Chiral Effective Field Theory and Nuclear Forces: concepts of chiral EFT

Kai Hebeler

Mainz, July 29, 2022

TALENT school @MITP:
Effective field theories in light nuclei

SFB 1245

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Basic ideas of effective theories: Multipole expansion

Consider electrostatic potential $\Phi(\mathbf{r})$ generated by a localized charge distribution $\rho(\mathbf{r})$:

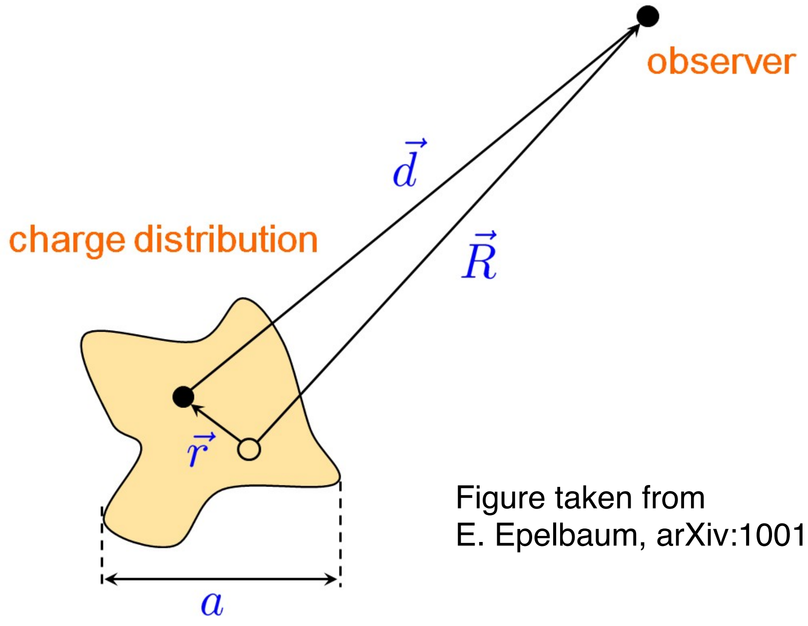

Figure taken from
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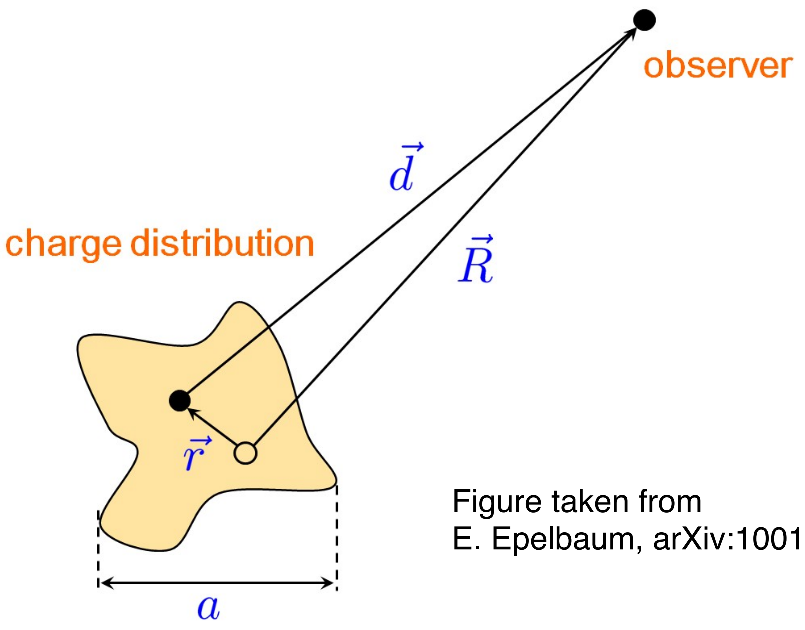

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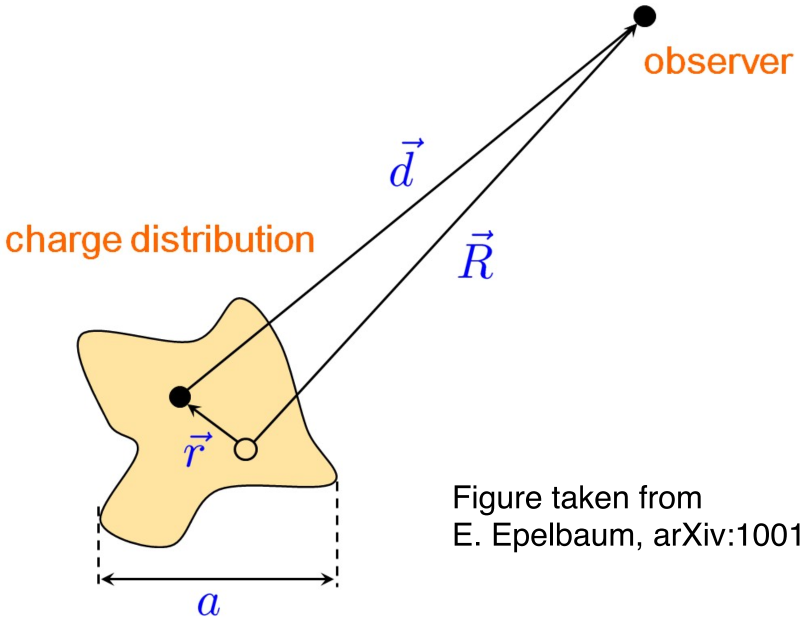

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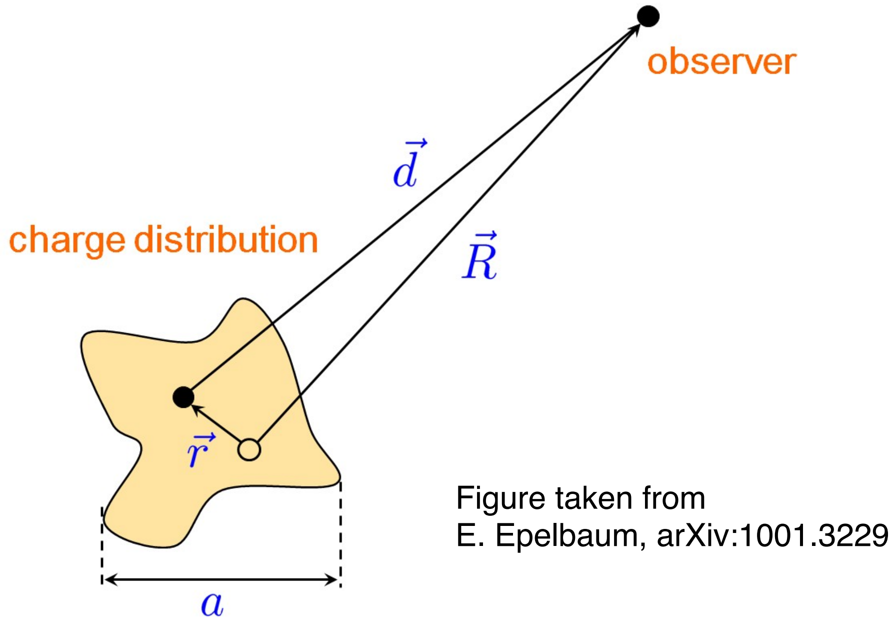

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Expand $\dfrac{1}{|\mathbf{R}-\mathbf{r}|}$:

$$\Phi(\mathbf{R}) = \frac{q}{R} + \frac{1}{R^3}\sum_i R_i P_i + \frac{1}{6R^5}\sum_{i,j}(3R_iR_j - \delta_{ij}R^2)Q_{ij} + \ldots$$

with:

$$\boxed{q = \int d^3\mathbf{r}\rho(\mathbf{r}),} \quad \boxed{P_i = \int d^3\mathbf{r}\rho(\mathbf{r})r_i,} \quad \boxed{Q_{ij} = \int d^3\mathbf{r}\rho(\mathbf{r})(3r_ir_j - \delta_{ij}r^2)}$$

Monopole Dipole Quadrupole

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Strategy (assuming that we don't even know Poisson's law!):

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3. Fixing constants: measure $\Phi(\mathbf{R})$ at several locations ($|\mathbf{R}| \gg a$), determine q, P_i, Q_{ij} and make predictions for other locations

Chiral effective field theory

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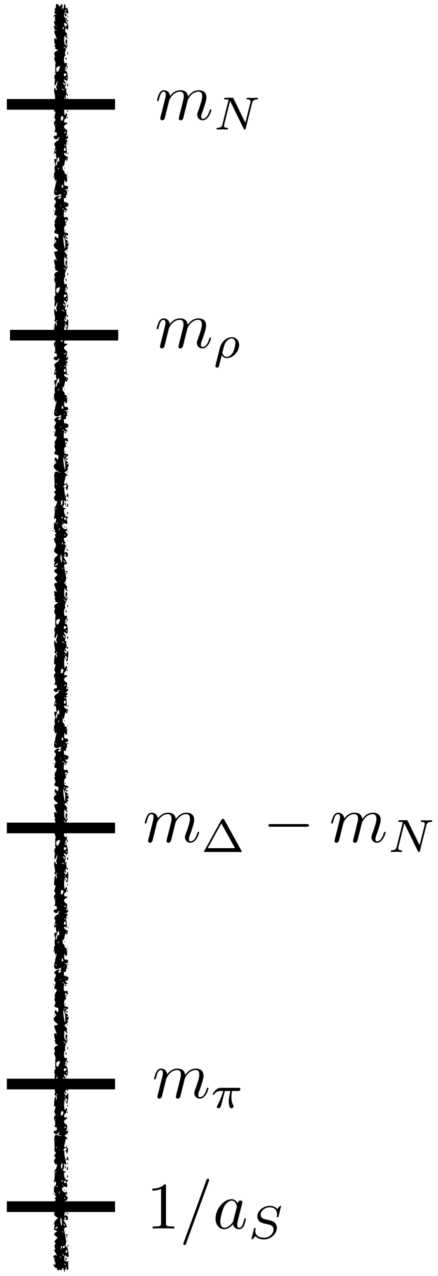
Energy scales in nuclear physics:



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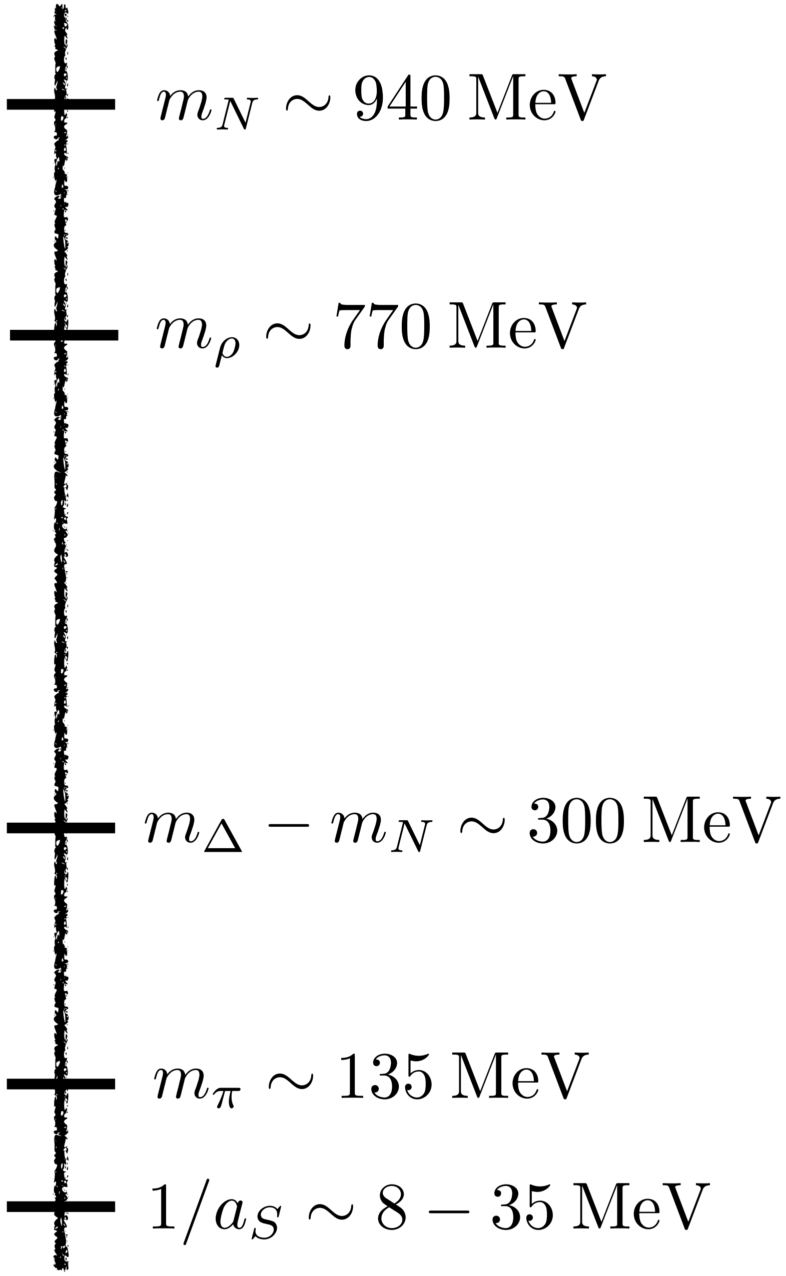

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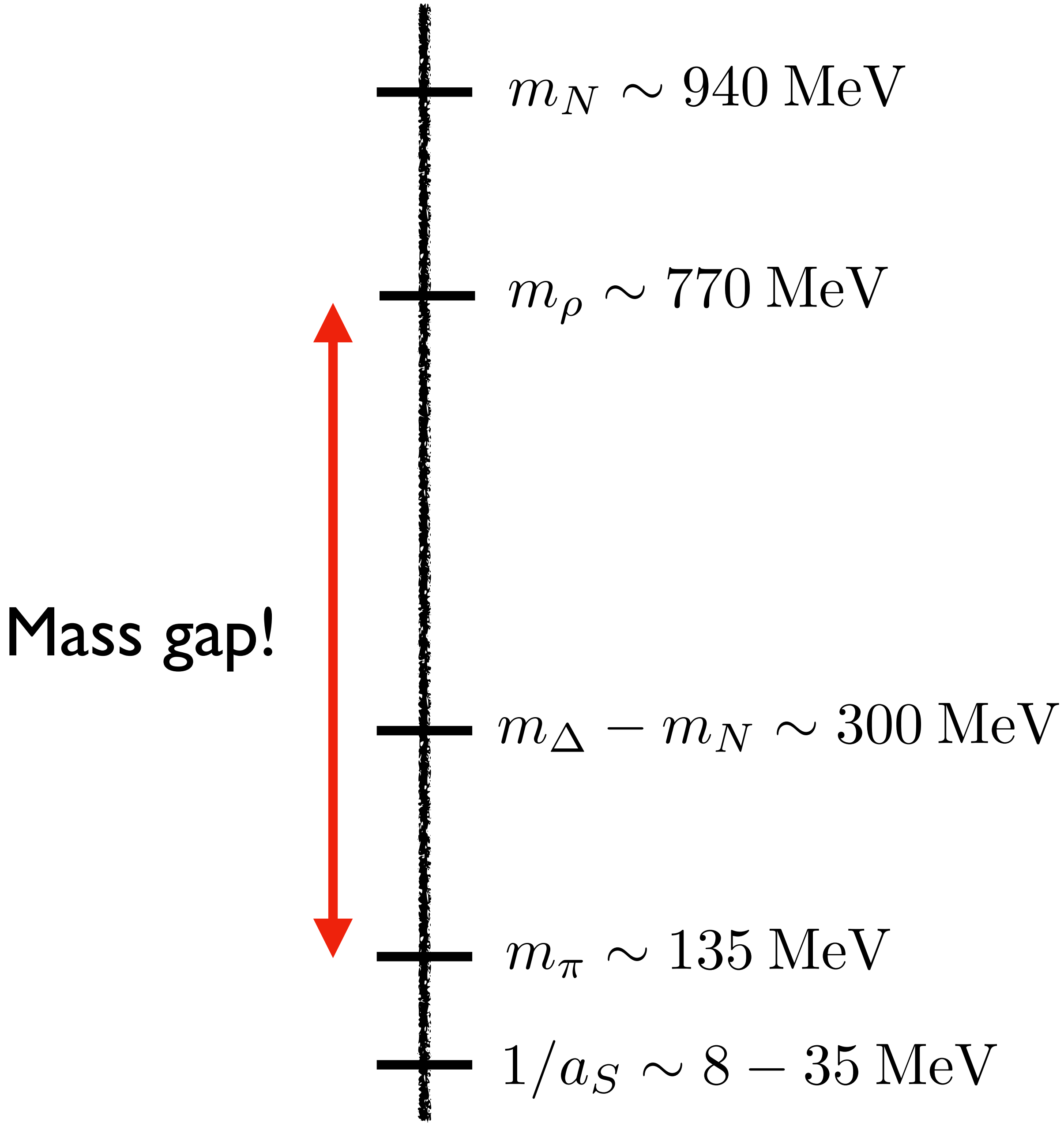

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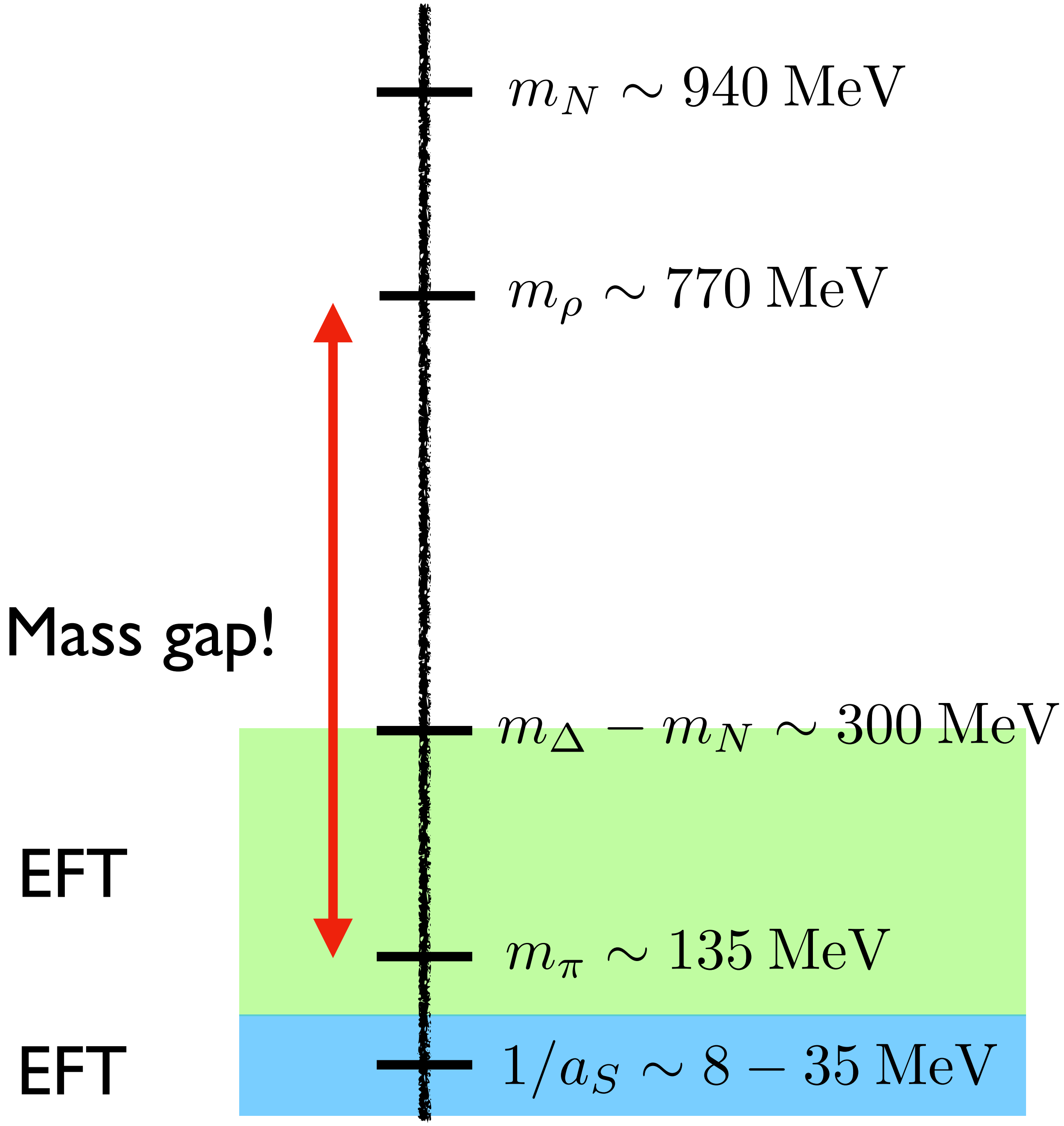

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Basic idea: utilize separation of energy scales!

$$\frac{Q}{\Lambda}$$

$\longleftarrow$ typical momenta of nucleons

$\longleftarrow$ breakdown scale of EFT (~500 MeV)

(compare multipole expansion in $\frac{a}{R}$)

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3. Fixing of low-energy constants: perform calculations at a given order in expansion, determine constants by matching to experimental data (e.g. NN scattering observables) and make predictions for other observables, in principle constants can be computed from QCD (cf. multipole expansion)

Symmetries of QCD - chiral symmetry

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{q}_f(i\gamma^\mu\partial_\mu - m_f)q_f + g\bar{q}\gamma^\mu T_a q A^a_\mu$$
$$\equiv -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{q}_f(i\gamma^\mu D_\mu - m_f)q_f$$

- f: flavour index, here we consider only light flavors ($f = u, d$)
- ‘covariant derivative’ (independent of flavor):

$$D_\mu = \partial_\mu - ig\bar{q}\gamma^\mu T_a q A^a_\mu$$

- introduce mass matrix (in flavour space):

$$\mathcal{M} = \mathrm{diag}(m_u, m_d)$$

Decompose quark fields (for each flavor) into left- and right-handed **chiral** components via:

$$q_L = P_L q,\ q_R = P_R q,\ P_{L/R} = \frac{1}{2}(1 \mp \gamma_5)\,,\ P_L + P_R = 1$$

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- for $\mathcal{M} = 0$ $\mathcal{L}_{QCD}$ is invariant under independent global rotations in flavour space ($SU(2)_L \times SU(2)_R$ symmetry):

$$q_L = \begin{pmatrix} q_{L,u} \\ q_{L,d} \end{pmatrix} \rightarrow q'_L = \exp\left(-i\boldsymbol{\theta}_L \cdot \boldsymbol{\tau}/2\right) q_L$$

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quark number

conservation

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quark number conservation	broken by anomaly

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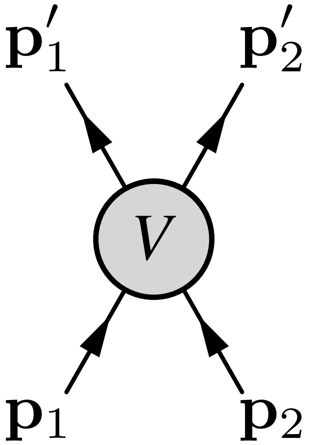
quark number	broken by	**spontaneously** broken down to subgroup $SU(2)_V$
conservation	anomaly	**explicitly** broken by finite $\mathcal{M}$

Regularization schemes for nuclear interactions (here: NN)

Separation of long- and short-range physics



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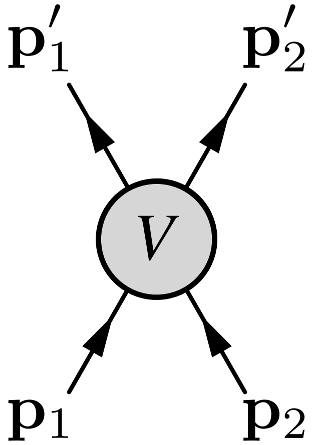

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Epelbaum, Glöckle, Meissner, NPA 747, 362 (2005)
Entem, Machleidt, PRC 68, 041001 (2003)

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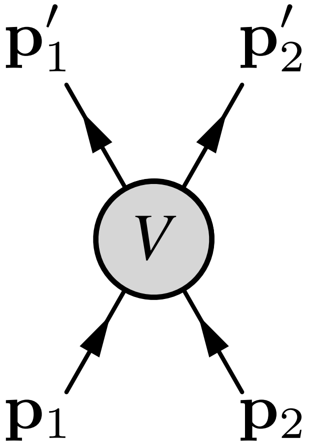

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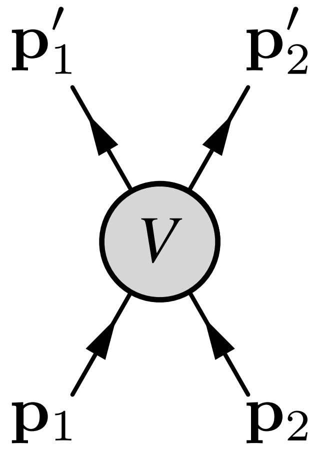
local
(momentum space)

$$V_{\mathrm{NN}}(\mathbf{q}) \to \exp\left[-\left(q^2/\Lambda^2\right)^n\right] V_{\mathrm{NN}}(\mathbf{q})$$

cf. Navratil, Few-body Systems 41, 117 (2007)

Regularization schemes for nuclear interactions (here: NN)

Separation of long- and short-range physics



$$\mathbf{p} = (\mathbf{p}_1 - \mathbf{p}_2)/2$$
$$\mathbf{p}' = (\mathbf{p}_1' - \mathbf{p}_2')/2$$
$$\mathbf{q} = (\mathbf{p}_1 - \mathbf{p}_1')$$

nonlocal

$$V_{\rm NN}(\mathbf{p},\mathbf{p}') \to \exp\left[-\left((p^2+p'^2)/\Lambda^2\right)^n\right] V_{\rm NN}(\mathbf{p},\mathbf{p}')$$

Epelbaum, Glöckle, Meissner, NPA 747, 362 (2005)
Entem, Machleidt, PRC 68, 041001 (2003)

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local
(coordinate space)

$$V_{\rm NN}^{\pi}(\mathbf{r}) \to \left(1 - \exp\left[-\left(r^2/R^2\right)^n\right]\right) V_{\rm NN}^{\pi}(\mathbf{r})$$
$$\delta(\mathbf{r}) \to \alpha_n \exp\left[-(r^2/R^2)^n\right]$$

Gezerlis et. al, PRL, 111, 032501 (2013)

Regularization schemes for nuclear interactions (here: NN)

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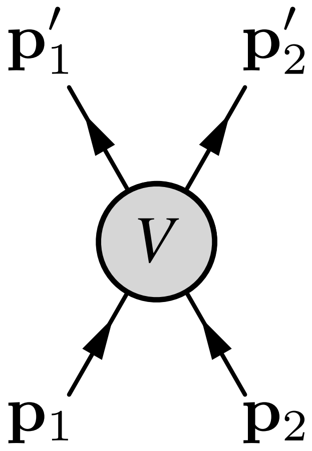

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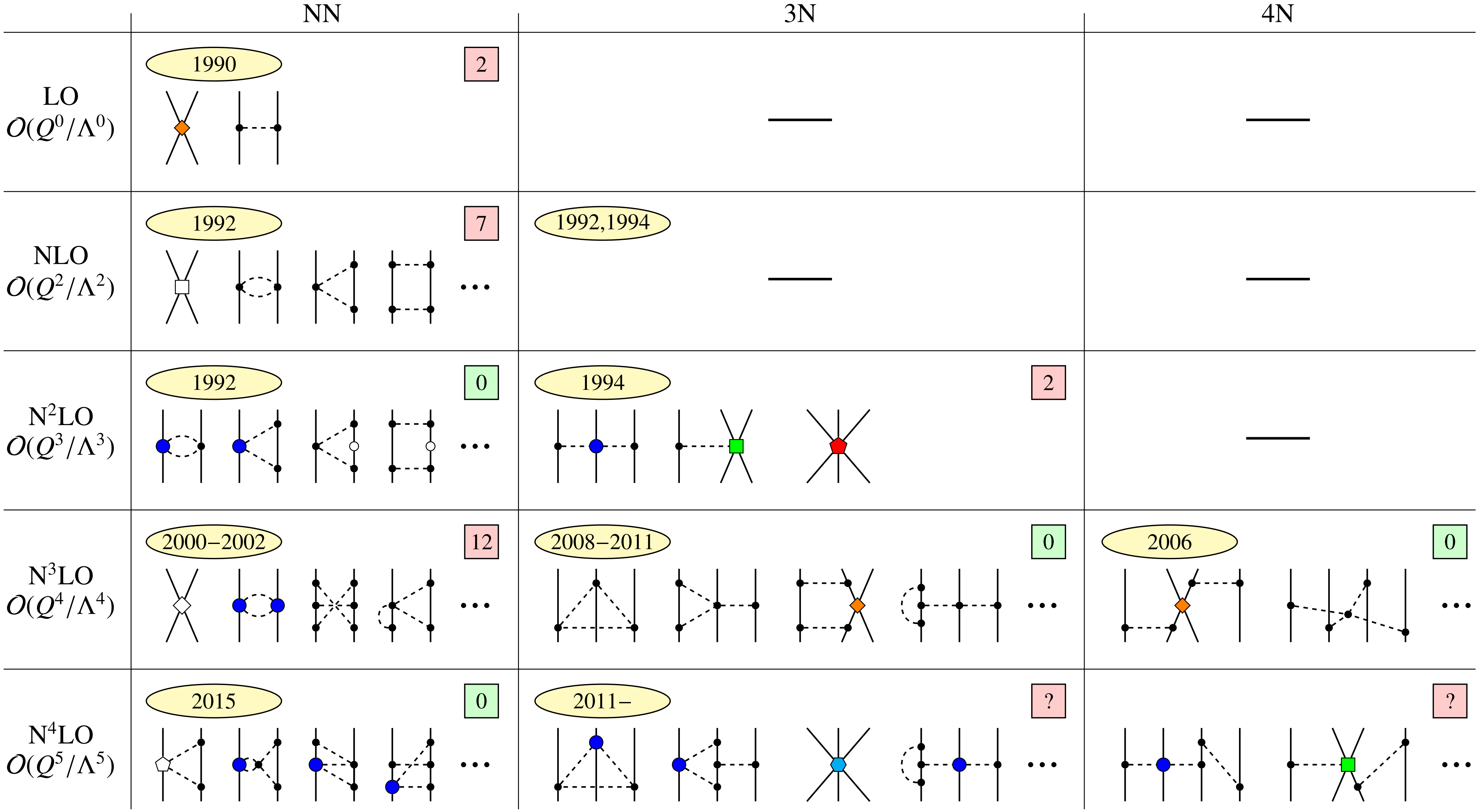
Gezerlis et. al, PRL, 111, 032501 (2013)

semi-local

$$V_{\mathrm{NN}}^{\pi}(\mathbf{r}) \to \left(1 - \exp\left[-\left(r^2/R^2\right)\right]\right)^n V_{\mathrm{NN}}^{\pi}(\mathbf{r})$$
$$\delta(\mathbf{r}) \to C \to \exp\left[-\left((p^2+p'^2)/\Lambda^2\right)^n\right] C$$

Epelbaum et. al, PRL, 115, 122301 (2015)

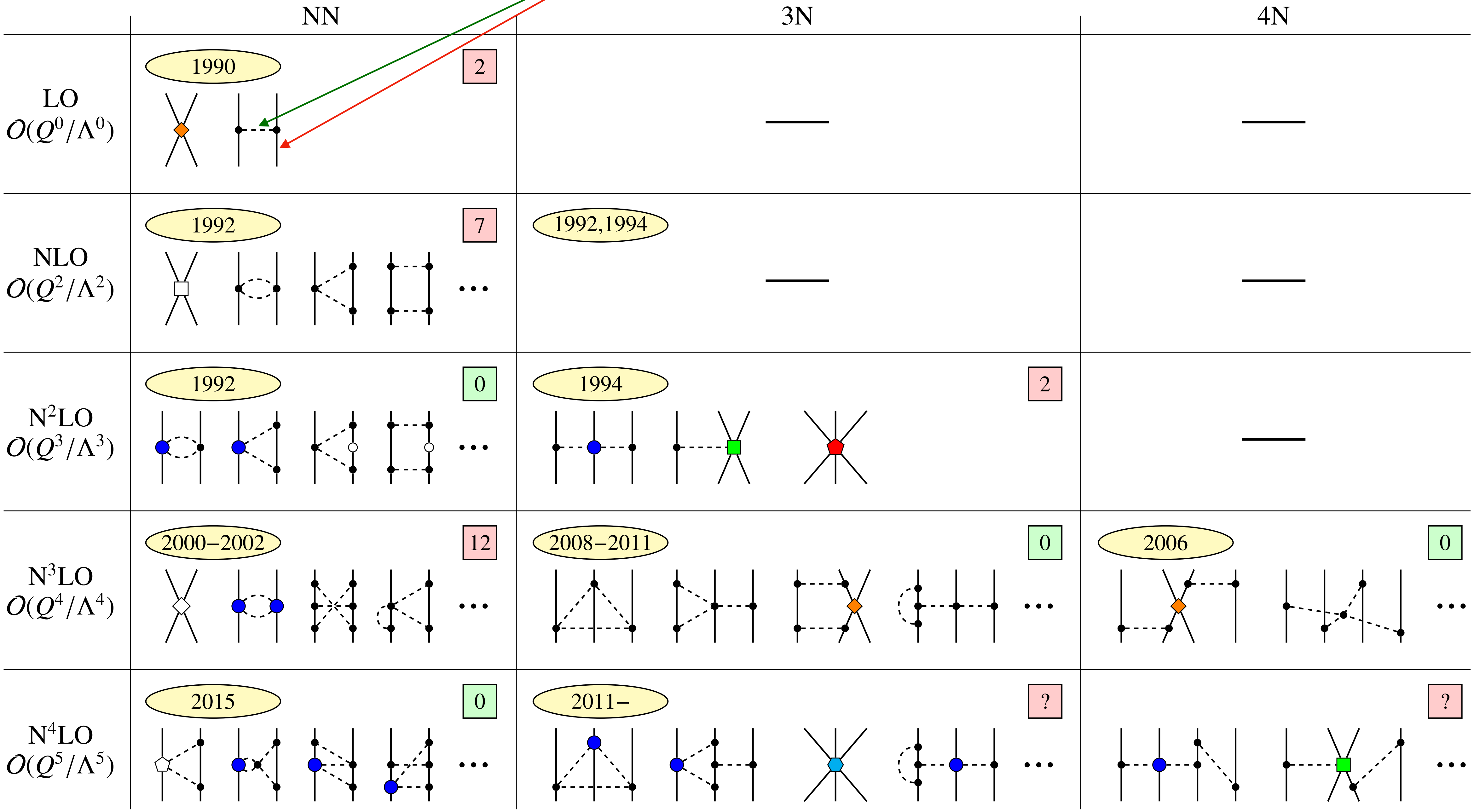
Chiral effective field theory for nuclear forces



	NN	3N	4N
LO $\mathcal{O}(Q^0/\Lambda^0)$	1990 2	—	—
NLO $\mathcal{O}(Q^2/\Lambda^2)$	1992 7 …	1992,1994 —	—
N^{2}LO $\mathcal{O}(Q^3/\Lambda^3)$	1992 0 …	1994 2	—
N^{3}LO $\mathcal{O}(Q^4/\Lambda^4)$	2000–2002 12 …	2008–2011 0 …	2006 0 …
N^{4}LO $\mathcal{O}(Q^5/\Lambda^5)$	2015 0 …	2011– ? …	? …

Chiral effective field theory for nuclear forces

degrees of freedom:
nucleons and pions



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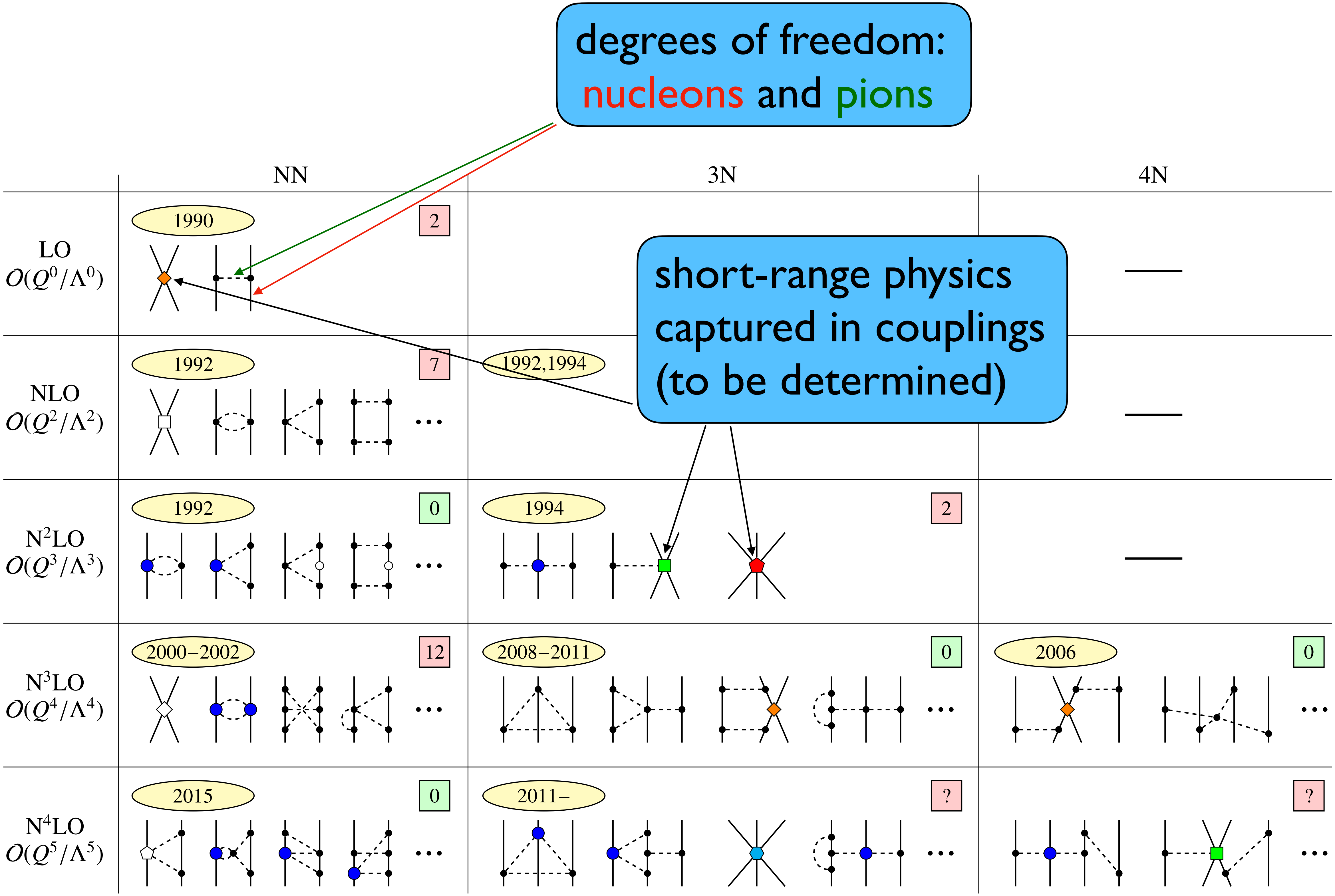

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captured in couplings
(to be determined)

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Chiral effective field theory for nuclear forces

power-counting:
expand in Q/Λ, error estimates!

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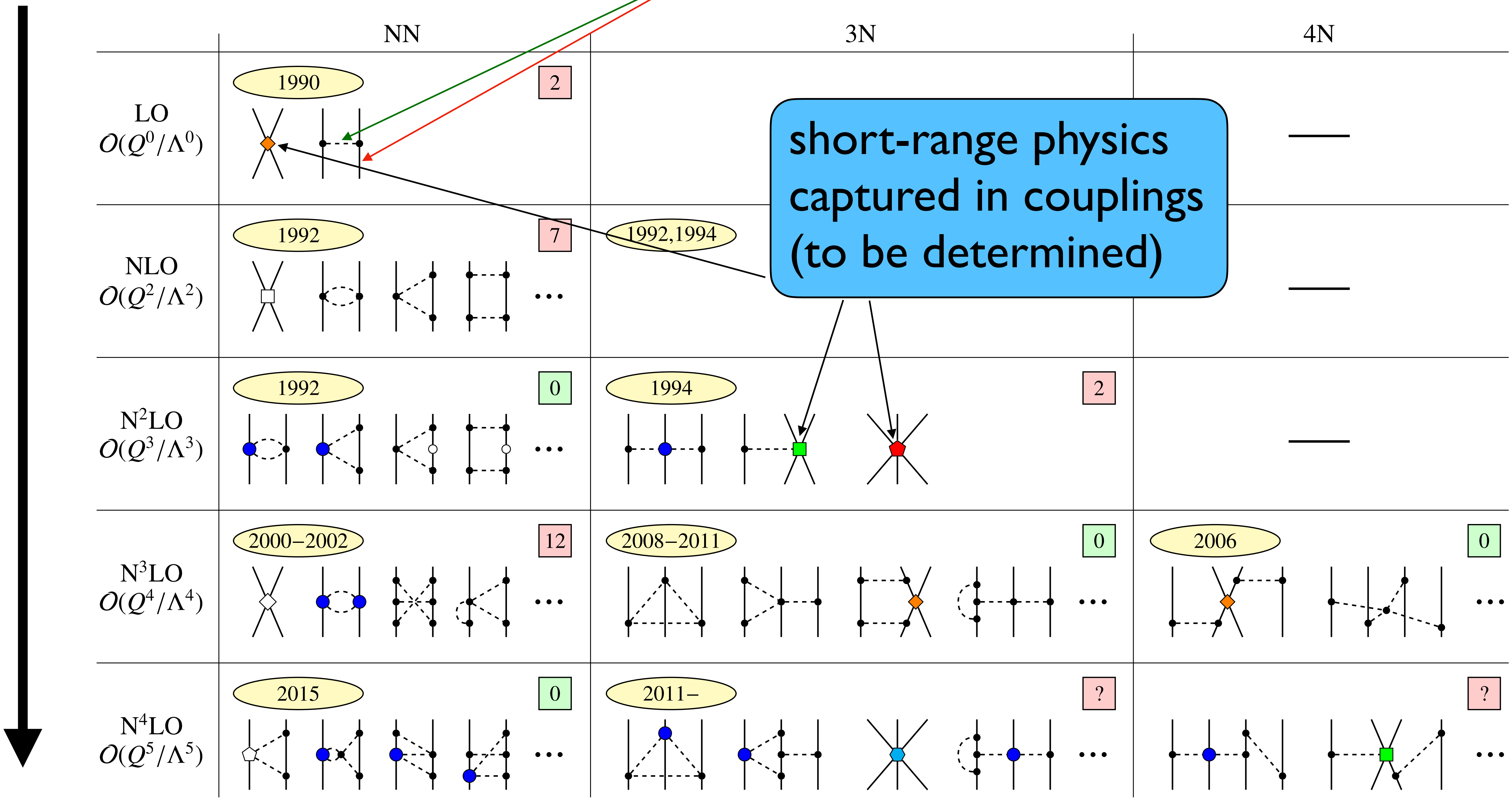

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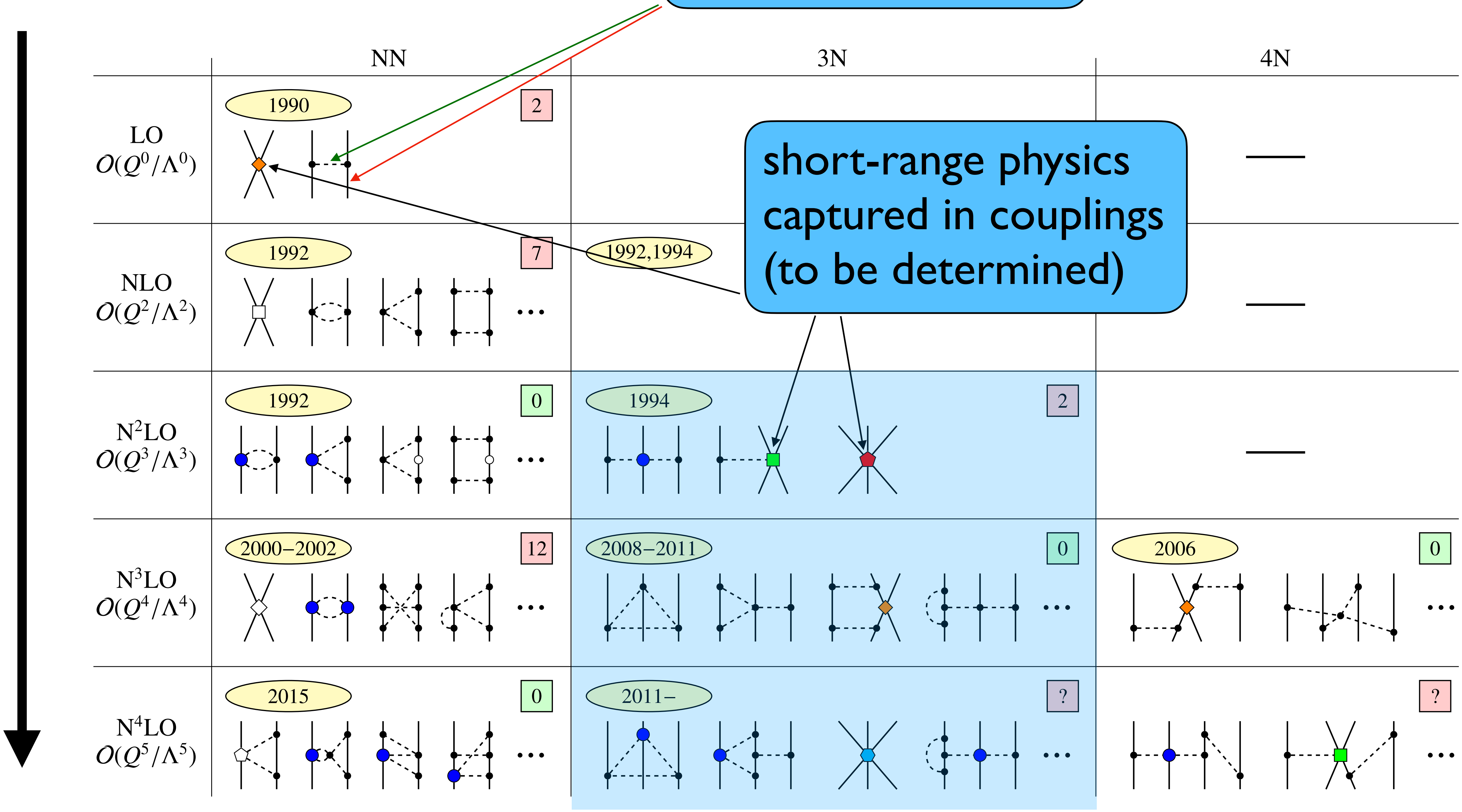

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short-range physics
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3N forces appear naturally

Chiral EFT uncertainty estimation

EFT expansion of an observable X at a momentum scale p:

$$X(p) = X_{\mathrm{ref}}(p) \sum_{n=0}^{\infty} c_n(p) Q^n$$

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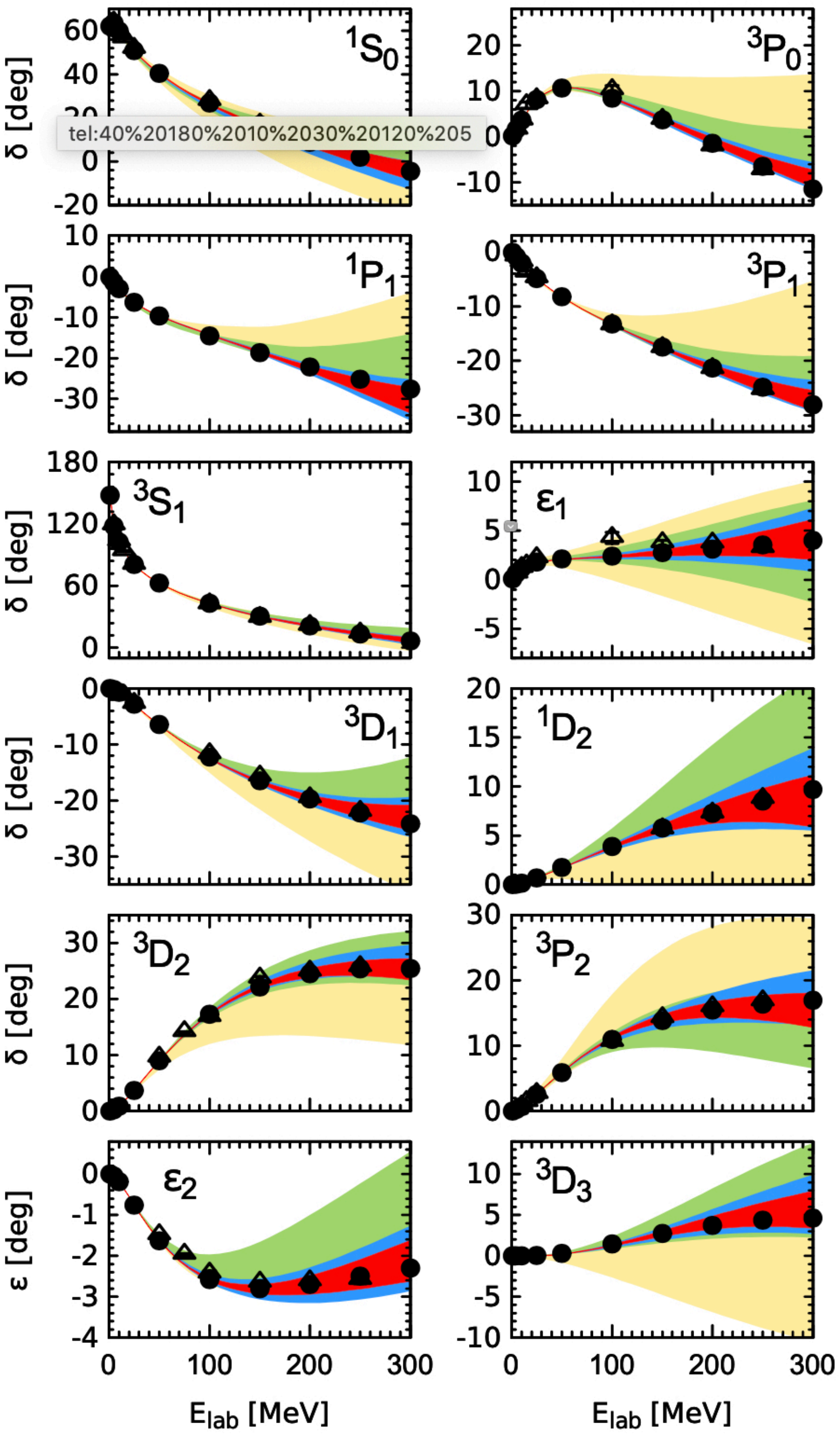
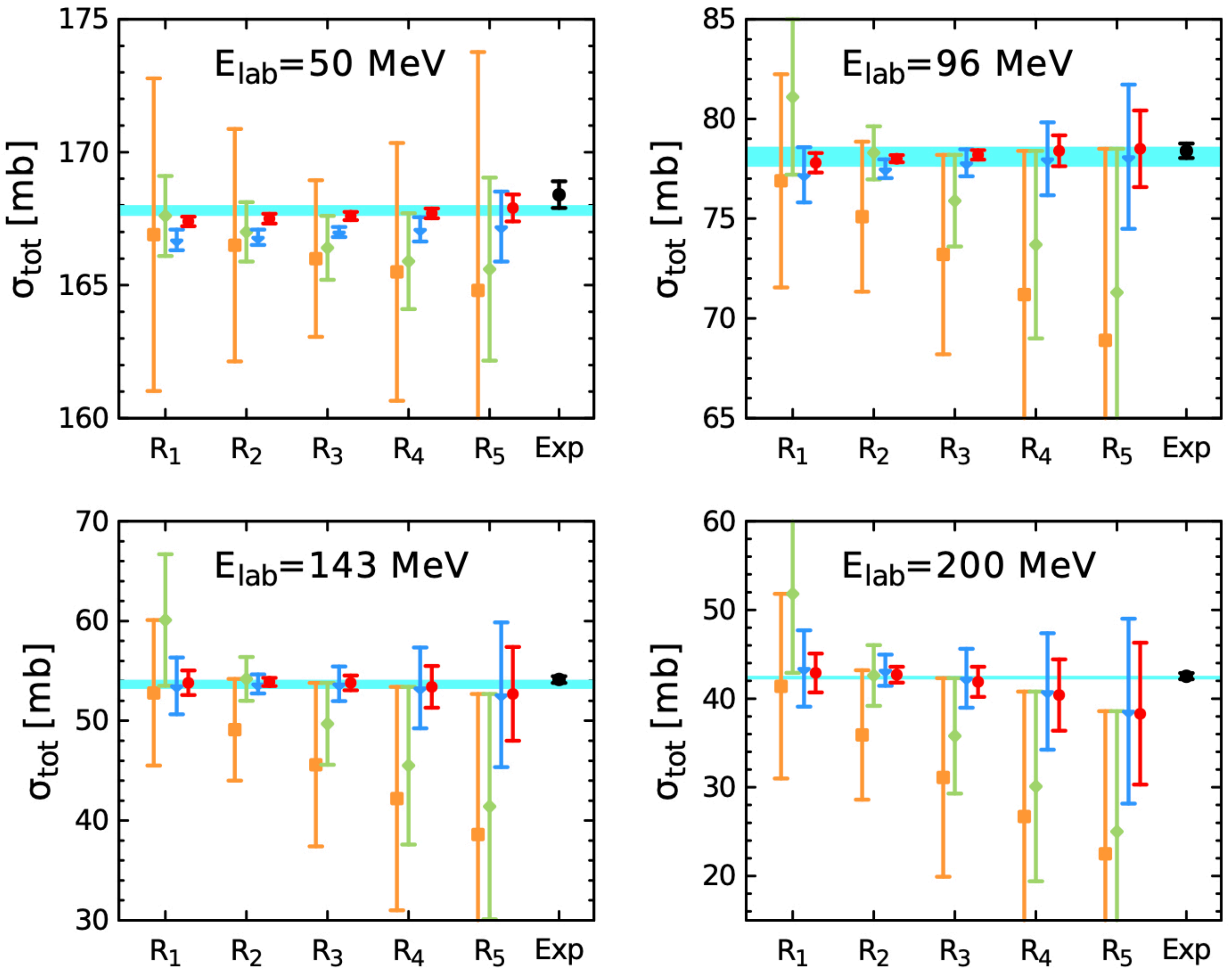
A conservative prescription:
$$\begin{aligned} \Delta X^{\mathrm{N^3LO}}(p) = \max \Big(& Q^5 \left| X^{\mathrm{LO}}(p) \right| , \\ & Q^3 \left| X^{\mathrm{LO}}(p) - X^{\mathrm{NLO}}(p) \right| , \\ & Q^2 \left| X^{\mathrm{NLO}}(p) - X^{\mathrm{N^2LO}}(p) \right| , \\ & Q \left| X^{\mathrm{N^2LO}}(p) - X^{\mathrm{N^3}LO}(p) \right| \Big) \end{aligned}$$

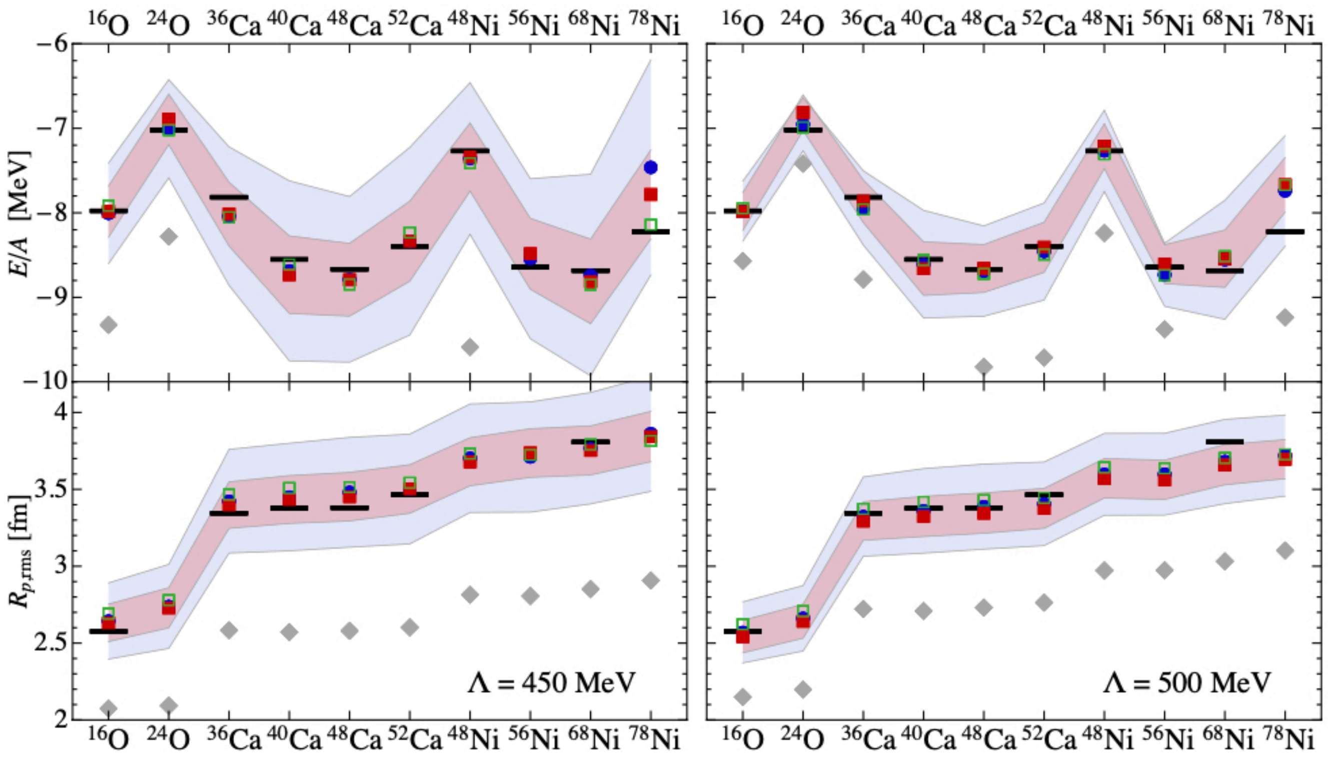
Chiral EFT uncertainty estimation: examples



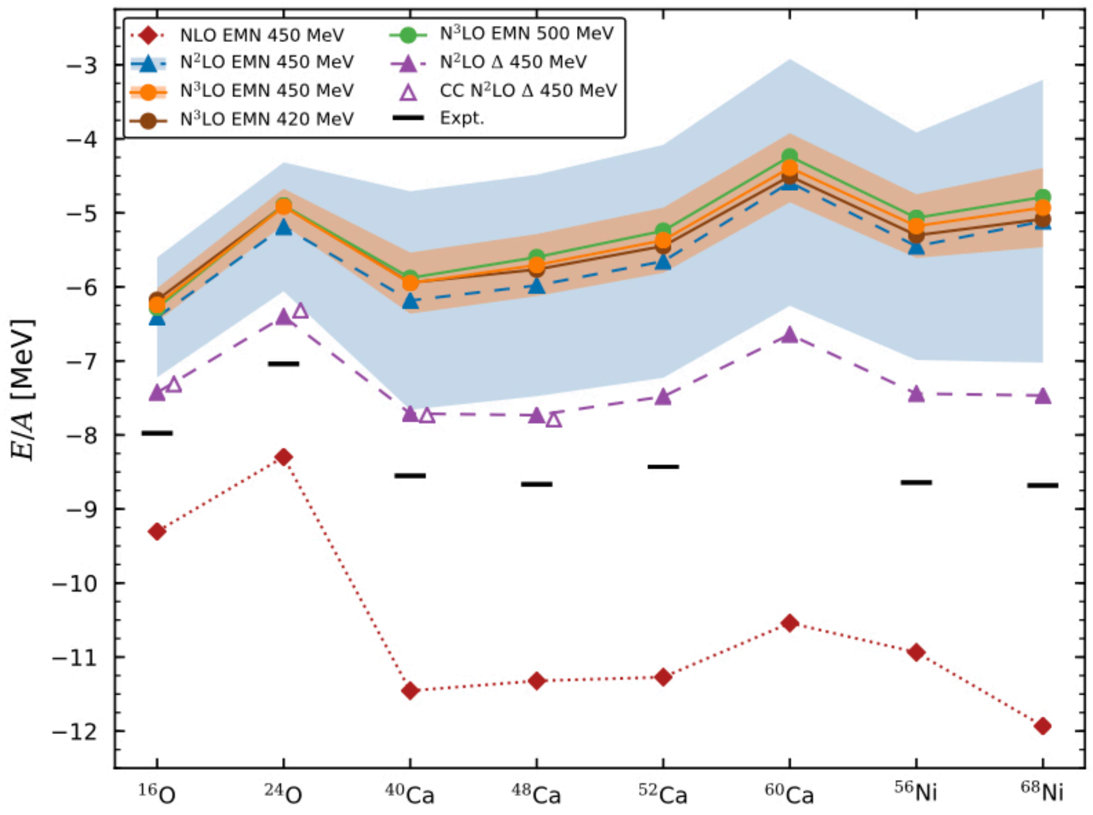


Epelbaum et al., PRL 115 (2015) 122301

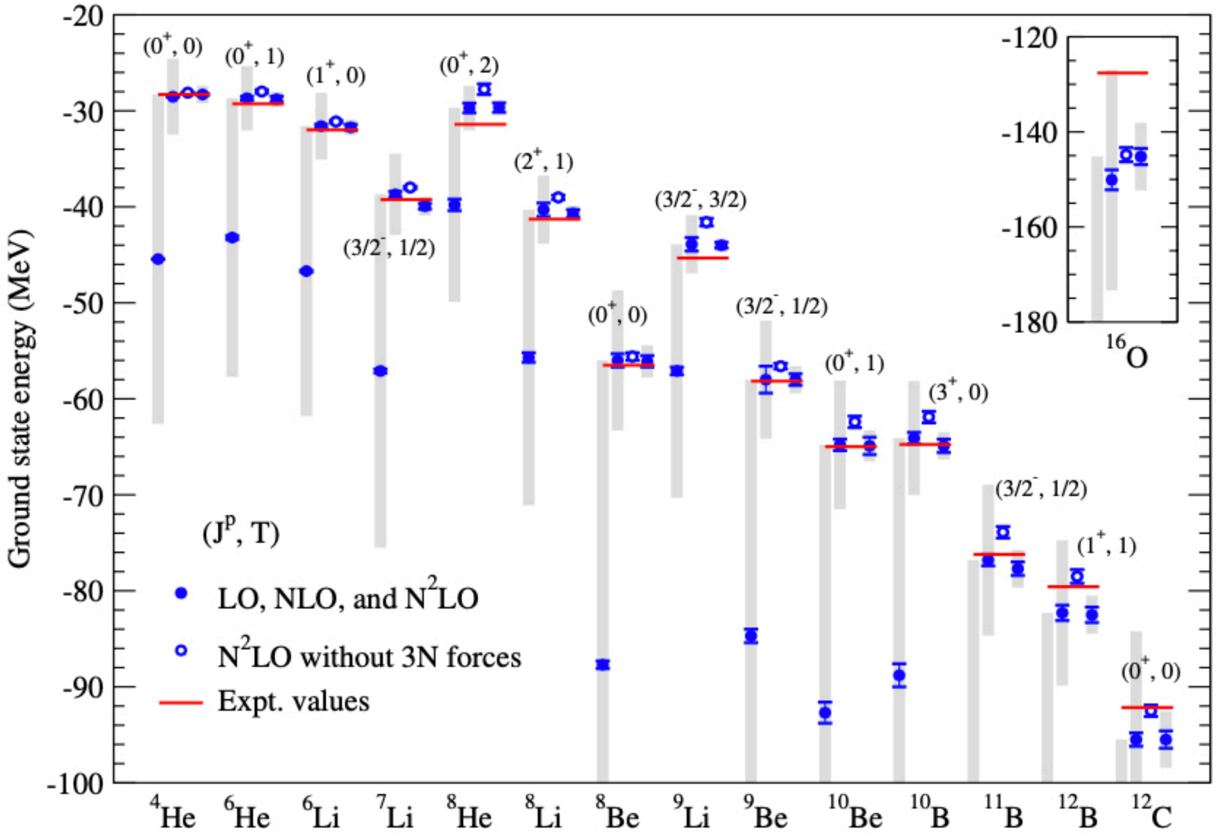
Chiral EFT uncertainty estimation: examples



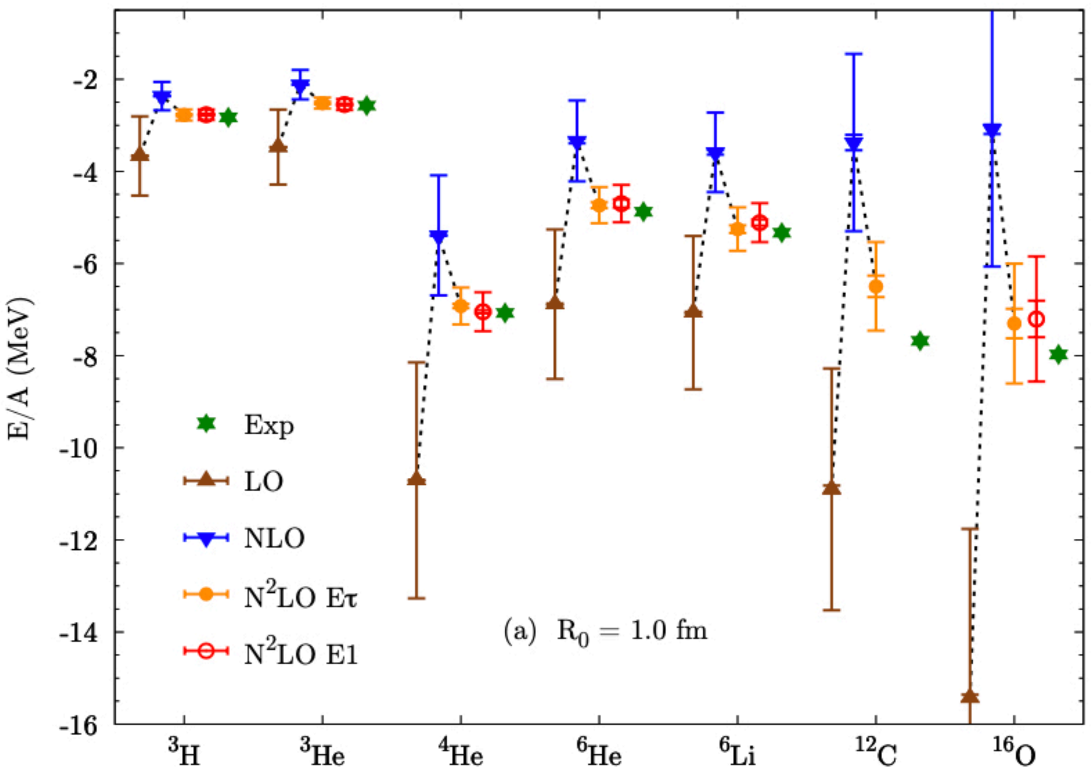
Hüther et al., PLB 808 (2020) 135651



Hoppe et al., PRC 100 (2019) 024318

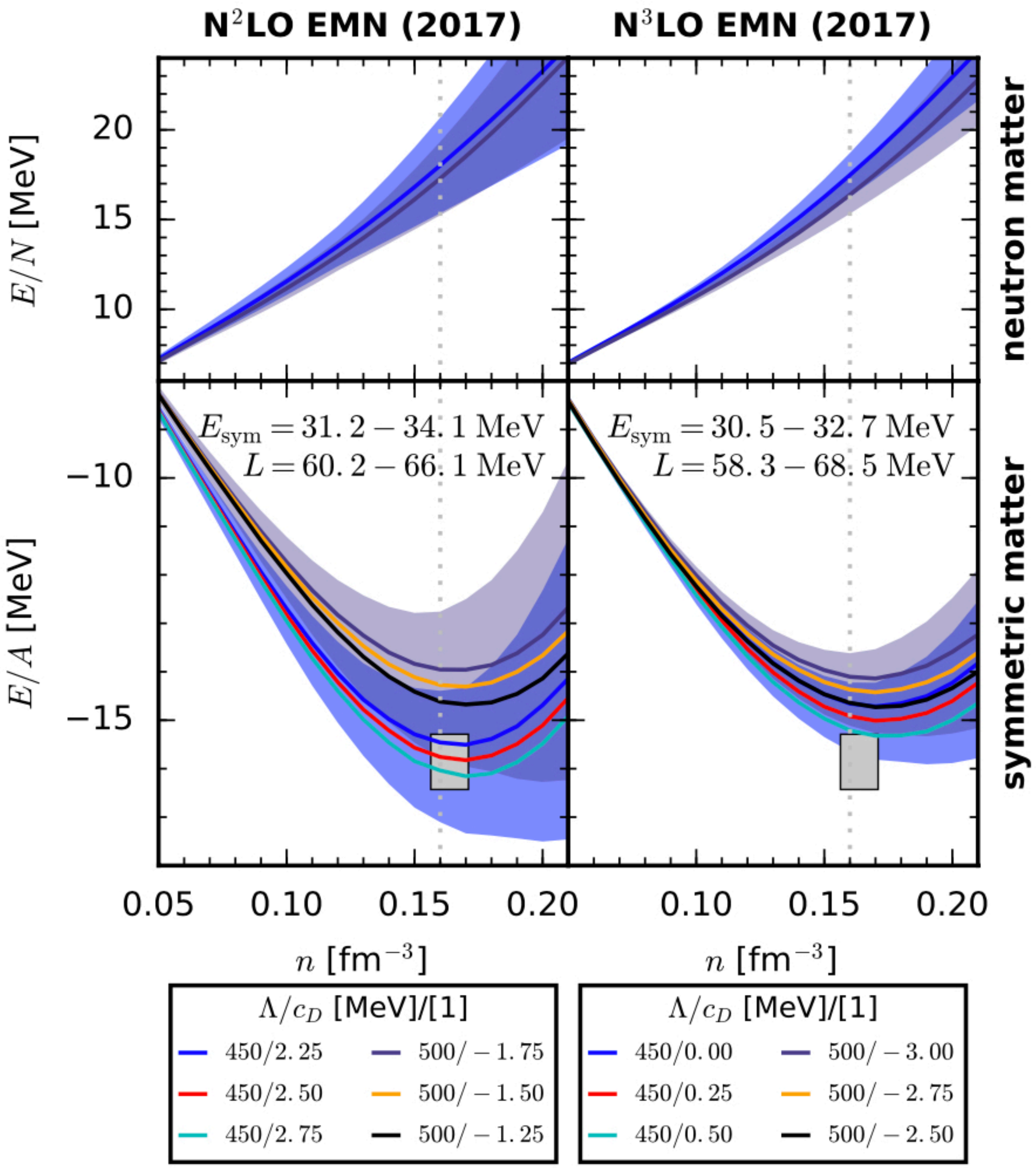


Epelbaum et al., PRC 99 (2019) 024313

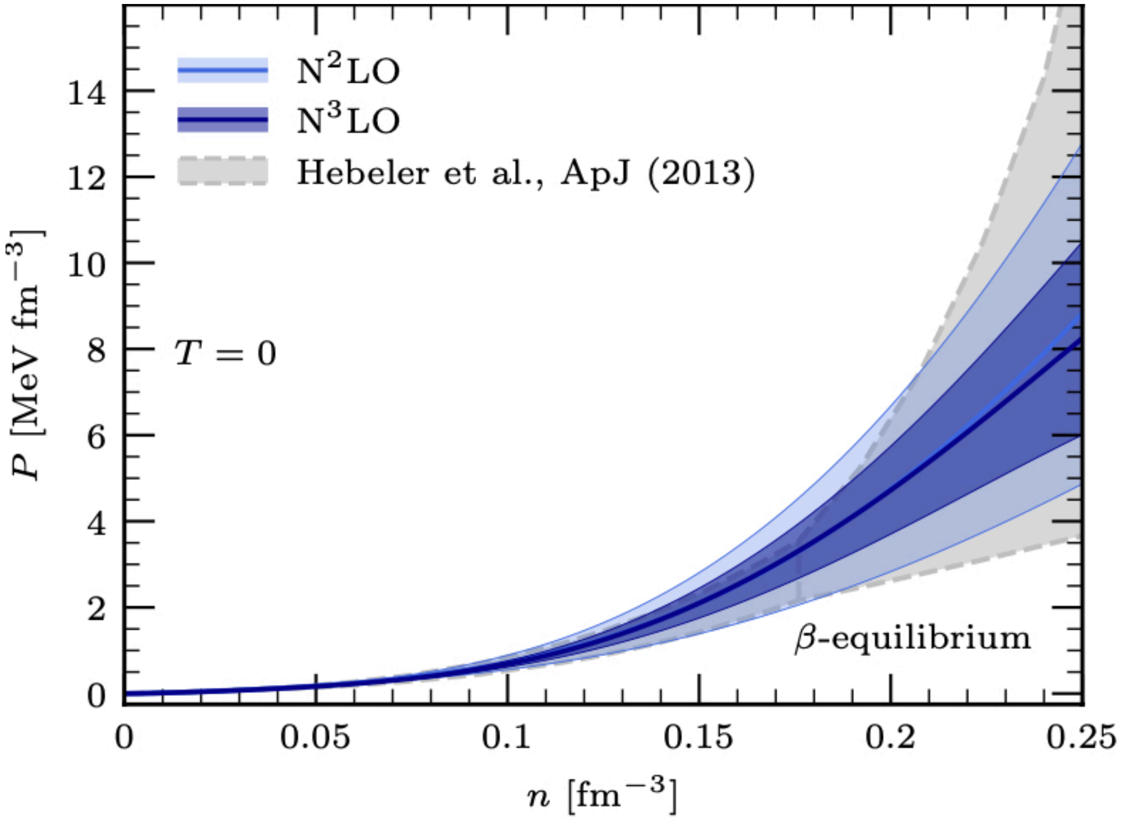


Lonardoni et al., PRC 97 (2018) 044318

Chiral EFT uncertainty estimation: examples

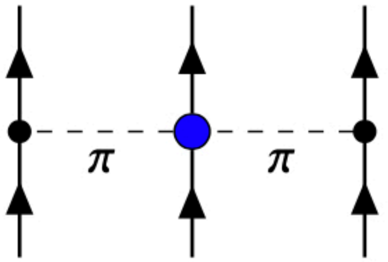


Drischler et al., PRL 122 (2019) 042501

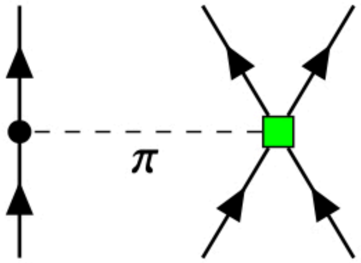


Keller et al., arXiv:2204.14016

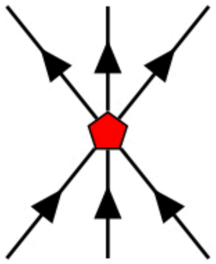
Fits of 3N LECs



$V_{3\mathrm{N}}^{2\pi}(c_1, c_3, c_4)$



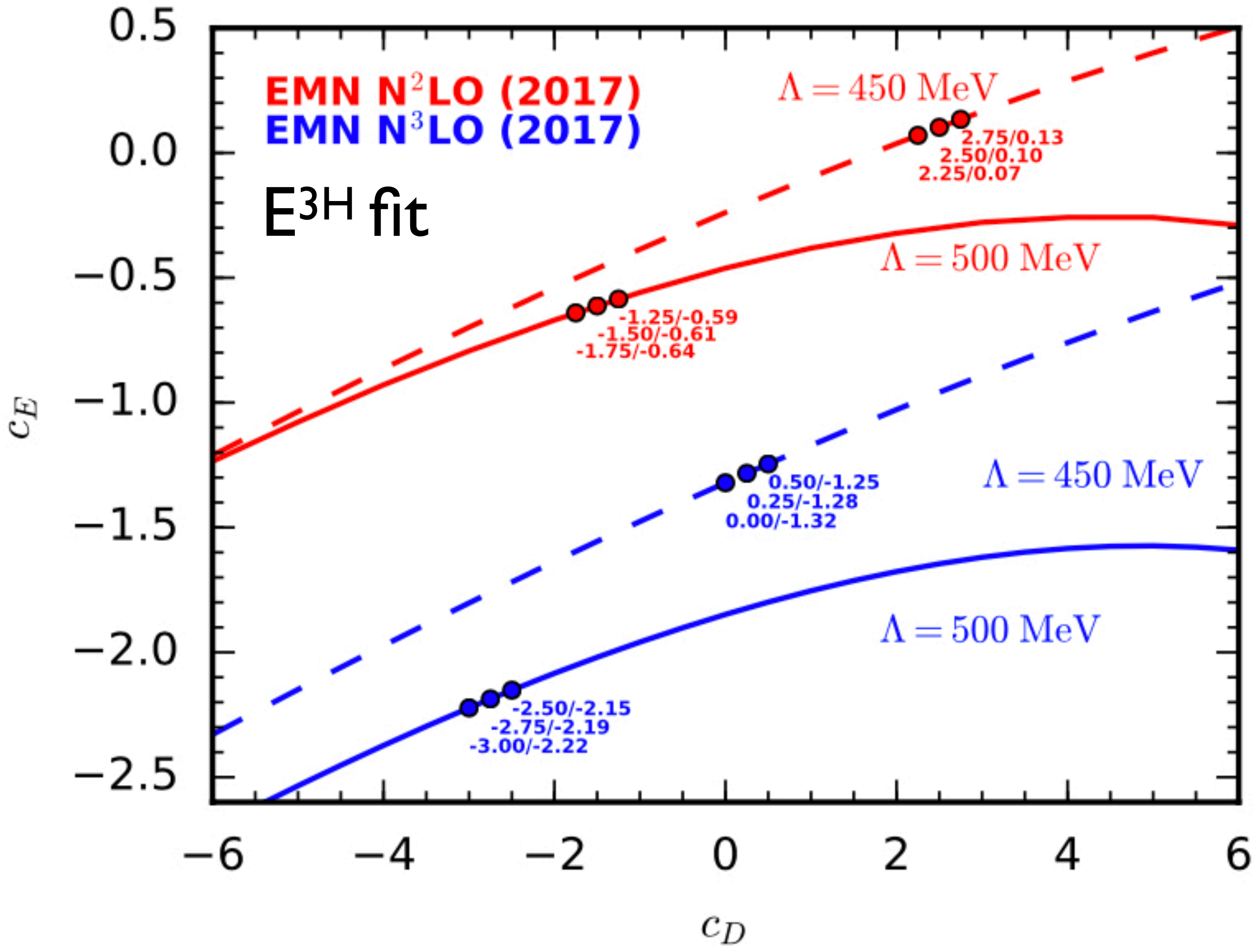
$V_{3\mathrm{N}}^{1\pi}(c_D)$



$V_{3\mathrm{N}}^{\mathrm{contact}}(c_E)$

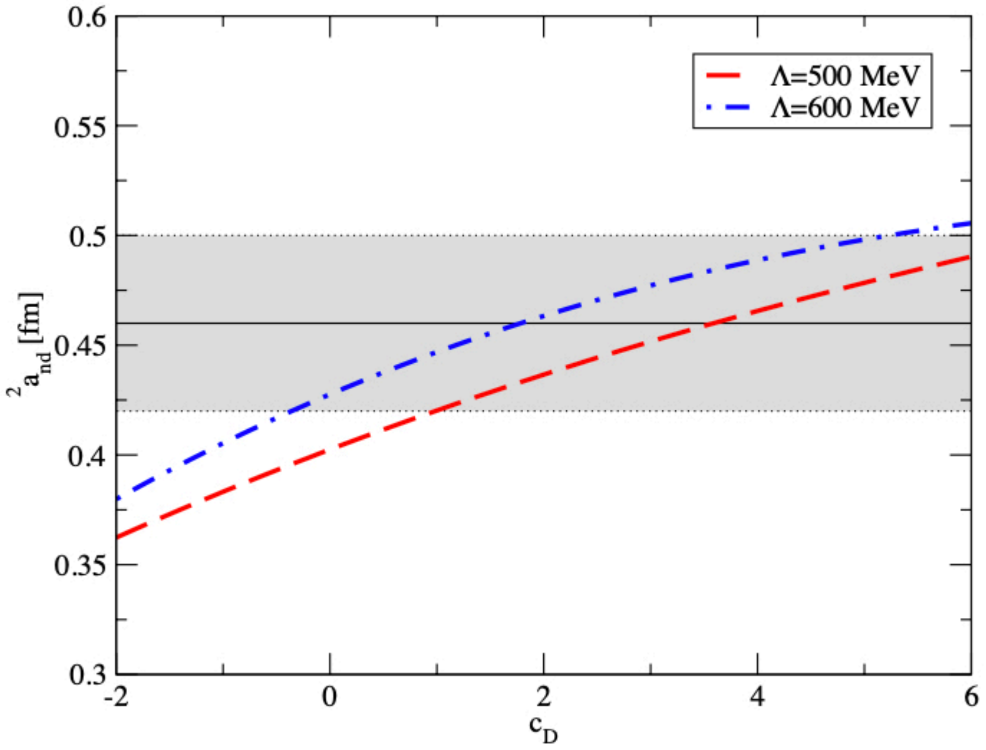
Constrained from πN scattering

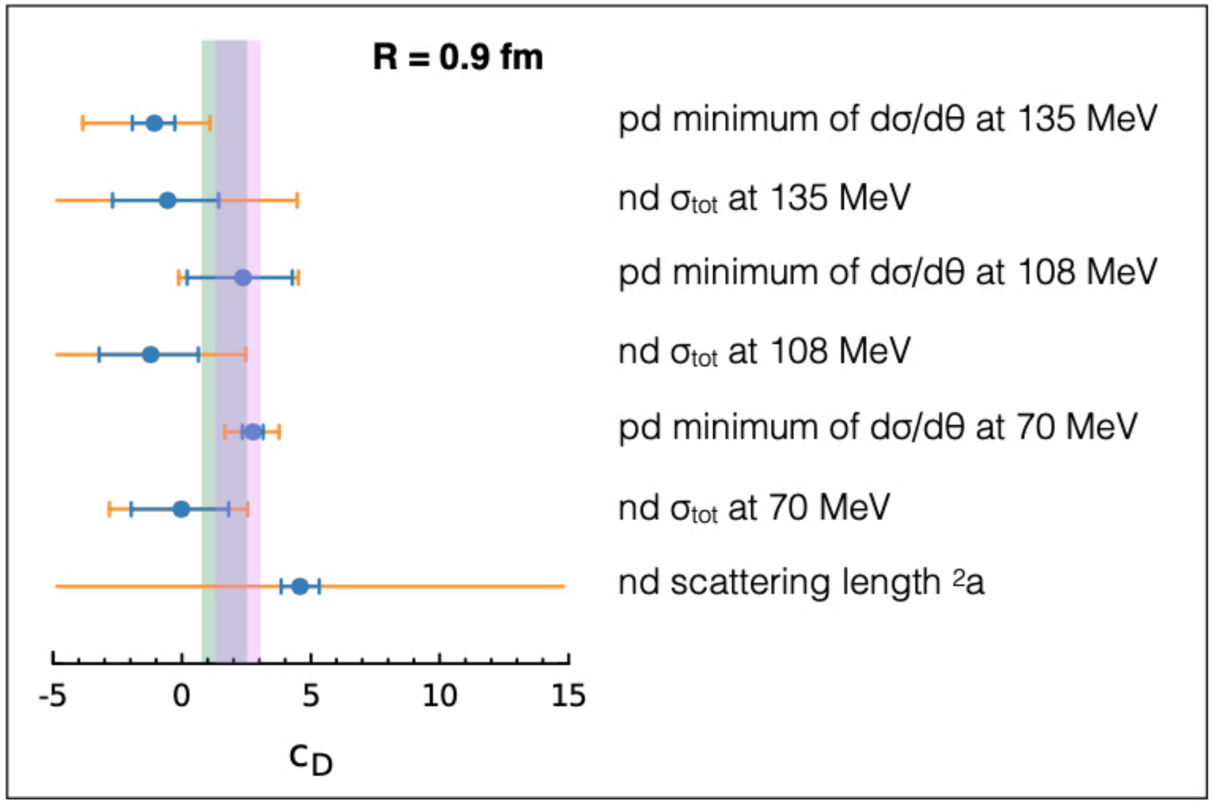
Hoferichter et al., Phys .Rept. 625 (2016) 1



Drischler et al., PRL 122 (2019) 042501

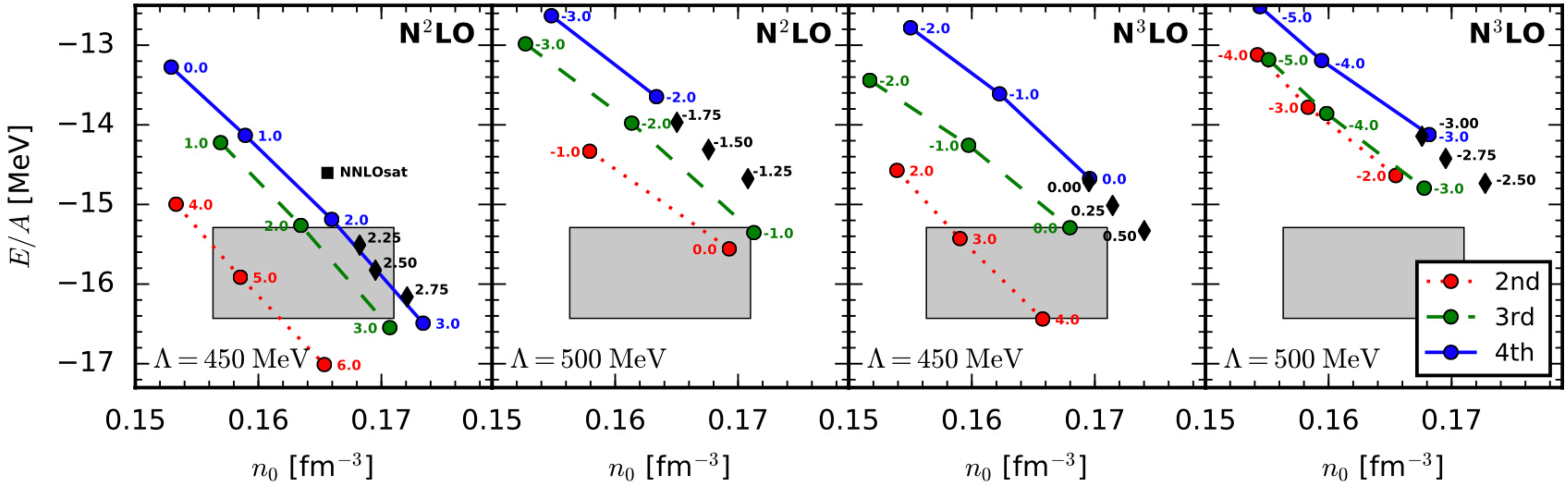
Fits of 3N LECs:
three-body scattering cross sections





- a single scattering observable not too constraining (correlated with E^{3H})
- a more global fit using several observables more robust

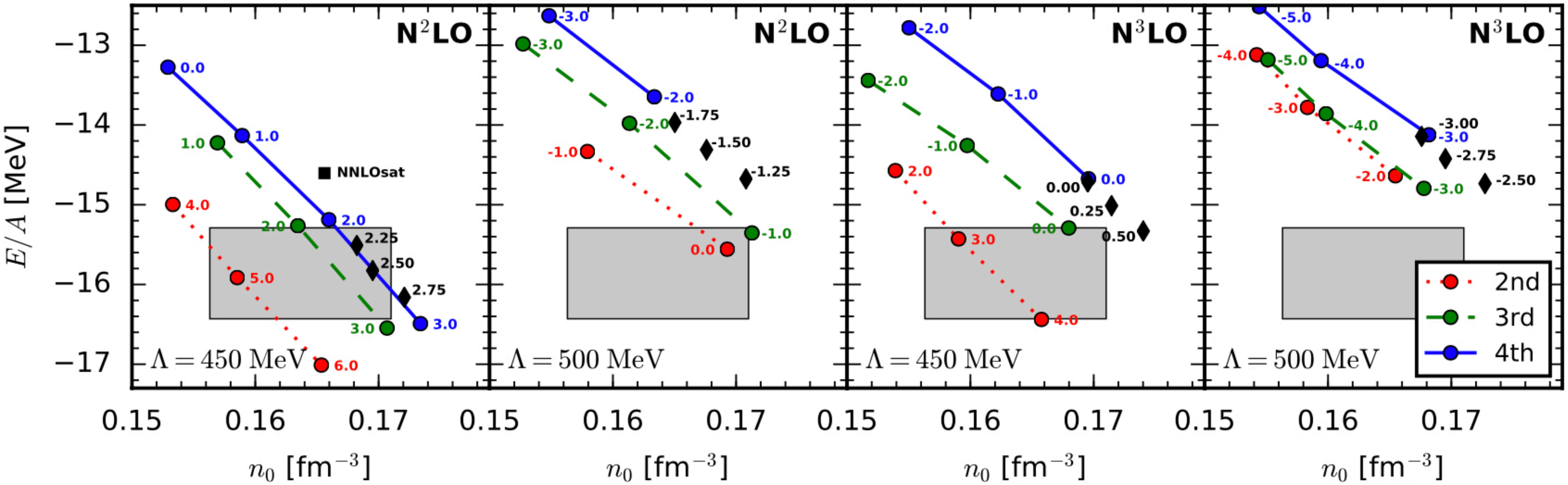
Determination of LECs: From nuclear matter saturation point



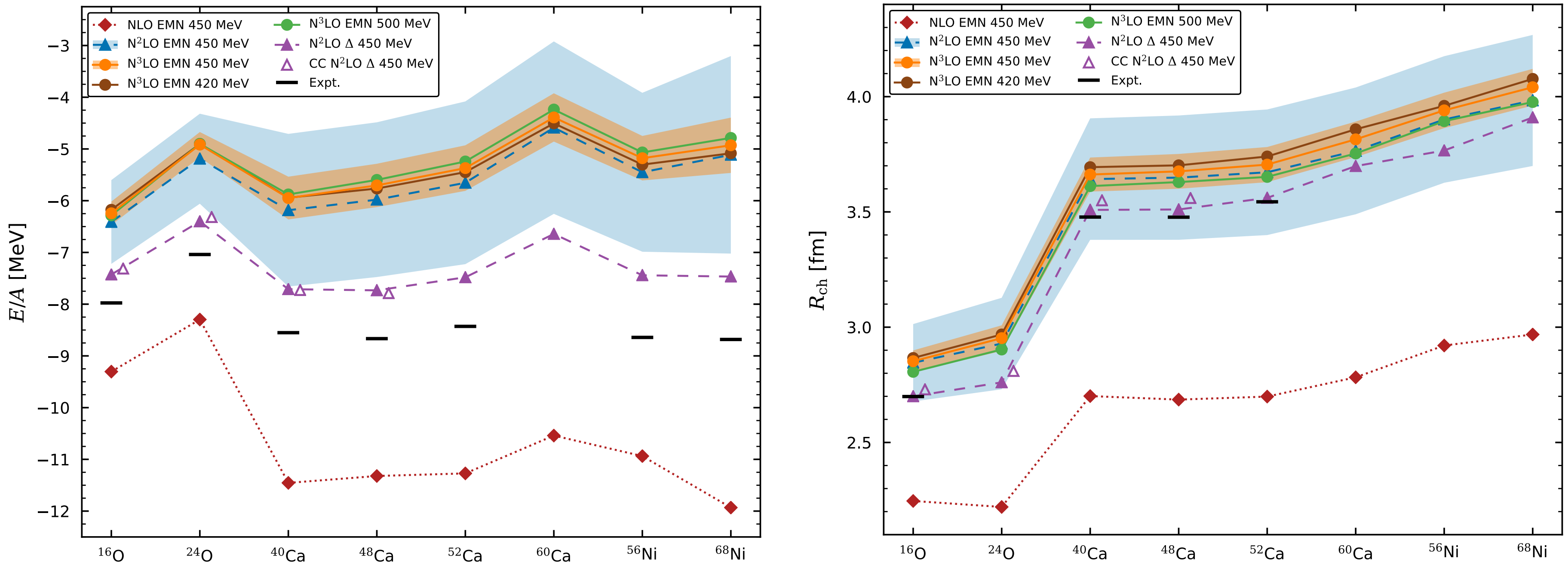
Drischler et al., PRL 122 (2019) 042501

- Use nuclear matter saturation energy and density to constrain LECs
- Reasonable reproduction of both quantities possible
- Results for medium-mass nuclei still not satisfactory

Determination of LECs:
From nuclear matter saturation point

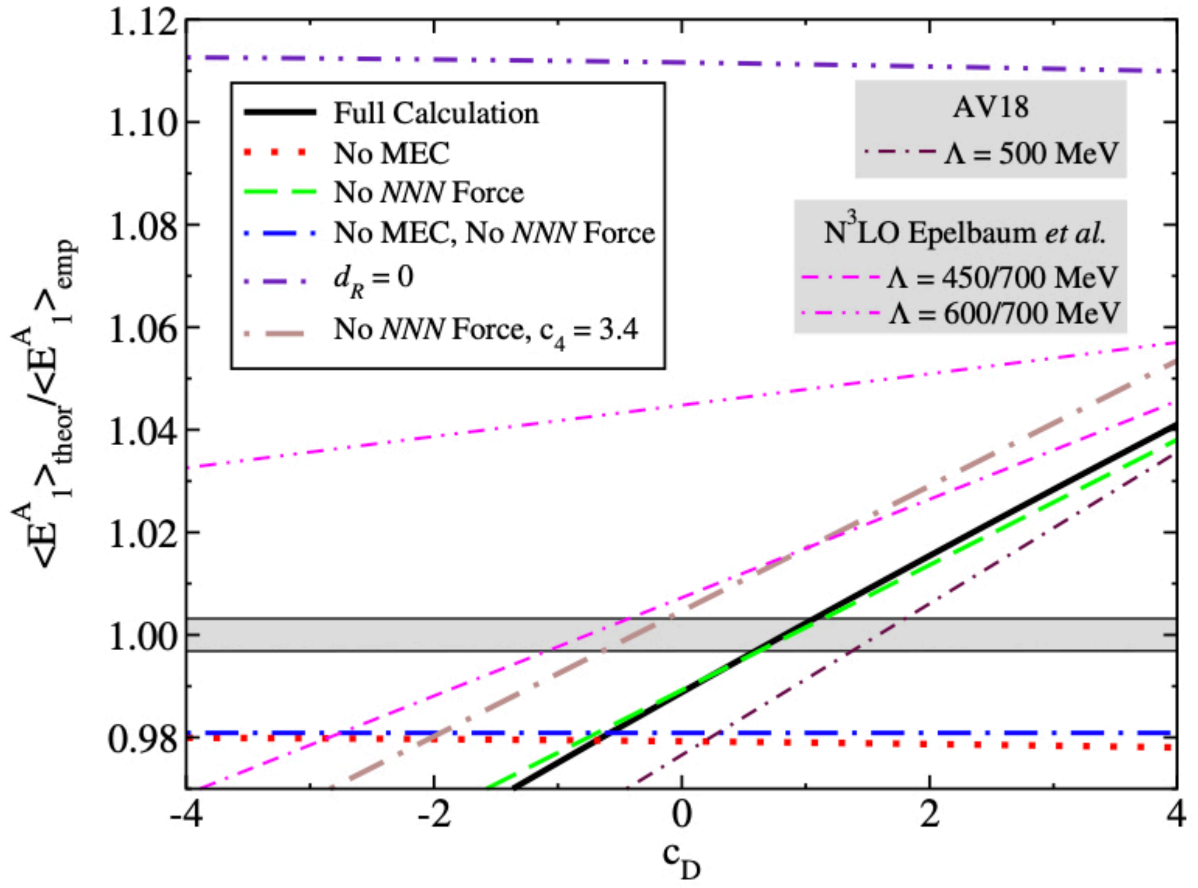


Drischler et al., PRL 122 (2019) 042501

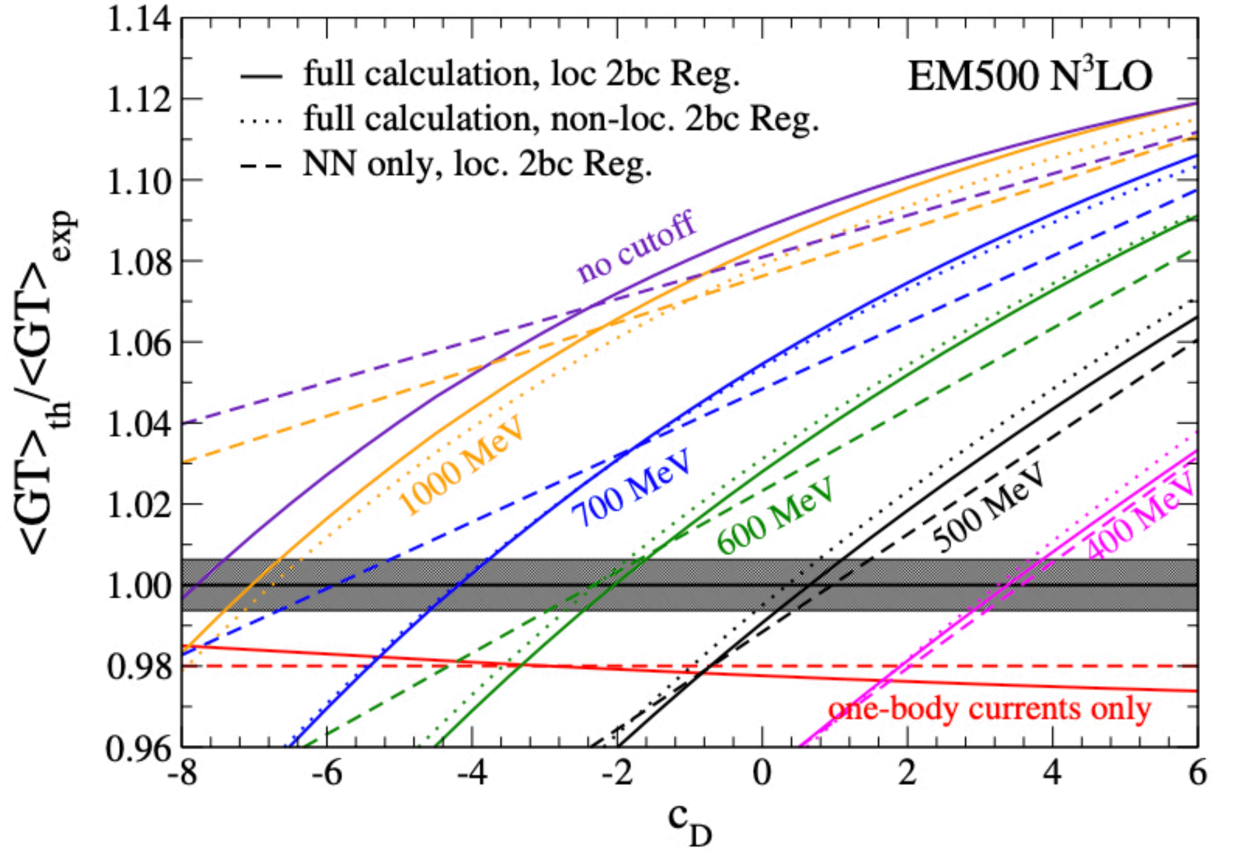


Hoppe et al.
PRC 100, 024318 (2019)

Determination of LECs: triton beta decay half life



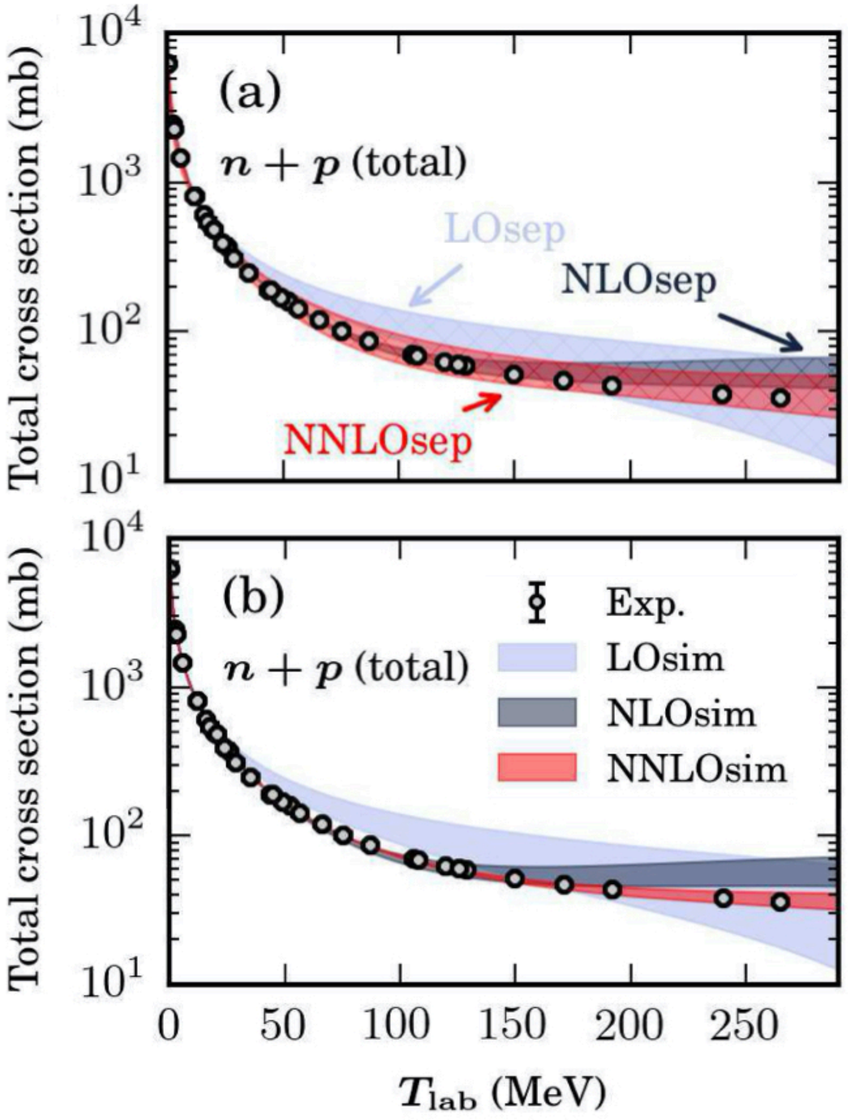
Gazit et al., PRL103 (2009) 102502

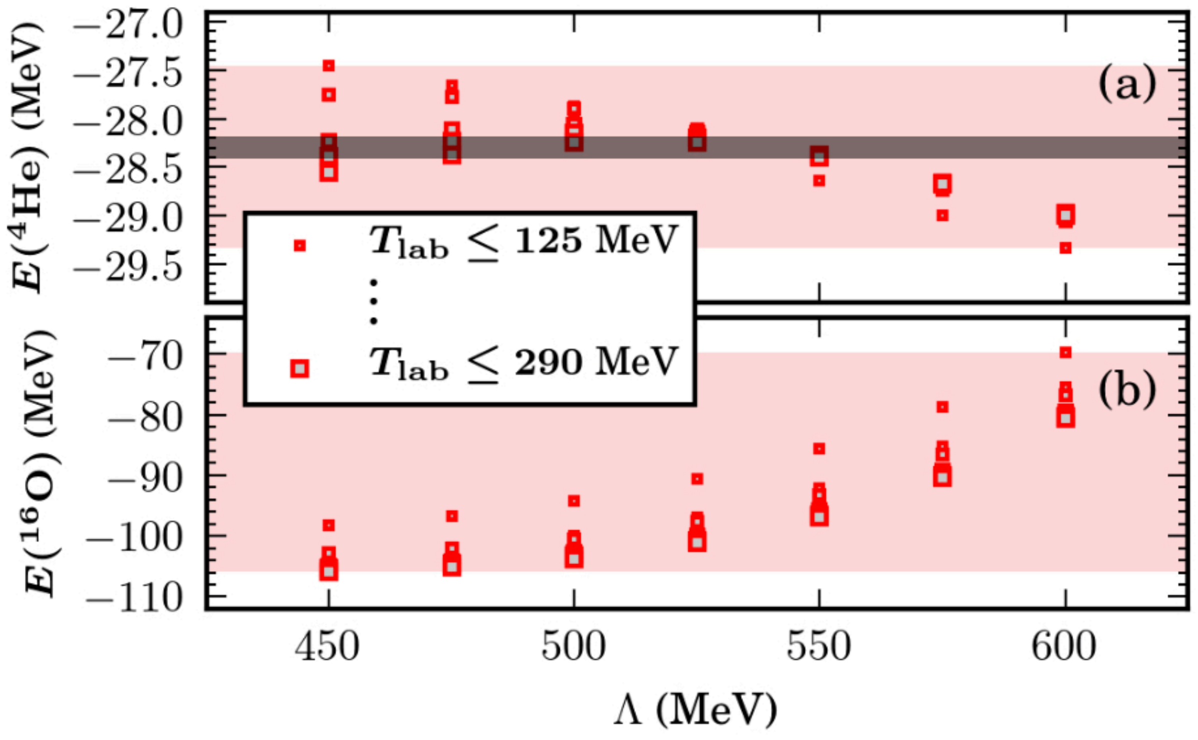


Klos et al., EPJA 53 (2017) 168

- utilize that electroweak 2b current contributions are proportional to c_D
- Triton beta decay half life much less correlated with E^{3H}
- how to choose the cutoffs consistently in currents and interaction (continuity equation?)

Determination of LECs: Simultaneous fit of NN and 3N interactions





- Automatized framework for fitting NN plus 3N interactions
- Computational challenging due to high dimension of parameter space
- Indications that simultaneous fits lead to more systematic EFT convergence
- Results for heavier systems not consistent with experimental results

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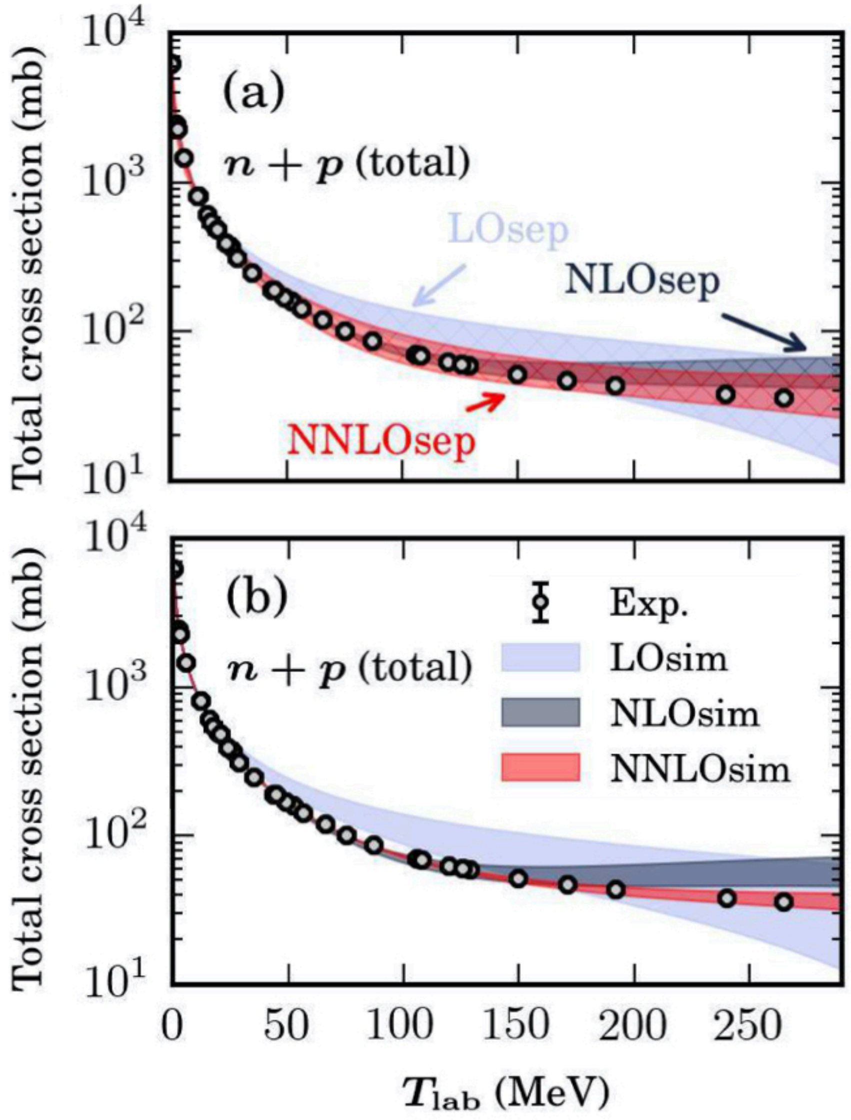
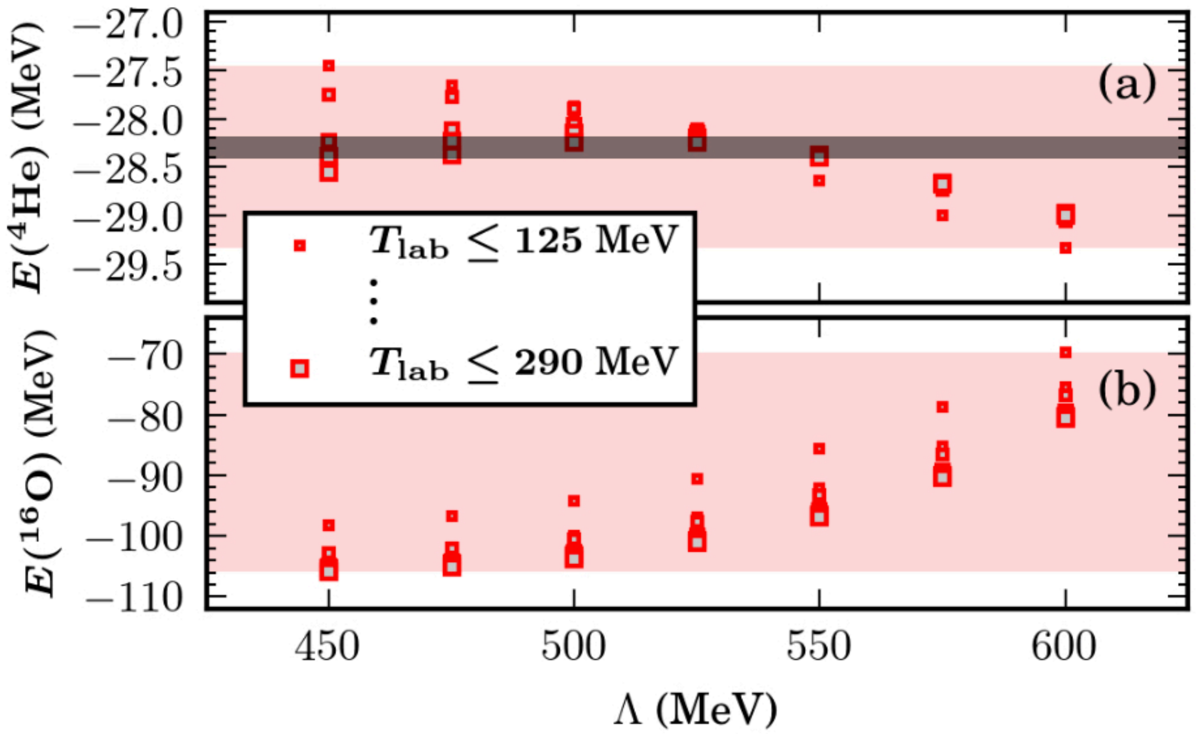


Carlsson et al.,
PRX 6, 011019 (2016)



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