Effective Field Theory for Halo Nuclei: Lecture 1

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Based on work done in collaboration with: Bijaya Acharya, Pierre Capel Hans-Werner Hammer, Chen Ji

Special thanks to Thomas Richardson

Further reading:
DP, Czech. J. Phys. 52, B49 (2002);
Hammer, Ji, Phillips, J. Phys. G 44, 103002 (2017)

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Ordinary vs. halo nuclei

Ordinary vs. halo nuclei

- In nuclei, each nucleon moves in the potential generated by the others
- The nuclear size grows as $A^{1/3}$; cross sections like $A^{2/3}$
- Nuclear binding energies are on the order of 8 MeV/nucleon

http://alternativephysics.org

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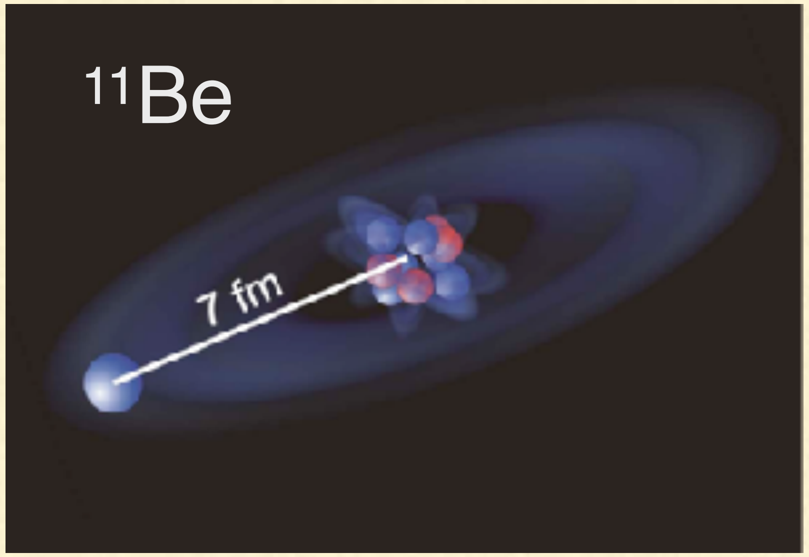

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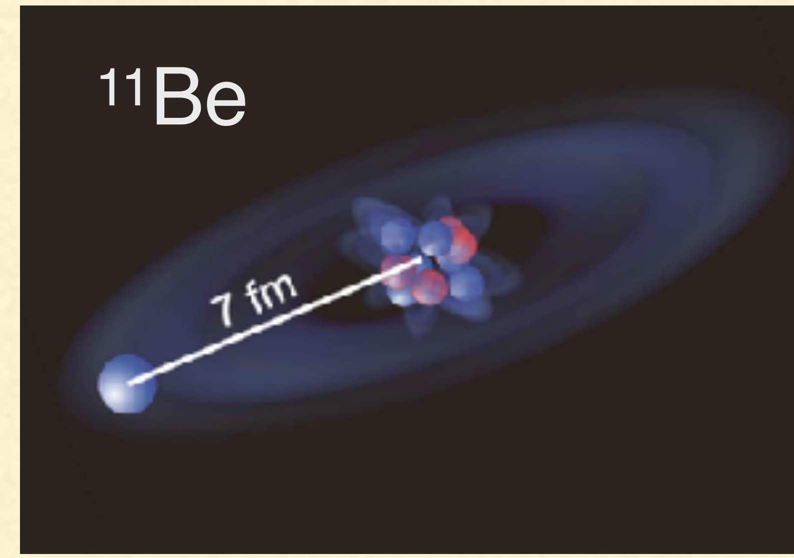

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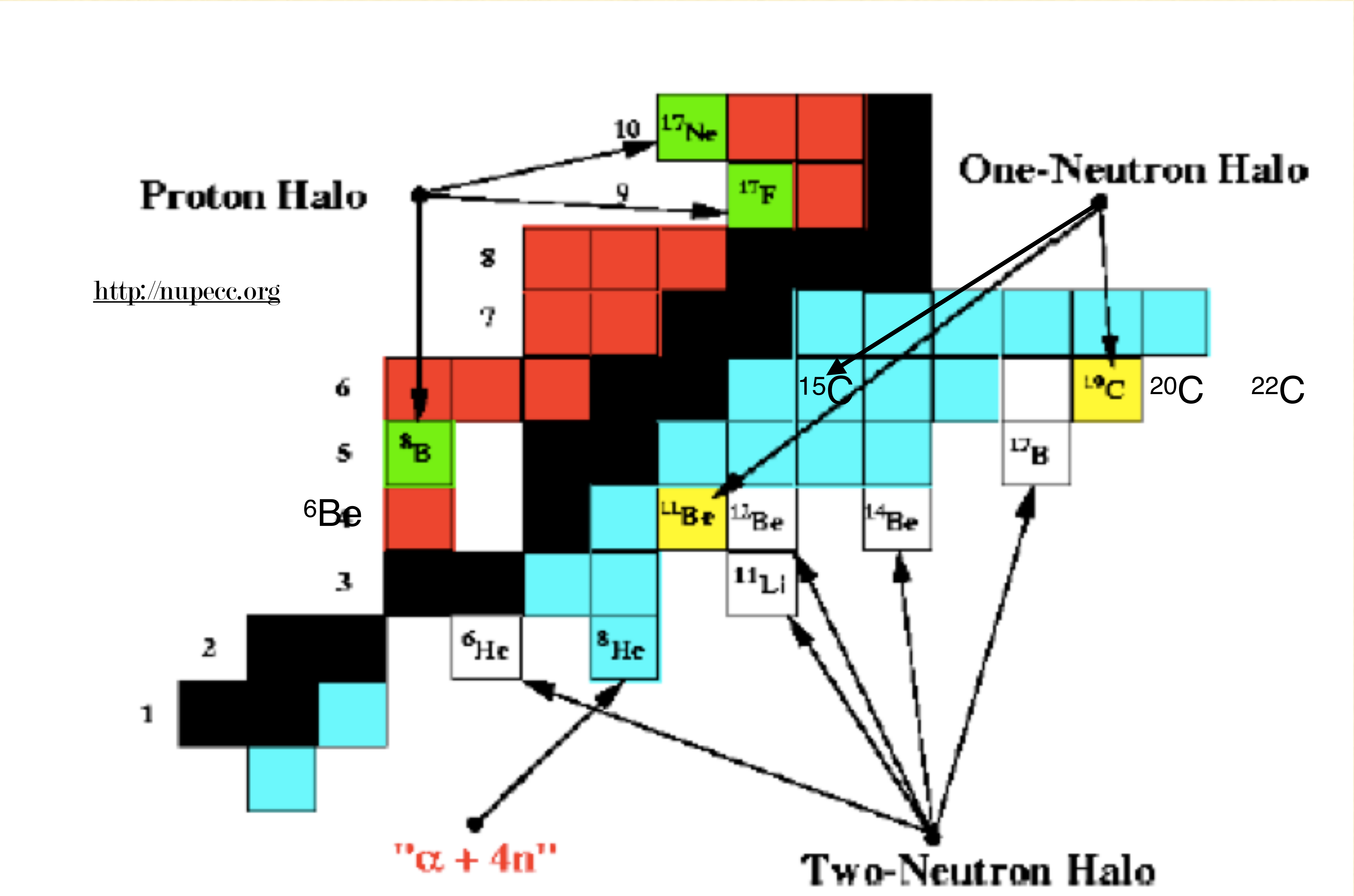
- Halo nuclei: the last few nucleons “orbit” far from the nuclear “core”
- Characterized by small nucleon binding energies, large radii, large interaction cross sections, large E1 transition strengths.

Halo nuclei: examples

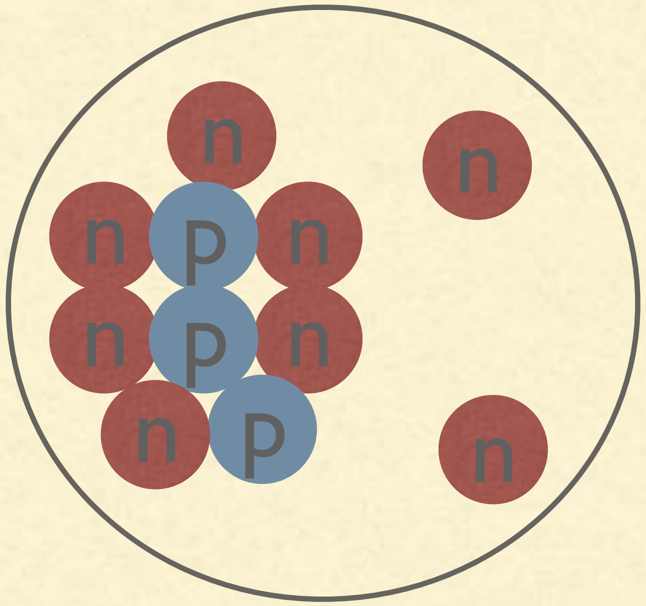


Question 1

"Orbit far from the nuclear core" sounds like it's based on a classical picture. Can you come up with a definition of a neutron halo that's not tied to classical concepts?

1. Halo wave function extends beyond core, and therefore beyond range of potential: systems in the tunneling regime
2. $<r_n^2>$ is large compared to that of protons
3. Separation energy of 1n is much smaller than separation energy of core
4. Sort of a two-body problem
5. The neutron wave function has significant strength at low momentum, which could be observed in, e.g., QE n knockout
6. Total cross section should be additive: $\sigma_T = \sigma_{core} + \sigma_{halo}$
7. Radius exceeds standard nuclear expectation of $r_0 A^{1/3}$
8. Weak form factor falls faster than EM form factor

Halo EFT



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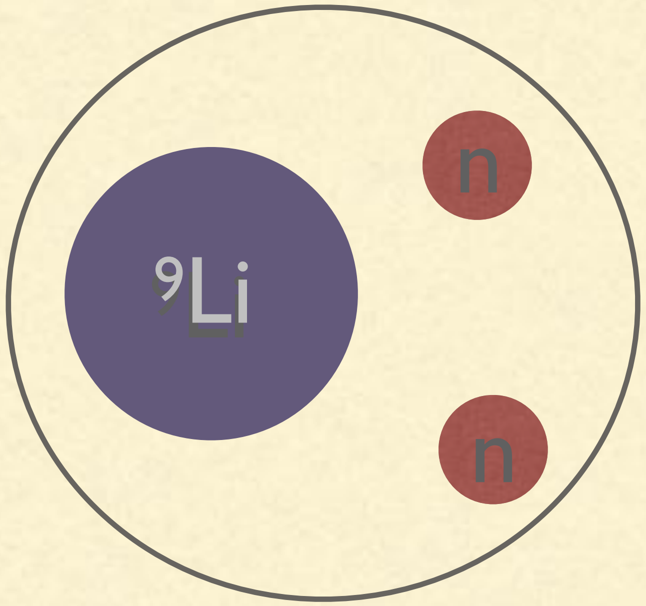
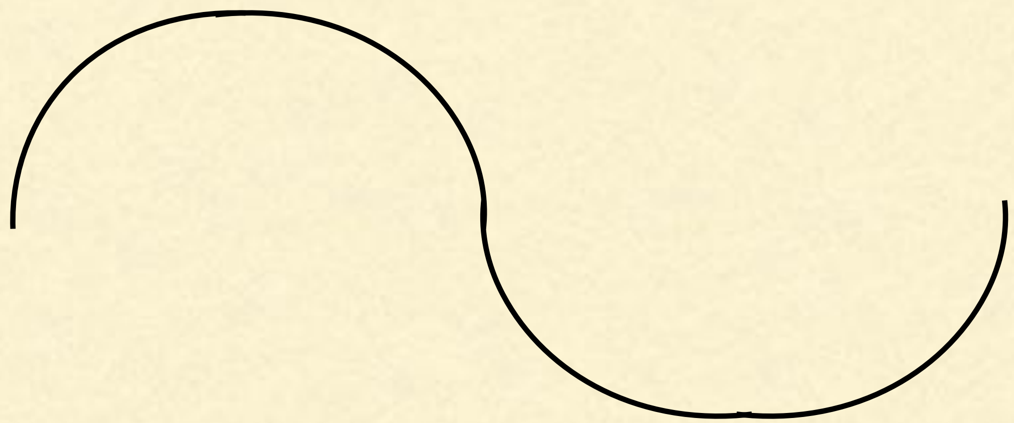
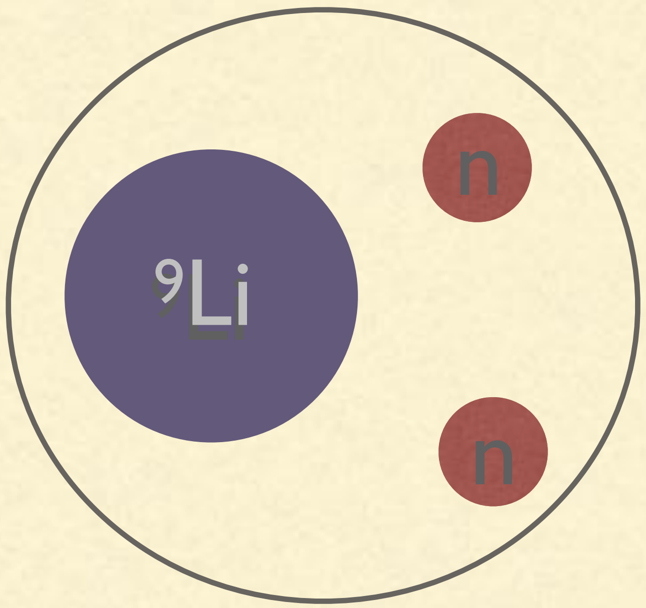


Halo EFT

$\lambda \gg R_{core}; \lambda \lesssim R_{halo}$





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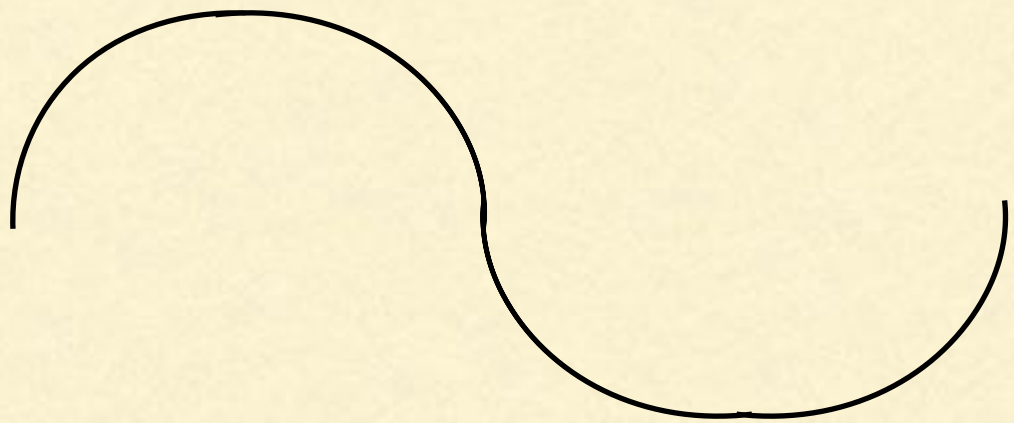

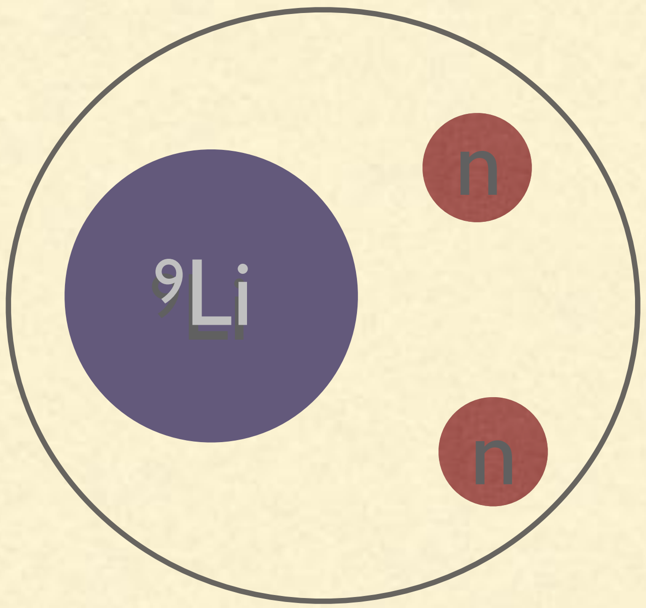

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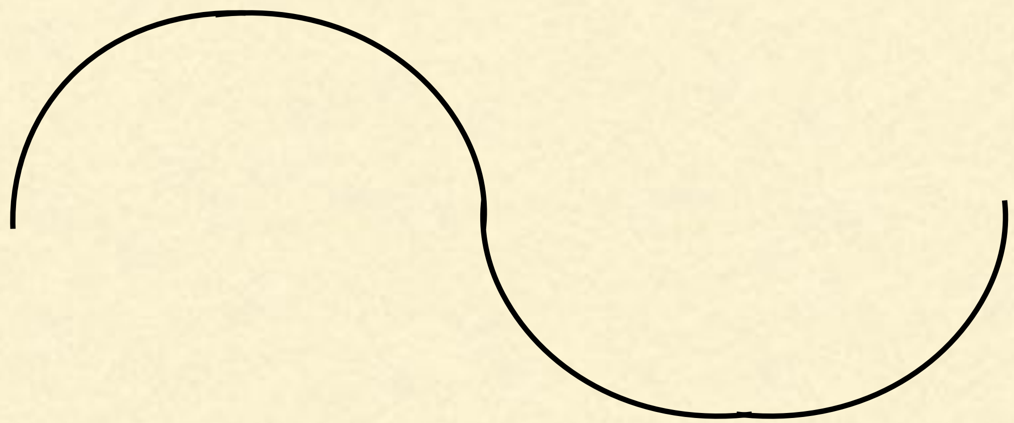

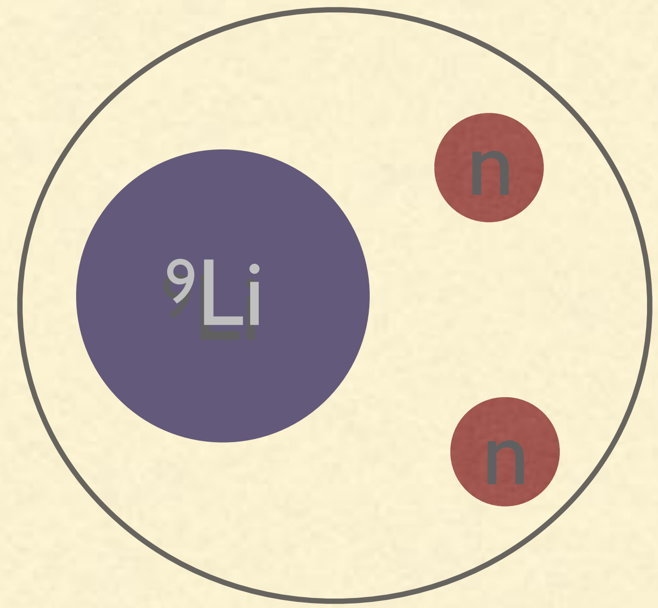

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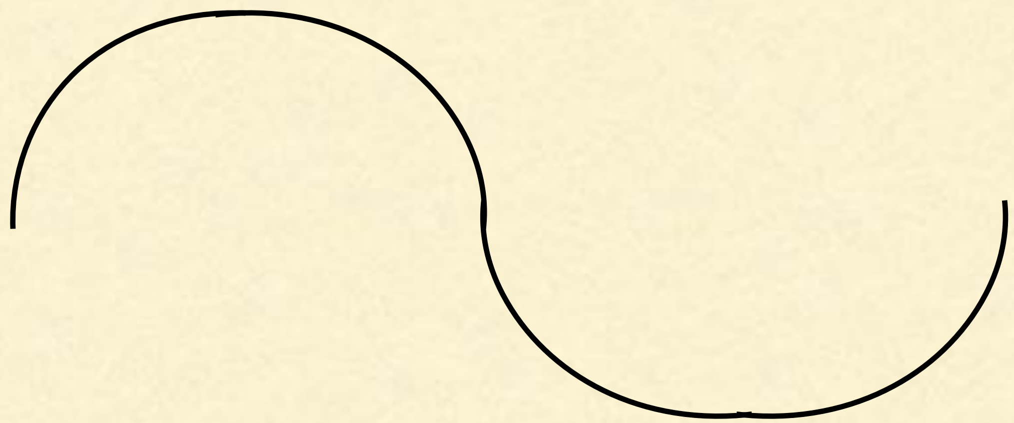

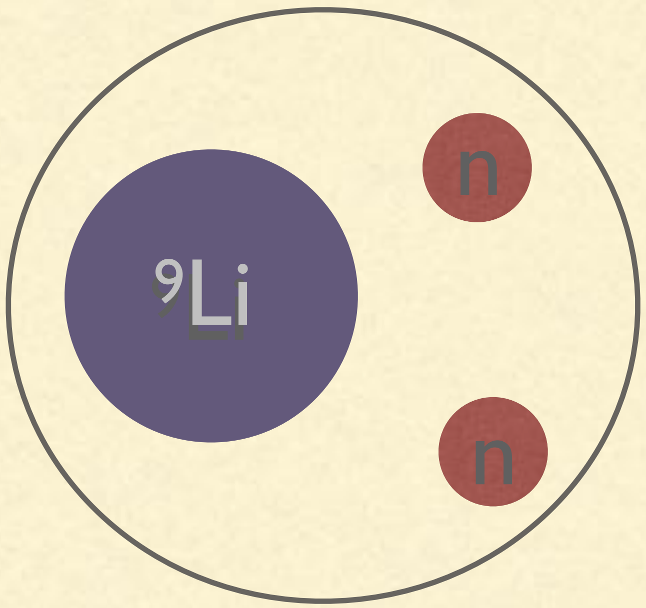

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Halo EFT

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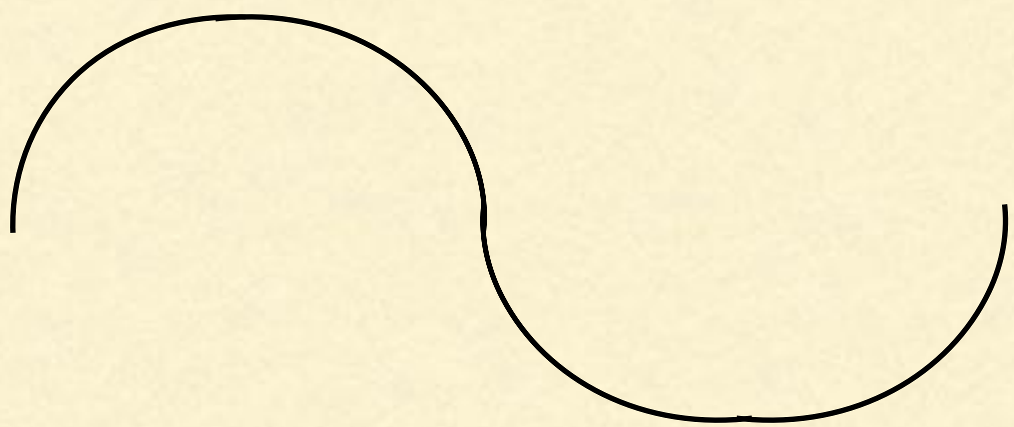

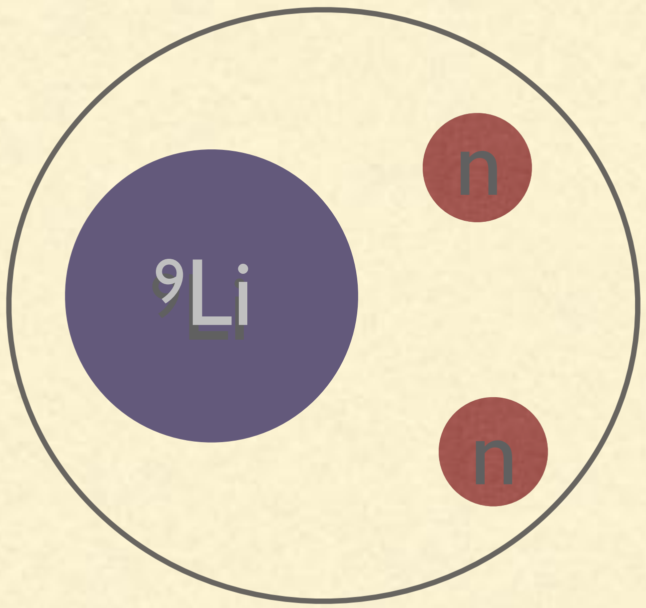

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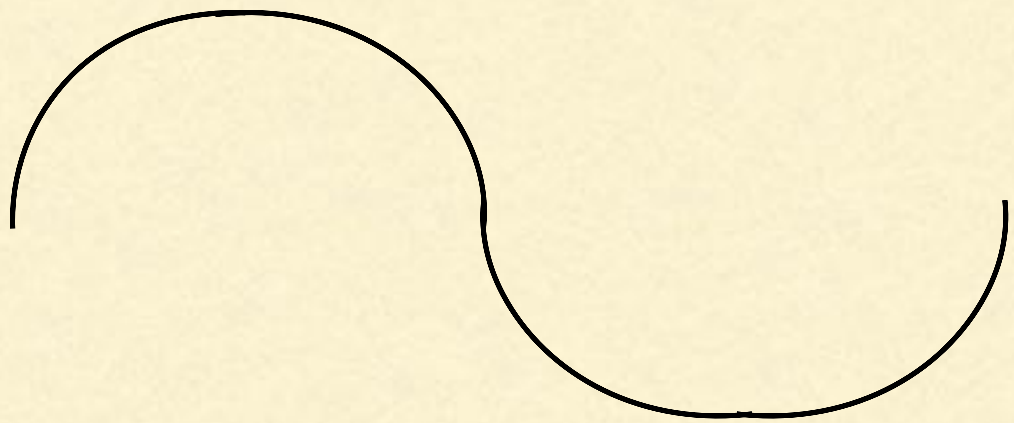

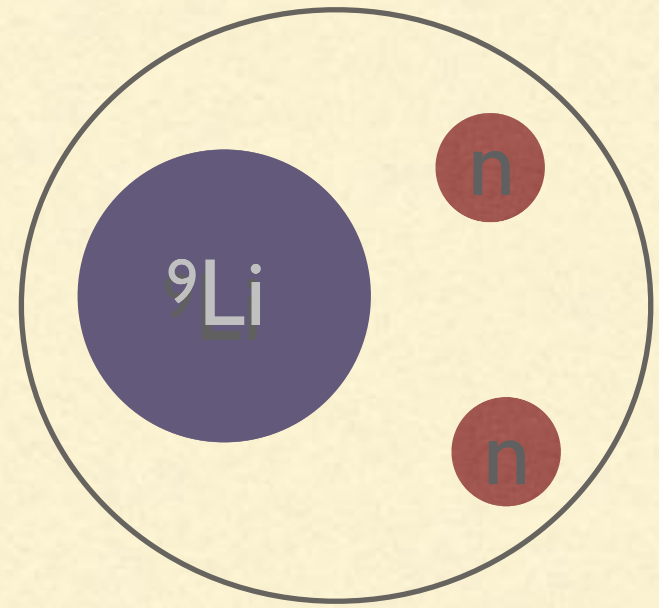

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- ^{11}Be, ^{19}C, and ^{2}H: all s-wave 1n halos
- ^{22}C, ^{11}Li, ^{12}Be, ^{62}Ca (hypothesized), and ^{3}H: all s-wave 2n halos

One-neutron halo properties

Table 1. Properties of one-neutron halos. S_{1n} is the one-neutron separation energy from AME2012 [78, 79]. The first core excitation energies E_c^* for $A > 1$ halos are taken from the TUNL database [80–82]. M_{halo} and M_{core} are estimated using S_{1n} and E_c^*, except for the deuteron, where we take $M_{\text{core}} = 140$ MeV.

	^{2}H	^{11}Be	^{15}C	^{19}C
Experiment				
J^P	1^+	$1/2^+$	$1/2^+$	$1/2^+$
S_{1n} (MeV)	2.224 573(2)	0.501 64(25)	1.2181(8)	0.58(9)
E_c^* (MeV)	293	3.368 03(3)	6.0938(2)	1.62(2)
$\langle r_{nc}^2 \rangle^{1/2}$ (fm)	3.936(12)[a]	6.05(23)[c]	4.15(50)[f]	6.6(5)[f]
	3.950 14(156)[b]	5.7(4)[d]	7.2 ± 4.0[g]	6.8(7)[h]
		5.77(16)[e]	4.5(5)[h]	5.8(3)[i]
EFT				
$M_{\text{halo}}/M_{\text{core}}$	0.33	0.39	0.45	0.6
$r_{0,\sigma}/a_{0,\sigma}$	0.32	0.38	0.43	0.33

Is ^{22}C (for example) a few-body system?

- At first sight, absolutely not!
- But $S_{2n} \leq 300$ keV; $S_{1n}(^{20}C)=3$ MeV
- Neutrons do not probe details at R_{core}
- Core can be treated as an effective degree of freedom provided we only care about low-momentum/long-distance structure of the halo
- Solve for dynamics of a few active degrees of freedom interacting via effective potentials

A comparison

	χEFT	Halo EFT
DEGREES OF FREEDOM	Nucleons and pions (sort-of in the case of π)	Core nucleus & halo nucleons
POTENTIALS	Pion exchanges and contact interactions	V_{cn} and V_{nn}: contact interaction
BREAK DOWN I	k/m_ρ of order 1	kR_{core} of order 1
BREAK DOWN II	$k/M_N<1$ $E/(M_\Delta-M_N)<1$	$E/E_c^*<1$

Lagrangian: shallow S-states

$$\mathscr{L} = c^\dagger \left(iD_t + \frac{D^2}{2M_c} \right) c + n^\dagger \left(iD_t + \frac{D^2}{2M_n} \right) n$$

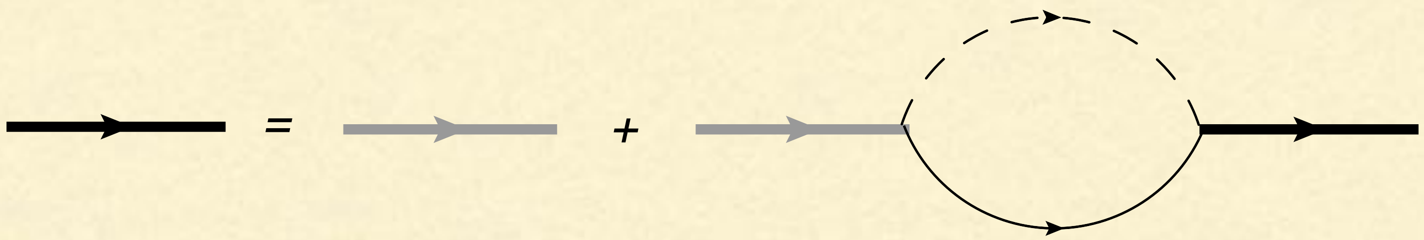
$$+ d^\dagger \left[\eta \left(iD_t + \frac{D^2}{2M_{nc}} \right) + \Delta \right] d - g(d^\dagger nc + n^\dagger c^\dagger d)$$

- c, n: "core", "neutron" fields. c: spin-0 boson, n: spin-1/2 fermion.
- d: S-wave "dimer" field
- Minimal substitution generates leading EM couplings

Dressing the s-wave state

Kaplan, Savage, Wise; van Kolck; Gegelia; Birse, Richardson, McGovern

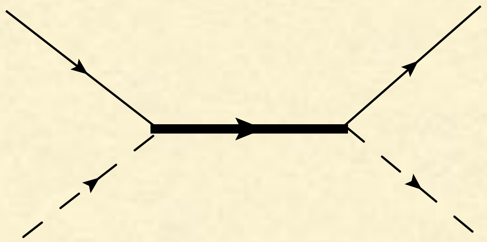
- dnc coupling g of order R_{halo}, nc loop of order $1/R_{halo}$. Justifies summing all bubbles:



$$D(p) = \frac{i}{\eta\left(p_0 - \frac{\mathbf{p}^2}{2M_{nc}}\right) + \Delta - \Sigma_d(p)}$$

$$\Sigma_d(p) = -\frac{g^2 m_R}{2\pi}\left[\frac{2\Lambda}{\pi} + i\sqrt{m_R\left(p_0 - \frac{\mathbf{p}^2}{2M_{nc}}\right)} + O\left(\frac{m_R p_0}{\Lambda}\right)\right] \qquad \text{(Cutoff } \Lambda\text{)}$$

Construct t, renormalize, take $\Lambda \to \infty$



$$t = \frac{2\pi}{m_R}\frac{1}{\frac{1}{a_0} - \frac{1}{2}r_0 k^2 + ik}$$

Do the matching!

$$t^{LO} = \frac{2\pi}{m_R} \frac{1}{\frac{1}{a_0} + ik}$$

$$\mathcal{M}(E, \mathbf{0}) = -g^2 \frac{1}{\Delta + \frac{m_R g^2}{2\pi}\left(\frac{2\Lambda}{\pi} + ik\right)}$$

1. Do $\mathcal{M}$ and t have the same sign?
2. What do you conclude about $\dfrac{\Delta}{g^2}$?

Two-body t beyond LO

$$t_0^{LO} = \frac{2\pi a_0}{m_R} \frac{1}{1 + i a_0 k}$$

$$t_0^{ERE}(E) = -\frac{2\pi}{m_R} \frac{1}{k \cot \delta(E) - ik}; k \cot \delta(E) = -\frac{1}{a} + \frac{1}{2} r k^2 + O(k^4 R^3)$$

Valid provided k~1/a

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- Expand t in r/a $t_0^{NLO}(E) = \frac{2\pi a}{m_R}\frac{1}{1+iak}\left[1 + \frac{1}{2}\frac{rk^2}{1/a+ik} + O\left(\frac{r^2}{a^2}\right)\right]$

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First corrections are of relative order kR, R/a: "higher order"

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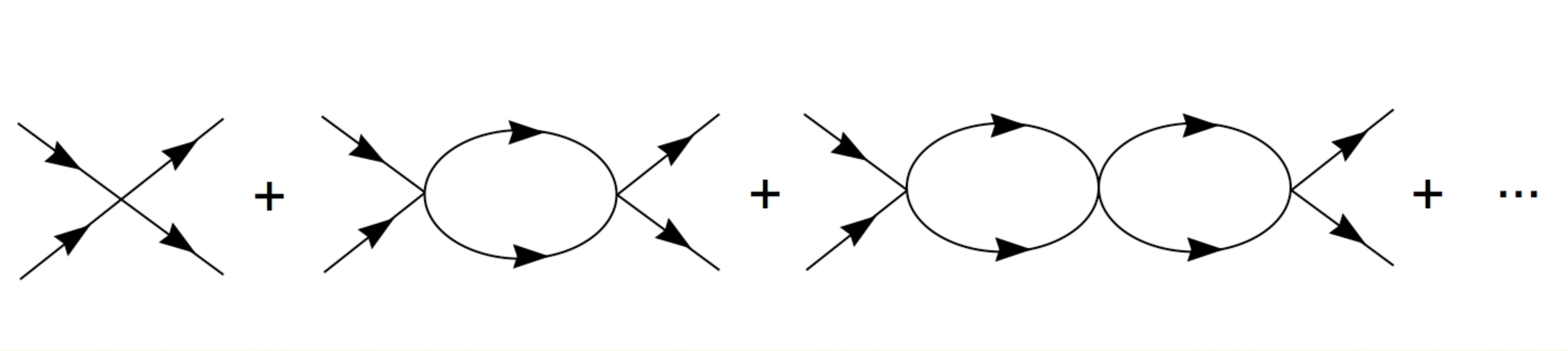
A quantum-mechanical implementation

$$-\frac{\hbar^2}{2m_R}\nabla^2\psi + V(\mathbf{r})\psi(\mathbf{r}) = E\psi(\mathbf{r})$$

- Details of force not important, so use something very simple: $V(\mathbf{r})=C_0\,\delta^{(3)}(r)$
- Coefficient C_0 set by the scattering length a

Quantum corrections essential

$$t_0^{2B} = \frac{2\pi a}{m_R}\frac{1}{1+iak}$$



To implement this numerically we use $V(\mathbf{r}) = C_0(R)\frac{1}{(2\pi R^2)^{3/2}}\exp\left(-\frac{r^2}{2R^2}\right)$

Lepage, "How to renormalize the Schrödinger equation", arXiv:nucl-th/9706029

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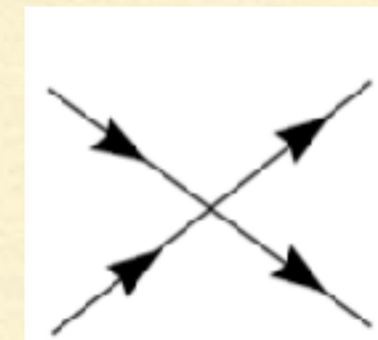

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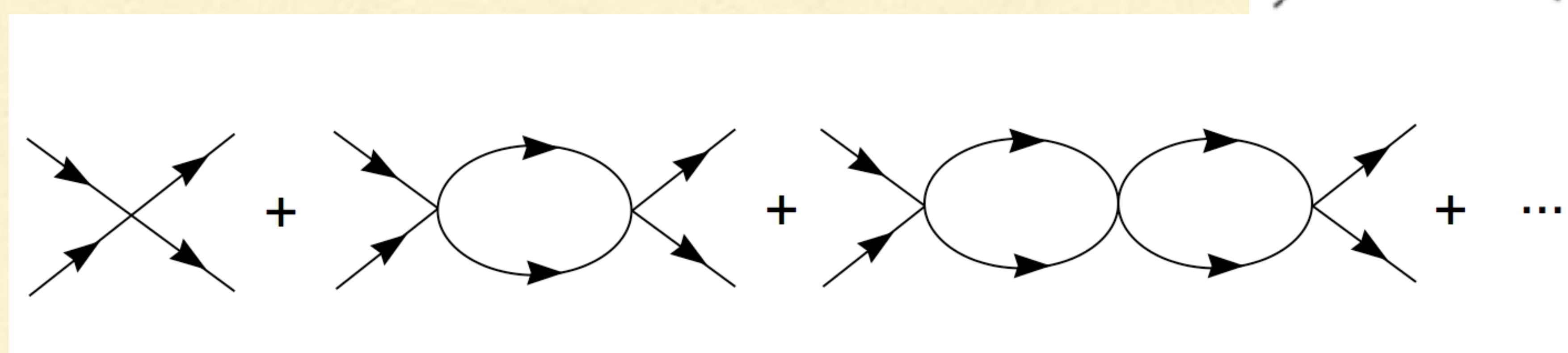

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Bonus material: DR with MS/PDS for Σ

- Dimensional regularization:

$$\Sigma_d(E,\mathbf{0}) = g^2 \left(\frac{\mu}{2}\right)^{4-d} \int \frac{d^{d-1}p}{(2\pi)^{d-1}} \frac{1}{E - \mathbf{p}^2/(2m_R) + i\epsilon}$$

$$\Rightarrow \Sigma_d(E,\mathbf{0}) = -\frac{m_R g^2}{2\pi} \left(\frac{-k^2 - i\epsilon}{4\pi}\right)^{\frac{d-3}{2}} \left(\frac{\mu}{2}\right)^{4-d} \Gamma\left(\frac{3-d}{2}\right)$$

- Minimal subtraction: no pole at d=4, so $\Sigma_d^{MS}(p) = -\dfrac{g^2 m_R}{2\pi} ik$

- Power-law divergence subtraction: subtract pole at d=3, then take limit d→4, so $\Sigma_d^{PDS}(p) = -\dfrac{g^2 m_R}{2\pi}(ik + \mu)$

- Subtraction scheme that simulates physics of a cutoff, at least at LO