1 Dimensional analysis I: Planck units

Construct the Planck length, mass, and time out of G, $\hbar$, c

$$[G] = \frac{m^3}{kg\, s^2}$$

(Newton's constant)

NR

$$kg\frac{m}{s^2} = [G]\frac{kg^2}{m^2}$$

$$[\hbar] = Js = \frac{kg\, m^2}{s}$$

$$[c] = \frac{m}{s}$$

$$\hookrightarrow \quad m_p = \left(\frac{\hbar}{G}c\right)^{1/2} \approx 2.2 \cdot 10^{-8}\, kg$$

Note: $m_p \approx 1.7 \cdot 10^{-27} kg$

$m_e \approx 9.1 \cdot 10^{-31} kg$

$$l_p = \left(\frac{\hbar G}{c^3}\right)^{1/2} \approx 1.6 \cdot 10^{-35}\, m$$

Note $r_e < 10^{-18}\, m$

$r_p \approx 10^{-15}\, m$

$$t_p = \left(\frac{\hbar G}{c^5}\right)^{1/2} \approx 5.4 \cdot 10^{-44}\, s$$

Note $\tau_\Delta \approx 5.6 \cdot 10^{-24}\, s$ strong

$\tau_{top} \approx 5 \cdot 10^{-25}\,s$ weak

$\tau_{\mu} \approx 2 \cdot 10^{-6}\,s$

$\tau_{\pi^{\pm}} \approx 2.6 \cdot 10^{-8}\,s$ weak

$\tau_{\pi^0} \approx 8 \cdot 10^{-17}\,s$ EM

$\tau_{\eta} \approx 5 \cdot 10^{-19}\,s$

At energies / lengths / times of order

m_p / ℓ_p / t_p

gravity becomes important.

$\longrightarrow$ quantum gravity required

2 Dimensional analysis II: hydrogen atom

Estimate E_0, a_0 (Bohr radius)

$m_p \gg m_e \longrightarrow$ set $m_p \approx \infty$

$\hookrightarrow$ only one dimensionful constant: m_e

$$\longrightarrow \quad E_0 \propto m_e$$

$$a_0 \propto 1/m_e$$

Dependence on fine structure constant: α

stronger coupling: – more binding
– smaller size

$$\longrightarrow \quad E_0 \propto m_e \alpha^n$$

$$a_0 \propto m_e^{-1} \alpha^{-n'} \quad , n>0, n'>0$$

Dynamics gives

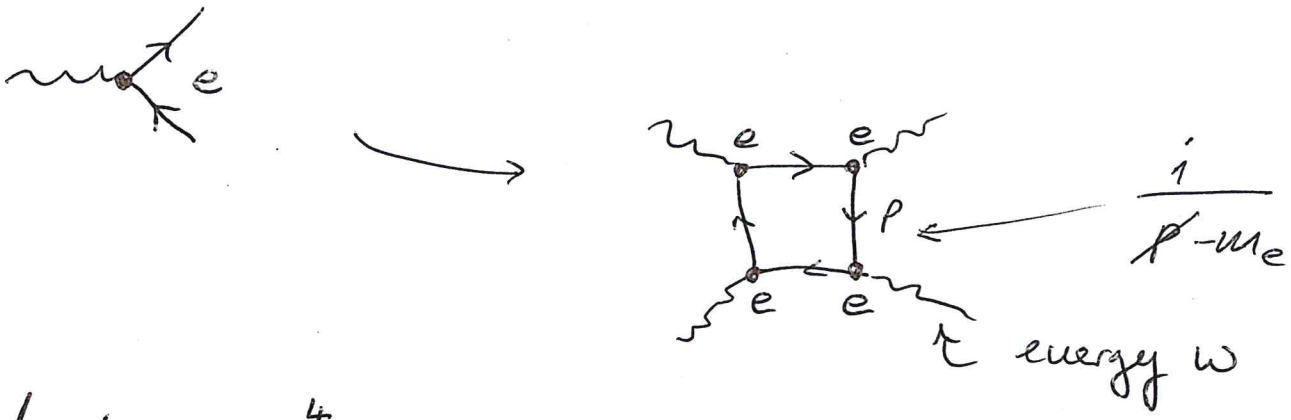
$$E_0 \propto \alpha^2 m_e$$

$$a_0 \propto \frac{1}{m_e \alpha}$$

3 Dimensional analysis III: photon-photon scattering

QED: no direct photon-photon scattering

$\hookrightarrow$ higher-order effect



Amplitude $\propto \frac{e^4}{m_e^4}$ for photon energy $\omega \ll m_e$

$\alpha = \frac{e^2}{4\pi}$

$A \propto \frac{\alpha^2}{m_e^4}$

photon flux

total cross section: $\sigma = \frac{1}{\dot{\partial}_\gamma} \int |A|^2 \, d\,\text{Phase space}$

can only add powers of photon energy ω

$[\sigma] = -2$

$\hookrightarrow \sigma \propto \frac{\alpha^4}{m_e^8} \cdot \omega^6$