# A7 Dimer formalism I

$$\mathcal{L}' = \psi^\dagger\left(i\partial_t + \frac{D^2}{2m}\right)\psi + \frac{g_2}{4} d^\dagger d - \frac{g_2}{4}\left(d^\dagger \psi^2 + (\psi^\dagger)^2 d\right) - \frac{g_3}{36} d^\dagger d \psi^\dagger \psi$$

EOM:
$$\partial_t \frac{\partial \mathcal{L}}{\partial(\partial_t d)} + \vec{\nabla} \frac{\partial \mathcal{L}}{\partial(\vec{\nabla} d)} - \frac{\partial \mathcal{L}}{\partial d} = 0$$

<u>Note:</u> $d$ static : no derivatives!

$$\Rightarrow \quad \frac{g_2}{4} d - \frac{g_2}{4}\psi^2 - \frac{g_3}{36} d\, \psi^\dagger\psi = 0$$

$$\Rightarrow \quad d\left(1 - \frac{g_3}{9 g_2}\psi^\dagger\psi\right) = \psi^2$$

$$\Rightarrow \quad \boxed{d = \frac{\psi^2}{1 - \frac{g_3}{9 g_2}\psi^\dagger\psi}}, \quad d^\dagger = \ldots$$

Now substitute back into $\mathcal{L}'$

$$\mathcal{L}' = \psi^\dagger\left(i\partial_t + \frac{D^2}{2m}\right)\psi + \frac{g_2}{4}(\psi^\dagger\psi)^2 \overbrace{\left(1 - \frac{g_3}{9 g_2}\psi^\dagger\psi\right)^{-2}}^{1 + \frac{2}{9}\frac{g_3}{g_2}(\psi^\dagger\psi) + \ldots}$$

$$- \frac{g_2}{4}(\psi^\dagger\psi)^2 \cdot 2 \left(1 - \frac{g_3}{9 g_2}\psi^\dagger\psi\right)^{-1} \rightarrow 1 + \frac{g_3}{9 g_2}\psi^\dagger\psi + \ldots$$

$$- \frac{g_3}{36}(\psi^\dagger\psi)^3 \left(1 - \frac{g_3}{9 g_2}\psi^\dagger\psi\right)^{-2} \rightarrow 1$$

$$= \psi^\dagger\left(i\partial_t + \frac{D^2}{2m}\right)\psi + \frac{g_2}{4}(\psi^\dagger\psi)^2 + \cancel{\frac{g_3}{18}(\psi^\dagger\psi)^3} - \frac{g_2}{2}(\psi^\dagger\psi)^2 - \cancel{\frac{g_3}{18}(\psi^\dagger\psi)^3}$$
$$- \frac{g_3}{36}(\psi^\dagger\psi)^3 + \ldots$$

$$\boxed{\mathcal{L}' = \psi^\dagger\left(i\partial_t + \frac{D^2}{2m}\right)\psi - \frac{g_2}{4}(\psi^\dagger\psi)^2 - \frac{g_3}{36}(\psi^\dagger\psi)^3 + \ldots = \mathcal{L}}$$

# (A8) Dimer formalism II

geometric series

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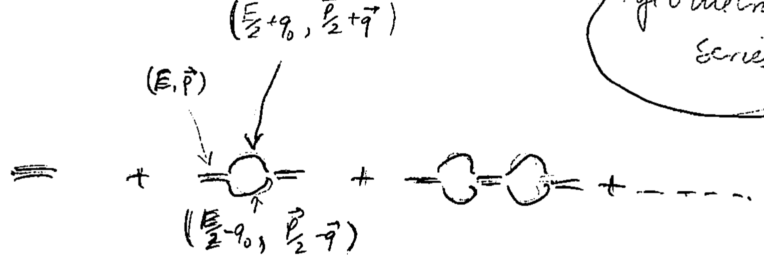
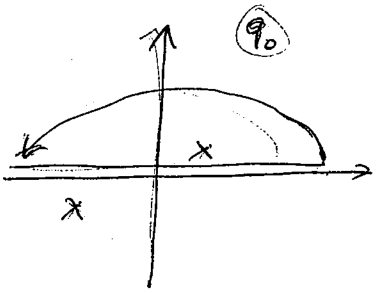
$i D_d(E,\vec{p}) =$




$$= \frac{4i}{g_2}\left(1 + \underbrace{\frac{4i}{g_2}\frac{1}{2}\int\frac{d^4q}{(2\pi)^4}\frac{(\cancel{-i}\,g_2/2)^2\,\cancel{i^2}}{\left(\frac{E}{2}+q_0-\frac{(\vec{p}/2+\vec{q})^2}{2m}+i\epsilon\right)\left(\frac{E}{2}-q_0-\frac{(\vec{p}/2+\vec{q})^2}{2m}+i\epsilon\right)}}+\ldots\right)$$




$$\underbrace{\frac{ig_2}{2}\int\frac{dq_0}{2\pi}\int\frac{d^3q}{(2\pi)^3}\frac{1}{\left(\frac{E}{2}+q_0-\frac{(\vec{p}/2+\vec{q})^2}{2m}+i\epsilon\right)\left(\frac{E}{2}-q_0-\frac{(\vec{p}/2-\vec{q})^2}{2m}+i\epsilon\right)}}$$

$$\underbrace{\frac{\cancel{i}g_2}{2}\frac{(-\cancel{2\pi i})}{\cancel{2\pi}}\int\frac{d^3q}{(2\pi)^3}\left[E-\frac{p^2}{4m}-q^2/m+i\epsilon\right]^{-1}}$$

$$-\frac{g_2}{2}\frac{m}{2\pi^2}\underbrace{\int_0^\Lambda q^2\,dq\left[q^2-mE+\frac{p^2}{4}-i\epsilon\right]^{-1}}_{\Lambda-\frac{\pi}{2}\sqrt{-mE+p^2/4-i\epsilon}\;+\;O(1/\Lambda)}$$

$$i D_d(E,\vec{p}) = \frac{4i}{g_2}\left(1-\frac{g_2 m}{4\pi^2}\left(\Lambda-\frac{\pi}{2}\sqrt{-mE+p^2/4-i\epsilon}\right)+\ldots\right)$$

$$= \frac{4i}{g_2}\left(1+\frac{g_2 m}{8\pi}\left(\frac{2}{\pi}\Lambda-\sqrt{-mE+p^2/4-i\epsilon}\right)\right)^{-1}$$

$$= \frac{-i\,32\pi}{m g_2^2}\left(\underbrace{-\left(\frac{8\pi}{m g_2(\Lambda)}+\frac{2}{\pi}\Lambda\right)}_{-1/a}+\sqrt{-mE+p^2/4-i\epsilon}\right)^{-1}$$

$$\Rightarrow \boxed{i D_d(E,\vec{p}) = \frac{-i\,32\pi}{m g_2^2}\,\frac{1}{-1/a+\sqrt{-mE+p^2/4-i\epsilon}}}$$

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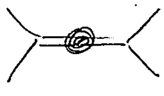
$i T(E) =$  $= \underbrace{(-ig_2/2)^2\left(\frac{-i\,32\pi}{m g_2^2}\right)}\frac{1}{-1/a+\sqrt{-mE+p^2/4-i\epsilon}}$

2-Teilchen-Streuung

$$i\,\frac{8\pi}{m}\,\frac{1}{-1/a+\sqrt{-mE+p^2/4-i\epsilon}}$$

## A9 Size of the dimer

Normalization $\quad 1 \overset{!}{=} \int d^3r \, |\psi_0(r)|^2$

$$= 4\pi \int_0^\infty r^2 dr \left( \frac{e^{-r/a}}{r} \right)^2 N^2 = N^2 \, 2\pi a$$

$$\Longrightarrow \quad N = \frac{1}{\sqrt{2\pi a}}$$

$$\langle r^2 \rangle = \frac{1}{2\pi a} \, 4\pi \underbrace{\int_0^\infty r^2 dr \, \cancel{r^2} \, \frac{e^{-2r/a}}{\cancel{r^2}}}_{a^3/4} = a^2/2$$

$$\Longrightarrow \quad \sqrt{\langle r^2 \rangle} = a/\sqrt{2}$$

$\Longrightarrow$ Size of the dimer is $a/\sqrt{2}$

Dimensional analysis: $\rightarrow a$

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R) $\int_0^\infty r dr \, e^{-br} = -\frac{d}{db} \int_0^\infty dr \, e^{-br} \qquad \int_0^\infty r^2 dr \, e^{-br} = \frac{d^2}{db^2} \underbrace{\int_0^\infty e^{-br} dr}_{1/b} = \frac{d}{db}\left(-\frac{1}{b^2}\right) = \frac{2}{b^3}$

here $b = 2/a$

$$\rightarrow \quad \frac{2}{(2/a)^3} = \frac{a^3}{4}$$

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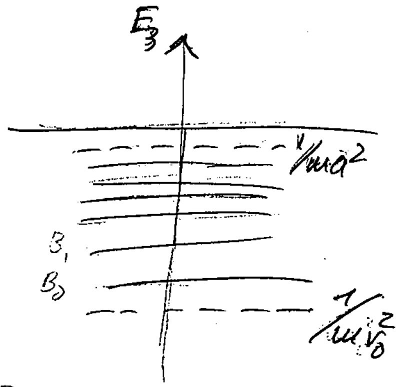
R)

$$1 = N^2 \, 4\pi \underbrace{\int_0^\infty dr \, e^{-2r/a}}_{-\frac{a}{2} e^{-2r/a} \big|^\infty} = N^2 \, 4\pi \, \frac{a}{2} \Rightarrow N = \frac{1}{\sqrt{2\pi a}}$$

(A11) Efimov effect:

geometrical spectrum of states




$$(*) \qquad \frac{B_3^{(n+1)}}{B_3^{(n)}} \to e^{-2\pi/s_0} \quad , \; s_0 = 1.00624\ldots$$

How many states between $\frac{1}{m r_0^2} \geq B \geq \frac{1}{m a^2}$ ?

(*) is approximately valid for $\frac{1}{m r_0^2} \geq B \geq \frac{1}{m a^2}$

$$\Rightarrow \quad \left(e^{2\pi/s_0}\right)^N \frac{1}{m a^2} \cong \frac{1}{m r_0^2}$$

$$\Rightarrow \quad e^{N\pi/s_0} \frac{1}{|a|} \cong \frac{1}{r_0}$$

$$\Rightarrow \quad \boxed{N \cong \frac{s_0}{\pi} \ln\left(|a|/r_0\right)}$$

only order of magnitude estimate !

spectrum not geometric near $\frac{1}{m|a|^2}$, $\frac{1}{m r_0^2}$