Few-body Reactions

Lecture 2

Electromagnetic Operators

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TALENT School @ MITP

EFFECTIVE FIELD THEORIES IN LIGHT NUCLEI:
from Structure to Reactions
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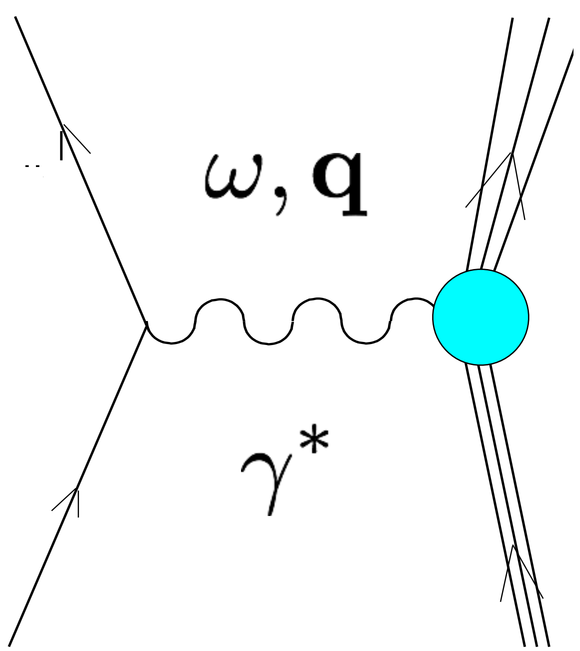
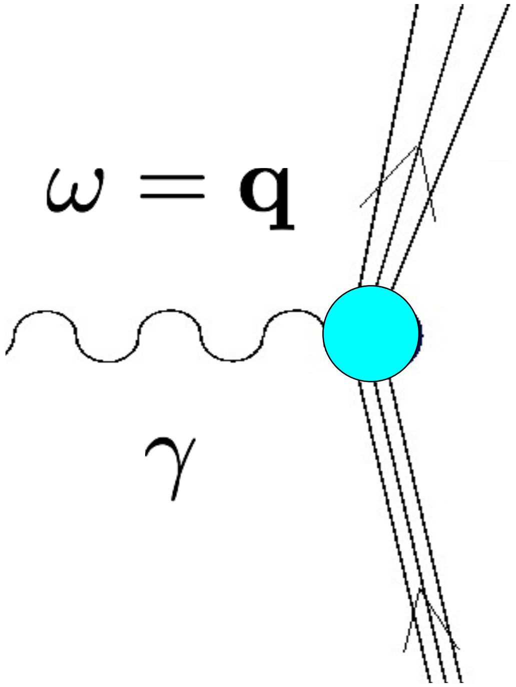
https://indico.mitp.uni-mainz.de/event/279/

Outline of Lecture 2

- Multipole decomposition of the charge operator
- Multipole decomposition of the current operator
- Siegert Theorem
- Extension to the weak sector

Electromagnetic processes





Electron scattering
(virtual photon)

Photoabsorption
(real photon)

Cross section involves the calculation of

$$\sigma_{em} \sim |\langle \Psi_f | \; \rho \text{ or } \mathbf{J} \; | \Psi_0 \rangle|^2$$

Multipole decomposition

Since the intrinsic states of the nucleus can be classified according to the total angular momentum (see lectures by Nir Barnea), it is very useful to perform a multipole decomposition of the charge and of the current operators, where each multipole transfers a definite angular momentum J.

$\langle \Psi_f | \mathcal{O}^J | \Psi_0 \rangle$ → selection rules

The advantage of this approach is also that one can use the Wigner-Eckart theorem, separating the geometrical aspects form the dynamical properties of the system, which remain in the reduced matrix element.

Reduces complexity of each nuclear matrix element

Charge operator

$$\rho\left(\mathbf{x}\right) = e\sum_i^A \frac{1+\tau_i^z}{2}\,\delta(\mathbf{x}-\mathbf{r}_i)$$

One body operator

$$\Big\downarrow\ \text{FF}$$

$$\rho(\mathbf{q}) = \int d^3x\; e^{i\mathbf{q}\cdot\mathbf{x}}\;\rho(\mathbf{x})$$

$$\rho(\mathbf{q}) = e\sum_i^A \frac{1+\tau_i^z}{2}\int d^3x\; e^{i\mathbf{q}\cdot\mathbf{x}}\;\delta(\mathbf{x}-\mathbf{r}_i)$$

$$= e\sum_i^A \frac{1+\tau_i^z}{2}\; e^{i\mathbf{q}\cdot\mathbf{r}_i}$$

Charge operator

Spatial part, single coordinate   omitting i-index

$e^{i\mathbf{q}\cdot\mathbf{r}}$ ⟶ scalar function, that depends on (r,θ,ϕ)

Any function that depends on angles can be expanded in spherical harmonics, as they are a complete set of basis states

$$f(\theta,\phi)=\sum_{J\mu} a_{J\mu} Y_\mu^{J*}(\theta,\phi)$$

with

$$a_{J\mu}=\int d\theta \int d\phi \; f(\theta,\phi) Y_\mu^{J} \;\; (\theta,\phi)$$

Charge operator

Spatial part, single coordinate omitting i-index

$e^{i\mathbf{q}\cdot\mathbf{r}}$ ⟶ scalar function, that depends on (r, θ, ϕ)

Any function that depends on angles can be expanded in spherical harmonics, as they are a complete set of basis states

$$f(r,\theta,\phi) = \sum_{J\mu} a_{J\mu}(r) Y_{\mu}^{J^*}(\theta,\phi)$$

with

$$a_{J\mu}(r) = \int d\theta \int d\phi \; f(r,\theta,\phi) Y_{\mu}^{J}\; (\theta,\phi)$$

Charge operator

Spatial part, single coordinate omitting i-index

$e^{i\mathbf{q}\cdot\mathbf{r}}$ ⟶ scalar function, that depends on (r,θ,ϕ)

Any function that depends on angles can be expanded in spherical harmonics, as they are a complete set of basis states

Plane wave expansion in spherical harmonics

$$e^{i\mathbf{q}\cdot\mathbf{r}} = 4\pi\sum_{J\mu} i^J \underset{\uparrow \text{ Radial part}}{j_J(qr)} Y^{J\,*}_{\mu}(\hat{q}) \underset{\uparrow \text{ Angular part}}{Y^J_\mu(\hat{r})}$$

Operator that carries angular momentum J

$$\Rightarrow \sum_{J\mu} j_J(qr) \quad Y^J_\mu(\hat{r}) \to \sum_{J\mu} C_{J\mu}(q)$$

Charge operator

Spatial part, single coordinate   omitting i-index

$e^{i\mathbf{q}\cdot\mathbf{r}}$ ⟶ scalar function, that depends on (r,θ,ϕ)

Any function that depends on angles can be expanded in spherical harmonics, as they are a complete set of basis states

Plane wave expansion in spherical harmonics

$$e^{i\mathbf{q}\cdot\mathbf{r}} = 4\pi \sum_{J\mu} i^J \underbrace{j_J(qr)}_{\text{Radial part}} Y_\mu^{J^*}(\hat{q}) \underbrace{Y_\mu^J(\hat{r})}_{\text{Angular part}}$$

Coulomb multipoles $C_{J\mu} \propto \sum_i^{A} \; j_J(qr_i) \; Y_\mu^J(\hat{r}_i)$

Illustrative Example ^{4}He

Inclusive cross section ^{4}He(e,e')X, after Rosenbluth separation (L/T)

To compare the experimental longitudinal response function with a calculation, we have to compute

$$R_L(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\int_f \left|\langle\Psi_f|\,\rho(\mathbf{q})\,|\Psi_0\rangle\right|^2 \delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$

charge operator

- For a given Hamiltonian, if you have a solver for $|\Psi_{0/f}\rangle$ (see lecture by Nir Barnea)
- Fix q on a certain grid and expand the charge operator into multipoles for every q
- Compute the individual multipoles (separating isoscalar and isovector)

  $\frac{1}{2}$ $\frac{\tau_i^3}{2}$

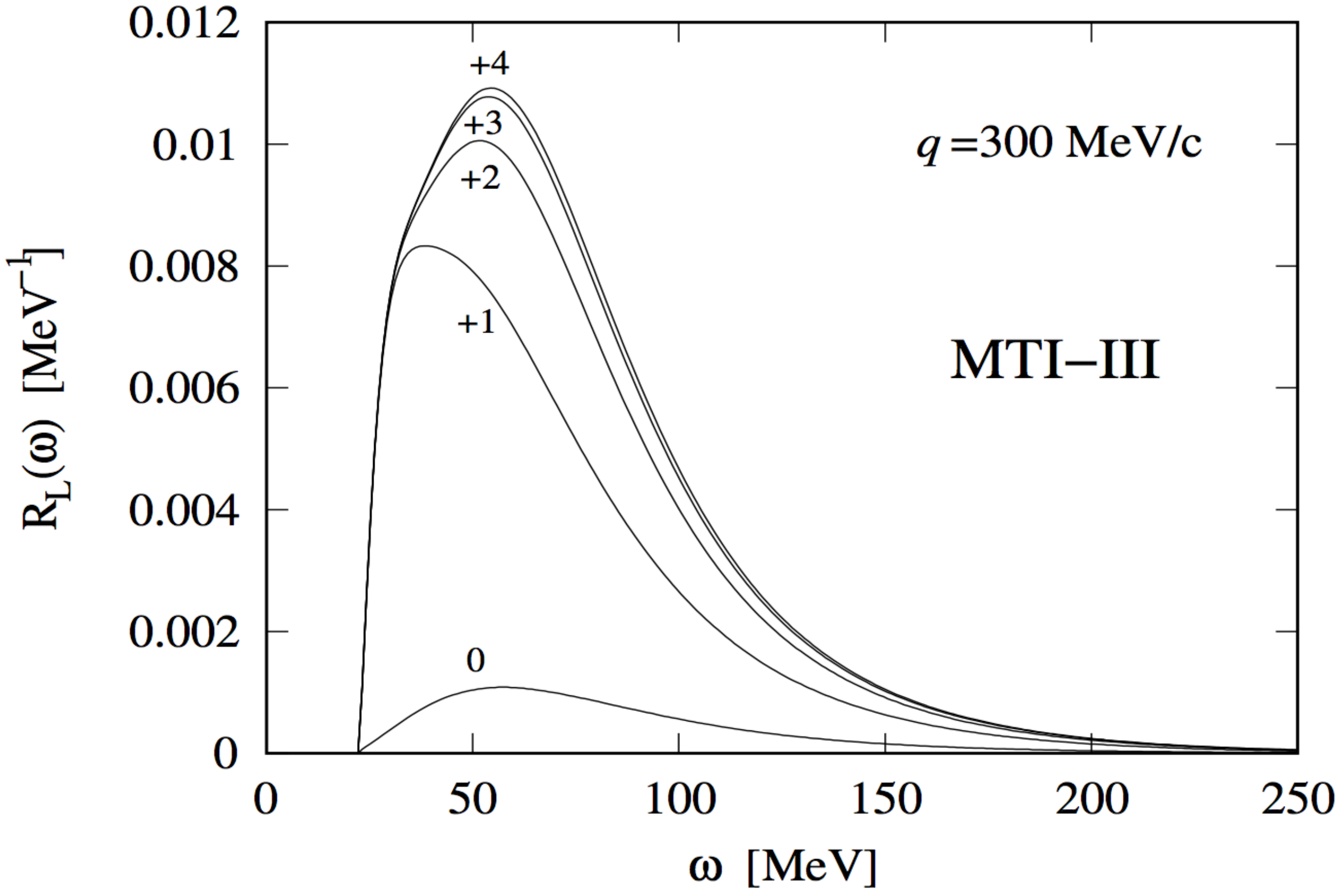
- Sum them up and compare to data

Illustrative Example ^{4}He

Recursive sum of Coulomb multipoles

S.B. *et al.*, PRC **76**, 014003 (2007)

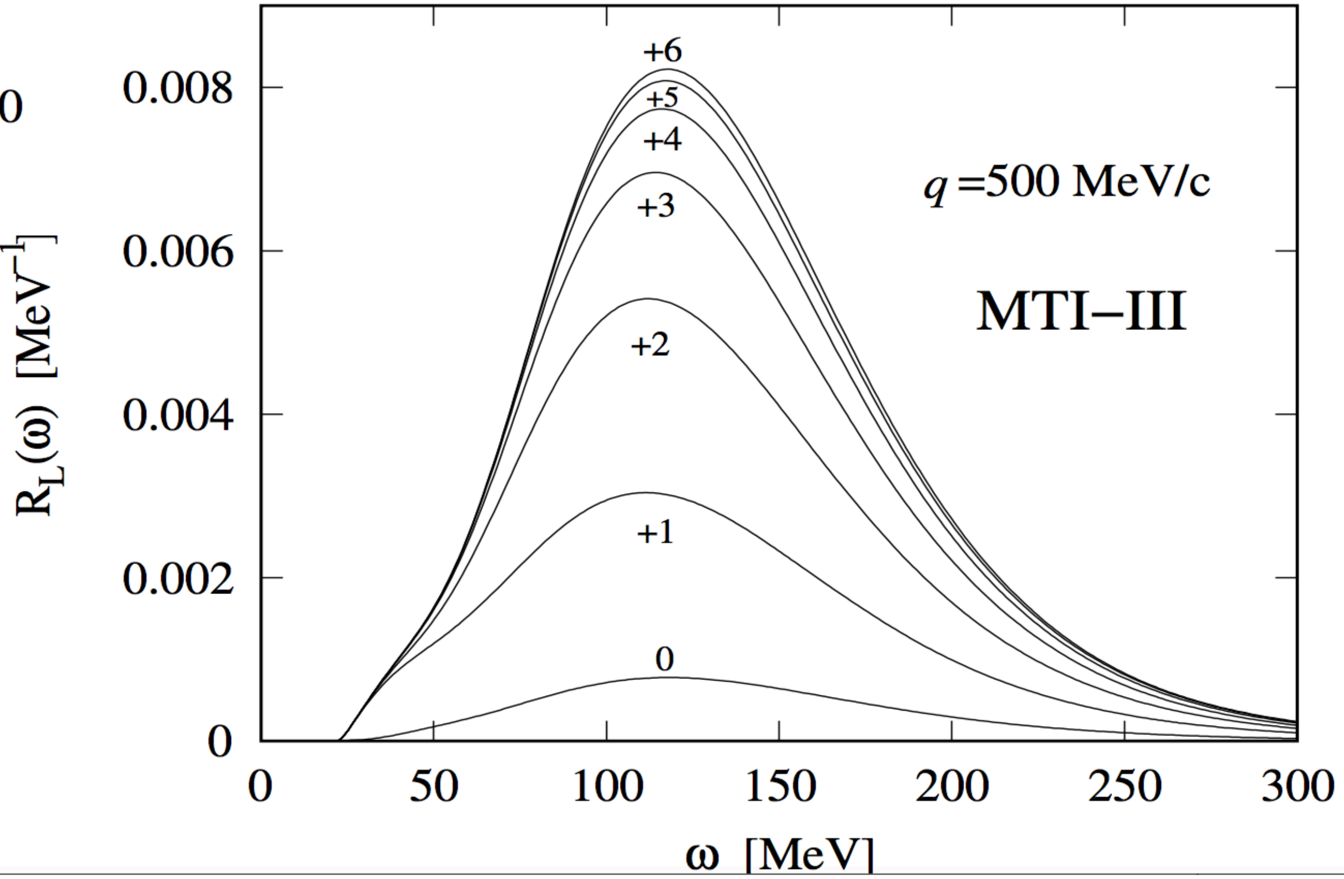
Isovector response



Calculations with hyperspherical harmonics

At low q the Coulomb dipole of oder 1 dominates

Isovector response

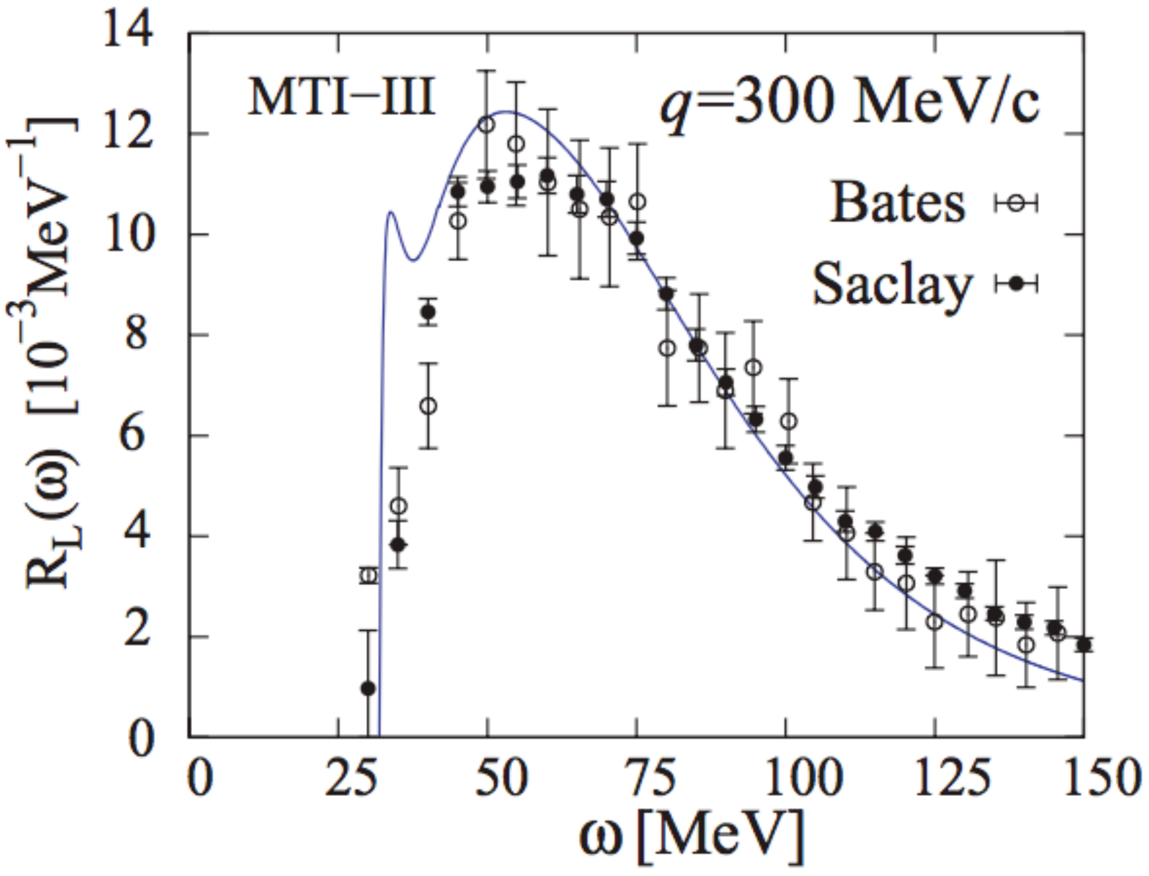


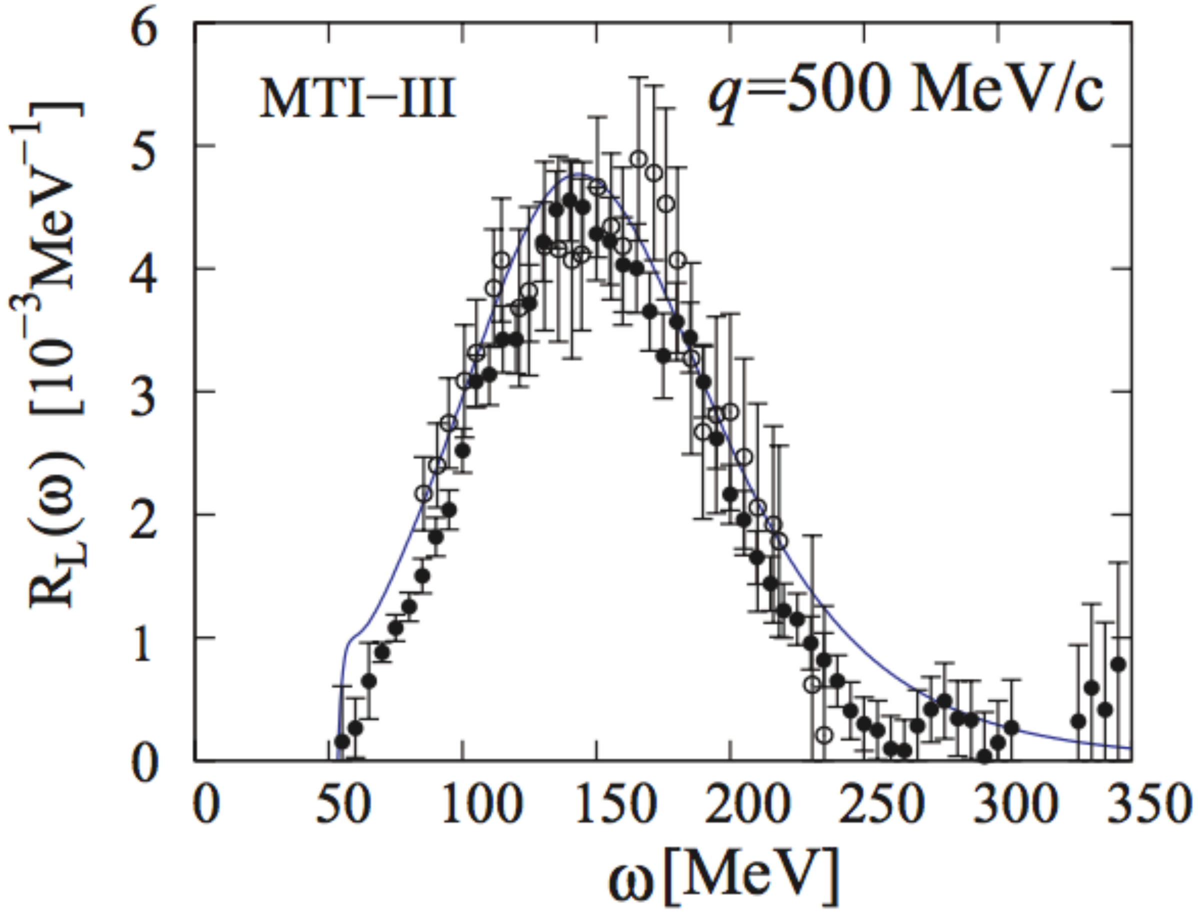
The higher the momentum transfer, the slower the multipole convergence

Illustrative Example ^{4}He

Comparison to experimental data

S.B. *et al.*, PRC **76**, 014003 (2007)





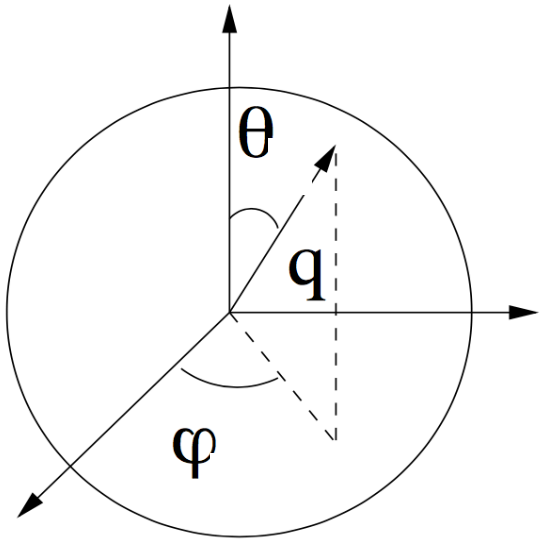
Agreement with experimental data is quite good!

Current operator

Since the current operator is a vector, the expansion is done in terms of the vector spherical harmonics

$$\mathbf{Y}^{\mu}_{Jl1}(\hat{q}) = \sum_{m\xi} \langle l1J|m\xi\mu\rangle\, Y^{l}_{m}(\hat{q})\mathbf{e}_{\xi}$$

Unit vector in the spherical basis



$$\mathbf{e}_1 = -\frac{1}{\sqrt{2}}(\mathbf{e}_x + i\mathbf{e}_y)$$

$$\mathbf{e}_0 = \mathbf{e}_z$$

$$\mathbf{e}_{-1} = \frac{1}{\sqrt{2}}(\mathbf{e}_x - i\mathbf{e}_y)$$

The vector spherical harmonics form a complete set on the unit sphere

$$\int d\hat{q}'\; \mathbf{Y}^{\mu'\,*}_{J'l'1}(\hat{q}') \cdot \mathbf{Y}^{\mu}_{Jl1}(\hat{q}') = \delta_{JJ'}\delta_{ll'}\delta_{\mu,\mu'}$$

JG|U

Current operator

Multipole expansion of the current operator

$$\mathbf{J}\left(\mathbf{q}\right)=4\pi\sum_{lJ\mu}J^{\mu}_{Jl}(q)\mathbf{Y}^{\mu *}_{Jl1}(\hat{q})$$

with $J^{\mu}_{Jl}(q)=\frac{1}{4\pi}\int d\hat{q}\,'\mathbf{J}\left(\mathbf{q}'\right)\cdot\mathbf{Y}^{\mu}_{Jl1}(\hat{q}\,')$

According to angular momentum rules *l*=J-1,J,J+1 ⇒ separate according to parity

$$\mathbf{J}\left(\mathbf{q}\right)=\sum_{J\mu}\left(\mathbf{J}^{el}_{J\mu}(\mathbf{q})+\mathbf{J}^{mag}_{J\mu}(\mathbf{q})\right)$$

$$\mathbf{J}^{el}_{J\mu}(\mathbf{q})=4\pi\left(J^{\mu}_{JJ-1}(q)\mathbf{Y}^{\mu *}_{JJ-11}(\hat{q})+J^{\mu}_{JJ+1}(q)\mathbf{Y}^{\mu *}_{JJ+11}(\hat{q})\right)$$

Electric multipoles

parity $(-1)^{J}$

$$\mathbf{J}^{mag}_{J\mu}(\mathbf{q})=4\pi J^{\mu}_{JJ}(q)\mathbf{Y}^{\mu *}_{JJ1}(\hat{q})$$

Magnetic multipoles parity $(-1)^{J+1}$

Current operator

The expression for the electric multipole can be rewritten as

$$\mathbf{J}^{el}_{J\mu}(\mathbf{q}) = 4\pi\left(J^{\mu}_{JJ-1}(q)\mathbf{Y}^{\mu *}_{JJ-11}(\hat{q}) + J^{\mu}_{JJ+1}(q)\mathbf{Y}^{\mu *}_{JJ+11}(\hat{q})\right)$$

$$J^{\mu}_{Jl}(q) = \frac{1}{4\pi}\int d\hat{q}'\,\mathbf{J}\left(\mathbf{q}'\right)\cdot\mathbf{Y}^{\mu}_{Jl1}(\hat{q}')$$

Using

$$\mathbf{Y}^{\mu}_{JJ-11}(\hat{q}) = \sqrt{\frac{J}{2J+1}}\,\hat{\mathbf{q}}Y^{J}_{\mu}(\hat{q}) - i\sqrt{\frac{J+1}{2J+1}}\,\hat{\mathbf{q}}\times\mathbf{Y}^{\mu}_{JJ11}(\hat{q})$$

(with $\hat{\mathbf{q}} = \frac{\mathbf{q}}{|\mathbf{q}|}$)

$$\mathbf{Y}^{\mu}_{JJ+11}(\hat{q}) = -\sqrt{\frac{J+1}{2J+1}}\,\hat{\mathbf{q}}Y^{J}_{\mu}(\hat{q}) - i\sqrt{\frac{J}{2J+1}}\,\hat{\mathbf{q}}\times\mathbf{Y}^{\mu}_{JJ11}(\hat{q})$$

we get

$$\mathbf{J}^{el}_{J\mu}(\mathbf{q}) = \hat{\mathbf{q}}Y^{J*}_{\mu}(\hat{q})\int d\hat{q}'\left(\hat{\mathbf{q}}'\cdot\mathbf{J}\left(\mathbf{q}'\right)\right)Y^{J}_{\mu}(\hat{q}') + \left(\hat{\mathbf{q}}\times\mathbf{Y}^{\mu *}_{JJ1}(\hat{q})\right)\int d\hat{q}'\left(\hat{\mathbf{q}}'\times\mathbf{Y}^{\mu}_{JJ1}(\hat{q}')\right)\mathbf{J}\left(\mathbf{q}'\right)$$

Longitudinal part of the current

Transverse part of the current

Current operator

Introducing longitudinal and transverse electric multipoles and magnetic multipoles

$$L^{el}_{J\mu}(q) = \frac{1}{4\pi}\int d\hat{q}'\,(\hat{\mathbf{q}}'\cdot\mathbf{J}\,(\mathbf{q}'))\,Y^{J}_{\mu}(\hat{q}')$$

$$T^{el}_{J\mu}(q) = \frac{i}{4\pi}\int d\hat{q}'\,(\hat{\mathbf{q}}'\times\mathbf{Y}^{\mu}_{JJ1}(\hat{q}'))\cdot\mathbf{J}\,(\mathbf{q}')$$

$$T^{mag}_{J\mu}(q) = \frac{1}{4\pi}\int d\hat{q}'\mathbf{J}\,(\mathbf{q}')\cdot\mathbf{Y}^{\mu}_{JJ1}(\hat{q}')$$

The magnetic multipoles are transverse only due to $\hat{\mathbf{q}}\cdot\mathbf{Y}^{\mu}_{JJ1}(\hat{q}) = 0$

NB: for every piece of em current (convection, spin, two-body current) one can calculate these multipoles

Current operator

Choosing the z-axis as the direction of propagation of the photon momentum

$\mathbf{q} = q\mathbf{e}_z = q\mathbf{e}_0$ then

$$\mathbf{Y}^{\mu}_{Jl1} = \langle l1J|0\mu\mu\rangle \frac{\hat{l}}{\sqrt{4\pi}}\mathbf{e}_\mu \qquad \text{with} \quad \hat{l} = \sqrt{2l+1}$$

Substitute all of these in the expression of the current in terms of longitudinal, electric and magnetic multipoles

$$\mathbf{J}(\mathbf{q}) = \sum_{J\mu} \sqrt{4\pi}\hat{J}\left[L^{el}_{J\mu}(q)\mathbf{e}_0 + \mu\langle J1J|0\mu\mu\rangle T^{el}_{J\mu}(q)\mathbf{e}^*_\mu\right] + \sum_{J\mu}\sqrt{4\pi}\hat{J}\langle J1J|0\mu\mu\rangle T^{mag}_{J\mu}(q)\mathbf{e}^*_\mu$$

As in the nuclear matrix elements typically we habe $\mathbf{e}_\lambda \cdot \mathbf{J}(\mathbf{q})$, then we rewrite as

$$\boxed{\mathbf{e}_\lambda \cdot \mathbf{J}(\mathbf{q}) = (-)^\lambda \sqrt{2\pi(1+\delta_{\lambda 0})}\sum_J \hat{J}\left[L^{el}_{J\lambda}(q)\delta_{\lambda 0} + \left(T^{el}_{J\lambda}(q) + \lambda T^{mag}_{J\lambda}(q)\right)\delta_{|\lambda|1}\right]}$$

Multipole decomposition of the current operator

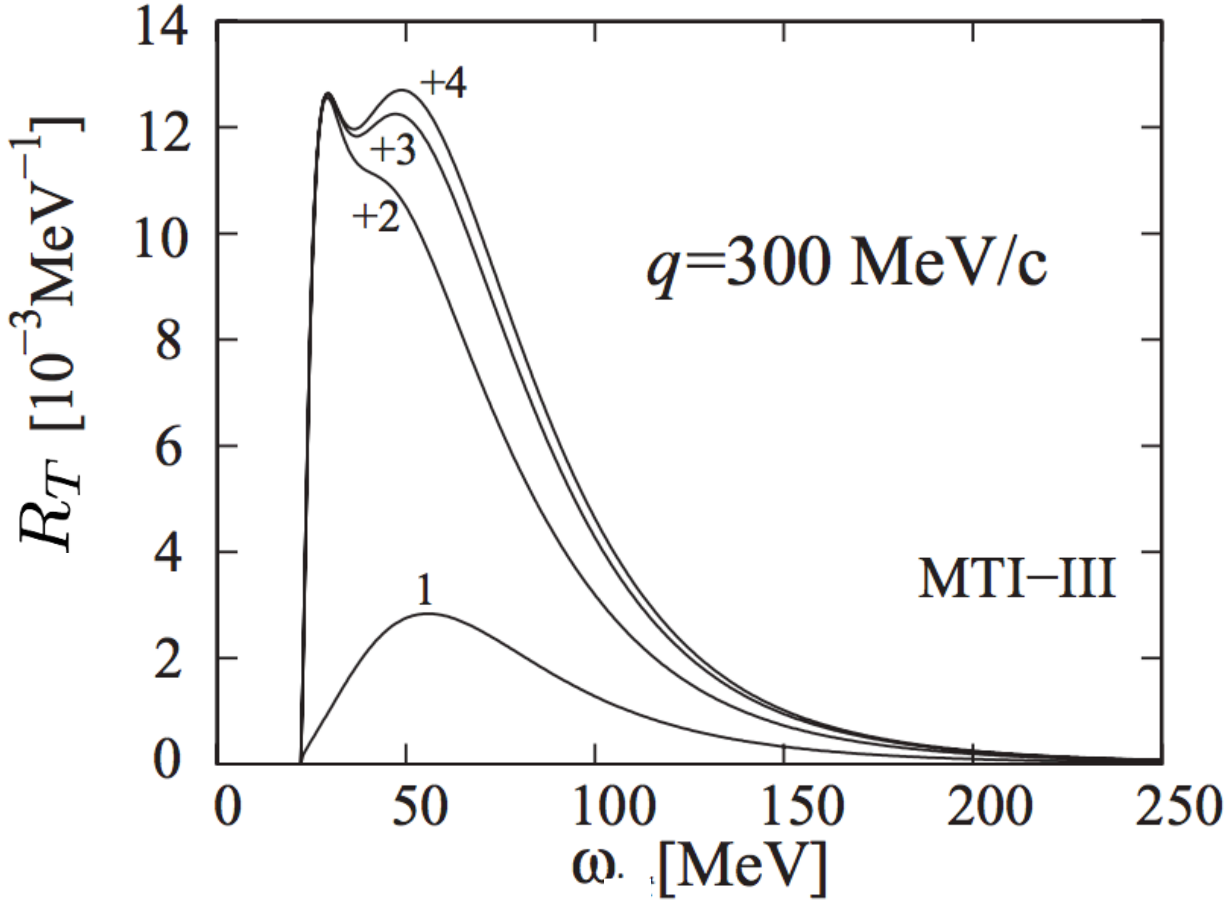
Illustrative Example ^{4}He

$$\mathbf{e}_\lambda \cdot \mathbf{J}\left(\mathbf{q}\right) = (-)^\lambda \sqrt{2\pi(1+\delta_{\lambda 0})}\sum_J \hat{J}\left[L^{el}_{J\lambda}(q)\delta_{\lambda 0} + \boxed{\left(T^{el}_{J\lambda}(q)+\lambda T^{mag}_{J\lambda}(q)\right)\delta_{|\lambda|1}}\right]$$

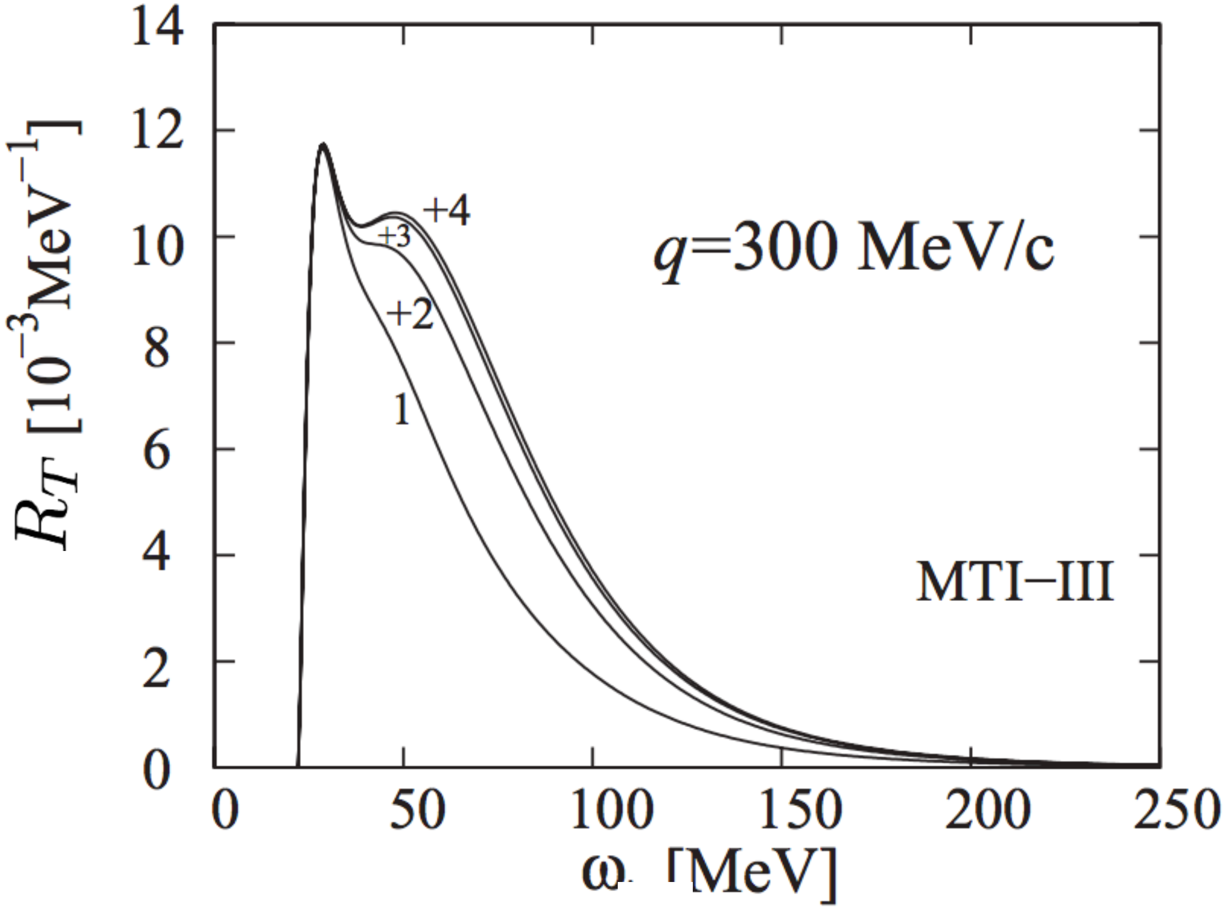
Recursive sum of transverse multipoles of spin and convection currents

S.B. *et al.*, PRC **76**, 014003 (2007)

Isovector *Magnetic multipoles*



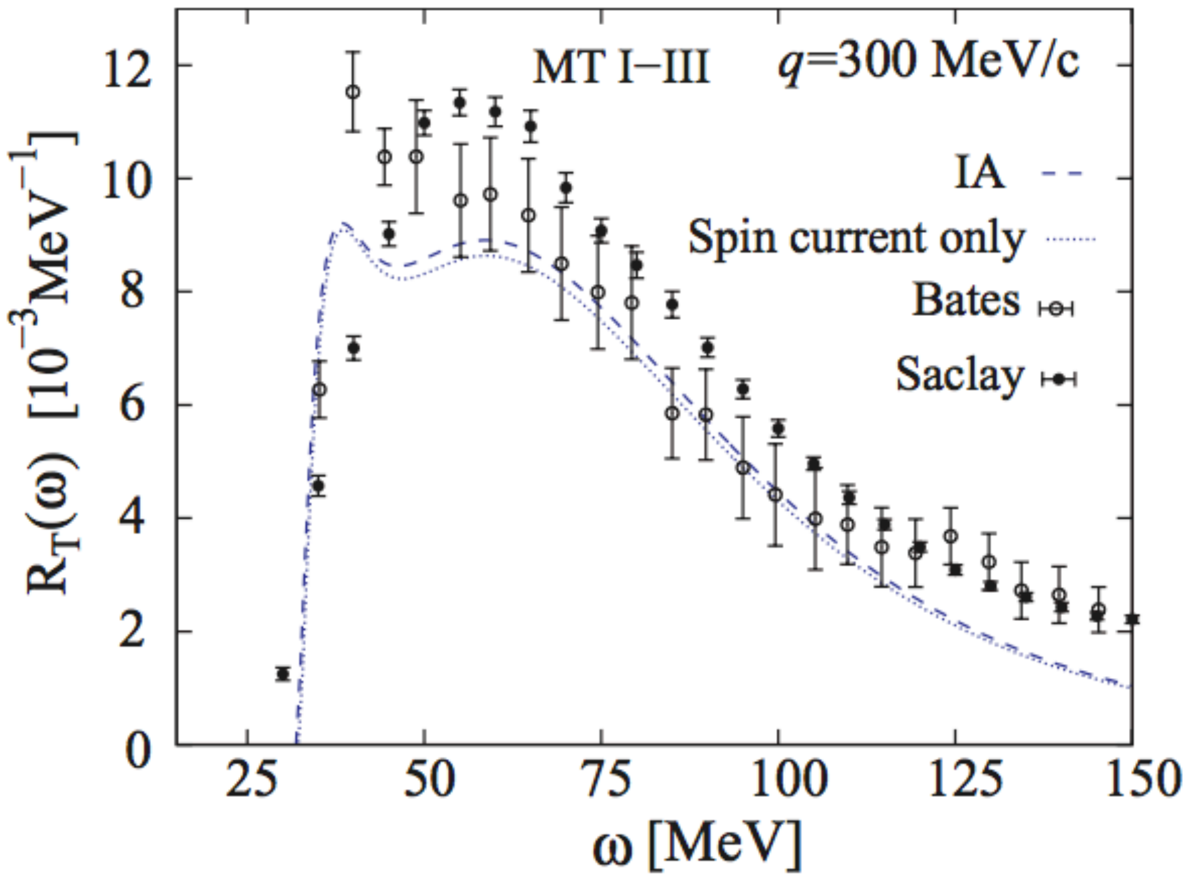
Isovector *Electric multipoles*

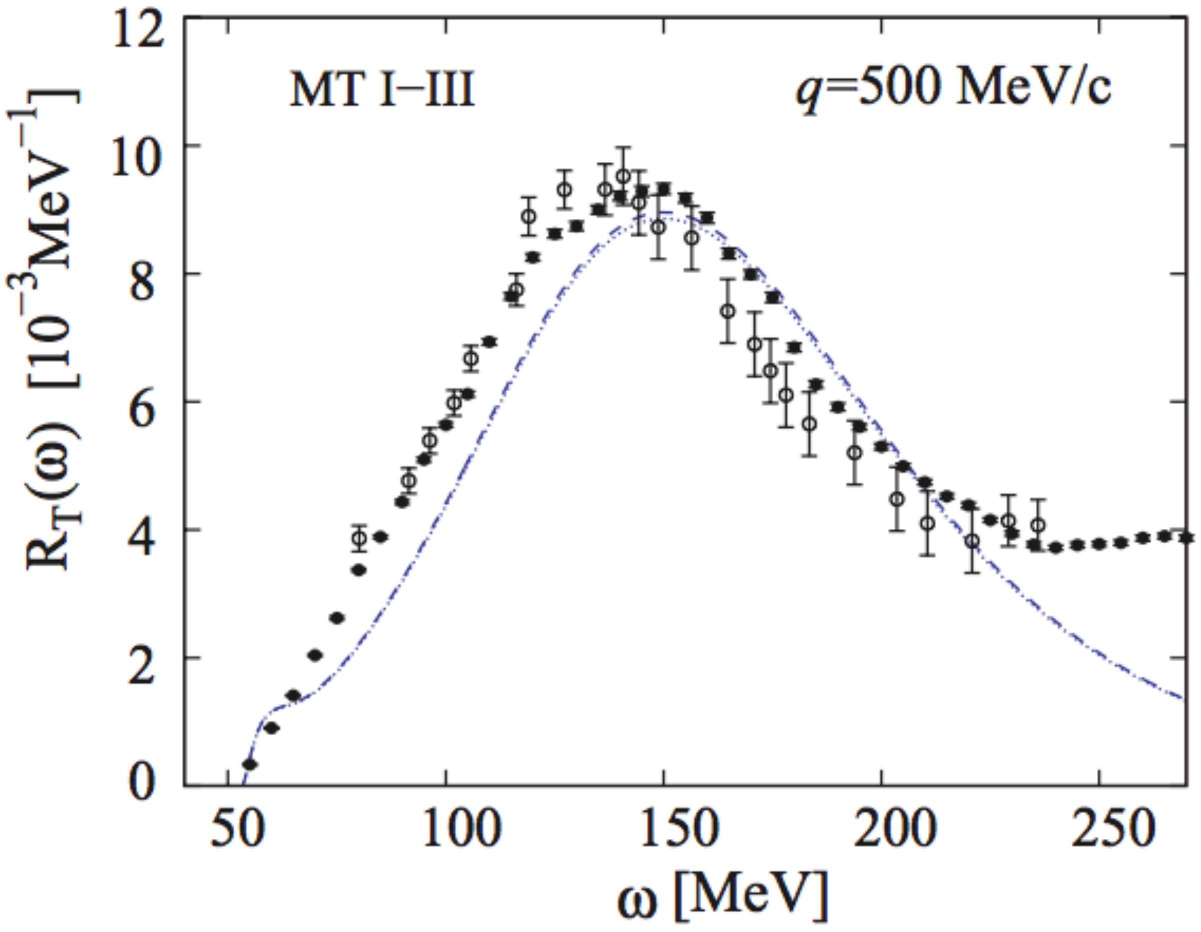


Illustrative Example ^4He

Comparison with experiment

S.B. *et al.*, PRC **76**, 014003 (2007)



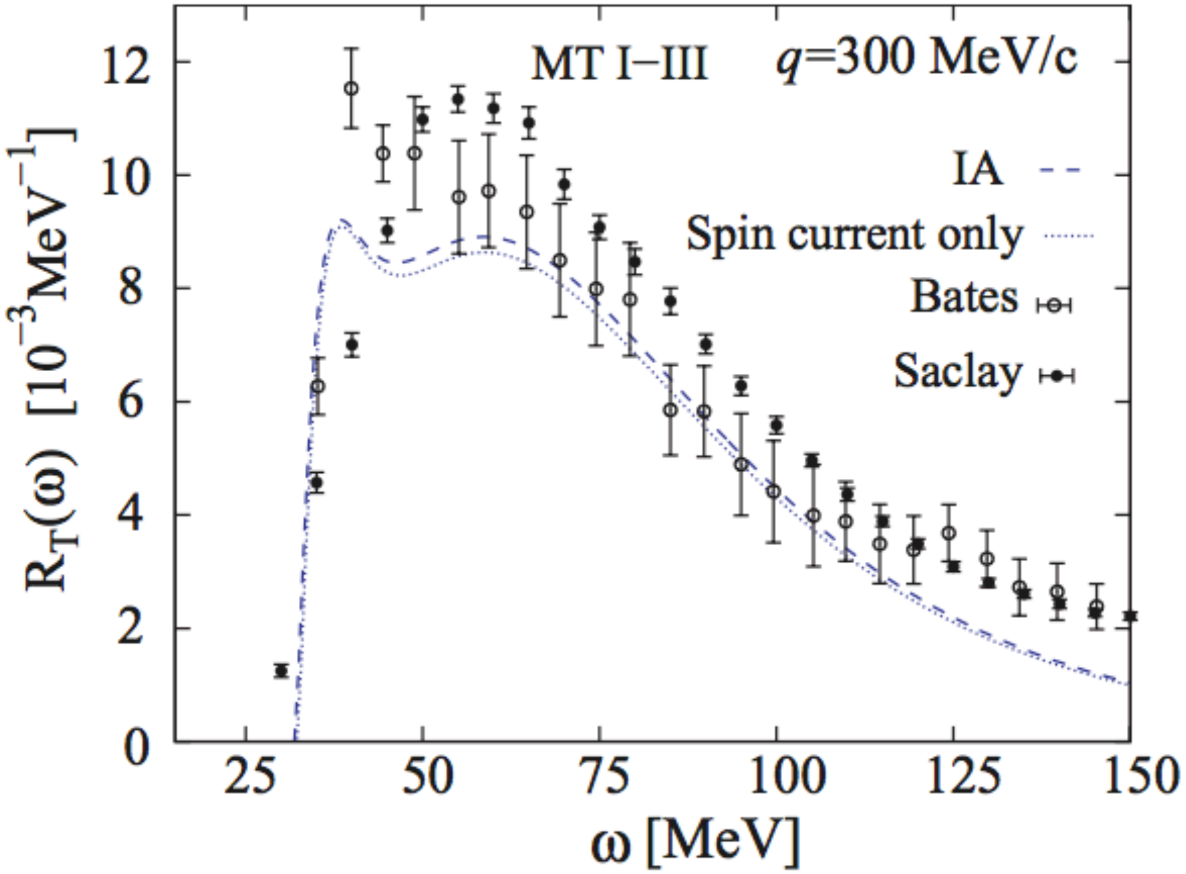


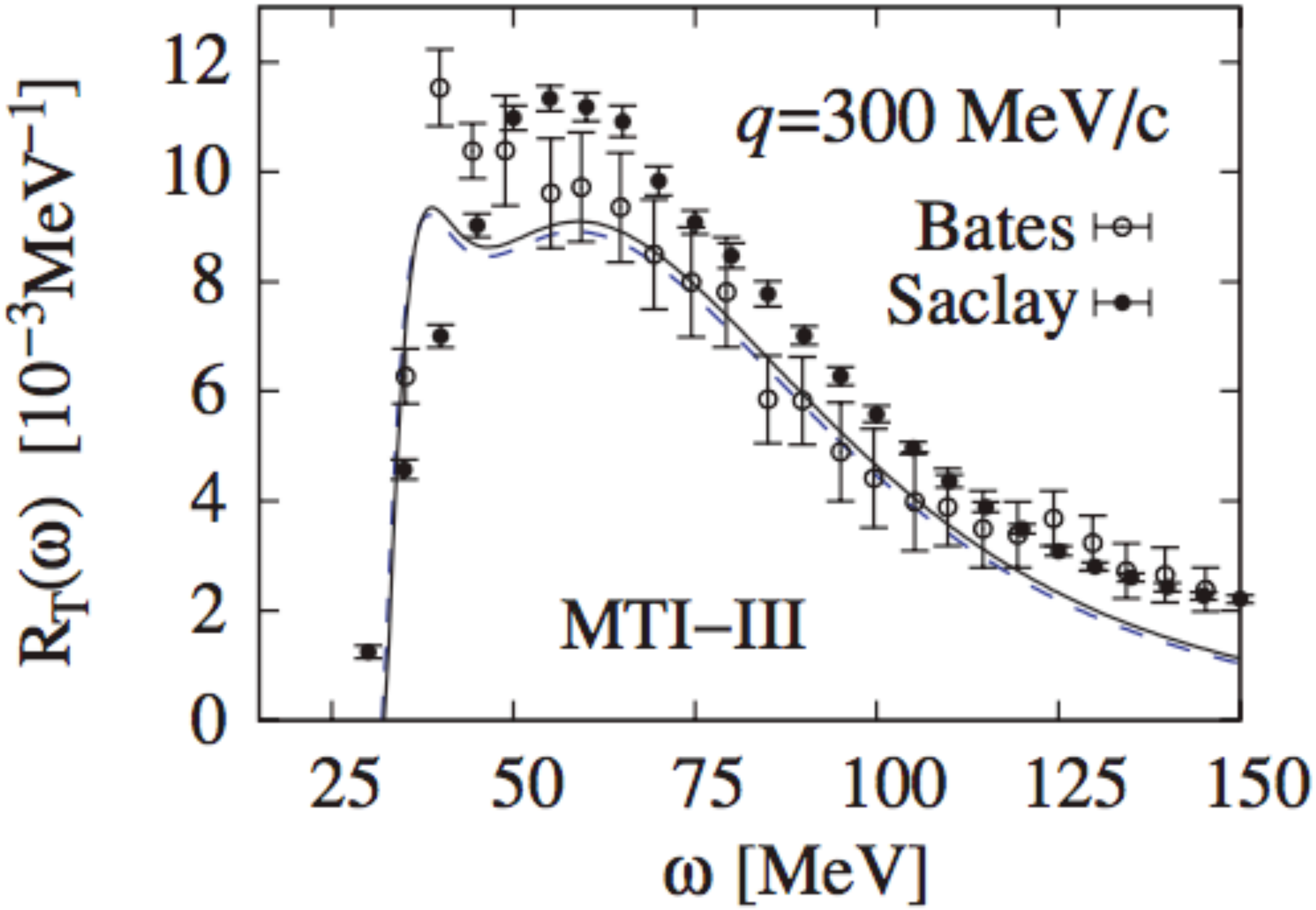
The agreement with experimental data is not good.
Missing strength, due to missing two-body currents.

Illustrative Example ^{4}He

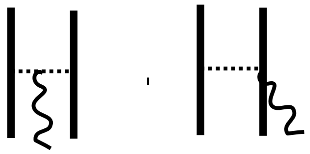
Comparison with experiment

S.B. *et al.*, PRC **76**, 014003 (2007)





In this semi-realistic case, consistent two-body currents (due to the exchange of a scalar meson) do not explain the data.



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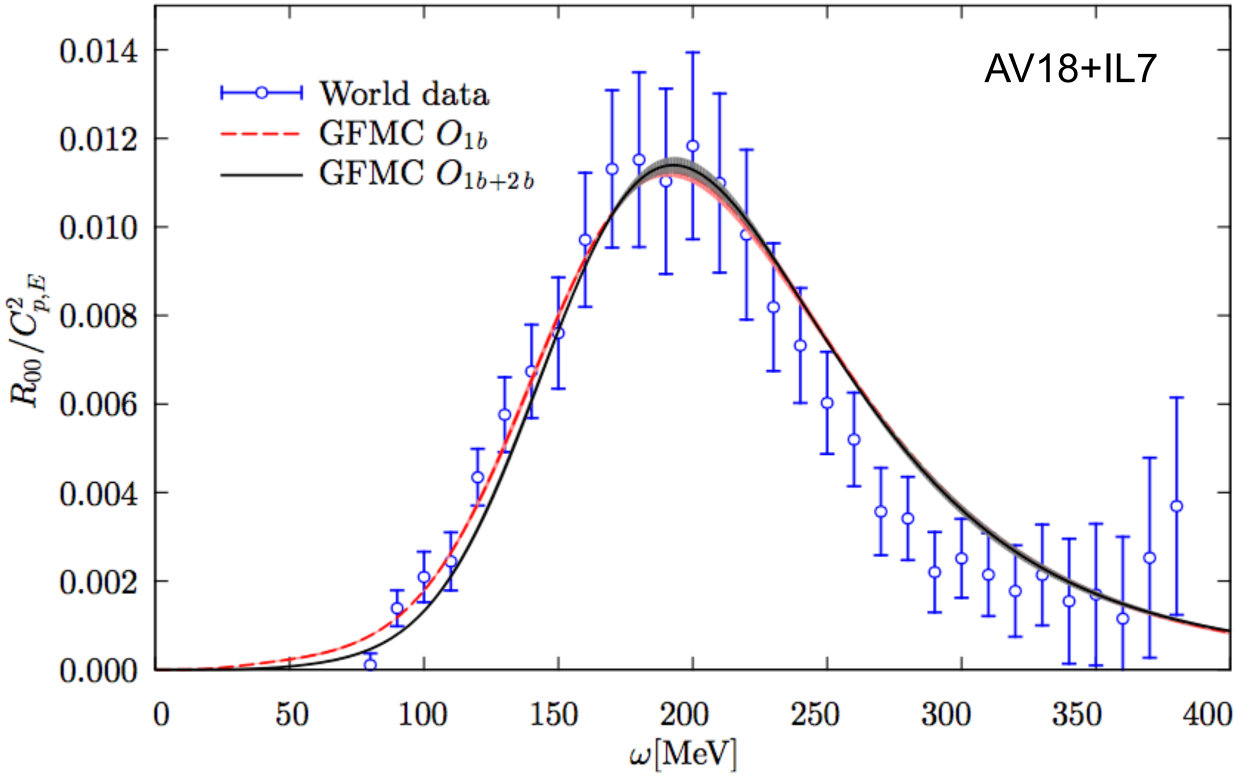
Realistic Calculation ^{4}He

With traditional potentials and two-body currents

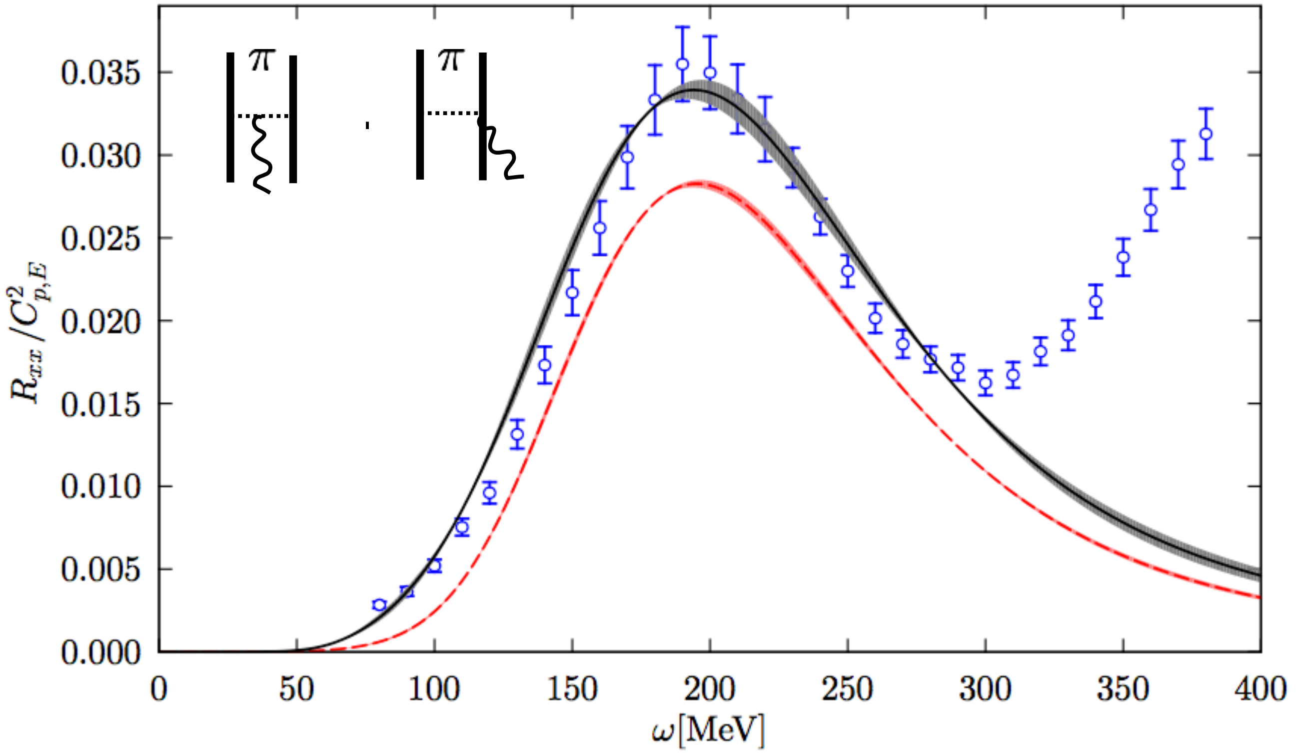
Lovato *et al.*, PRC **91**, 062501 (2015)

Calculations with Green's Function Monte Carlo

Longitudinal, q=600 MeV/c

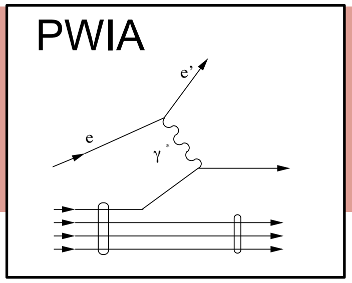


Transverse, q=600 MeV/c



Realistic two-body currents are necessary to obtain agreement with data in the transverse response

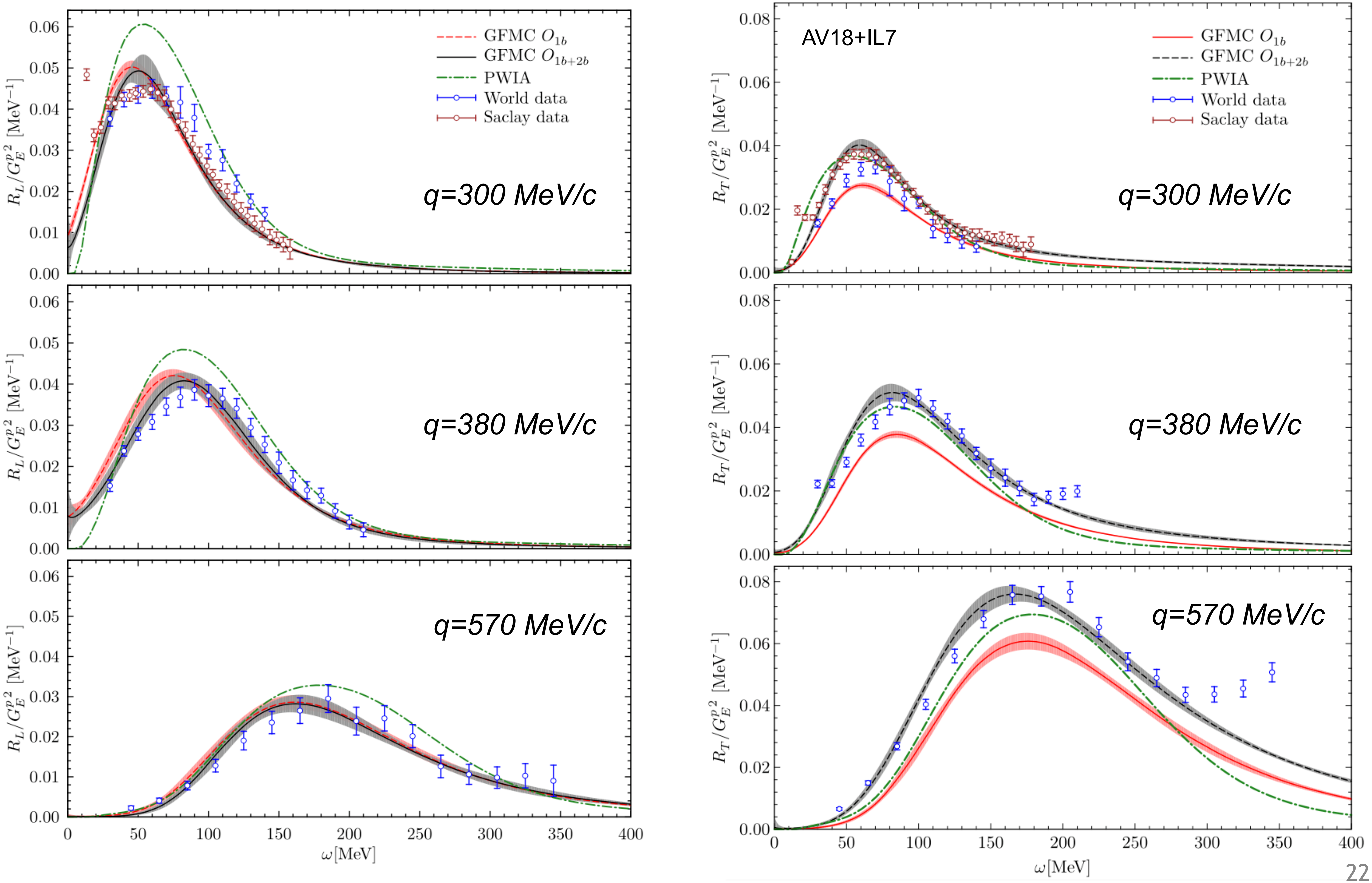
Extension to ^{12}C



Lovato *et al.*, PRL **117,** 082501 (2017)

Longitudinal

Transverse

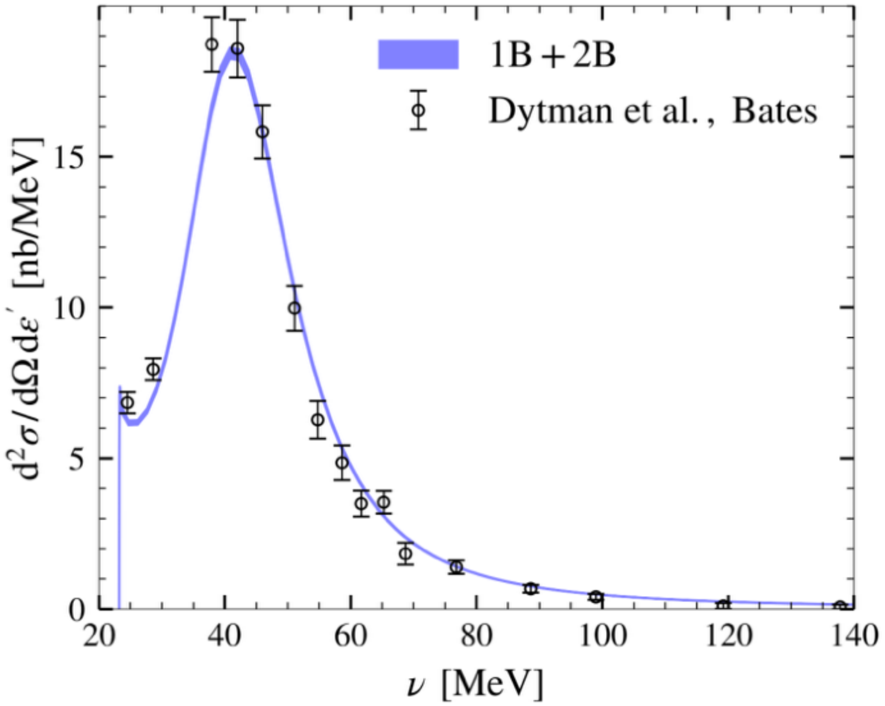


What about calculations with chiral effective field theory dynamical ingredients (potential and currents)?

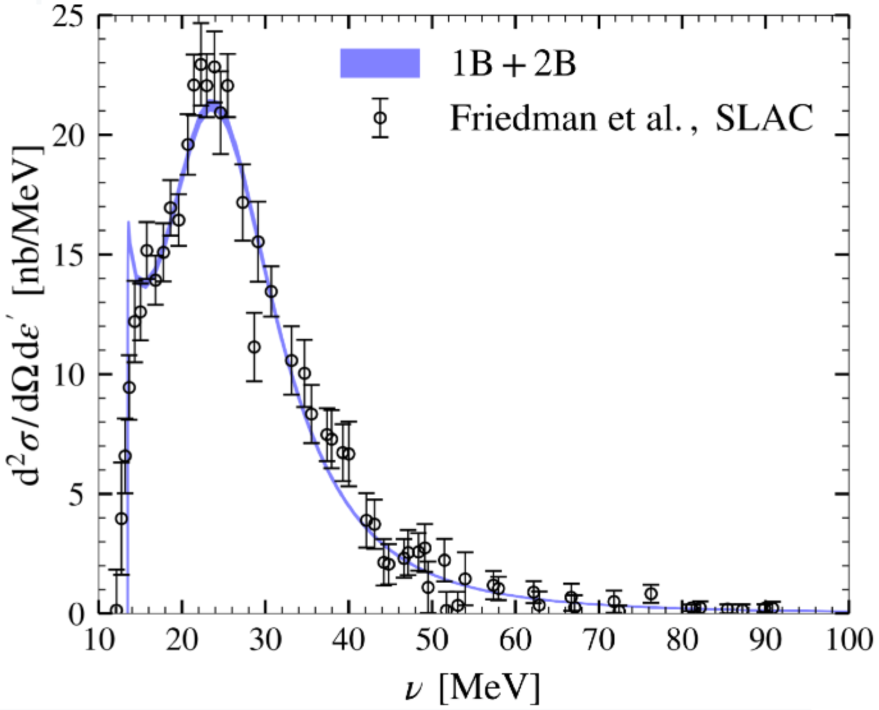
Chiral EFT potentials and currents

Deuteron electrodisintegration at N2LO

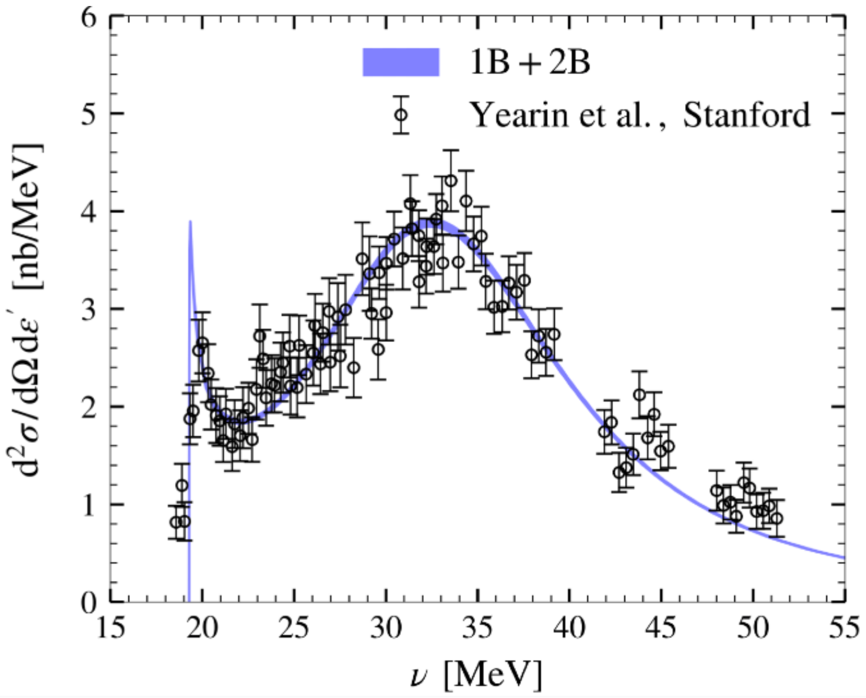
Acharya, Lensky, SB, Gorchtein, Vanderhaeghen et al., Phys. Rev. C ***103*** *(2021) 024001*



$\epsilon = 292.8$ MeV, $\theta = 60^o$



$\epsilon = 175$ MeV, $q = 206$ MeV



$\epsilon = 147$ MeV, $\theta = 135^o$

The Siegert theorem

Let us look again at the form of the transverse electric multipoles

$$T^{el}_{J\mu}(q) = \frac{i}{4\pi}\int d\hat{q}'\,(\hat{\mathbf{q}}'\times \mathbf{Y}^{\mu}_{JJ1}(\hat{q}'))\cdot \mathbf{J}\,(\mathbf{q}')$$

using the property: $\hat{\mathbf{q}}\times \mathbf{Y}^{\mu}_{JJ1}(\hat{q}) = i\sqrt{\frac{J+1}{J}}\hat{\mathbf{q}}Y^{J}_{\mu}(\hat{q}) + i\frac{\hat{J}}{\sqrt{J}}\mathbf{Y}^{\mu}_{JJ+11}(\hat{q})$

$$T^{el}_{J\mu}(q) = \boxed{-\frac{1}{4\pi}\int d\hat{q}'\left[\sqrt{\frac{J+1}{J}}\hat{\mathbf{q}}'\cdot \mathbf{J}(\mathbf{q}')\,Y^{J}_{\mu}(\hat{q}')\right.} + \left.\frac{\hat{J}}{\sqrt{J}}\mathbf{Y}^{\mu}_{JJ+11}(\hat{q}')\cdot \mathbf{J}(\mathbf{q}')\right]$$

Siegert operator

Can be related to a Coulomb multipole via the use of the continuity equation

Correction to the Siegert operator

$$\mathbf{q}\cdot \mathbf{J}(\mathbf{q}) = \omega\rho(\mathbf{q})$$ see lecture 1

Siegert theorem

$$\boxed{T^{el}_{J\mu}(q) = -\frac{1}{4\pi}\sqrt{\frac{J+1}{J}}\frac{\omega}{q}\int d\hat{q}'\rho(\mathbf{q}')Y^{J}_{\mu}(\hat{q}')} - \frac{1}{4\pi}\frac{\hat{J}}{\sqrt{J}}\int d\hat{q}'\,\mathbf{Y}^{\mu}_{JJ+11}(\hat{q}')\cdot \mathbf{J}(\mathbf{q}')$$

Coulomb multipole $C_{J\mu}$

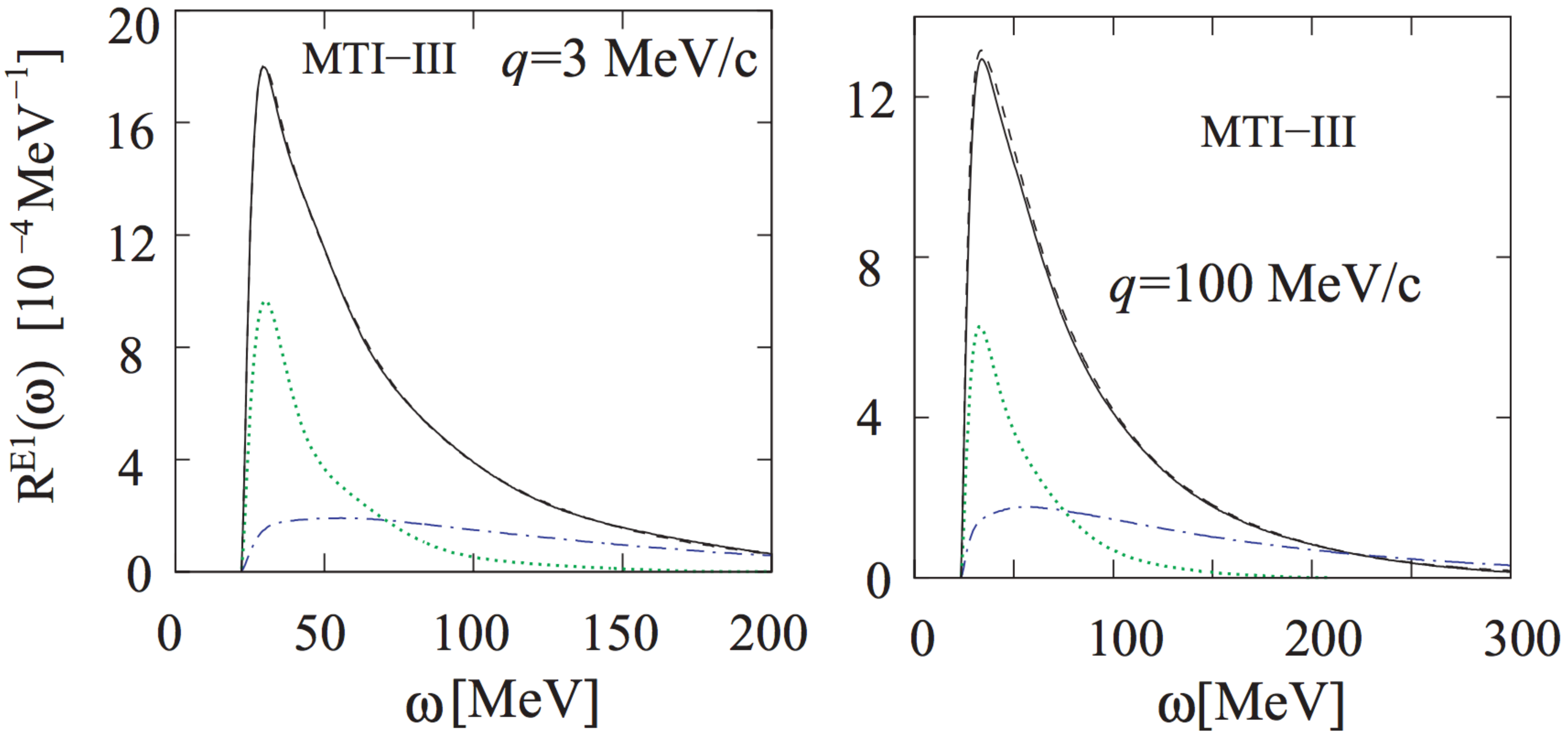
Negligible at low q

Practical Example J=1 ^{4}He

$$T^{el}_{J\mu}(q) = \boxed{-\frac{1}{4\pi}\int d\hat{q}' \left[\sqrt{\frac{J+1}{J}}\,\hat{\mathbf{q}}'\cdot\mathbf{J}(\mathbf{q}')\;Y^J_\mu(\hat{q}')\right.}$$

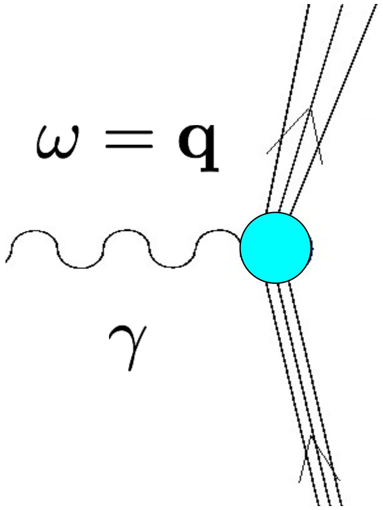
Siegert operator	——	via continuity equation, relate to $C_{1\mu}$
Convection current	········	$\hat{\mathbf{q}}'\cdot\mathbf{J}_c(\mathbf{q}')$
MEC	–·–·–	$\hat{\mathbf{q}}'\cdot\mathbf{J}_{\rm MEC}(\mathbf{q}')$
MEC+convection	- - - -	$\hat{\mathbf{q}}'\cdot(\mathbf{J}_c(\mathbf{q}')+\mathbf{J}_{\rm MEC}(\mathbf{q}'))$

S.B. *et al.*, PRC **76**, 014003 (2007)



Photoabsorption

Recap:



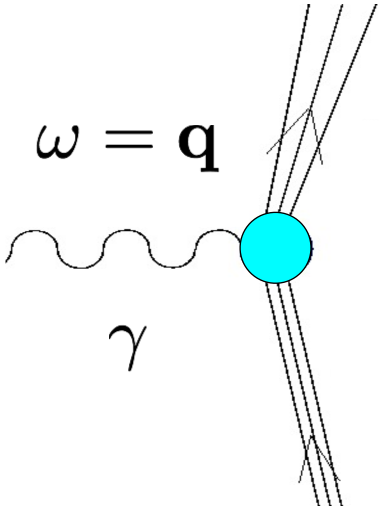
$$\sigma = \sigma_M \left[\xcancel{\frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q})} + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2 \frac{\theta}{2} \right) \overset{\omega = \mathbf{q}}{R_T(\omega, \mathbf{q})} \right]$$

~~Longitudinal part:~~
~~Photon polarization along momentum axis~~

Transverse part: $\omega = \mathbf{q}$
Photon polarization transverse to momentum axis

Photoabsorption

Recap:



$$R_T(\omega = \mathbf{q}) \rightarrow |\langle \Psi_f | J_T(q) | \Psi_0 \rangle|^2 = \sum_{\lambda = \pm 1} |\langle \Psi_f | J_\lambda(q) | \Psi_0 \rangle|^2$$

Now we can use the multipole decomposition of the current that we just derived

Current operator

General multipole decomposition of the current

$$\mathbf{e}_\lambda \cdot \mathbf{J}(\mathbf{q}) = (-)^\lambda \sqrt{2\pi(1+\delta_{\lambda 0})} \sum_J \hat{J} \left[L^{el}_{J\lambda}(q)\delta_{\lambda 0} + \left(T^{el}_{J\lambda}(q) + \lambda T^{mag}_{J\lambda}(q) \right) \delta_{|\lambda| 1} \right]$$

Real photons: no longitudinal polarization possible
only transverse polarization $\lambda = \pm 1$

$$\mathbf{e}_\lambda \cdot \mathbf{J}(\mathbf{q}) \longrightarrow (-)^\lambda \sqrt{2\pi} \sum_J \hat{J} \left[\left(\underset{\underset{j_J(qr)}{\uparrow}}{T^{el}_{J\lambda}}(q) + \lambda \underset{\underset{j_{J+1}(qr)}{\uparrow}}{T^{mag}_{J\lambda}}(q) \right) \delta_{|\lambda| 1} \right]$$

See when explicitly calculating
multipole of a current operator

Low momentum transfer:
Only lowest multipole prevails J=1 and electric multipole dominates over magnetic

$$J_\lambda(q) \longrightarrow T^{el}_J \xrightarrow{\text{Siegert}} \rightarrow C_{J=1} \xrightarrow{\text{low q and q // z}} qr\sqrt{\frac{3}{4\pi}}\cos(\theta) \rightarrow \omega z$$

Photoabsorption

Thus, photoabsorption at low energy can be calculated simply from a dipole response function

$$\sigma(\omega) = \frac{4\pi^2\alpha}{2J_0+1}\omega R(\omega)$$

$$R(\omega) = \sum_f |\langle\Psi_f|\, D_z \,|\Psi_0\rangle|^2 \,\delta(E_f - E_0 - \omega)$$

$$D_z = \sum_i^A z_i \left(\frac{1+\tau_i^z}{2}\right)$$

Photoabsorption

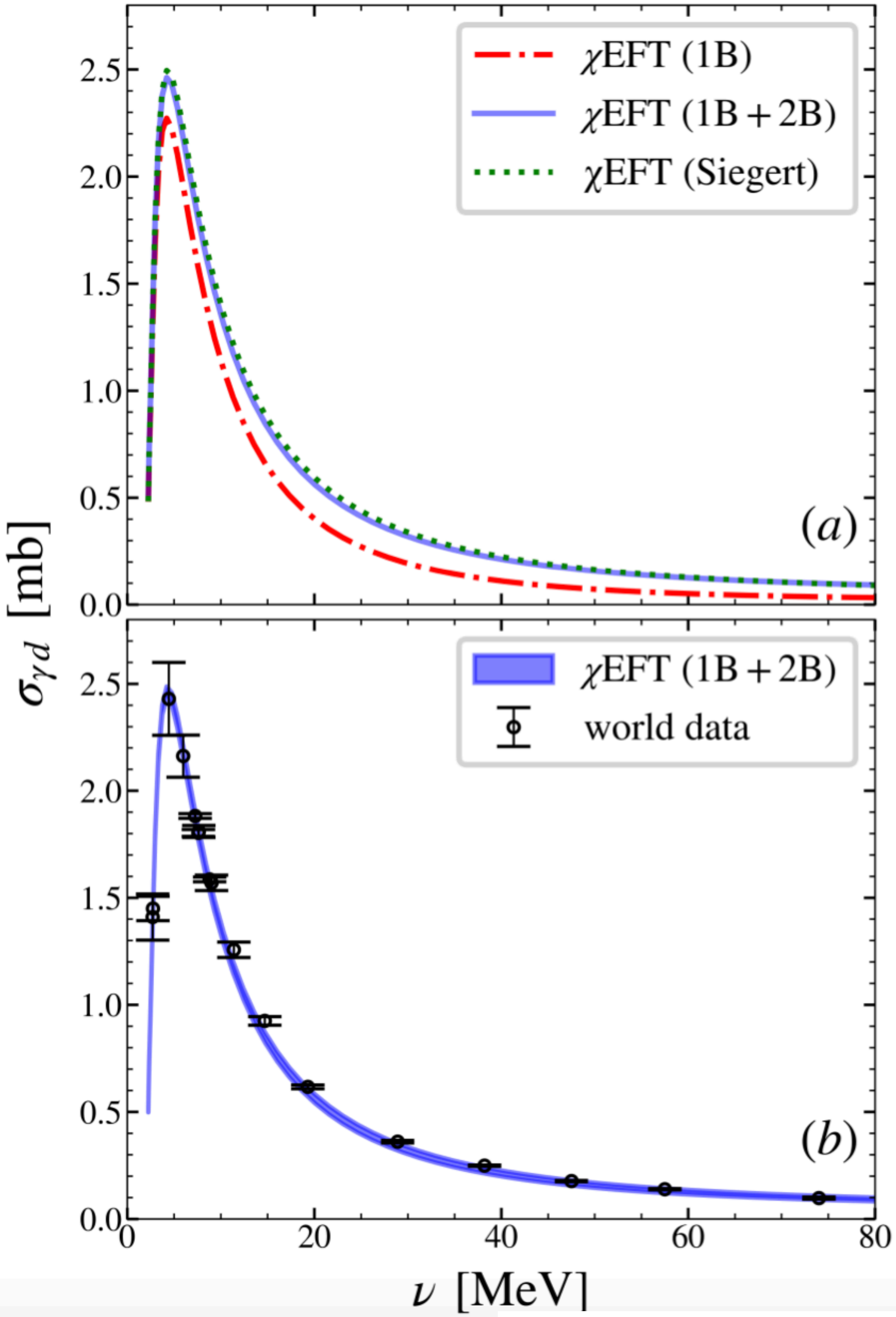
Comparing calculations in which one uses the dipole operator (Siegert theorem)

$$|\langle \Psi_f | D_z | \Psi_0 \rangle|^2$$

with calculations where one explicitly insert the transverse current (1-body + 2-body, etc.)

$$\sum_{\lambda=\pm 1} |\langle \Psi_f | J_\lambda(q) | \Psi_0 \rangle|^2$$

Practical Example: Deuteron



Using the explicit one-body current only it is not enough.

Using the Siegert theorem is equivalent as using explicit one- and two-body currents.

Perfect agreement with experiment for the deuteron.

Acharya, Lensky, SB, Gorchtein, Vanderhaeghen et al., Phys. Rev. C ***103*** *(2021) 024001*

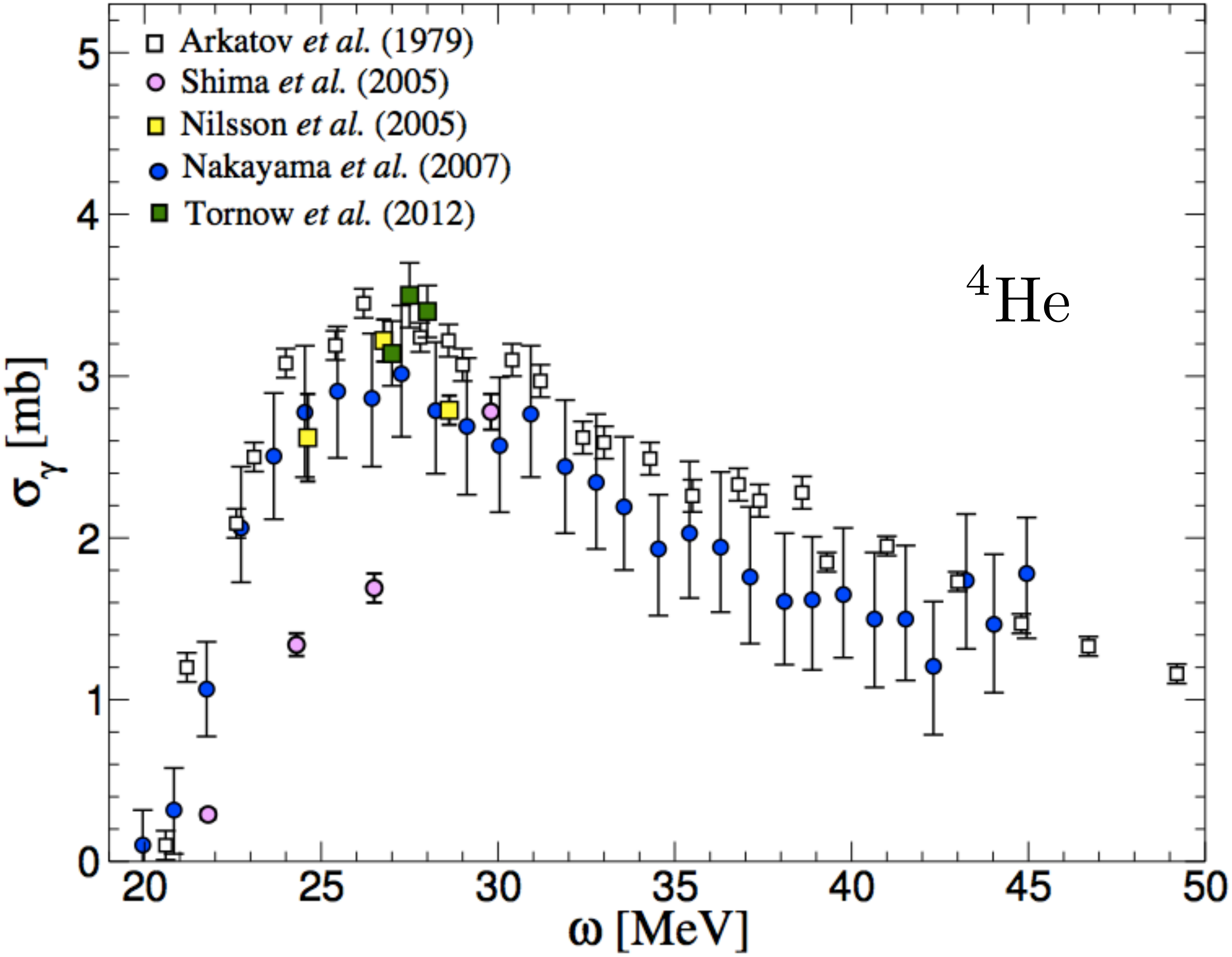
Photoabsorption

Thus, photoabsorption at low energy can be calculated simply from a dipole response function

We will see several few- and many-body applications after we have explained how to deal with wave functions in the continuum.

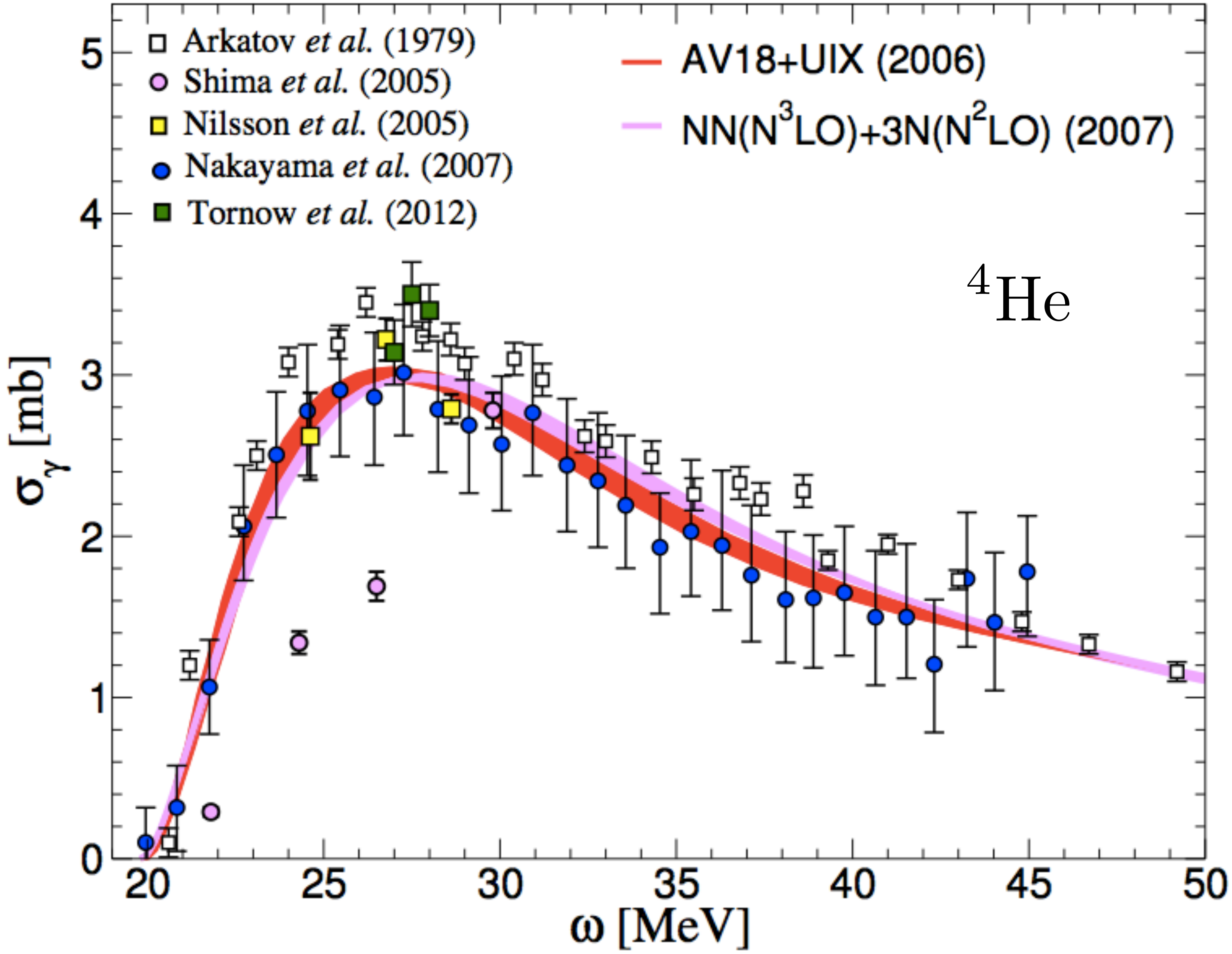
For now, let us just have a look at the ^{4}He case.

Photoabsorption



SB and Saori Pastore, Journal of Physics G.: Nucl. Part. Phys. **41**, 123002 (2014)

Photoabsorption

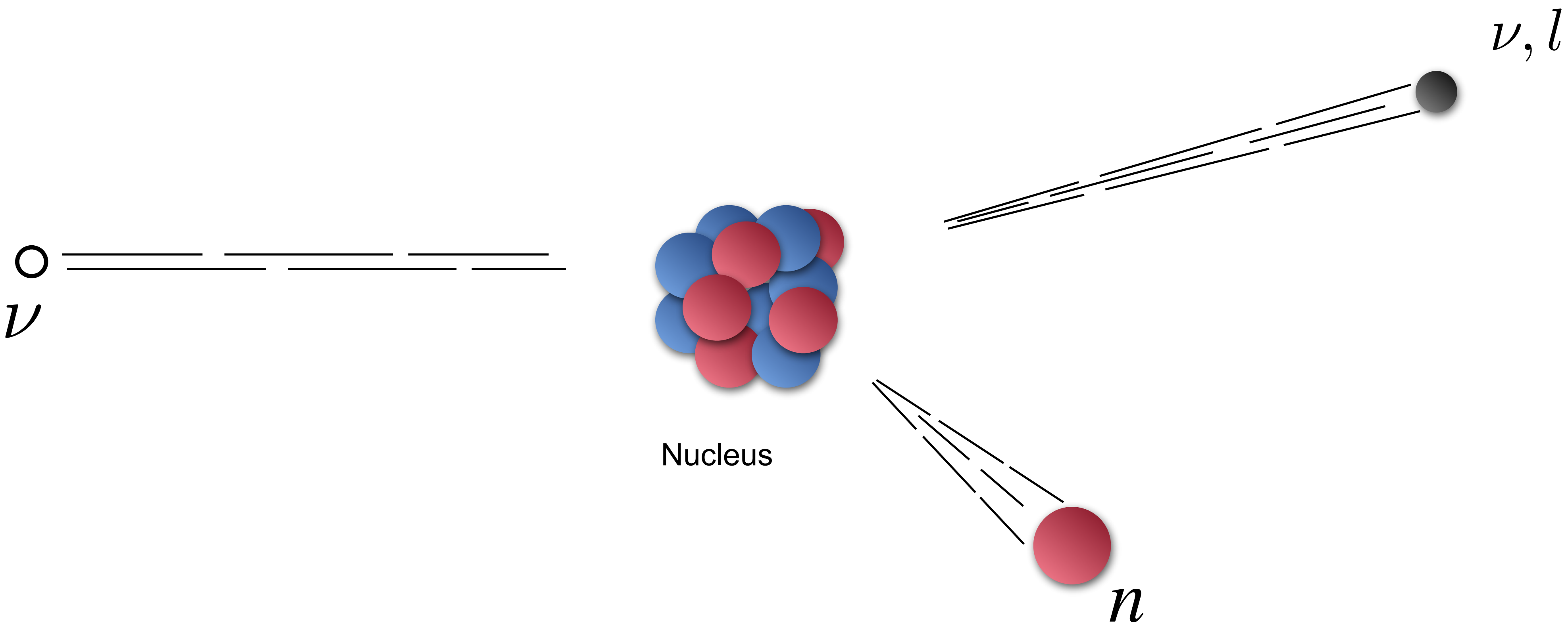


SB and Saori Pastore, Journal of Physics G.: Nucl. Part. Phys. **41**, 123002 (2014)

Extension to weak sector

Extension to the weak sector

Need it for example if you want to study neutrino scattering



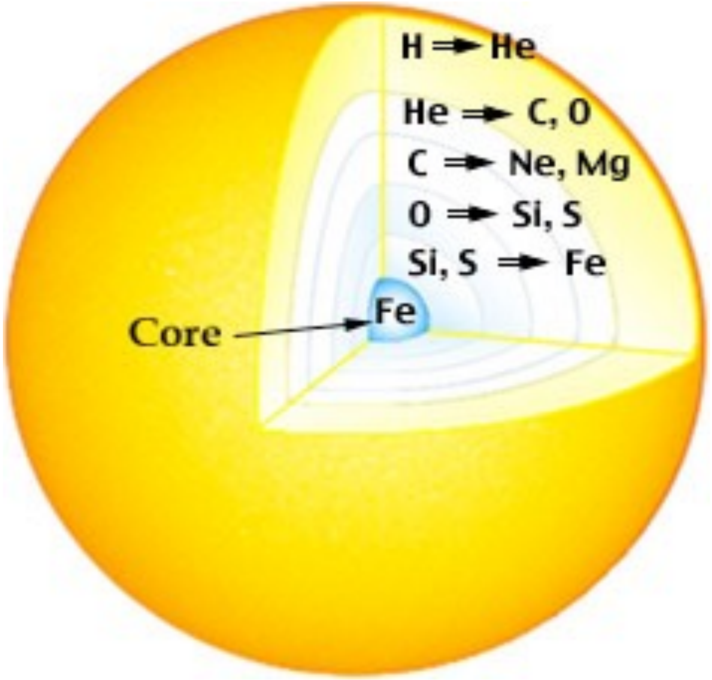
Nucleons can be kicked out

Neutrino scattering in astrophysics

- Core collapse supernovae are gigantic explosions of massive stars
- 99% of the released energy is carried by neutrinos in all flavors

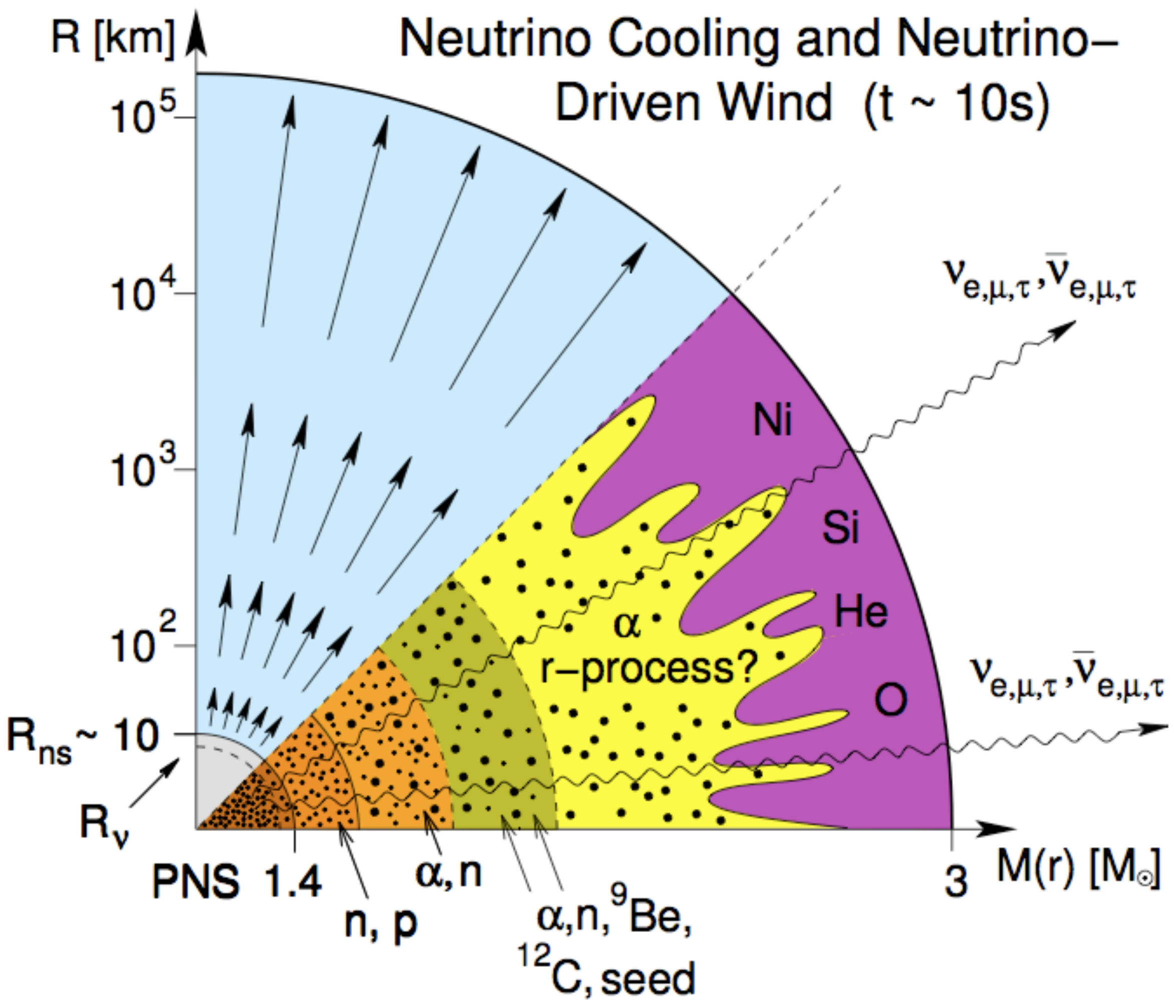
→ Phenomena inside the SN are sensitive to neutrino interaction with matter

- The progenitor presents an onion skin structure
- In the iron core the burning process stops and it becomes gravitationally unstable → the core collapses
- Nuclear forces halt the collapse, and drive an outgoing shock, which loses energy due to dissociation, neutrino radiation.
- The shock stalls … possibly revived by neutrino heating



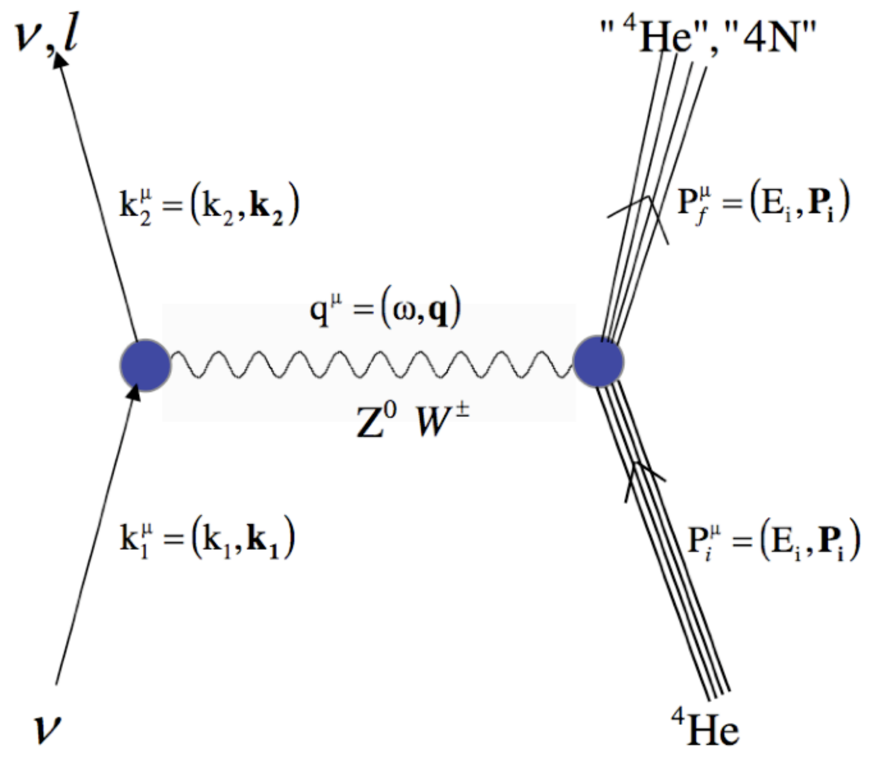
Neutrino scattering in astrophysics

What inelastic neutrino scattering with are relevant in SN?



$$^4\mathrm{He}(\nu,\nu')X$$

Microscopic calculations can be achieved

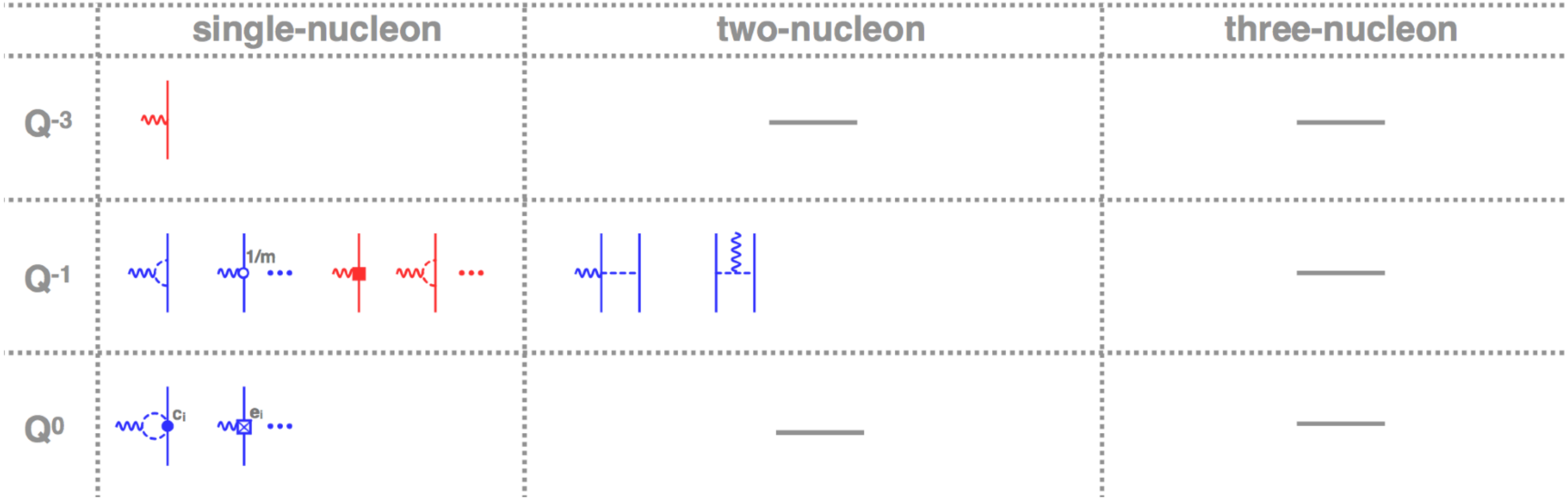


SN neutrinos have energies below 50 Mev.

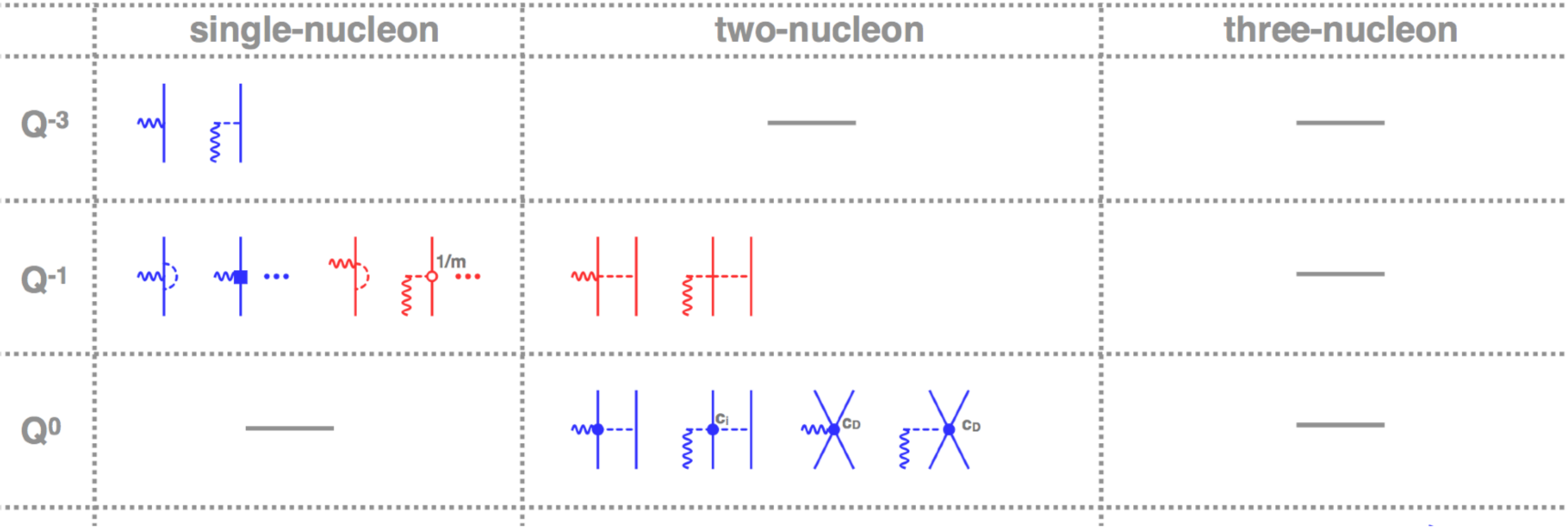
Chiral currents (em/weak)

From E.Epelbaum, Mainz Workshop, October 2018

Chiral expansion of the electromagnetic **current** and **charge** operators



Chiral expansion of the axial **current** and **charge** operators

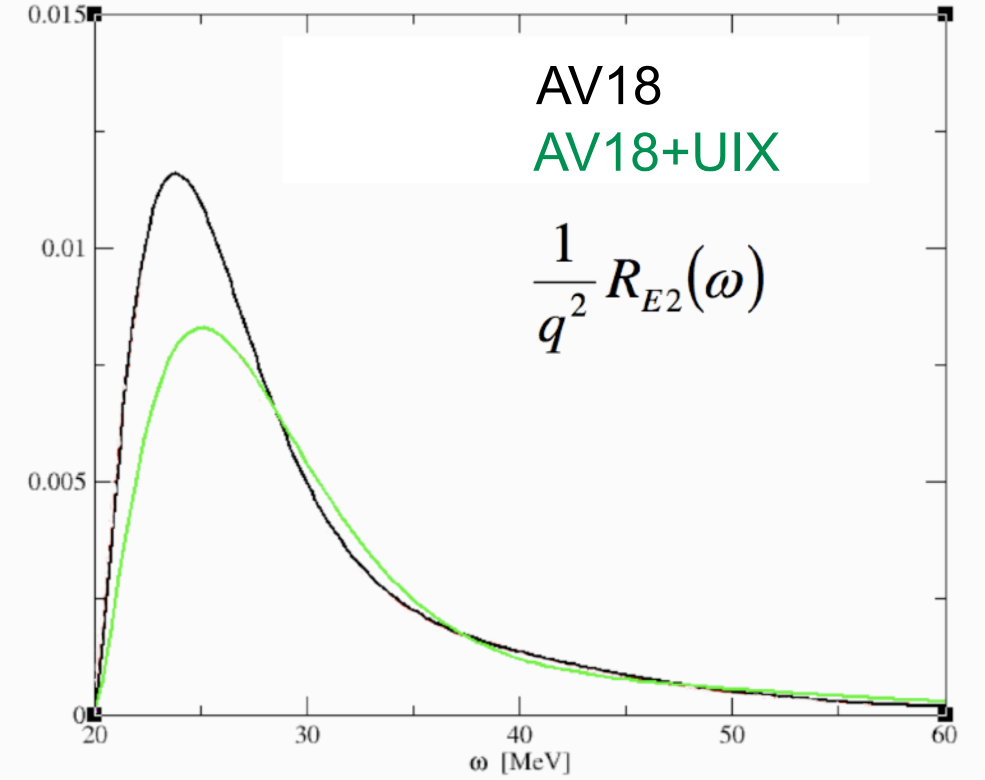


Neutrino scattering in astrophysics

Gazit, Barnea, PRL 98 (2007) 192501

Multipole decomposition and low-q expansion of the currents

Sensitivity to the nuclear Hamiltonian

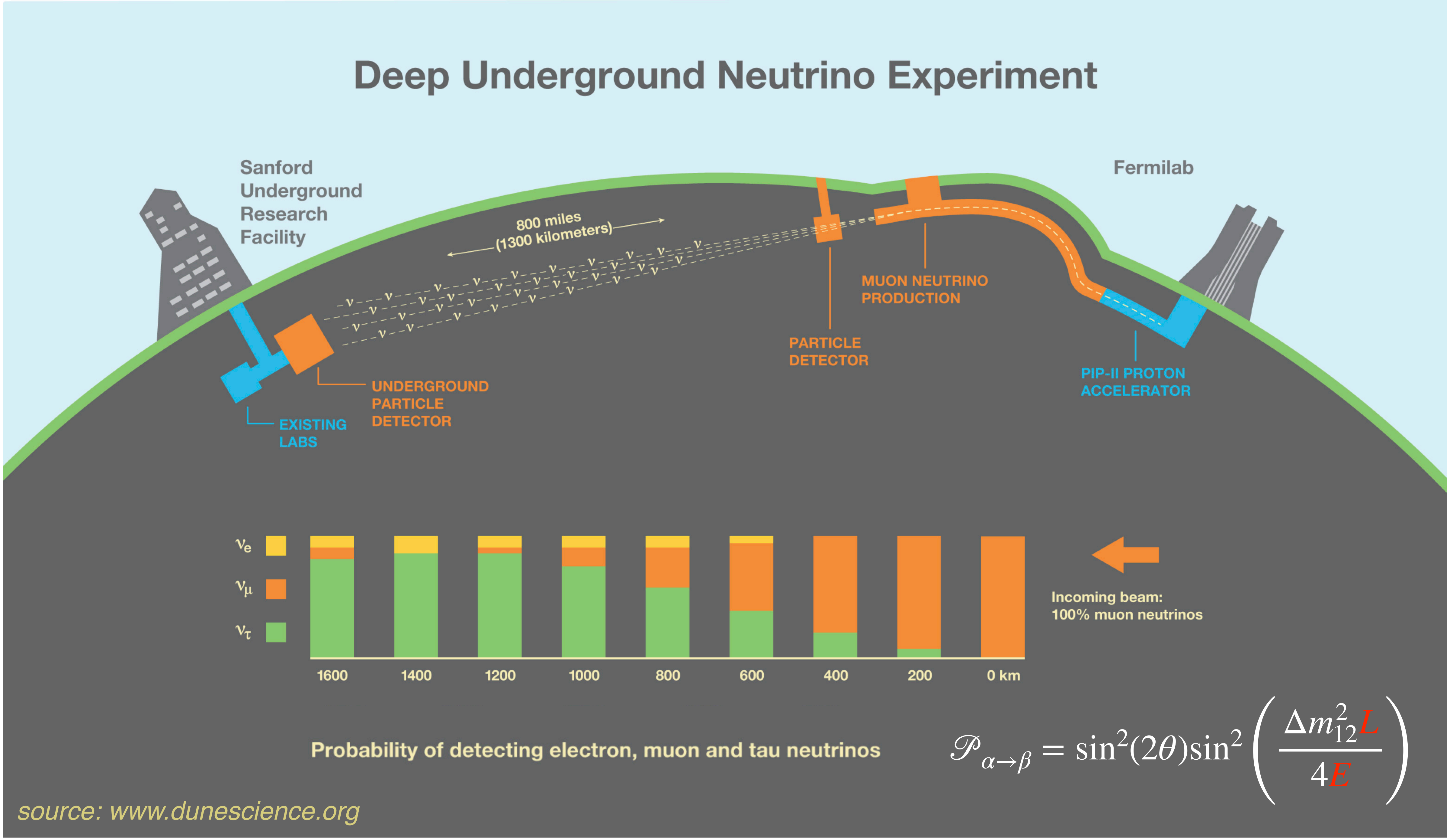


Temperature averaged $\langle\sigma\rangle_T$ neutral cross section

T [MeV]	AV18	AV18 + UIX	AV18 + UIX + MEC (from EFT)
4	2.31×10^{-3}	1.63×10^{-3}	1.66×10^{-3}
6	4.30×10^{-2}	3.17×10^{-2}	3.20×10^{-2}
8	2.52×10^{-1}	1.91×10^{-1}	1.92×10^{-1}
10	8.81×10^{-1}	6.77×10^{-1}	6.82×10^{-1}
12	2.29	1.79	1.80
14	4.53	3.91	3.93

Large 3NF effect and small contribution of two-body currents

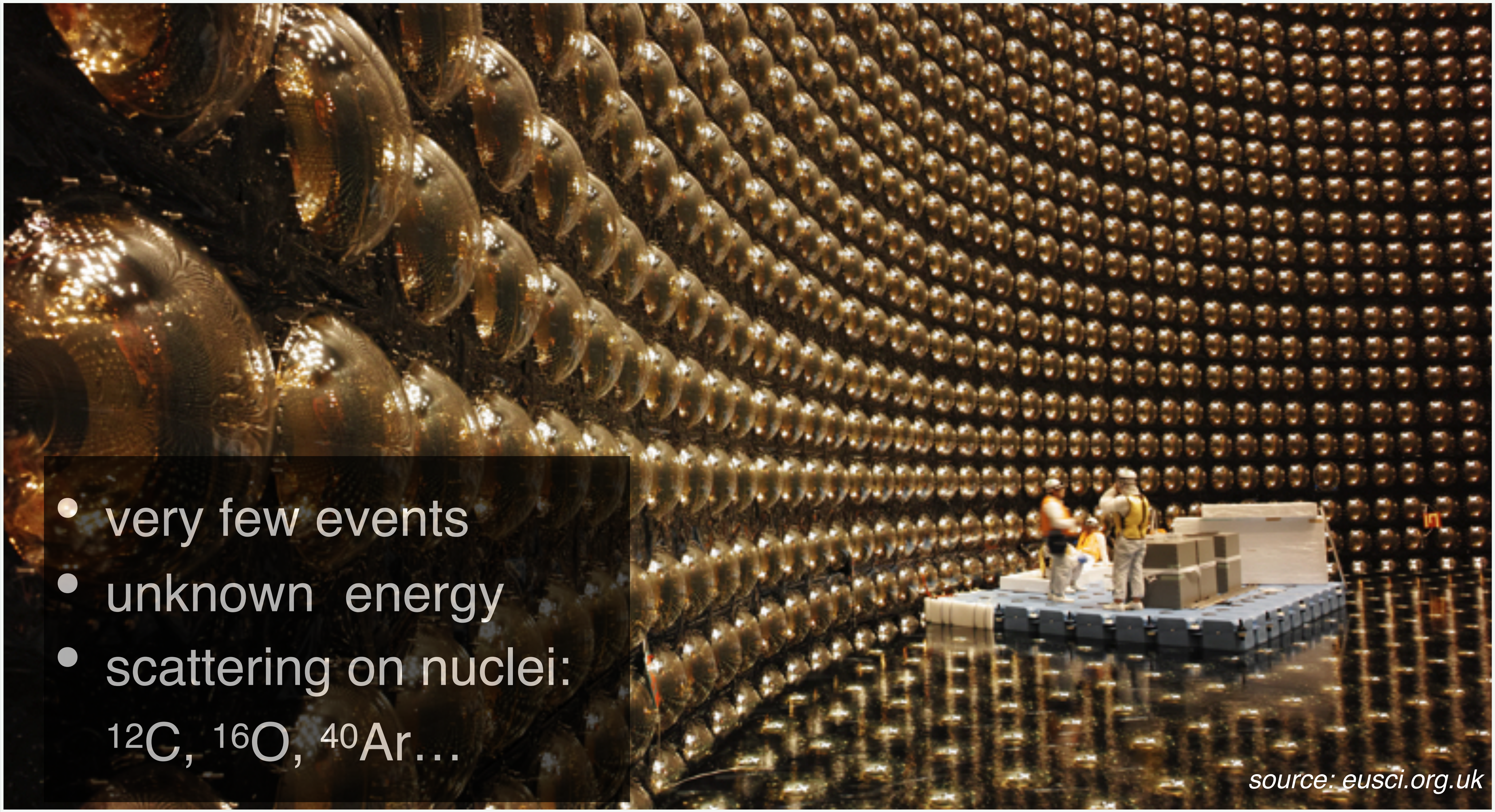
Neutrino Oscillations



$$\mathscr{P}_{\alpha\to\beta} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2_{12} L}{4E}\right)$$

source: www.dunescience.org

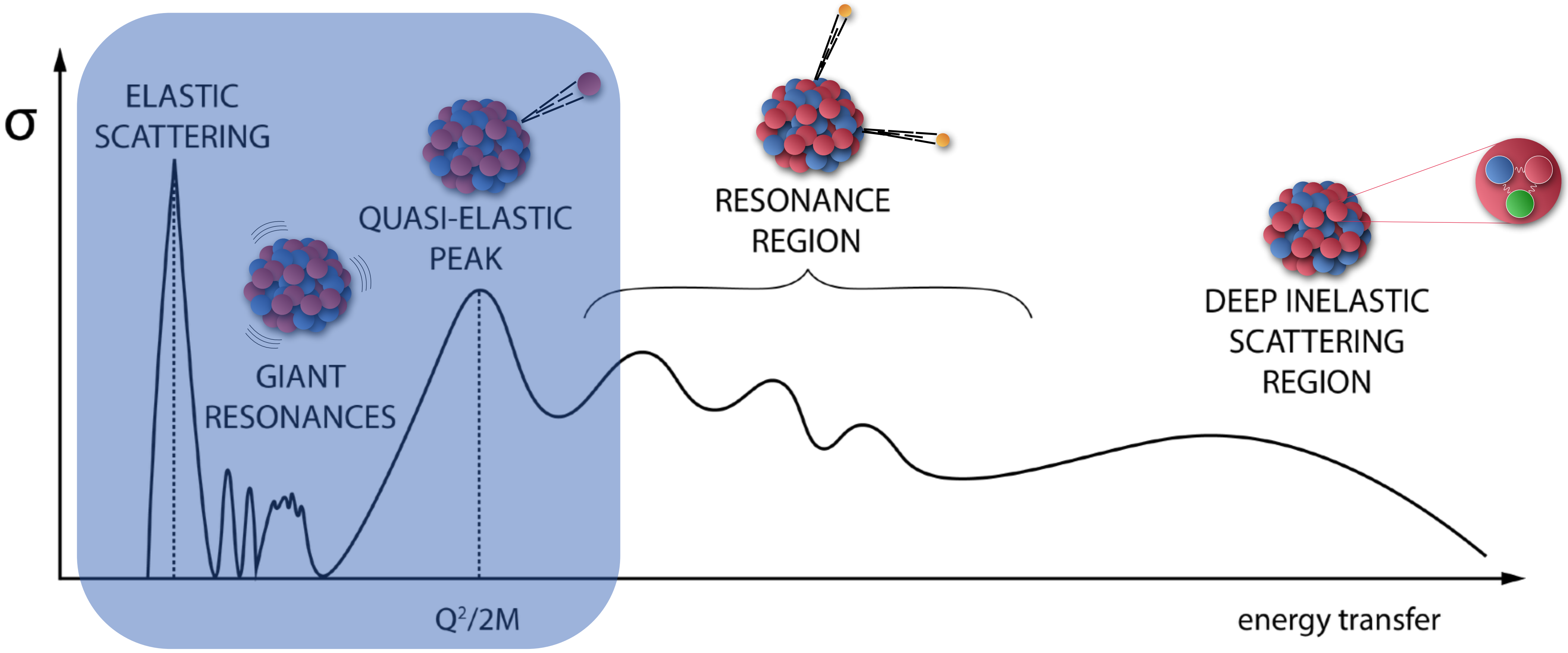
Neutrino Oscillations



- very few events
- unknown energy
- scattering on nuclei: ^{12}C, ^{16}O, ^{40}Ar…

source: eusci.org.uk

Nuclear Physics Input



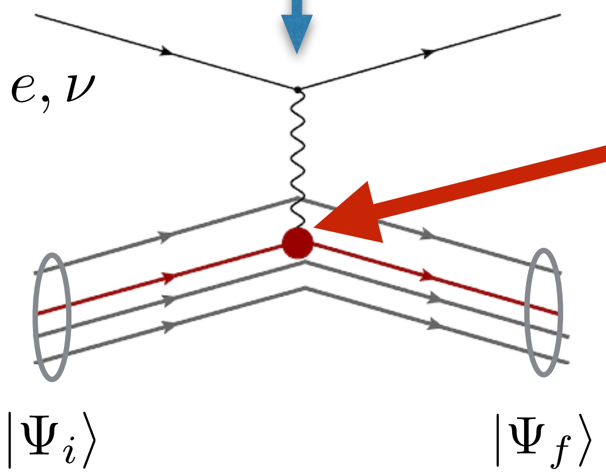
Energy range one can study with ab-initio methods

Electrons vs neutrinos

ν-A scattering
$$\left.\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\,\mathrm{d}\omega}\right|_{\nu/\bar{\nu}} = \sigma_0\left[\ell_{CC}R_{CC} + \ell_{CL}R_{CL} + \ell_{LL}R_{LL} + \ell_T R_T \pm \ell_{T'}R_{T'}\right]$$

e-A scattering
$$\left.\frac{d^2\sigma}{d\Omega d\omega}\right|_e = \sigma_M\left[\frac{Q^4}{q^4}R_L + \left(\frac{Q^2}{2q^2} + \tan^2\frac{\theta_e}{2}\right)R_T\right]$$

Response function
$$R_{\alpha\beta}(q,\omega) = \sum_f \langle\Psi_0|J_\alpha(q)|\Psi_f\rangle\langle\Psi_f|J_\beta(q)|\Psi_i\rangle\ \delta(\omega + E_I - E_F)$$



$F_A(q), F_{E/M}(q)$

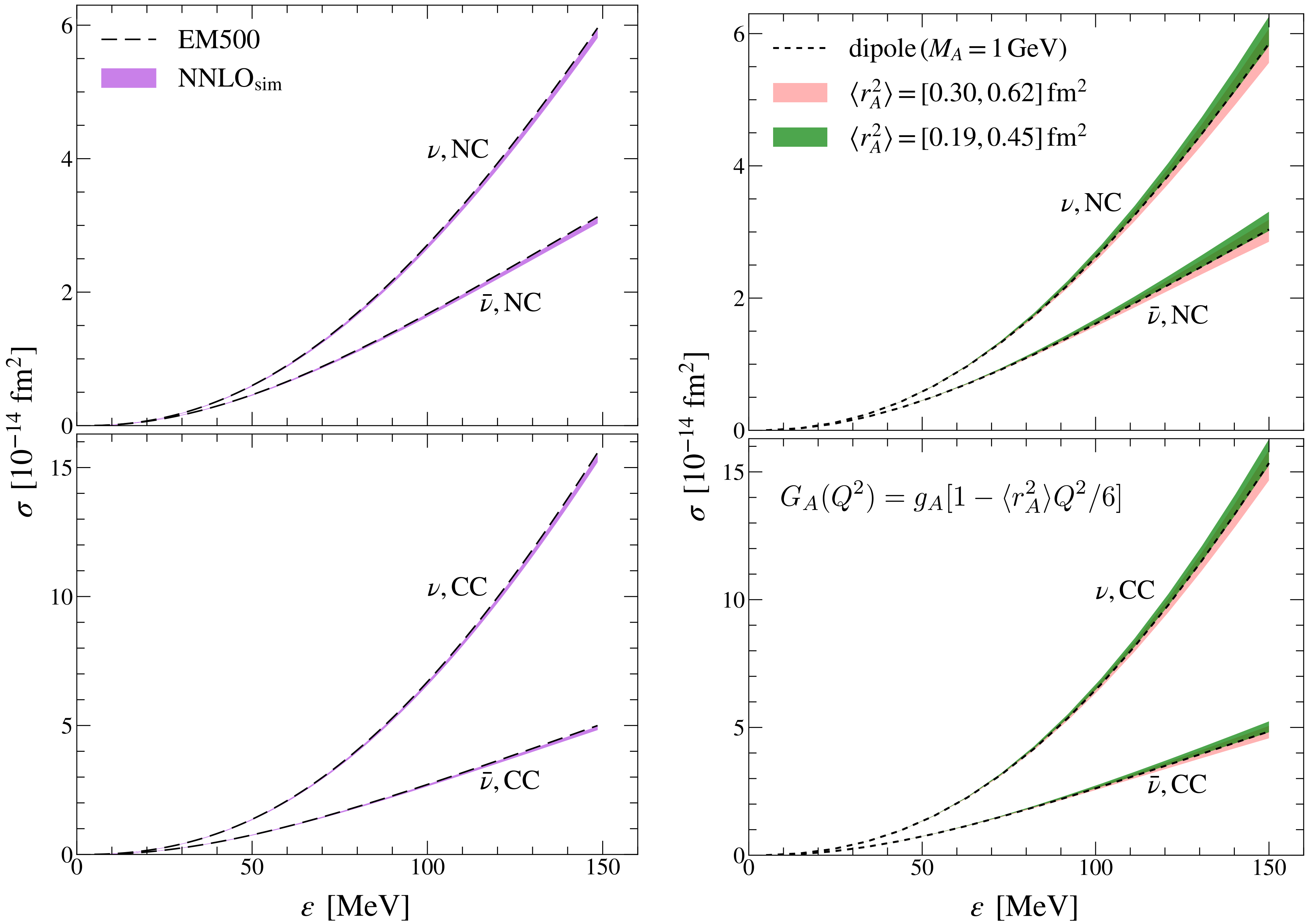
Single-nucleon dynamics
(lattice QCD)

Few-nucleon dynamics

(Anti)neutrino-deuteron scattering

B. Acharya and SB, PRC **101**, 015505 (2020)

Chiral effective field theory calculations at N2LO

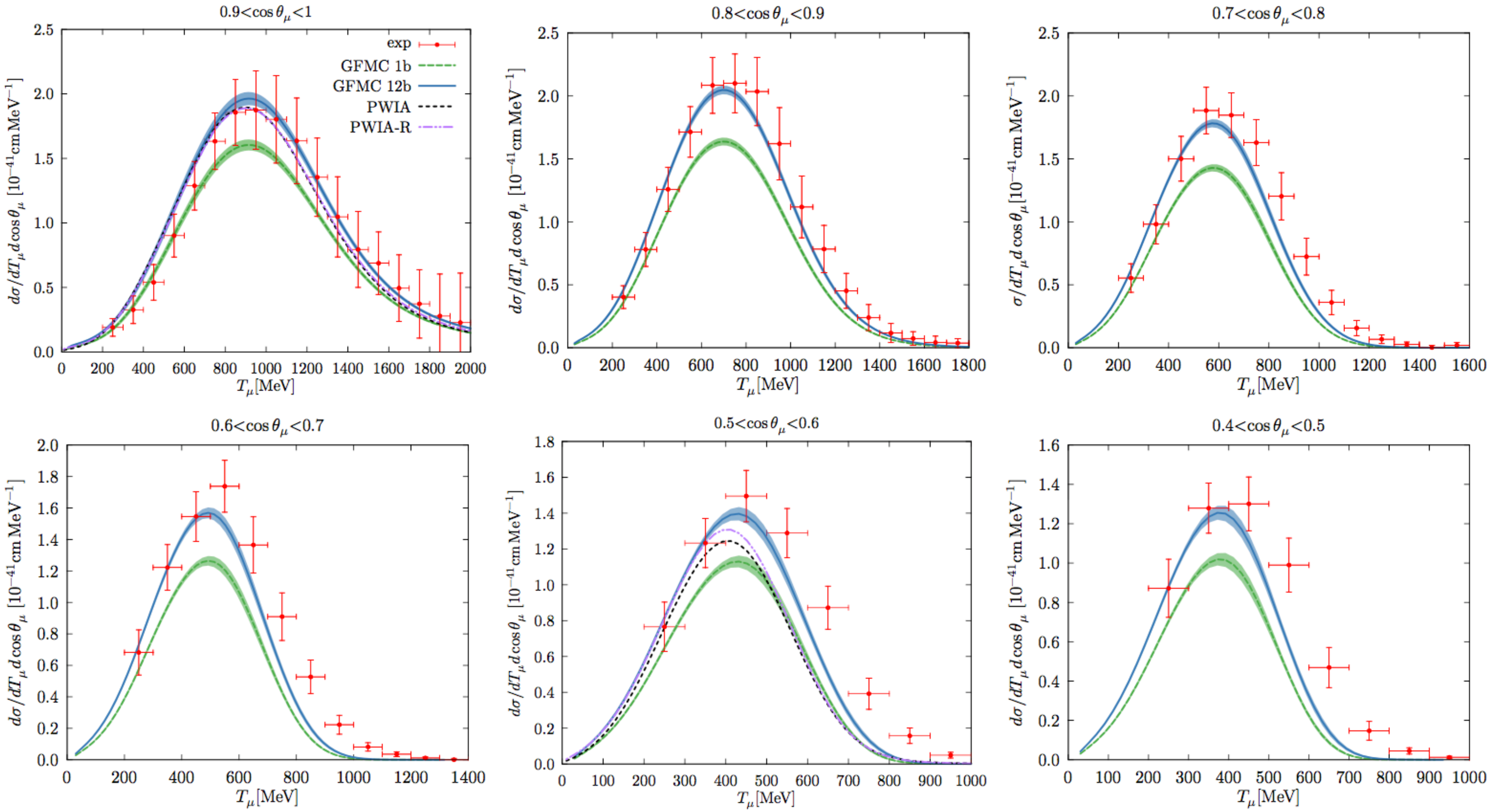


Neutrino-^{12}C scattering

Lovato *et al.*, PRX **10** (2020) 3, 031068

Calculations with traditional potentials

ν_μ CCQE, comparison to MiniBoone



Two-body currents are important also in neutrino scattering

Future Goals

Study ^{16}O and ^{40}Ar nuclei

Stay tuned!