Previously in Breakup-Reaction Theory. . .

On Monday, we've seen

- how to include Coulomb in scattering problems
- how to account for ~~closed~~ open channels in a simple effective way : optical potentials

  i.e. with imaginary part that absorbs flux from elastic channel
- how to explicitly include the breakup channel for 2-b projectiles

  $\Rightarrow$ need to solve a three-body Schrödinger equation

Yesterday, we've seen various methods to solve that 3-b problem

- CDCC
- Time-dependent approach
- Eikonal approximation and its variations
  - DEA
  - CCE (used in the project)

You've started the breakup-reaction project

- develop the c-f interaction (e.g. within Halo-EFT)
- find optical potentials for the c-T and f-T interactions
- pay attention to the numerical convergence of the scheme

Breakup-Reaction Theory
Part III: Reaction Dynamics and Application to Nuclear Astrophysics

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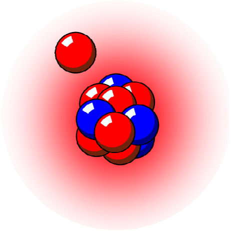
10 August 2022

Structure of ^{15}C



^{14}C + n

$5/2^+$	-0.478	$0d5/2$
$1/2^+$	-1.218	$1s1/2$

^{15}C spectrum

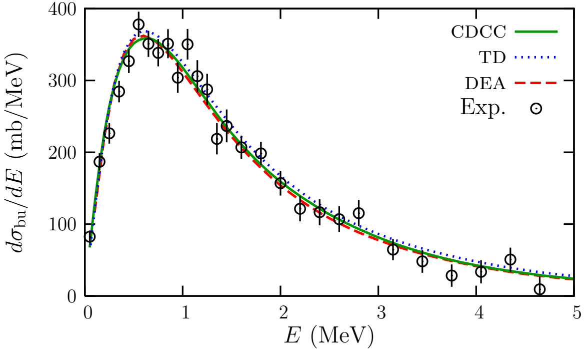
$$^{15}\mathrm{C} \equiv {}^{14}\mathrm{C}(0^+) + \mathrm{n}$$

^{14}C cluster assumed in 0^+ ground state (extreme shell model) (see Daniel's classes)

$\Rightarrow$ spin and parity of ^{15}C states fixed by angular momenta l and j of n :

- $1/2^+$ ground state in $s1/2$
- $5/2^+$ excited (bound) state in $d5/2$

^{15}C+Pb @ $68A$MeV : energy distribution

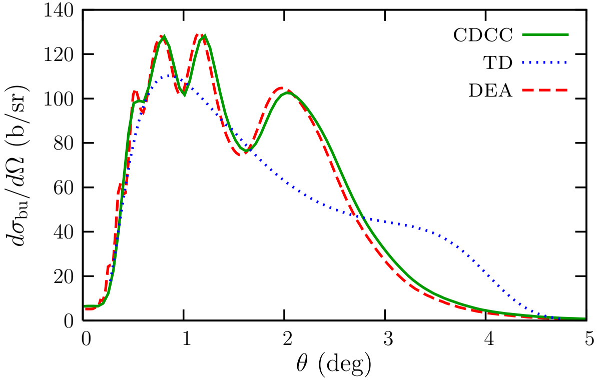


- Excellent agreement between all three methods
  [P.C., Esbensen and Nunes, PRC 85, 044604 (2012)]
- Excellent agreement with experiment
  [Nakamura *et al.* PRC 79, 035805 (2009)]

⇒ Confirms the validity of the approximations

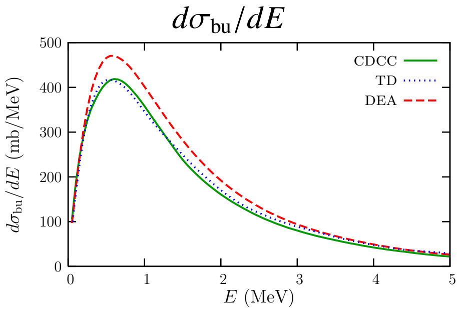
. . . and the two-body structure of ^{15}C

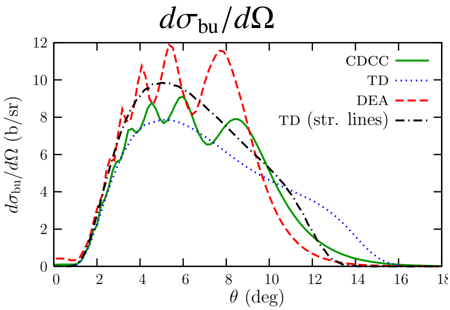
^{15}C+Pb @ $68A$MeV : angular distribution



- TD lacks quantum interferences
  but reproduces the general trend at small θ
- DEA exhibits quantum interferences
  though much less time consuming than CDCC

^{15}C + Pb @ 20AMeV





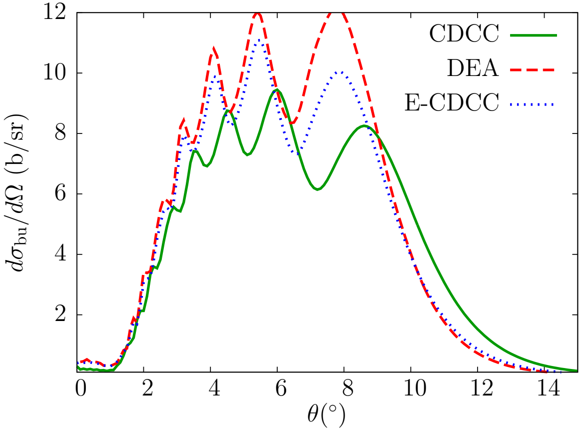
TD ≡ CDCC
DEA too high

TD gives trend of CDCC
(lacks oscillations)
DEA peaks too early

DEA≠CDCC due to Coulomb deflection
Eikonal is a high-energy approximation
Could an Eikonal-CDCC model solve the problem ?

[Ogata *et al.* PRC 68, 064609 (2003)]

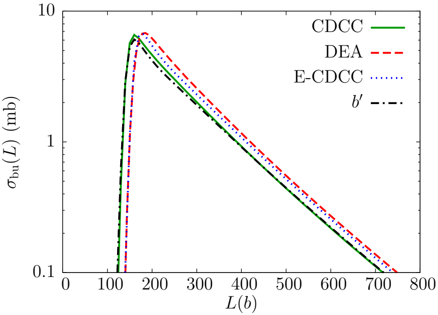
Semiclassical correction

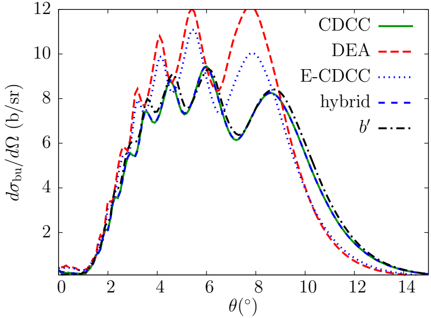


[Fukui, Ogata, P.C. PRC 90, 034617 (2014)]

- E-CDCC also too high and too forward

Semiclassical correction

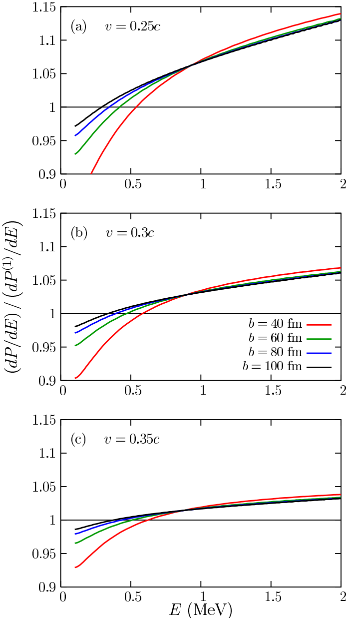




[Fukui, Ogata, P.C. PRC 90, 034617 (2014)]

- E-CDCC also too high and too forward
  Shift in $L \Rightarrow$ correction $b \rightarrow b'$ (classical closest approach)
- hybrid solution : CDCC at low L (b) and eikonal at large L (b)
  $\Rightarrow$ excellent agreement with full CDCC
- Improve eikonal using Coulomb correction : $b \rightarrow b'$

Dynamical vs. First-Order Calculations

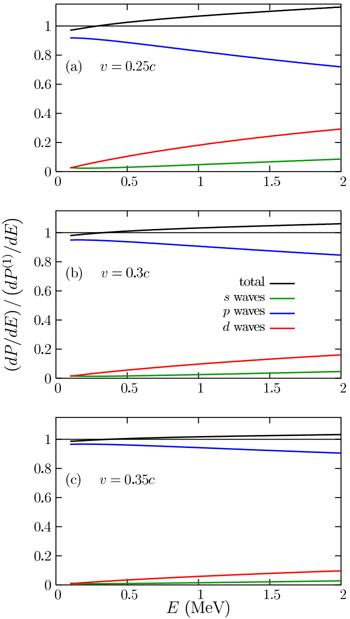


[P. C., Baye, PRC 71, 044609 (2005)]

- Comparison exact TD/FO for breakup of ^{11}Be on Pb at different v and b
- Relative agreement between first-order and exact solution
- Accuracy of first order improves at large v
- But systematic distortion even at high b

See also [Typel, Baur, PRC 64, 024601 (2001)] [Esbensen, Bertsch, Snover PRL 94, 042502 (2005)]

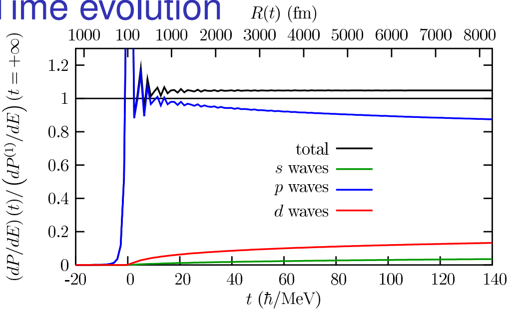
Partial-wave analysis

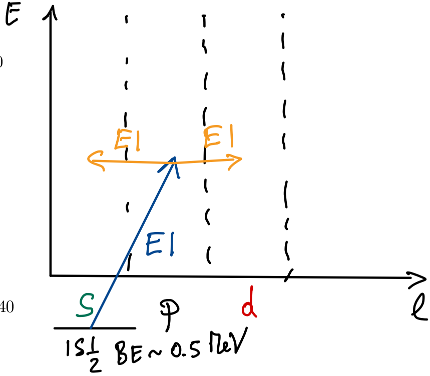


- dP_{bu}/dE decomposed in partial-wave contributions
- First order predicts mainly E1 transitions
  $\Rightarrow$ p waves from s bound state
- In exact solution :
  - Total close to first-order
  - p waves indeed dominate
  - But s and d components which are not first-order

$\Rightarrow$ higher-order effects

Time evolution





- At $t < 0$ no breakup
- At closest approach $t = 0$ steep rise
  Only p waves : E1 transition from s ground state
- At $t > 0$ p waves depleted towards s and d
  But total remains constant

$\Rightarrow$ Significant couplings in the continuum

with $\Delta l = 1$ and $\Delta E \approx 0 \Rightarrow$ mostly E1 coupling $\approx \frac{\hbar}{R^2}$

[P. C., Baye, PRC 71, 044609 (2005)]

$\Rightarrow$ FO does not capture the whole reaction mechanism

Introduction : a bit of history

Where do we come from ?
Where was produced the matter that surrounds us ?

The answer came from astrophysics...

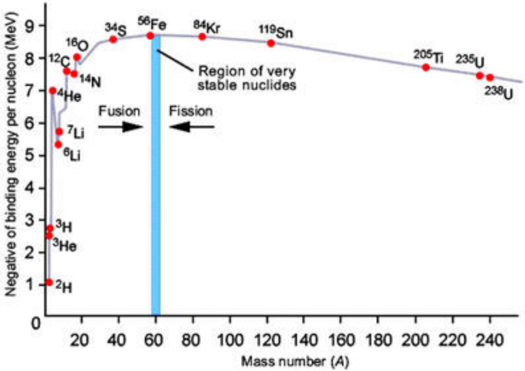
In 1920 A. Eddington : stars are nuclear powered
In 1929 R. Atkinson and F. Houtermans : fusion of light elements produces energy
e.g. fusion of 4 protons into ^{4}He

$$4\,\mathrm{p} \rightarrow \ ^4\mathrm{He} + 2\mathrm{e}^+ + 2\nu_\mathrm{e} + 26.73\ \mathrm{MeV}$$

In 1938-39, H. Bethe and C. Critchfield : pp chain and CNO cycles (H. Bethe got NP in 1967)
In 1957, seminal paper of Burbidge, Burbidge, Fowler and Hoyle on nucleosynthesis in stars [Rev. Mod. Phys. **29**, 257]

Introduction : nucleosynthesis in a nutshell

By fusion of light elements we can reach the Fe-Ni region because reactions are exoenergetic and Coulomb repulsion is small



Beyond, processes based on n or p capture lead to heavy nuclei : s, r, p, rp processes. . .

pp chain

$$p + p \longrightarrow d + e^+ + \nu_e$$

$$d + p \longrightarrow {}^3\mathrm{He} + \gamma$$

85% 15%

$${}^3\mathrm{He} + {}^3\mathrm{He} \longrightarrow {}^4\mathrm{He} + p + p$$

(pp chain I)

$${}^3\mathrm{He} + {}^4\mathrm{He} \longrightarrow {}^7\mathrm{Be} + \gamma$$

15% 0.02%

$${}^7\mathrm{Be} + e \xrightarrow{EC} {}^7\mathrm{Li} + \nu_e$$

$${}^7\mathrm{Be} + p \longrightarrow {}^8\mathrm{B} + \gamma$$

$${}^7\mathrm{Li} + p \longrightarrow {}^4\mathrm{He} + {}^4\mathrm{He} + \gamma$$

$${}^8\mathrm{B} \xrightarrow{\beta} {}^8\mathrm{Be} + e^+ + \nu_e$$

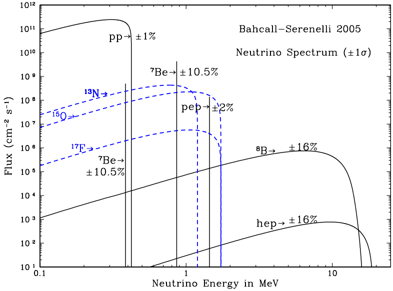
$$\hookrightarrow {}^4\mathrm{He} + {}^4\mathrm{He}$$

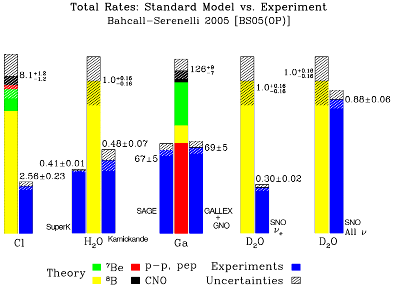
Ray Davis' Experiment

In 1964, Ray Davis measures the solar neutrino flux using

$$^{37}\mathrm{Cl} + \nu_e \longrightarrow \; ^{37}\mathrm{Ar} + e^-$$

threshold $E_{\nu_e} = 0.8$ MeV $\Rightarrow$ sensitive mostly to ^{8}B neutrinos





The measured flux does not fit the prediction of the Solar Model...

Solved by SNO and Super Kamiokande : neutrino oscillations

NP : Davis in 2002 and Mc Donald and Kajita in 2015

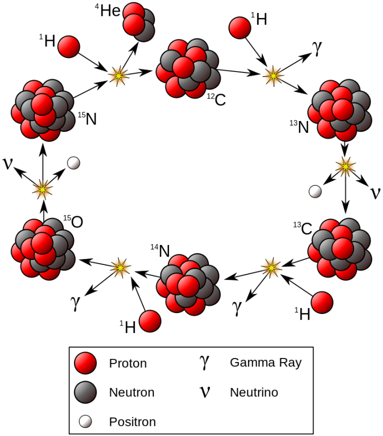
CNO cycle(s)

If the star contains C, N or O they can be used as catalyst to synthesise ^{4}He from 4 p e.g. CNO C cycle :

$$
\begin{aligned}
{}^{12}\mathrm{C} + \mathrm{p} &\rightarrow {}^{13}\mathrm{N} + \gamma \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C} + \mathrm{e}^+ + \nu_\mathrm{e} \\
{}^{13}\mathrm{C} + \mathrm{p} &\rightarrow {}^{14}\mathrm{N} + \gamma \\
{}^{14}\mathrm{N} + \mathrm{p} &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + \mathrm{e}^+ + \nu_\mathrm{e} \\
{}^{15}\mathrm{N} + \mathrm{p} &\rightarrow {}^{12}\mathrm{C} + \alpha
\end{aligned}
$$

Summary :

$$4\mathrm{p} \rightarrow {}^4_2\mathrm{He} + 2\mathrm{e}^+ + 2\nu_\mathrm{e} + 25\mathrm{MeV}$$



CNO C cycle

Other cycles

Other cycles are possible

- CNO N cycle using ^{14}N as catalyst :

$$
\begin{aligned}
{}^{14}\mathrm{N} + \mathrm{p} &\rightarrow {}^{15}\mathrm{O} + \gamma \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \\
{}^{15}\mathrm{N} + \mathrm{p} &\rightarrow {}^{16}\mathrm{O} + \gamma \\
{}^{16}\mathrm{O} + \mathrm{p} &\rightarrow {}^{17}\mathrm{F} + \gamma \\
{}^{17}\mathrm{F} &\rightarrow {}^{17}\mathrm{O} + \mathrm{e}^{+} + \nu_{\mathrm{e}} \\
{}^{17}\mathrm{O} + \mathrm{p} &\rightarrow {}^{14}\mathrm{N} + \alpha
\end{aligned}
$$

- NeNaMg cycles
- …

Reaction rate

We consider the radiative-capture reaction : $\mathrm{b} + \mathrm{c} \rightarrow \mathrm{a} + \gamma$
The reaction rate is the number of reactions occurring per unit time and volume

$$r = N_b N_c \, \sigma \, v$$

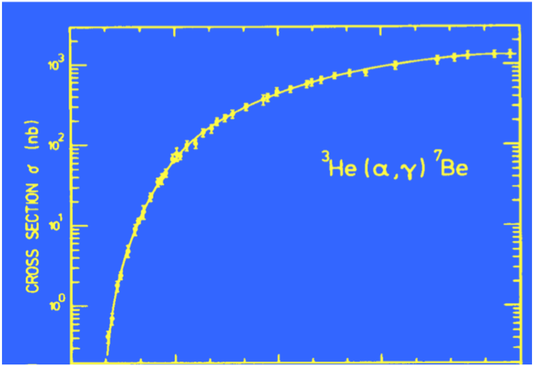
The velocity v is distributed according to Maxwell-Boltzmann

$$\begin{aligned} \phi(\boldsymbol{v}) &\propto e^{-E/kT} \\ \Rightarrow \langle \sigma \, v \rangle &= 4\pi \int \phi(\boldsymbol{v}) \, \sigma(v) \, v^3 \, dv \\ &\propto \int e^{-E/kT} \, \sigma(E) \, E \, dE \end{aligned}$$

$\sigma(E)$ at low energy

Due to Coulomb barrier σ plummets at low E
because reaction takes place only through tunneling

$$^3\mathrm{He} + \alpha \to {}^7\mathrm{Be} + \gamma \text{ also noted } {}^3\mathrm{He}(\alpha,\gamma)\,{}^7\mathrm{Be}$$



Astrophysical S factor

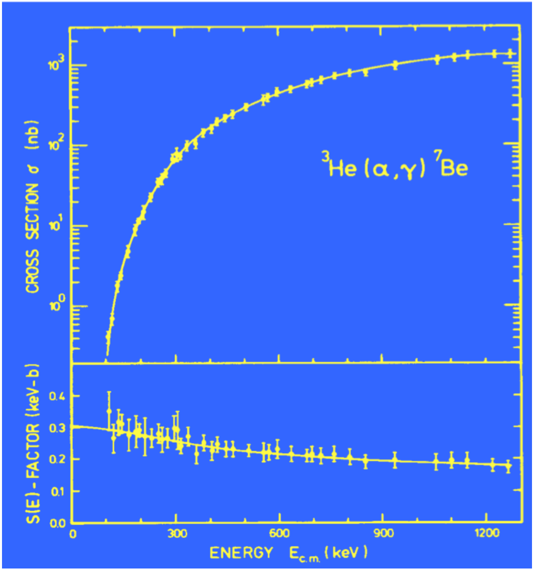
The rapid drop explained by the Gamow factor $e^{-2\pi\eta}$,

$$\eta = \frac{Z_b Z_c e^2}{4\pi\epsilon_0 \hbar v}$$

is Sommerfeld parameter

$$\Rightarrow \sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta}$$

The astrophysical S factor varies smoothly with E



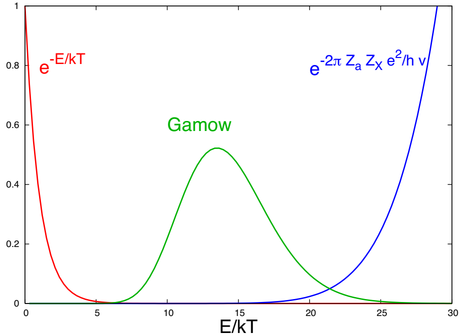
Gamow peak

$$\langle \sigma v \rangle \propto \int e^{-E/kT}\, \sigma(E)\, E\, dE$$

$$= \int e^{-E/kT}\, e^{-2\pi\eta}\, S(E)\, dE$$

$\Rightarrow S$ must be known only in the Gamow peak

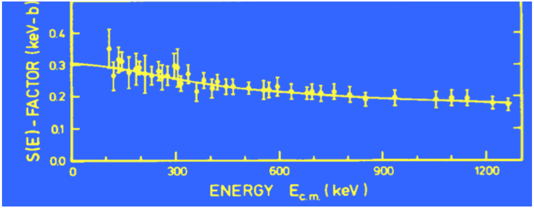
$$g(E) = e^{-E/kT}\, e^{-2\pi\eta}$$



Example

For the reaction $^3\mathrm{He}(\alpha,\gamma)\,^7\mathrm{Be}$ in the sun

$Z_b = 2, A_b = 3$
$Z_c = 2, A_c = 4$
$T = 0.015\ T_9$
Gamow peak
at $E_0 \simeq 20$ keV



$\Rightarrow$ difficult to measure due to background

Solutions

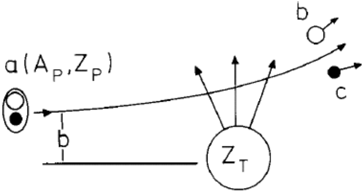
- Rely on theory to extrapolate down to astrophysical energies
- Go to an underground laboratory to reduce background
  e.g. LUNA collaboration
- Use indirect techniques, e.g. Coulomb breakup

Coulomb Breakup Method

Coulomb breakup : projectile breaks up colliding with a heavy target

$$a + T \rightarrow b + c + T$$

Coulomb dominated ⇒ due to exchange of virtual photons



Baur and Rebel Ann. Rev. Nucl. Part. Sc. 46, 321 (1996)

⇒ seen as the time-reversed reaction of the radiative capture

⇒ use Coulomb breakup to infer radiative-capture cross section

[Baur, Bertulani and Rebel NPA458, 188 (1986)]

Radiative Capture Cross Section

Radiative capture :

electromagnetic transition b-c continuum $\rightarrow a \equiv b + c$ bound state

At low energy, dominated by $\mathrm{E1}$ transitions

$$\sigma_{\mathrm{cap}}^{(\mathrm{E1})}(E) = \frac{2\pi^3}{3}\frac{E}{\hbar c}\frac{dB(\mathrm{E1})}{dE} \propto \frac{d\sigma_{\mathrm{bu}}^{(1)}(\mathrm{E1})}{dE}$$

$\Rightarrow$ Infer σ_{cap} from $d\sigma_{\mathrm{bu}}/dE$ [Baur, Bertulani and Rebel NPA458, 188 (1986)]

- easier to measure (above Coulomb barrier)
- higher cross sections

But :

- Nuclear interaction must be negligible
- Coulomb breakup must take place at first order
- and be dominated by E1 transitions

$^8\mathrm{B}$

$^8\mathrm{B}$ has only one 2^+ loosely-bound state with $S_\mathrm{p} = 137$ keV
Often considered as a one-proton halo nucleus

Described as

$$\left|{}^8\mathrm{B}(2^+)\right\rangle = \left|{}^7\mathrm{Be}(3/2^-) \otimes \mathrm{p}(\mathrm{p}3/2)\right\rangle$$

Model of Esbensen & Bertsch [NPA 600, 37 (1996)] :

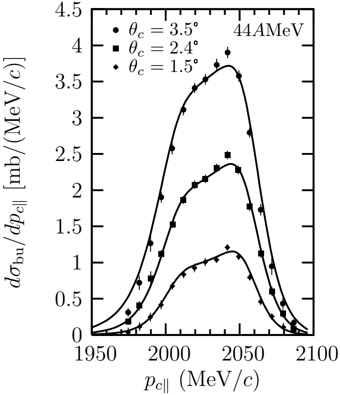
- $^7\mathrm{Be}$ assumed spherical, its spin is neglected
- $^7\mathrm{Be}$-p potential has Woods-Saxon form factor (plus spin-orbit)

Parallel-momentum distribution

Parallel-momentum distribution is best to test this

see [Esbensen, Bertsch NPA 600, 37 (1996)]

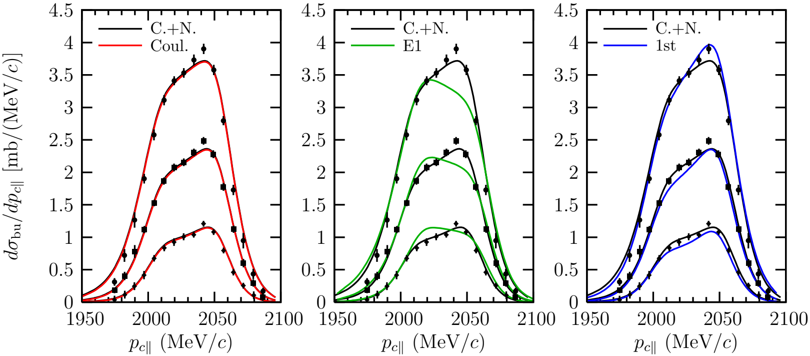
$^8\mathrm{B} + \mathrm{Pb}$ @ 44AMeV   Exp : [Davids *et al.* PRL 81, 2209 (2001)]



Th : DEA [Goldstein, P.C., Baye, PRC 76, 064608 (2007)]

Excellent agreement with exp. (no fitting parameter)

Reaction Dynamics

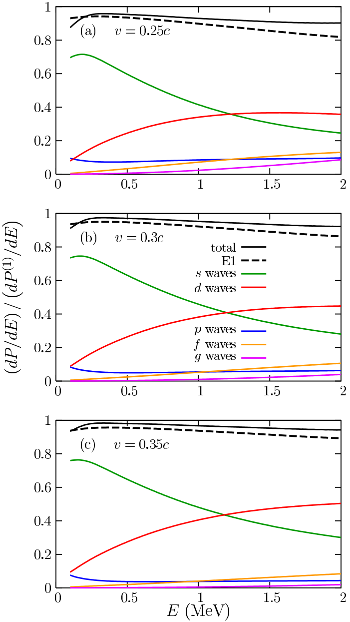


Nuclear interaction negligible at forward angles

Significant E2 contribution (asymmetry)

First-order : more asymmetric ⇒ higher-order

Analysis of the Dynamics of ^{8}B Breakup on Pb



- Both E1 and E2 are significant
- Higher-order effects :
  - presence of g components from a p ground state
  - $p + f >$ first-order E2
- Exact solution < first-order
  $\Rightarrow$ destructive interferences
  e.g. E1-E1 vs E2

$\Rightarrow$ be careful with first-order :
interesting qualitative tool
but inaccurate quantitative results

Interpretation

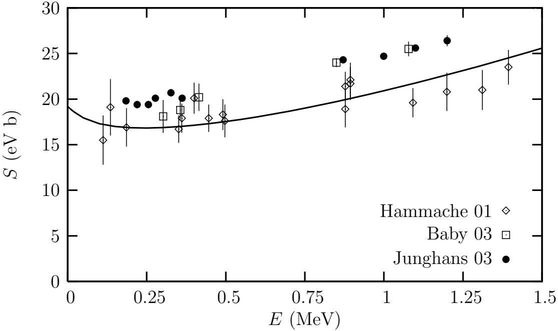
These results suggest the following mechanism

- at forward angle, reaction dominated by Coulomb
  $\Rightarrow$ removes sensitivity to nuclear interaction
- not only one-step E1 to continuum
- also one-step E2
- and two-step E1-E1
  which interfere with E2

$\Rightarrow$ direct extraction of $\sigma_{\rm capt}$ from $\sigma_{\rm bu}$ not that simple

S_{17}

Using this ^{8}B description, the $^{7}\mathrm{Be}(\mathrm{p},\gamma)^{8}\mathrm{B}$ S_{17} is



We obtain $S_{17} = 19.2$ b eV at $E = 0$

Good agreement with Hammache [PRL 86, 3985 (2001)]

Too low but good shape compared to Junghans [PRC 68, 065803 (03)]

Summers and Nunes suggest another idea ...

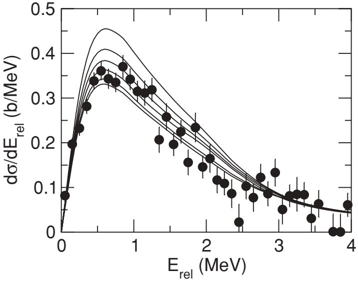
[Summers, Nunes PRC 78, 011601 (2008)]

Analysis by Summers & Nunes

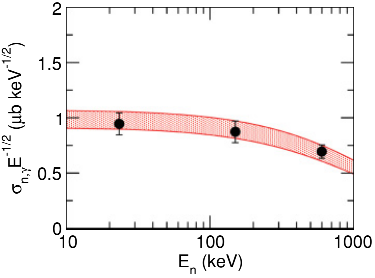
Summers and Nunes have calculated

$$^{15}\mathrm{C} + \mathrm{Pb} \rightarrow {}^{14}\mathrm{C} + \mathrm{n} + \mathrm{Pb} \quad \text{at } 68A\text{MeV}$$

within CDCC using different $V_{^{14}\mathrm{C-n}}$ [PRC 78, 011601 (2008)]



Exp. : Nakamura *et al.*
Th. : Summers, Nunes



Exp. : Reifarth *et al.*
Th. : Summers, Nunes

Significant dynamical effects ⇒ requires an accurate reaction model

From a χ^2 fit to the data, they extract an ANC they use to get $\sigma_{\mathrm{n},\gamma}$

Astrophysical Application of Coulomb Breakup

Initial idea : [Baur, Bertulani and Rebel NPA458, 188 (1986)]

See Coulomb breakup as time-reverse of radiative capture :

$$\sigma_{\text{cap}}(E) \propto \frac{dB(\text{E1})}{dE} \propto \frac{d\sigma^{(1)}_{\text{bu}}(\text{E1})}{dE}$$

But :

- Nuclear interaction (negligible at forward angles)
- E2 transitions
- higher orders

⇒ requires accurate reaction model

Nevertheless both reactions

- sensitive to same input (projectile description : ANC and δ_l)
- dominated by same interaction (Coulomb)

⇒ use breakup to constrain projectile model

from which to calculate capture