Breakup-Reaction Theory
Part I: An Introduction

Pierre Capel

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

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Outline

1. Monday : Introduction to Breakup Reactions
    - Rutherford Scattering
    - Optical Model
    - Model of Breakup Reaction of Halo Nuclei
    - PM : Setup of the project on breakup reactions
2. Tuesday : Computing Breakup Reactions
    - CDCC
    - Time-Dependant Approach
    - Eikonal Approximation and related models
3. Wednesday : Dynamics of Breakup Reactions
    - Couplings to and within the Continuum
    - Coulomb-Nuclear interferences
    - Application to Nuclear Astrophysics
4. Thursday : Extension of Breakup Models
    - 4-body CDCC (Breakup of Borromean Nuclei)
    - Core Excitation
5. Friday : Extension of Breakup Models
    - AM : Presentations of the Project Outcomes
    - PM : Wrap up

1. Charged Case
   - Rutherford Scattering
   - Coulomb + Nuclear Scattering
2. Optical Model
   - Reaction Cross Section
   - Optical Potential
3. Model of Breakup of Halo Nuclei
   - Halo nuclei
   - Reactions with Halo Nuclei
   - Breakup reaction
4. Reaction Project

Coulomb Scattering

We assumed $r^2 V(r) \underset{r\to\infty}{\longrightarrow} 0$, which excludes Coulomb $V_C(r) = \frac{Z_a Z_b e^2}{4\pi\epsilon_0 r}$

Coulomb requires special treatment, but similar results are obtained

Defining the Sommerfeld parameter $\eta = \frac{Z_a Z_b e^2}{4\pi\epsilon_0 \hbar v}$, v: relative a-b velocity

Schrödinger equation for a and b scattered by Coulomb reads

$$\left(\Delta - \frac{2\eta k}{r} + k^2\right)\Psi_C(\boldsymbol{r}) = 0,$$

which can be solved exactly and

$$\Psi_C(\boldsymbol{r}) \underset{r\to\infty}{\longrightarrow} (2\pi)^{-3/2}\left(e^{i\left[kz+\eta\ln k(r-z)\right]} + f_C(\theta)\frac{e^{i\left[kr-\eta\ln 2kr\right]}}{r}\right),$$

with $f_C(\theta) = -\frac{\eta}{2k\sin^2(\theta/2)} e^{2i\left[\sigma_0-\eta\ln\sin(\theta/2)\right]}$ $[\sigma_0 = \arg\Gamma(1+i\eta)]$

the Coulomb scattering amplitude

Rutherford cross section

The same analysis can be done defining $\boldsymbol{j}_i$ and $\boldsymbol{j}_s$
to define the Coulomb elastic scattering cross section
or Rutherford cross section :

$$\begin{aligned} \frac{d\sigma_R}{d\Omega} &= |f_C(\theta)|^2 \\ &= \left(\frac{Z_a Z_b e^2}{4\pi\epsilon_0}\right)^2 \frac{1}{16E^2 \sin^4(\theta/2)} \end{aligned}$$

Note that it diverges at $\theta = 0$

Partial-wave analysis

We can again separate the angular from the radial part solution of

$$\left(\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - \frac{2\eta k}{r} - \frac{2\mu}{\hbar^2} V_N(r) + k^2\right) u_{kl}(r) = 0$$

If additional (nuclear) term $r^2 V_N(r) \underset{r\to\infty}{\longrightarrow} 0$, $u_{kl}(r) \underset{r\to\infty}{\longrightarrow} u_{kl}^{\mathrm{as}}(r)$:

$$\begin{aligned} u_{kl}^{\mathrm{as}}(r) &= A\, F_l(\eta, kr) + B\, G_l(\eta, kr) \\ &\underset{r\to\infty}{\longrightarrow} A \sin(kr - l\pi/2 - \eta \ln kr + \sigma_l) \\ &\quad + B \cos(kr - l\pi/2 - \eta \ln kr + \sigma_l) \end{aligned}$$

where F_l and G_l are regular and irregular Coulomb functions
and $\sigma_l = \arg \Gamma(l + 1 + i\eta)$ is the Coulomb phaseshift

pose $A = C \cos \delta_l$ $\quad$ $B = C \sin \delta_l$

$$\Rightarrow u_{kl}^{\mathrm{as}}(r) \underset{r\to\infty}{\longrightarrow} C \sin(kr - l\pi/2 - \eta \ln kr + \sigma_l + \delta_l)$$

δ_l is an additional phaseshift,
which contains all information about the nuclear interaction V_N

(Additional) scattering amplitude

The stationary scattering states have now the asymptotic behaviour

$$\Psi(\boldsymbol{r}) \xrightarrow[r\to\infty]{} \Psi_C(\boldsymbol{r}) + (2\pi)^{-3/2} f_{\text{add}}(\theta) \frac{e^{i(kr-\eta \ln kr)}}{r}$$

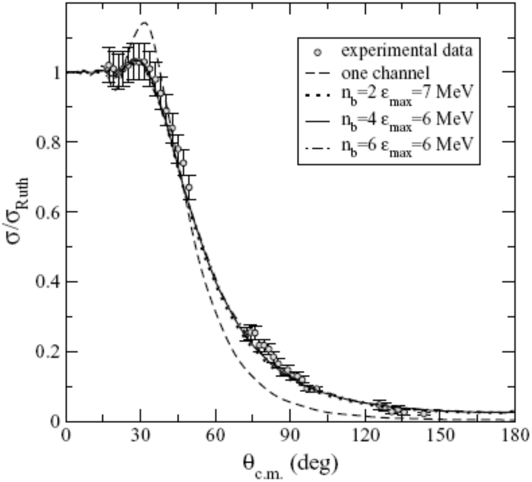
with $$f_{\text{add}}(\theta) = \frac{1}{2ik} \sum_{l=0}^{\infty} (2l+1) e^{2i\sigma_l} (e^{2i\delta_l} - 1) P_l(\cos\theta)$$

the additional scattering amplitude
The total scattering amplitude $f(\theta) = f_C(\theta) + f_{\text{add}}(\theta)$
gives the elastic-scattering cross section

$$\boxed{\frac{d\sigma}{d\Omega} = |f_C(\theta) + f_{\text{add}}(\theta)|^2}$$

At forward angles ($\theta \ll 1$), $f_C \gg f_{\text{add}}$, and $d\sigma/d\Omega \approx d\sigma_R/d\Omega$
$\Rightarrow$ usually $(d\sigma/d\Omega)/(d\sigma_R/d\Omega)$ is plotted

Example : $^{6}He + {}^{64}Zn$ @ 14MeV



[Rodrìguez-Gallardo *et al.* PRC 77, 064609 (2008)]

Reaction cross section

So far we have described only elastic scattering

Other channels can be open, like transfer : $a + b \to d + e$

We can define a differential cross section for these other channels

$$\frac{d\sigma}{d\Omega}(a + b \to d + e) = \lim_{r\to\infty} \frac{r^2 j_{d+e}}{j_i}$$

The sum of all channels but elastic scattering (inelastic, transfer, breakup,. . .) gives the reaction cross section

$$\sigma_r = \sum_{\text{channel}\backslash a+b} \sigma(a + b \to \text{channel})$$

The interaction cross section corresponds to all channels but elastic and inelastic scattering

$$\sigma_I = \sum_{\text{channel}\backslash(a+b)\cup(a+b^*)\cup(a^*+b)\cup(a^*+b^*)} \sigma(a + b \to \text{channel})$$

Optical Model

Using real scattering potential V implies that $\nabla \boldsymbol{j} = 0$

$\Leftrightarrow$ flux stays in elastic channel

To simulate other channels, use complex potential

$$U_{\mathrm{opt}}(\boldsymbol{r}) = V(\boldsymbol{r}) + iW(\boldsymbol{r}) \quad \Rightarrow \quad -\frac{\hbar^2}{2\mu}\Delta\Psi + U_{\mathrm{opt}}\Psi = E\Psi$$

$$\vec{\nabla}\vec{j} = \vec{\nabla}\frac{1}{\mu} \mathrm{Re}\left\{\psi^*(-i\hbar)\vec{\nabla}\psi\right\}$$

$$= \frac{\hbar}{2\mu}\vec{\nabla}\left(\psi^*(-i)\vec{\nabla}\psi + \psi\, i\vec{\nabla}\psi^*\right)$$

$$= \frac{-i\hbar}{2\mu}\left[\cancel{(\vec{\nabla}\psi^*)\vec{\nabla}\psi} + \psi^*\Delta\psi - \cancel{(\vec{\nabla}\psi)\vec{\nabla}\psi^*} - \psi\Delta\psi^*\right]$$

$$= -\frac{i\cancel{\hbar}}{\cancel{2\mu}}\frac{\cancel{2\mu}}{\hbar^{\cancel{2}}}\left[\psi^*(U_{\mathrm{opt}} \cancel{- E})\psi - \psi(U^*_{\mathrm{opt}} \cancel{- E})\psi^*\right]$$

$$\vec{\nabla}\vec{j} = -\frac{i}{\hbar}\left[\psi^* V_{opt}\psi - \psi V_{opt}^*\psi^*\right]$$

$$= -\frac{i}{\hbar}\, 2i\, W(\vec{r})\, |\psi(\vec{r})|^2$$

$$= \frac{2}{\hbar}\, W(\vec{r})\, |\psi(\vec{r})|^2$$

$$\leq 0 \quad \forall \vec{r} \quad \Rightarrow \quad W(\vec{r}) \leq 0 \quad \forall \vec{r}$$

Partial-wave expansion

The optical potential U_{opt} leads to a complex phaseshift :

$$\delta_l = \Re(\delta_l) + i\Im(\delta_l)$$

with $\Im(\delta_l) \geq 0$ (because $W \leq 0$)

$$\Rightarrow S_l = \eta_l \, e^{2i\Re(\delta_l)}$$
$$\text{where} \quad \eta_l = e^{-2\Im(\delta_l)} < 1$$

simulates the absorption from the elastic channel in

$$u_{kl}(r) \underset{r\to\infty}{\longrightarrow} \propto \left[e^{-i(kr-l\pi/2)} - \underbrace{\eta_l}_{<1} \, e^{2i\Re(\delta_l)} \; e^{i(kr-l\pi/2)} \right]$$

outgoing amplitude is reduced compared to incoming wave
there is a loss of flux simulating the other (open) channels

The name optical model comes from optics, where a complex refraction index simulates the absorption of light by the medium

Absorption cross section

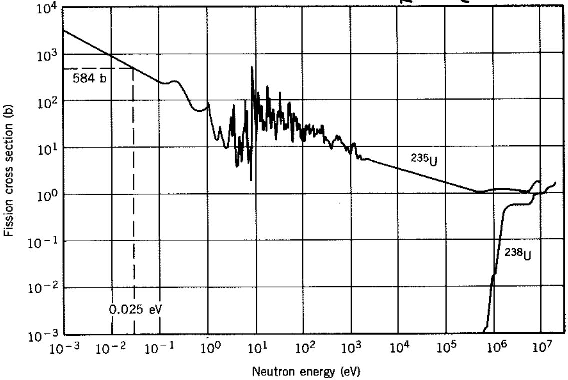
The absorption cross section σ_a
corresponds to all other channels simulated by U_{opt} :

$$
\begin{aligned}
\sigma_a &= \frac{-\int \nabla \boldsymbol{j}\, d^3\boldsymbol{r}}{j_i} \propto \int W(\boldsymbol{r})\, |\Psi(\boldsymbol{r})|^2\, d\boldsymbol{r} \\
&= \frac{-\lim_{r\to\infty} \oint \boldsymbol{j} \cdot \hat{\boldsymbol{r}}\, r^2 d\Omega}{j_i} \\
&= \frac{\pi}{k^2} \sum_{l=0}^{\infty} (2l+1)\left(1-\eta_l^2\right)
\end{aligned}
$$

It can be compared to the reaction cross section σ_r

Induced Fission (n,f)

$$\sigma_a = \frac{\pi}{k^2} \sum_{\ell} (2\ell+1)(1-\eta_\ell^2)$$



At low neutron energy ($E_n < 0.1$ eV),
the fission cross section of ^{235}U exhibits a simple behaviour

- What is the mathematical expression of that behaviour? $\sigma_{nf} \propto \frac{1}{\sqrt{E}}$
- Assuming that, in addition to elastic scattering, only the induced fission channel (n,f) is open, explain that behaviour on theoretical grounds

- $\sigma_{r,f} \propto \dfrac{1}{\sqrt{E}}$
- $\sigma_{r,f} \simeq \sigma_a = \dfrac{\pi}{k^2} \sum_\ell (2\ell+1)\left(1 - \eta_\ell^2\right)$

low energy $\Rightarrow$ 1) only $\ell = 0$ matters

$$\Rightarrow \sigma_a = \frac{\pi}{k^2}\left(1 - \eta_0^2\right)$$

$\eta_0 = e^{-2\,\mathrm{Im}\,\delta_0}$ and since $\delta_0 \xrightarrow[k \to 0]{} -a\,k$

$\simeq e^{-2k\,\mathrm{Im}\,a} \simeq 1 - 2k\,\mathrm{Im}\,a$

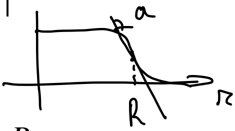
$$\Rightarrow |\eta_0|^2 \simeq 1 - 4k\,\mathrm{Im}\,a + \mathcal{O}(k^2)$$

$$\Rightarrow \sigma_a \simeq -\frac{4\pi}{k}\,\mathrm{Im}\,a \propto \frac{1}{\sqrt{E}}$$

Shape of Optical Potentials

Most optical potentials are expressed in a Woods-Saxon form

$$
\begin{aligned}
U(r) &= V f(r, R_R, a_R) + iW f(r, R_I, a_I) \\
&\quad + iW_D\, a_D \left| \frac{\partial}{\partial r} f(r, R_D, a_D) \right| + V_C(r, R_C)
\end{aligned}
$$



with $\quad f(r, R, a) = \left[1 + \exp\left(\frac{r - R}{a} \right) \right]^{-1}$

and $\quad V_C(r, R_C) = \begin{cases} \frac{Z_a Z_b}{2R_C} \frac{e^2}{4\pi\epsilon_0} \left(3 - \frac{r^2}{R_C^2} \right) & r < R_C \\ \frac{Z_a Z_b}{r} \frac{e^2}{4\pi\epsilon_0} & r \geq R_C \end{cases}$

Often radii are parametrised as $R_i = r_i\, A^{1/3}$ or $R_i = r_i\, (A_a^{1/3} + A_b^{1/3})$

V, W, and $W_D < 0$ (nuclear interaction is attractive and absorptive)

Usually parameters (depths, radii and diffusenesses) fitted to data

$\Rightarrow$ accurate reproduction of experiment

But no predictive power : you need data for the exact collision
and at the exact energy you want

An Example

Bonin et al. in NPA 445, 381 (1985) have fitted optical potentials to reproduce α scattering on various data (Ni, Sn and Pb)

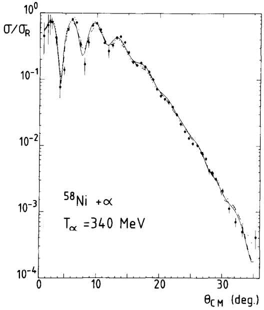


Fig. 3. Comparison of the fits for ^{58}Ni at 340 MeV obtained with the parameter set 1 (dotted line), 2 (dashed line), and 3 (full line) of table 2.

TABLE 2

Optical potentials

Set	T_α [MeV]	V [MeV]	W [MeV]	r_R [fm]	r_I [fm]	a_R [fm]	a_I [fm]	β_2	β_3
				(a) $\alpha + {}^{58}$Ni system					
1	288	55.9	19.0	1.000	1.010	0.799	0.789		
2		71.0	31.0	0.887	0.883	0.983	0.756		
3		71.8	24.2	0.935	1.000	0.875	0.820	0.213	
1	340	51.2	22.3	1.000	1.010	0.828	0.808		
2		67.7	34.4	0.900	0.932	0.962	0.793		
3		65.6	26.8	0.935	1.000	0.875	0.820	0.204	
1	480	43.0	34.5	0.980	0.960	0.872	0.783		
2		59.0	46.0	0.894	0.927	0.893	0.803		
3		50.9	43.4	0.935	0.929	0.875	0.820	0.205	
1	699	44.2	28.9	0.920	0.890	0.861	0.819		
2		53.6	49.4	0.844	0.783	0.946	0.879		
3		44.8	42.3	0.935	0.898	0.875	0.820	0.191	

All three sets reproduce the data well, but with different values
Showing these potentials are by no means unique

(potentials are not observables)

Where to find optical potentials ?

Somewhere in the literature…

In the past, there were compilations, e.g.
Perey and Perey, At. Data Nucl. Data Tables 17, 1 (1976)

For some projectiles (p, n, d, α…) global optical potentials exist : parameters are expressed as functions of energy, N and Z of target

- p,n :
  - Becchetti and Greenlees Phys. Rev. 182, 1190 (1969)
  - Chapel Hill : Varner et al. Phys. Rep. 201, 57 (1991)
  - Koning, Delaroche, NPA 713, 231 (2003)
- α
  - Nolte et al. PRC 36, 1312 (1987)

Collection of optical potentials that computes the parameters :
`https://sites.google.com/view/opticalpotentials/`

Before using them, check their range of validity (target, energy etc.)

Know the sensitivity of your calculations to that choice

Modern Optical Potentials

Recently : efficient optical potentials derived from first principles

For nucleon-nucleus, starting from χEFT NN interactions :

- Rotureau et al. PRC 98, 044625 (2018)
- Vorabbi et al. PRC 98, 064602 (2018)
- Idini et al. PRL 123, 092501 (2019)

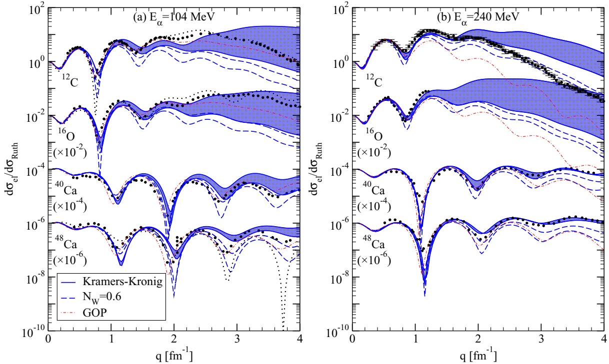
For nucleus-nucleus, using a double-folding technique :

- Chamon et al. PRC 66, 014610 (2002)
- Furutomo et al. PRC 85, 044607 (2012)
- Khoa et al. PRC 94, 034612 (2016)
- Durant et al. PLB 782, 668 (2018)

They are produced in a numerical form

An Example of Double-Folding Optical Potential

Victoria Durant et al. in PRC 105, 014606 (2022) have developed α-nucleus optical potentials from double folding of N^{2}LO χEFT $V_{\rm NN}$



Very good agreement with data (no parameter fitting)

Sensitivity to cutoff R_0 mostly at large angle and on light targets

$\Rightarrow$ stronger predictive power than phenomenological potentials

Bibliography

The following books are good references for more details on low-energy nuclear-reaction theory :

- C. J. Joachain *Quantum Collision Theory* (North-Holland, Amsterdam, 1975)
- C. Cohen-Tannoudji, B. Diu & F. Laloë *Quantum Mechanics*, Vol.2 (John Wiley & Sons, Paris, 1977)
- C. A. Bertulani & P. Danielewicz *Introduction to Nuclear Reations* (Institute of Physics, London, 2004)
- I. J. Thompson & F. M. Nunes *Nuclear Reactions for Astrophysics : Principles, Calculation and Applications of Low-Energy Reactions* (Cambridge University Press, 2009)
- J. R. Taylor *Scattering Theory : The Quantum Theory of Nonrelativistic Collisions* (Dover, New York, 1972)

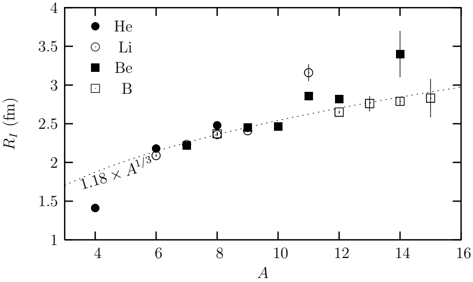
(One of the) First Experiments with Unstable Nuclei...

In the mid-80s, Isao Tanihata used RIBs to measure interaction cross sections of light exotic nuclei.

[I. Tanihata *et al.* PRL 55, 2676 (1985)]

In a simple geometrical model

$$\sigma_I(P,T) = \pi[R_I(P) + R_I(T)]^2$$



Some nuclei appear larger : ^{6}He, ^{11}Be, ^{11}Li,...

$\Rightarrow$ large collective deformation or exotic structure ?

Role of valence neutrons

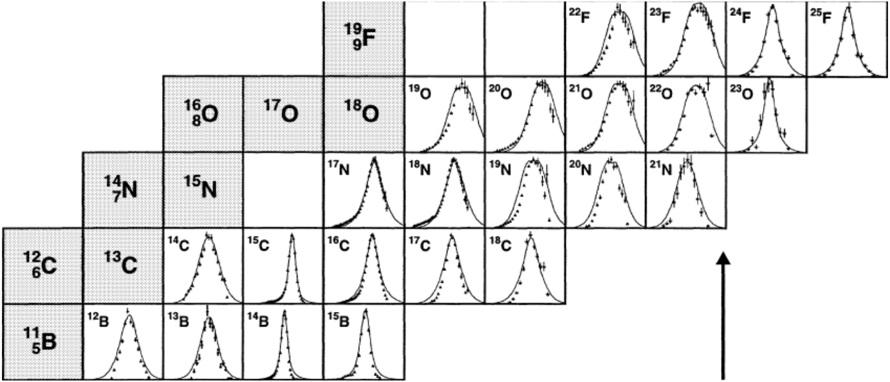
The large σ_I is due to valence neutrons :

Nucleus	core	$\sigma_I - \sigma_I(c)$	$\sigma_{-\mathrm{n}}$ or $\sigma_{-2\mathrm{n}}$
^{11}Be	^{10}Be	129 ± 18 mb	169 ± 4 mb
^{11}Li	^{9}Li	251 ± 46 mb	213 ± 21 mb

[I. Tanihata, J. Phys. G, 22, 157 (1996)]

Parallel-momentum distributions

These nuclei exhibit also a narrow parallel-momentum distribution in one-neutron removal reaction

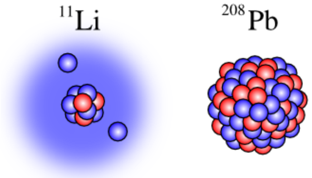


[E. Sauvan *et al.* PLB, 491, 1 (2000)]

Sign of an extended spacial core-neutron distribution

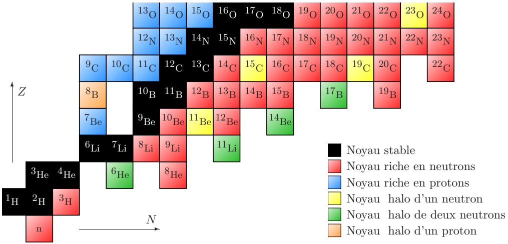
Halo nuclei

- Light, neutron-rich nuclei
- small S_n or S_{2n}
- low-ℓ orbital



One-neutron halo
^{11}Be ≡ ^{10}Be + n
^{15}C ≡ ^{14}C + n

Two-neutron halo
^{6}He ≡ ^{4}He + n + n
^{11}Li ≡ ^{9}Li + n + n



Proton halœs are possible but less probable : ^{8}B, ^{17}F

Two-neutron halo nuclei are Borromean. . .

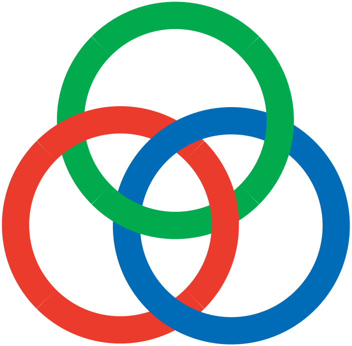
c+n+n is bound but not two-body subsystems

e.g. ^{6}He bound but not ^{5}He or ^{2}n

Borromean nuclei

Named after the Borromean rings. . .

[M. V. Zhukov *et al.* Phys. Rep. 231, 151 (1993)]



Reactions with Halo Nuclei

Halo nuclei exhibit a very exotic structure

However difficult to study because of short lifetime : $\tau_{1/2}(^{11}\mathrm{Be}) = 13$ s

$\Rightarrow$ often studied through reactions :

- elastic scattering

$$^{11}\mathrm{Be} + \mathrm{Zn} \rightarrow \ ^{11}\mathrm{Be} + \mathrm{Zn}$$

- knockout, e.g., one-neutron removal :

$$^{11}\mathrm{Be} + \mathrm{C} \rightarrow \ ^{10}\mathrm{Be} + \mathrm{X}$$

- transfer, e.g. (d,p) :

$$^{10}\mathrm{Be} + \mathrm{d} \rightarrow \ ^{11}\mathrm{Be} + \mathrm{p}$$

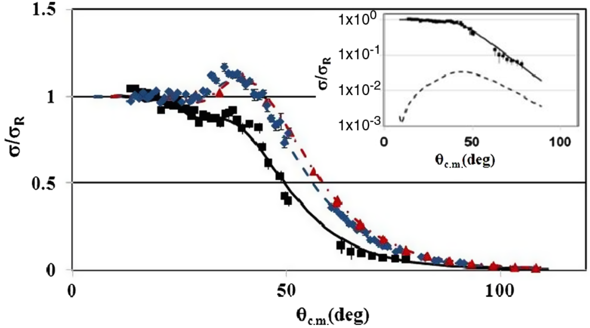
- (elastic) breakup :

$$^{11}\mathrm{Be} + \mathrm{Pb} \rightarrow \ ^{10}\mathrm{Be} + \mathrm{n} + \mathrm{Pb}$$

Elastic Scattering

Di Pietro et al. PRL 105, 022701 (2010) have measured the scattering (elastic and inelastic) @ $E_{\mathrm{cm}} \approx 24.5$ MeV

$$^{9,10,11}\mathrm{Be} + {}^{64}\mathrm{Zn} \rightarrow {}^{9,10,11}\mathrm{Be} + {}^{64}\mathrm{Zn}$$

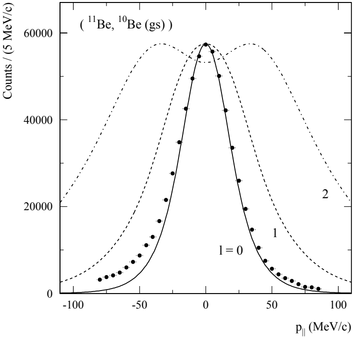


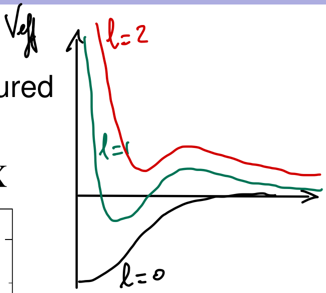
- ^{9,10}Be scattering cross sections exhibit usual behaviours can be reproduced by usual optical potentials
- ^{11}Be scattering cross section seems depleted @ maximum optical potential needs long-ranged imaginary term to fit data $\Rightarrow$ effect of halo ?

Knockout

Aumann et al. PRL 85, 35 (2000) have measured the one-neutron KO @ $E = 60A$ MeV

$$^{11}\text{Be} + {}^{9}\text{Be} \rightarrow {}^{10}\text{Be} + \gamma + \text{X}$$



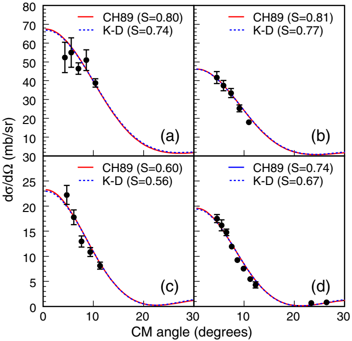


- Measurement well described by eikonal calculation
- Indicates that $^{11}\text{Be} \equiv {}^{10}\text{Be}(0^+) \otimes \text{n}(s_{1/2}) \Rightarrow$ confirms halo
- $l = 1, 2$ would give broader distributions   Why ?

Transfer

Schmitt et al. PRL 108, 192701 (2012) have measured the transfer @ E_d = (a) 12, (b) 15, (c) 18, (d) 21.4 MeV

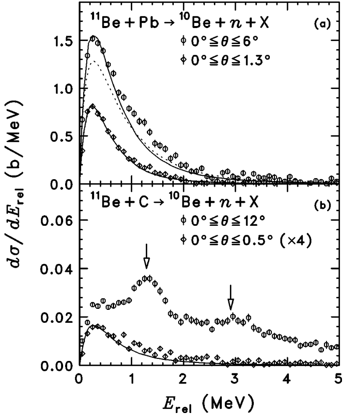
$$^{10}\mathrm{Be} + \mathrm{d} \rightarrow\ ^{11}\mathrm{Be} + \mathrm{p}$$



- Measurement well described by DWBA calculation
- Confirms that $^{11}\mathrm{Be} \equiv\ ^{10}\mathrm{Be}(0^+) \otimes \mathrm{n}(s_{1/2}) \Rightarrow$ halo

Breakup

Fukuda et al. PRC 70, 054606 (2004) have measured the breakup @ $E =$ (a) $69A$ MeV and (b) $67A$ MeV



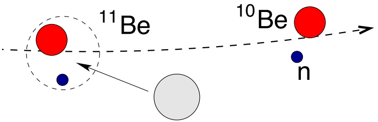
- Large cross sections well described by 1st-order calculations
  $\Rightarrow$ confirms the halo structure in $^{11}\mathrm{Be}$
- Some bumps on C target corresponding to resonances ?
- Goal : learn more on few-body model of breakup

Breakup reaction

Breakup $\equiv$ dissociation of projectile in constituent clusters by interaction with target

$$^{11}\mathrm{Be} + {}^{12}\mathrm{C} \rightarrow {}^{10}\mathrm{Be} + \mathrm{n} + {}^{12}\mathrm{C}$$
$$^{8}\mathrm{B} + {}^{208}\mathrm{Pb} \rightarrow {}^{7}\mathrm{Be} + \mathrm{p} + {}^{208}\mathrm{Pb}$$

The target T acts differently on projectile P contituents
$\Rightarrow$ tidal force $\rightarrow$ breakup



[Figure by A. Moro]

Breakup reaction

Breakup ≡ dissociation of projectile in constituent clusters
by interaction with target

$$
\begin{aligned}
{}^{11}\mathrm{Be} + {}^{12}\mathrm{C} &\rightarrow {}^{10}\mathrm{Be} + \mathrm{n} + {}^{12}\mathrm{C} \\
{}^{8}\mathrm{B} + {}^{208}\mathrm{Pb} &\rightarrow {}^{7}\mathrm{Be} + \mathrm{p} + {}^{208}\mathrm{Pb}
\end{aligned}
$$

The target T acts differently on projectile P contituents
$\Rightarrow$ tidal force $\rightarrow$ breakup

Used to

- study cluster structure in nuclei e.g. halo nuclei
- infer reaction rates of astrophysical interest

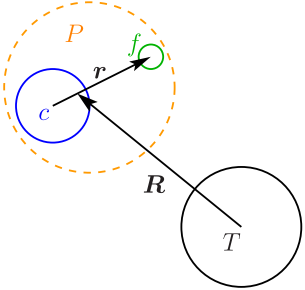
Need a good understanding of the reaction mechanism
i.e. an accurate theoretical description of reaction
coupled to a realistic model of projectile

Elastic breakup ≡ all clusters measured in coincidence
(exclusive measurement)

Framework

Projectile (P) modelled as a two-body system :
core (c)+loosely bound fragment (f) described by

$$H_0 = T_r + V_{cf}(\boldsymbol{r})$$



V_{cf} adjusted to reproduce
P spectrum

Target T seen as
structureless particle

P-T interaction simulated by optical potentials

$\Rightarrow$ breakup reduces to three-body scattering problem :

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$

Projectile Hamiltonian H_0

$$H_0 = -\frac{\hbar^2 \Delta_r}{2\mu_{cf}} + V_{cf}(r)$$

V_{cf} has usually a Woods-Saxon form factor

$$V_{cf}(r) = \frac{V_0}{1 + e^{(r-R_0)/a}}$$

Halo-EFT is a more efficient alternative... (see Daniel Phillips' classes)

c-f relative motion described by H_0 eigenstates

- $E_{nl} < 0$: discrete set of bound states $H_0\ \phi_{nlm}(\boldsymbol{r}) = E_{nl}\ \phi_{nlm}(\boldsymbol{r})$
- $E > 0$: c-f continuum $\equiv$ broken up projectile

$H_0\ \phi_{klm}(\boldsymbol{r}) = E\ \phi_{klm}(\boldsymbol{r})$ where $E = \hbar^2 k^2/2\mu_{cf}$

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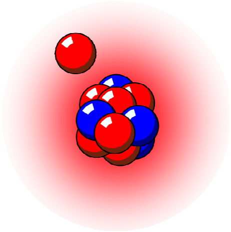
Breakup ≡ transition from bound state to continuum
through interaction with target (Coulomb and nuclear)

Breakup can take place in one or more steps
will be sensitive to both bound and continuum states

Example : $^{11}\mathrm{Be}$



5/2$^+$	1.274	$d5/2$
	$^{10}\mathrm{Be} + \mathrm{n}$	
1/2$^-$	-0.184	$0p1/2$
1/2$^+$	-0.504	$1s1/2$

$^{11}\mathrm{Be}$ spectrum

$$^{11}\mathrm{Be} \equiv {}^{10}\mathrm{Be}(0^+) + \mathrm{n}$$

$^{10}\mathrm{Be}$ cluster assumed in 0^+ ground state (extreme shell model) (see Daniel's classes)

$\Rightarrow$ spin and parity of $^{11}\mathrm{Be}$ states fixed by angular momenta l and j of n :

- $1/2^+$ ground state in $s1/2$
- $1/2^-$ excited state in $p1/2$
- $5/2^+$ resonance in $d5/2$

$\Rightarrow$ fit V_{cf} in $s1/2$, $p1/2$ and $d5/2$ waves (but not in $p3/2\ldots$)

Projectile-target interaction : V_{cT} and V_{fT}

The breakup channel is now included in the collision description

However other channels not included :

- absorption of the fragment by the target
- breakup of the core
- …

c-T and f-T interactions described by optical potentials V_{cT} and V_{fT}
Their imaginary parts simulate the other channels

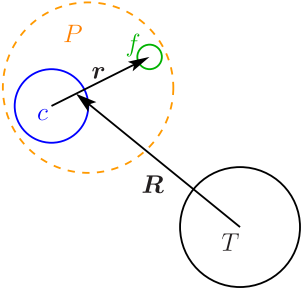
Usually chosen in the literature

- V_{cT} : problematic if c-T scattering not measured
  $\Rightarrow$ extrapolate what exists or use folding technique
- V_{fT} : many N-T global potentials exist
  [Becchetti and Greenlees, Phys. Rev. 182, 1190 (1969)]
  Chapel Hill : [Varner et al. Phys. Rep. 201, 57 (1991)]
  [Koning and Delaroche NPA 713, 231 (2003)]

Three-body Scattering Problem

Within this framework breakup reduces to three-body problem

$$\left[T_R + H_0 + V_{cT} + V_{fT}\right]\Psi(\boldsymbol{r},\boldsymbol{R}) = E_T\Psi(\boldsymbol{r},\boldsymbol{R})$$



with the initial condition

$$\Psi(\boldsymbol{r},\boldsymbol{R}) \underset{Z\to-\infty}{\longrightarrow} e^{iKZ+\cdots}\phi_{n_0 l_0 m_0}(\boldsymbol{r})$$

$\Leftrightarrow$ P in its ground state $\phi_{n_0 l_0 m_0}$ impinging on T

Various methods developed to solve that equation

[Review : Baye, P.C., Lecture Notes in Physics 848, 121 (2012) ; on Indico]

- Coupled-channel method with discretised continuum (CDCC)
- Time-dependent approach (TD) (semiclassical)
- Eikonal approximation
- ...

Reaction Project

During the exercise sessions, you'll be computing and analysing breakup cross sections of halo nuclei

- You'll be using the Fortran code `Chaconne.f` (see Indico)
  That code implements the Coulomb Corrected Eikonal (CCE)
  - runs fast (a few minutes at most)
  - accounts for the P-T interaction at all orders
  - includes a 1st order correction of the Coulomb interaction
- You pick one (or two, or all...) of the reactions
  - ^{11}Be + Pb → ^{10}Be + n + Pb @$69A$ MeV
    [Fukuda *et al.* PRC 70, 054606 (2004)]
  - ^{11}Be + C → ^{10}Be + n + C @$67A$ MeV
    [Fukuda *et al.* PRC 70, 054606 (2004)]
  - ^{15}C + Pb → ^{15}C + n + Pb @$68A$ MeV
    [Nakamura *et al.* PRC 79, 035805 (2009)]
  - ^{19}C + Pb → ^{19}C + n + Pb @$67A$ MeV
    [Nakamura *et al.* PRL 83, 1112 (1999)]

Reaction Project

- Study the reaction :
  - develop a V_{cf} interaction (within Halo EFT)
    (use the code `Boscos.f` to fit the interaction, see Indico)
  - find suitable optical potentials V_{cT} and V_{fT}
  - check the convergence
  - compare to existing data (available on Indico)
    There are energy and angular distributions
    Don't forget to account for the experimental resolution
  - analyse the agreement/differences with experiment
- Work in groups of ~~4~~ 3
  (make sure that one of you has a computer to run the code)
- Friday morning, present the results of your study to the others
- This afternoon session is to decide on the system and set V_{cf}

Resources on Indico

- Codes `Boscos.f` (structure) and `Chaconne.f` (reaction)
  with short user's manuals
  and examples of input files (`*.dat` files)
  and output files (`*.dep` and `*.sdE` files)
- Experimental data (`*.dat` and `*.rtf` files)
  - projectile and target are self-explanatory
  - `erel_*.*` are energy distributions ($d\sigma_{\rm bu}/dE$) obtained after integration over angular range
  - `angle_*.*` are angular distributions ($d\sigma_{\rm bu}/d\Omega$) obtained after integration over a definite energy range

  Details about the beam energy, experimental resolution etc.
  can be found in the original articles,
  which are provided in that same folder.