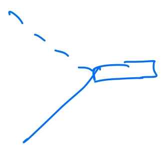
Lecture 1: Additional notes

Feynman Rules

$(E, \vec{p})$

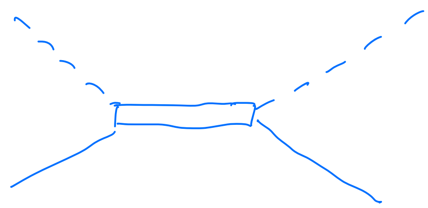
$$\frac{i}{\Delta + \eta\left(E - \vec{p}^{\,2}/2M_{nc}\right)}$$



$$-ig$$

$$M_{nc} = M_n + M_c$$

nc tree-level scattering amplitude



$$i\mathcal{M} = (-ig)\,\frac{i}{\Delta + \eta\left(E - \vec{p}^{\,2}/2M_{nc}\right)}\,(-ig)$$

$$\Rightarrow \quad \mathcal{M} = \frac{-g^2}{\Delta + \eta\left(E - \vec{p}^{\,2}/2M_{nc}\right)}$$

$\eta = 0$: LO tree-level amplitude

$$\mathcal{M}^{LO}_{tree} = -\frac{g^2}{\Delta} = -C_0 \quad (\eta = 0)$$

LO loops

$$D^{LO}_d(E, \vec{p}) = \frac{i}{\Delta} + \frac{i}{\Delta}(-i\Sigma)\frac{i}{\Delta}$$

$$+ \frac{i}{\Delta}(-i\Sigma)\frac{i}{\Delta}(-i\Sigma)\frac{i}{\Delta}$$

$$\cdot\ \frac{i}{\Delta}(-i\Sigma)\frac{i}{\Delta}(-i\Sigma)\frac{i}{\Delta} + \cdots$$

$$= \frac{i}{\Delta}\left[1 + \frac{\Sigma}{\Delta} + \frac{\Sigma^2}{\Delta^2} + \cdots\right]$$

$$= \frac{i}{\Delta}\,\frac{1}{1-\Sigma/\Delta}$$

$$= \frac{i}{\Delta - \Sigma}$$

$\Sigma(E, \vec{p})$: a function of $E - \dfrac{\vec{p}^{\,2}}{2M_{nc}}$

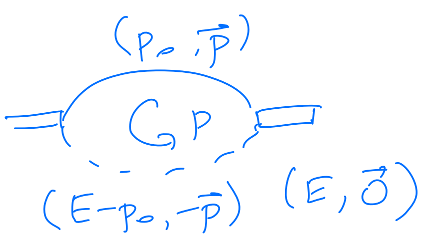
$\rightarrow$ Gallilean invariance

$$\Sigma(E,\vec{p}) \overset{\shortparallel}{\vphantom{x}} \quad \text{@ } \vec{p}=0:\quad \Sigma(E, \vec{0})$$

Feynman Rules

$\overline{\qquad\qquad}$ p $\qquad \dfrac{i}{p_0 - \dfrac{\vec{p}^{\,2}}{2M_n} + i\varepsilon}$

$- - - -$ p $\qquad \dfrac{i}{p_0 - \dfrac{\vec{p}^{\,2}}{2M_c} + i\varepsilon}$



$$-i\Sigma(E,\vec{0}) = (-ig)^2 \int \frac{d^4p}{(2\pi)^4}\, \frac{i}{E - p_0 - \dfrac{\vec{p}^{\,2}}{2M_c} + i\varepsilon}$$

UHP $\leftarrow$

$$\frac{i}{p_0 - \frac{\vec{P}^2}{2M_n} + i\varepsilon}$$

LHP

$$\Rightarrow \Sigma = i g^2 \frac{(-2\pi i)}{2\pi} \int \frac{d^3p}{(2\pi)^3} \frac{1}{E - \frac{\vec{p}^2}{2m_R} + i\varepsilon}$$

$$\frac{1}{m_R} \equiv \frac{1}{M_n} + \frac{1}{M_c}$$

$$\Rightarrow \quad \Sigma = 2 m_R g^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{k^2 - p^2 + i\varepsilon}$$

$$k^2 = 2 m_R E$$

$$= \frac{2 m_R g^2}{2\pi^2} \int_0^\Lambda dp \, \frac{p^2}{k^2 - p^2 + i\varepsilon}$$

$$\frac{1}{x - x_0 + i\varepsilon} = P \frac{1}{x - x_0} - i\pi\,\delta(x - x_0)$$

$$= \frac{m_R g^2}{\pi^2} \left[-\Lambda - i\frac{\pi k}{2} + O\left(\frac{k^2}{\Lambda}\right) \right]$$

NLO

$$D_d(E, \vec{P}) = \frac{i}{\Delta + \eta\left(E - \frac{\vec{P}^2}{2M_{nc}}\right) - \Sigma(E, \vec{P})}$$

Appears at

NLO

$$\mathcal{M}(E,\vec{0}) = -\frac{2\pi}{m_R}\frac{1}{\frac{\Delta \cdot 2\pi}{m_R g^2} + \frac{2\pi\eta}{m_R g^2}E + \frac{2\Lambda}{\pi} + ik}$$

$$= -\frac{2\pi}{m_R}\frac{1}{\underbrace{\frac{\delta \cdot 2\pi}{m_R g^2} + \frac{2\Lambda}{\pi}}_{=\frac{1}{a_0}} + \underbrace{\frac{2\pi\eta}{m_R^2 g^2}}_{=-r_0}\frac{k^2}{2} + ik}$$

$$r_0 = -\frac{2\pi\eta}{m_R^2 g^2}$$

$\eta = -1$ for $r_0 > 0$

$\eta = 0$: switch off NLO effects

$\eta = -1$: "ghost field"

Don't need η as a parameter because

$$\frac{\Delta}{g^2} \longleftrightarrow \frac{1}{a_0}$$

$$\frac{1}{g^2} \rightarrow r_0 \quad ,$$

i.e. already have two parameters in dimer description.