TALENT School — Johannes Gutenberg Universität
"Effective Field Theories in Light Nuclei: From Structure to Reactions" — 25 July 2022

Introduction to Scattering Theory
Notion of Phase Shift and Scattering Cross Sections

1. We consider the following square-well potential

$$V(r) = \begin{cases} -V_0 & r < r_0 \\ 0 & r \geq r_0 \end{cases}, \tag{1}$$

where V_0 is a positive constant.

(a) We propose to solve the radial Schrödinger equation for a positive energy E in the s wave ($l = 0$). From this solution, deduce an analytical expression for the phase shift δ_0.

- What is the behaviour of δ_0 at low energy E?
How can the different cases relate to the presence of a bound state hosted by the potential (1) in the s wave?
- What is the behaviour of δ_0 at high energy E?

(b) Extend the previous discussion to any partial wave $l > 0$.
Indications: relate the radial Schrödinger equation to the spherical Bessel equation and use the properties of the spherical Bessel functions j_l and n_l seen during the lecture (and available on Wikipedia).

2. An experimental cross section is well described by the following formula

$$\frac{d\sigma}{d\Omega} = a + b\cos\theta + c\cos^2\theta, \tag{2}$$

where a, b, and c are numerical parameters fitted to the data. This parametrisation remains valid at lower beam energies.

(a) What can be deduced about the phaseshifts at that energy?

(b) Propose a way to test this hypothesis.

(c) What is the total scattering cross section?
Find the lowest number of measurements needed to infer it from the differential scattering cross section (assuming a perfect measurement without any uncertainty).

(d) Show that the formula (2) remains valid when $\delta_0 \gg \delta_1 \gg \delta_2$, while δ_2 is not negligible.
In that case, find a way to infer an experimental value for δ_0 and δ_1.