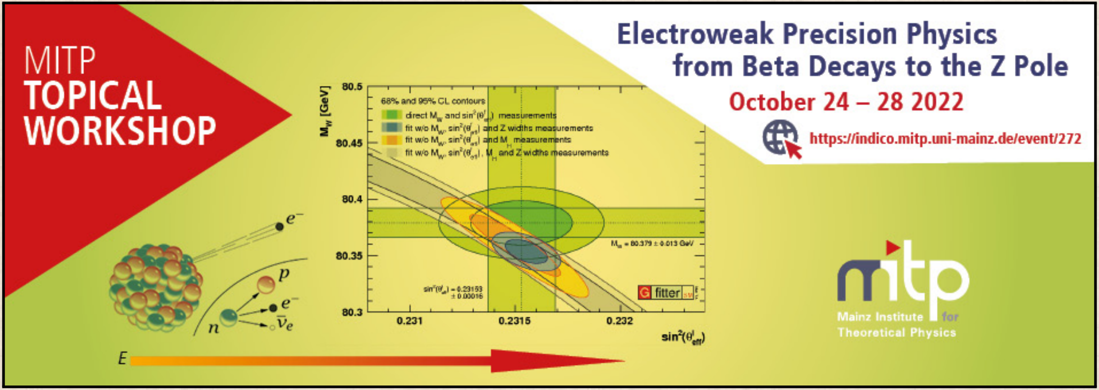

Prospects for |Vus| determinations from tau decays

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$|V_{us}|$ calculation using tau branching fractions

$|V_{us}|$ from tau inclusive

- $$|V_{us}|_{\tau s} = \sqrt{R(\tau \to X_{\text{strange}}\,\nu) / \left[\frac{R(\tau \to X_{\text{non-strange}}\,\nu)}{|V_{ud}|^2} - \delta R_{\tau,\text{SU3 breaking}} \right]} \qquad \boxed{\tau \to X_s \nu}$$
  - $R(\tau \to X\,\nu) = \mathcal{B}(\tau \to X\,\nu)/\mathcal{B}(\tau \to e\bar{\nu}\nu)_{\text{universality improved}}$
  - $\delta R_{\tau,\text{SU3 breaking}}$ computed using tau spectral functions and perturbative QCD (OPE)
  - Gamiz,Jamin,Pich,Prades,F.Schwab, Nucl.Phys.Proc.Suppl.169 (2007) 85, arXiv:hep-ph/0612154

$|V_{us}|$ from tau exclusive

- $$\frac{\mathcal{B}(\tau^- \to K^-\nu_\tau)}{\mathcal{B}(\tau^- \to \pi^-\nu_\tau)} = \left(\frac{f_{K\pm}}{f_{\pi\pm}}\right)^2 \frac{|V_{us}|^2_{\tau K/\pi}}{|V_{ud}|^2} \frac{\left(m_\tau^2 - m_K^2\right)^2}{\left(m_\tau^2 - m_\pi^2\right)^2} (1 + \delta R_{\tau K/\tau\pi}) \qquad \boxed{\tau \to K/\tau \to \pi}$$
- $$\mathcal{B}(\tau^- \to K^-\nu_\tau) = \frac{1}{16\pi}\left(\frac{G_F}{\hbar^3 c^3}\right)^2 |V_{us}|^2_{\tau K} f_{K\pm}^2 \frac{\tau_\tau}{\hbar} m_\tau^3 c^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW}^{m_\tau}(1 + \delta R_{\tau K}) \qquad \boxed{\tau \to K}$$

$|V_{us}|$ calculation notes

tau branching fractions

- HFLAV tau BR fit, includes published and additional estimated correlations
- arXiv:2206.07501 (accepted by PRD), HFLAV Tau Winter 2022 web report

radiative corrections definitions

- $\Gamma(\tau^- \to \pi^- \nu_\tau) = \Gamma_{\text{th,LO}}(\tau^- \to \pi^- \nu_\tau) S_{EW}^{m_\tau} (1 + \delta R_{\tau\pi})$
- $\Gamma(\tau^- \to K^- \nu_\tau) = \Gamma_{\text{th,LO}}(\tau^- \to K^- \nu_\tau) S_{EW}^{m_\tau} (1 + \delta R_{\tau K})$
- $\frac{\Gamma(\tau^- \to K^- \nu_\tau)}{\Gamma(\tau^- \to \pi^- \nu_\tau)} = \frac{\Gamma_{\text{th,LO}}(\tau^- \to K^- \nu_\tau)}{\Gamma_{\text{th,LO}}(\tau^- \to \pi^- \nu_\tau)} \frac{S_{EW}^{m_\tau}(1 + \delta R_{\tau K})}{S_{EW}^{m_\tau}(1 + \delta R_{\tau\pi})} = \frac{\Gamma_{\text{th,LO}}(\tau^- \to K^- \nu_\tau)}{\Gamma_{\text{th,LO}}(\tau^- \to \pi^- \nu_\tau)} (1 + \delta R_{\tau K/\tau\pi})$

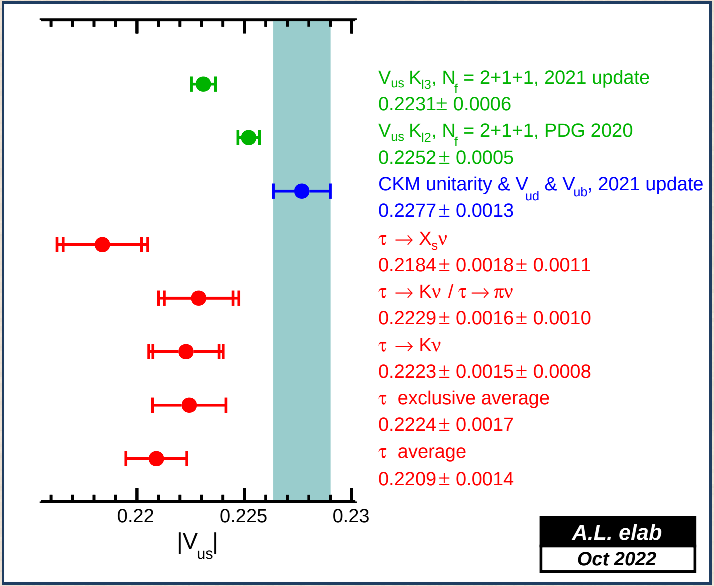
recent new radiative corrections for tau hadronic decays

- [M.A.Arroyo-Ureña, G.Hernández-Tomé, G.López-Castro, P.Roig, I.Rosell, PRD 104 (2021) L091502]
- $\delta R_{\tau\pi} = (-0.24 \pm 0.56)\%$, $\delta R_{\tau K} = (-0.15 \pm 0.57)\%$, $\delta R_{\tau K/\tau\pi} = (0.10 \pm 0.80)\%$
- larger uncertainties than previous ones [*] but more reliable
  [*] R.Decker, M.Finkemeier, PLB 334 (1994) 199; NPB 438 (1995) 17; NPB 40 (1995) 453 (P.S.)

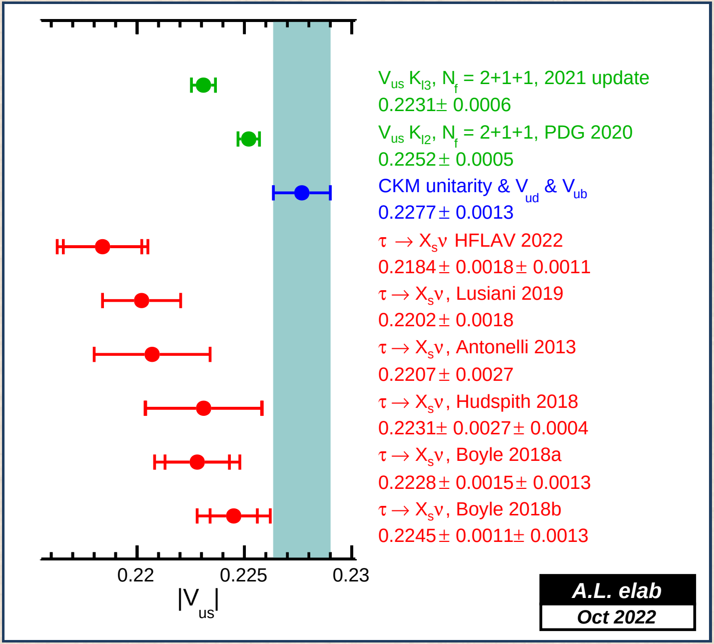
$\mathcal{B}(\tau \to X_s \nu)$ from HFLAV Winter 2022 fit

Tau decay mode	Branching fraction (%)
$K^- \nu_\tau$	0.6957 ± 0.0096
$K^- \pi^0 \nu_\tau$	0.4322 ± 0.0148
$K^- 2\pi^0 \nu_\tau$ (ex.K^0)	0.0634 ± 0.0219
$K^- 3\pi^0 \nu_\tau$ (ex.K^0, η)	0.0465 ± 0.0213
$\pi^- \bar{K}^0 \nu_\tau$	0.8375 ± 0.0139
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.3810 ± 0.0129
$\pi^- \bar{K}^0 2\pi^0 \nu_\tau$ (ex.K^0)	0.0234 ± 0.0231
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.0222 ± 0.0202
$K^- \eta \nu_\tau$	0.0155 ± 0.0008
$K^- \pi^0 \eta \nu_\tau$	0.0048 ± 0.0012
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0094 ± 0.0015
$K^- \omega \nu_\tau$	0.0410 ± 0.0092
$K^- \phi(K^+ K^-) \nu_\tau$	0.0022 ± 0.0008
$K^- \phi(K^0_S K^0_L) \nu_\tau$	0.0015 ± 0.0006
$K^- \pi^- \pi^+ \nu_\tau$ (ex.K^0, ω)	0.2924 ± 0.0068
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex.K^0, ω, η)	0.0387 ± 0.0142
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex.K^0)	0.0001 ± 0.0001
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex.K^0)	0.0001 ± 0.0001
$X_s^- \nu_\tau$	2.9076 ± 0.0478

$|V_{us}|$ from tau


V_{us} K_{l3}, N_f = 2+1+1, 2021 update
0.2231 ± 0.0006
V_{us} K_{l2}, N_f = 2+1+1, PDG 2020
0.2252 ± 0.0005
CKM unitarity & V_{ud} & V_{ub}, 2021 update
0.2277 ± 0.0013
$\tau \to X_s \nu$
0.2184 ± 0.0018 ± 0.0011
$\tau \to K\nu$ / $\tau \to \pi\nu$
0.2229 ± 0.0016 ± 0.0010
$\tau \to K\nu$
0.2223 ± 0.0015 ± 0.0008
τ exclusive average
0.2224 ± 0.0017
τ average
0.2209 ± 0.0014
0.22
0.225
0.23
$|V_{us}|$
A.L. elab
Oct 2022

$|V_{us}|$ from tau inclusive determinations



$|V_{us}|$ from $\tau \to X_s \nu$ determinations

HFLAV 2022

- uses $\delta R_{\tau,\mathrm{SU3\ breaking}}$ from Gamiz,Jamin,Pich,Prades,F.Schwab, Nucl.Phys.Proc.Suppl.169 (2007) 85
- Gamiz,Jamin,Pich,Prades,Schwab, PoS KAON (2008) 008 provide $\delta R_{\tau,\mathrm{SU3\ breaking}}$ twice more precise
- second group (K.Maltman and others) provides alternative determinations, with more heavy and complex use of tau spectral functions and lattice QCD calculations
- no consensus between groups, and in the community, about recommended $|V_{us}|$ calculation
- today $\Delta\,|V_{us}|\,(\tau)_{\mathrm{exp}} \sim 2{\times}\Delta\,|V_{us}|\,(\tau)_{\mathrm{th}}$
- HFLAV reports $|V_{us}|$ with Gamiz *et al.* calculations because it can be easily updated, while K.Maltman *et al.* calculations are much more complex and would require assistance of the authors

$|V_{us}|$ from $\tau \to X_s\nu$ determinations

M. Antonelli *et al.*, JHEP 10 (2013) 76

- predict tau BRs $\mathcal{B}(\tau \to K\nu)$, $\mathcal{B}(\tau \to K\pi^0\nu)$, $\mathcal{B}(\tau \to K_s^0\pi\nu)$ from kaon BRs $\mathcal{B}(K \to \ell\nu)$, $\mathcal{B}(K \to \ell\pi^0\nu)$
- replace measurements of above tau branching fractions their predictions
- compute $|V_{us}|$ with Gamiz *et al.* technique
- other tau branching fractions from HFLAV 2012

A.L., SciPost Phys. Proc. 1 (2019) 1

- use Antonelli 2013 predictions of 3 tau branching fractions, but rather than replacing the respective tau measurements, statistically combine predictions and measurements in modified HFLAV tau BRs fit
- compute $|V_{us}|$ with Gamiz *et al.* technique
- other tau branching fractions from:
  - HFLAV Spring 2017
  - *BABAR* ICHEP 2018 results (6 channels), *BABAR* 2018 paper (1 channel)

$|V_{us}|$ from $\tau \to X_s\nu$ determinations

J. Hudspith *et al.*, PLB 781 (2018) 206

- revised technique, uses also tau spectral functions
  "a combination of continuum and lattice results is shown to suggest a new implementation of the flavor-breaking sum rule approach in which not only |Vus|, but also D>4 effective condensates, are fit to data."
- replace tau BRs $\begin{matrix}\mathcal{B}(\tau \to K\pi^0\nu) \\ \mathcal{B}(\tau \to K^0_s\pi\nu)\end{matrix}$ with Antonelli 2013 predictions
- other tau branching fractions from HFLAV Spring 2017

P. Boyle *et al.*, PRL 121 (2018) 202003

- compute $|V_{us}|$ from tau inclusive with a nove technique using
  - tau spectral functions
  - lattice QCD
- capitalizes on LQCD work for muon $g-2$ hadronic contribution
- two $|V_{us}|$ results:

  Boyle 2018a: using HFLAV Spring 2017 results

  Boyle 2018b: HFLAV Spring 2017 replacing $\begin{matrix}\mathcal{B}(\tau \to K\pi^0\nu) \\ \mathcal{B}(\tau \to K^0_s\pi\nu)\end{matrix}$ with Antonelli 2013

Uncertainties

method	experiment [%]	theory [%]	lattice QCD [%]	rad.corr. [%]
$\tau \to X_s \nu$	0.84	0.49		
$\tau \to K / \tau \to \pi$	0.72		0.18	0.40
$\tau \to K$	0.69		0.19	0.29

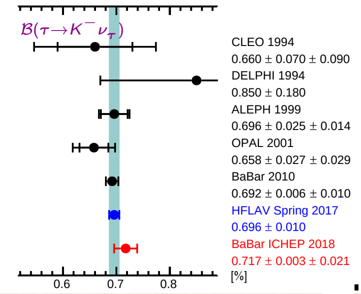
uncertainties prospects

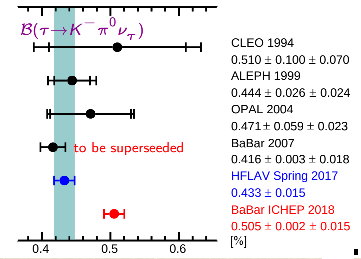
- experiment and rad.corr. uncertainties had minor or no improvements since LEP 1 times
- lattice QCD uncertainties decreased substantially in recent years and are now sub-leading
- recent activity on theory uncertainties, but lack of community consensus on recommended calculation

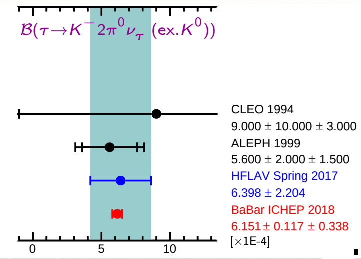
Esperimental precision prospects

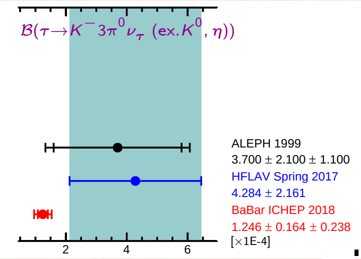
- best large tau BRs from LEP 1 measurements, especially from ALEPH
- B-factories have much larger samples but worse conditions and larger systematics
- B factories improved several small tau BRs where LEP 1 was statistically limited
- Belle II goal is 100x *BABAR* and 50x Belle statistics
  - may provide up to 10x improvement w.r.t. *BABAR*
  - hard work since *BABAR* relevant measurements already systematically limited
  - guestimate: 3x improvement on Cabibbo-suppressed tau BRs
- FCCee(Z) goal is 1300x ALEPH statistics
  - may provide up to 36x improvement on all tau BRs
  - guestimate: 15x improvement on all tau BRs
- Super charm-tau factories may contribute especially in two-body tau BRs, but less probable to happen
- 10x precision improvement $\Rightarrow \sim$0.1% precision will require improvements in PHOTOS for radiation in decay and tau pair generator improvements in sinulation of IFR, FSR, higher order effects

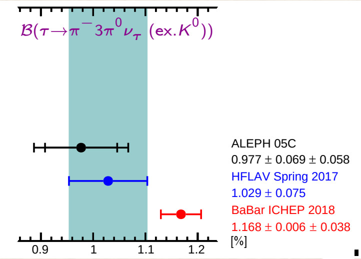
BABAR ICHEP 2018 preliminary tau BR measurements

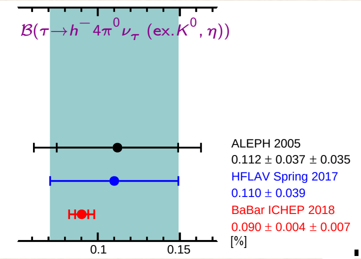




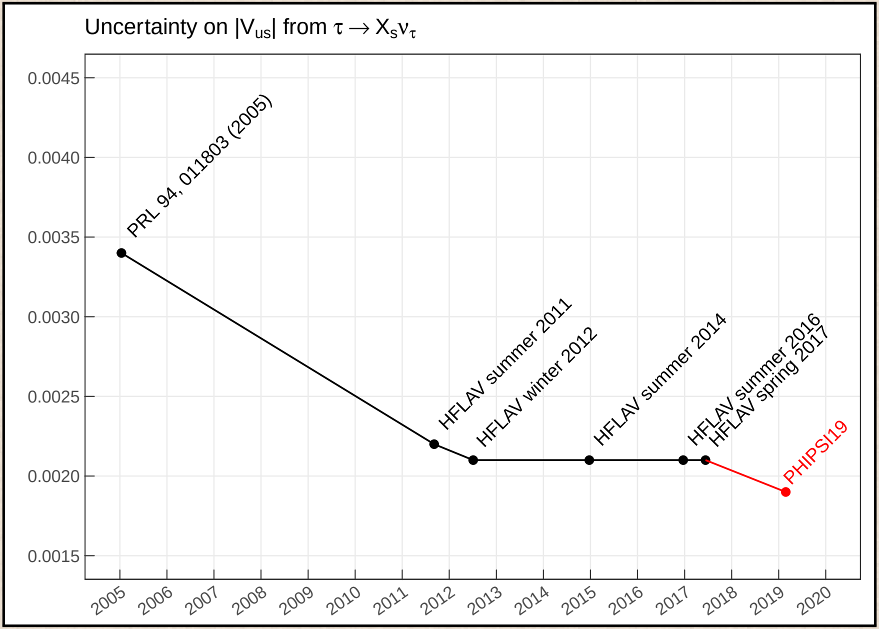








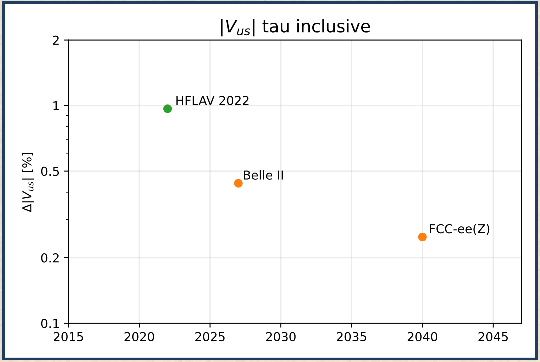
Precision of $|V_{us}|$ from $\tau \to X_s \nu$ over time

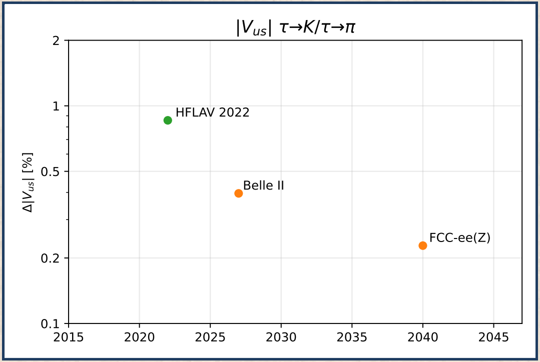


Theory, rad.corr. and lattice QCD prospects

- don't expect near future significant improvements
- guestimate a reduction of 30% (Belle II time) or 50% (FCC-ee)

$|V_{us}|$ from tau prospects





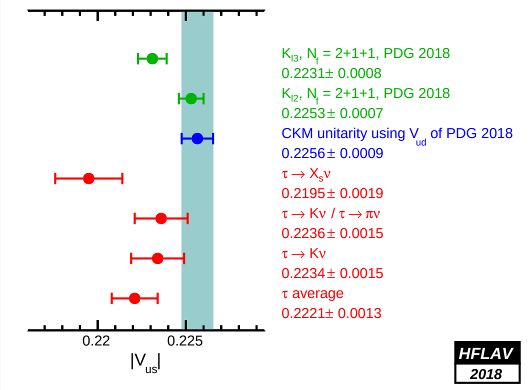
Backup Slides

CKM matrix first row unitarity test: PDG 2018 and HFLAV 2018

- PDG 2018 $|V_{ud}|$- $|V_{us}|$ review
  - $|V_{us}|$ from kaons ∼consistent with unitarity
- HFLAV 2018 report, PRL 93 (2004) 231803
  - $|V_{us}|$ from tau inclusive $>3\,\sigma$ discrepancy
  - $|V_{us}|$ from tau exclusive ∼consistent

note

- alternative $|V_{us}|$ determinations from $\tau \to X_s \nu$ exist, which are more consistent with kaons and CKM unitarity
  - Hudspith, Lewis, Maltman & Zanotti 2018
  - Boyle *et al.* 2018



- $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ ∼consistent with unitarity except when using $|V_{us}|$ from tau inclusive

CKM matrix first row unitarity test: PDG 2020 and HFLAV 2022

$|V_{ud}|$

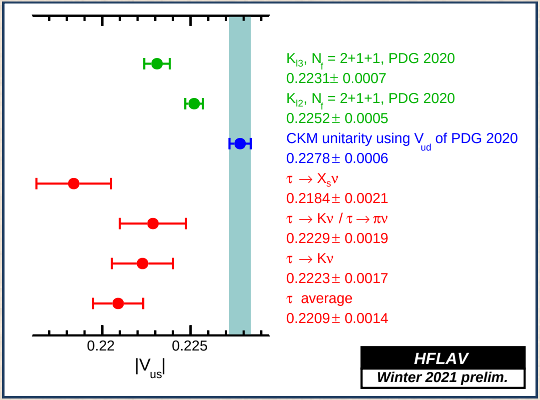
- new dispersive calculation of Δ_R^V inner or universal electroweak radiative corrections (RC) to superallowed nuclear beta decays
  Seng, Gorchtein & Ramsey-Musolf, Phys. Rev. D 100, 013001 (2019)
  - $\sim 2\times$ more precise
  - significant shift

$|V_{us}|$ from kaons

- updated more precise lattice QCD constants

$|V_{us}|$ from tau

- updated lattice QCD constants (minor)
- numerical typo fixed on $|V_{us}|$ from $\tau \to K\nu$



- $|V_{ud}|$ - $|V_{us}|_K$ anomaly $\sim 3\,\sigma$

  (scale factor = 2 on $|V_{us}|_K$ because of difference on $|V_{us}|_K$ from $K_{\ell 3}$ and K_μ)

CKM matrix first row unitarity test: 2021 $|V_{ud}|$ - $|V_{us}|$ update and HFLAV 2022

$|V_{ud}|$

- J.C.Hardy & I.S.Towner, PRC 102, 045501 (2020)
  - revised experimental inputs
  -

Marciano and Sirlin 2006	2.361 ± 0.038
Seng et al. 2018/2019	2.467 ± 0.022
Czarnecki, Marciano and Sirlin 2019	2.426 ± 0.032
Adopted value for Δ_R^V	2.454 ± 0.019

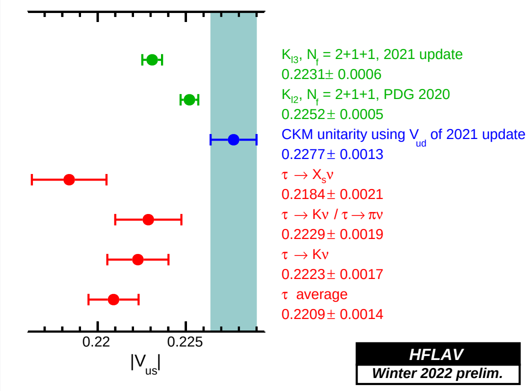
  - increased systematic uncertainty (new nuclear corrections)

$|V_{us}|$ from kaons

- improved K_{e3} radiative corrections, Seng, Gorchtein & Ramsey-Musolf, arXiv:2103.04843 [hep-ph]
- new calculation of $|V_{us}|_{K\ell3}$ Seng, Galviz, Marciano, Meißner, arXiv:2107.14708 [hep-ph]

$|V_{us}|$ from tau

- using 2021 update $|V_{ud}|$ (minor)



- $|V_{ud}|$ - $|V_{us}|_K$ anomaly $\sim 3\,\sigma$
  - no scale factor on $|V_{us}|_K$
  - $\sim 5\,\sigma$ without increased $|V_{ud}|$ systematics

$|V_{us}|$ determinations from kaons

- $\Gamma(K \to \pi\ell\bar{\nu}_\ell[\gamma]) = \frac{G_F^2 m_K^5}{192\pi^3} C_K^2\, S_{\mathrm{EW}}^K \left(|V_{us}| f_+^{K\pi}(0)\right)^2 I_K^\ell \left(1 + \delta_{\mathrm{EM}}^{K\ell} + \delta_{\mathrm{SU(2)}}^{K\pi}\right)^2$ $\boxed{K_{\ell 3}}$
- $\frac{\Gamma(K^- \to \ell^-\bar{\nu}_\ell)}{\Gamma(\pi^- \to \ell^-\bar{\nu}_\ell)} = \frac{|V_{us}|^2}{|V_{ud}|^2} \left(\frac{f_{K\pm}}{f_{\pi\pm}}\right)^2 \frac{m_K(1 - m_\ell^2/m_K^2)^2}{m_\pi(1 - m_\ell^2/m_\pi^2)^2} (1 + \delta_{\mathrm{EM}})$ $\boxed{K_{\ell 2}}$