

TALENT School@MITP 2022

DMC python exercise

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Open the python script `DMC_exercise.py`. This script is prepared for an implementation of the DMC method to obtain the ground state energy and wave-function of the Schrödinger equation with a local interaction.

Harmonic oscillator exercise :

We first try to calculate the ground state of the 1D harmonic oscillator Hamiltonian, which in natural units $m = \hbar = 1$ reads

$$H = -\frac{1}{2} \frac{d^2}{dx^2} + \frac{1}{2} x^2. \quad (1)$$

In the script you can find the function `dmc_step(poten,xold,eold,ntarget,dtau)` the parameters of the routine are: Note that in the code `h2m = \hbar^2/m`.

Table 1: DMC step subroutine

| input | <code>xold</code> | old (current) walkers positions |
|--------|----------------------|--------------------------------------|
| | <code>eold</code> | old (current) energy estimate |
| | <code>poten</code> | the interaction, e.q V_HO or V_Morse |
| | <code>ntarget</code> | desired number of walkers |
| | <code>dtau</code> | time step |
| output | <code>xnew</code> | new walkers positions |
| | <code>enew</code> | new energy estimate |

1. Complete the `step_dmc` function calculating the evolution of the walkers population via the DMC method.
2. After completion of `step_dmc` function use the python script to calculate the one dimensional HO ground state energy and wave function.

3. Introduce some smoothing procedure to extract the “best” estimate for the $E, \Delta E$.
4. Repeat the calculation changing `nwalkers` and plot the convergence of the energy as a function of `nwalkers`.
5. Repeat the calculation changing `tau` and plot the convergence of the energy as a function of τ .
6. Repeat the calculation for the Morse potential

$$V(x) = \frac{1}{2}(e^{-2x} - 2e^{-x}) \quad (2)$$

given in the code. What is the ground state energy in this case?