Few-body Reactions

Lecture 1

Electromagnetic Processes

Sonia Bacca

References

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Outline of Lecture 1

- Motivations
- Minimal coupling
- Continuity equation and two-body currents
- Electron scattering and photodisintegration
- Examples

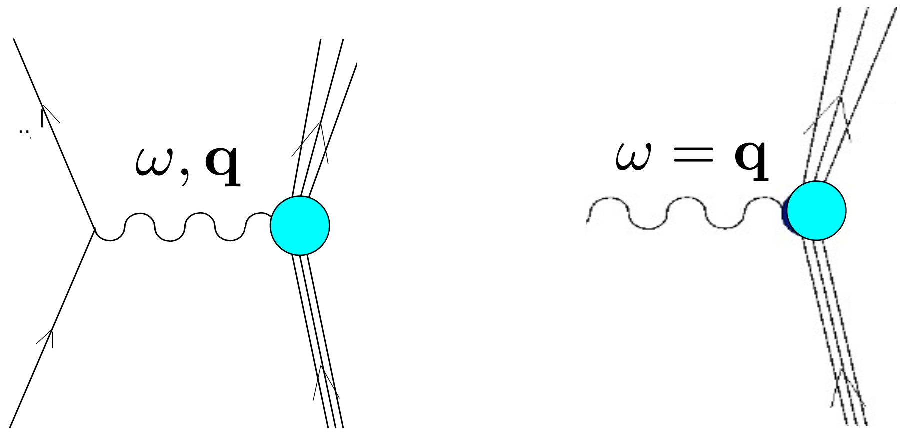
Electromagnetic probes

"*With the electromagnetic probe, we can immediately relate the cross section to the transition matrix element of the current operator, thus to the structure of the target itself*"
[De Forest-Walecka, Ann. Phys. 1966]

$$\sigma \propto \left| \langle \Psi_f | J^\mu | \Psi_0 \rangle \right|^2$$

Can be treated perturbatively $\alpha = \dfrac{1}{137} \ll 1$

Use Born approximation ⇒ single photon exchange



Electromagnetic probes

"With the electromagnetic probe, we can immediately relate the cross section to the transition matrix element of the current operator, thus to the structure of the target itself"
[De Forest-Walecka, Ann. Phys. 1966]

$$\sigma \propto |\langle \Psi_f | J^\mu | \Psi_0 \rangle|^2$$

Can be used to:

- Understand the nuclear dynamics
- Connect nuclear physics to other fields of research

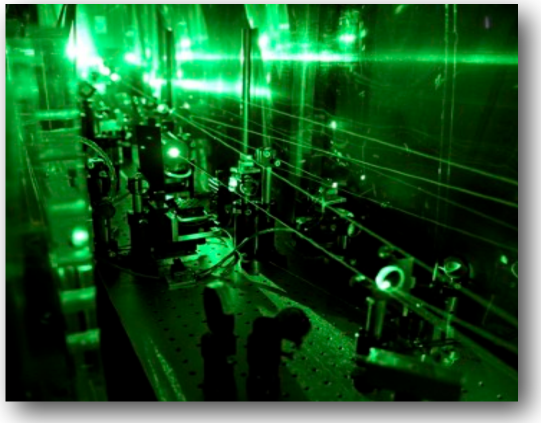
Electromagnetic probes

Provide important informations in other fields of physics, where nuclear physics plays a crucial role:

- Astrophysics

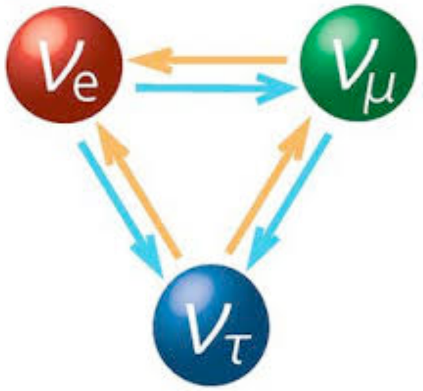
Example: radiative capture reactions

- Atomic physics



Example: muonic atoms

- Particle physics

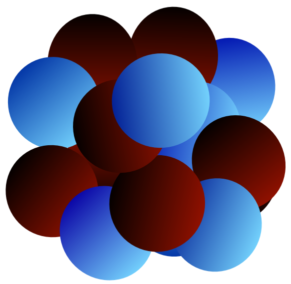


Example: neutrino experiment extending electromagnetic to electro-weak

Electromagnetic probes

Here we will consider a study of electromagnetic observables in nuclei described in the ab-initio approach using nucleons as degrees of freedom

"Ab-initio" approach



- Start from neutrons and protons as building blocks (centre of mass coordinates, spins, isospins)

- Solve the non-relativistic quantum mechanical problem of A-interacting nucleons

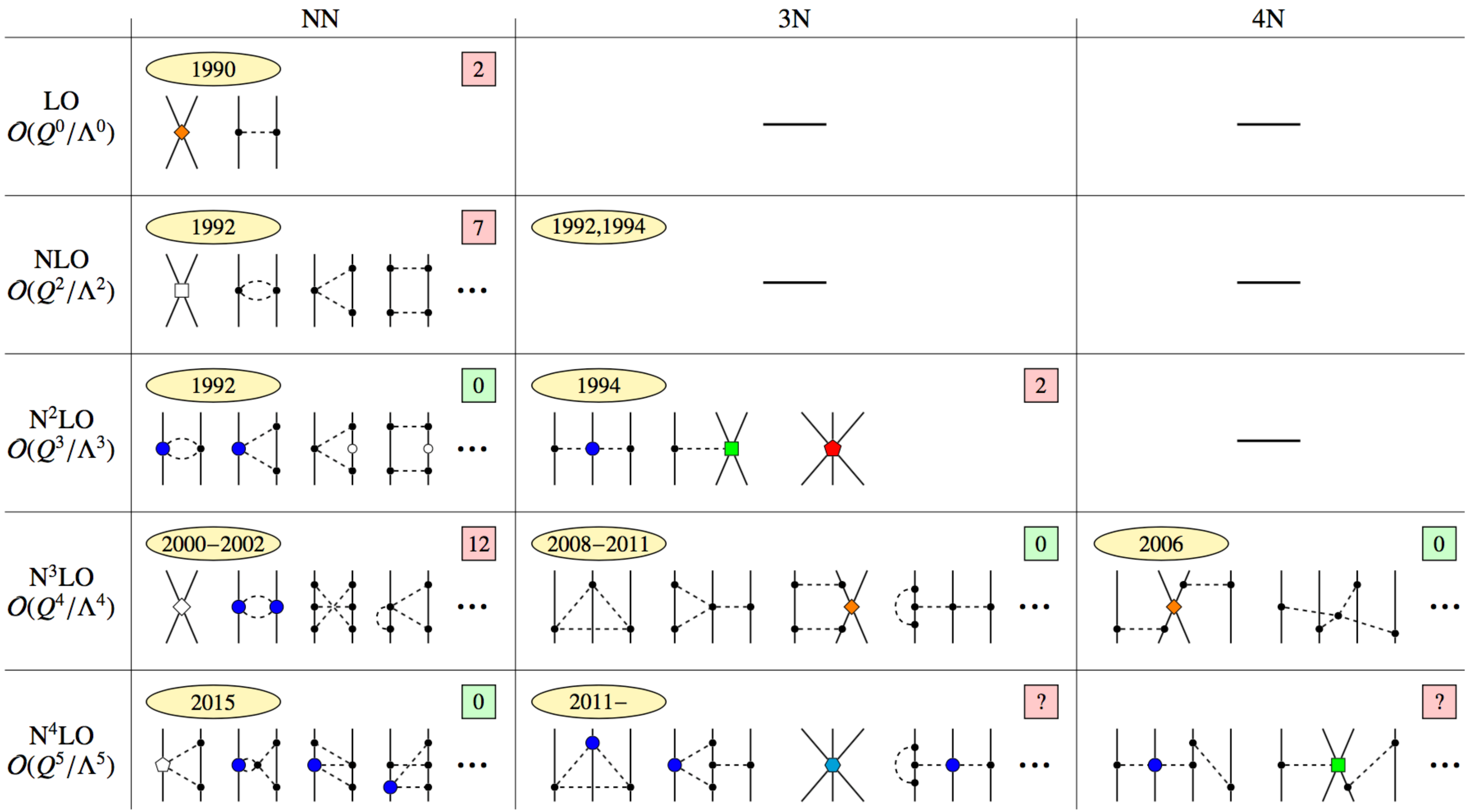
$$H|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \dots$$

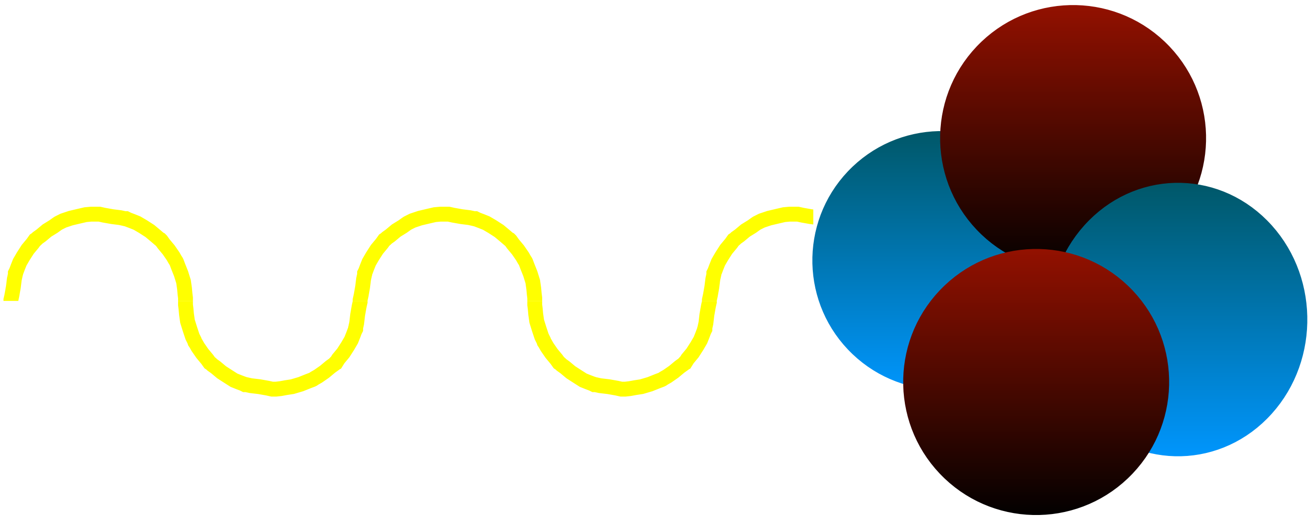
- Find numerical solutions with no approximations or controllable approximations

Chiral effective field theory

See lectures by Kai Hebeler



Coupling to the em field



Coupling to the em field

Let us visualize the electromagnetic perturbation in a nucleus by first considering the simple case of a point particle carrying charge e and no magnetic moment (for now)

In the absence of an em field, the Hamiltonian would be just the kinetic energy

$$H = \frac{\mathbf{p}^2}{2m}$$

In the presence of an em field, the momentum is modified by the following transformation

$$\mathbf{p} \to \mathbf{p} - \frac{e}{c}\mathbf{A}$$

Where $\mathbf{A}$ is the vector potential for the electromagnetic field

The hamiltonian for the charged particle now take the form

$$H' = \frac{(\mathbf{p} - \frac{e}{c}\mathbf{A})^2}{2m}$$

Coupling to the em field

$$H' = \frac{(\mathbf{p} - \frac{e}{c}\mathbf{A})^2}{2m}$$

$$H' = \frac{\mathbf{p}^2}{2m} - \frac{e}{mc}\mathbf{p}\cdot\mathbf{A} + \cancelto{0}{\frac{1}{2m}\frac{\mathbf{A}^2 e^2}{c^2}}$$

The interaction hamiltonian at leading order is

$$H_{int} = -\frac{e}{mc}\mathbf{p}\cdot\mathbf{A}$$

and if we define the em current as $\mathbf{J} = \mathbf{v} = \dfrac{\mathbf{p}}{m}$ for a unit of charge,

The interaction hamiltonian becomes

$$H_{int} = -\frac{e}{c}\mathbf{J}\cdot\mathbf{A}$$

Coupling to the em field

If we have spacial charge distribution

$$e \rightarrow e\rho(\mathbf{x})$$

Then you have to integrate on that spacial distribution

$$H_{int} = -\frac{e}{c} \int d^3x \; \mathbf{J}(\mathbf{x}) \cdot \mathbf{A}(\mathbf{x})$$

Finally, in a quantum field theory language, you will use the four-vector notation and set e=c=1

$$\boxed{H_{int} = \int d^3x \; J_\mu(\mathbf{x}) A^\mu(\mathbf{x})}$$

perturbative interaction hamiltonian

$$J_\mu = (\rho, -\mathbf{J})$$

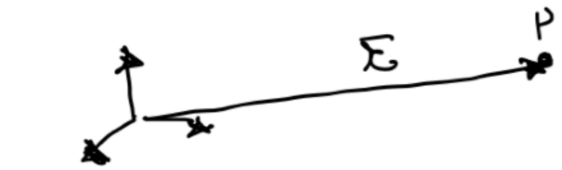
The current can be obtained by taking the derivative of H_{int} w.r.t. $\mathbf{A}$

$$J_\mu(\mathbf{x}) = \left. \frac{\delta H_{int}(A)}{\delta A^\mu(\mathbf{x})} \right|_{A\equiv 0} .$$

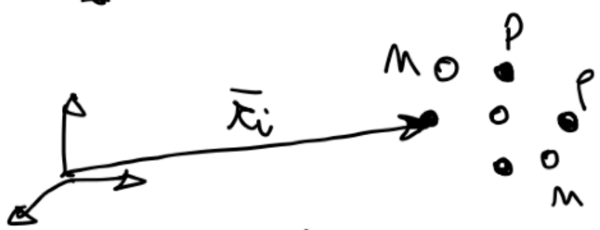
Minimal Coupling/Minimal Substitution

Derive currents using MC

CHARGE



$$\rho(\bar{x}) = e\,\delta(\bar{x} - \bar{r})$$



A = nucleons

$$\boxed{\rho(\bar{x}) = e \sum_{i=1}^{A} \delta(\bar{x} - \bar{r}_i)\left(\frac{1+\tau_i^z}{2}\right)}$$

CURRENT OPERATOR

$$H = \frac{p^2}{2m} \; ; \; \bar{p} \to \bar{p} - e\bar{A}(\bar{r})$$

$$H' = \frac{(\bar{p} - e\bar{A})^2}{2m} = \frac{p^2}{2m} \underbrace{- \frac{e}{2m}\left(\bar{p}\cdot\bar{A}(\bar{r}) + \bar{A}(\bar{r})\cdot\bar{p}\right)}_{H_{int}} + \mathcal{O}(A^2)$$

$$H_{int} = -\frac{e}{2m}\left(\bar{p}\cdot\bar{A}(\bar{r}) + \bar{A}(\bar{r})\cdot\bar{p}\right)$$

Remember $J_\mu(x) = \left.\frac{\delta H_{int}(A)}{\delta A^\mu(x)}\right|_{A=0}$

$$-\bar{J} = \frac{\delta H_{int}}{\delta \bar{A}(\bar{r})} \frac{\delta \bar{A}(\bar{r})}{\delta \bar{A}(\bar{x})} = -\frac{e}{2m}\left(\bar{p}\cdot\delta(\bar{x}-\bar{r}) + \delta(\bar{x}-\bar{r})\bar{p}\right)$$

$$\Rightarrow \; J = \frac{e}{2m}\left\{\bar{p}, \delta(\bar{x}-\bar{r})\right\} \quad 1 \text{ particles}$$

$$J_\mu = (\rho, -\bar{J})$$

Derive currents using MC

$$\boxed{\bar{J}^c = \frac{e}{2m}\sum_i^A \{\bar{P}_i, \delta(\bar{x}-\bar{r}_i)\}\left(\frac{1+\tau_i^z}{2}\right)}$$

CONVECTION CURRENT

anti-comm

SPIN CURRENT

$$\boxed{\bar{J}^s = \frac{ie}{2m}\sum_{i=1}^A \mu_i \bar{\sigma}_i \times [\bar{P}_i, \delta(x-\bar{r}_i)]\left(\frac{1+\tau_i^z}{2}\right)}$$

magnetic moment of nucleon

comut.

$\bar{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$

Spin is ul.

$$H = \frac{p^2}{2m} = \frac{(\bar{\sigma}\cdot\bar{p})^2}{2m}$$

$$(\bar{\sigma}\cdot\bar{a})(\bar{\sigma}\cdot\bar{b}) = \bar{a}\cdot\bar{b} + i\bar{\sigma}\cdot(\bar{a}\times\bar{b})$$

$$\Rightarrow (\bar{\sigma}\cdot\bar{p})(\bar{\sigma}\cdot\bar{p}) = p^2 + i\bar{\sigma}\cdot\underbrace{(\bar{p}\times\bar{p})}_{0}$$

Add to H this vanishing term

$$H = \frac{i\mu}{2m}\bar{\sigma}\cdot(\bar{p}\times\bar{p}) \xrightarrow{} MC \quad \text{get spin current}$$

Fermi golden rule

This perturbative Hamiltonian will act on the nucleus and perturbed it.
Any transition rate generated by the electromagnetic perturbation can be calculated starting from the Fermi golden rule (perturbation theory)

Henley-Garcia, Chapter 10

Density of states

$$w_{I,F} = \frac{2\pi}{\hbar} |\langle F | H_{int} | I \rangle|^2 \, d(E)$$

Final state

Initial state

We now understand that the cross section of an em process will involve the calculation of

$$\sigma^{em} \sim |\langle F| \; \rho \;\; \text{or} \;\; \mathbf{J} \; |I\rangle|^2 \, d(E)$$

Nuclear charge and current operators

As the nuclear potential admit an expansion into many-body operators, also the electromagnetic charge and current operators do

$$\rho = \rho_{(1)} + \rho_{(2)} + \cdots = \sum_{i}^{A} \rho_i + \sum_{i<j}^{A} \rho_{ij} + \ldots$$ charge operator

$$\mathbf{J} = \mathbf{J}_{(1)} + \mathbf{J}_{(2)} + \cdots = \sum_{i}^{A} \mathbf{J}_i + \sum_{i<j}^{A} \mathbf{J}_{ij} + \ldots$$ current operator

Indexes are running one the number of nucleons, in general.
NB: neutrons, even though do not have a total charge, they have a magnetic moment

Nuclear charge and current operators

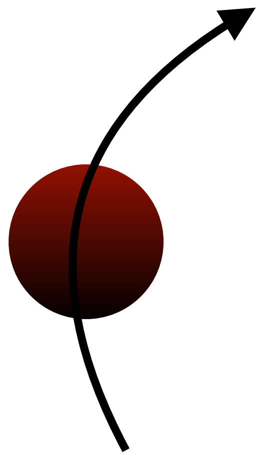
One-body part, for point-like objects

$$\rho_{(1)}(\mathbf{x}) = e \sum_i^A \frac{1+\tau_i^z}{2} \, \delta(\mathbf{x}-\mathbf{r}_i)$$

NB: isospin operator effectively projects on protons

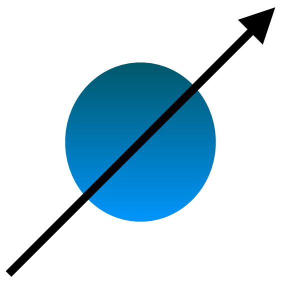
Convection current

$$\mathbf{J}^c_{(1)}(\mathbf{x}) = \frac{e}{2m} \sum_i^A \frac{1+\tau_i^z}{2} \{\mathbf{p}_i, \delta(\mathbf{x}-\mathbf{r}_i)\}$$



Spin current

$$\mathbf{J}^s_{(1)}(\mathbf{x}) = i \frac{e}{2m} \sum_i^A \mu_i \frac{1+\tau_i^z}{2} \boldsymbol{\sigma}_i \times [\mathbf{p}_i, \delta(\mathbf{x}-\mathbf{r}_i)]$$



The use of one-body operators only is called Impulse Approximation

Continuity Equation

Since the Hamiltonian is invariant under a gauge transformation, then according to Noether's theorem, there is a conserved quantity, the electromagnetic current.

The electromagnetic current conservation reads:

$$\boxed{\partial^\mu J_\mu(\mathbf{x}) = 0}$$

Continuity equation

Which can be written as $\frac{\partial}{\partial t}\rho(\mathbf{x}) - \boldsymbol{\nabla}\cdot\mathbf{J}(\mathbf{x}) = 0 \Rightarrow \boxed{\frac{\partial}{\partial t}\rho(\mathbf{x}) = \boldsymbol{\nabla}\cdot\mathbf{J}(\mathbf{x})}$

in coordinate space

The continuity equation can also be written in momentum space

$$\boxed{q^\mu J_\mu(\mathbf{q}) = 0}$$

$$\omega\rho(\mathbf{q}) - \mathbf{q}\cdot\mathbf{J}(\mathbf{q}) = 0 \Rightarrow \boxed{\omega\rho(\mathbf{q}) = \mathbf{q}\cdot\mathbf{J}(\mathbf{q})}$$

we will use this later on

Charge and current operators are not independent on each other

Continuity Equation

We shall write the continuity equation yet in a another way starting from

$$\frac{\partial}{\partial t}\rho(\mathbf{x}) = \boldsymbol{\nabla}\cdot\mathbf{J}(\mathbf{x})$$

And by realizing that operators always need to be sandwiched with w.f. in expectation values, e.g.

$$\langle\ \rho\ \rangle(t) = \langle\Psi(\mathbf{x},t)|\ \rho\ |\Psi(\mathbf{x},t)\rangle$$

In the time-independent Schrödinger equation, the time-dependence in the w.f. is trivial and cancels out in the expectation value

$$0 = \frac{d}{dt}\langle\ \rho\ \rangle = \frac{1}{i}\langle[\rho,H]\rangle + \langle\ \frac{\partial}{\partial t}\rho\ \rangle$$

Generalized Ehrenfest theorem

$$\langle\ \frac{\partial}{\partial t}\rho\ \rangle = -\frac{1}{i}\langle[\rho,H]\rangle = \frac{1}{i}\langle[H,\rho]\rangle = -i\langle[H,\rho]\rangle$$

Equating and omitting the bra-kets

$$\Rightarrow \quad \boxed{\boldsymbol{\nabla}\cdot\mathbf{J}(\mathbf{x}) = -i[H,\rho(\mathbf{x})]}$$

Charge and current operators are not independent on each other and they are related to the Hamiltonian

Continuity Equation

$$\boxed{\boldsymbol{\nabla} \cdot \mathbf{J}(\mathbf{x}) = -i[H, \rho(\mathbf{x})]}$$

Now, if we take the one-body charge and current operators, we will see that

$$\boldsymbol{\nabla} \cdot \mathbf{J}_{(1)}(\mathbf{x}) = -i\,[T, \rho_{(1)}(\mathbf{x})]$$

See this in the exercise session

But $\quad [V, \rho_{(1)}(\mathbf{x})] \neq 0$

Thus, there has to be a two-body current operator such that

$$\boldsymbol{\nabla} \cdot \mathbf{J}_{(2)}(\mathbf{x}) = -i[V, \rho_{(1)}(\mathbf{x})]$$

NB: The continuity equation constraints only the divergence of the current, not the curl.
So you can always add an arbitrary part of the current so that the curl is zero
Unless you rely on a microscopic theory to guide you in the construction of the current.

Continuity Equation

What kind of potential do I need so that there must exist a two-body current?

If $V = V(|r_i - r_j|)$ then $[V, \rho] = 0$

But, even if $V = V(|r_i - r_j|)\, \vec{\tau}_1 \cdot \vec{\tau}_2$ then $[V, \rho] \neq 0$

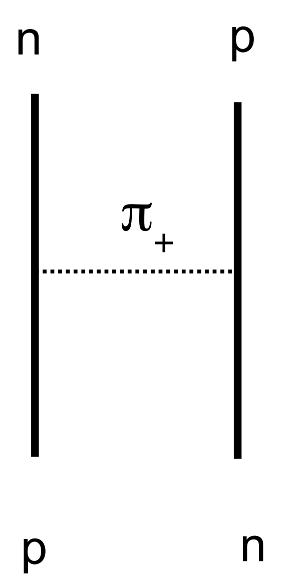
$\Rightarrow$ there must exist a two-body current, whose isospin part goes like $(\vec{\tau}_1 \times \vec{\tau}_2)^z$

See this in the exercise session

Two-body currents

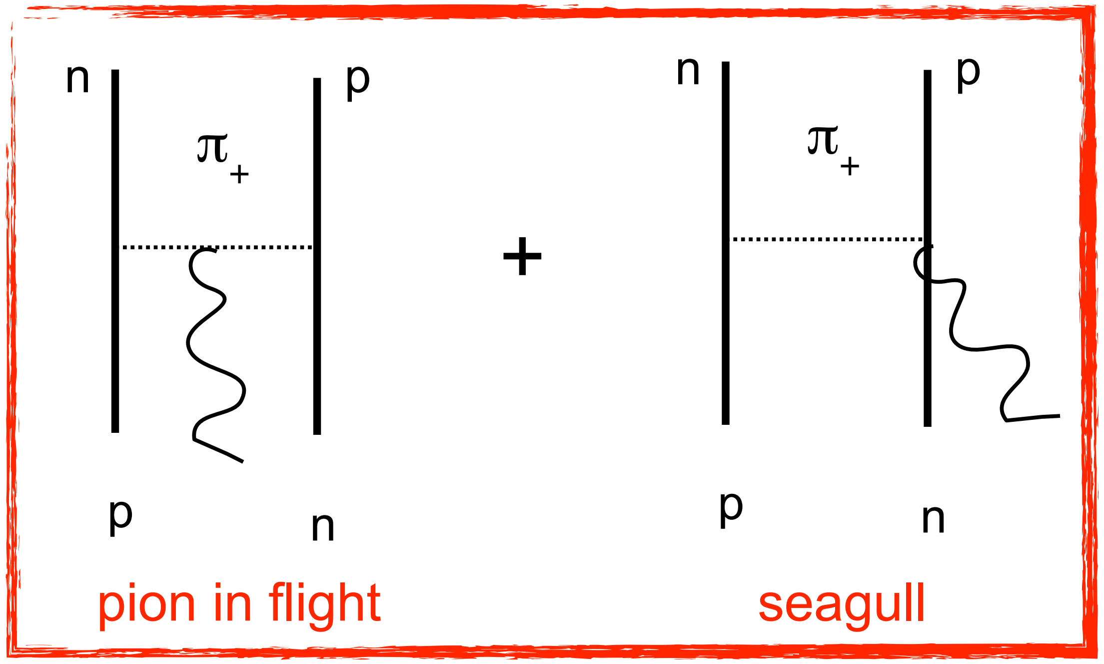
Clearly these two-body currents are related to the fact that nucleons are not free particles but interact with each other.

They interact mostly via a one-pion exchange



If the pion is charged, it can interact with a photon

Diagram of a two-body current



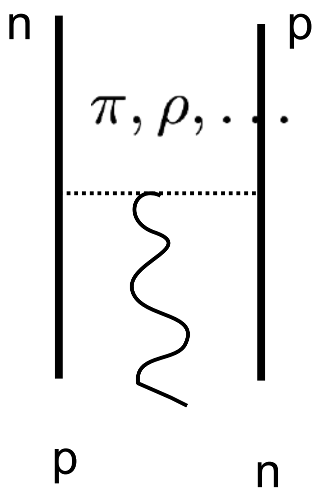
Leading term of two-body currents OPE

Two-body currents

Clearly these two-body currents have to do with subnuclear degrees of freedom

In the past, meson-exchange theory could be used to construct currents

Meson-exchange currents



Today, this is done in the language of **chiral effective field theory**

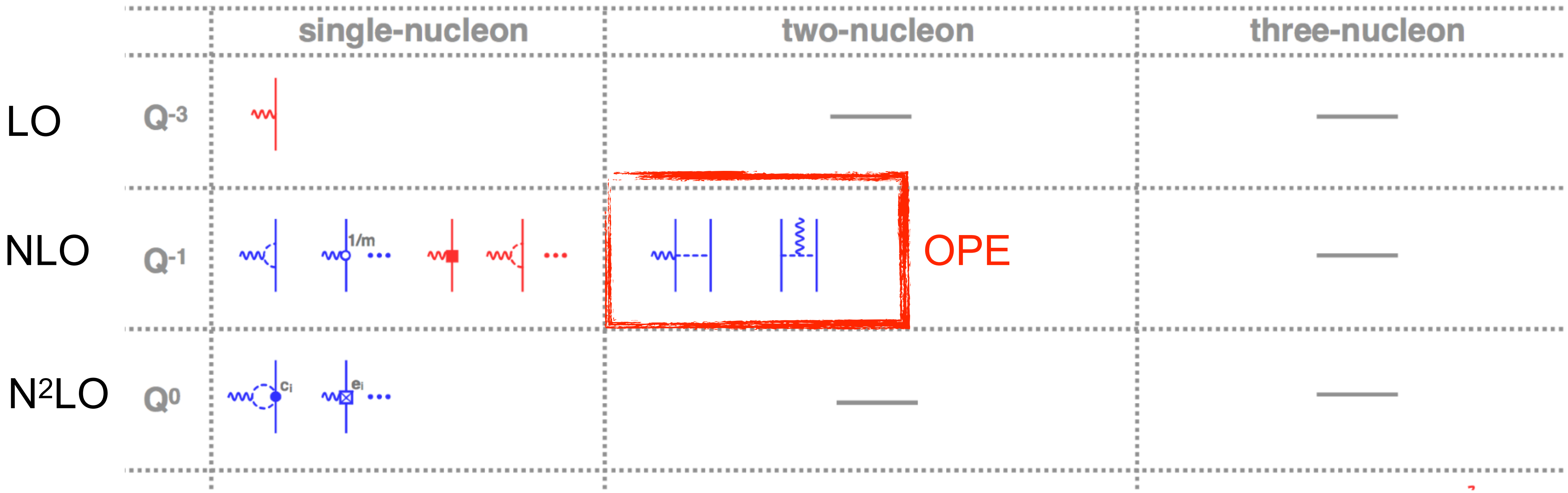
Pastore, Schiavilla, Epelbaum….

Two-body currents

Two-body currents in chiral EFT

From E.Epelbaum, Mainz Workshop, October 2018

Chiral expansion of the electromagnetic **current** and **charge** operators



Two-body operators appear at NLO for the em current

Two-body operators do not appear for the em charge in the first three orders

Two-body currents

How do we get a handle on these currents?

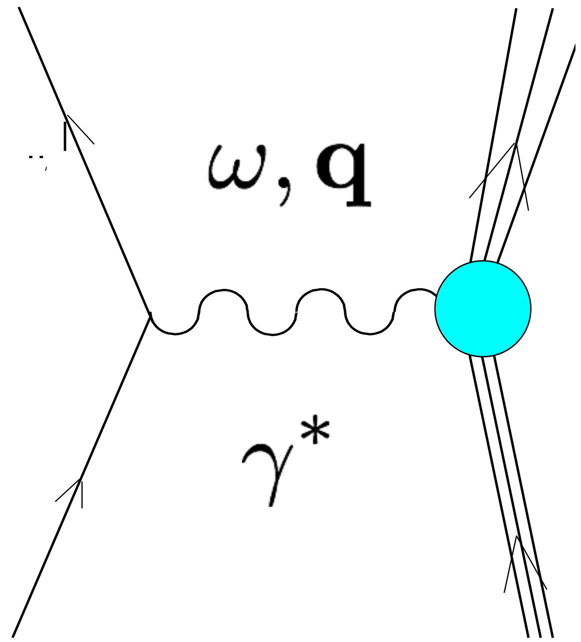
By studying electromagnetic observables

Em probes can be used to:

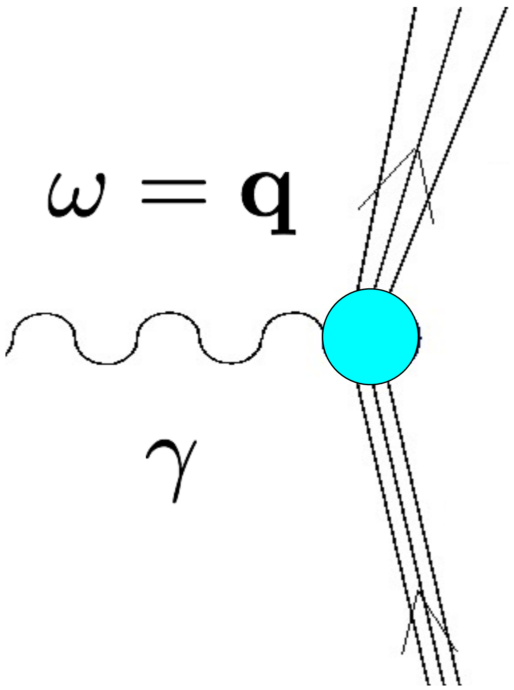
and subnuclear

- Understand the nuclear dynamics
- Connect nuclear physics to other fields of research

Electromagnetic probes



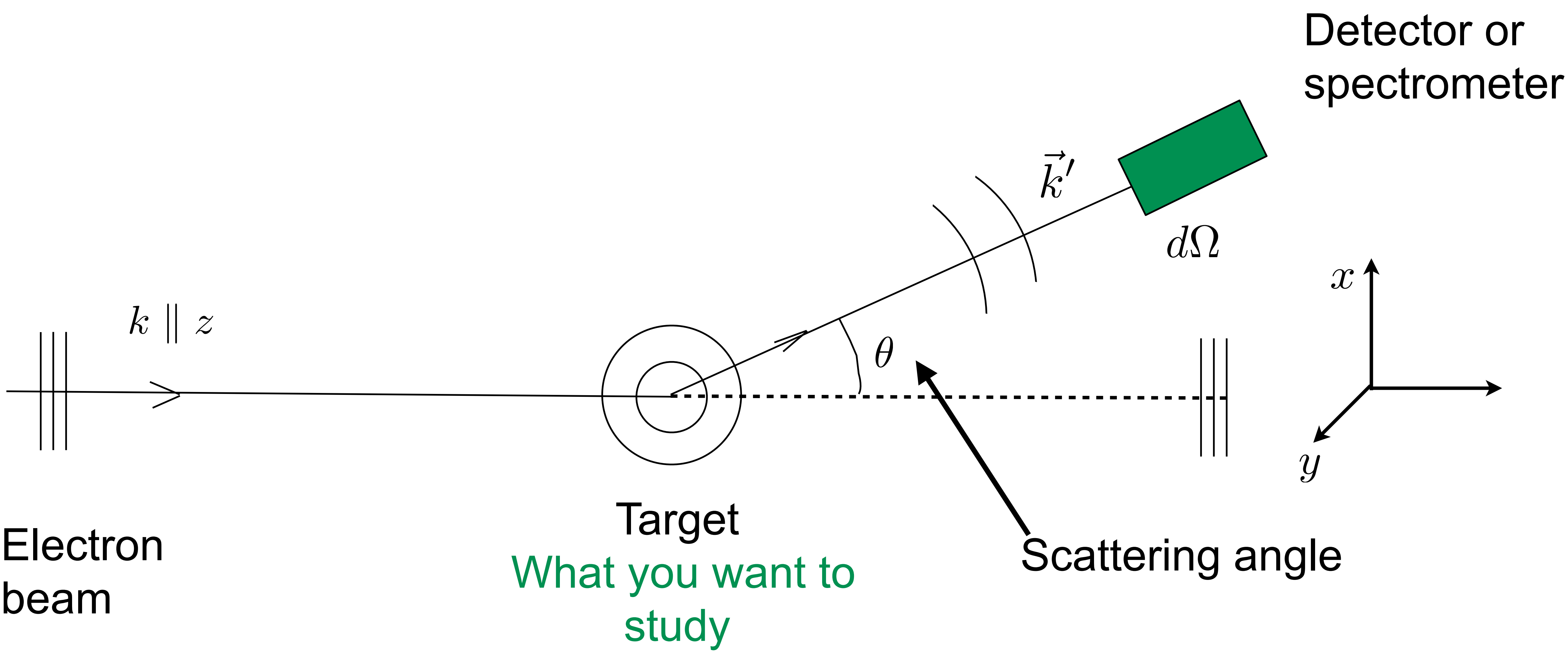
Electron scattering
(virtual photon)



Photoabsorption
(real photon)

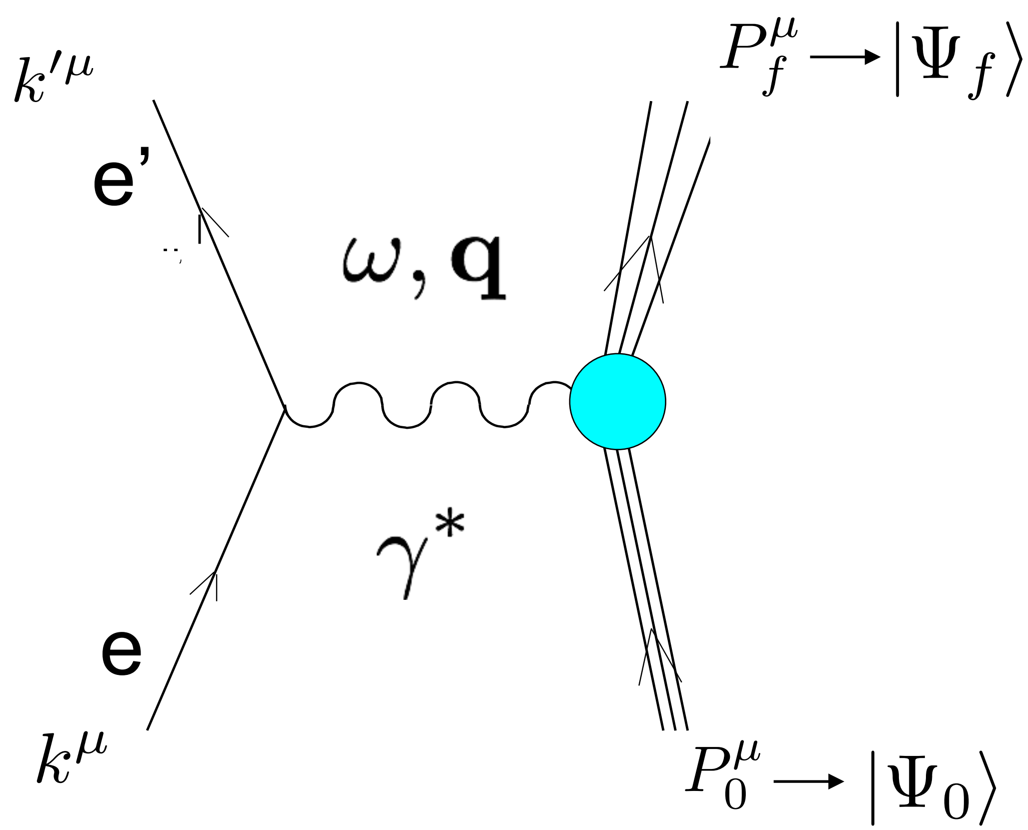
The real photon case can be seen as a special case of the virtual photon

Electron scattering



You can vary the beam energy and the scattering angle

Electron scattering



$$k^{\mu} = (\epsilon, \mathbf{k})$$

$$k'^{\mu} = (\epsilon', \mathbf{k}')$$

$$\omega = \epsilon - \epsilon'$$ Energy transfer

$$\mathbf{q} = \mathbf{k} - \mathbf{k}'$$ Momentum transfer

$$\epsilon = \sqrt{\mathbf{k}^2 + m^2} \simeq |\mathbf{k}|$$

$$\epsilon' = \sqrt{\mathbf{k}'^2 + m^2} \simeq |\mathbf{k}'|$$

$$\Rightarrow$$

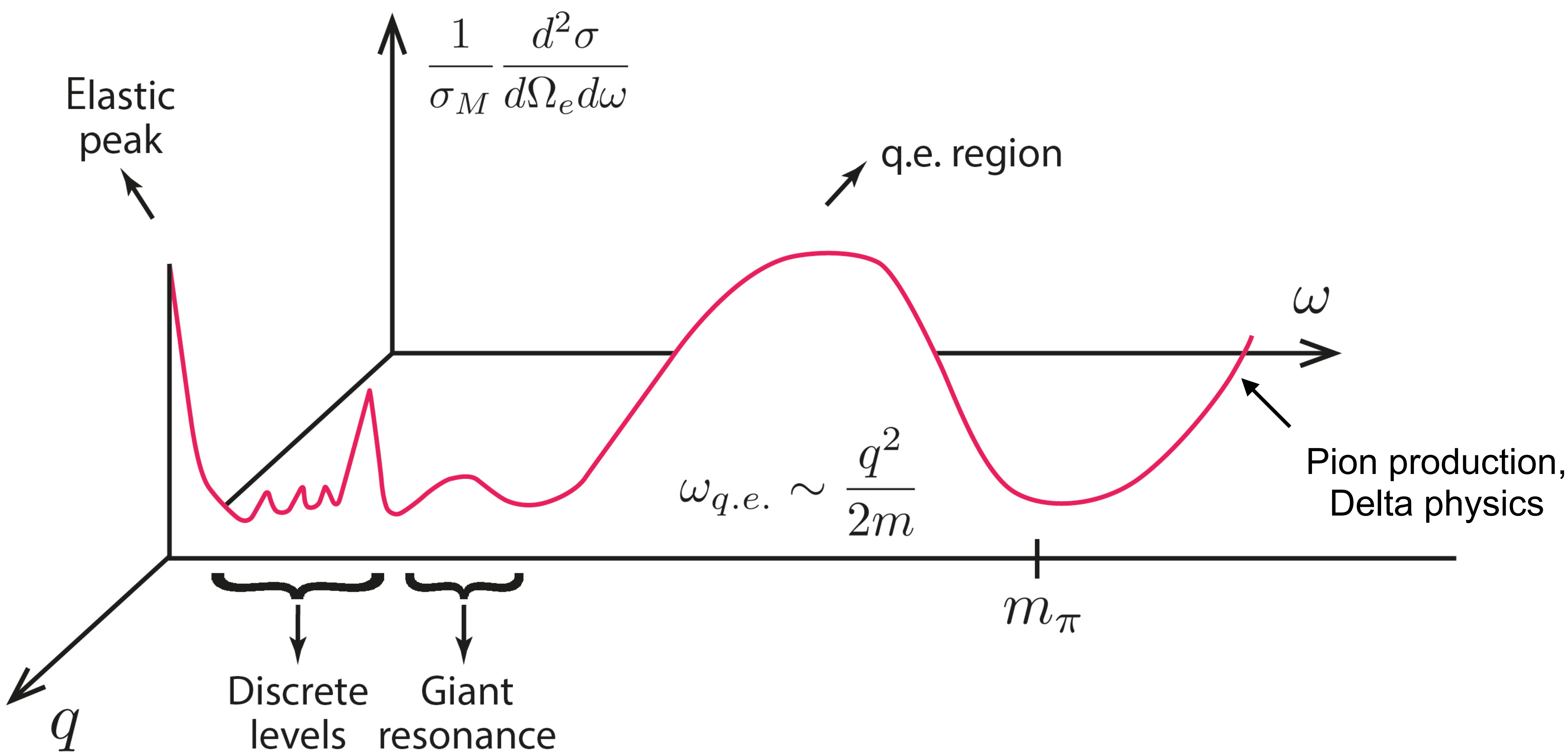
$$\omega^2 = |\mathbf{k}|^2 + |\mathbf{k}'|^2 - 2|\mathbf{k}||\mathbf{k}'|$$

$$|\mathbf{q}|^2 = |\mathbf{k}|^2 + |\mathbf{k}'|^2 - 2\ \mathbf{k} \cdot \mathbf{k}'$$

$$|\mathbf{q}|^2 = |\mathbf{k}|^2 + |\mathbf{k}'|^2 - 2\ |\mathbf{k}||\mathbf{k}'| \cos\theta$$

$$\boxed{\Rightarrow \mathbf{q}^2 \geq \omega^2}$$

Electron scattering



Inclusive electron scattering

(e,e')

Only final electron is detected

Relatively simple kind of experiment

Energy

- Elastic peak: all the energy transferred to the nucleus converts into recoil (kinetic energy) and the nucleus remains in the round state $\Rightarrow$ study nuclear g.s. properties
- Excitation of low-lying discrete inelastic states $\Rightarrow$ study nuclear excited states
- Collective excitations of the nucleus (giant resonances)
- Quasi-elastic peak: the dominant process is the scattering off one nucleon $\Rightarrow$ study single particle properties, momentum and energy distributions of nuclei inside the nucleus
- Delta and pion production …

Inclusive cross section

$$\sigma \sim \langle \overbrace{f\, k'}^{\langle F|} \,|\, J_\mu A^\mu \,|\, \overbrace{0\, k}^{|I\rangle} \rangle\, |^2 \,\delta(\omega - E_F + E_I)$$

experiment

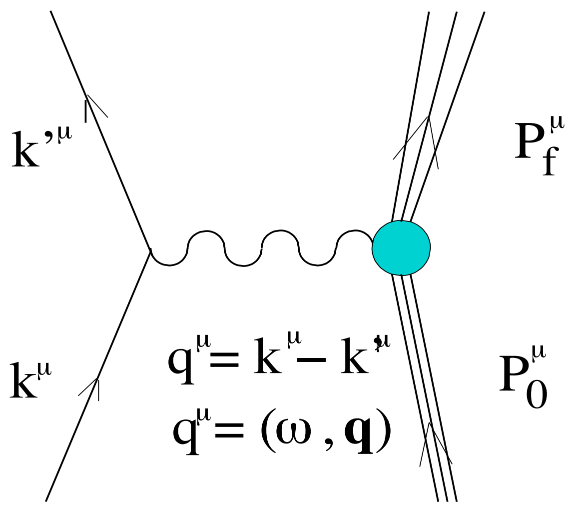
External field: electron
Known!

Hadronic properties. Unknown.
HERE IS THE INTERESTING PHYSICS!!!

Work out: the lepton and nuclear information can be separated in the cross section

See notes by Nir Barnea on electron scattering for people interested in more details in the derivation

Inclusive cross section



$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2\frac{\theta}{2} \right) R_T(\omega, \mathbf{q}) \right]$$

with $Q^2 = -q_\mu^2 = \mathbf{q}^2 - \omega^2$ and θ scattering angle

and σ_M Mott cross section $\sigma_M = \left\{ \frac{\alpha \cos\theta_e/2}{2\epsilon \sin^2\theta_e/2} \right\}^2 .$

Elastic scattering of a spin 1/2 particle from a Coulomb field
(analogous to Rutherford scattering for alpha particle, but with spin)

This expression is valid in Born approximation, i.e. where distortions of the Coulomb field can be neglected (light nuclei).

Inclusive cross section

Unit vector in the spherical basis

$$\mathbf{e}_1 = -\frac{1}{\sqrt{2}}(\mathbf{e}_x + i\mathbf{e}_y)$$

$$\mathbf{e}_0 = \mathbf{e}_z$$ ← direction of $\mathbf{q}$

$$\mathbf{e}_{-1} = \frac{1}{\sqrt{2}}(\mathbf{e}_x - i\mathbf{e}_y)$$

$R_L(\omega, \mathbf{q})$

Longitudinal:

Photon polarization along direction of momentum propagation

$\mathbf{e}_0$

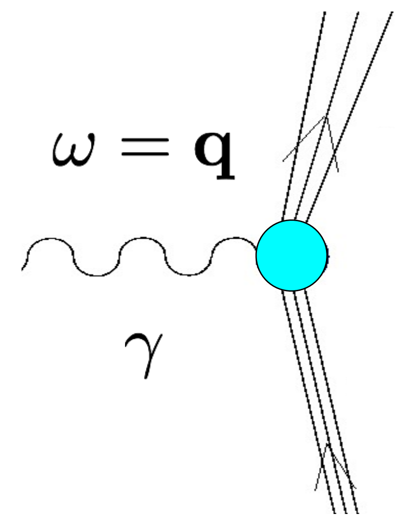
$R_T(\omega, \mathbf{q})$

Transverse:

Photon polarization transverse to direction of momentum propagation

$\mathbf{e}_\pm$

Photoabsorption



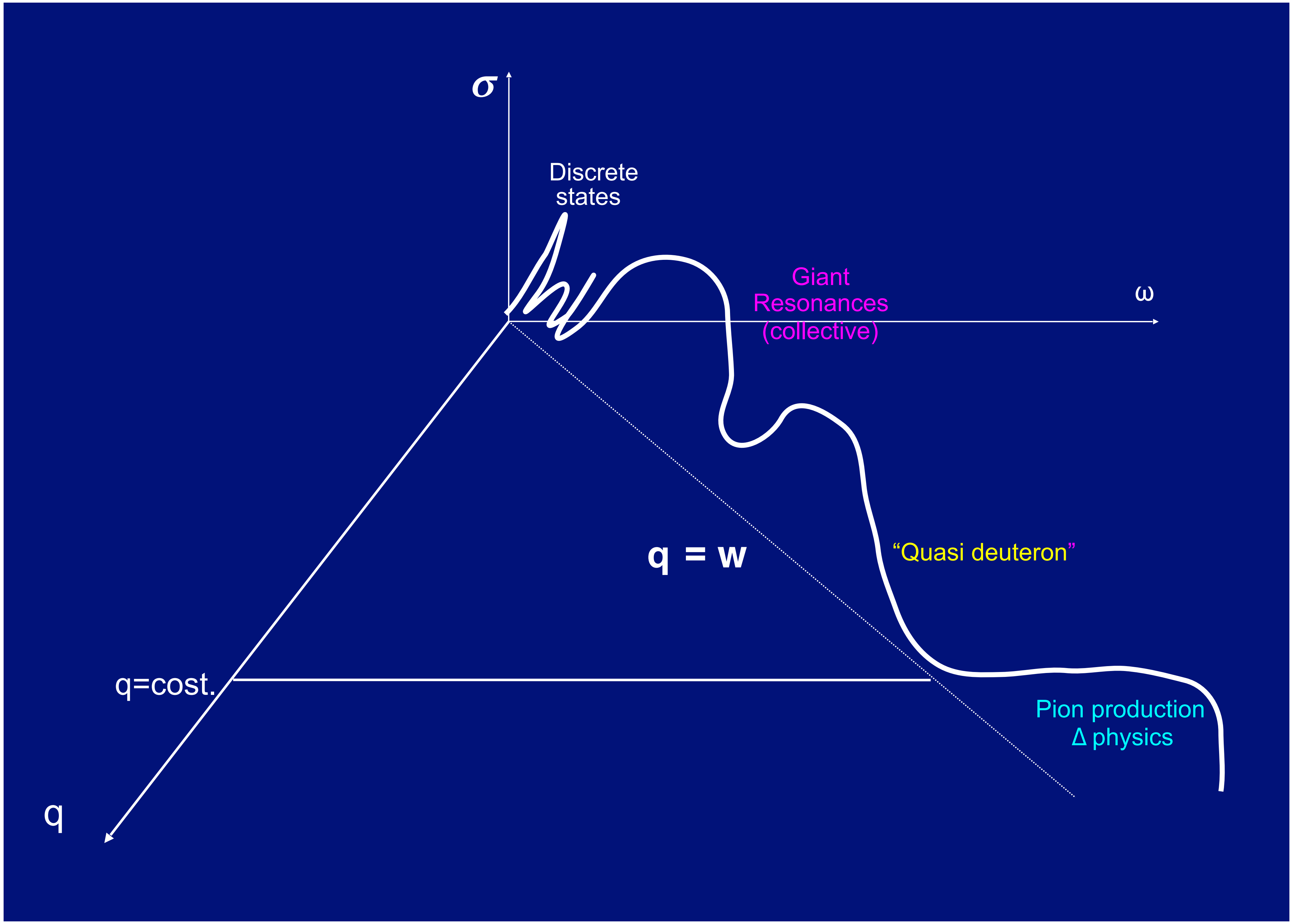
$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\cancel{\frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q})} + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2\frac{\theta}{2} \right) \overset{\omega = \mathbf{q}}{R_T(\omega, \mathbf{q})} \right]$$

~~Longitudinal part:~~
~~Photon polarization along momentum axis~~

Transverse part: $\omega = \mathbf{q}$
Photon polarization transverse to momentum axis

Photoabsorption

Real photons



Response Functions

Inclusive cross section A(e,e')X

Nuclear part

$$R_L(\omega, \mathbf{q}) = \sum\!\!\!\!\!\!\!\int_f |\langle\Psi_f|\,\rho(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$ charge operator

$$R_T(\omega, \mathbf{q}) = \sum\!\!\!\!\!\!\!\int_f |\langle\Psi_f|\,J_T(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$ current operator

Response Functions

Inclusive cross section A(e,e')X

Nuclear part

$$R_L(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\int_f |\langle\Psi_f|\,\rho(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$ charge operator

$$R_T(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\int_f |\langle\Psi_f|\,J_T(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$ current operator

Operators

$$\rho(\mathbf{q}) = \int d^3x\; e^{i\mathbf{q}\cdot\mathbf{x}}\;\rho(\mathbf{x})$$

$$\mathbf{J}(\mathbf{q}) = \int d^3x\; e^{i\mathbf{q}\cdot\mathbf{x}}\;\mathbf{J}(\mathbf{x})$$

Response Functions

Inclusive cross section A(e,e')X

Nuclear part

$$R_L(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\!\int_f |\langle\Psi_f|\,\rho(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$ charge operator

$$R_T(\omega,\mathbf{q}) = \sum\!\!\!\!\!\!\!\int_f |\langle\Psi_f|\,J_T(\mathbf{q})\,|\Psi_0\rangle|^2\,\delta\left(E_f - E_0 - \omega + \frac{\mathbf{q}^2}{2M}\right)$$ current operator

Longitudinal part $\Rightarrow$ information on the charge density

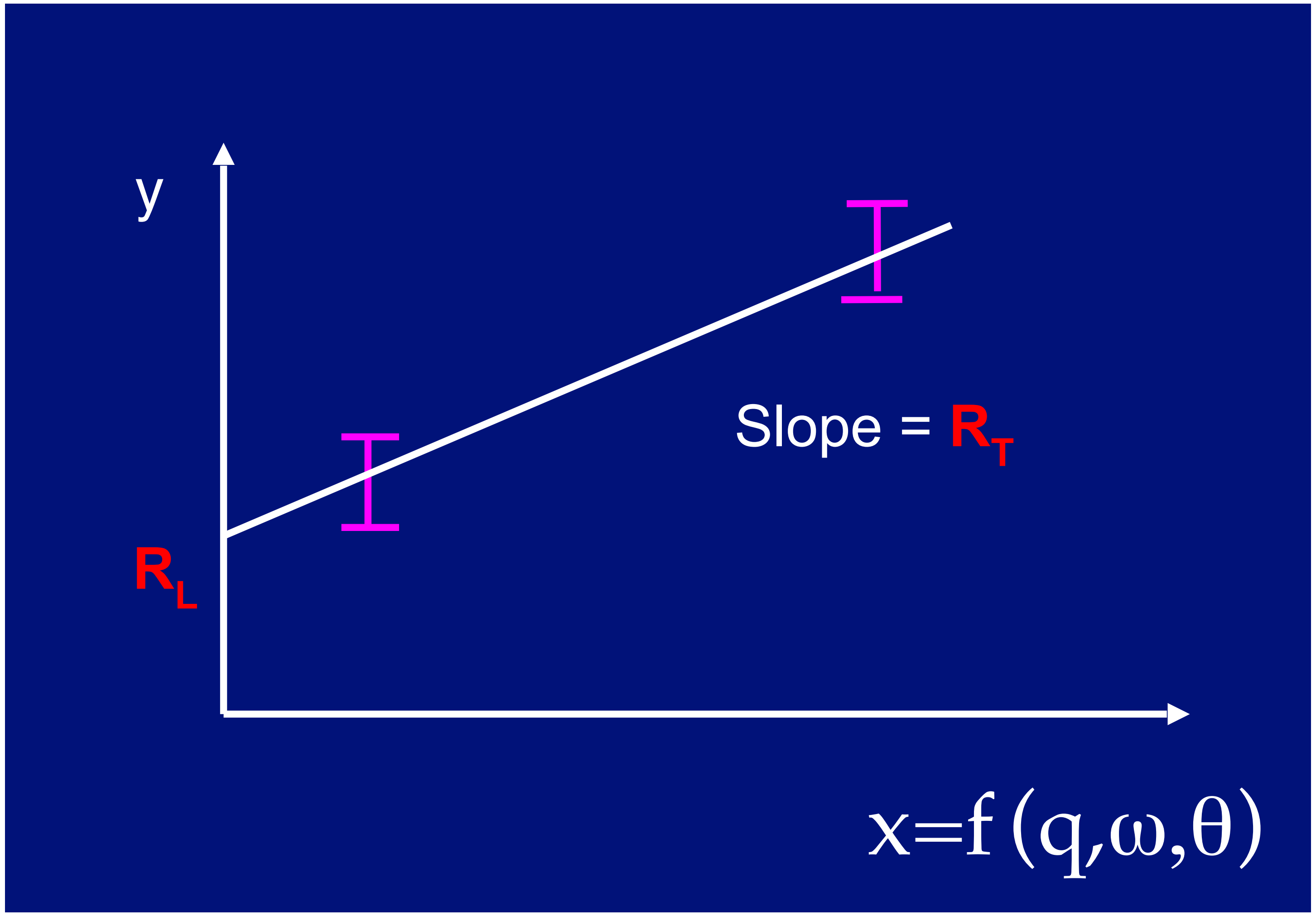
Transverse part $\Rightarrow$ information on the current density

Rosenbluth separation

$$\frac{d^2\sigma}{d\Omega d\omega} = \sigma_M \left[\frac{Q^4}{\mathbf{q}^4} R_L(\omega, \mathbf{q}) + \left(\frac{Q^2}{2\mathbf{q}^2} + \tan^2\frac{\theta}{2} \right) R_T(\omega, \mathbf{q}) \right]$$

$$\frac{\frac{d^2\sigma}{d\Omega d\omega}}{\sigma_M} \frac{\mathbf{q}^4}{Q^4} = R_L + R_T f(q, \omega, \theta)$$

$$y = a + bx$$



Elastic scattering

$\omega = 0$ No energy transfer, only momentum

$f = 0$ Nucleus stays in ground-state

$$R_L(\omega, \mathbf{q}) \rightarrow \boxed{F_L(q)} \rightarrow |\langle \Psi_0 | \rho(q) | \Psi_0 \rangle|^2$$

Charge distribution: sensitive to protons

$$R_T(\omega, \mathbf{q}) \rightarrow \boxed{F_T(q)} \rightarrow |\langle \Psi_0 | \mathbf{J}_T(q) | \Psi_0 \rangle|^2$$

Current distribution: sensitive also to neutrons because of magnetic moment and spin currents

Form factors

Nuclear charge radius

How do you measure the nuclear charge radius?

From elastic electron scattering off a nucleus

$$\frac{d^2\sigma}{d\Omega} = \sigma_M \left[|F_L(q^2)|^2 + \left(\frac{1}{2} - \tan^2\frac{\theta}{2} \right) |F_T(q^2)|^2 \right]$$

⇒ Rosenbluth separation to obtain the longitudinal or charge form factor

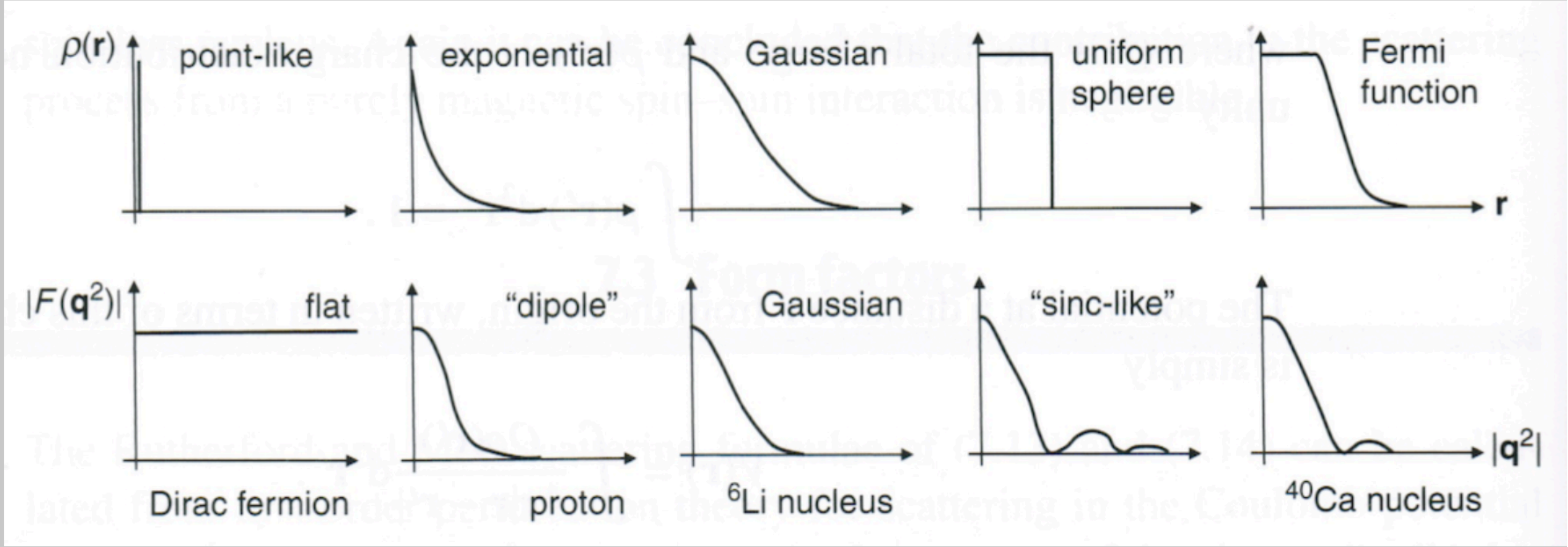
$$|F_L(q^2)|^2 = |F(q^2)|^2 \rightarrow F(q^2)$$

Nuclear charge radius

Fourier transform of the charge distribution

$$F(q^2) = \int d^3x e^{i\mathbf{q}\cdot\mathbf{x}} \rho(\mathbf{x})$$

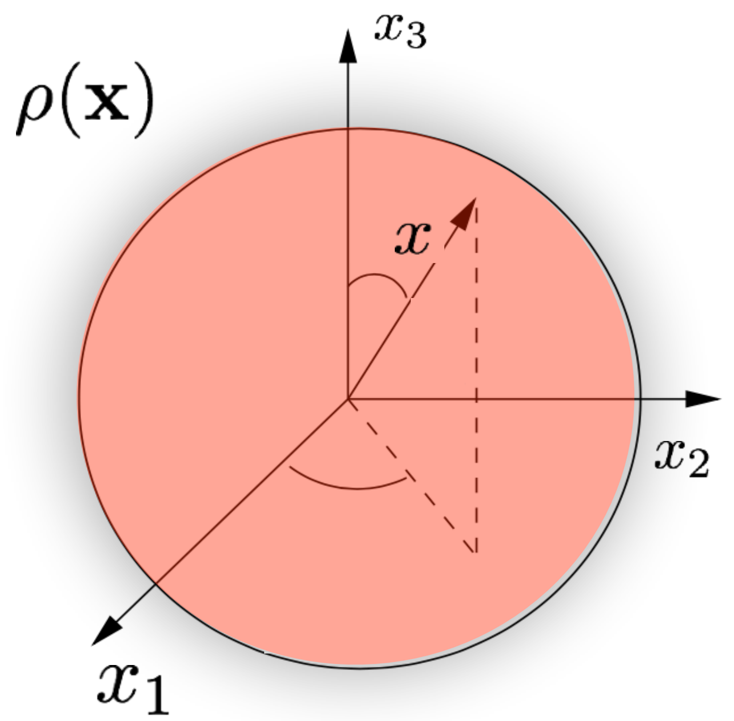
To be intended as the expectation
value of the the charge operator in coordinate
space, not an operator here



Nuclear charge radius

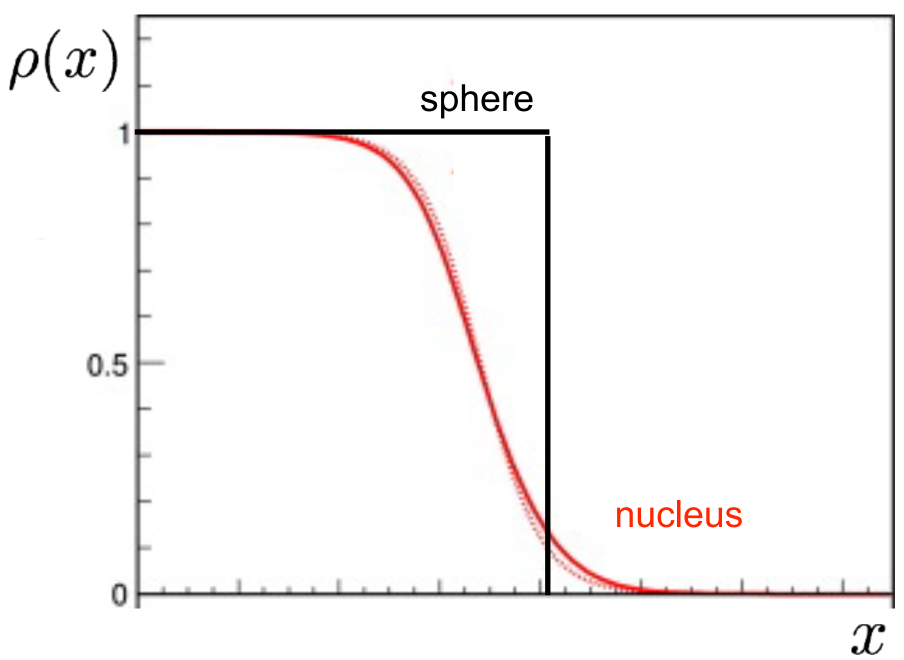
Fourier transform of the charge distribution

$$F(q^2) = \int d^3x e^{i\mathbf{q}\cdot\mathbf{x}} \rho(\mathbf{x})$$



Assuming spherical symmetry, only interesting part is the radial dependence

$$\begin{aligned} F(q^2) &= \int dx x^2 \rho(x) \int d\theta d\phi \sin(\theta) e^{iqx\cos\theta} \\ &= 2\pi \int dx x^2 \rho(x) \int d\theta \sin(\theta) e^{iqx\cos\theta} \\ &= \frac{4\pi}{q} \int dx x \rho(x) \sin qx \end{aligned}$$



Nuclear charge radius

Now consider low-q limit

$$F(q^2) = \frac{4\pi}{q}\int dx x\rho(x)\sin qx = \frac{4\pi}{q}\int dx x\rho(x)\left(qx - \frac{(qx)^3}{3!}\right) + \dots$$

$$= 4\pi\int dx x^2\rho(x) - \frac{q^2}{6}4\pi\int dx x^4\rho(x) + \dots$$

$$= Z(1 - \frac{q^2}{6}\langle r^2\rangle + \dots)$$

Nuclear charge radius

Now consider low-q limit

$$F(q^2) = \frac{4\pi}{q}\int dx x\rho(x)\sin qx = \frac{4\pi}{q}\int dx x\rho(x)\left(qx - \frac{(qx)^3}{3!}\right) + \dots$$

$$= 4\pi\int dx x^2\rho(x) - \frac{q^2}{6}4\pi\int dx x^4\rho(x) + \dots$$

$$= Z(1 - \frac{q^2}{6}\langle r^2\rangle + \dots)$$

Typically F is then normalized to 1, i.e. divided by Z

$$\boxed{\langle r^2\rangle = -6\frac{dF(q^2)}{dq^2}\Big|_{q^2=0}}$$

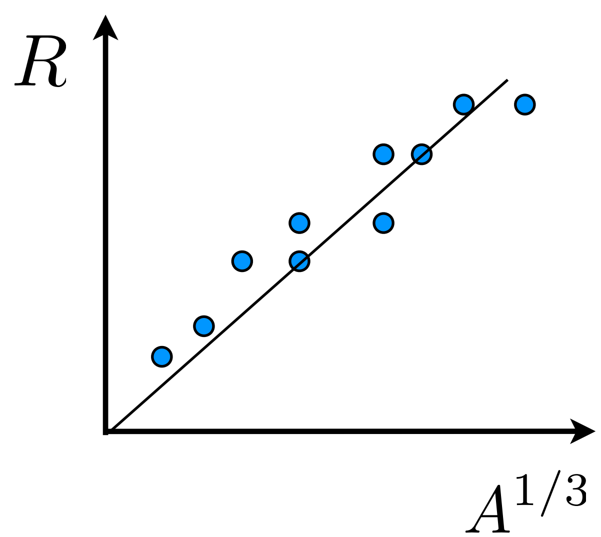
If you measure the form factor at low q and then take the derivative with respect to q^2 you obtain the charge radius.

Nuclear charge radius

From electron scattering you obtain the charge radius and the charge distributions

$$R = \sqrt{\langle r^2 \rangle}$$

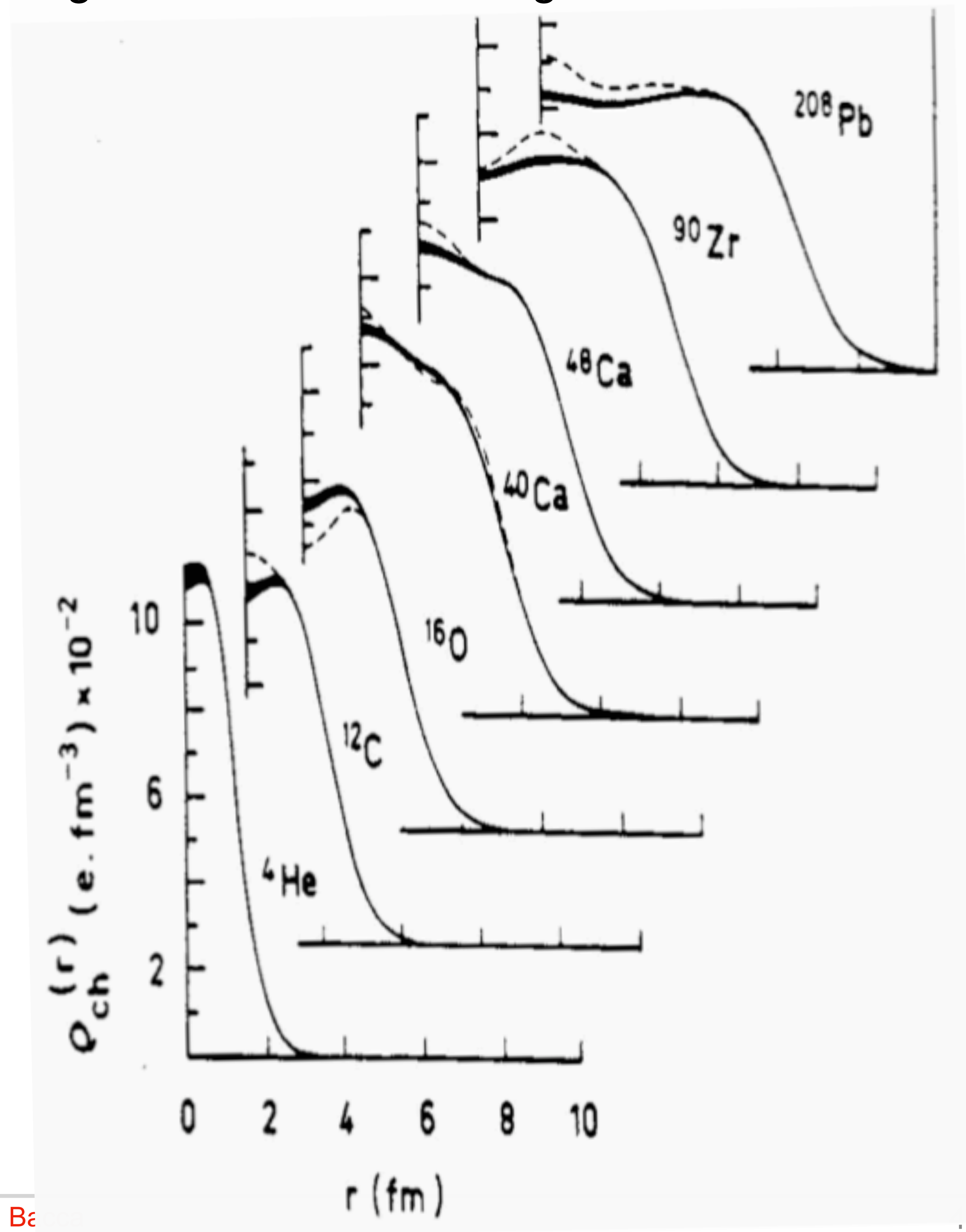
stable nuclei



$$R \simeq r_0 A^{1/3}$$

$$r_0 = 1 - 1.25\text{fm}$$

Behaviour that is disobeyed by
exotic nuclei, such as halo nuclei



Elastic scattering

Physics studies:

Compare between N=Z and Neutron-rich isotopes

how does the charge distribution change because of the presence of the NEUTRONS?

how does the current distribution change as a function of the number of NEUTRONS?

how does the current distribution change because of two-body currents?

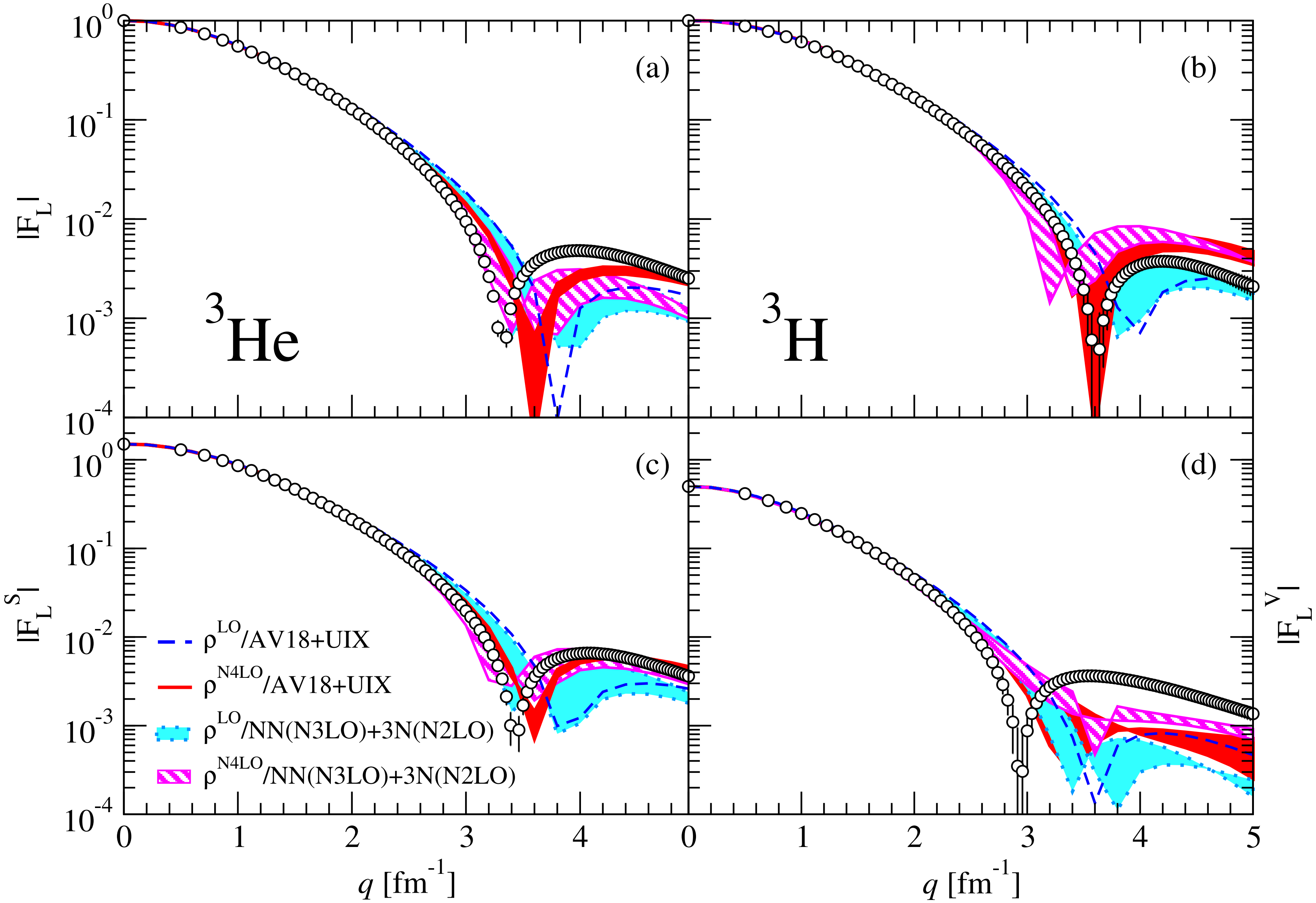
which two-body currents are relevant in one case and in the other?

A few examples

Elastic Electron Scattering

M. Piarulli, L. Girlanda, L. Marcucci, S. Pastore, R. Schiavilla, *et al.*, Phys. Rev. C87, 014006 (2013)

Calculations with hyper-spherical harmonics



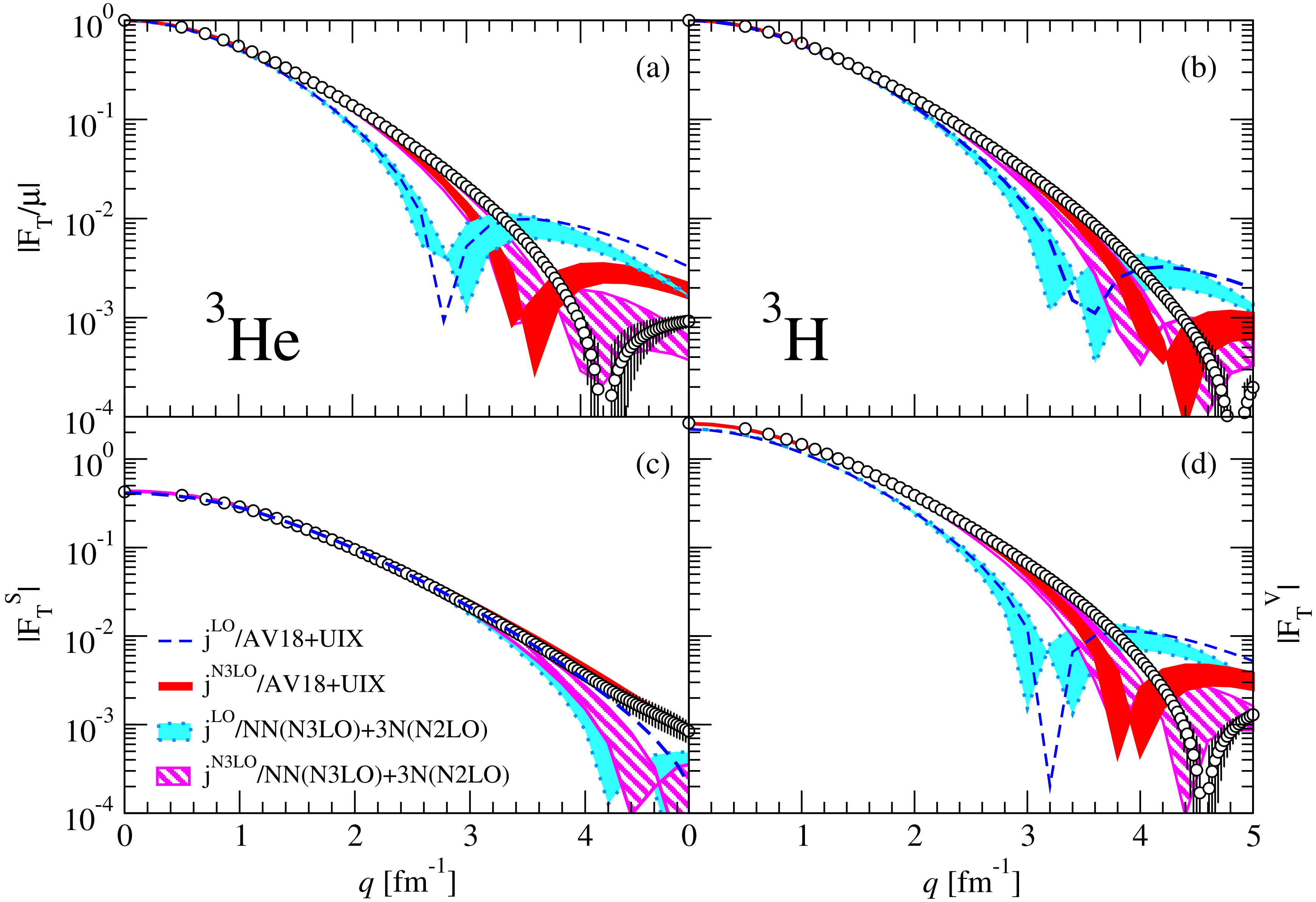
$$F_L^{S,V}(q) = \frac{1}{2}[2F_L(q,{}^3\,\mathrm{He}) \pm F_L(q,{}^3\,\mathrm{H})]$$

Traditional nuclear physics and chiral EFT agree
Two-body currents not important in longitudinal FF

Elastic Electron Scattering

M. Piarulli, L. Girlanda, L. Marcucci, S. Pastore, R. Schiavilla, *et al.*, Phys. Rev. C87, 014006 (2013)

Calculations with hyper-spherical harmonics



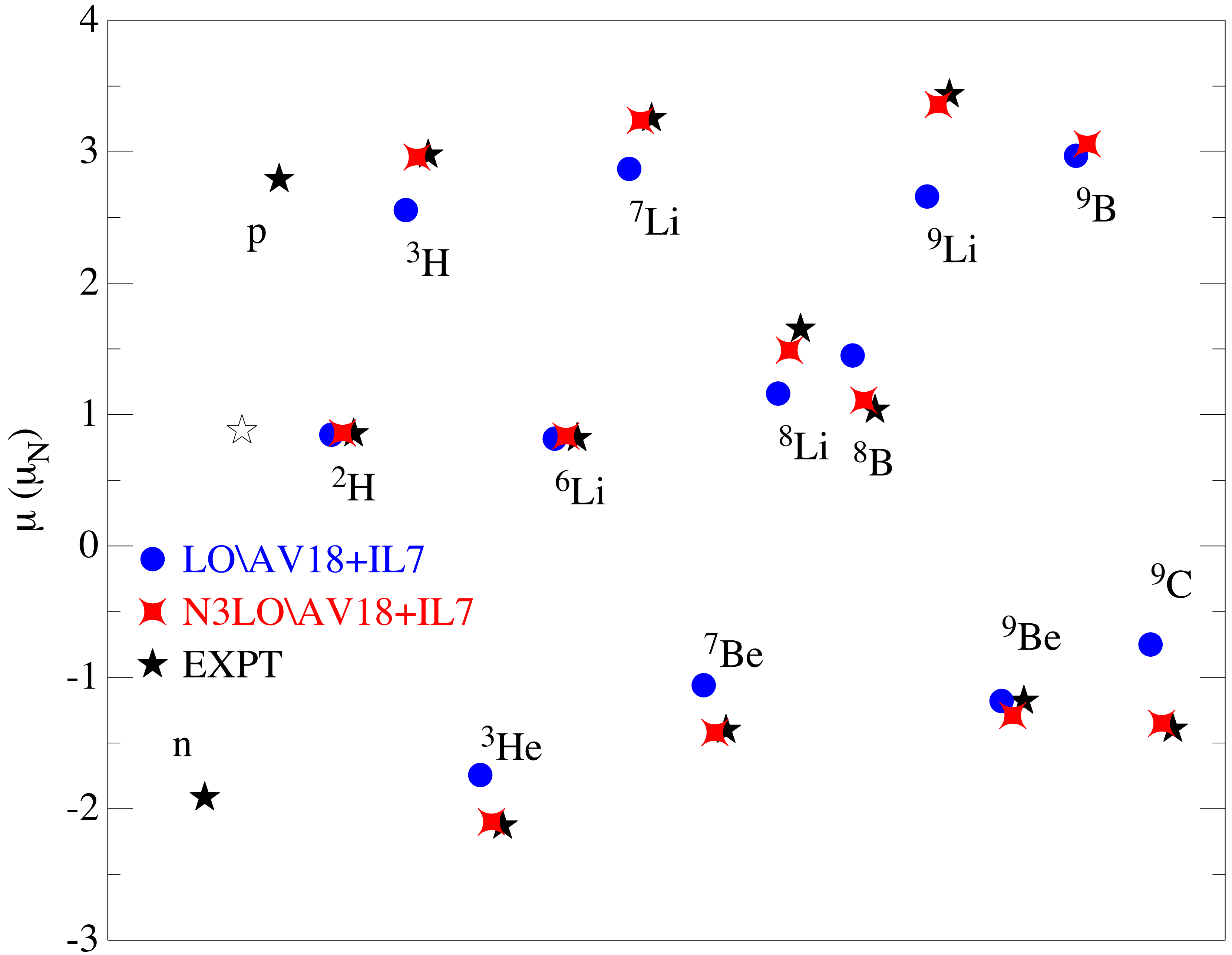
$$F_T^{S,V}(q) = \frac{1}{2}[\mu(^3\mathrm{He})F_T(q,^3\mathrm{He}) \pm \mu(^3\mathrm{H})F_T(q,^3\mathrm{H})]$$

Traditional nuclear physics and chiral EFT slightly different
Two-body currents important in transverse/magnetic FF

Magnetic moments

S. B. and S. Pastore, J. Phys. G: Nucl. Part. Phys. **41** 123002 (2014).

Calculations with Monte Carlo methods



Inelastic scattering

$\omega \neq 0$	Energy and momentum transferred
$f \neq 0$	Nucleus does not stay in ground-state

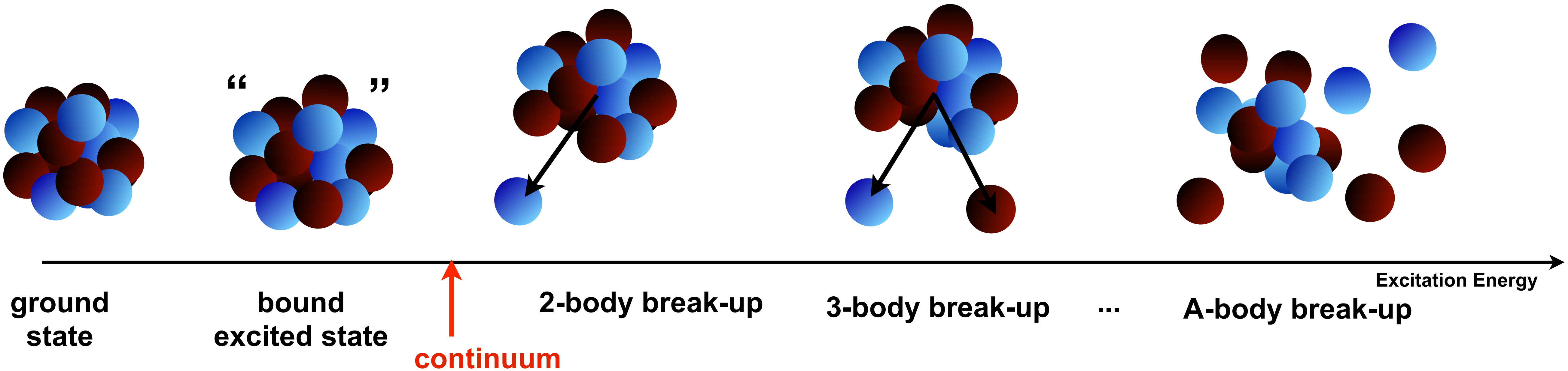
Much richer!

Information is about both ground state and excited states

Not only “static” properties but also
“dynamical” properties e.g. collective motions

More complicated, though.
Need to be able to calculate final states also in the continuum

Inelastic scattering



$$R(\omega) \propto |\langle \Psi_f | J^\mu | \Psi_0 \rangle|^2$$

Exact knowledge limited in energy and mass number

We will discuss this important issue later and first better understand how to practically deal with the charge and current operators