Breakup-Reaction Theory
Part II: Breakup Methods

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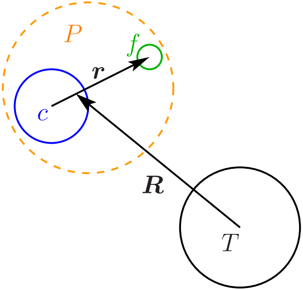
1. CDCC
2. Time-dependent approach
   - Semiclassical Approximation
   - First-Order of Perturbation Theory
3. Eikonal approximation
   - DEA and E-CDCC
   - Usual Eikonal
   - CCE
   - S-DEA

Three-body Scattering Problem

Within this framework breakup reduces to three-body problem

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r}, \boldsymbol{R}) = E_T\Psi(\boldsymbol{r}, \boldsymbol{R})$$



with the initial condition

$$\Psi(\boldsymbol{r}, \boldsymbol{R}) \underset{Z\to-\infty}{\longrightarrow} e^{iKZ+\cdots}\phi_{n_0l_0m_0}(\boldsymbol{r})$$

⇔ P in its ground state $\phi_{n_0l_0m_0}$ impinging on T

Various methods developed to solve that equation

[Review : Baye, P.C., Lecture Notes in Physics 848, 121 (2012) ; on Indico]

- Coupled-channel method with discretised continuum (CDCC)
- Time-dependent approach (TD)
  (semiclassical)
- Eikonal approximation

(for simplicity spin will be neglected)

Coupled-Channel Method

The eigenstates of H_0 $\{|\phi_i\rangle\}$ are a basis in $\boldsymbol{r}$: $H_0|\phi_i\rangle = E_i|\phi_i\rangle$

Idea : expand Ψ on that basis : $\Psi(\boldsymbol{r},\boldsymbol{R}) = \sum_i \chi_i(\boldsymbol{R})\langle \boldsymbol{r}|\phi_i\rangle$

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

$$\Rightarrow \sum_i \left[T_R + H_0 + V_{cT} + V_{fT}\right] \chi_i(\vec{R})\,\phi_i(\vec{r}) = E_T \sum_i \chi_i(\vec{R})\,\phi_i(\vec{r})$$

$$\Leftrightarrow \sum_i \phi_i(\vec{r})\, T_R\, \chi_i(\vec{R}) + \chi_i(\vec{R})\, E_i\, \phi_i(\vec{r})$$

$$+ \left(V_{cT} + V_{fT}\right) \chi_i(\vec{R})\,\phi_i(\vec{r}) = E_T \sum_i \chi_i(\vec{R})\,\phi_i(\vec{r})$$

$\langle \phi_j | \left(\right.$

$$T_R\, \chi_j(\vec{R}) \underbrace{\langle \phi_j | \phi_i \rangle}_{=\delta_{ij}} + E_j\, \chi_j(\vec{R}) \langle \phi_j | \phi_i \rangle$$

$$\sum_i \langle \phi_j | V_{cT} + V_{fT} | \phi_i \rangle \chi_i(\vec{R})$$

$$= E_T\, \chi_j(\vec{R}) \langle \phi_j | \phi_i \rangle$$

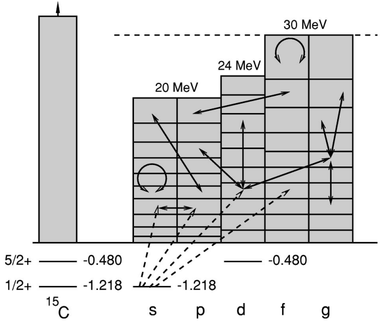
$$\Rightarrow T_R\, \chi_j(\vec{R}) + \sum_i \langle \phi_j | V_{cT} + V_{fT} | \phi_i \rangle \chi_i(\vec{R}) = (E_T - E_j)\, \chi_j(\vec{R})$$

Coupled Equation

This leads to a set of coupled equations in $\chi_i(\boldsymbol{R})$

$$T_R \chi_j(\boldsymbol{R}) + \sum_i \langle \phi_j | V_{cT} + V_{fT} | \phi_i \rangle \chi_i(\boldsymbol{R}) = \left(E_T - E_j\right) \chi_j(\boldsymbol{R})$$

where the coupling terms are $\langle \phi_j | V_{cT} + V_{fT} | \phi_i \rangle$
i.e. connect the various projectile states through the P-T interaction



[Tostevin *et al.* PRC 66, 024607 (2002)]

Model of breakup requires description of continuum tractable in computations, i.e. discrete

ϕ_{klm} with $k \in \mathbb{R}^+ \rightarrow \phi_{ilm}$ with $i \in \mathbb{N}$

[Rawitscher, PRC 9, 2210 (1974)]

Discretising the Continuum

Various methods exist :

- mid-point : divide continuum in bins $[E_i - \Delta E_i/2, E_i + \Delta E_i/2]$ and choose $\phi_{ilm}(\boldsymbol{r}) = \phi_{k_i lm}(\boldsymbol{r})$ to describe bin i
- average the wave function over the bin

$$\phi_{ilm}(\boldsymbol{r}) = \frac{1}{\sqrt{W_i}} \int_{E_i - \frac{\Delta E_i}{2}}^{E_i + \frac{\Delta E_i}{2}} f_i(E)\, \phi_{klm}(\boldsymbol{r})\, dE \quad \text{with } W_i = \int_{E_i - \frac{\Delta E_i}{2}}^{E_i + \frac{\Delta E_i}{2}} |f_i(E)|^2\, dE$$

$\Rightarrow$ square-integrable wave functions ϕ_{ilm} : binning technique

[Austern *et al.* , Phys. Rep. 154, 125 (1987)]

- pseudo-states : solve $H_0\, \phi_{ilm} = E\, \phi_{ilm}$ on finite basis or in a box
  $\Rightarrow$ square-integrable wave functions ϕ_{ilm} but E_i not chosen

  [Druet *et al.* NPA 845 88 (2010)]
- THO : Transformed Harmonic Oscillator
  Map the (discrete) states of HO onto the continuum

  [Pérez-Bernal *et al.* PRA 63, 052111 (2001)]

Solving the Coupled Equations

Expanding χ into spherical harmonics

$$\chi_j(\boldsymbol{R}) = \frac{1}{R}\sum_L i^L u_{jL}(R)\; Y_L^0(\Omega)$$

…and coupling l and L into J, the coupled equations read

$$-\frac{\hbar^2}{2\mu_{PT}}\left(\frac{d^2}{dR^2}-\frac{L(L+1)}{R^2}\right)u_c^J(R)+\sum_{c'}V_{cc'}^J(R)u_{c'}^J(R) \quad = \quad (E_T-E_j)u_c^J(R)$$

with $c \equiv \{j, L\}$ and $\boldsymbol{J}_T = \boldsymbol{L}+\boldsymbol{l}$

These equations are solved assuming the asymptotic behaviour

$$u_c^J(R) \underset{R\to\infty}{\longrightarrow} \left[\delta_{c0}\; I_L(\eta, KR) - S_{c0}^J\; O_L(\eta, KR)\right]$$

$$\text{where}\quad I_L = G_L - iF_L \quad \text{incoming Coulomb function}$$
$$= O_L^* \quad \text{outgoing Coulomb function}$$

The S matrix is used to compute the breakup cross sections

CDCC breakup cross sections

Starting from ground state $0 l_0 m_0$ impinging on target with $\boldsymbol{K} = K\hat{Z}$, the scattering amplitudes into state $c \equiv jlm$

scattered with momentum $\boldsymbol{K}'$

$$\mathcal{F}_{m\,m_0}(\boldsymbol{K}') = \frac{2\pi}{iK}\sqrt{\frac{K'}{K}} \sum_{L_0\,L\,J} (L_0 l_0 0 m_0 | J m_0)(L l (m_0 - m) m | J m_0) \times e^{i(\sigma_{L_0}+\sigma_L)} S^{\,J}_{c0} Y^0_{L_0}(\hat{K}) Y^{m_0-m}_{L}(\hat{K}')$$

with $E_T = \hbar^2 K^2/2\mu + E_0 = \hbar^2 K'^2/2mu + E_j$

Breakup cross section to bin energy E_j in direction $\Omega \equiv \hat{K}'$

$$\frac{d\sigma_{\rm bu}(E_j)}{dEd\Omega} = \frac{1}{2l_0+1} \sum_{m_0} \sum_{l,m} \left|\mathcal{F}_{m\,m_0}(\boldsymbol{K}')\right|^2$$

CDCC

Continuum Discretised Coupled-Channel : CDCC
[Austern *et al.* , Phys. Rep. 154, 125 (1987)]
[Tostevin, Nunes, Thompson, PRC 63, 024617 (2001)]
Recent review : [Yahiro *et al.* , PTEP 2012 01A206 (2012)]

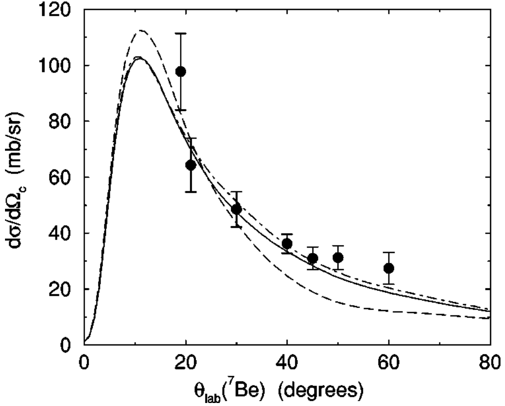
Fully quantal approximation
No approximation on P-T motion, nor restriction on energy
But expensive computationally (at high energies)

Various codes have been written to solve these coupled equations
FRESCO written by Ian Thompson is free on `www.fresco.org.uk`

Example : ^{8}B breakup

$$^{8}\mathrm{B} + {}^{58}\mathrm{Ni} \rightarrow {}^{7}\mathrm{Be} + \mathrm{p} + {}^{58}\mathrm{Ni} \quad @26\mathrm{MeV}$$

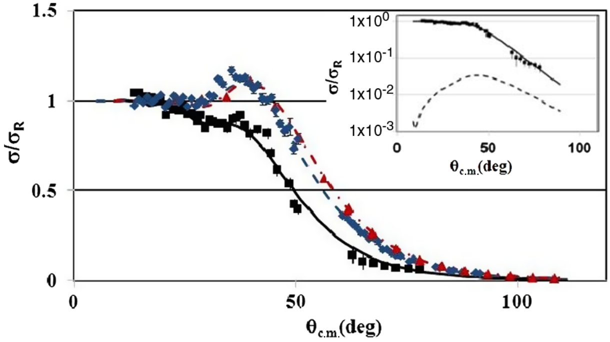
Exp. :[V. Guimarães *et al.* PRL 84, 1862 (2000)]



Th. :[Tostevin *et al.* PRC 63, 024617 (2001)]

Influence of breakup on the elastic channel

^{9,10,11}Be + Zn @ 24.5MeV

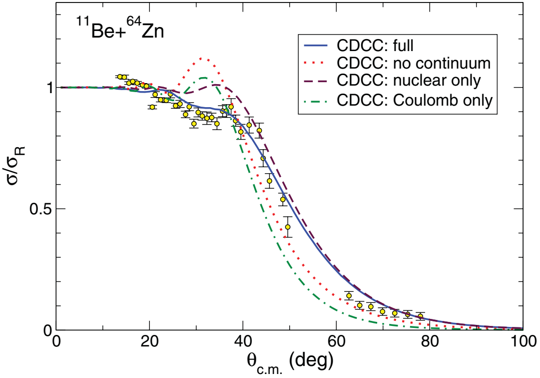


Exp. : [A. Di Pietro *et al.* PRL 105, 022701 (2010)]

- ^{9,10}Be elastic scattering reproduced with usual optical potentials
- ^{11}Be elastic scattering strongly affected by breakup channel

Influence of breakup on the elastic channel

^{9,10,11}Be + Zn @ 24.5MeV

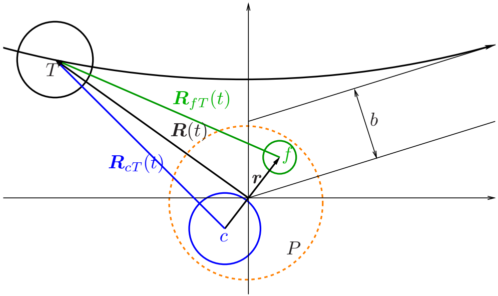


[A. Di Pietro *et al.* PRC 85, 054607 (2012)]

- ^{9,10}Be elastic scattering reproduced with usual optical potentials
- ^{11}Be elastic scattering strongly affected by breakup channel
  Confirmed by CDCC calculations

Time-dependent model

P-T motion described by classical trajectory $\boldsymbol{R}(t)$ defined by $V_{\rm traj}(\boldsymbol{R})$



P structure described quantum-mechanically by H_0
Time-dependent potentials simulate P-T interaction

$\Rightarrow$ time-dependent (TD) Schrödinger equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},\boldsymbol{b},t) = [H_0 + V_{cT}(t) + V_{fT}(t) - V_{\rm traj}(t)]\Psi(\boldsymbol{r},\boldsymbol{b},t)$$

Solved for each b with initial condition $\Psi^{(m_0)} \underset{t\to-\infty}{\longrightarrow} \phi_{n_0 l_0 m_0}$

Numerical resolution of the TD Schrödinger equation

Time-step evolution approximating the evolution operator

$$\Psi^{(m_0)}(\boldsymbol{r}, b, t+\Delta t) \quad = \quad U(t+\Delta t, t)\,\Psi^{(m_0)}(\boldsymbol{r}, b, t)$$

with $U(t', t) = \exp[\frac{i}{\hbar}\int_t^{t'} H(\tau)d\tau]$ and $\Psi^{(m_0)}(\boldsymbol{r}, b, t \to -\infty) = \phi_{n_0 l_0 m_0}(\boldsymbol{r})$

$$U(t+\Delta t, t) = e^{-\frac{i\Delta t}{\hbar} H_0}\, e^{-\frac{i\Delta t}{\hbar} V(t)} + O(\Delta t^2)$$

$$V(t) = \langle l' m' | V(t) | l m \rangle$$

Padé approximation

$$e^{-\frac{i\Delta t}{\hbar} A} \approx \left(1 + \frac{i\Delta t}{2\hbar} A\right)\left(1 - \frac{i\Delta t}{2\hbar} A\right)^{-1} + O(\Delta t^3)$$

Numerical resolution of the TD Schrödinger equation

- Faster computation compared to CDCC
  because each trajectory treated separately
- Lacks quantum interferences between trajectories

Many codes developed to solve TD

- Partial-wave expansion of Ψ :
  [Kido, Yabana, and Suzuki, PRC 50, R1276 (1994)]
  [Esbensen, Bertsch and Bertulani, NPA 581, 107 (1995)]
  [Typel and Wolter, Z. Naturforsch.A 54, 63 (1999)]
- Expansion on a 3D spherical mesh :
  [P. C., Melezhik and Baye, PRC 68, 014612 (2003)]
- Expansion on 3D cubic lattice : [Fallot *et al.* NPA700, 70 (2002)]

Semicassical breakup cross sections

For each trajectory (b) a breakup probability can be computed

$$\frac{dP_{\mathrm{bu}}(b)}{dE} = \frac{\mu_{cf}}{\hbar^2 k}\frac{1}{2l_0+1}\sum_{m_0}\sum_{l,m}|\langle\phi_{klm}|\Psi^{(m_0)}(b,t\to\infty)\rangle|^2$$

We can build an angular distribution from $b \leftrightarrow \theta$

$$\frac{d\sigma_{\mathrm{bu}}}{dE\,d\Omega} = \frac{d\sigma_{\mathrm{el}}}{d\Omega}\,\frac{dP_{\mathrm{bu}}[b(\theta)]}{dE},$$

where $d\sigma_{\mathrm{el}}/d\Omega$ is obtained from V_{traj}

And an energy distribution

$$\frac{d\sigma_{\mathrm{bu}}}{dE} = 2\pi\int_0^\infty \frac{dP_{\mathrm{bu}}(b)}{dE}\,b\,db$$

Initially TD equation solved perturbatively

At the first-order [Alder and Winther *Electromagnetic Excitation* (1975)]

At the first order of the perturbation theory

$$\langle \phi_{klm} | \Psi(\vec{r},t) \rangle \overset{F.O.}{=} \frac{e^{-\frac{iEt}{\hbar}}}{i\hbar} \int_{-\infty}^{t} e^{i\omega t'} \langle \phi_{klm} | V(t') | \phi_0 \rangle \, dt'$$

where $\omega = \dfrac{E-E_0}{\hbar}$

For purely Coulomb interaction

$$V(t) = \frac{Z_C Z_T e^2}{4\pi\varepsilon_0} \left[\frac{1}{R_{CT}(t)} - \frac{1}{R(t)} \right]$$

$$\vec{R}_{CT}(t) = \vec{R}(t) - \frac{m_f}{m_p} \vec{r}$$

$$\frac{1}{R_{CT}(t)} = \frac{1}{R(t)} \sum_{\lambda=0}^{\infty} \left[\frac{\frac{m_f}{m_p} r}{R(t)} \right]^{\lambda} P_\lambda(\cos\alpha)$$

if $r \ll R(t)$

$$\frac{1}{\left|\vec{R}(t) - \frac{m_f}{m_p}\vec{r}\right|} = \frac{1}{R(t)} 4\pi \sum_{\lambda=0}^{\infty} \frac{1}{2\lambda+1} \sum_{\mu=-\lambda}^{+\lambda} (-1)^{\mu} Y_{\lambda}^{-\mu}\left[\Omega_R(t)\right] Y_{\lambda}^{\mu}(\Omega_r) \left[\frac{\frac{m_f}{m_p} r}{R(t)}\right]^{\lambda}$$

If $\lambda = 1$ dominates (dipole approximation)

$$V(t) = \frac{4\pi}{3} \frac{Z_T \frac{m_f}{m_p} Z_c e^2}{4\pi\varepsilon_0} \frac{r}{R^2(t)} \sum_{\mu=-1}^{+1} (-1)^{\mu} Y_1^{-\mu}(\Omega_R(t)) Y_1^{\mu}(\Omega_r)$$

structure: $\langle \phi_{k\ell m} | \underbrace{r Y_1^{\mu}(\Omega_r)}_{E1} | \phi_0 \rangle$

reaction: $\displaystyle\int_{-\infty}^{+\infty} e^{i\omega t} \frac{Y_1^{-\mu}(\Omega_R(t))}{R^2(t)} dt$

First-Order Perturbation Theory

At the first-order of the perturbation theory

[Alder and Winther *Electromagnetic Excitation* (1975)]

$$\langle\phi_{klm}|\Psi^{(m_0)}(t\to\infty)\rangle = \int_{-\infty}^{\infty} e^{i\omega t}\left\langle\phi_{klm}\left|V_{cT}(t)+V_{fT}(t)-V_{\mathrm{traj}}(t)\right|\phi_{n_0l_0m_0}\right\rangle dt$$

with $\omega=(E-E_0)/\hbar$

For purely Coulomb P-T interaction the E1 contribution to breakup

$$\frac{dB(\mathrm{E1})}{dE} = \frac{\mu_{cf}}{\hbar^2 k}\frac{\left(\frac{m_f}{m_P}Z_c e\right)^2}{4\pi\epsilon_0}\frac{1}{2l_0+1}\sum_{m_0}\sum_{l,m}\left|\left\langle\phi_{klm}\left|rY_1^{m-m_0}\right|\phi_{n_0l_0m_0}\right\rangle\right|^2$$

$$\frac{d\sigma^{(1)}_{\mathrm{bu}}(\mathrm{E1})}{dE} = \frac{32\pi^2}{9}\frac{1}{4\pi\epsilon_0}\left(\frac{Z_T e}{\hbar v}\right)^2 x_{\min}K_0(x_{\min})K_1(x_{\min})\frac{dB(\mathrm{E1})}{dE}$$

with $x_{\min}=\omega b_{\min}/v$ and $v=\hbar K/\mu_{PT}$ the P-T relative velocity

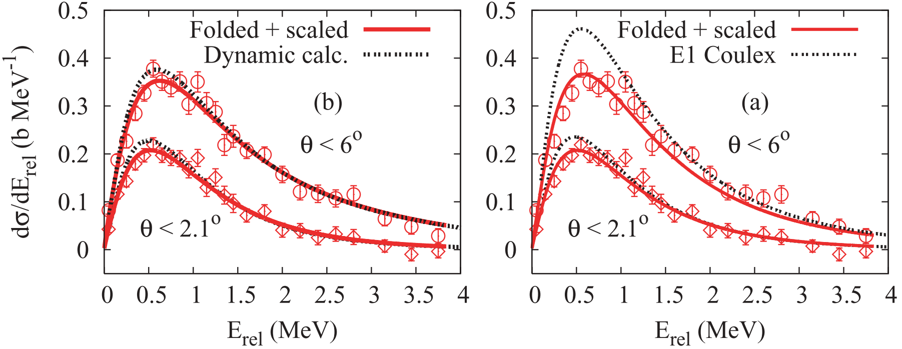
K_0 and K_1 are modified Bessel functions

Example : ^{15}C Coulomb breakup

$^{15}\mathrm{C} \equiv {}^{14}\mathrm{C}(0^+) + \mathrm{n}$

$$^{15}\mathrm{C} + {}^{208}\mathrm{Pb} \rightarrow {}^{14}\mathrm{C} + \mathrm{n} + {}^{208}\mathrm{Pb} \quad @68A\mathrm{MeV}$$

Exp. :[Nakamura *et al.* PRC 79, 035805 (2009)]



Th. :[Esbensen, PRC 80, 024608 (2009)]

Eikonal approximation

Three-body scattering problem :

$$\Big[T_R + H_0 + V_{cT} + V_{fT}\Big]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

with condition $\Psi^{(m_0)} \underset{Z\to-\infty}{\longrightarrow} e^{iKZ}\phi_{n_0 l_0 m_0}$

$$\psi^{(m_0)} = e^{iKZ}\,\hat{\psi}^{(m_0)}$$

$$\cancel{e^{iKZ}}\left[\frac{\hbar^2K^2}{2\mu_{PT}} - \frac{2i\hbar^2K}{2\mu_{PT}}\frac{\partial}{\partial Z} - \underbrace{\cancel{\frac{\hbar^2\Delta_R}{2\mu_{PT}}}}_{\text{negl.}}\right]\hat{\psi}^{(m_0)}$$

$$+\ \cancel{e^{iKZ}}\Big[H_0 + V_{cT} + V_{fT}\Big]\hat{\psi}^{(m_0)} = E_T\,\cancel{e^{iKZ}}\,\hat{\psi}^{(m_0)}$$

eikonal appr.

$$\Rightarrow \quad \Delta_R \ll K\frac{\partial}{\partial Z}$$

$$i\hbar v\frac{\partial}{\partial Z}\hat{\psi}^{(m_0)} = \Big[(H_0 - E_0) + V_{cT} + V_{fT}\Big]\hat{\psi}^{(m_0)}$$

where $E_T = \dfrac{\hbar^2K^2}{2\mu_{PT}} + E_0$, $v = \dfrac{\hbar K}{\mu_{PT}}$

Dynamical Eikonal Approximation (DEA)

$$i\hbar v\frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z) = [H_0 - E_{n_0 l_0} + V_{cT} + V_{fT}]\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z)$$

solved for each $\boldsymbol{b}$ with condition $\widehat{\Psi}^{(m_0)} \underset{Z\to-\infty}{\longrightarrow} \phi_{n_0 l_0 m_0}(\boldsymbol{r})$

This is the dynamical eikonal approximation (DEA)

[Baye, P. C., Goldstein, PRL 95, 082502 (2005)]

Equation is mathematically equivalent to TD with straight line trajectories $(Z = vt)$

⇒ we know how to solve it

A similar development can be done within CDCC

Leading to the Eikonal-CDCC (E-CDCC)

[Ogata *et al.* PRC 68, 064609 (2003) & PRC 73, 024605 (2006)]

Eikonal cross section

After some mathematical developments…

[Goldstein, Baye, P.C. PRC 73, 024602 (2006)]

$$\frac{d\sigma_{\rm bu}}{dEd\Omega} \propto \frac{1}{2l_0+1} \sum_{m_0} \sum_{lm} \left| \int_0^\infty J_{|m_0-m|}(qb)\, S^{(m_0)}_{klm}(b)\; bdb \right|^2 ,$$

$S^{(m_0)}_{klm}(b) = \langle \phi_{klm} | \widehat{\Psi}^{(m_0)}(Z \to \infty) \rangle$ are breakup amplitudes

$$\frac{d\sigma_{\rm bu}}{dEd\Omega} \xrightarrow{\int d\Omega} \frac{d\sigma_{\rm bu}}{dE}$$

$\Rightarrow$ Dynamical eikonal extends TD

takes into account interferences between *trajectories*

(sum of breakup amplitudes)

Usual Eikonal

$$i\hbar v \frac{\partial}{\partial Z}\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z) = [H_0 - E_{n_0 l_0} + V_{cT} + V_{fT}]\widehat{\Psi}(\boldsymbol{r},\boldsymbol{b},Z)$$

The usual eikonal uses adiabatic approx. $H_0 - E_{n_0 l_0} \sim 0$

$$\widehat{\Psi}^{(m_0)}_{\rm eik}(\boldsymbol{r},\boldsymbol{b},Z) = \exp\left\{-\frac{i}{\hbar v}\int_{-\infty}^{Z} dZ' \left[V_{cT}(\boldsymbol{r},\boldsymbol{b},Z') + V_{fT}(\boldsymbol{r},\boldsymbol{b},Z')\right]\right\}\phi_{n_0 l_0 m_0}(\boldsymbol{r})$$

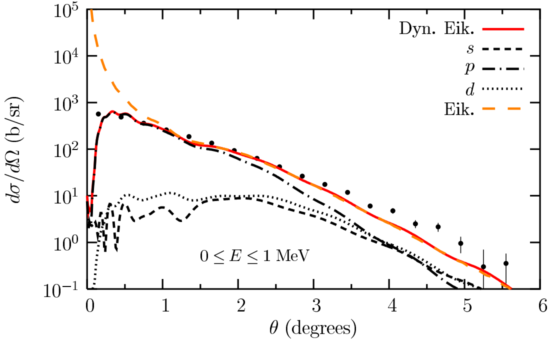
- Easy to interpret and implement
- Neglects internal dynamics of projectile

$\Rightarrow$ dynamical eikonal generalises eikonal

Example : ^{11}Be Coulomb breakup

$$^{11}\mathrm{Be} + {}^{208}\mathrm{Pb} \to {}^{10}\mathrm{Be} + \mathrm{n} + {}^{208}\mathrm{Pb} \quad @69A\mathrm{MeV}$$

Exp. :[Fukuda *et al.* PRC 68, 054606 (2004)]



Th. :[Goldstein, Baye, P.C. PRC 73, 024602 (2006)]

- DEA exhibits interferences (oscillations)
- Usual eikonal diverges at forward angles   Why ?

Coulomb Correction to the Eikonal approximation (CCE)

For a one-neutron halo nucleus, the eikonal Coulomb phase reads

$$\chi_C(\boldsymbol{r}, b) = -\eta \int_{-\infty}^{\infty} \frac{1}{R_{cT}} dZ \propto \frac{1}{b}$$

$\Rightarrow\ e^{i\chi_C} = 1 + i\chi_C - \frac{1}{2}\chi_C^2 + \ldots$ diverges when $\int db$ to compute $\sigma_{\rm bu}$

However, only because of $i\chi_C$ term : higher-order terms converge

Idea : replace $i\chi_C$ by $i\chi_{\rm FO}$ from perturbation theory

[Margueron, Bonaccorso, and Brink, NPA 720, 337 (2003)]

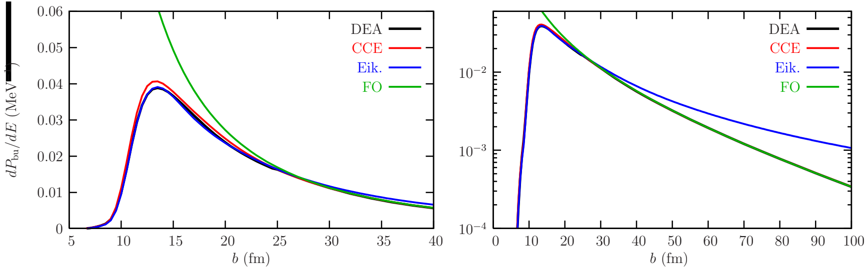
$$\chi_{\rm FO}(\boldsymbol{r}, b) = -\eta \int_{-\infty}^{\infty} e^{i\omega Z} \frac{1}{R_{cT}} dZ \propto \frac{e^{-\omega b}}{b},$$

which has the correct asymptotics

That Coulomb Correction to the Eikonal approximation (CCE) reads

$$e^{i\chi} = e^{i\chi_N} \left(e^{i\chi_C} - i\chi_C + i\chi_{\rm FO} \right)$$

^{11}Be Breakup on Pb @ $69A$ MeV

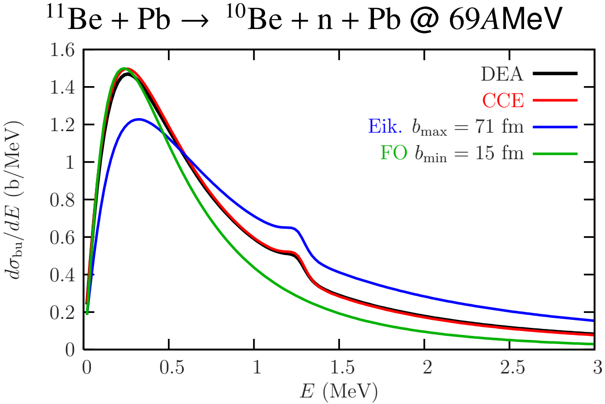


- Good agreement between CCE and DEA at all bs
- Eikonal ok at small b (nuclear) but $\propto 1/b$ at large b
- FO good asymptotic (Coulomb), but no nuclear

$\Rightarrow$ CCE combines advantages of eikonal and FO

[P. C., Baye and Suzuki, PRC 78, 054602 (2008)]

Energy Distribution

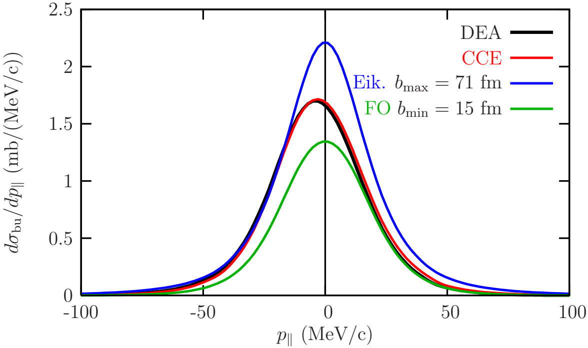


- Excellent agreement between CCE and DEA
- Eikonal needs cutoff at large b ; wrong shape
- FO needs cutoff at small b ; lacks nuclear

⇒ Confirms the validity of the Coulomb correction

Parallel-Momentum Distribution

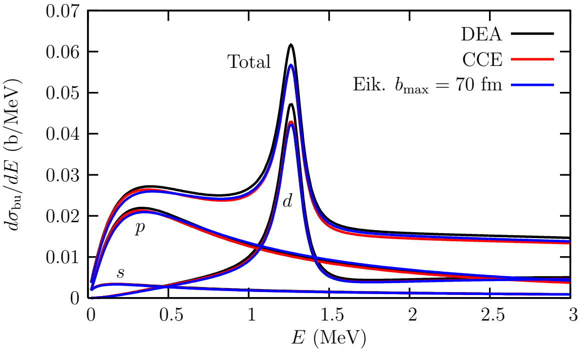
$^{11}\mathrm{Be} + \mathrm{Pb} \rightarrow {}^{10}\mathrm{Be} + \mathrm{n} + \mathrm{Pb}$ @ $69A$MeV



- Excellent agreement between CCE and DEA
  In particular the asymmetry in the distribution
- Eikonal too high and symmetric
- FO too low and symmetric

$\Rightarrow$ Coulomb correction restore dynamical effects

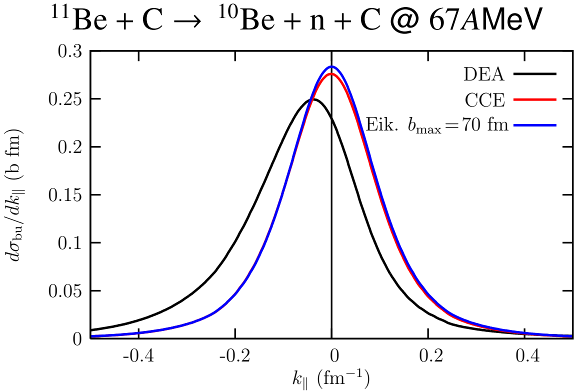
^{11}Be Breakup on C @ $67A$ MeV



- Very good agreement between all eikonal approximations (DEA, CCE, Eik.)
- Usual eikonal well suited for light targets
- Coulomb correction is minor in nuclear-dominated reactions

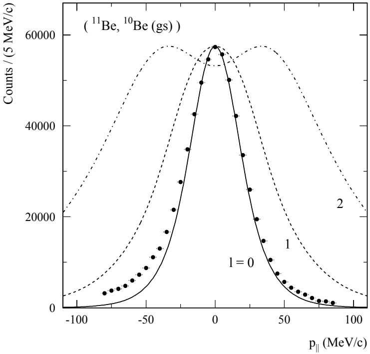
[P. C., Baye and Suzuki, PRC 78, 054602 (2008)]

Parallel-Momentum Distribution



- CCE and Eikonal are both symmetric
- They miss the asymmetry of DEA distribution
- Confirms the minor effect of Coulomb correction

Parallel-Momentum Distribution



[Aumann *et al.* PRL 85, 35 (2000)]

- CCE and Eikonal are both symmetric
- They miss the asymmetry of DEA distribution
- Confirms the minor effect of Coulomb correction
- Seen in analysis of KO data

$\Rightarrow$ missing dynamical effects on light targets

Simplified Dynamical Eikonal Approximation (S-DEA)

Idea : use χ_{FO} approximation instead of χ
and do that for both Coulomb and nuclear interactions

[Hebborn & Baye PRC 101, 054609 (2020)]

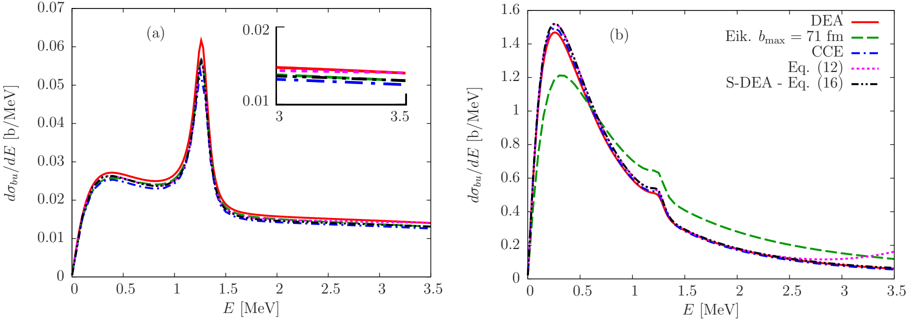
$$e^{i\chi^C} e^{i\chi^N} \rightarrow e^{i\chi^C_{\mathrm{FO}}} e^{i\chi^N_{\mathrm{FO}}}$$

with

$$\chi^N_{\mathrm{FO}} = -\frac{1}{\hbar v} \int_{-\infty}^{\infty} e^{i\frac{\omega Z}{v}} \left\{ e^{-i\frac{\omega m_c}{v m_P} z} V^N_{fT}(R_{fT}) + e^{i\frac{\omega m_f}{v m_P} z} V^N_{cT}(R_{cT}) \right\} dZ$$

^{11}Be breakup on C and Pb @ $\sim 70A$MeV

$^{11}\text{Be} + \text{C} \rightarrow {}^{10}\text{Be} + \text{n} + \text{C}$ @ $67A$MeV | $^{11}\text{Be} + \text{Pb} \rightarrow {}^{10}\text{Be} + \text{n} + \text{Pb}$ @ $69A$MeV



[Hebborn & Baye PRC 101, 054609 (2020)]

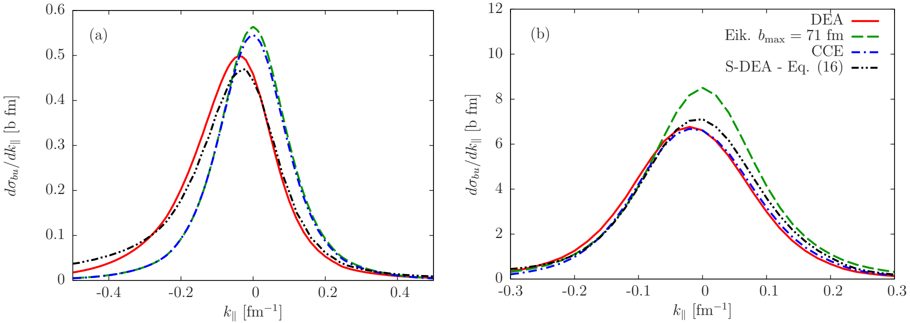
- All corrections reproduce DEA fairly well
- Problem at large energy with full optical potential [Eq. (12)]
- The S-DEA solves that issue using instead [Eq. (16)]

$$\chi^N_{\text{S-DEA}} = \Im\left\{\chi^N\right\} - \frac{1}{\hbar v}\int_{-\infty}^{\infty} e^{i\frac{\omega Z}{v}}\left\{e^{-i\frac{\omega m_c}{v m_P}z}\Re\left\{V^N_{fT}(R_{fT})\right\} + e^{i\frac{\omega m_f}{v m_P}z}\Re\left\{V^N_{cT}(R_{cT})\right\}\right\} dZ$$

Parallel-Momentum Distributions

$^{11}\text{Be} + \text{C} \rightarrow {}^{10}\text{Be} + \text{n} + \text{C}$ @ $67A$MeV

$^{11}\text{Be} + \text{Pb} \rightarrow {}^{10}\text{Be} + \text{n} + \text{Pb}$ @ $69A$MeV



[Hebborn & Baye PRC 101, 054609 (2020)]

- On C, S-DEA reproduces DEA
- Not on Pb, but problem is less dire

⇒ S-DEA could be a good approximation to study

- KO
- more complex nuclear structure (3-b projectiles)

Summary

- Breakup can be included assuming cluster structure of P
- Two-body structure leads to a three-body scattering probem
- CDCC : Ψ expanded over H_0 eigenstates
  - fully quantal model $\Rightarrow$ valid at all energies
  - requires continuum discretisation
  - heavy computationally
- Time-dependent : collision simulated by trajectory
  - semiclassical approximation $\Rightarrow$ no quantal interferences
  - simple interpretation and light numerically
- Eikonal : high-energy approximation
  - DEA : includes interferences and dynamic
    only valid at high energy
  - Usual eikonal : add adiabatic approximation
    $\Rightarrow$ not valid for Coulomb breakup
  - CCE and S-DEA offer efficient corrections
    for Coulomb treatment and dynamical effects

$\Rightarrow$ important to know the range of validity of the models you use