Effective Field Theories

Literature

- Braaten, Hammer, "Universality in few-body systems with large scattering length", Phys. Rep. 428 (2006) 259, arXiv: cond-mat/0410417
  → Sec. 8

- Kaplan, "Effective Field Theories", NNPS '95
  arXiv: nucl-th/9506035

- Lepage, "How to renormalize the Schrödinger equation", Swieca '97, arXiv: nucl-th/9706029

- Lepage, "What is renormalization", TASI '89
  arXiv: hep-ph/0506330

1. Introduction

1.1 Scales in Physics

- Why are the methods in physics so different from biology?

  - DNA molecule: many energy scales of comparable magnitude
  - Hydrogen atom: few separated scales w/ hierarchy
    $$m_e \ll m_p \ll m_Z \ll \ldots$$

- Physical quantities have unit
  $\longleftrightarrow$ dimensional analysis

e.g. Hydrogen atom:

$$E_0 = m_e \, f\left(\alpha, \frac{m_e}{m_p}, \frac{m_e}{m_Z}, \dots\right)$$

dimensional analysis   dynamics

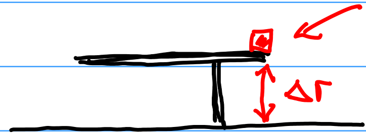
$$E_0 = \frac{\alpha^2 m_e}{2}\left(1 + \mathcal{O}\left(\frac{m_e}{m_p}\right) + \mathcal{O}\left(\frac{m_e}{m_Z}\right)\right)$$

EFT: systematic method to exploit separation of scales and calculate $f(\dots)$ in power series expansion

1.2 Simple Effective Theories

a) Newtonian Gravity

Eraser dropping from table



$$\vec{F} = m\vec{g}$$

Why effective theory? expand Newton's law of gravity

$$F/m = g = G\frac{M_E}{(\Delta r + R_E)^2} = \underbrace{\frac{G M_E}{R_E^2}}_{g}\left(1 - 2\frac{\Delta r}{R_E} + 3\left(\frac{\Delta r}{R_E}\right)^2 + \dots\right)$$

$$= g\left(1 - 2x + 3x^2 + \dots\right), \quad x = \frac{\Delta r}{R_E}$$

- expansion in $x = \frac{1\,\mathrm{m}}{6800\,\mathrm{km}} \approx 10^{-7}$
- breakdown scale: $x \sim 1$ $(\Delta r \sim R_E)$
- Note: underlying theory is itself effective theory

General philosophy

- identify relevant scales: R_E, Δr
- expand in small quantities: $x = \Delta r / R_E$
- preserve (approximate) symmetries of underlying theory ($\rightarrow$ rotational symmetry)
- Few constants capture complicated dynamics

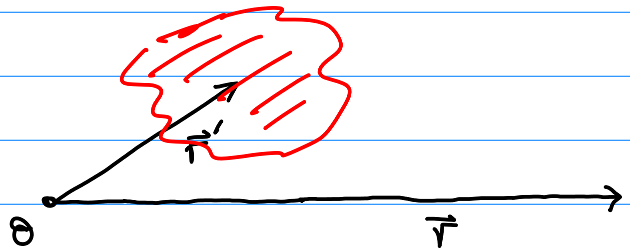
$$F(x) = \underset{g = 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}}{mg}\,(1 + \underset{-2}{c_1}\,x + \underset{3}{c_2}\,x^2 + \underset{\cdots}{\cdots})$$

$\hookrightarrow$ becomes inefficient at some point (# of parameters grows fast)

- Naturalness: dimensionless coefficients are typically $O(1)$, if not: fine tuning
- controlled expansion $\Leftrightarrow$ power counting
  - + work to given accuracy
  - + error estimates

b) Multipole decomposition

- consider a localized charge distribution



$\Longrightarrow$ potential at $\vec r$ can be expanded in multipoles

$$\Phi(\vec r) = \int d^3\vec r\,' \, \frac{\rho(\vec r\,')}{|\vec r - \vec r\,'|}$$

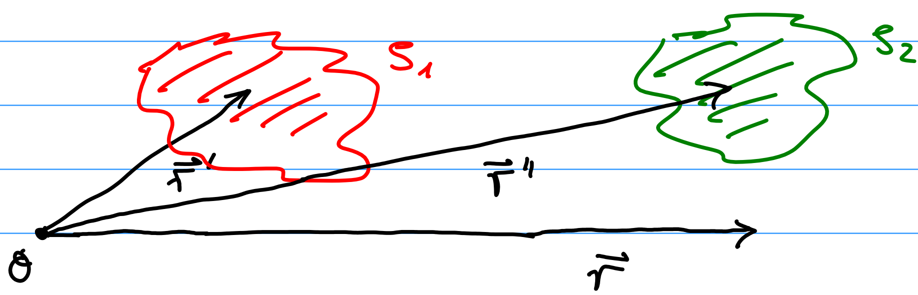
$r_< / r_> \hat{=}$ smaller / larger of $|\vec r|, |\vec r\,'|$

remember $$\frac{1}{|\vec r - \vec r\,'|} = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} \frac{4\pi}{2\ell+1} \frac{r_<^{\ell}}{r_>^{\ell+1}} Y_{\ell m}^*(\theta',\varphi') \, Y_{\ell m}(\theta,\varphi)$$

$$\hookrightarrow \Phi(\vec r) = \sum_{\ell,m} \frac{4\pi}{2\ell+1} \frac{1}{r^{\ell+1}} Y_{\ell m}(\theta,\varphi) \underbrace{\int d^3\vec r\,' \rho(\vec r\,') \, Y_{\ell m}^*(\theta',\varphi') \, (r')^{\ell}}_{q_{\ell m}}$$

Multipole moments $q_{\ell m}$

- now add second charge distribution:



cannot be expanded in multipoles

$$\Phi(\vec r) = \sum_{\ell,m} \frac{4\pi}{2\ell+1} Y_{\ell m}(\theta,\varphi) \frac{q_{\ell m}}{r^{\ell+1}} + \int d\vec r\,'' \, \frac{\rho_2(\vec r\,'')}{|\vec r - \vec r\,''|}$$

- Separation of scales: $|\vec{r}| \gg |\vec{r}'|$ but <u>NOT</u> $|\vec{r}| \gg |\vec{r}''|$
- "short-range" physics: can be expanded in r'/r
- "long-range" physics: <u>treat explicitly</u>!

1.3 Effective theories in Quantum mechanics

- Remember: wave character of quantum phenomena ($\hbar = 1$)

$$\psi(x) = N \int_{-\infty}^{\infty} \tilde{\psi}(p)\, e^{i p \cdot x / \hbar}\, dp$$

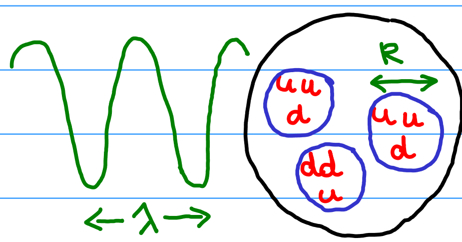
main contribution
$p \cdot x \simeq 1 = \hbar$

large momentum / energy $\Longleftrightarrow$ small distance

small momentum / energy $\Longleftrightarrow$ large distance

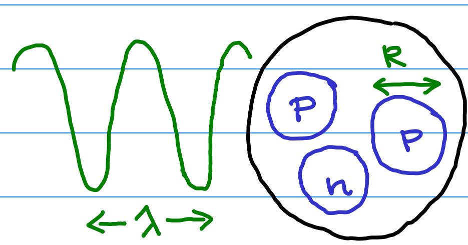
a) limited resolution w/ large wavelengths

resolution is of the order of the de Broglie wave length



quarks
gluons

EFT



nucleons
(pions)

$\Rightarrow$ expansion in R/λ

or using $\frac{2\pi}{\lambda} = k$: expansion in kR

Long wavelengths $\Longleftrightarrow$ low-energy experiments

- as in previous examples:
  - include short-range physics in low-energy constants (quarks, gluons $\longrightarrow$ nucleons, (pions))
  - include long-range physics explicitly (interactions of nucleons)

b) Construction of "effective potential" $V_{eff}(r)$:

- consider 2 particles interacting via $V(\vec{r})$
- interested in low-energy observables:

$\longrightarrow$ $E = \frac{k^2}{2\mu}$ close to threshold $E = 0$

- "fundamental" potential $V(\vec{r})$

long-distance

$$V(\vec{r}) = \begin{cases} \text{known accurately} & \text{for } r \geq S_0 \\ \text{not well-known or complicated} & \text{for } r < S_0 \end{cases}$$

short distance

- Examples:
  - finite-range potential: $V(\vec{r}) \equiv 0$, $r > S_0$

- von der Waals pot. : $V(\vec{r}) = -\dfrac{C_6}{r^6}$, $r > \rho_0$

- one-pion exchange: $V(\vec{r}) = \dfrac{g_{\pi NN}^2}{4\pi m_\pi} (\vec{\tau}_1 \cdot \vec{\tau}_2)(\vec{\sigma}_2 \cdot \vec{\nabla})(\vec{\sigma}_1 \cdot \vec{\nabla}) \dfrac{e^{-m_\pi r}}{r}$

- Idea: replace $V(\vec{r})$ by effective potential $V_{eff}(\vec{r})$ with adjustable parameters $\overbrace{c_1, c_2, \ldots, c_n}^{c}$ which coincides with $V(\vec{r})$ for $r \geq \rho_0$

↪ But tuning $\overbrace{c_1, \ldots, c_n}^{c}$, low-energy observables can be determined up to errors that scale as E^n

- In particular, for the scattering phase shift

$$\hookrightarrow \quad \delta(k) - \delta_{eff}(k, c) \sim \mathcal{O}(k^{2n-1})$$

Phase shift for $V(r)$ Phase shift for $V_{eff}(r; c)$

- Further reading:
  - Sec. 8.1 of Braaten & Hammer
  - Kaplan lecture notes