Snowmass report on « Weak Decays of Strange and Light Quarks »

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Electroweak Precision Physics from Beta Decays to the Z Pole
Mainz Institute for Theoretical Physics, October 24 – 29, 2022

Based on Snowmass report: ArXiv: 2209.07156 [hep-ex]
Goudzovski et al.
Outline

1. Introduction and Motivation
2. First row CKM unitarity
3. Charged pion decays
4. Conclusion and Outlook
1. Introduction and Motivation
1.1 Weak Decays of Strange and Light Quarks
Rare Frontier (RF2)

- Flavor physics experiments probe both very high mass scales, and feebly interacting hidden sectors.

- RF2: precision measurements of kaon, hyperon, $\pi$ and $\eta(')$ decays
  - CKM parameter measurements and unitary tests; symmetry tests;
    lepton flavor/number conservation tests; lepton universality tests
  - Heavy new physics: sensitivity up to the PeV mass scale
  - Hidden sectors: leading sensitivity below the GeV mass scale
1.1 Weak Decays of Strange and Light Quarks
Rare Frontier (RF2)

- **Vibrant experimental activities**
  - Ultra-rare kaon decays at NA62 and KOTO (+ future projects)
  - CPV in hyperon decays at BESIII (+ future super charm-tau factories)
  - Kaon and hyperon decays at LHCb
  - LFU and $V_{ud}$ in pion decays at PIONEER
  - Symmetry tests at $\eta(')$ factories: JEF + REDTOP proposal

- **Significant advances in theory and lattice QCD:** crucial for progress.

- **Medium-scale initiatives** (many centered in Europe and Asia)
  - powerful physics insights
  - relatively short time scales
  - superb training opportunities
  - modest investment

It would be great if the **US community** could join
1.2 Contributions

- 8 white papers were submitted:
  - Rare kaon decays: theory *arXiv:2203.09524*
  - Kaon decays: lattice computations *arXiv:2203.10998*
  - Kaon decays: experiments *arXiv:2204.13394*
  - Rare $\pi^+$ decays: *PIONEER* at PSI *arXiv:2203.05505*
  - *Belle II* *arXiv:2204.13394*
  - Rare $\eta(')$ decays: *REDTOP* *arXiv:2203.07651*
  - CPV in hyperon decays at *BESIII* and *SCTF* *arXiv:2203.03035*

- 23 Lols were submitted
2. First row CKM unitarity
2.1 Status on $V_{us}$ and $V_{ud}$ Cabibbo angle anomaly

\[ |V_{ud}| = 0.97373(31) \]
\[ |V_{us}| = 0.2231(6) \]
\[ |V_{us}|/|V_{ud}| = 0.2311(5) \]

Fit results, no constraint

- $V_{ud} = 0.97365(30)$
- $V_{us} = 0.22414(37)$

$\chi^2/ndf = 6.6/1 \ (1.0\%)$

$\Delta_{\text{CKM}} = -0.0018(6)$

$-2.7\sigma$

Negligible $\sim 2x10^{-5}$

(B decays)
2.1 Paths to $V_{ud}$ and $V_{us}$

- From kaon, pion, baryon and nuclear decays

<table>
<thead>
<tr>
<th>$V_{ud}$</th>
<th>$0^+ \rightarrow 0^+$</th>
<th>$\pi^\pm \rightarrow \pi^0 e^\nu_e$</th>
<th>$n \rightarrow pe^\nu_e$</th>
<th>$\pi \rightarrow lv_l$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{us}$</td>
<td>$K \rightarrow \pi lv_l$</td>
<td>$\Lambda \rightarrow pe^\nu_e$</td>
<td>$K \rightarrow lv_l$</td>
<td></td>
</tr>
</tbody>
</table>

$$\Gamma_k = (G_F^{(\mu)})^2 \times |V_{ij}|^2 \times |M_{had}|^2 \times (1 + \delta_{RC}) \times F_{kin}$$

Channel-dependent effective CKM element

Hadronic matrix element

Radiative corrections

Recent progress on
1) Hadronic matrix elements from lattice QCD
2) Radiative corrections from dispersive methods + Lattice QCD

Seng, Gorchtein, Patel, Ramsey-Musolf’18,’19, Feng et al’20, Seng et al.’21
2.2 $V_{us}$ from $K_{l3}$ ($K \rightarrow \pi l \nu_l$)

- Master formula for $K \rightarrow \pi l \nu_l$: $K = \{K^+, K^0\}$, $l = \{e, \mu\}$

\[
\Gamma\left(K \rightarrow \pi l \nu_l [\gamma]\right) = Br\left(K_{l3}\right) / \tau = C_K^2 \frac{G_F^2 m_K^5}{192 \pi^3} S_{EW}^K \left| V_{us} \right|^2 \left| f_+^{K^0 \pi^-}(0) \right|^2 I_{KI} \left(1 + 2 \Delta_{EM}^{KI} + 2 \Delta_{SU(2)}^{K \pi}\right)
\]

Average and work by Flavianet Kaon WG Antonelli et al’11 and then by M. Moulson, see e.g. Moulson.@CKM2021

Theoretically
- Update on long-distance EM corrections for $K_{e3}$ Seng et al.’21
- Improvement on Isospin breaking evaluation due to more precise dominant input: quark mass ratio from $\eta \rightarrow 3\pi$ Colangelo et al.’18
- Progress from lattice QCD on the $K \rightarrow \pi$ FF

\[
\langle \pi^-(p) \left| \bar{s} \gamma_\mu u \right| K^0(p) \rangle = f_+^{K^0 \pi^-}(0) \left[ (P + p)_\mu \bar{f}_+^{K^0 \pi^-}(t) + (P - p)_\mu \bar{f}_-^{K^0 \pi^-}(t) \right]
\]
**Recent progress on Lattice QCD for determining $f_+(0)$**

- **FLAG2021**
  - $f_+(0)$
    - $f_+(0)_{N_f=2+1+1}^{FLAG21} = 0.9698(17)$
    - 0.18% uncertainty
  - to be compared to
    - $f_+(0)_{N_f=2+1+1}^{FLAG16} = 0.9704(32)$
    - $f_+(0)_{N_f=2+1}^{2010} = 0.959(50)$

Uncertainty divided by ~2 w/ 2016 and by 25 w/ 2011!

- Lattice uncertainties at the same level as exp.
- $-3.2\sigma$ away from unitarity!

2011: $V_{us} = 0.2254(5)_{exp(11)}_{lat} \rightarrow V_{us} = 0.2231(4)_{exp(4)}_{lat}$
2.3 $V_{us}/V_{ud}$ from $K_{l2}/\pi_{l2}$

$$\frac{|V_{us}| f_K}{|V_{ud}| f_\pi} = \left( \frac{\Gamma_{K_{\mu2(\gamma)}}}{\Gamma_{\pi_{\mu2(\gamma)}}} \frac{m_{\pi^\pm}}{m_{K^\pm}} \right)^{1/2} \frac{1 - m_{\mu}^2/m_{\pi^\pm}^2}{1 - m_{\mu}^2/m_{K^\pm}^2} \left( 1 - \frac{1}{2} \delta_{EM} - \frac{1}{2} \delta_{SU(2)} \right)$$

- Recent progress on radiative corrections computed on lattice:

**First lattice calculation of EM corrections to $P_{l2}$ decays**

- Ensembles from ETM
- $N_f = 2+1+1$ Twisted-mass Wilson fermions

$\delta_{SU(2)} + \delta_{EM} = -0.0122(16)$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld ’11:

$\delta_{SU(2)} + \delta_{EM} = -0.0112(21)$

Update, extended description, and systematics of Giusti et al.

$\delta_{SU(2)} + \delta_{EM} = -0.0126(14)$
2.3 $V_{us}/V_{ud}$ from $K_{l2}/\pi_{l2}$

\[
\frac{|V_{us}| f_K}{|V_{ud}| f_\pi} = \left( \frac{\Gamma_{K\mu_2(y)} m_{\pi^\pm}}{\Gamma_{\pi\mu_2(y)} m_{K^\pm}} \right)^{1/2} \left( \frac{1 - m_{\mu^2}/m_{\pi^\pm}}{1 - m_{\mu^2}/m_{K^\pm}} \right) \left( 1 - \frac{1}{2} \delta_{EM} - \frac{1}{2} \delta_{SU(2)} \right)
\]

- Recent progress on radiative corrections computed on lattice:

  \textit{Di Carlo et al.'19}

- Main input hadronic input: $f_K/f_\pi$

- In 2011: $V_{us}/V_{ud} = 0.2312(4)_{\text{exp}}^{12}_{\text{lat}}$

- In 2021: $V_{us}/V_{ud} = 0.2311(3)_{\text{exp}}^{4}_{\text{lat}}$ the lattice error is reducing by a factor of 3 compared to 2011! It is now of the same order as the experimental uncertainty.

  \textbf{1.8$\sigma$ away from unitarity}
2.3 $V_{us}/V_{ud}$ from $K_{l2}/\pi_{l2}$

Progress since 2018: new results from ETM’21 and CalLat’20

Now Lattice collaborations include SU(2) IB corr.

For $N_f=2+1+1$, FLAG2021

$$\frac{f_{K^\pm}/f_{\pi^\pm}}{}$$

0.18% uncertainty

Results have been stable over the years

For average subtract IB corr.

$$\frac{f_K/f_\pi}{1.1967(18)}$$

In 2011: $f_K/f_\pi = 1.193(6)$

$V_{us}/V_{ud} = 0.23108(29)_{\text{exp}}(42)_{\text{lat}}$
2.4 Experimental Prospects for $V_{us}$

On Kaon side

- **NA62** could measure several BRs: $K_{\mu 3}/K_{\mu 2}$, $K \rightarrow 3\pi$, $K_{\mu 2}/K \rightarrow \pi\pi$
- Note that the high precision measurement of BR($K_{\mu 2}$) (0.3%) comes only from a single experiment: KLOE. It would be good to have another measurement at the same level of accuracy

- **LHCb**: could measure BR($K_S \rightarrow \pi\mu\nu$) at the < 1% level?
  $K_S \rightarrow \pi\mu\nu$ measured by KLOE-II but not competitive
  $\tau_S$ known to 0.04% (vs 0.41% for $\tau_L$, 0.12% for $\tau_\pm$)

- $V_{us}$ from Tau decays at **Belle II**:

  Belle II with 50 ab$^{-1}$ and $\sim 4.6 \times 10^{10}$ $\tau$ pairs will improve $V_{us}$ extraction from $\tau$ decays
  Inclusive measurement is an opportunity to have a complete independent extraction of $V_{us}$ not easy as you have to measure many channels

$$|V_{us}| = 0.2184 \pm 0.0018_{\text{exp}} \pm 0.0011_{\text{th}}$$

To be competitive theory error will have to be improved as well

*Cirigliano et al’22*
2.5 $V_{us}$ from Hyperon decays

$V_{us}$ can be measured from Hyperon decays:

- $\Lambda \rightarrow p e \nu_e$ Possible measurement at BESIII, Super $\tau$-Charm factory? 
- Possibilities at LHCb?

<table>
<thead>
<tr>
<th>Channel</th>
<th>$R$</th>
<th>$\epsilon_L$</th>
<th>$\epsilon_D$</th>
<th>$\sigma_L$(MeV/c$^2$)</th>
<th>$\sigma_D$(MeV/c$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_S^0 \rightarrow \mu^+\mu^-$</td>
<td>1</td>
<td>1.0 (1.0)</td>
<td>1.8 (1.8)</td>
<td>$\sim$ 3.0</td>
<td>$\sim$ 8.0</td>
</tr>
<tr>
<td>$K_S^0 \rightarrow \pi^+\pi^-$</td>
<td>1</td>
<td>1.1 (0.30)</td>
<td>1.9 (0.91)</td>
<td>$\sim$ 2.5</td>
<td>$\sim$ 7.0</td>
</tr>
<tr>
<td>$K_S^0 \rightarrow \pi^0\mu^+\mu^-$</td>
<td>1</td>
<td>0.93 (0.93)</td>
<td>1.5 (1.5)</td>
<td>$\sim$ 35</td>
<td>$\sim$ 45</td>
</tr>
<tr>
<td>$K_S^0 \rightarrow \gamma\mu^+\mu^-$</td>
<td>1</td>
<td>0.85 (0.85)</td>
<td>1.4 (1.4)</td>
<td>$\sim$ 60</td>
<td>$\sim$ 60</td>
</tr>
<tr>
<td>$K_S^0 \rightarrow \mu^+\mu^-\mu^+\mu^-$</td>
<td>1</td>
<td>0.37 (0.37)</td>
<td>1.1 (1.1)</td>
<td>$\sim$ 1.0</td>
<td>$\sim$ 6.0</td>
</tr>
<tr>
<td>$K_L^0 \rightarrow \mu^+\mu^-$</td>
<td>$\sim$ 1</td>
<td>2.7 (2.7) $\times 10^{-3}$</td>
<td>0.014 (0.014)</td>
<td>$\sim$ 3.0</td>
<td>$\sim$ 7.0</td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+\pi^+\pi^-$</td>
<td>$\sim$ 2</td>
<td>9.0 (0.75) $\times 10^{-3}$</td>
<td>41 (8.6) $\times 10^{-3}$</td>
<td>$\sim$ 1.0</td>
<td>$\sim$ 4.0</td>
</tr>
<tr>
<td>$K^+ \rightarrow \pi^+\mu^+\mu^-$</td>
<td>$\sim$ 2</td>
<td>6.3 (2.3) $\times 10^{-3}$</td>
<td>0.030 (0.014)</td>
<td>$\sim$ 1.5</td>
<td>$\sim$ 4.5</td>
</tr>
<tr>
<td>$\Sigma^+ \rightarrow p\mu^+\mu^-$</td>
<td>$\sim$ 0.13</td>
<td>0.28 (0.28)</td>
<td>0.64 (0.64)</td>
<td>$\sim$ 1.0</td>
<td>$\sim$ 3.0</td>
</tr>
<tr>
<td>$\Lambda \rightarrow p\pi^-\bar{\nu}_\mu$</td>
<td>$\sim$ 0.45</td>
<td>0.41 (0.075)</td>
<td>1.3 (0.39)</td>
<td>$\sim$ 1.5</td>
<td>$\sim$ 5.0</td>
</tr>
<tr>
<td>$\Xi^- \rightarrow \Lambda\mu^-\bar{\nu}_\mu$</td>
<td>$\sim$ 0.04</td>
<td>39 (5.7) $\times 10^{-3}$</td>
<td>0.27 (0.09)</td>
<td>$\sim$ 1.0</td>
<td>$\sim$ 3.0</td>
</tr>
<tr>
<td>$\Xi^- \rightarrow \Sigma^0\mu^-\bar{\nu}_\mu$</td>
<td>$\sim$ 0.03</td>
<td>24 (4.9) $\times 10^{-3}$</td>
<td>0.21 (0.068)</td>
<td>$\sim$ 1.0</td>
<td>$\sim$ 3.0</td>
</tr>
<tr>
<td>$\Xi^- \rightarrow p\pi^-\pi$</td>
<td>$\sim$ 0.03</td>
<td>0.41 (0.05)</td>
<td>0.94 (0.20)</td>
<td>$\sim$ 3.0</td>
<td>$\sim$ 9.0</td>
</tr>
<tr>
<td>$\Xi^0 \rightarrow p\pi^-$</td>
<td>$\sim$ 0.03</td>
<td>1.0 (0.48)</td>
<td>2.0 (1.3)</td>
<td>$\sim$ 5.0</td>
<td>$\sim$ 10</td>
</tr>
<tr>
<td>$\Omega^- \rightarrow \Lambda\pi^-$</td>
<td>$\sim$ 0.001</td>
<td>95 (6.7) $\times 10^{-3}$</td>
<td>0.32 (0.10)</td>
<td>$\sim$ 7.0</td>
<td>$\sim$ 20</td>
</tr>
</tbody>
</table>

- To be able to extract $V_{us}$ one needs to compute form factors precisely

Lattice effort from RBC/UKQCD

Talk by Dettori@FPCP20
2.6 Theoretical Prospects for $V_{us}$

- Lattice Progress on hadronic matrix elements: decay constants, FFs

- Full QCD+QED decay rate on the lattice, for **Leptonic decays of kaons and pions**
  - Inclusion of EM and IB corrections:
    - Perturbative treatment of QED on lattice established
    - Formalism for $K_{l2}$ worked out

- Application of the method for **semileptonic Kaon ($K_{l3}$) and Baryon decays**
  - Aim: Per mille level within 10 years
2.7 $|V_{ud}|$ from $0^+ \rightarrow 0^+$ superallowed $\beta$ decays

PDG 2018:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9994(4)_{V_{ud}}(2)_{V_{us}}$$

PDG 2020:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(3)_{V_{ud}}(4)_{V_{us}}$$

Recent improvement on the theoretical RCs + Nuclear Structure Corrections

Use of a data driven dispersive approach

Seng et al.'18'18, Gorshteyn’18

See Colloquium by C-Y Seng on Wednesday
2.8 \( |V_{ud}| \) from Neutrons

- **Master Formula:**
  \[
  \left| V_{ud} \right|^2 = \frac{5024.7 s}{\tau_n (1 + 3 \lambda^2)(1 + \Delta_R)}
  \]
  Lifetime \( \lambda = g_A / g_V \)

- Needs \( \delta \lambda / \lambda \approx 3 \times 10^{-4} \) and \( \delta \tau_n \approx 0.3 \text{ s} \) to compete with \( 0^+ \rightarrow 0^+ \) transitions.
- Theoretically, the radiative corrections are under control (same as for \( 0^+ \rightarrow 0^+ \))
- Recent progress:
  - New Perkeo III result: **PERKEO III** result improves world-average of beta asymmetry by factor 5! Half of it is due to the reduction of the scale factor
    \[
    A = -0.11958(21), \ S = 1.2 \quad \lambda_A = -1.2757(5)
    \]
  - Tension with **aSPECT** result:
    \[
    \lambda_{\text{avg}} = -1.2754(13), \ S = 2.7
    \]
2.8 $|V_{ud}|$ from Neutrons

- **Master Formula:**

$$|V_{ud}|^2 = \frac{5024.7s}{\tau_n \left(1 + 3\lambda^2 \right) \left(1 + \Delta_R \right)}$$

- Lifetime

- $\lambda = g_A / g_V$

- Needs $\delta \lambda/\lambda \approx 3 \times 10^{-4}$ and $\delta \tau_n \approx 0.3$ s to compete with $0^+ \rightarrow 0^+$ transitions.

- Theoretically, the radiative corrections are under control (same as for $0^+ \rightarrow 0^+$)

- Recent progress:
  - New Perkeo III result: **PERKEO III** result improves world-average of beta asymmetry by factor 5! Half of it is due to the reduction of the scale factor
    $$A = -0.11958(21), \quad S = 1.2 \quad \lambda_A = -1.2757(5)$$
  - New result for Lifetime from **UCN**
    $$\tau_n = 877.75 \pm 0.28^{+0.22}_{-0.16} \text{ s}$$

- Improvement by a factor of 2.25 compared to previous result

---

*See Talk by Chen Yu Liu this afternoon*
2.9 \(|V_{ud}| \) from pion β decay: \(\pi^+ \rightarrow \pi^0 e^+\nu\)

- Theoretically cleanest method to extract \(V_{ud}\): corrections computed in SU(2) ChPT

- Present result: PIBETA Experiment (2004) \(\rightarrow\) Uncertainty: 0.64%

\[
B(\pi^+ \rightarrow \pi^0 e^+\nu) = (1.036 \pm 0.004_{\text{stat}} \pm 0.004_{\text{syst}} \pm 0.003_{\pi e2}) \times 10^{-8} (\pm 0.6\%)
\]

\[
|V_{ud}| = 0.9739(28)_{\exp} \left(1\right)_{\text{th}}
\]
to be compared to \(|V_{ud}| = 0.97373(31)|

- Reduction of the theory error thanks to a new lattice calculation for RC Feng et al’20

- Next generation experiment PIONEER Phase II and III measurement at 0.02% level \(\rightarrow\) will be competitive with current \(0^+ \rightarrow 0^+\) extraction

- Would be completely independent check! No nuclear correction and different RCs compared to neutron decay

- Opportunity to extract \(V_{us}/V_{ud}\) from \[
\frac{B(K \rightarrow \pi l\nu)}{B(\pi^+ \rightarrow \pi^0 e^+\nu)}
\]

\(\text{EW Rad. Corr. cancel}\)

Improve precision on \(B(\pi^+ \rightarrow \pi^0 e^+\nu)\) by x3 \(\rightarrow\) \(V_{us}/V_{ud} < \pm 0.2\%\)
3. Charged pion decays
3.1 Pion decays and LFU tests

- Lepton Flavor Universality test in

  \[ R_{e/\mu}^{\text{theory}} = \frac{\Gamma(\pi \rightarrow e\nu(\gamma))}{\Gamma(\pi \rightarrow \mu\nu(\gamma))} \]

  (dominated by PIENU expt.)

  - Early insight into the V–A structure of weak interactions
  - Exceptional precision of the SM prediction using ChPT

  \[ R_{e/\mu}(\text{SM}) = 1.23524(015) \times 10^{-4} \]

  *Cirigliano & Rosell’07*

  - World average (mainly PIENU at TRIUMF):

  \[ R_{e/\mu}(\text{Exp}) = 1.23270(230) \times 10^{-4} \]

  15 times worse than theory!

Goal of PIONEER: reduce unc. by a factor of 10 ! by far most precise test of LFU

\[ \frac{g_e}{g_\mu} = 0.9990 \pm 0.0009 \ (\pm 0.09\%) \]
3.2 PIONEER (Phase-I)

PIONEER (Phase-I) approved at PSI, physics starting in ~2029

- Goal: matching the SM precision on $R_{e/\mu}$
  - Test of New Physics at 1 PeV scale

- Stopped $\pi^+$ at high rate (300 kHz), focus on reduction of systematics.

- Detectors: highly-segmented LGAD active target, positron tracker, LXe calorimeter

- Collection of $2 \times 10^8 \ \pi^+ \rightarrow e^+\nu_e$ events in three years.

- Key point: control of the $\pi^+ \rightarrow e^+\nu_e$ signal tail in the calorimeter to a $10^{-4}$ precision

PIONEER Phase II,III:

- $V_{ud}$ from $\pi^+ \rightarrow \pi^0 e^+\nu_e$ decays to a 0.02% level
3.3 Example: Constraints on Heavy Neutral Leptons

- Strongest $|U_{e4}|^2$ limits below 400 MeV: $K^+, \pi^+ \rightarrow e^+N$ from NA62 & PIENU.
- Also important limits on $|U_{\mu4}|^2$ from E949, NA62 and PIENU.
- NA62/E949 limits are complementary to HNL decay searches at T2K.
- Next-generation $K^+$ and $p^+$ experiments (NA62++, PIONEER) to improve by up to factor 10, reaching the seesaw bound.

Electron coupling

Muon coupling

[arXiv:2201.07805]
4. Conclusion and Outlook
Conclusion and Outlook

• Recent precision determinations of $V_{us}$ and $V_{ud}$ enable unprecedented tests of the SM and constraints on possible NP models.

• Tensions in unitarity of 1st row of CKM matrix have reappeared!

• We need to work hard to understand where they come from:
  - On experimental side:
    For $V_{us}$, new measurements in kaons ($NA62$: $K_{\mu 3}/K_{\mu 2}$, $LHCb$?) but mainly in tau decays from $Belle$ II $V_{us}$ from hyperon decays? $BESSIII$, $LHCb$?
    - For $V_{ud}$, understand the situation of the neutron lifetime, beta decay of pion? $PIONEER$ Consider $R_V = \Gamma (K \rightarrow \pi l \nu(\gamma)) / \Gamma (\pi^+ \rightarrow \pi^0 e^+ \nu(\gamma))$ $Czarnecki$, Marciano, Sirlin’20
  - On theory side:
    Calculate very precisely radiative corrections, isospin breaking effects and matrix elements
    Be sure the uncertainties are under control
    - If these tensions are confirmed what do they tell us?

• Interesting time ahead of us!
5. Back-up
2.1 $V_{us}$ from $K_{l3}$

Progress since 2018:

- First experimental measurement of BR of $K_S \rightarrow \pi \mu \nu$
  \[ \text{BR}(K_S \rightarrow \pi \mu \nu) = (4.56 \pm 0.20) \times 10^{-4} \]

- Theoretically update on long-distance EM corrections:
  \[ \Gamma \rightarrow e^2 p^2 + \text{model estimate for the LECs} \]

Up to now computation at fixed order $e^2 p^2$ + model estimate for the LECs

New calculation of complete EW RC using hybrid current algebra and ChPT (Sirlin’s representation) with resummation of largest terms to all chiral orders
  - Reduced uncertainties at $O(e^2 p^4)$
  - Lattice evaluation of QCD contributions to $\gamma W$ box diagrams

\[ \text{Cirigliano et al. ’08} \]
\[ \text{Seng et al. ’21} \]
2.1 \( V_{us} \) from \( K_{l3} \)

Progress since 2018:

- First experimental measurement of BR of \( K_S \rightarrow \pi \mu \nu \)
  \[
  \text{BR}(K_S \rightarrow \pi \mu \nu) = (4.56 \pm 0.20) \times 10^{-4}
  \]

- Theoretically update on long-distance EM corrections:

Only \( K_{e3} \) at present
For \( K_{\mu 3} \) modes continue to use Cirigliano et al. ’08

<table>
<thead>
<tr>
<th></th>
<th>Cirigliano et al. ’08</th>
<th>Seng et al. ’21</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta_{EM}(K^0_{e3}) ) [%]</td>
<td>0.50 ( \pm ) 0.11</td>
<td>0.580 ( \pm ) 0.016</td>
</tr>
<tr>
<td>( \Delta_{EM}(K^+_{e3}) ) [%]</td>
<td>0.05 ( \pm ) 0.13</td>
<td>0.105 ( \pm ) 0.024</td>
</tr>
<tr>
<td>( \rho )</td>
<td>+0.081</td>
<td>-0.039</td>
</tr>
</tbody>
</table>
2.1 \( V_{us} \) from \( K_{l3} \)

Progress since 2018:

- Theoretical progress on isospin breaking correction

\[
\Delta^{SU(2)} = \frac{f_+(0) K^+ \pi^0}{f_+(0) K^0 \pi^-} - 1
\]

\[
= 3 \left[ \frac{1}{4} \frac{M_K^2}{M_{\pi}^2} + \frac{\chi_p^4}{2} \left( 1 + \frac{m_s}{\hat{m}} \right) \right] \quad Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}
\]

\[
\chi_p^4 = 0.252 \quad \text{NLO in strong interaction}
\]

\[
\epsilon_{EM}^{(4)} \sim 10^{-6}
\]

Cirigliano et al., ’02; Gasser & Leutwyler, ’85

\[
= +2.61(17)\% \quad \text{Calculated using:}
\]

- \( Q = 22.1(7) \)
- \( m_s/\hat{m} = 27.23(10) \)
- \( M_K = 494.2(3) \)
- \( M_{\pi} = 134.8(3) \)

Test by evaluating \( V_{us} \) from \( K^\pm \) and \( K^0 \) data with no corrections:

Equality of \( V_{us} \) values would require \( \Delta^{SU(2)} = 2.86(34)\% \)
2.1 $V_{us}$ from $K_{l3}$

Previous to recent results for $Q$, uncertainty on $\Delta_{SU(2)}$ was leading contributor to uncertainty on $V_{us}$ from $K^{\pm}$ decays

Reference value of $Q$ from dispersion relation analyses of $\eta \rightarrow 3\pi$ Dalitz plots
Colangelo et al., ’18
$Q = 22.1 \pm 0.7$

Lattice results for $Q$ somewhat higher than analytical results
But, lattice results have finite correction to LO expectation:

$$Q_M^2 \equiv \frac{\hat{M}_K^2 - \hat{M}_\pi^2}{\hat{M}_K^2 - \hat{M}_K^0 - \hat{M}_K^{2+}}$$

Low-energy theorem: $Q$ has no correction at NLO

E. Passemard, CD 2021
**$V_{us}$ from Tau decays**

- Belle II with 50 ab$^{-1}$ and $\sim 4.6 \times 10^{10}$ $\tau$ pairs will improve $V_{us}$ extraction
- Inclusive measurement is an opportunity to have a complete independent measurement of $V_{us}$ not easy as you have to measure many channels

### Summary of $|V_{us}|$ results
- $|V_{us}|$ from kaon and tau falls short of CKM unitarity value by $\sim 3\sigma$
- $|V_{us}|$ from inclusive tau decays independent of Lattice errors used for kaons
- New physics affecting 3rd generation only affects $|V_{us}|$ from taus
- Tau decays at Belle II offers unique and complementary insight

### Preliminary Results
- $V_{us}$ $K_{13}, N_f = 2+1+1, 2021$ update
  - $0.2231 \pm 0.0006$  
  - $-3.2\sigma$
- $V_{us}$ $K_{12}, N_f = 2+1+1, PDG 2020$
  - $0.2252 \pm 0.0005$  
  - $-2.7\sigma$
- CKM unitarity & $V_{ud}$ & $V_{ub}$
  - $0.2277 \pm 0.0013$  
  - $-3.7\sigma$
  - $\tau \rightarrow X_{s}\nu$
  - $0.2184 \pm 0.0021$  
  - $-2.1\sigma$
  - $\tau \rightarrow K\nu / \tau \rightarrow \pi\nu$
  - $0.2229 \pm 0.0019$  
  - $-2.6\sigma$
  - $\tau \rightarrow K\nu$
  - $0.2219 \pm 0.0017$  
  - $-2.5\sigma$
  - $\tau$ exclusive average
  - $0.2222 \pm 0.0017$  
  - $-3.5\sigma$
  - $\tau$ average
  - $0.2207 \pm 0.0014$  
  - $-3.2\sigma$
**V_{us} from Tau decays**

HFLAV 2021 \( \tau \) branching fractions to strange final states:

<table>
<thead>
<tr>
<th>Branching fraction</th>
<th>HFLAV 2021 fit (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K^- \nu_\tau )</td>
<td>0.6957 ± 0.0096</td>
</tr>
<tr>
<td>( K^- \pi^0 \nu_\tau )</td>
<td>0.4322 ± 0.0148</td>
</tr>
<tr>
<td>( K^- 2 \pi^0 \nu_\tau ) (ex.( K^0 ))</td>
<td>0.0634 ± 0.0219</td>
</tr>
<tr>
<td>( K^- 3 \pi^0 \nu_\tau ) (ex.( K^0, \eta ))</td>
<td>0.0465 ± 0.0213</td>
</tr>
<tr>
<td>( \pi^- K^0 \nu_\tau )</td>
<td>0.8375 ± 0.0139</td>
</tr>
<tr>
<td>( \pi^- \bar{K}^0 \pi^0 \nu_\tau )</td>
<td>0.3810 ± 0.0129</td>
</tr>
<tr>
<td>( \pi^- \bar{K}^0 2 \pi^0 \nu_\tau ) (ex.( K^0 ))</td>
<td>0.0234 ± 0.0231</td>
</tr>
<tr>
<td>( \bar{K}^0 h^- h^- h^+ \nu_\tau )</td>
<td>0.0222 ± 0.0202</td>
</tr>
<tr>
<td>( K^- \eta \nu_\tau )</td>
<td>0.0155 ± 0.0008</td>
</tr>
<tr>
<td>( K^- \pi^0 \eta \nu_\tau )</td>
<td>0.0048 ± 0.0012</td>
</tr>
<tr>
<td>( \pi^- \bar{K}^0 \eta \nu_\tau )</td>
<td>0.0094 ± 0.0015</td>
</tr>
<tr>
<td>( K^- \omega \nu_\tau )</td>
<td>0.0410 ± 0.0092</td>
</tr>
<tr>
<td>( K^- \phi(K^+K^-) \nu_\tau )</td>
<td>0.0022 ± 0.0008</td>
</tr>
<tr>
<td>( K^- \phi(K^0\bar{K}^0) \nu_\tau )</td>
<td>0.0015 ± 0.0006</td>
</tr>
<tr>
<td>( K^- \pi^- \pi^+ \nu_\tau ) (ex.( K^0, \omega ))</td>
<td>0.2924 ± 0.0068</td>
</tr>
<tr>
<td>( K^- \pi^- \pi^+ \pi^0 \nu_\tau ) (ex.( K^0, \omega, \eta ))</td>
<td>0.0387 ± 0.0142</td>
</tr>
<tr>
<td>( K^- 2 \pi^- 2 \pi^+ \nu_\tau ) (ex.( K^0 ))</td>
<td>0.0001 ± 0.0001</td>
</tr>
<tr>
<td>( K^- 2 \pi^- 2 \pi^+ \pi^0 \nu_\tau ) (ex.( K^0 ))</td>
<td>0.0001 ± 0.0001</td>
</tr>
<tr>
<td>( X_s^- \nu_\tau )</td>
<td>2.9076 ± 0.0478</td>
</tr>
</tbody>
</table>

HFLAV'21

\[
R_{\tau} \equiv \frac{\Gamma(\tau^{-} \rightarrow \nu_{\tau} + \text{hadrons})}{\Gamma(\tau^{-} \rightarrow \nu_{\tau} e^- \bar{\nu}_e)} \approx N_c
\]

partron model prediction

\[
\delta R_{\tau} \equiv \frac{R_{\tau,NS}}{|V_{ud}|^2} - \frac{R_{\tau,S}}{|V_{us}|^2}
\]

**SU(3) breaking** quantity, strong dependence in \( m_s \) computed from OPE (L+T) + phenomenology

\[
\delta R_{\tau,\text{th}} = 0.0242(32)
\]

Gamiz et al'07, Maltman’11

\[
|V_{us}|^2 = \frac{R_{\tau,S}}{R_{\tau,NS}} \left( \frac{\delta R_{\tau,\text{th}}}{|V_{ud}|^2} \right) - \delta R_{\tau,\text{th}}
\]

2.9σ away from unitarity!

\[
|V_{us}| = 0.2184 \pm 0.0018_{\text{exp}} \pm 0.0011_{\text{th}}
\]