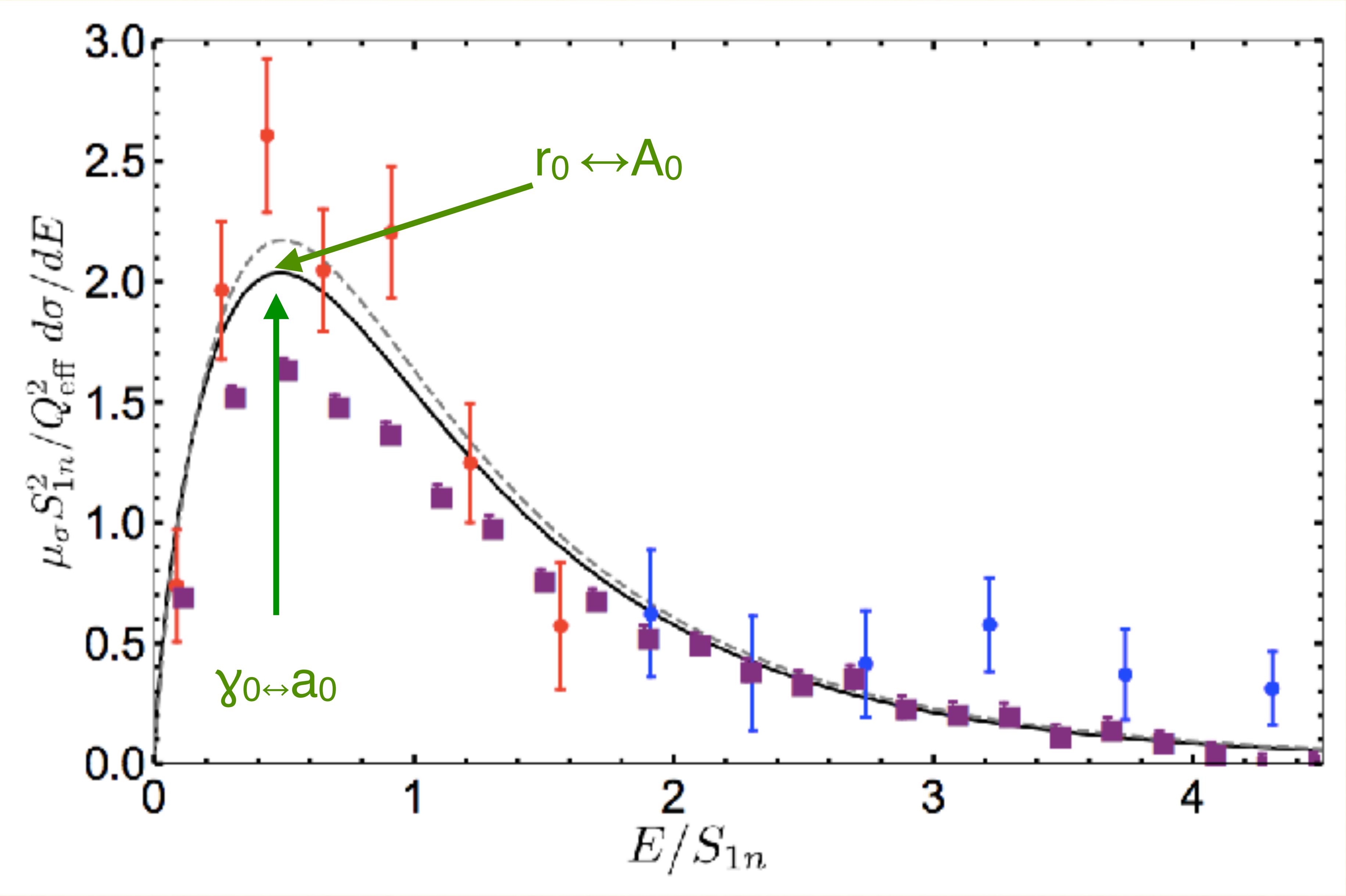
- For ^{19}C:

$$\begin{aligned} a &= (7.75 \pm 0.35(\text{stat.}) \pm 0.3(\text{EFT}))\ \text{fm}; \\ r_0 &= (2.6^{+0.6}_{-0.9}(\text{stat.}) \pm 0.1(\text{EFT}))\ \text{fm}. \end{aligned}$$

Results



^{19}C

^{11}Be: what's up?

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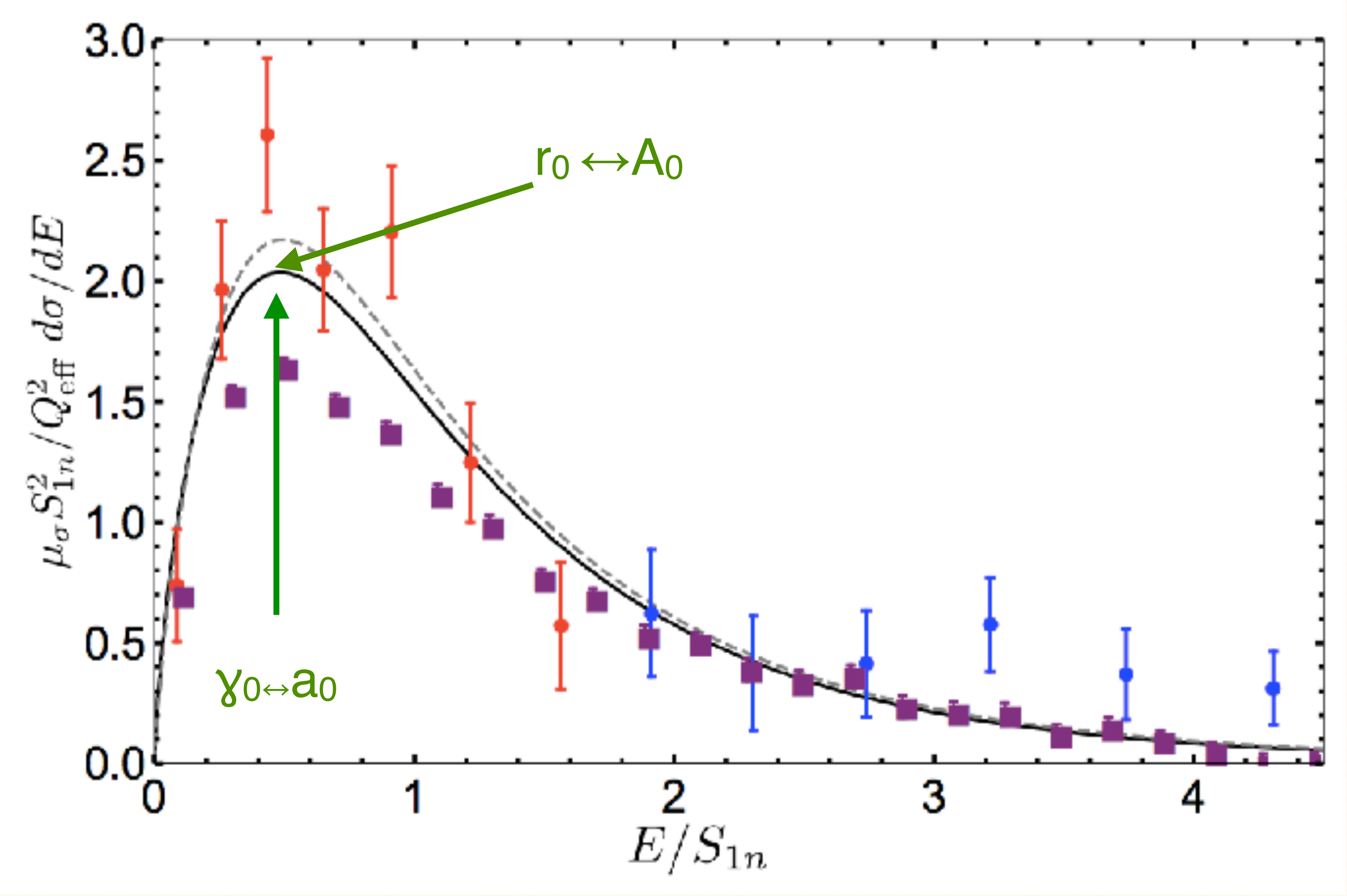

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Combine Halo EFT with reaction theory: Capel, Hammer, Hebborn, Moschini, Yang, DP, et al.

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The three-body scattering problem

- Consider the Schrödinger equation for the quantum-mechanical three-body scattering problem. Let's start with just pairwise potentials:

$$\left[-\frac{\nabla_1^2}{2m_1}-\frac{\nabla_2^2}{2m_2}-\frac{\nabla_3^2}{2m_3}+V_{12}(\mathbf{r}_{12})+V_{23}(\mathbf{r}_{23})+V_{31}(\mathbf{r}_{31})\right]\Psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)=E\Psi$$

- Two (related) problems:
  - Disconnected diagrams are present in solution for total wave function Ψ
  - What boundary condition should we impose?

Solving problem I

- Let's define 2B t-matrices that are embedded in the 3B Hilbert space
- Or, equivalently, think about states $|\psi_3^{(+)}\rangle \otimes |\mathbf{q}_3\rangle$, where $|\psi_3^{(+)}\rangle$ is the solution of the Schrödinger equation in which particle 3 is a "spectator",while the (12) wave function has spherical outgoing wave boundary conditions

$$\left[-\frac{\nabla_1^2}{2m_1} - \frac{\nabla_2^2}{2m_2} + V_{12}(\mathbf{r}_{12})\right] \psi_3(\mathbf{r}_{12}) = E_{12}\psi_3(\mathbf{r}_{12})$$

- Questions:
  1. If $|\psi_3^{(+)}\rangle \otimes |\mathbf{q}_3\rangle$ is a three-body eigenstate of energy E what is the energy E_{12} that appears in this Schrödinger equation
  2. Is the (12) system in its center-of-mass?
  3. If not, write the equivalent one-body Schrödinger equation, i.e., the equation for $\psi_3(\mathbf{r}_{12})$ in the co-ordinate $\mathbf{r}_{12}$

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- So we're going to need to specify different pieces of Ψ in order to specify what kind of reaction we're interested in

The Faddeev equations

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- Where $|\Psi_i\rangle = G_0 V_i |\Psi\rangle$ and $|\Phi\rangle$ is a three-body plane wave; separate wave function according to the *last* interaction before particles go to the detector: that defines three separate outgoing boundary conditions
- Show that the resulting $|\Psi\rangle$ solves the 3B Schrödinger equation $(E - H_0)|\Psi\rangle = (V_1 + V_2 + V_3)|\Psi\rangle$
- Manipulate $|\Psi_1\rangle = G_0 V_1 (|\Phi\rangle + |\Psi_1\rangle + |\Psi_2\rangle + |\Psi_3\rangle)$ into a formal equation for $|\Psi_1\rangle$

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$$|\Psi_i\rangle = G_0 V_i |\Phi\rangle \equiv G_0 T_i |\Phi\rangle \Rightarrow T_i = t_i + t_i \sum_{j \neq i} G_0 T_j$$