Snowmass report on « Weak Decays of Strange and Light Quarks »

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Electroweak Precision Physics from Beta Decays to the Z Pole
Mainz Institute for Theoretical Physics, October 24 – 29, 2022

Based on *Snowmass report: ArXiv: 2209.07156 [hep-ex]*
Goudzovski et al.

Outline

1. Introduction and Motivation
2. First row CKM unitarity
3. Charged pion decays
4. Conclusion and Outlook

1. Introduction and Motivation

1.1 Weak Decays of Strange and Light Quarks Rare Frontier (RF2)

- Flavor physics experiments probe both *very high mass scales*, and *feebly interacting hidden sectors*.

- RF2: precision measurements of kaon, hyperon, π and η(') decays
  - ➢ CKM parameter measurements and unitary tests; symmetry tests; lepton flavor/number conservation tests; lepton universality tests
  - ➢ Heavy new physics: sensitivity up to the PeV mass scale
  - ➢ Hidden sectors: leading sensitivity below the GeV mass scale

1.1 Weak Decays of Strange and Light Quarks Rare Frontier (RF2)

- Vibrant experimental activities
  - Ultra-rare kaon decays at *NA62* and *KOTO* (+ future projects)
  - CPV in hyperon decays at *BESIII* (+ future super charm-tau factories)
  - Kaon and hyperon decays at *LHCb*
  - LFU and V_{ud} in pion decays at *PIONEER*
  - Symmetry tests at $\eta(')$ factories: *JEF* + *REDTOP* proposal

- Significant advances in theory and lattice QCD: crucial for progress.

- Medium-scale initiatives (many centered in Europe and Asia)
  - powerful physics insights
  - relatively short time scales
  - superb training opportunities
  - modest investment

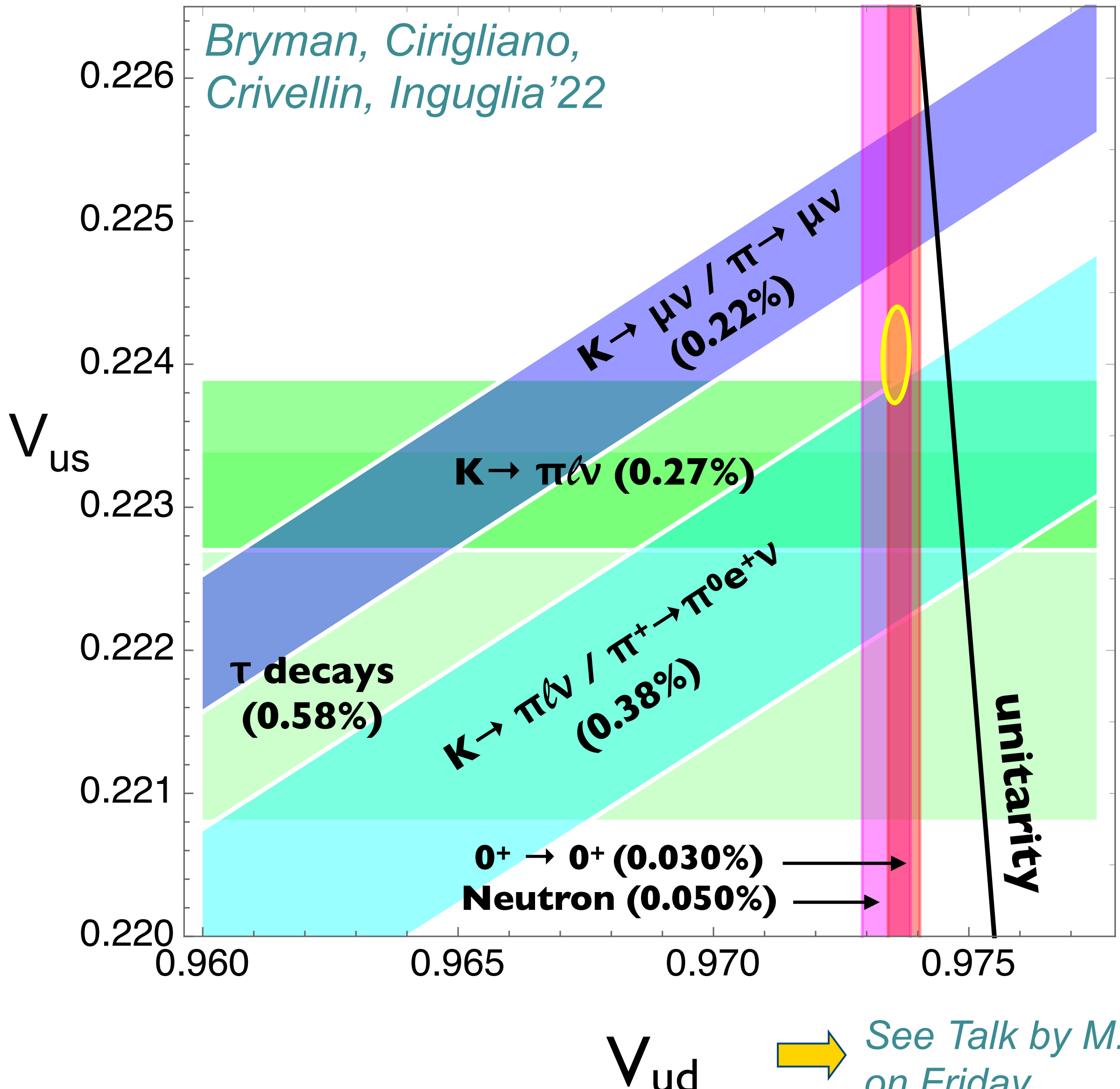
⇒ It would be great if the *US community* could join

1.2 Contributions

- 8 white papers were submitted:
  - Rare kaon decays: theory *arXiv:2203.09524*
  - Kaon decays: lattice computations *arXiv:2203.10998*
  - Kaon decays: experiments *arXiv:2204.13394*
  - Rare π^+ decays*: PIONEER* at PSI *arXiv:2203.05505*
  - *Belle II arXiv:2204.13394*
  - Rare η(') decays: *REDTOP arXiv:2203.07651*
  - CPV in hyperon decays at *BESIII* and *SCTF arXiv:2203.03035*
- 23 LoIs were submitted

2. First row CKM unitarity

2.1 Status on V_{us} and V_{ud} Cabibbo angle anomaly



Moulson@CKM2021

$|V_{ud}| = 0.97373(31)$

$|V_{us}| = 0.2231(6)$

$|V_{us}|/|V_{ud}| = 0.2311(5)$

Fit results, no constraint

$V_{ud} = 0.97365(30)$

$V_{us} = 0.22414(37)$

$\chi^2/\text{ndf} = 6.6/1\ (1.0\%)$

$\Delta_{CKM} = -0.0018(6)$

−2.7σ

$$|V_{ud}|^2 + |V_{us}|^2 + \cancel{|V_{ub}|^2} = 1 + \Delta_{CKM}$$

Negligible ~2x10⁻⁵ (B decays)

See Talk by M.Moulson on Friday

2.1 Paths to V_{ud} and V_{us}

- From kaon, pion, baryon and nuclear decays

V_{ud}	$0^+ \to 0^+$ $\pi^\pm \to \pi^0 e \nu_e$	$n \to p e \nu_e$	$\pi \to l \nu_l$
V_{us}	$K \to \pi l \nu_l$	$\Lambda \to p e \nu_e$	$K \to l \nu_l$

$$\Gamma_k = (G_F^{(\mu)})^2 \times |V_{ij}|^2 \times |M_{\text{had}}|^2 \times (1 + \delta_{RC}) \times F_{\text{kin}}$$

Channel-dependent effective CKM element

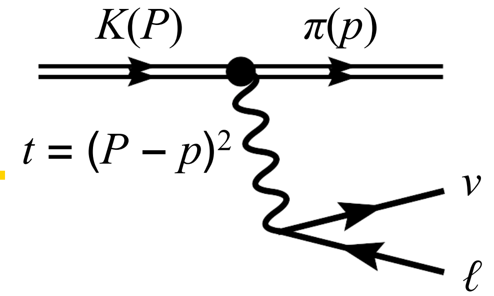
Hadronic matrix element

Radiative corrections

Recent progress on 1) Hadronic matrix elements from lattice QCD
2) Radiative corrections from dispersive methods + Lattice QCD

Seng, Gorchtein, Patel, Ramsey-Musolf'18,'19, Feng et al'20, Seng et al.'21

2.2 V_{us} from K_{l3} ($K \to \pi l \nu_l$)



- Master formula for $K \to \pi l \nu_l$: $K = \{K^+, K^0\}$, $l=\{e,\mu\}$

$$\boxed{\Gamma\left(K \to \pi l \nu\left[\gamma\right]\right) = Br(K_{l3})/\tau = C_K^2 \frac{G_F^2 m_K^5}{192\pi^3} S_{EW}^K \left|V_{us}\right|^2 \left|f_+^{K^0\pi^-}(0)\right|^2 I_{Kl}\left(1 + 2\Delta_{\mathrm{EM}}^{Kl} + 2\Delta_{\mathrm{SU(2)}}^{K\pi}\right)}$$

Average and work by Flavianet Kaon WG *Antonelli et al'11* and then by *M. Moulson*, see e.g. *Moulson.@CKM2021*

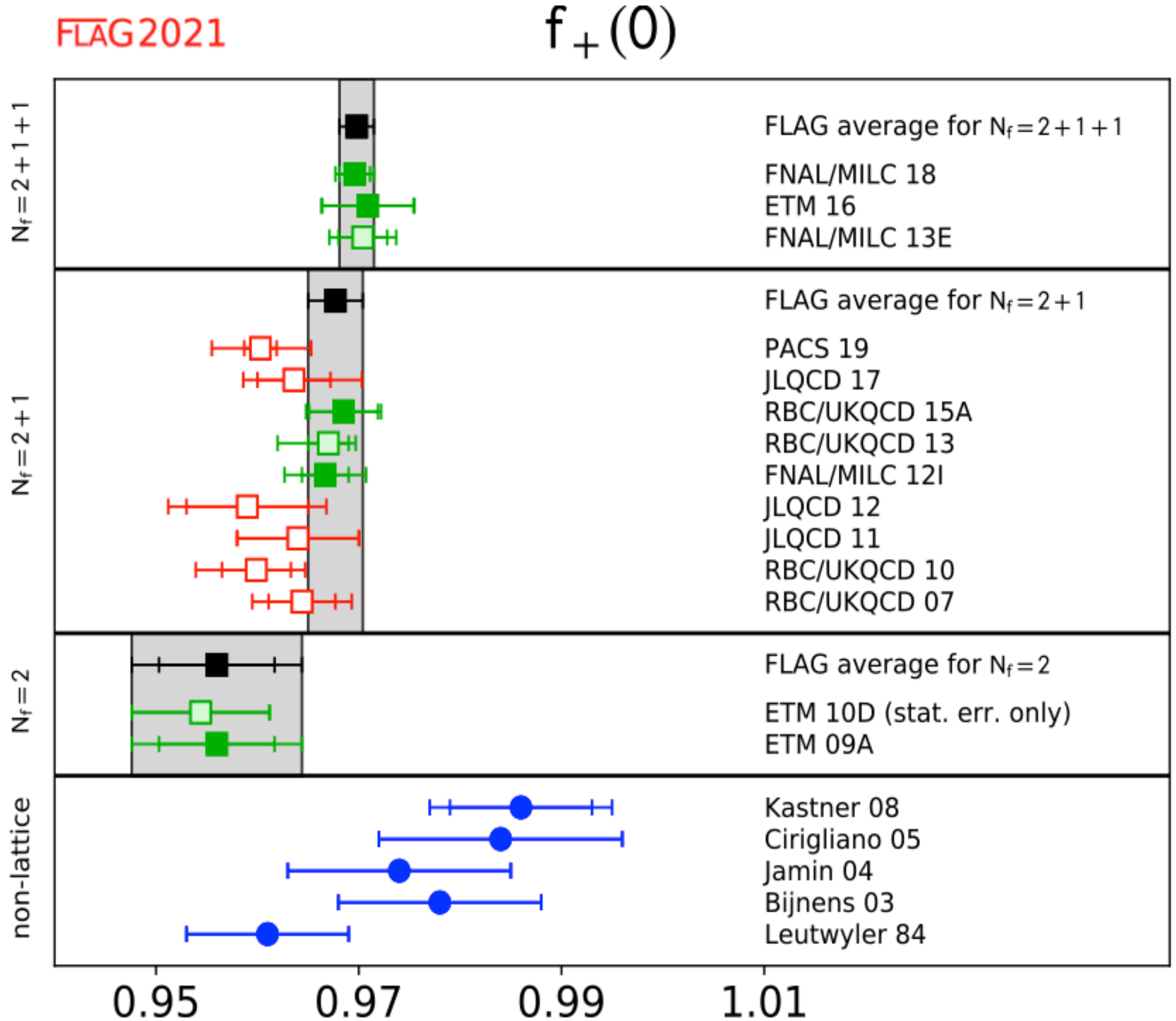
Theoretically

- Update on long-distance EM corrections for K_{e3} *Seng et al.'21*
- Improvement on Isospin breaking evaluation due to more precise dominant input: quark mass ratio from $\eta \to 3\pi$ *Colangelo et al.'18*
- Progress from lattice QCD on the $K \to \pi$ FF

$$\boxed{\left\langle \pi^-(p)\right| \bar{s}\gamma_\mu u \left|K^0(P)\right\rangle = f_+^{K^0\pi^-}(0)\left[\left(P+p\right)_\mu \bar{f}_+^{K^0\pi^-}(t) + \left(P-p\right)_\mu \bar{f}_-^{K^0\pi^-}(t)\right]}$$

$f_+(0)$ from lattice QCD

- Recent progress on Lattice QCD for determining $f_+(0)$



$$f_+(0)^{FLAG21}_{N_f=2+1+1} = \mathbf{0.9698(17)}$$

0.18% uncertainty

to be compared to

$$f_+(0)^{FLAG16}_{N_f=2+1+1} = \mathbf{0.9704(32)}$$

$$f_+(0)^{2010}_{N_f=2+1} = \mathbf{0.959(50)}$$

Uncertainty divided by ~2 w/ 2016 and by 25 w/ 2011!

Lattice uncertainties at the same level as exp.

-3.2σ away from unitarity!

2011: $V_{us} = 0.2254(5)_{exp}(11)_{lat}$ → $V_{us} = 0.2231(4)_{exp}(4)_{lat}$

2.3 V_{us}/V_{ud} from K_{l2}/π_{l2}

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1 - m_\mu^2/m_{\pi^\pm}^2}{1 - m_\mu^2/m_{K^\pm}^2}\left(1 - \frac{1}{2}\delta_{\mathrm{EM}} - \frac{1}{2}\delta_{SU(2)}\right)$$

- Recent progress on radiative corrections computed on lattice:

First lattice calculation of EM corrections to P_{l2} decays

- Ensembles from ETM
- N_f = 2+1+1 Twisted-mass Wilson fermions

Giusti et al.'18

$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0122(16)}$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld '11:

$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0112(21)}$

Update, extended description, and systematics of Giusti et al.

$\boldsymbol{\delta_{SU(2)} + \delta_{\mathrm{EM}} = -0.0126(14)}$

Di Carlo et al.'19

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- Recent progress on radiative corrections computed on lattice:

Di Carlo et al.'19

- Main input hadronic input: f_K/f_π

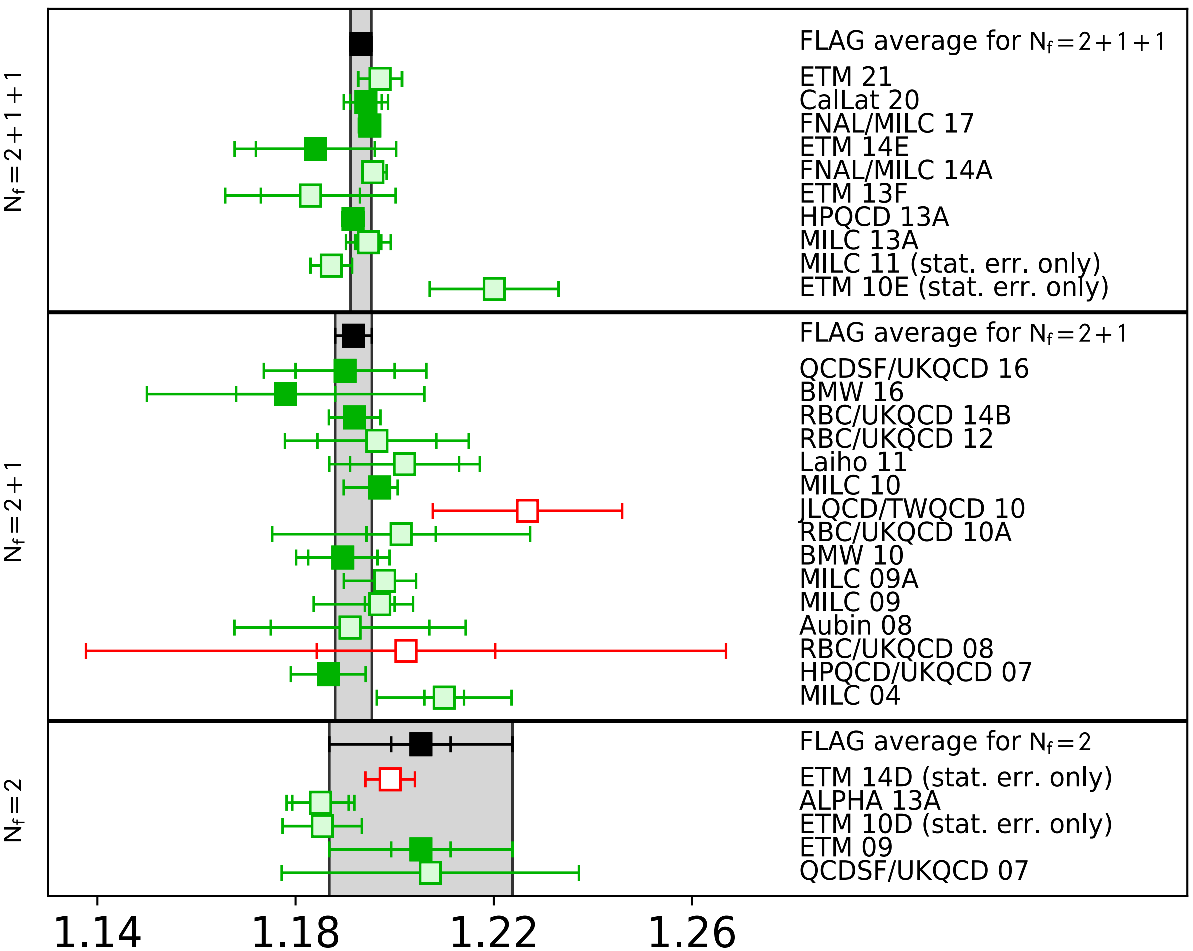
- In 2011: V_{us}/V_{ud} = 0.2312(4) $_{exp}$(12)$_{lat}$

- In 2021: V_{us}/V_{ud} = 0. 2311(3)$_{exp}$(4)$_{lat}$ the lattice error is reducing by a factor of 3 compared to 2011! It is now of the same order as the experimental uncertainty.

-1.8σ away from unitarity

2.3 V_{us}/V_{ud} from K_{l2}/π_{l2}

Progress since 2018: ⇨ new results from *ETM'21* and *CalLat'20*



Now Lattice collaborations include SU(2) IB corr.
For N_f=2+1+1, FLAG2021

$$f_{K^+}/f_{\pi^+} = 1.1932(21)$$

0.18% uncertainty

Results have been stable over the years

For average substract IB corr.

$$f_K/f_\pi = 1.1967(18)$$

In 2011: $f_K/f_\pi = 1.193(6)$

⇨ V_{us}/V_{ud} = 0. 23108(29)$_{exp}$(42)$_{lat}$

2.4 Experimental Prospects for V_{us}

On Kaon side

Cirigliano et al'22

- *NA62* could measure several BRs: $K_{\mu 3}/K_{\mu 2}$, $K \to 3\pi$, $K_{\mu 2}/K \to \pi\pi$
- Note that the high precision measurement of BR($K_{\mu 2}$) (0.3%) comes only from a single experiment: KLOE. It would be good to have another measurement at the same level of accuracy

- *LHCb* : could measure BR($K_S \to \pi\mu\nu$) at the < 1% level?
  $K_S \to \pi\mu\nu$ measured by KLOE-II but not competitive
  τ_S known to 0.04% (vs 0.41% for τ_L, 0.12% for $\tau_\pm$)

- V_{us} from Tau decays at *Belle II*:

  Belle II with 50 ab^{-1} and ~4.6 x 10^{10} τ pairs will improve V_{us} extraction from τ decays
  Inclusive measurement is an opportunity to have a complete independent extraction of V_{us} ⇨ not easy as you have to measure many channels

⇨ $$\boxed{|V_{us}| = 0.2184 \pm 0.0018_{exp} \pm 0.0011_{th}}$$

To be competitive theory error will have to be improved as well

HFLAV'21

2.5 V_{us} from Hyperon decays

V_{us} can be measured from Hyperon decays:

- **$\Lambda \to pe\nu_e$** Possible measurement at *BESIII, Super τ-Charm factory?*

- Possibilities at *LHCb*?

Talk by Dettori@FPCP20

Channel	$\mathcal{R}$	ϵ_L	ϵ_D	$\sigma_L(\mathrm{MeV}/c^2)$	$\sigma_D(\mathrm{MeV}/c^2)$
$K^0_S \to \mu^+\mu^-$	1	1.0 (1.0)	1.8 (1.8)	~ 3.0	~ 8.0
$K^0_S \to \pi^+\pi^-$	1	1.1 (0.30)	1.9 (0.91)	~ 2.5	~ 7.0
$K^0_S \to \pi^0\mu^+\mu^-$	1	0.93 (0.93)	1.5 (1.5)	~ 35	~ 45
$K^0_S \to \gamma\mu^+\mu^-$	1	0.85 (0.85)	1.4 (1.4)	~ 60	~ 60
$K^0_S \to \mu^+\mu^-\mu^+\mu^-$	1	0.37 (0.37)	1.1 (1.1)	~ 1.0	~ 6.0
$K^0_L \to \mu^+\mu^-$	~ 1	2.7 (2.7) $\times 10^{-3}$	0.014 (0.014)	~ 3.0	~ 7.0
$K^+ \to \pi^+\pi^+\pi^-$	~ 2	9.0 (0.75) $\times 10^{-3}$	41 (8.6) $\times 10^{-3}$	~ 1.0	~ 4.0
$K^+ \to \pi^+\mu^+\mu^-$	~ 2	6.3 (2.3) $\times 10^{-3}$	0.030 (0.014)	~ 1.5	~ 4.5
$\Sigma^+ \to p\mu^+\mu^-$	~ 0.13	0.28 (0.28)	0.64 (0.64)	~ 1.0	~ 3.0
$\Lambda \to p\pi^-$	~ 0.45	0.41 (0.075)	1.3 (0.39)	~ 1.5	~ 5.0
$\Lambda \to p\mu^-\bar{\nu}_\mu$	~ 0.45	0.32 (0.31)	0.88 (0.86)	–	–
$\Xi^- \to \Lambda\mu^-\bar{\nu}_\mu$	~ 0.04	39 (5.7) $\times 10^{-3}$	0.27 (0.09)	–	–
$\Xi^- \to \Sigma^0\mu^-\bar{\nu}_\mu$	~ 0.03	24 (4.9) $\times 10^{-3}$	0.21 (0.068)	–	–
$\Xi^- \to p\pi^-\pi^-$	~ 0.03	0.41(0.05)	0.94 (0.20)	~ 3.0	~ 9.0
$\Xi^0 \to p\pi^-$	~ 0.03	1.0 (0.48)	2.0 (1.3)	~ 5.0	~ 10
$\Omega^- \to \Lambda\pi^-$	~ 0.001	95 (6.7) $\times 10^{-3}$	0.32 (0.10)	~ 7.0	~ 20

R = ratio of production

ϵ = ratio of efficiencies

- To be able to extract V_{us} one needs to compute form factors precisely

  ⇨ Lattice effort from *RBC/UKQCD*

2.6 Theoretical Prospects for V_{us}

- Lattice Progress on hadronic matrix elements: decay constants, FFs

- Full QCD**+**QED decay rate on the lattice,for **Leptonic decays of kaons and pions** ⇨ Inclusion of EM and IB corrections :
  - Perturbative treatment of QED on lattice established
  - Formalism for K_{l2} worked out

- Application of the method for **semileptonic Kaon** (K_{l3}) **and Baryon decays**

⇨ **Aim: Per mille level within 10 years**

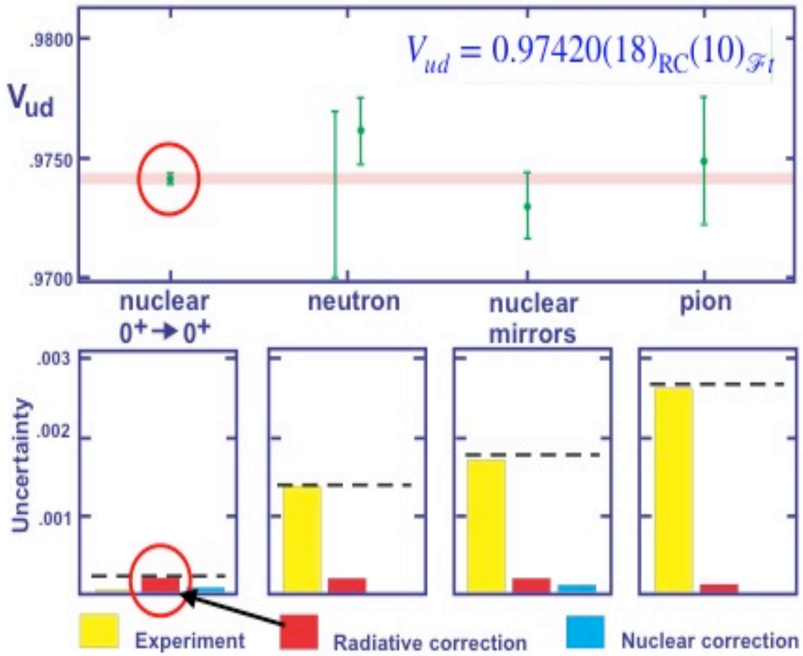
2.7 $|V_{ud}|$ from $0^+ \rightarrow 0^+$ superallowed β decays

See Talk by Misha Gorshteyn @CKM2021

Figure adapted from J. Hardy

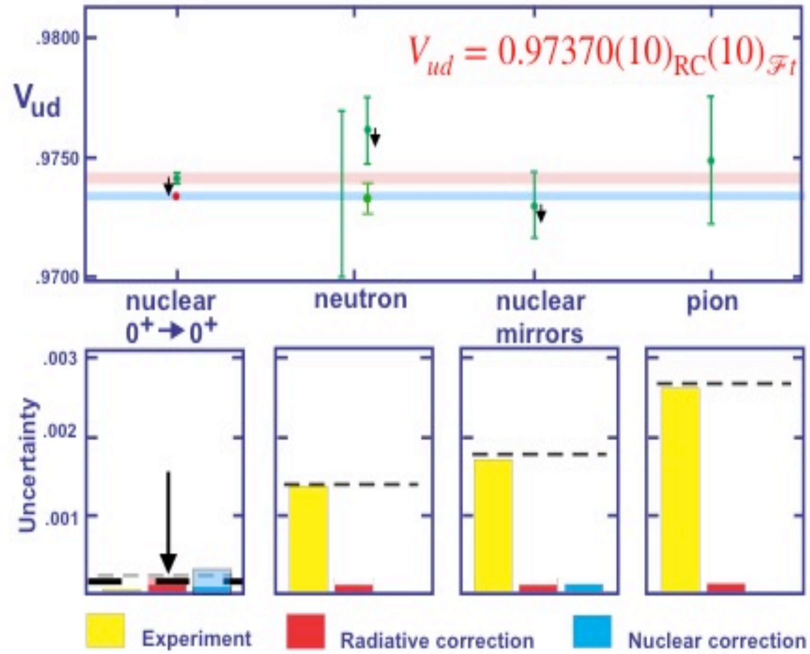
PDG 2018:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(4)_{V_{ud}}(2)_{V_{us}}$$



PDG 2020:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(3)_{V_{ud}}(4)_{V_{us}}$$



Recent improvement on the theoretical RCs +Nuclear Structure Corrections

➡ Use of a data driven dispersive approach *Seng et al.'18'18, Gorshteyn'18*

➡ *See Colloquium by C-Y Seng on Wednesday*

2.8 $|V_{ud}|$ from Neutrons

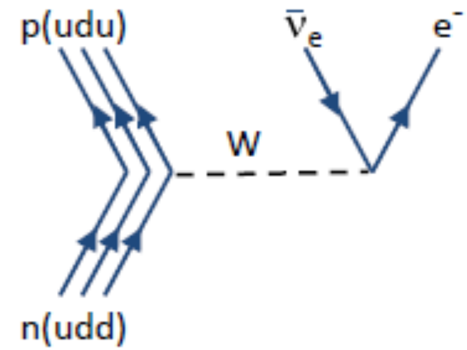
See Talk by Chen Yu Liu this afternoon

- Master Formula:

$$\boxed{\left|V_{ud}\right|^2 = \frac{5024.7s}{\tau_n\left(1+3\lambda^2\right)\left(1+\Delta_R\right)}}$$

Lifetime $\qquad$ $\lambda = g_A / g_V$

$$n \to p + e^- + \overline{\nu}_e$$



- Needs $\delta\lambda/\lambda \approx 3 \times 10^{-4}$ and $\delta\tau_n \approx 0.3$ s to compete with $0^+ \to 0^+$ transitions.
- Theoretically, the radiative corrections are under control (same as for $0^+ \to 0^+$)
- Recent progress :
  - New Perkeo III result: *PERKEO III* result improves world-average of beta asymmetry by factor 5! Half of it is due to the reduction of the scale factor

$$\boxed{A = -0.11958(21),\ S = 1.2 \quad \lambda_A = -1.2757(5)}$$

  - Tension with *aSPECT* result: $\boxed{\lambda_{\text{avg}} = -1.2754(13),\ S = 2.7}$

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Lifetime $\quad \lambda = g_A / g_V$

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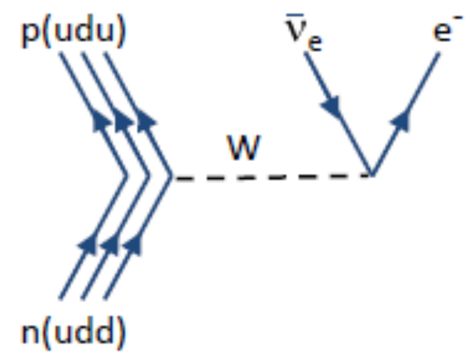

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    ⇨ $\boxed{A = -0.11958(21),\ S = 1.2 \quad \lambda_A = -1.2757(5)}$

  - New result for Lifetime from *UCNτ* $\tau_n = 877.75 \pm 0.28^{+0.22}_{-0.16}$ s

    ⇨ improvement by a factor of 2.25 compared to previous result

2.9 $|V_{ud}|$ from pion β decay: $\pi^+ \to \pi^0 e^+ \nu$

- Theoretically cleanest method to extract V_{ud} : corrections computed in SU(2) ChPT
  Sirlin'78, Cirigliano et al.'03, Passera et al'11
- Present result: *PIBETA* Experiment (2004) → **Uncertainty: 0.64%**

$$B(\pi^+ \to \pi^0 e^+ \nu) = (1.036 \pm 0.004_{\rm stat} \pm 0.004_{\rm syst} \pm 0.003_{\pi e2})\ \mathrm{x}10^{-8}\ (\pm 0.6\%)$$

⇒ $\boxed{|V_{ud}| = \mathbf{0.9739(28)_{exp}(1)_{th}}}$ to be compared to $|V_{ud}| = \mathbf{0.97373(31)}$

- Reduction of the theory error thanks to a new lattice calculation for RC *Feng et al'20*
- Next generation experiment PIONEER Phase II and III measurement at 0.02% level ⇒ will be competitive with current $0^+ \to 0^+$ extraction
- Would be completely independent check! No nuclear correction and different RCs compared to neutron decay
  Czarnecki, Marciano, Sirlin'20
- **Opportunity to extract V_{us}/V_{ud}** from $\dfrac{B(K \to \pi l \nu)}{B(\pi^+ \to \pi^0 e^+ \nu)}$ EW Rad. Corr. cancel

Improve precision on $B(\pi^+ \to \pi^0 e^+ \nu)$ by x3 ⇒ $V_{us}/V_{ud} < \pm 0.2\%$

3. Charged pion decays

3.1 Pion decays and LFU tests

- Lepton Flavor Universality test in $R^{theory}_{e/\mu} = \dfrac{\Gamma(\pi \to e\nu(\gamma))}{\Gamma(\pi \to \mu\nu(\gamma))}$
  - Early insight into the **V−A** structure of weak interactions
  - Exceptional precision of the SM prediction using ChPT

$$R_{e/\mu}(\text{SM}) = 1.23524(015) \times 10^{-4}$$

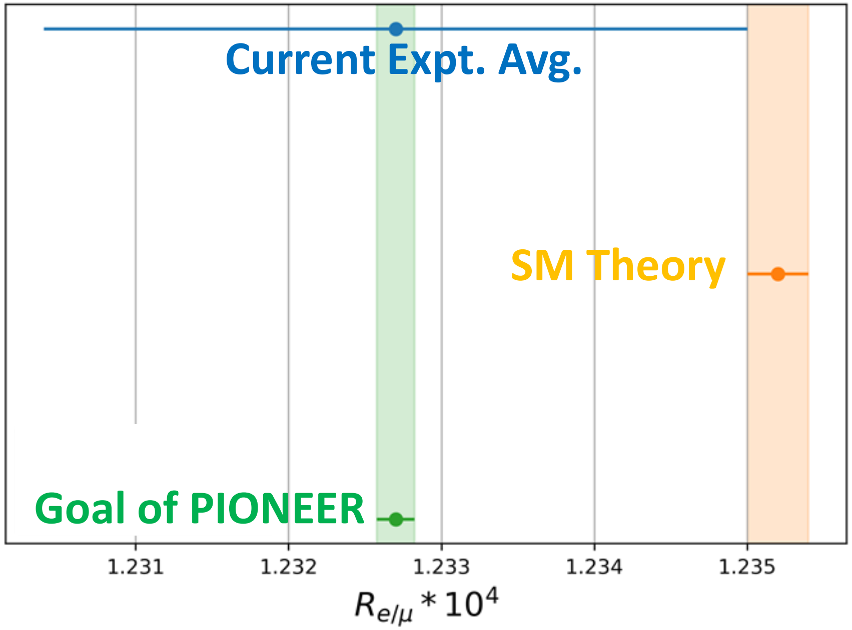
Cirigliano & Rosell'07

  - World average (mainly *PIENU* at TRIUMF):

$$R_{e/\mu}(\text{Exp}) = 1.23270(230) \times 10^{-4}$$

15 times worse than theory!

(dominated by PIENU expt.)



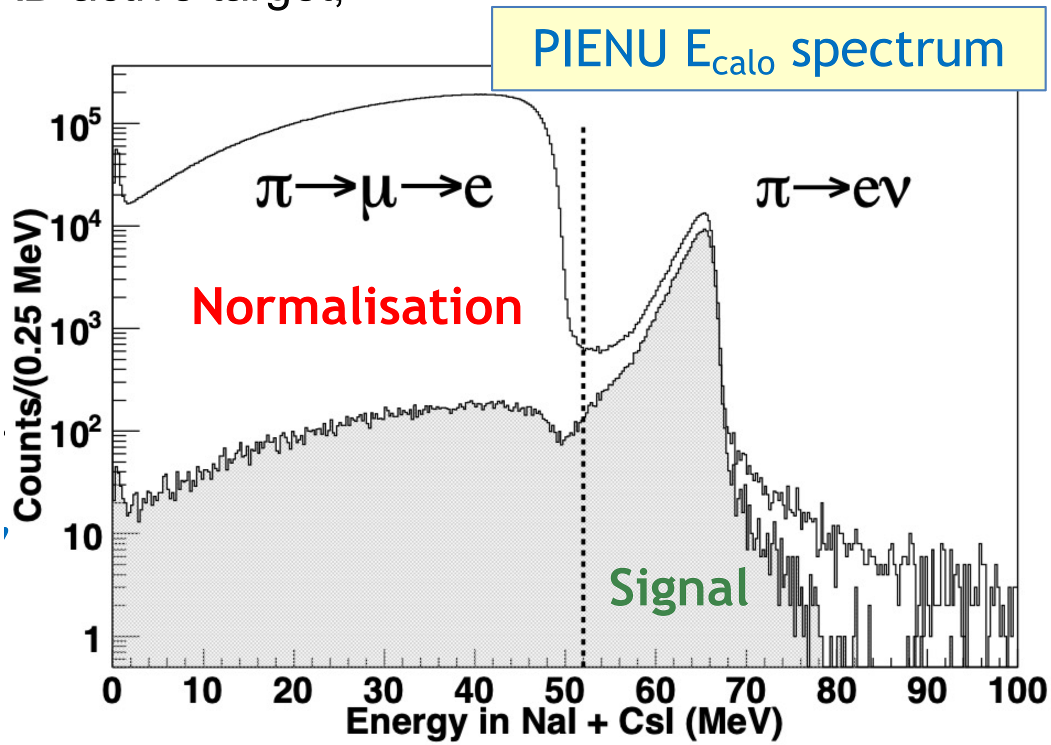
$$\frac{g_e}{g_\mu} = 0.9990 \pm 0.0009 \quad (\pm 0.09\%)$$

Goal of PIONEER: reduce unc. by a factor of 10 ! ⇨ by far most precise test of LFU

3.2 PIONEER (Phase-I)

PIONEER (Phase-I) approved at PSI, physics starting in ~2029

- Goal: matching the SM precision on $R_{e/\mu}$
  ⇨ Test of New Physics at *1 PeV scale*
- Stopped π^+ at high rate (300 kHz), focus on reduction of systematics.
- Detectors: highly-segmented LGAD active target, positron tracker, LXe calorimeter
- Collection of 2×10^8 $\pi^+\to e^+\nu_e$ events in three years.
- Key point: control of the $\pi^+\to e^+\nu_e$ signal tail in the calorimeter to a 10^{-4} precision

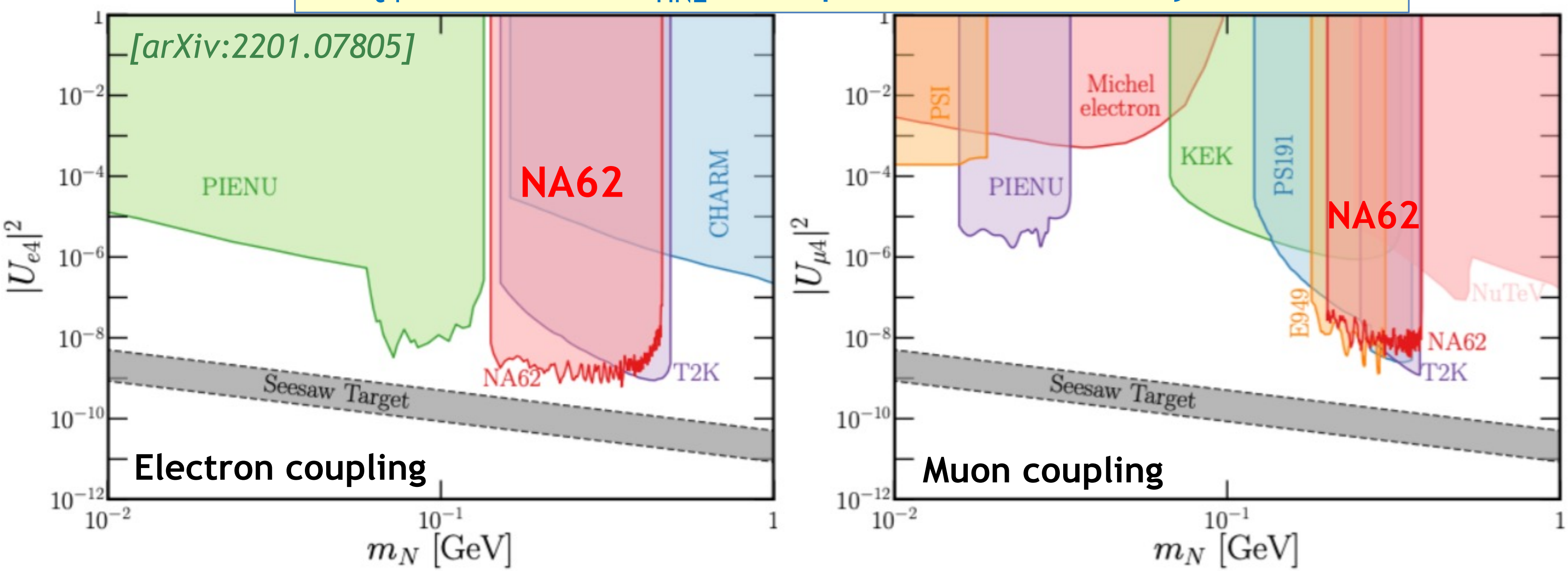


PIONEER Phase II,III:

V_{ud} from $\pi^+ \to \pi^0 e^+\nu_e$ decays to a *0.02%* level

3.3 Example: Constraints on Heavy Neutral Leptons

$|U_{\ell 4}|^2$ limits vs m_{HNL} from production & decay searches



- Strongest **$|U_{e4}|^2$** limits below **400 MeV**: **$K^+, \pi^+ \rightarrow e^+N$** from *NA62* & *PIENU*.
- Also important limits on **$|U_{\mu4}|^2$** from *E949*, *NA62* and *PIENU*.
- *NA62/E949* limits are complementary to HNL *decay* searches at T2K.
- Next-generation **K^+** and p**+** experiments (*NA62^{++}*, *PIONEER*) to improve by up to factor **10**, reaching the seesaw bound.

4. Conclusion and Outlook

Conclusion and Outlook

- Recent precision determinations of V_{us} and V_{ud} enable unprecedented tests of the SM and constraints on possible NP models
- Tensions in unitarity of 1st row of CKM matrix have reappeared!
- We need to work hard to understand where they come from:
  - On experimental side:
    For V_{us}, new measurements in kaons (*NA62:* $K_{\mu 3}/K_{\mu 2}$, *LHCb?*)
    but mainly in tau decays from *Belle II*
    V_{us} from hyperon decays? ⇨ *BESSIII*, *LHCb*?
  - For V_{ud}, understand the situation of the neutron lifetime, beta decay of pion? ⇨ *PIONEER* Consider $R_V = \Gamma\left(K \to \pi l \nu(\gamma)\right) / \Gamma\left(\pi^+ \to \pi^0 e^+ \nu(\gamma)\right)$

    Czarnecki, Marciano, Sirlin'20
  - On theory side:
    Calculate very precisely radiative corrections, isospin breaking effects and matrix elements
    Be sure the uncertainties are under control
  - If these tensions are confirmed ⇨ what do they tell us?
- Interesting time ahead of us!

5. Back-up

2.1 V_{us} from K_{l3}

Matthew Moulson, Chien Yeah Seng

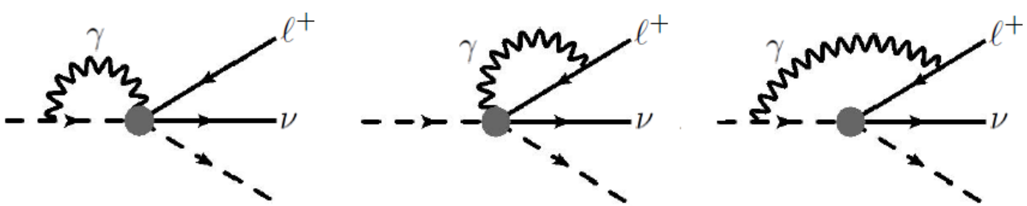
Progress since 2018:

- First experimental measurement of BR of $K_S \to \pi\mu\nu$

  BR($K_S \to \pi\mu\nu$) = (4.56 ± 0.20) ×10^{-4}

KLOE-2
PLB 804 (2020)

- Theoretically update on long-distance EM corrections:



Up to now computation at fixed order e^2p^2 + model estimate for the LECs

Cirigliano et al. '08

New calculation of complete EW RC using hybrid current algebra and ChPT (Sirlin's representation) with resummation of largest terms to all chiral orders

– Reduced uncertainties at O(e^2p^4)

– Lattice evaluation of QCD contributions to γW box diagrams

Seng et al.'21

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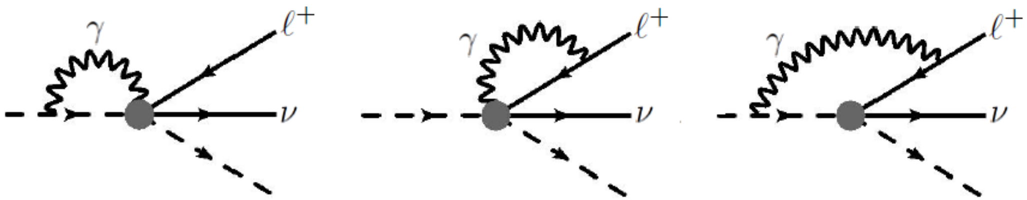


Only K_{e3} at present
For $K_{\mu3}$ modes continue to use **Cirigliano et al. '08**

	Cirigliano et al. '08	Seng et al. '21
$\Delta^{EM}(K^0{}_{e3})$ [%]	0.50 ± 0.11	**0.580 ± 0.016**
$\Delta^{EM}(K^+{}_{e3})$ [%]	0.05 ± 0.13	**0.105 ± 0.024**
ρ	+0.081	**–0.039**

2.1 V_{us} from K_{l3}

Progress since 2018:

- Theoretical progress on isospin breaking correction

$$\Delta^{SU(2)} \equiv \frac{f_+(0)^{K^+\pi^0}}{f_+(0)^{K^0\pi^-}} - 1$$

Strong isospin breaking

Quark mass differences, η-π^0 mixing in $K^+\pi^0$ channel

$$= \frac{3}{4}\frac{1}{Q^2}\left[\frac{\overline{M}_K^2}{\overline{M}_\pi^2} + \frac{\chi_{p^4}}{2}\left(1+\frac{m_s}{\hat{m}}\right)\right] \qquad Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

$\chi_p{}^4 = 0.252$
NLO in strong interaction
O(e^2p^2) term $\varepsilon_{EM}{}^{(4)} \sim 10^{-6}$

Cirigliano et al., ’02; Gasser & Leutwyler, ’85

$=$ **+2.61(17)%** **Calculated using:**

$Q = 22.1(7)$ Colangelo et al. ’18, avg. from $\eta \to 3\pi$

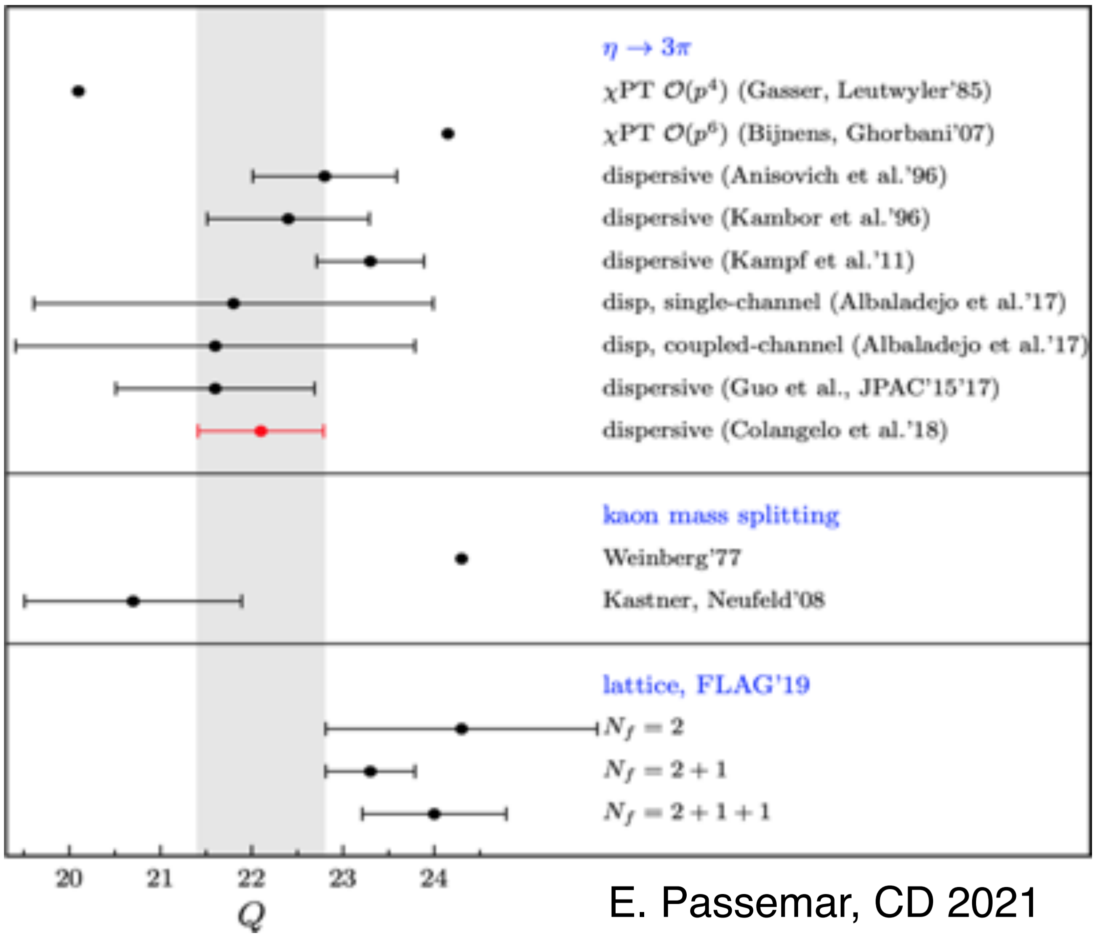
$m_s/\hat{m} = 27.23(10)$ FLAG ’20, $N_f = 2+1+1$ avg.

$M_K = 494.2(3)$
$M_\pi = 134.8(3)$] Isospin-limit meson masses from FLAG ’17

Test by evaluating V_{us} from $K^\pm$ and K^0 data with **no** corrections:
Equality of V_{us} values would require **$\Delta^{SU(2)} = 2.86(34)\%$**

2.1 V_{us} from K_{l3}

Previous to recent results for Q, uncertainty on $\Delta^{SU(2)}$ was leading contributor to uncertainty on V_{us} from $K^{\pm}$ decays



E. Passemar, CD 2021

Reference value of Q from dispersion relation analyses of $\eta \to 3\pi$ Dalitz plots

Colangelo et al., '18

$Q = 22.1 \pm 0.7$

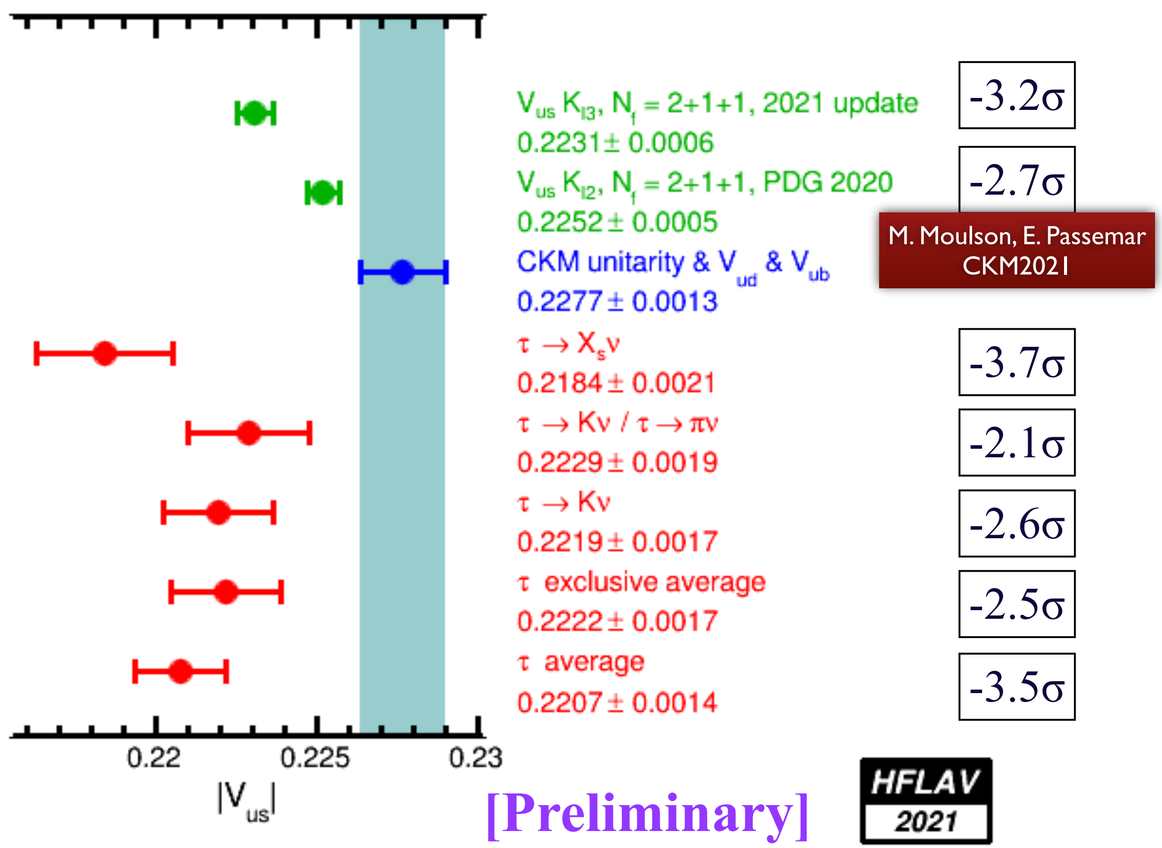
Lattice results for Q somewhat higher than analytical results

But, lattice results have finite correction to LO expectation:

$$Q_M^2 \equiv \frac{\hat{M}_K^2}{\hat{M}_\pi^2} \frac{\hat{M}_K^2 - \hat{M}_\pi^2}{\hat{M}_{K^0}^2 - \hat{M}_{K^+}^2}$$

Low-energy theorem: Q has no correction at NLO

V_{us} from Tau decays



- Belle II with 50 ab^{-1} and ~4.6 x 10^{10} τ pairs will improve V_{us} extraction
- Inclusive measurement is an opportunity to have a complete independent measurement of V_{us} ⇒ not easy as you have to measure many channels

V_{us} from Tau decays

e 13: HFLAV 2021 τ branching fractions to strange final states.

Branching fraction	HFLAV 2021 fit (%)
$K^-\nu_\tau$	0.6957 ± 0.0096
$K^-\pi^0\nu_\tau$	0.4322 ± 0.0148
$K^-2\pi^0\nu_\tau$ (ex.K^0)	0.0634 ± 0.0219
$K^-3\pi^0\nu_\tau$ (ex.K^0,η)	0.0465 ± 0.0213
$\pi^-\overline{K}^0\nu_\tau$	0.8375 ± 0.0139
$\pi^-\overline{K}^0\pi^0\nu_\tau$	0.3810 ± 0.0129
$\pi^-\overline{K}^0 2\pi^0\nu_\tau$ (ex.K^0)	0.0234 ± 0.0231
$\overline{K}^0 h^- h^- h^+\nu_\tau$	0.0222 ± 0.0202
$K^-\eta\nu_\tau$	0.0155 ± 0.0008
$K^-\pi^0\eta\nu_\tau$	0.0048 ± 0.0012
$\pi^-\overline{K}^0\eta\nu_\tau$	0.0094 ± 0.0015
$K^-\omega\nu_\tau$	0.0410 ± 0.0092
$K^-\phi(K^+K^-)\nu_\tau$	0.0022 ± 0.0008
$K^-\phi(K^0_S K^0_L)\nu_\tau$	0.0015 ± 0.0006
$K^-\pi^-\pi^+\nu_\tau$ (ex.K^0,ω)	0.2924 ± 0.0068
$K^-\pi^-\pi^+\pi^0\nu_\tau$ (ex.K^0,ω,η)	0.0387 ± 0.0142
$K^-2\pi^-2\pi^+\nu_\tau$ (ex.K^0)	0.0001 ± 0.0001
$K^-2\pi^-2\pi^+\pi^0\nu_\tau$ (ex.K^0)	0.0001 ± 0.0001
$X_s^-\nu_\tau$	2.9076 ± 0.0478

HFLAV'21

$$\boxed{R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau e^- \overline{\nu}_e\right)} \approx N_C}$$

parton model prediction

$$\boxed{\delta R_\tau \equiv \frac{R_{\tau,NS}}{\left|V_{ud}\right|^2} - \frac{R_{\tau,S}}{\left|V_{us}\right|^2}}$$

SU(3) breaking quantity, strong dependence in m_s computed from OPE (L+T) + phenomenology

$$\delta R_{\tau,th} = 0.0242(32)$$

Gamiz et al'07, Maltman'11

$$\boxed{\left|V_{us}\right|^2 = \frac{R_{\tau,S}}{\dfrac{R_{\tau,NS}}{\left|V_{ud}\right|^2} - \delta R_{\tau,th}}}$$

2.9σ away from unitarity!

$$\Rightarrow \quad \boxed{\left|V_{us}\right| = 0.2184 \pm 0.0018_{\text{exp}} \pm 0.0011_{\text{th}}}$$