TALENT school on "Effective Field Theories in Light Nuclei: from Structure to Reactions", Week 3, Exercise Sheet 1

Date: Monday, August 8

In this sheet we will construct a set of Gaussian potentials which reproduce the central values of the scattering length and effective range inferred for ^{18}C-neutron scattering by Acharya and Phillips in *Nucl. Phys.* **A913**, 103 (2013). The pertinent data are:

$$a_0 = 7.75 \text{ fm}; \quad r_0 = 2.6 \text{ fm}. \tag{1}$$

The mass of the ^{18}C nucleus is 16.79 GeV/c^2 [1]. The neutron mass is 0.9396 GeV/c^2.

LO analysis

We first consider Gaussians of the form

$$V_{LO}(r; R) = C_0(R)\frac{1}{(2\pi R^2)^{3/2}} \exp\left(-\frac{r^2}{2R^2}\right). \tag{2}$$

The standard deviation of the Gaussian, R, acts as a regulator. In the limit $R \to 0$ this becomes a three-dimensional δ-function.

1. For the different radii $R = 1.5$ 2, 2.5, and 3 fm adjust the strength of the Gaussian $C_0(R)$ so that, no matter which of those radii you pick, you get the a_0 listed above.
2. What is the effective range for these different potentials?
3. Can you also find $C_0(R)$'s that give the right a_0 if $R = 0.5$ and 1 fm? Bonus points (okay, so we're not giving points, so not really) if you can also do it at $R = 0.1$ fm.
4. Plot $C_0(R)$ as a function of R.
5. What do you notice about the effective range in these cases?
6. Plot the phase shift $\delta(E)$ for s-wave ^{18}C-neutron scattering for all of the LO potentials you've constructed. What do you notice?

NLO analysis

Now we add a second-order (in the EFT sense) term to the potential. The NLO form of the "Halo EFT potential" is (note that $\tilde{C}_0 \neq C_0$ in general):

$$V_{NLO}(r; R) = \frac{1}{(2\pi R^2)^{3/2}}\left[\tilde{C}_0(R) \exp\left(-\frac{r^2}{2R^2}\right) + C_2(R) r^2 \exp\left(-\frac{r^2}{2R^2}\right)\right]. \tag{3}$$

1. Show that $\nabla^2 \exp\left(-\frac{r^2}{2R^2}\right) = \frac{r^2}{R^4}\exp\left(-\frac{r^2}{2R^2}\right) - \frac{1}{R^2}\exp\left(-\frac{r^2}{2R^2}\right)$. This justifies Eq. (3).
2. For the different radii $R = 2$ fm, $R = 2.5$ fm and $R = 3$ fm adjust the parameters $\tilde{C}_0(R)$ and $C_2(R)$ so that, no matter which of those radii you pick, you get both the a_0 and r_0 listed above.
3. What happens if you try to do this for $R = 1.5$ fm?
4. Plot the phase shift $\delta(E)$ for s-wave ^{18}C-neutron scattering for all of the NLO potentials you've constructed. What do you notice?

[1] Atomic mass taken from AME 2020 and corrected for presence of six electrons