Precision determination of g_A from lattice QCD & not-yet determined isospin breaking corrections

MITP Workshop: Precision EW Physics from Beta-Decays to the Z-pole
October 24-28

André Walker-Loud

A precise determination of g_A with lattice QCD - why?

- A precise calculation of g_A is important to benchmark lattice QCD calculations
  - do we have all of our systematic uncertainties under control?

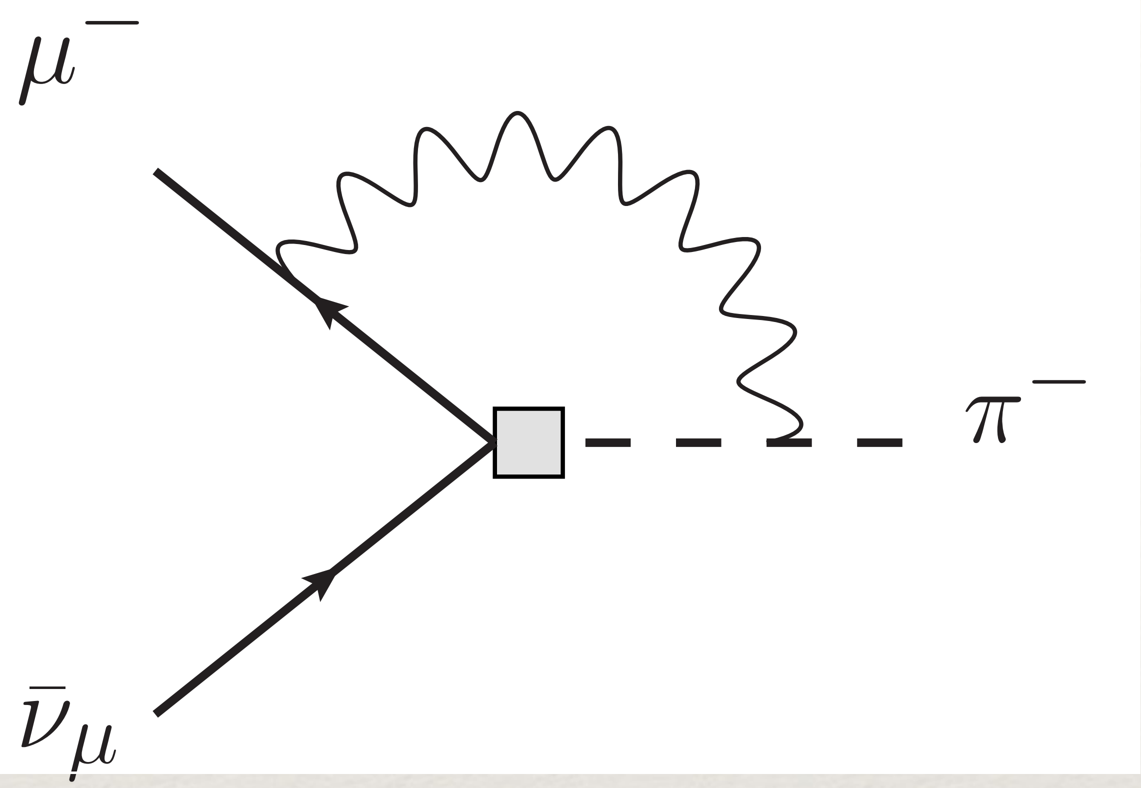
- A precise calculation of g_A, combined with
  experimental measurements
  radiative QED corrections
  can be used to constrain BSM right-handed currents

- A precise calculation of g_A is a stepping stone to
  - Calculating g_A in QCD+QED
  - A full lattice QCD+QED calculation of $n \to p e \bar{\nu}$

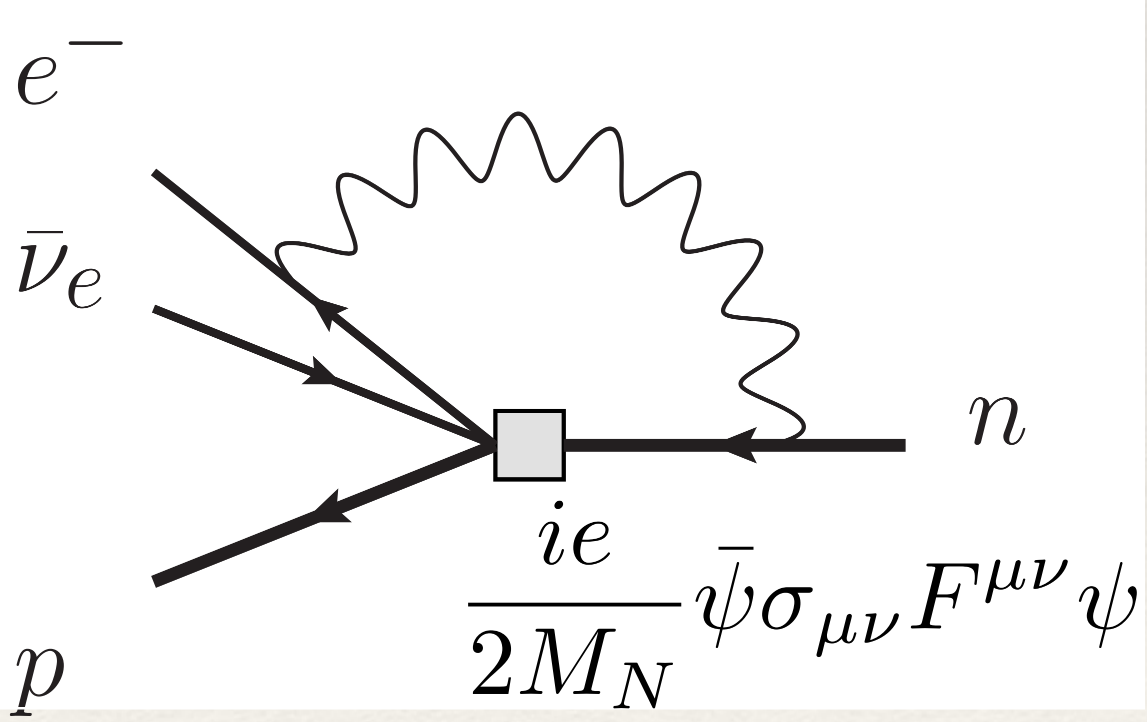
A precise determination of g_A with lattice QCD - why?

- Strictly speaking, g_A is not uniquely determined when QED is included

- Similar to pion decay:
  UV divergences mix weak and EM

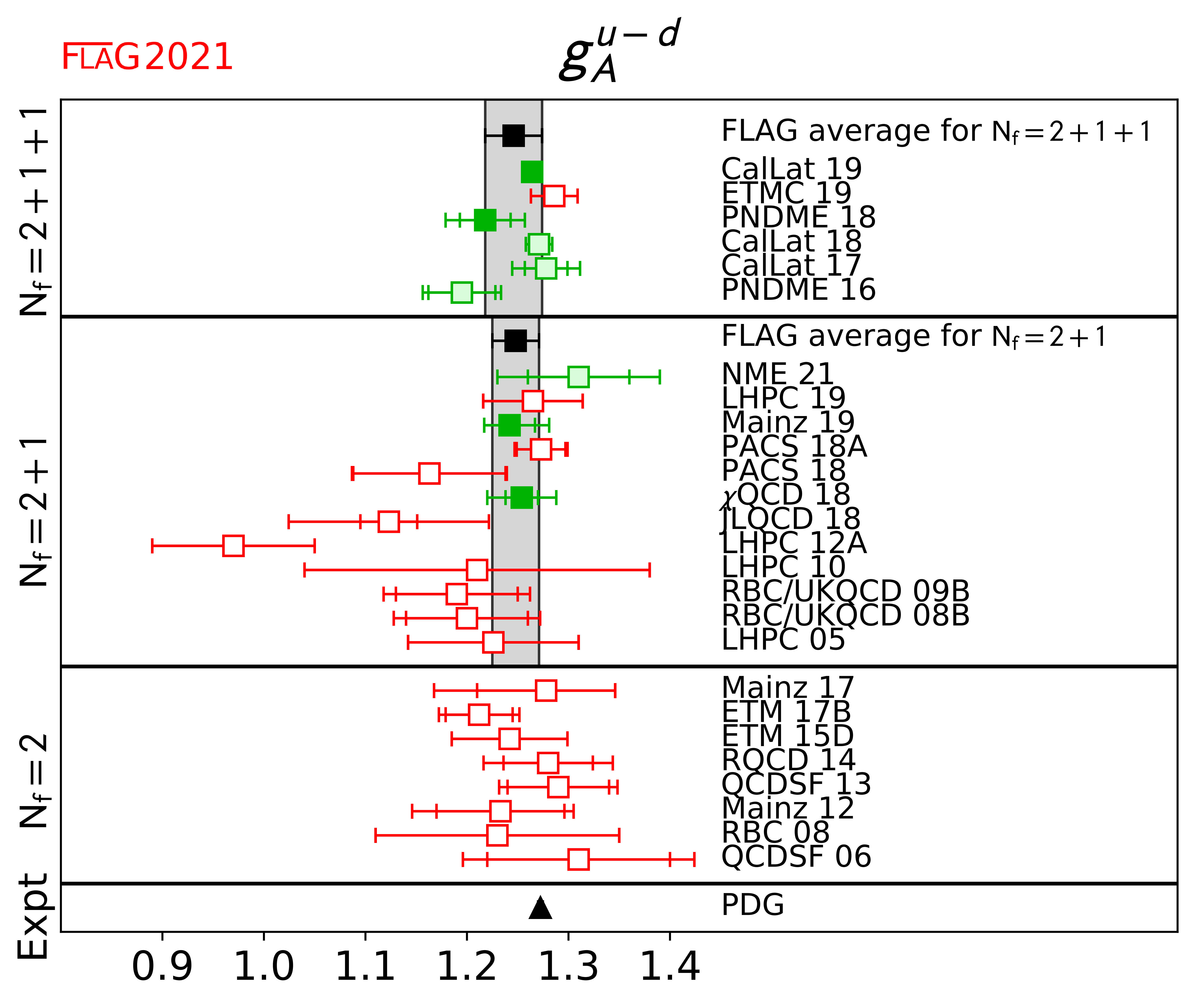


- In the case of the nucleon, these issues are $1/M_N$ suppressed in the matrix-element



- They are still important for the renormalization

Status of lattice QCD results for g_A – 2021



- Now many groups obtaining values of g_A fully extrapolated to the physical point (green)
  - physical pion mass
  - continuum
  - infinite volume

- The CalLat results are noticeably more precise than others
  - I'll describe these results in some detail

Background on my perspective: LQCD + EFT

- I grew up doing chiral perturbation theory (XPT) for lattice QCD (LQCD)
  - It was my “bible” for how to understand low-energy QCD

- In my first postdoc, I began doing numerical LQCD calculations
  - I had to confront my XPT expectations real LQCD results

    Never the twain shall meet

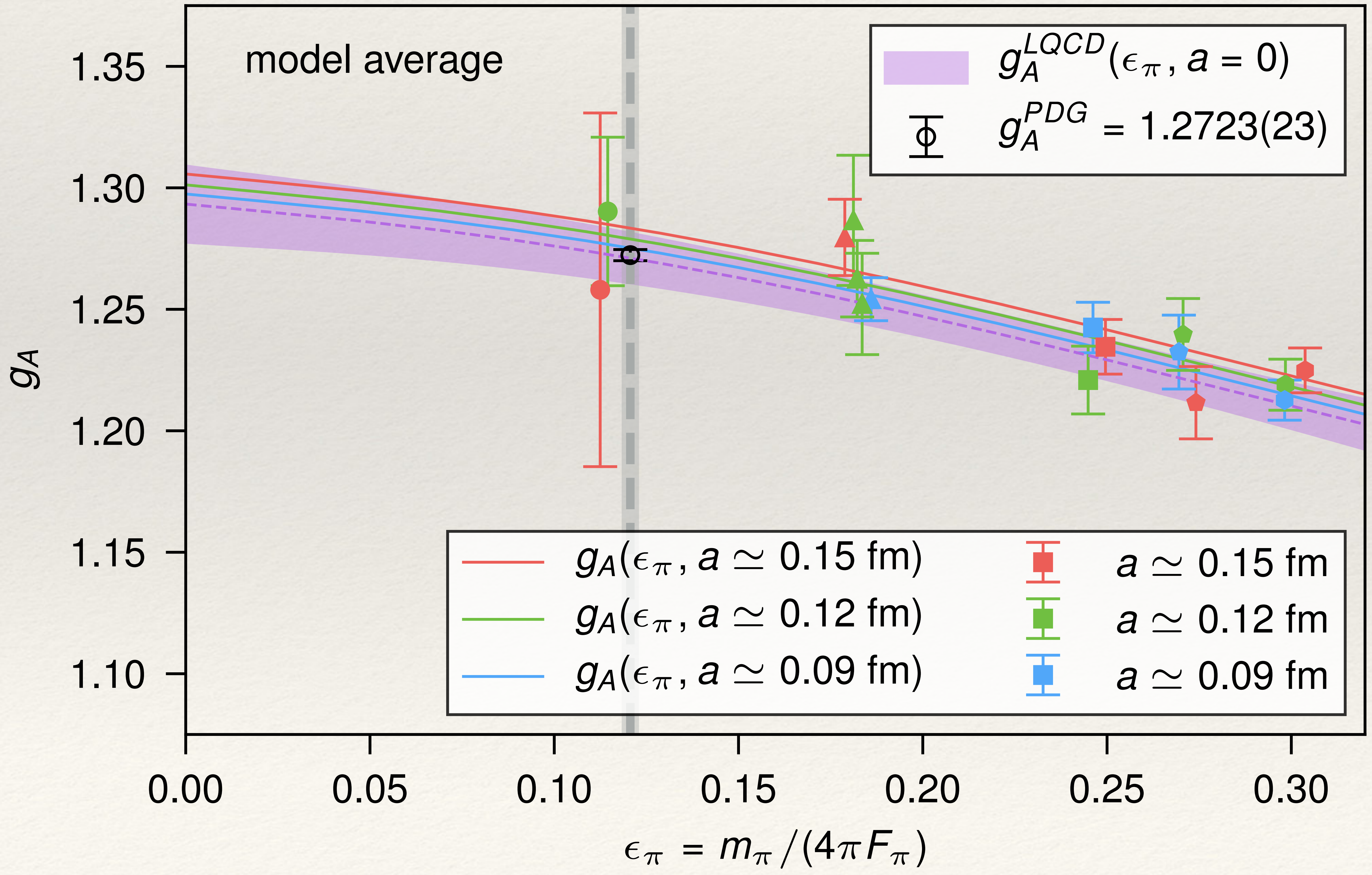
- I am not motivated to “bash” XPT

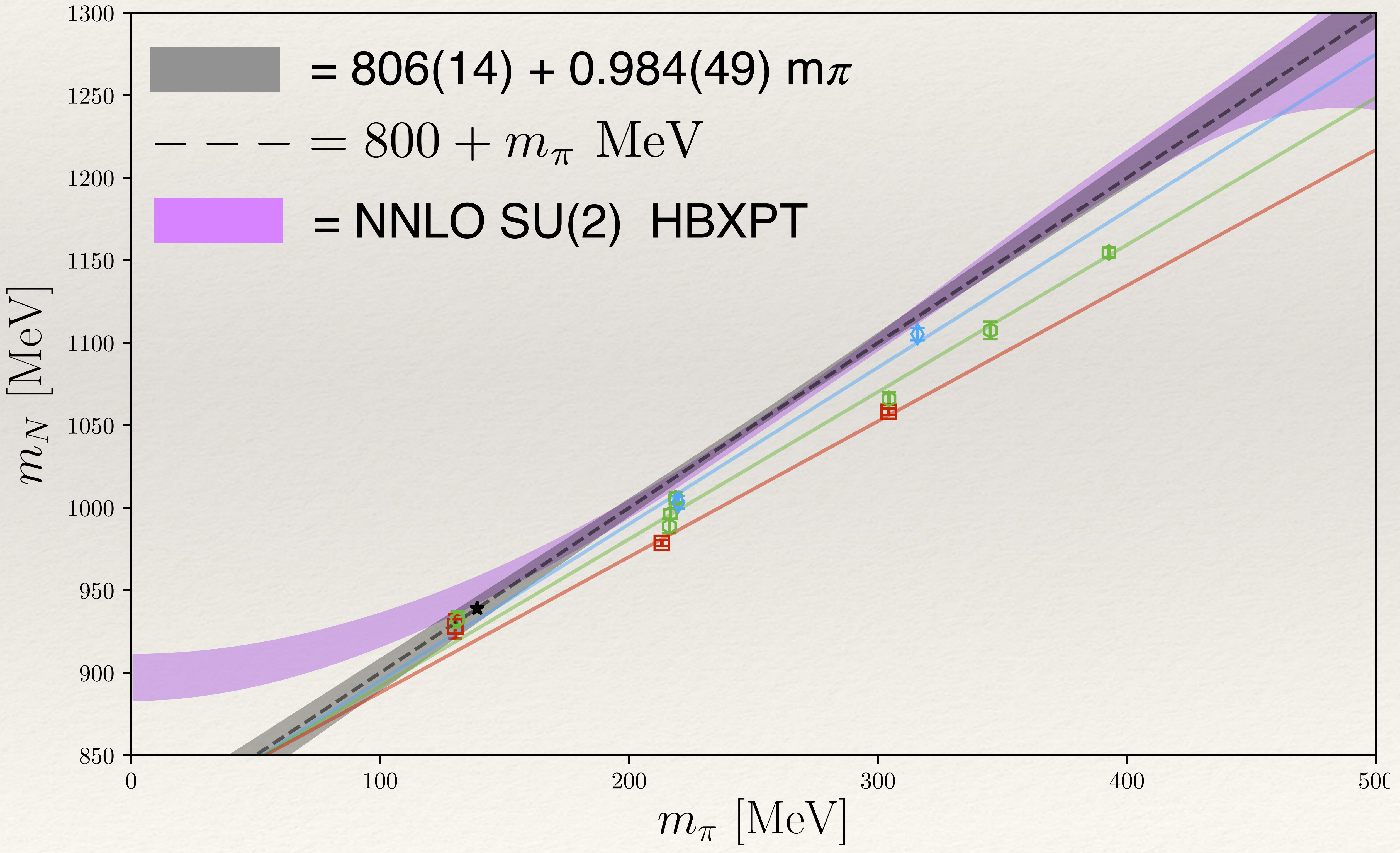
- I am motivated to carefully scrutinize what we learn from LQCD about XPT and carefully check that XPT is healthy
  - it is our foundation for understanding low-energy nuclear physics

- I’ve come to view XPT is a great qualitative guide, but not a reliable tool for precision & accuracy — at least not without input from LQCD

Background on my perspective: LQCD + EFT

- I've come to view XPT is a great qualitative guide, but not a reliable tool for precision & accuracy — at least not without input from LQCD
- 2 examples where strong cancellation between different orders are required to explain results





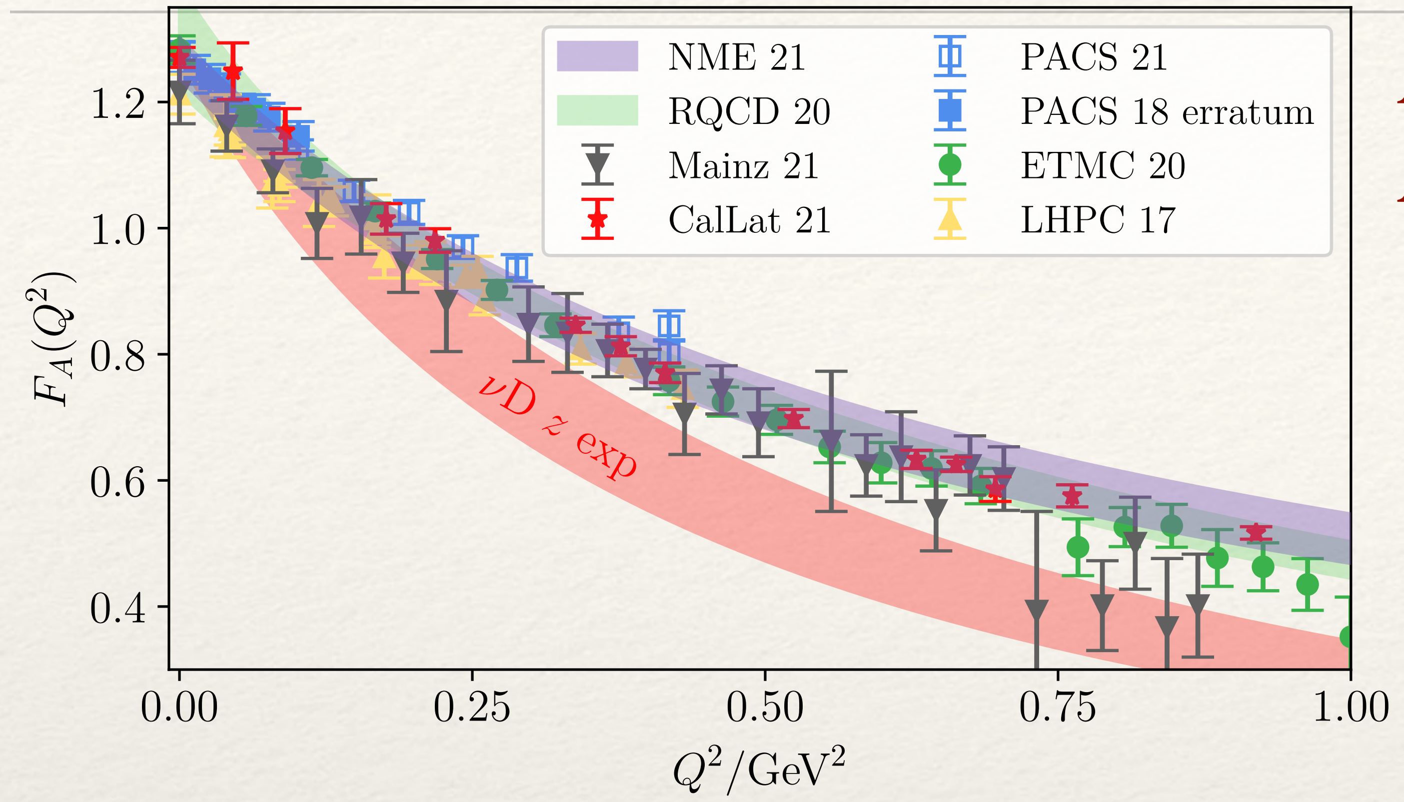
Background on my perspective: LQCD must be justified

- $\rightarrow$ exascale era, LQCD calculations are becoming quite expensive
  - human resources (many people)
  - computing resources (expensive computers, Frontier @ ORNL ~ $600M US)
  - Critical to maximize science output per flop
  - Critical to obtain robust results

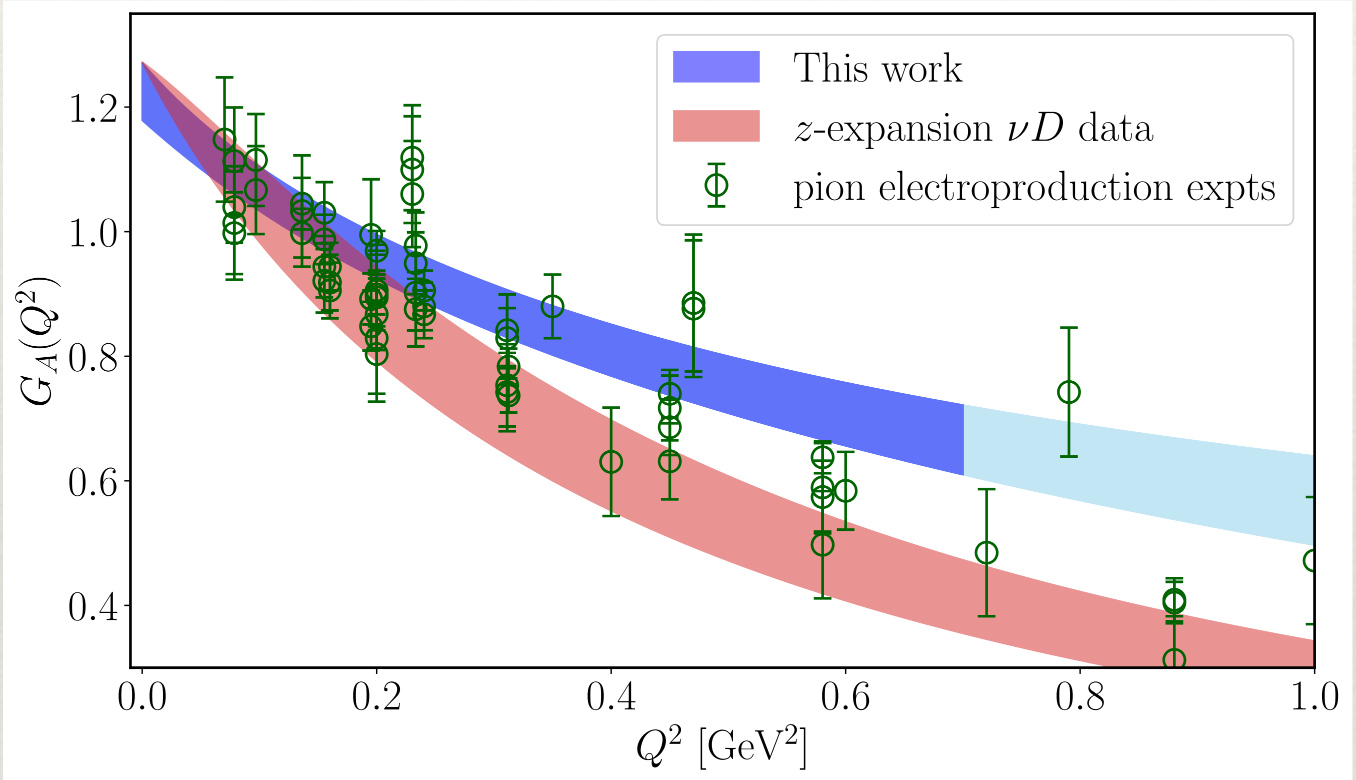
- We should be critical of our own work (more than anyone else) as well as others work
  - "Hold our feet to the fire"
  - Make sure we do not over-promise and under-deliver

- Given the significant cost of LQCD calculations - we need to balance two goals
  - Be aggressive and obtain the smallest uncertainty possible from a given data set
  - Obtain robust results that are stable under all possible variations of analysis choices

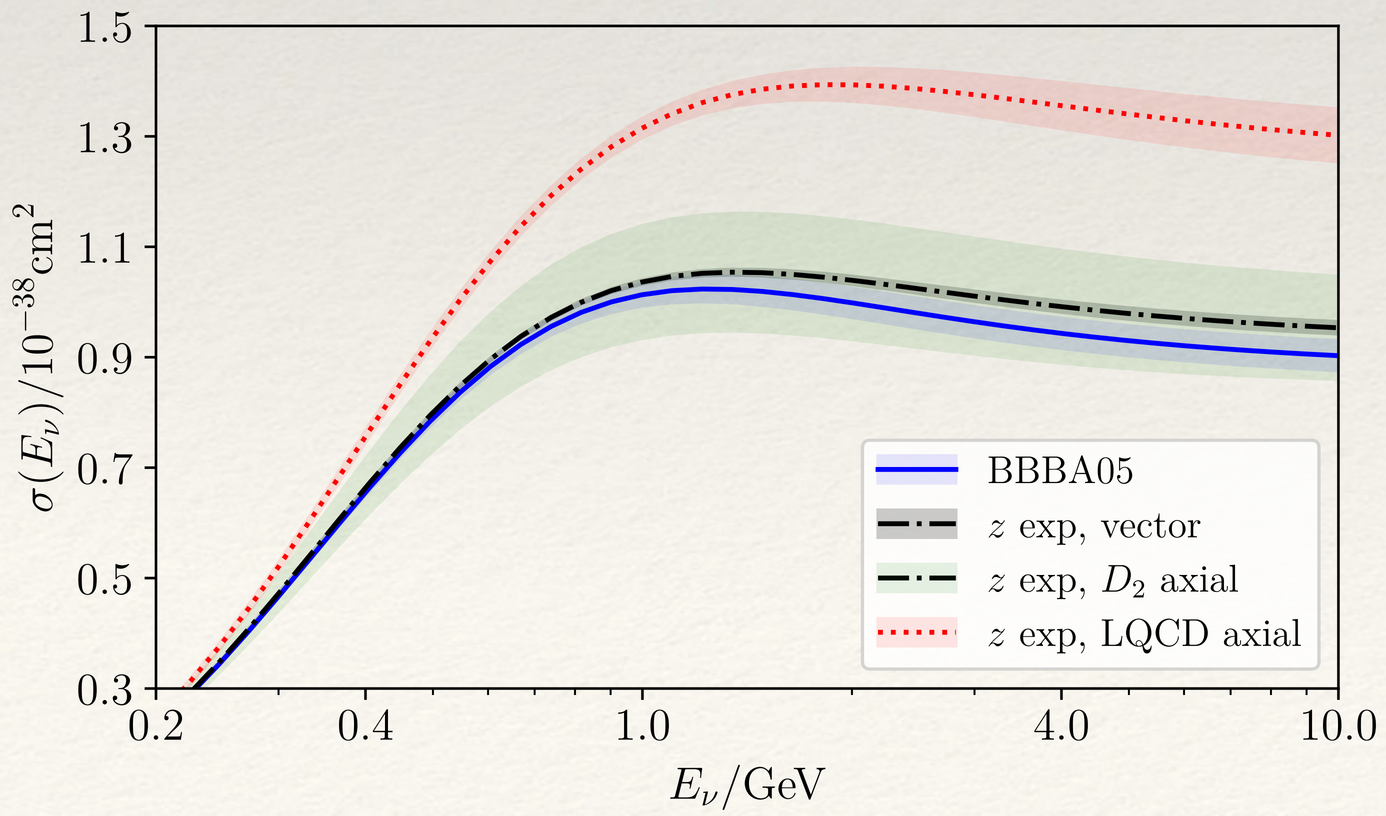
Background on my perspective: example of importance



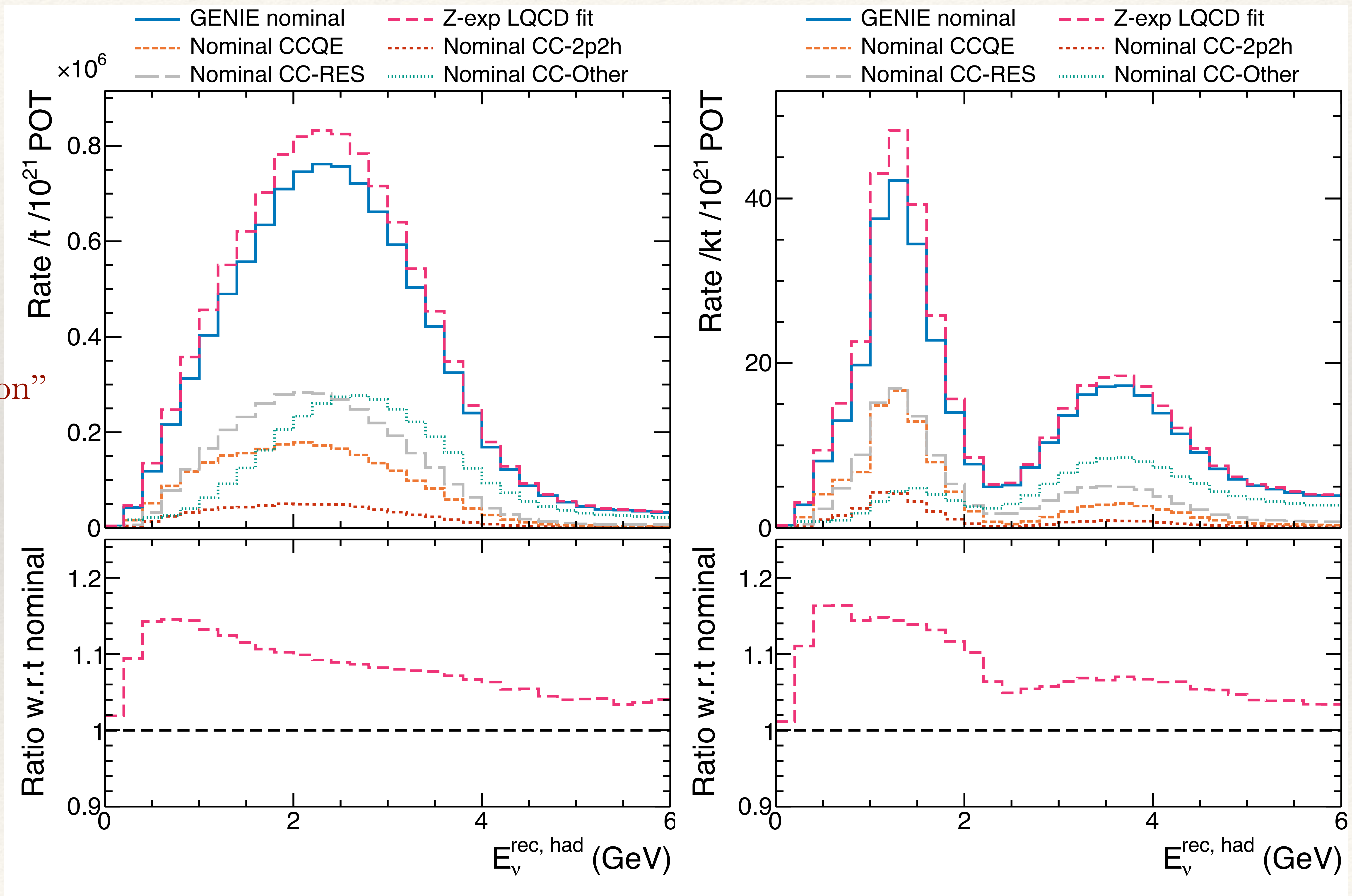
A.S. Meyer, A. Walker-Loud, C. Wilkinson
Ann.Rev.Nucl.Part.Sci. 72 (2022)



D. Djukanovic et al.
"Mainz Collaboration"
PRD 106 (2022)



Near Detector DUNE Far Detector



Background on my perspective: Open Science

- To support these goals - we make publicly available
  - our production LQCD code
  - our LQCD results (raw correlators)
  - our correlator analysis code
  - our extrapolation analysis code

- To the best of my knowledge, we are the only group to make all four of these publicly open

- This fully open-science model enables any interested party or person to scrutinize our results

Why are the LQCD calculations so difficult?

(and why might there be discrepancies?)

Systematic Uncertainties

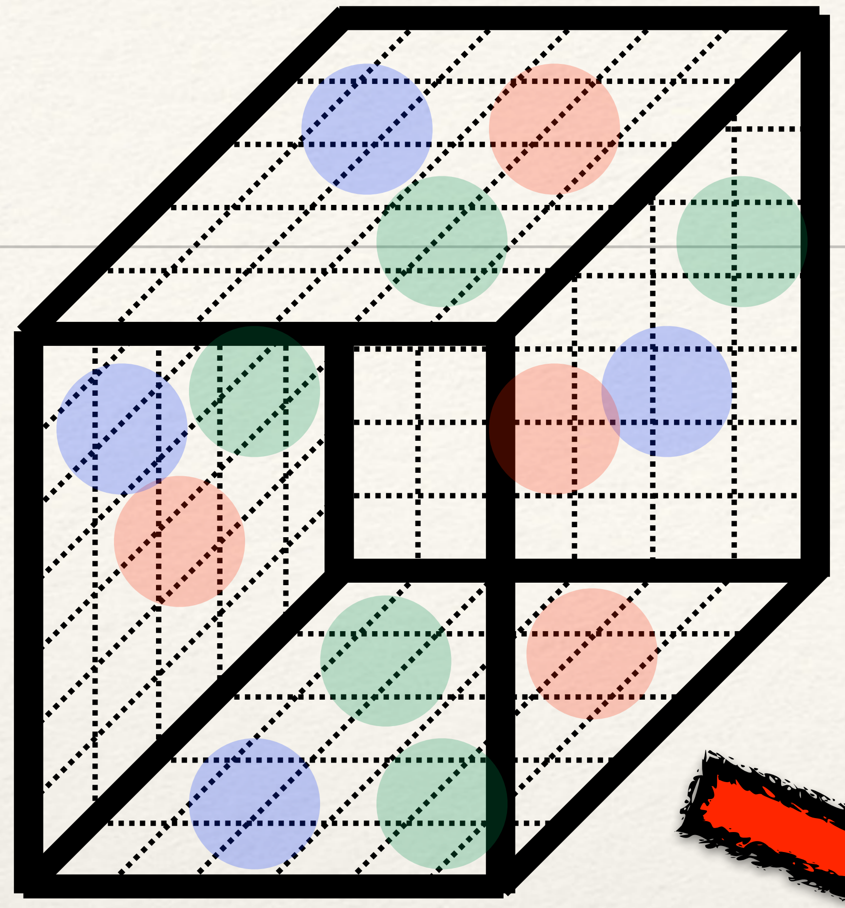
Standard LQCD systematics

- ❑ Physical pion mass
- ❑ continuum limit
- ❑ infinite volume

extra systematics with nucleons

- ❑ Signal-to-Noise (S/N)
- ❑ Excited state contamination

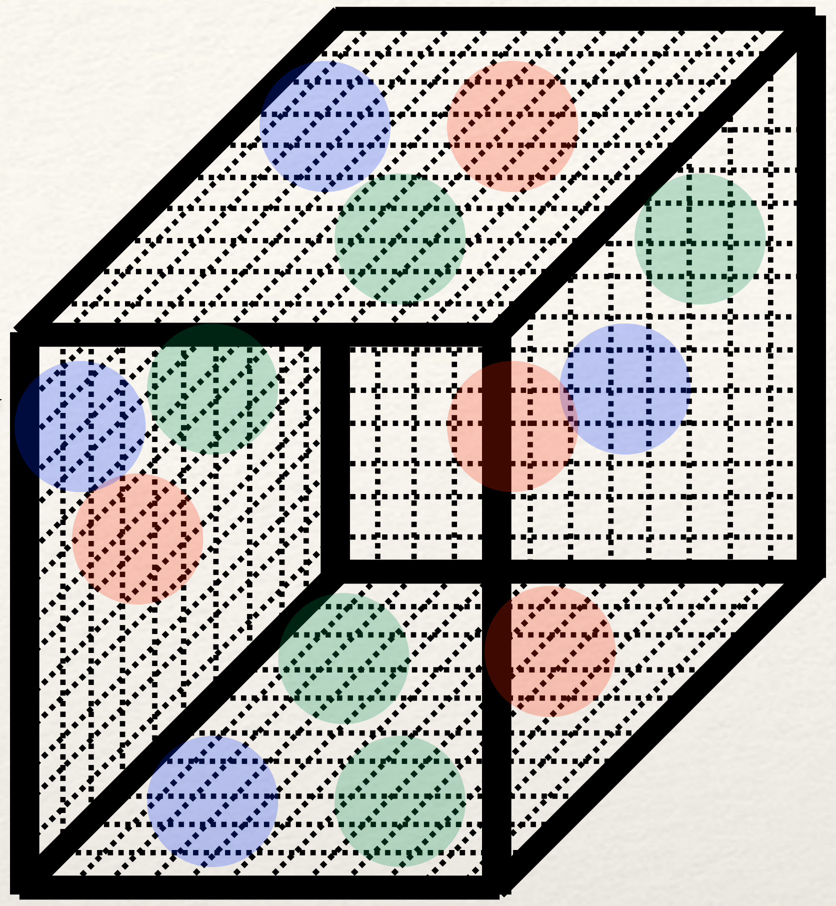
What does it mean to have a LQCD result?



continuum limit

need 3 or more lattice spacings

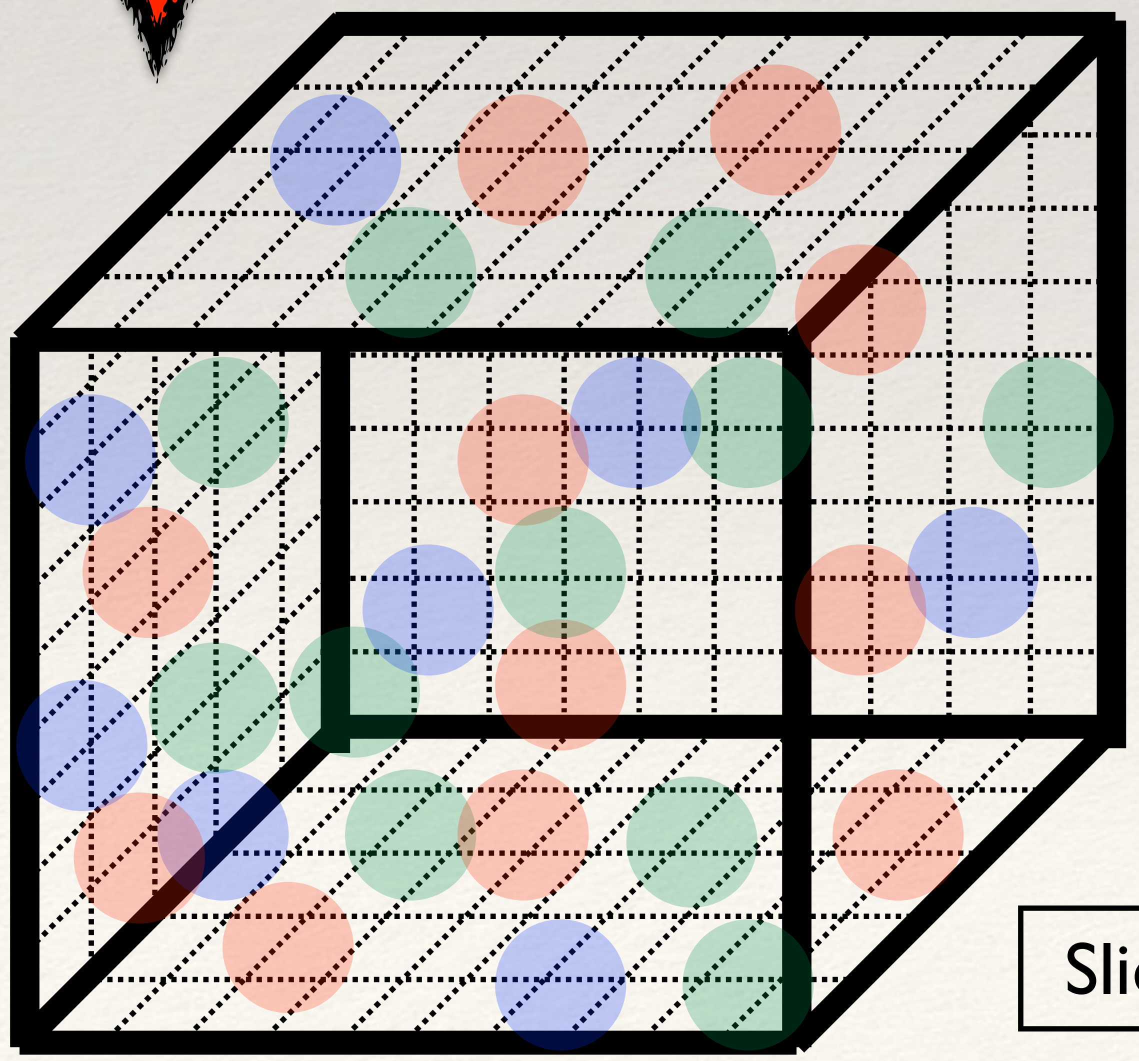
$$t_{comp} \propto \frac{1}{a^6}$$



infinite volume limit

$$t_{comp} \propto V^{5/4}$$

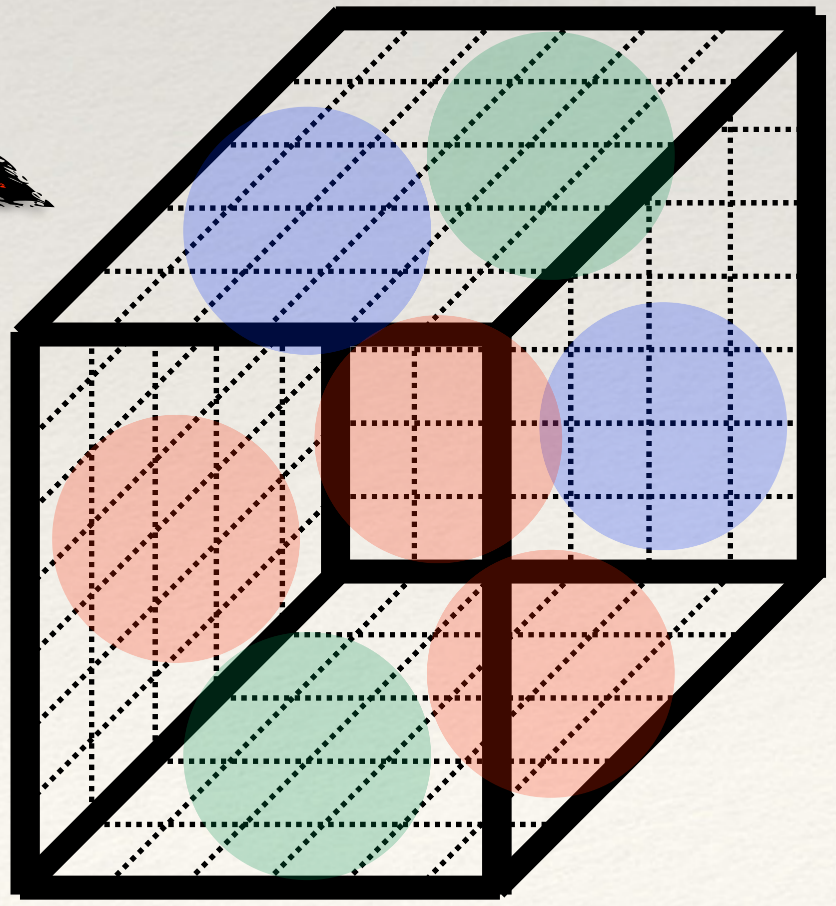
$$V = N_L^3 \times N_T$$



Slide adapted from E. Berkowitz

physical pion masses

exponentially bad signal-to-noise problem



Introduction to LQCD

$$C(t) = \langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle = \frac{1}{\mathcal{Z}}\int D\psi D\bar{\psi} DU \mathcal{O}(t)\mathcal{O}(0) e^{iS_M[\bar{\psi},\psi,U]}$$

$$= \frac{1}{\mathcal{Z}}\int DU \det\left(\mathrm{i}\not{D} - \mathrm{M}\right) \mathcal{O}(\mathrm{t})\mathcal{O}(0) \mathrm{e}^{\mathrm{i}\mathrm{S}_\mathrm{M}[\bar{\psi},\psi,\mathrm{U}]}$$

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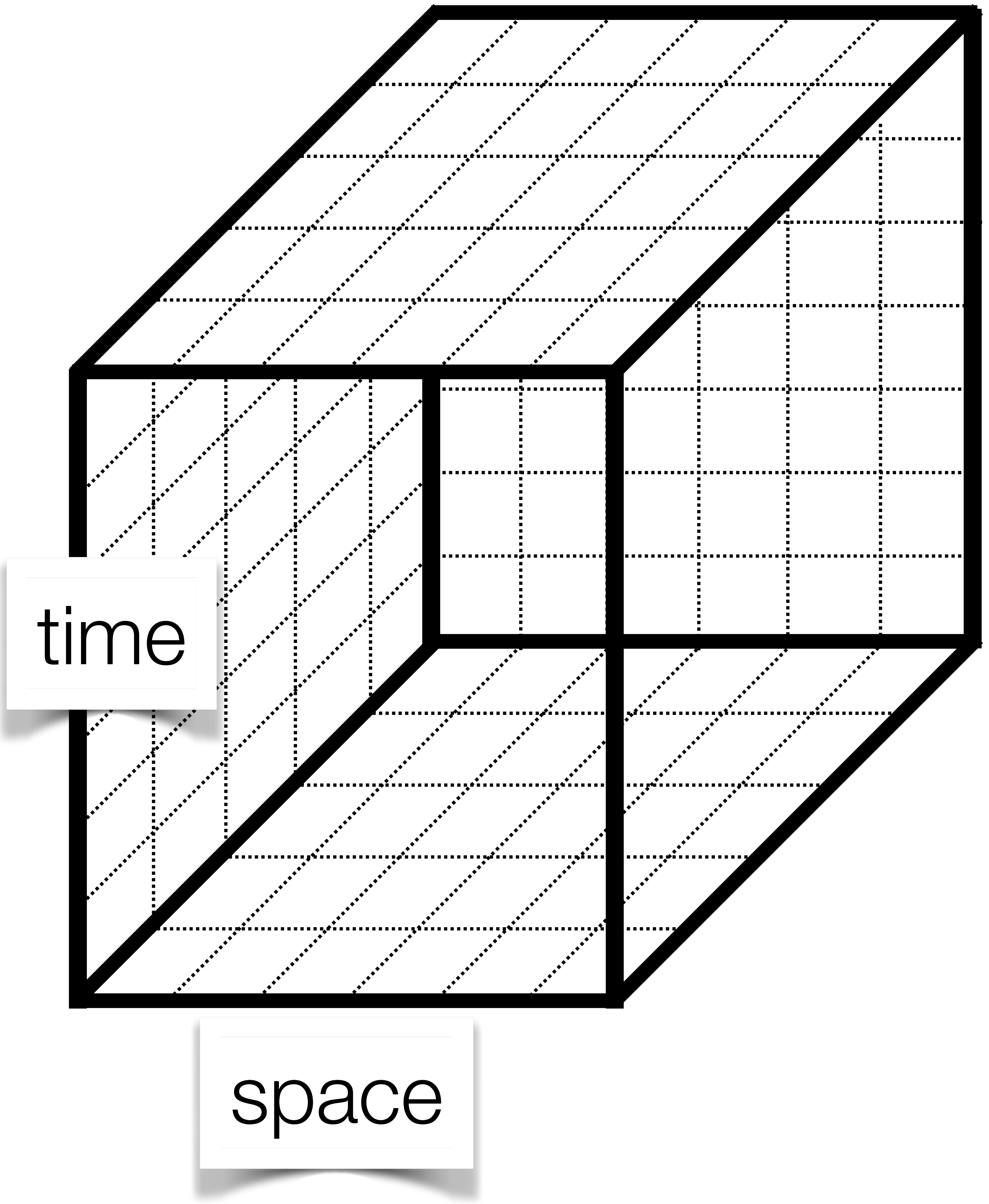

lattice
finite volume

Introduction to LQCD

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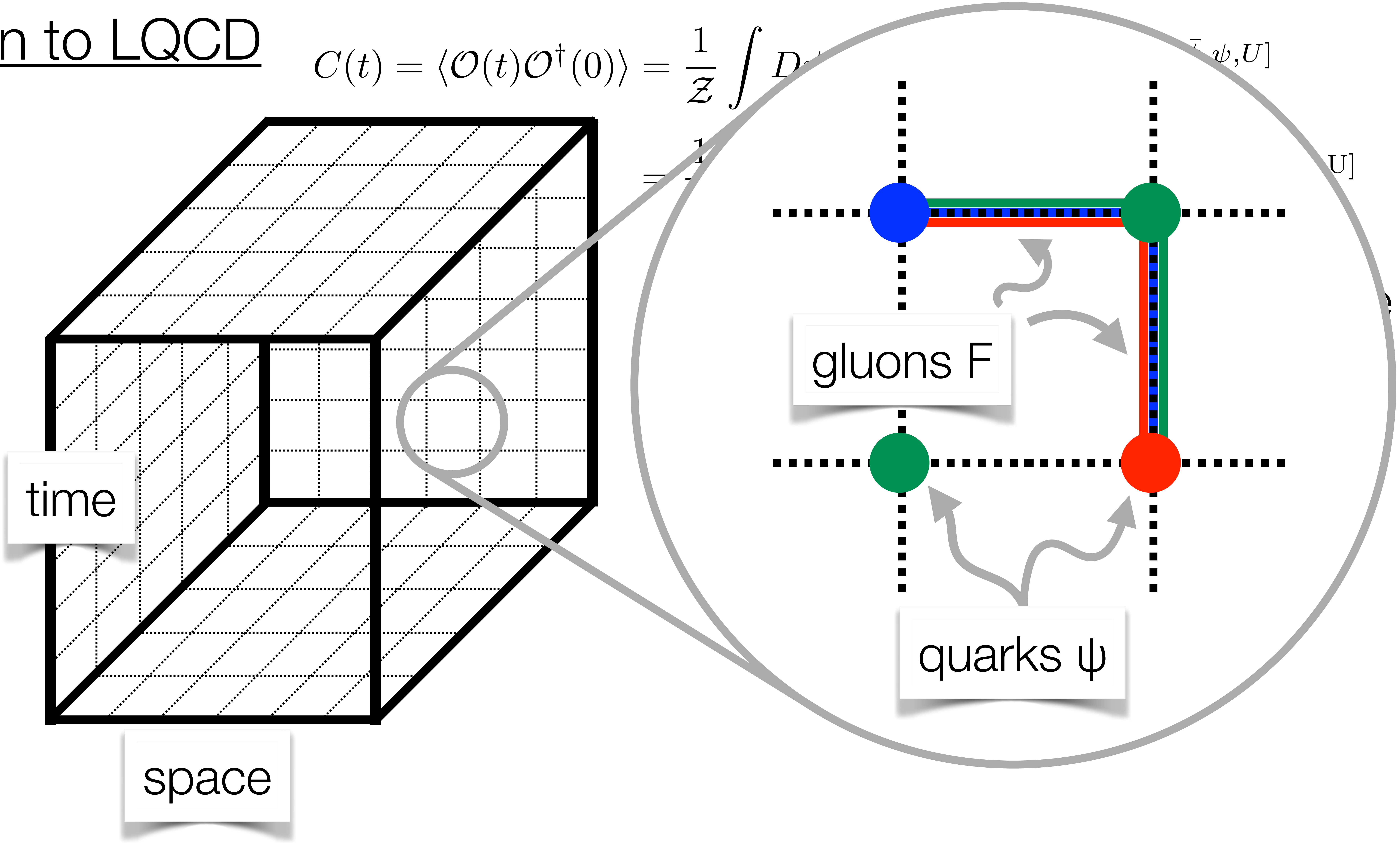

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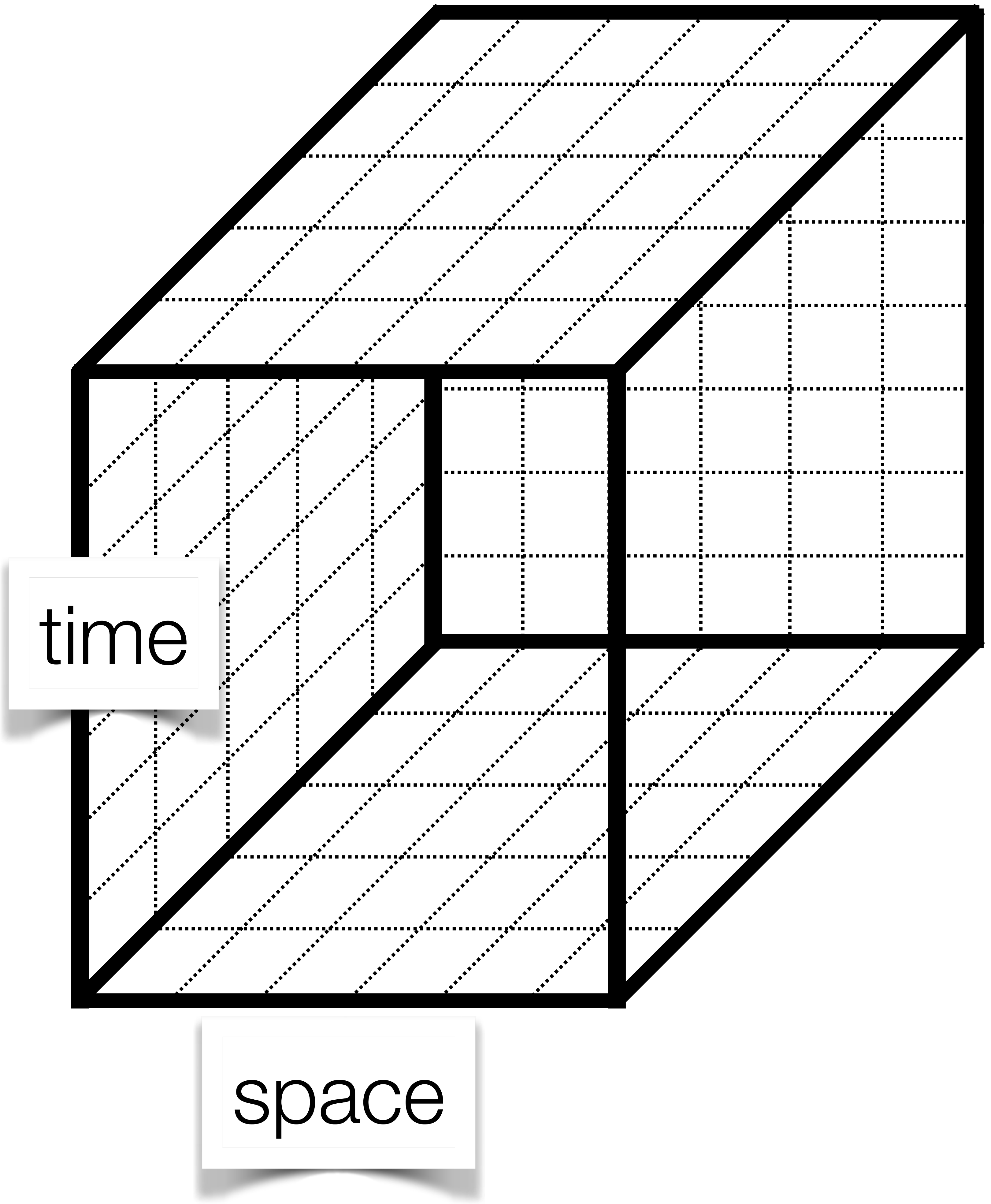

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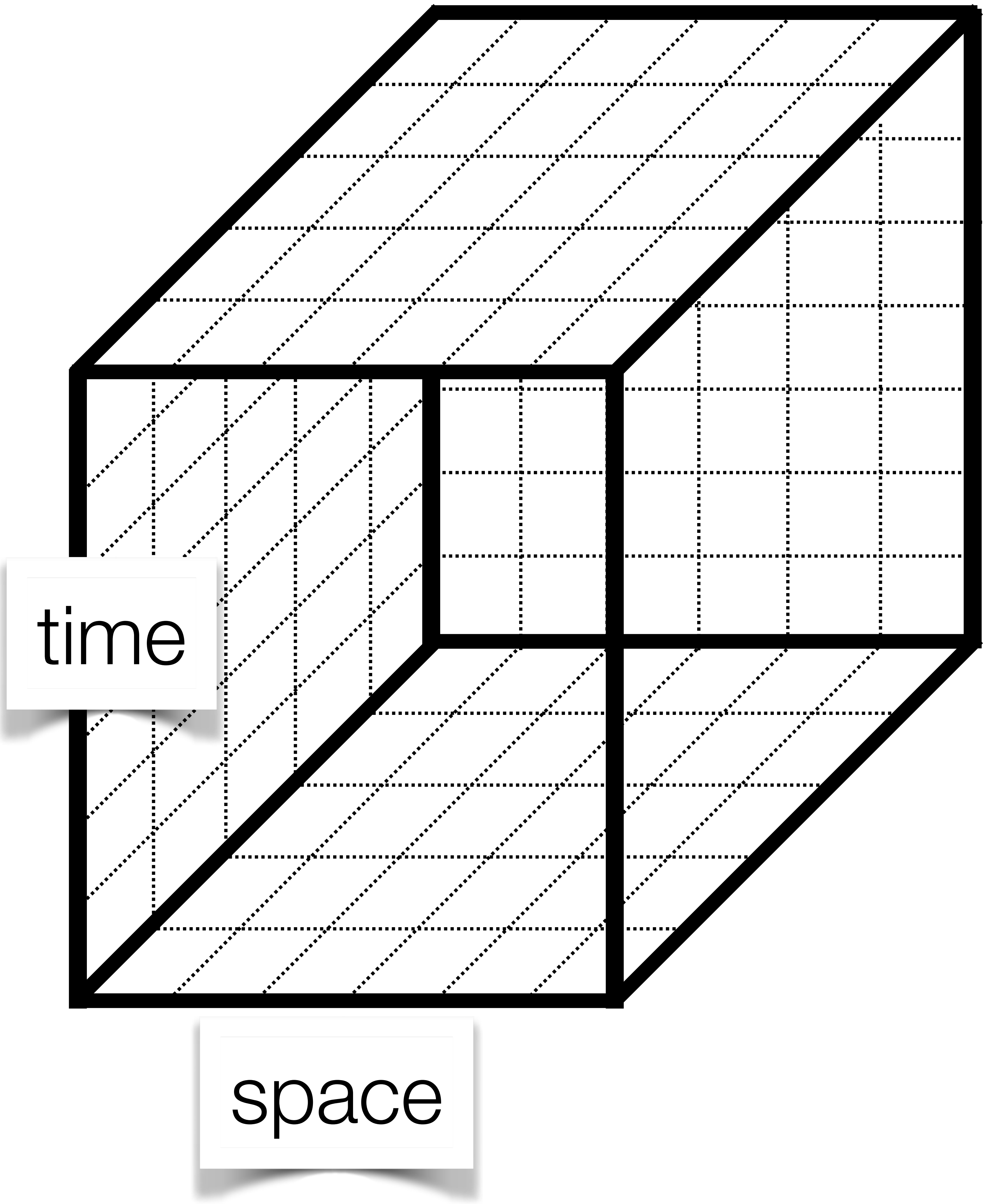
$$= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \det\left(\not{D} + M\right) e^{-S[U]}\ \mathcal{O}(t)\mathcal{O}^\dagger(0)$$



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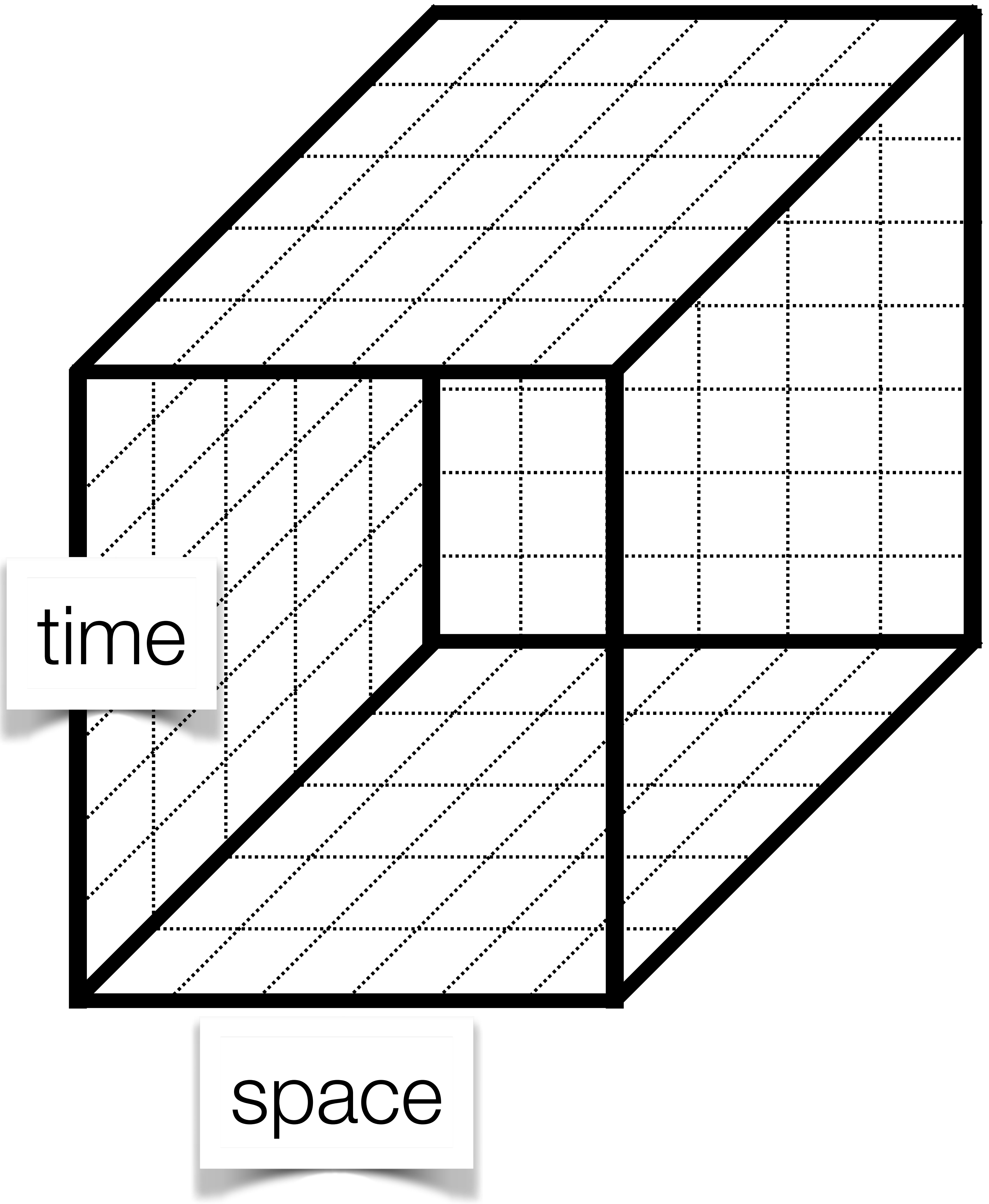

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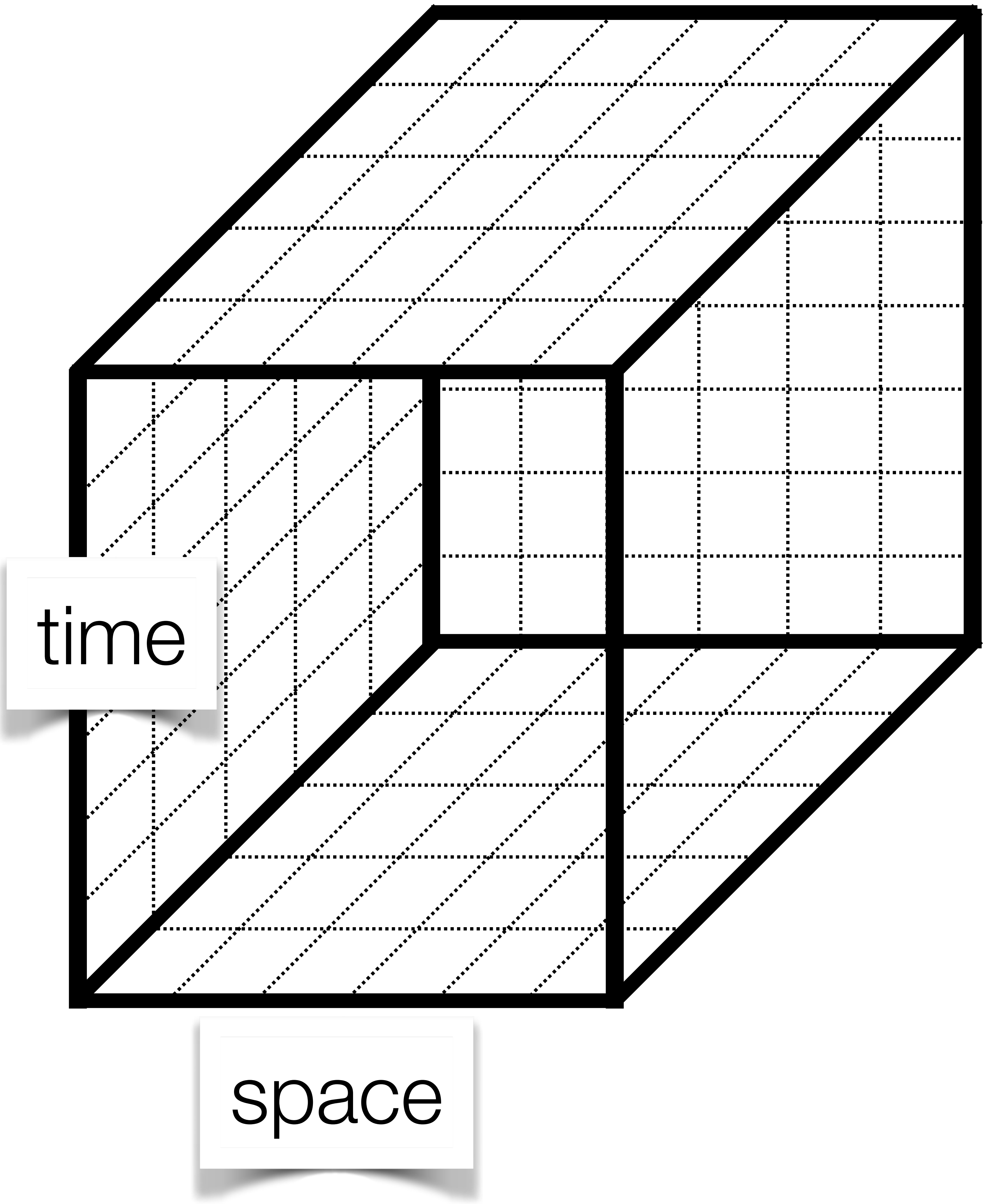

$$\{U_1, U_2, U_3, \ldots, U_N\}$$

Markov Chain Monte Carlo

Slide adapted from E. Berkowitz

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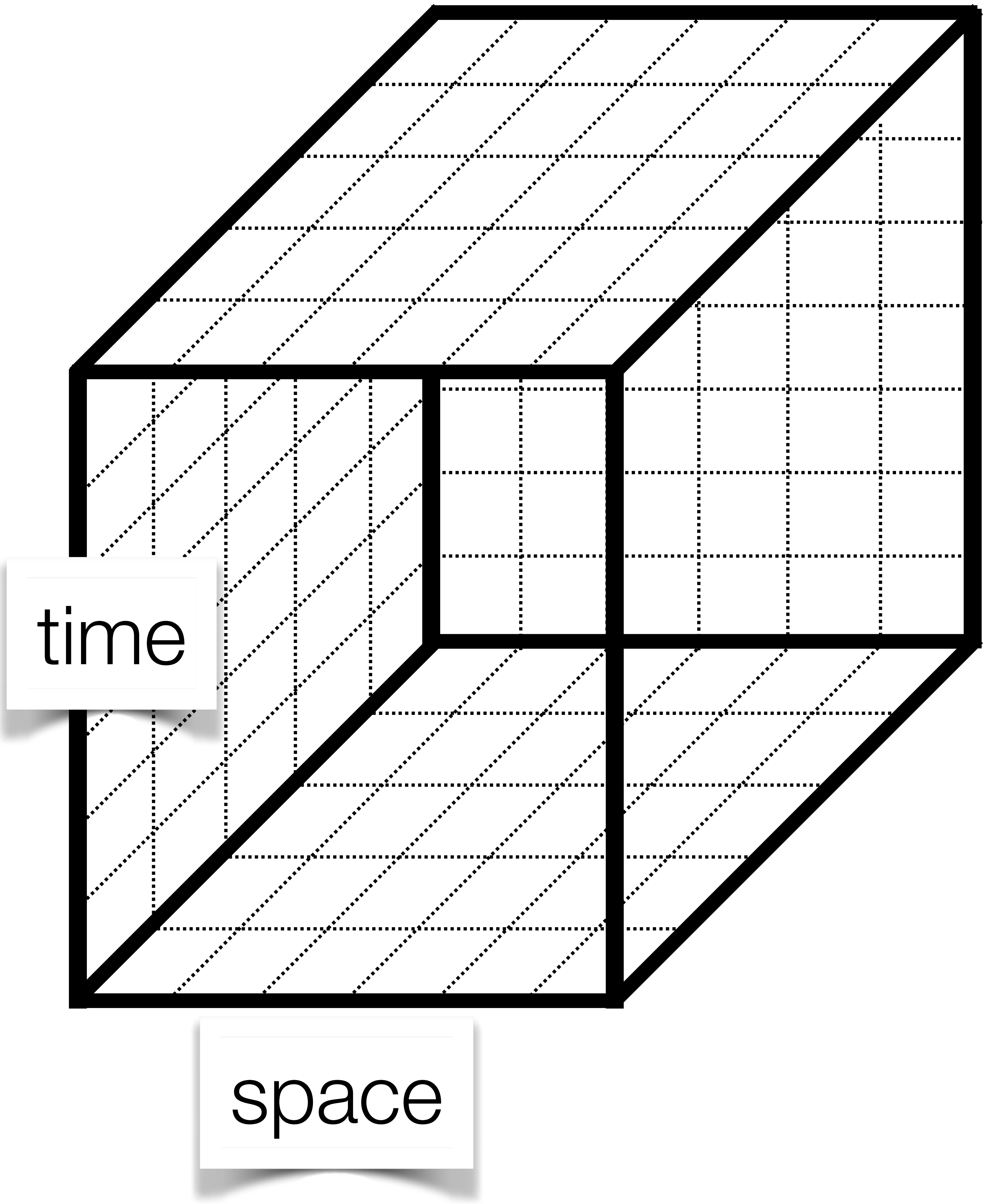

$$\{U_1, U_2, U_3, \ldots, U_N\}$$

Markov Chain Monte Carlo

$$\approx \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}(t)\mathcal{O}^\dagger(0)[U_i]$$

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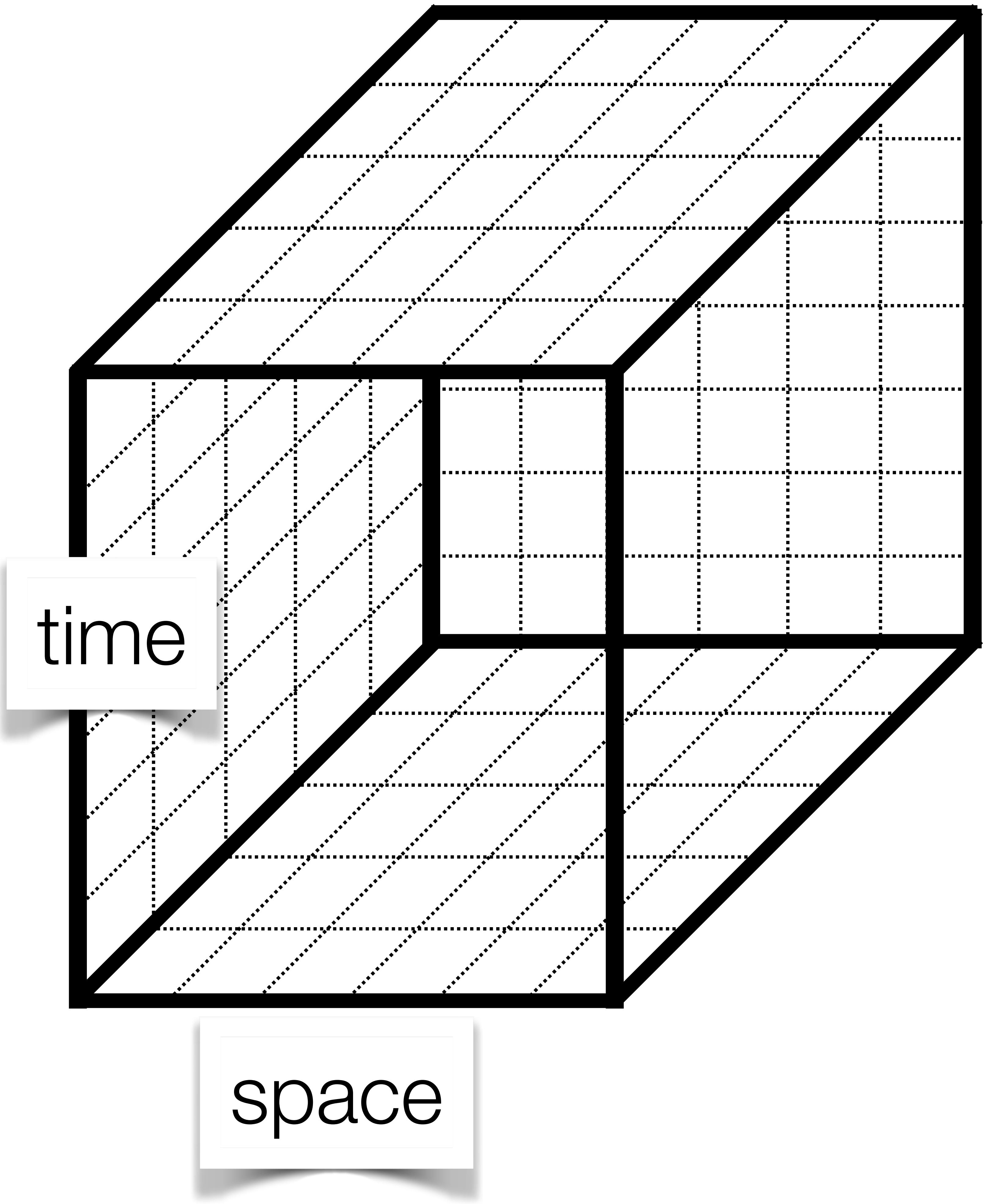

$$\{U_1, U_2, U_3, \ldots, U_N\}$$

Markov Chain Monte Carlo

$$\approx \frac{1}{N}\sum_{i=1}^{N}\mathcal{O}(t)\mathcal{O}^\dagger(0)[U_i]\;+O\left(\frac{1}{\sqrt{N}}\right)$$

Slide adapted from E. Berkowitz

Introduction to LQCD

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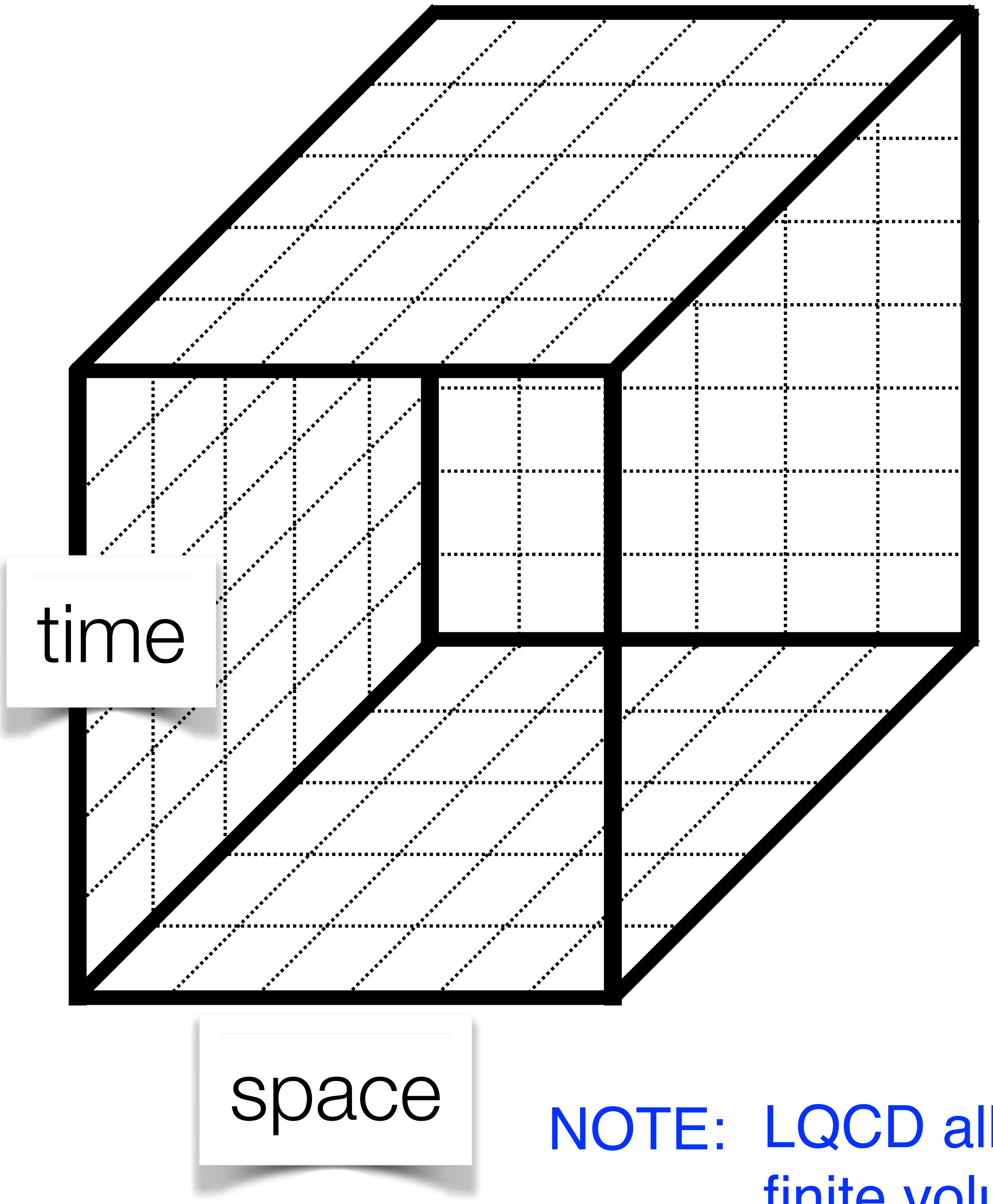

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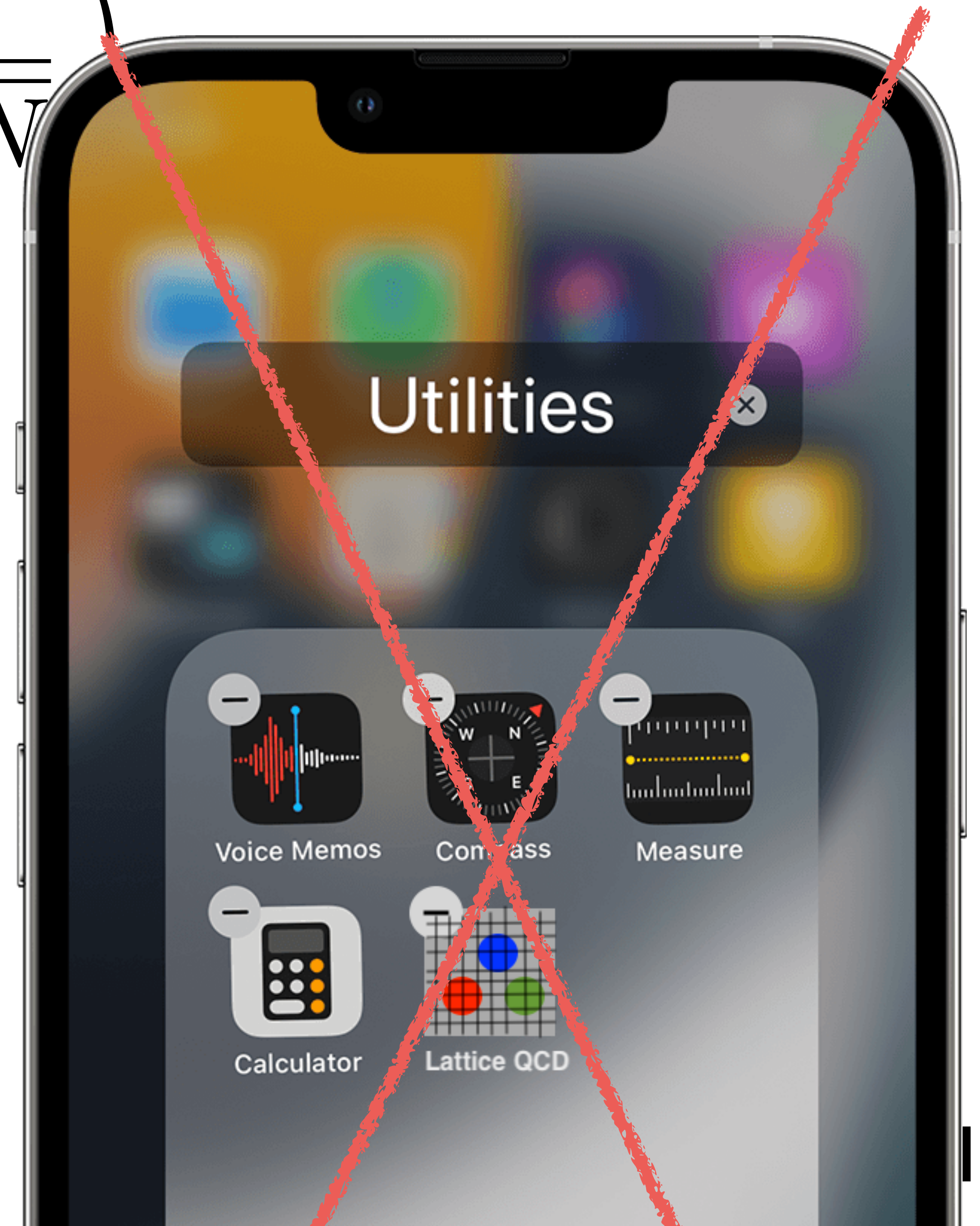


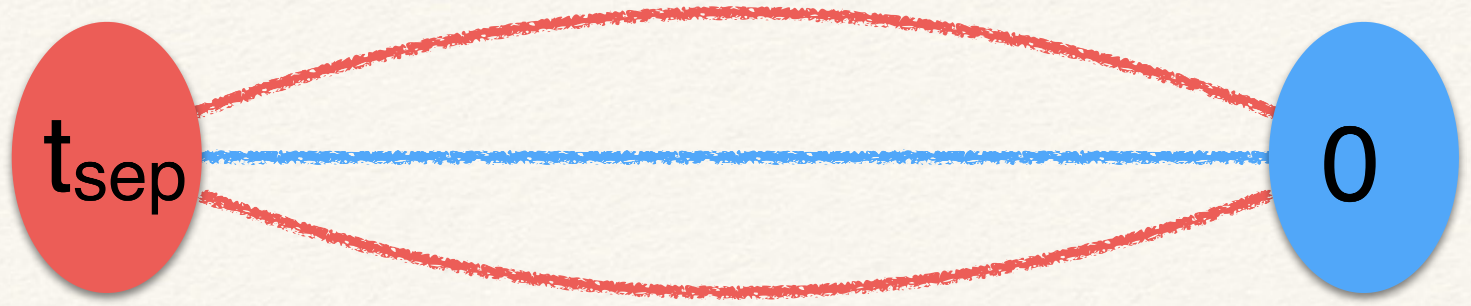
NOTE: LQCD allows us to compute Euclidean space, finite volume, correlation functions

Non-trivial numerical analysis (and sometimes formalism) to extract spectrum, matrix elements, form factors, …

Slide adapted from E. Berkowitz

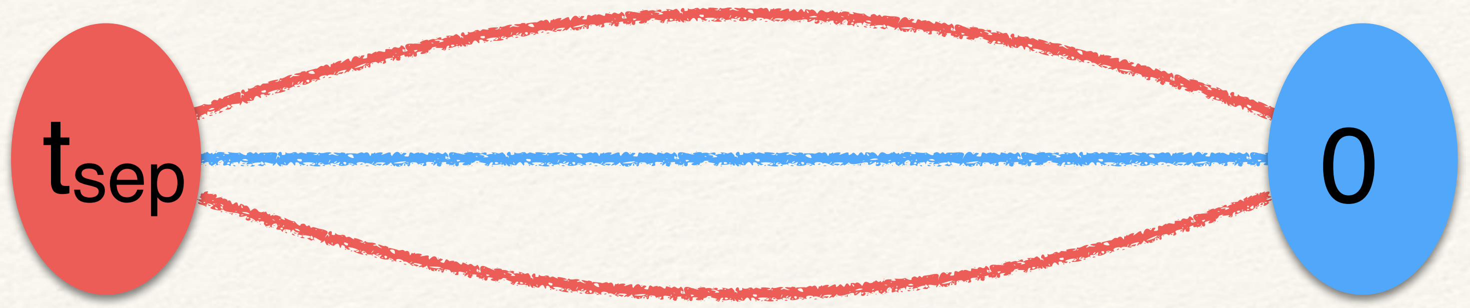
LQCD: 2 point functions

$$C(t,\mathbf{p}) = \sum_{\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{x}}\langle\Omega|O(t,\mathbf{x})O^\dagger(0,\mathbf{0})|\Omega\rangle$$



$$\begin{aligned}
C(t) &= \sum_{\mathbf{x}}\langle\Omega|O(t,\mathbf{x})O^\dagger(0,\mathbf{0})|\Omega\rangle && \text{focus on 0-momentum}\\
&= \sum_{\mathbf{x}}\langle\Omega|e^{\hat{H}t}O(0,\mathbf{x})e^{-\hat{H}t}O^\dagger(0,\mathbf{0})|\Omega\rangle && \text{time-evolve operator}\\
&= \sum_{n}\sum_{\mathbf{x}}\langle\Omega|e^{\hat{H}t}O(0,\mathbf{x})e^{-\hat{H}t}|n\rangle\langle n|O^\dagger(0,\mathbf{0})|\Omega\rangle && \text{multiply by 1,} \quad 1 = \sum_n |n\rangle\langle n|\\
&= \sum_{n} e^{-E_n t}\sum_{\mathbf{x}}\langle\Omega|O(0,\mathbf{x})|n\rangle\langle n|O^\dagger(0,\mathbf{0})|\Omega\rangle && \text{define vacuum to have 0-energy}\\
&= \sum_{n} e^{-E_n(\mathbf{p}=0)t}\langle\Omega|O(0)|n,\mathbf{p}=0\rangle\langle n,\mathbf{p}=0|O^\dagger(0)|\Omega\rangle\\
&= \sum_{n} e^{-E_n t}z_n z_n^\dagger && \text{sum of exponentials}
\end{aligned}$$

LQCD: 2 point functions



$$C(t, \mathbf{p}=0) = \sum_{\mathbf{x}} \langle \Omega | N(t, \mathbf{x}) N^\dagger(0, \mathbf{0}) | \Omega \rangle$$

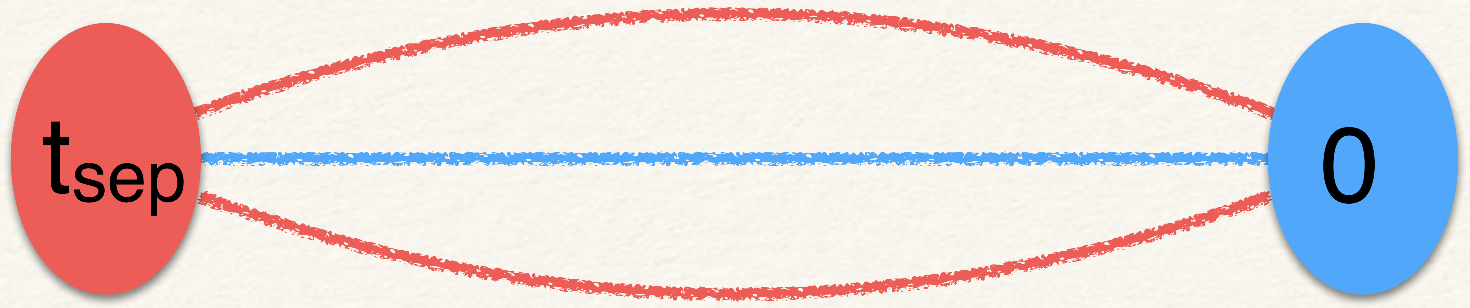
$$= A_0 e^{-E_0 t} \left(1 + \sum_{n>0} r_n e^{-\Delta_{n0} t} \right)$$

$$\Delta_{n0} = E_n - E_0$$

$$m_{\text{eff}}(t) = \ln \left(\frac{C(t)}{C(t+1)} \right)$$

$$= E_0 + \ln \left(\frac{1 + \sum_{n>0} r_n e^{-\Delta_{n0} t}}{1 + \sum_{n>0} r_n e^{-\Delta_{n0}(t+1)}} \right)$$

LQCD: 2 point functions



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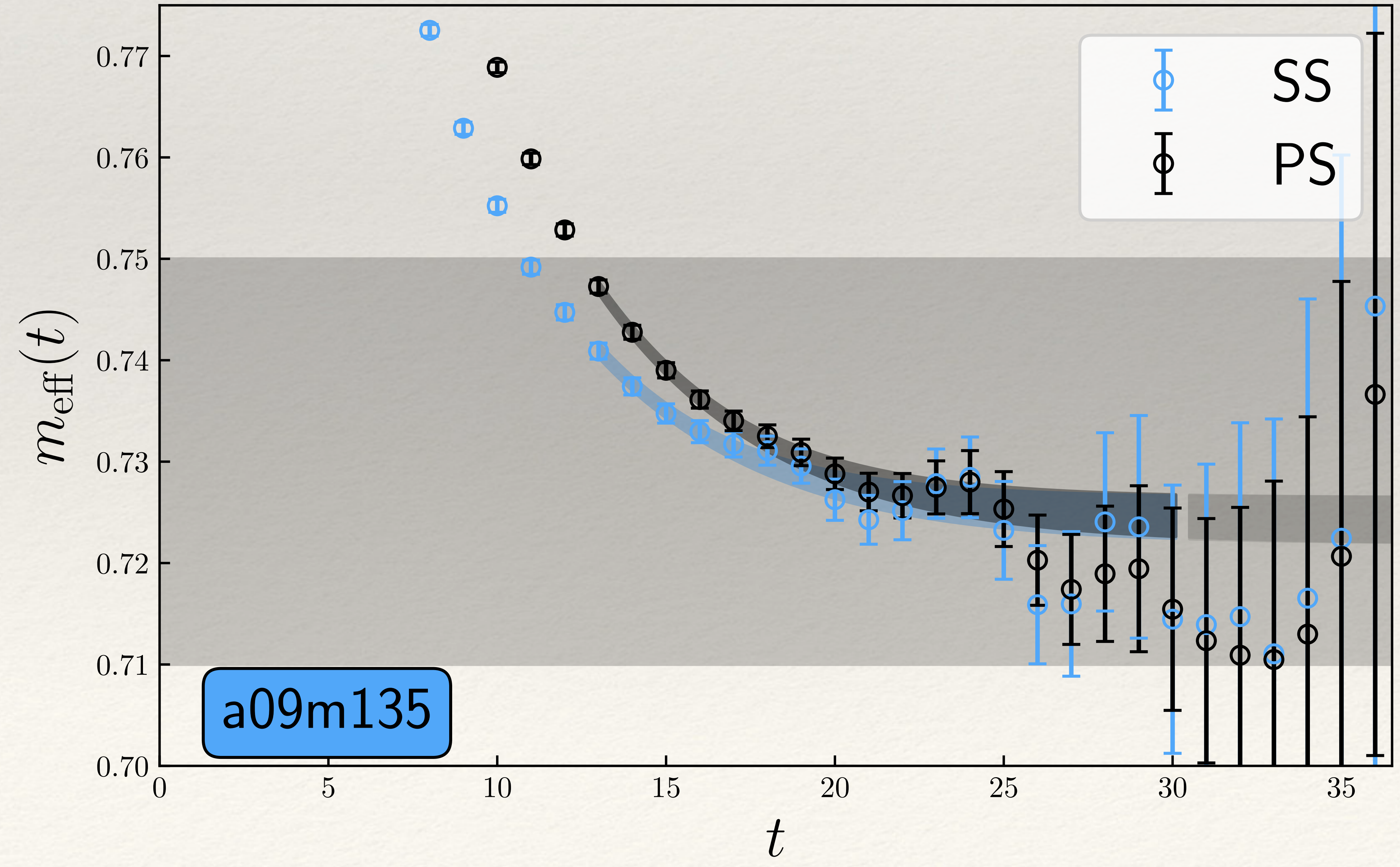
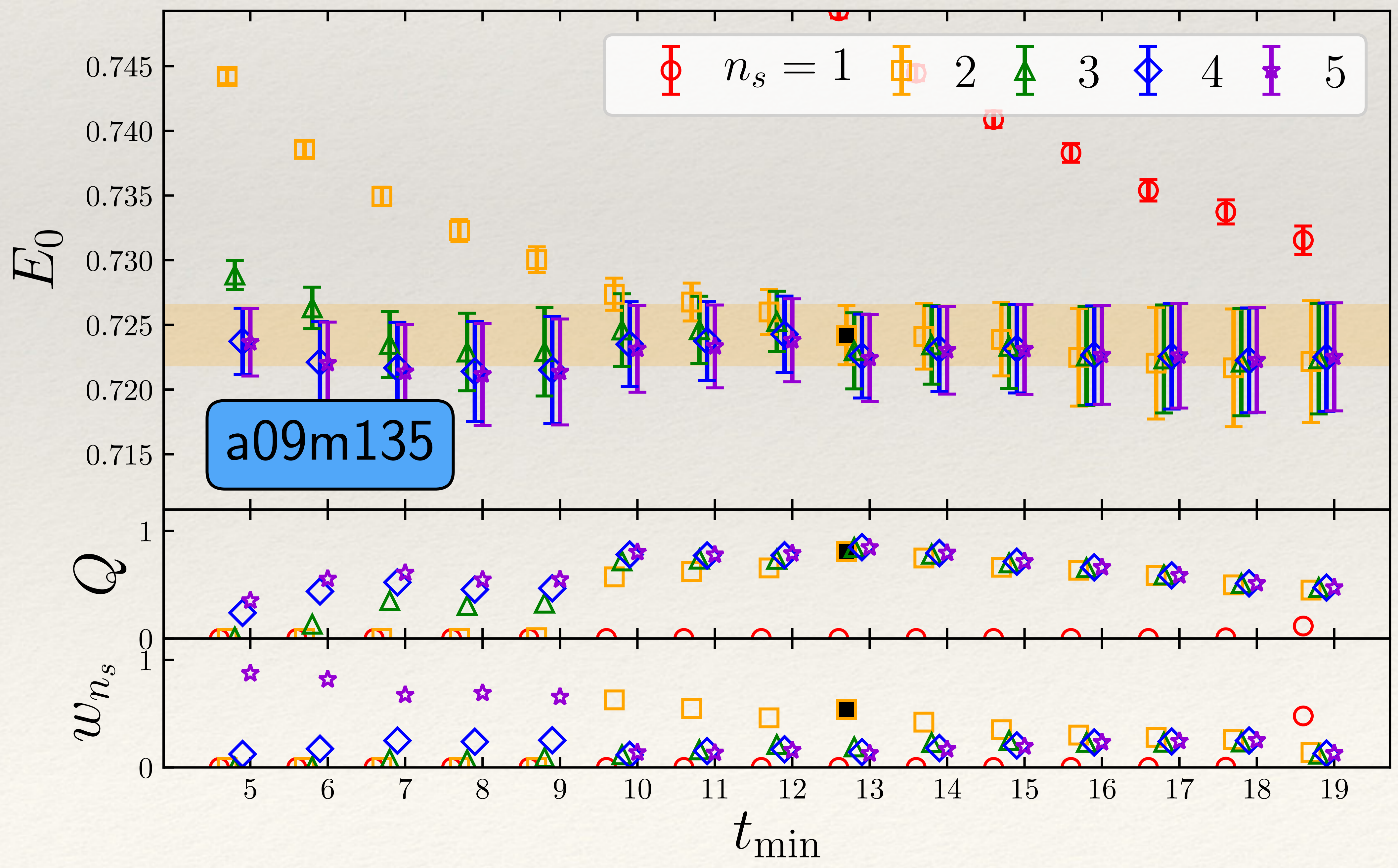
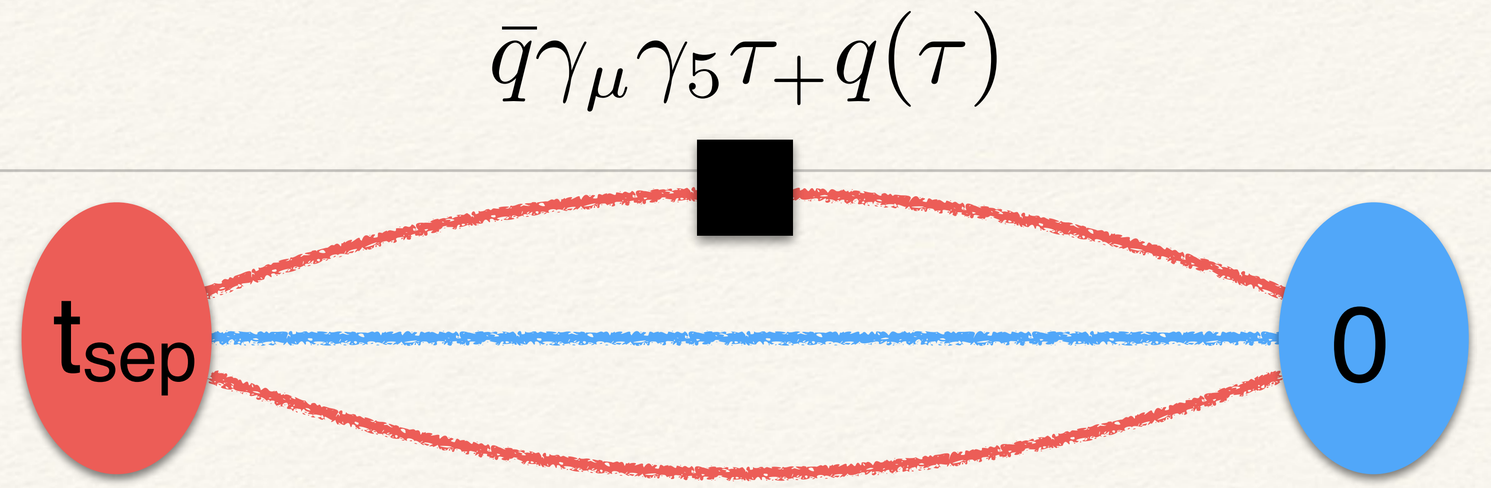
but… signal-to-noise - can not simply “wait till long time” to get ground state (g.s.)





LQCD: 3 point functions

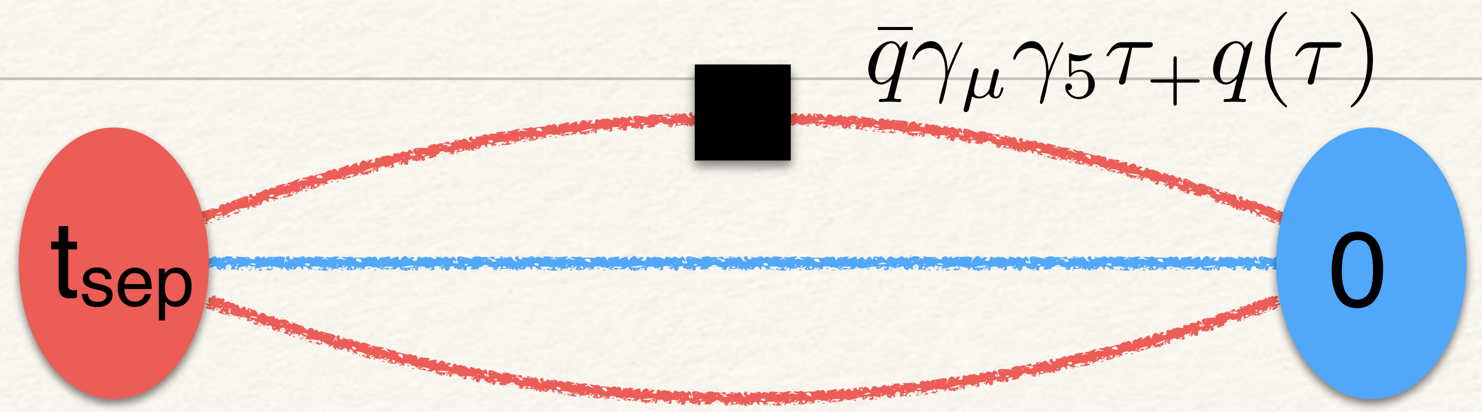


- The most common method (sub-optimal) to compute nucleon matrix elements
  - For a few values of t, compute the 3-point function for all τ

$$C_\Gamma(t,\tau,\mathbf{p},\mathbf{q}) = \sum_{\mathbf{y},\mathbf{x}} e^{-i\mathbf{p}\cdot\mathbf{y}} e^{i\mathbf{q}\cdot\mathbf{x}} \langle\Omega|N(t,\mathbf{y})j_\Gamma(\tau,\mathbf{x})N^\dagger(0,\mathbf{0})|\Omega\rangle$$

  - Each choice of t is a new, expensive computation
  - Ideally, $t \sim 2\ t_{2pt\text{-}gs}$, but, S/N prevents that
  - The g.s. matrix-element/form-factor must be determined through an extrapolation in t and τ after numerical analysis

LQCD: 3 point functions



- Consider zero-momentum (p=0) and zero momentum transfer (q=0)

use "multiply by 1" trick

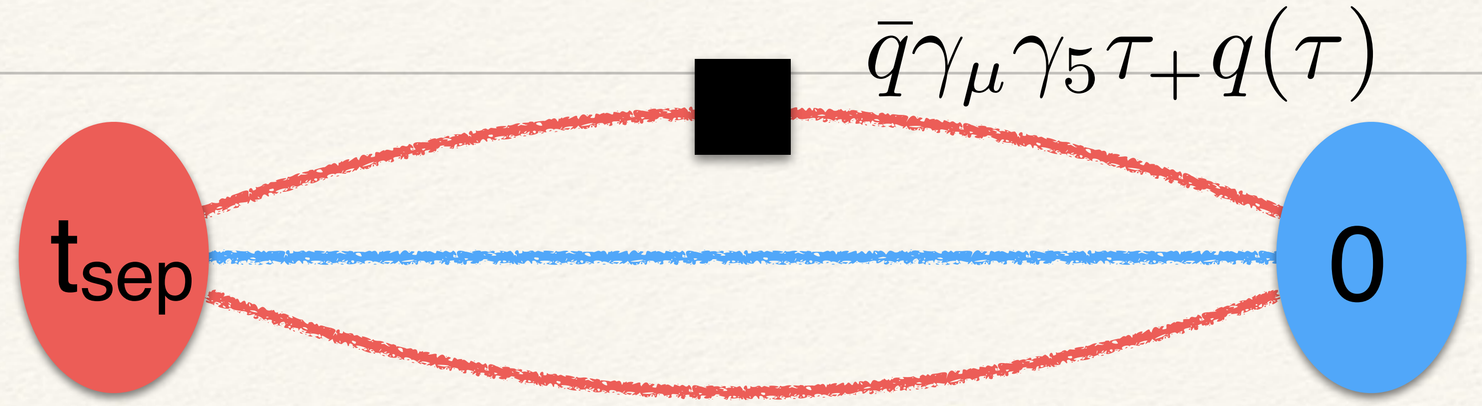
$$1 = \sum_n |n\rangle\langle n|$$

$$C_\Gamma(t,\tau) = \sum_{\mathbf{y},\mathbf{x}} \langle\Omega|N(t,\mathbf{y})j_\Gamma(\tau,\mathbf{x})N^\dagger(0,\mathbf{0})|\Omega\rangle$$

$$= |z_0|^2 g^\Gamma_{00} e^{-E_0 t} + \sum_{n>0} |z_n|^2 g^\Gamma_{nn} e^{-E_n t} + 2\sum_{n<m} z_n z_m^\dagger g^\Gamma_{nm} e^{-(E_n+\frac{\Delta_{mn}}{2}t)} \cosh\left[\Delta_{mn}\left(\tau - \frac{t}{2}\right)\right]$$

$$R_\Gamma(t_{sep},\tau) = \frac{C_\Gamma(t_{sep},\tau)}{C_2(t_{sep})}$$

LQCD: 3 point functions



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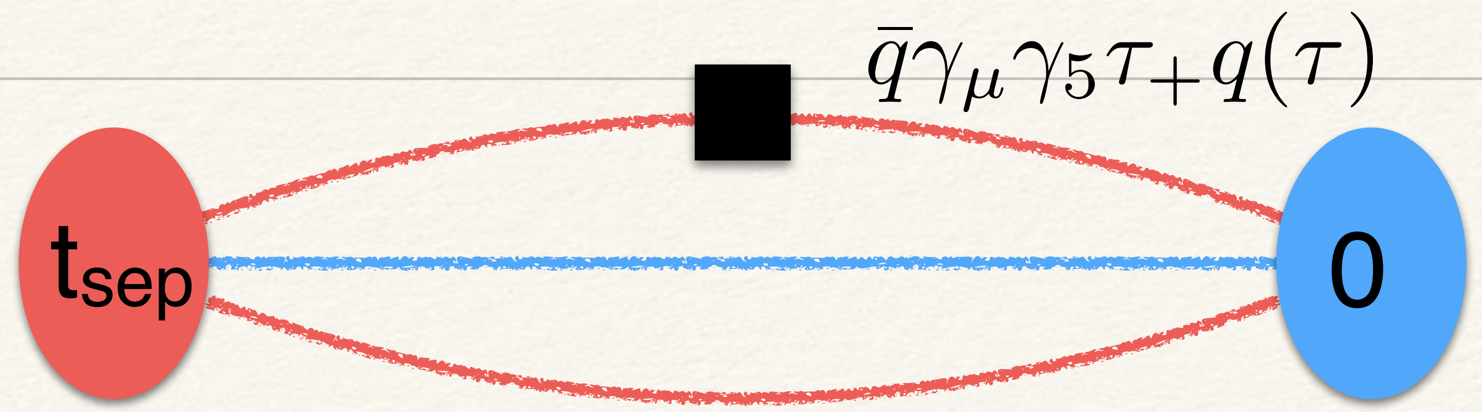
“scattering” (sc) excited states

“transition” (tr) excited states

$$R_\Gamma(t_{\text{sep}},\tau) = \frac{C_\Gamma(t_{\text{sep}},\tau)}{C_2(t_{\text{sep}})}$$

LQCD: 3 point functions



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use "multiply by 1" trick

$$1 = \sum_n |n\rangle\langle n|$$

$$C_\Gamma(t,\tau) = \sum_{\mathbf{y},\mathbf{x}} \langle\Omega|N(t,\mathbf{y})j_\Gamma(\tau,\mathbf{x})N^\dagger(0,\mathbf{0})|\Omega\rangle$$

$$= |z_0|^2 g_{00}^\Gamma e^{-E_0 t} + \underbrace{\sum_{n>0} |z_n|^2 g_{nn}^\Gamma e^{-E_n t}} + \underbrace{2\sum_{n<m} z_n z_m^\dagger g_{nm}^\Gamma e^{-(E_n+\frac{\Delta_{mn}}{2}t)} \cosh\left[\Delta_{mn}\left(\tau - \frac{t}{2}\right)\right]}$$

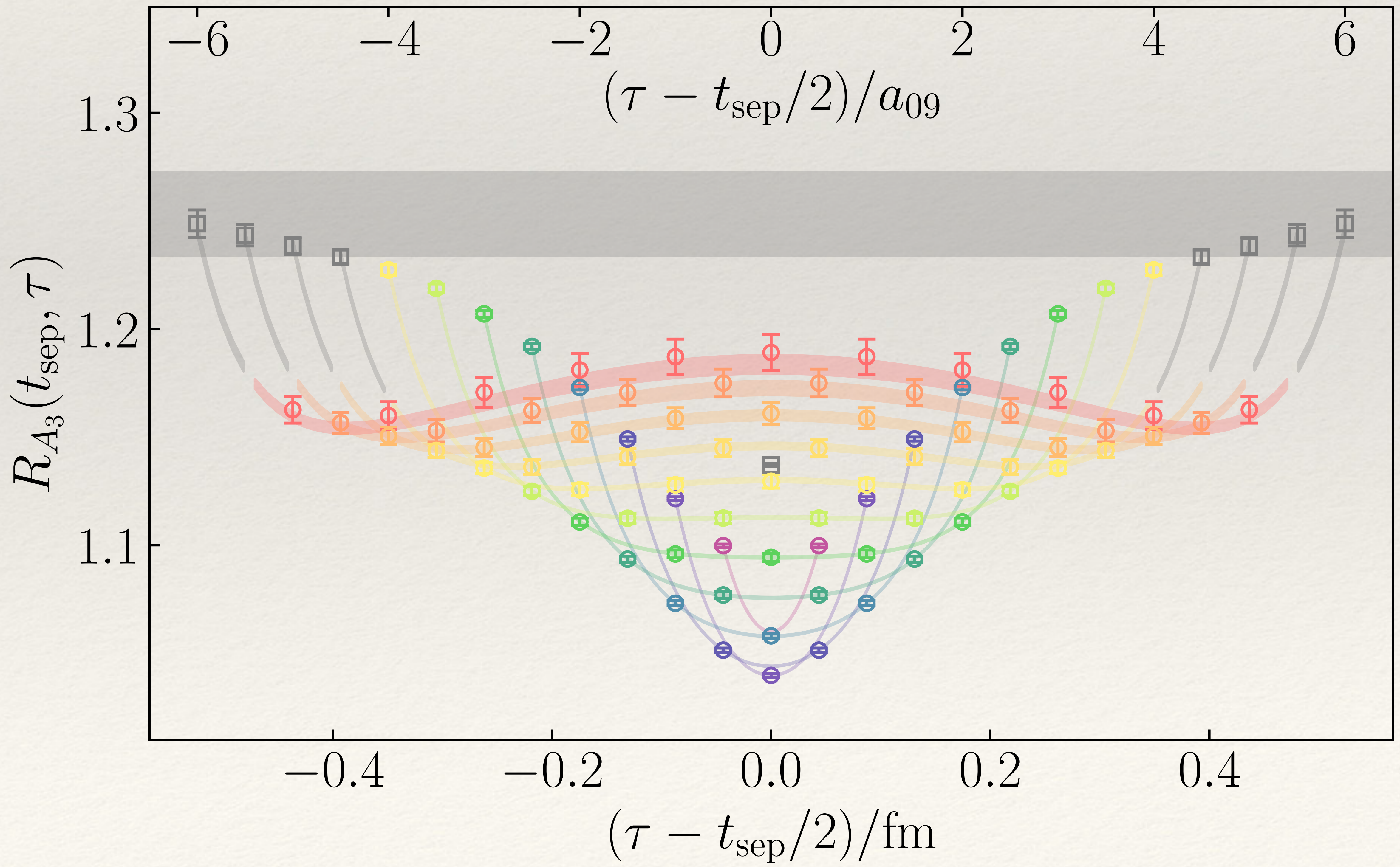
"scattering" (sc) excited states

"transition" (tr) excited states

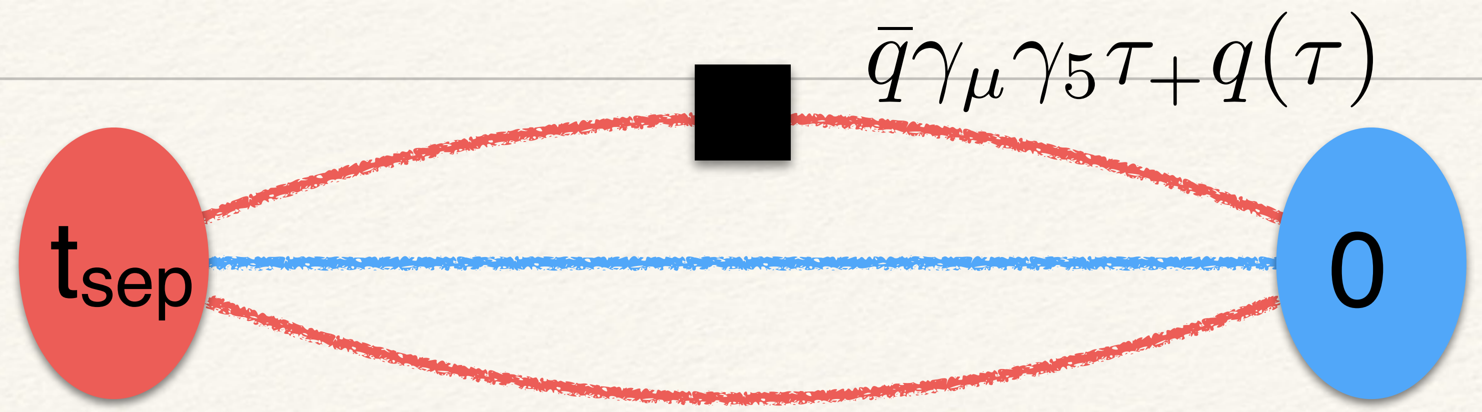
- scattering excited states only depend on t
- transition excited states depend on t and τ

$$R_\Gamma(t_{sep},\tau) = \frac{C_\Gamma(t_{sep},\tau)}{C_2(t_{sep})}$$

- NOTE: for intermediate t, there is a conspiracy of excited states that give the appearance of no excited state contamination

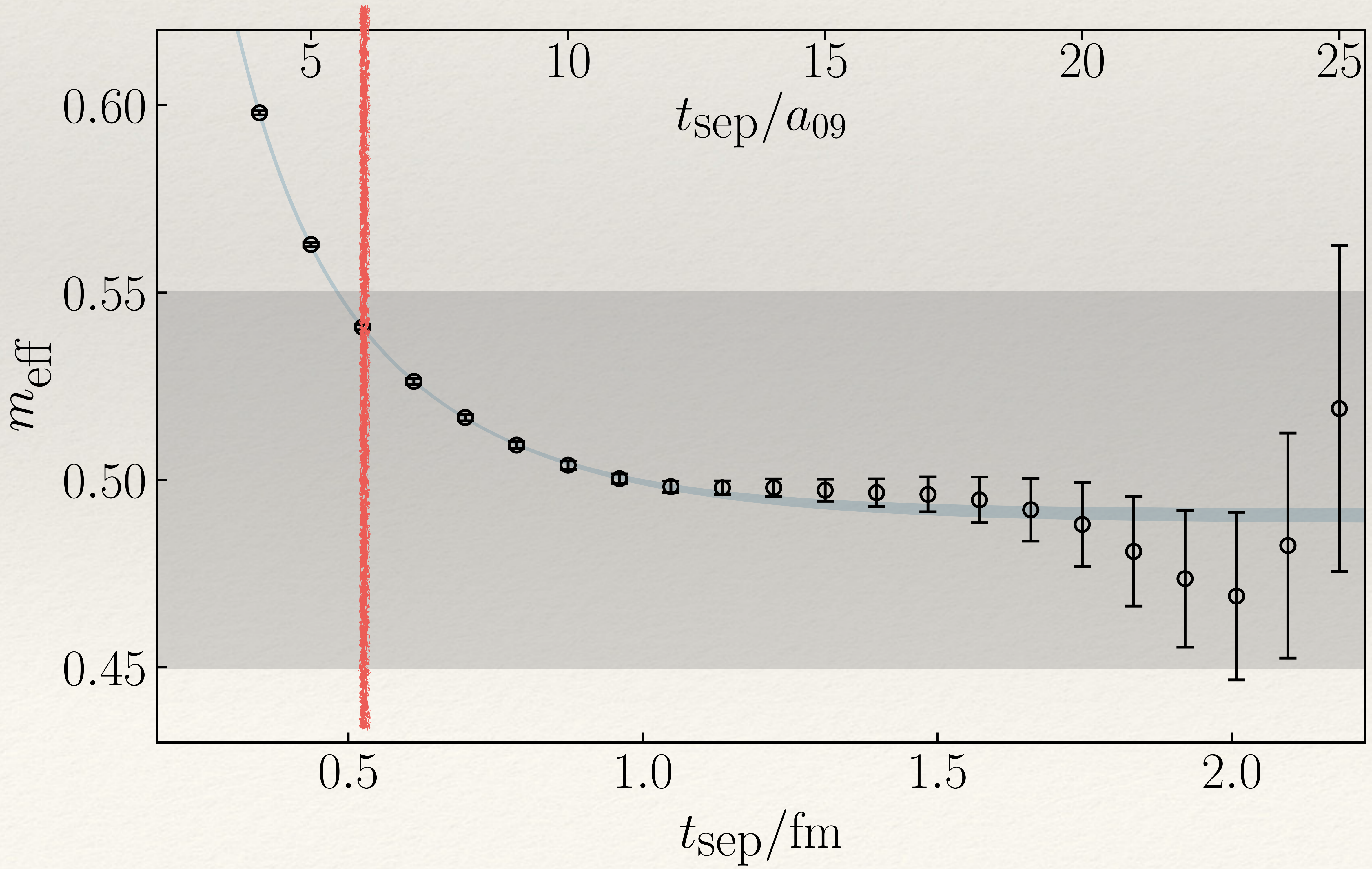


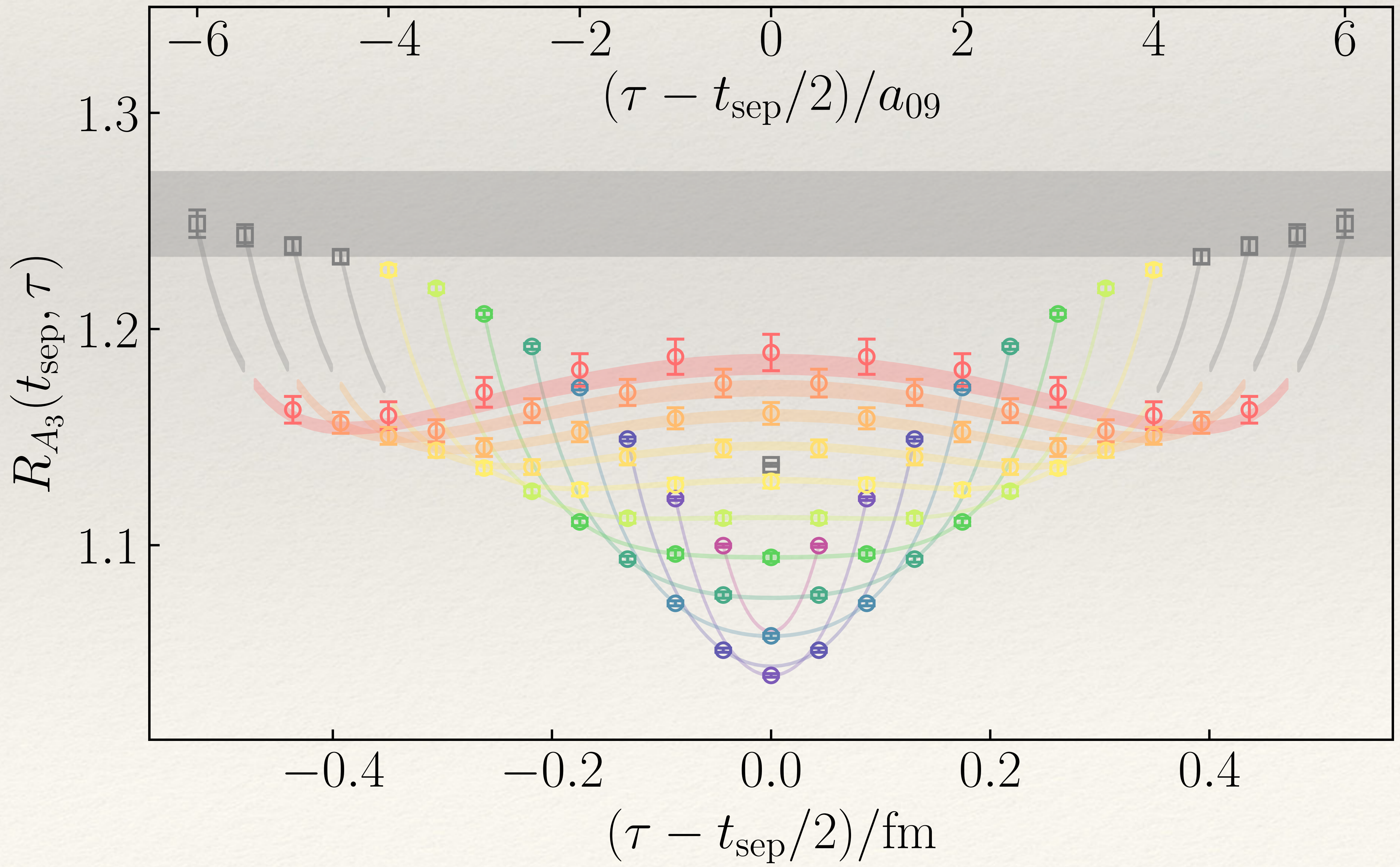
LQCD: 3 point functions



- Consider zero-momentum (p=0) and zero momentum transfer (q=0)

- NOTE: 2pt at t=6 (similar to 3pt with t=12, τ=6) has significant excited state contamination

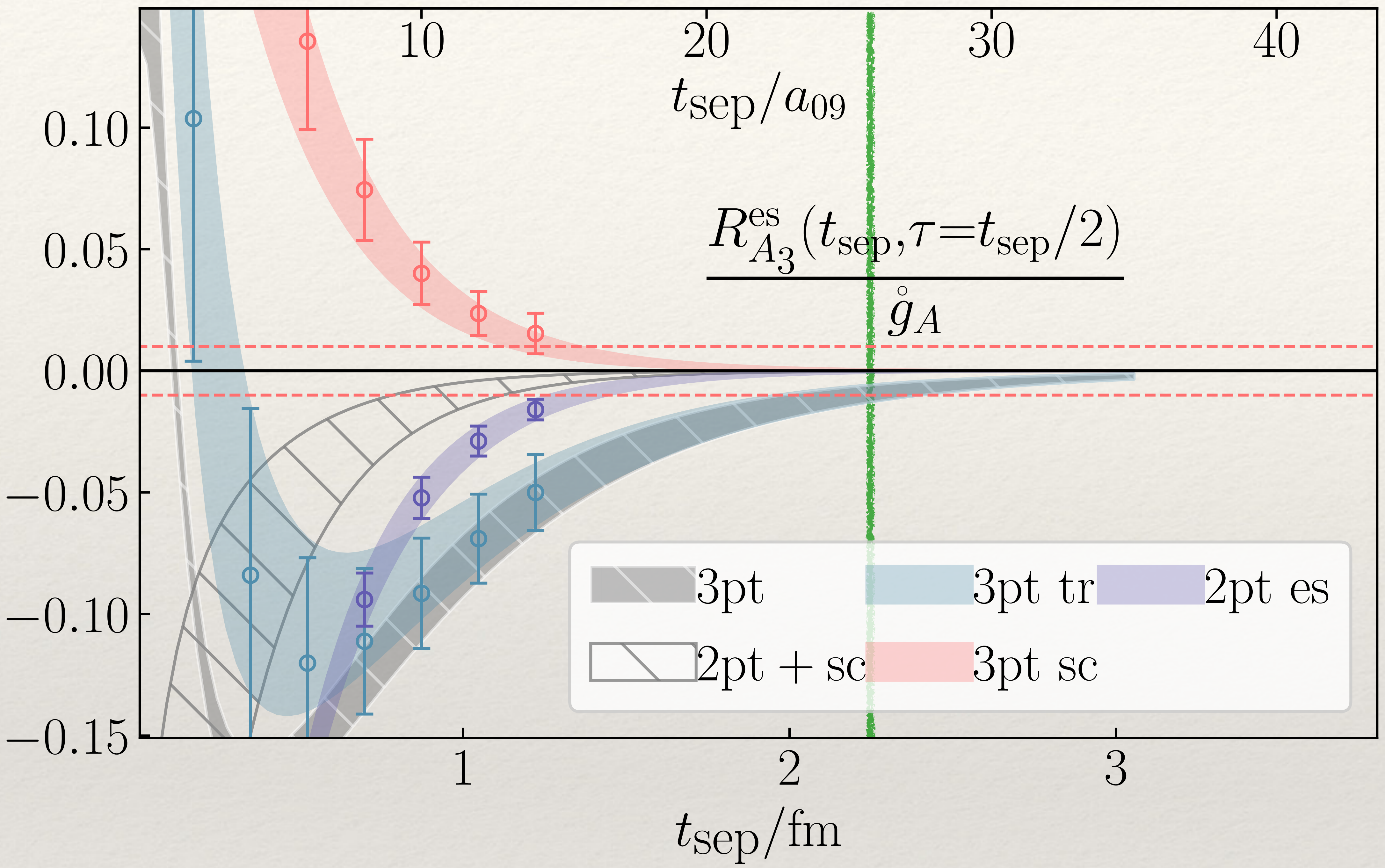


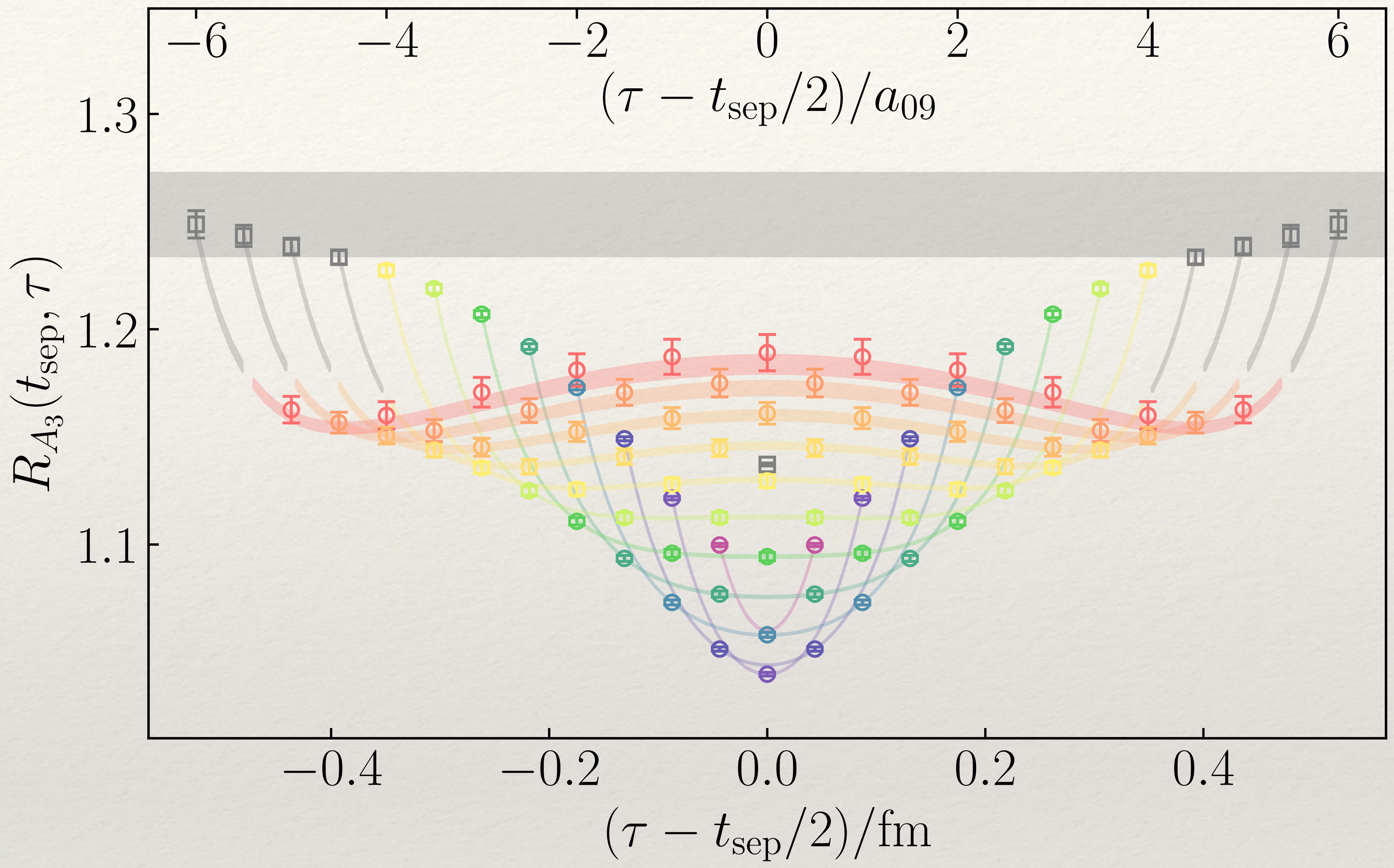


Computational & Analysis Strategies

❑ Disentangle excited states

CalLat: arXiv:2104.05226 5-state simultaneous fit





$$R_\Gamma(t_{\rm sep},\tau) = \frac{C^{\rm gs}_\Gamma(t_{\rm sep}) + C^{\rm sc}_\Gamma(t_{\rm sep}) + C^{\rm tr}_\Gamma(t_{\rm sep},\tau)}{C_2(t_{\rm sep})}$$

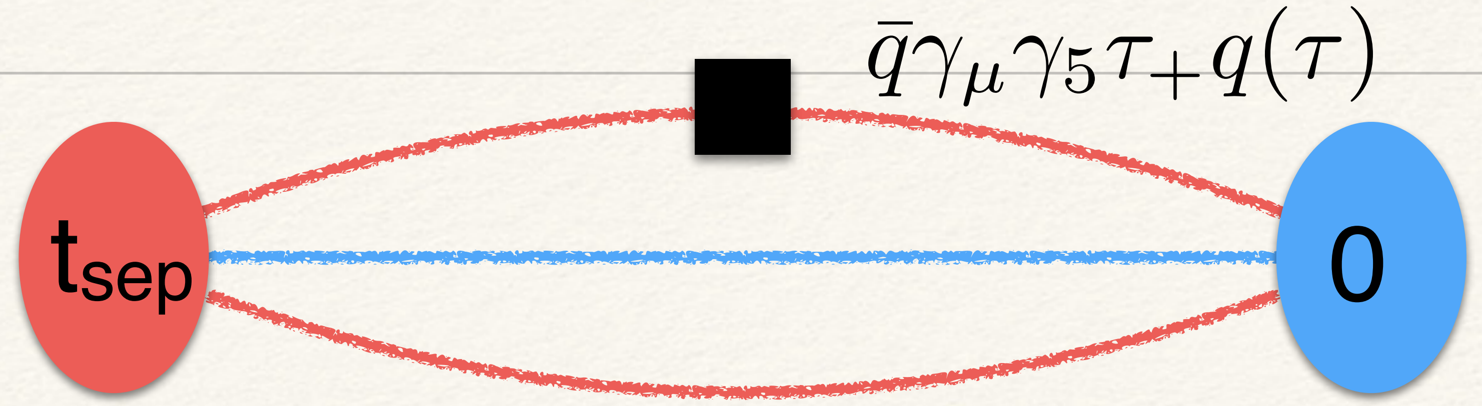
$$= g^\Gamma_{00} + \frac{C^{\rm sc}_\Gamma(t_{\rm sep}) - g^\Gamma_{00} C^{\rm es}_2(t_{\rm sep})}{C_2(t_{\rm sep})} + \frac{C^{\rm tr}_\Gamma(t_{\rm sep},\tau)}{C_2(t_{\rm sep})}$$

$$= g^\Gamma_{00} + \frac{\sum_{n\geq 1}(g^\Gamma_{nn} - g^\Gamma_{00})|z_n|^2 e^{-E_n t_{\rm sep}}}{C_2(t_{\rm sep})} + \frac{2\sum_{n<m} z_n z^\dagger_m g^\Gamma_{nm} e^{-(E_n+\frac{\Delta_{mn}}{2})t_{\rm sep}} \cosh\left[\Delta_{mn}\left(\tau - \frac{t_{\rm sep}}{2}\right)\right]}{C_2(t_{\rm sep})}$$

We must go to $t_{\rm sep} \gtrsim 2.2$ fm for e.s. $\lesssim 1\%$

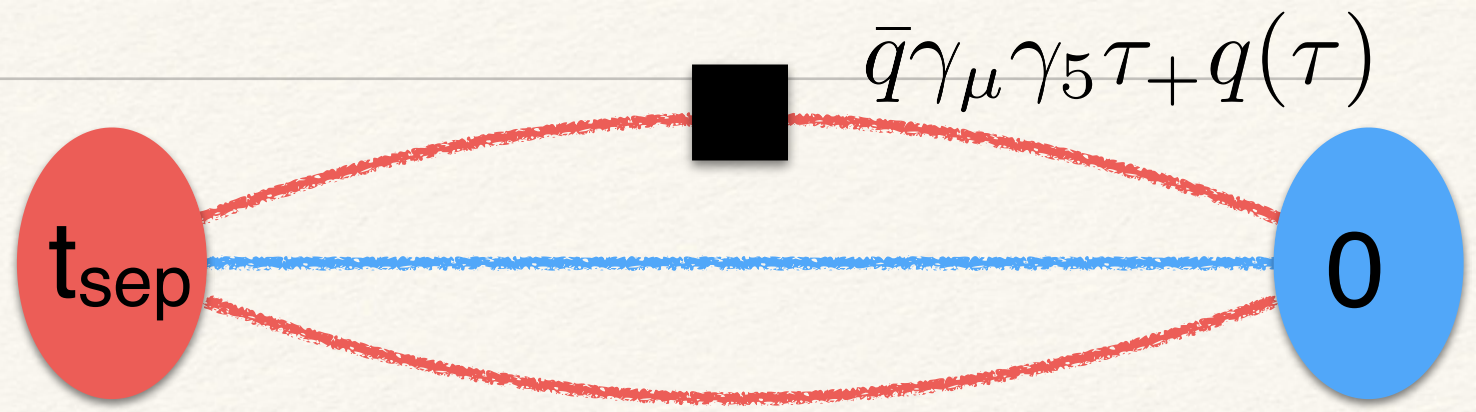
Signal “dies” $t_{\rm sep} \sim 1.5$ fm

LQCD: 3 point functions



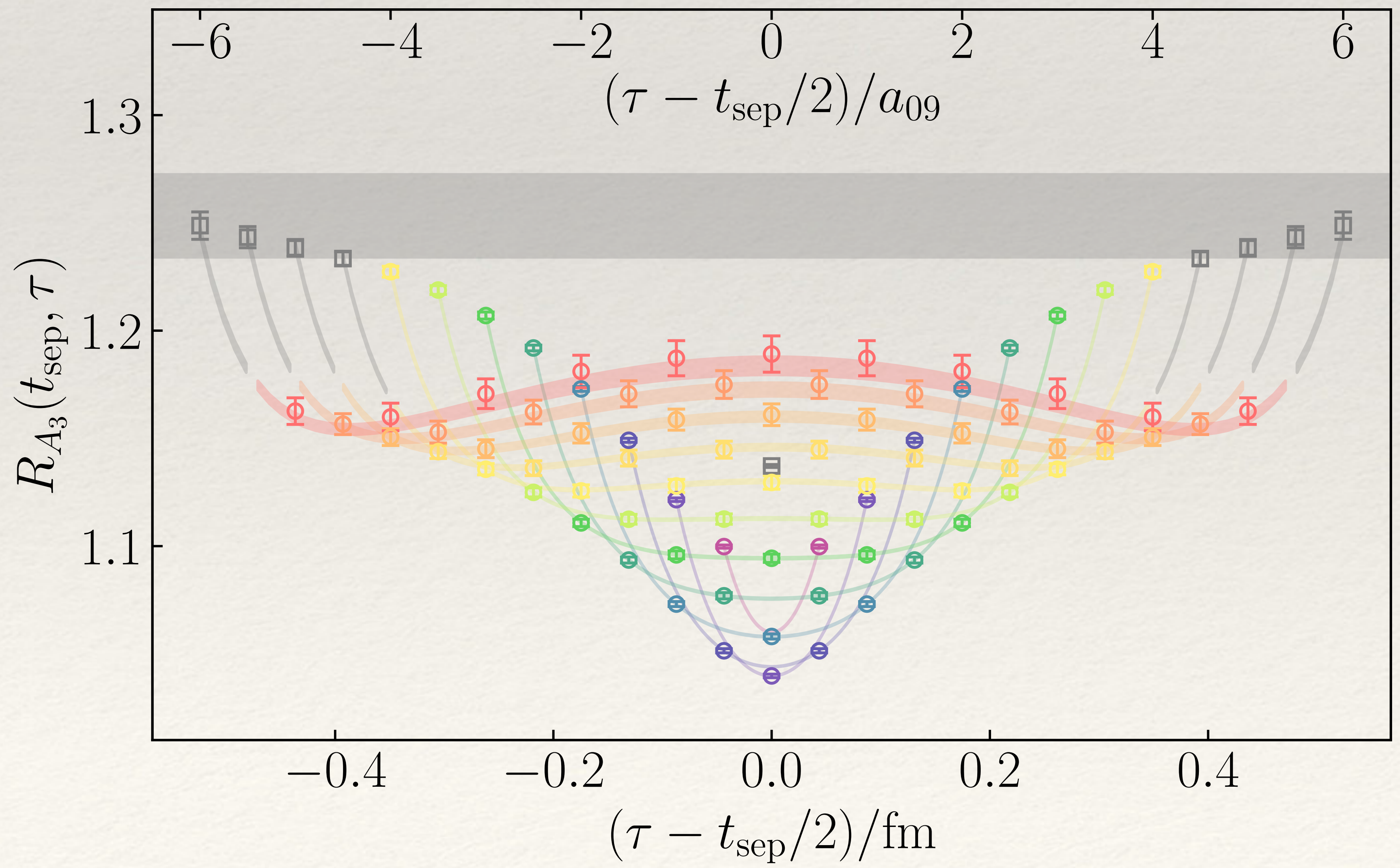
- Lessons learned:
  - Excited state (e.s.) contamination was the main cause of the g_A deficiency
  - It is not practical to pull source/sink far enough apart to eliminate e.s. contamination
  - Need at least 3 values of t (t_{sep}) to control extrapolation
    - most groups advocate high statistics, O($5e^5$), at large t_{sep} values
    - some groups advocate medium statistics, O($2e^4$), with O(10) t_{sep} values
  - Numerical analysis is critical:
    large, highly correlated data sets with under-sampled covariance
    insufficient to get spectrum from 2pt and use that in 3pt analysis - need global analysis
  - The operators we use are sub-optimal to eliminate e.s. contamination
    Some analysis tricks can suppress e.s. for forward matrix elements

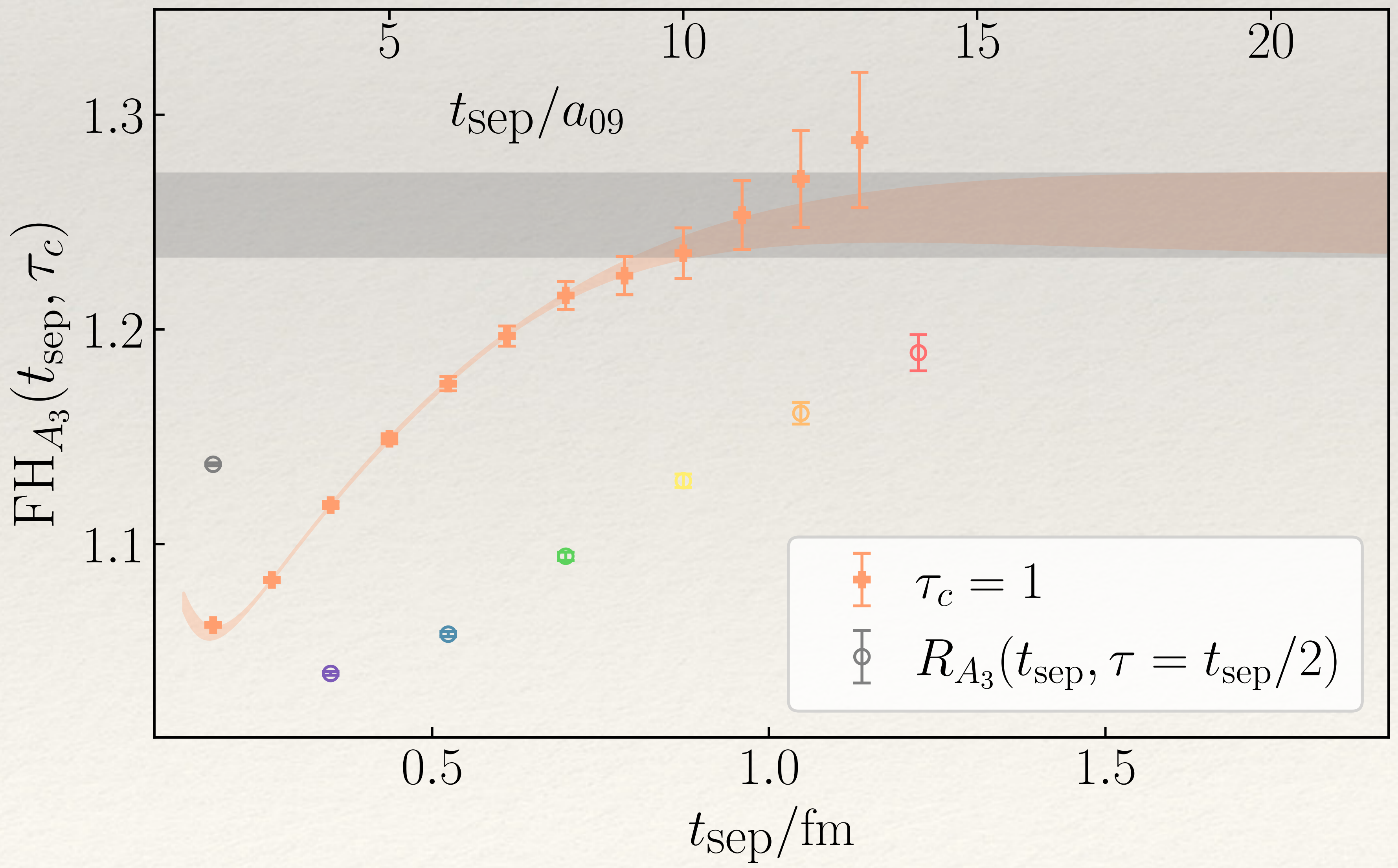
LQCD: 3 pt Feynman-Hellmann Method



- Feynman-Hellmann Method
  follows from an application of the Feynman-Hellmann Theorem of the effective mass
  Bouchard, Chang, Kurth, Orginos, Walker-Loud, PRD 96 (2017) [arXiv:1612.06963]
  see also Maiani, Martinelli, Paciello, Taglienti, Nucl.Phys.B293 (1987)
  Capitani, Della Morte, von Hippel, Jäger, Jüttner, Knippschild, Meyer, Wittig, PRD86 (2012) [1205.0180]

$$FH_\Gamma(t_{\rm sep}) = \frac{d}{dt_{\rm sep}} \int d\tau R_\Gamma(t_{\rm sep},\tau) = \sum_\tau \frac{R_\Gamma(t_{\rm sep}+dt,\tau) - R_\Gamma(t_{\rm sep},\tau)}{dt} = g_{00}^\Gamma + \text{ excited states}$$





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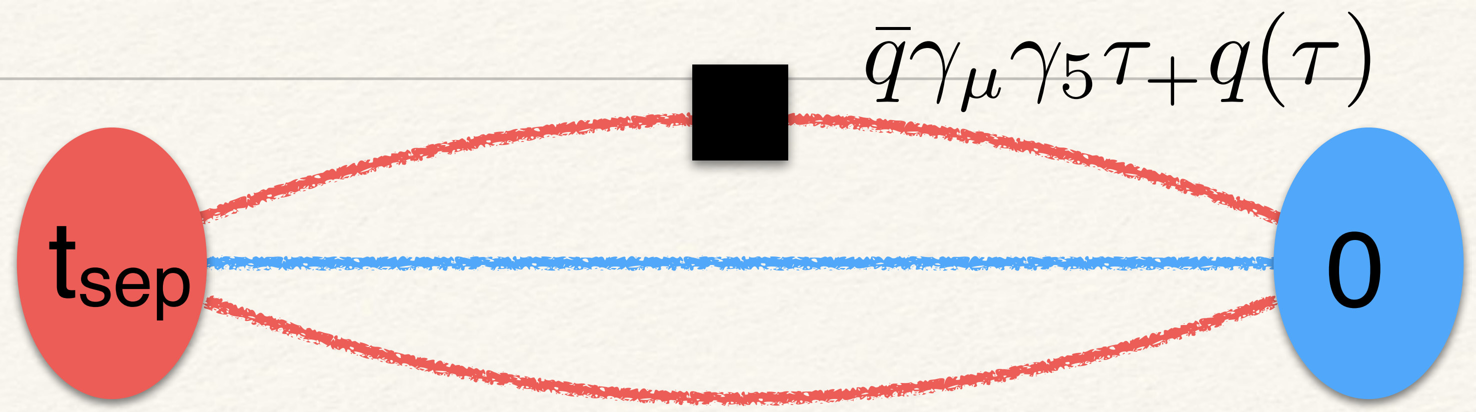

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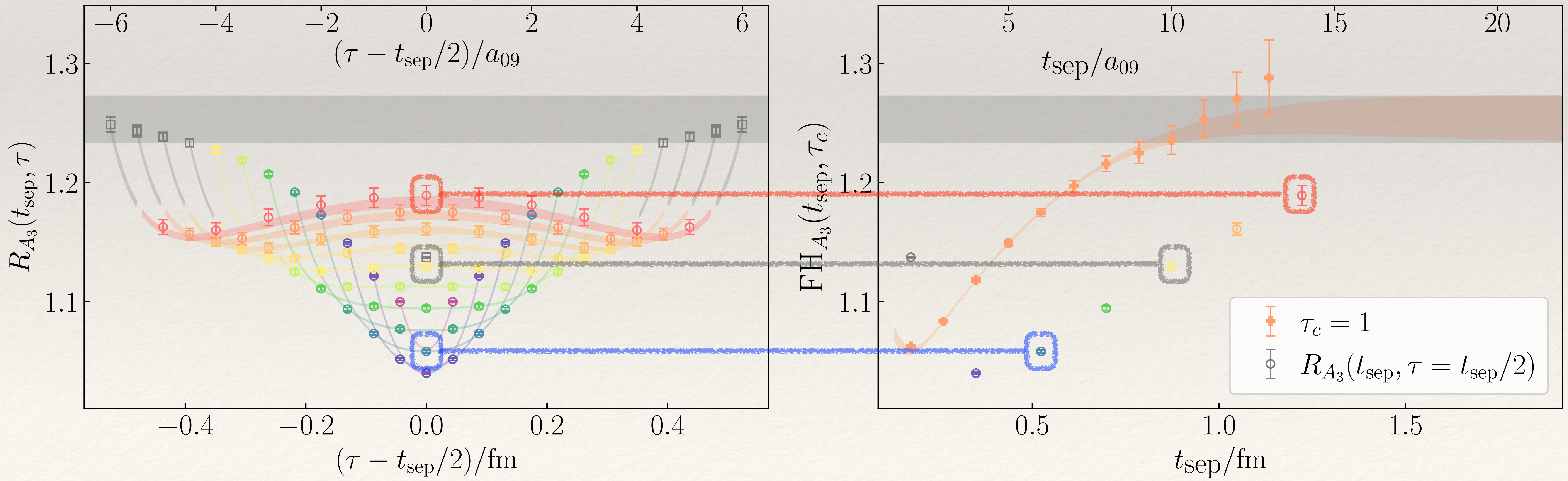

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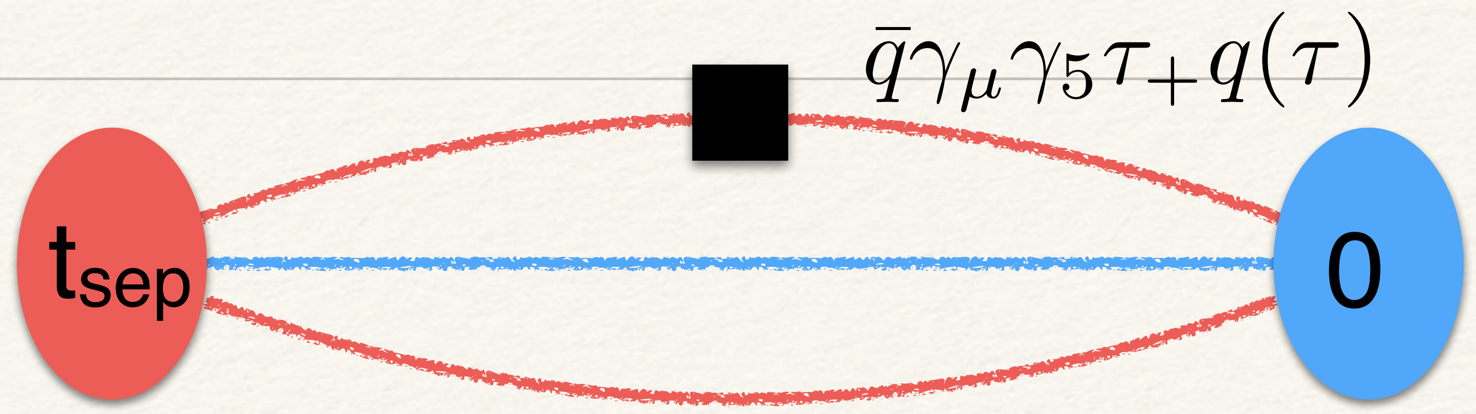

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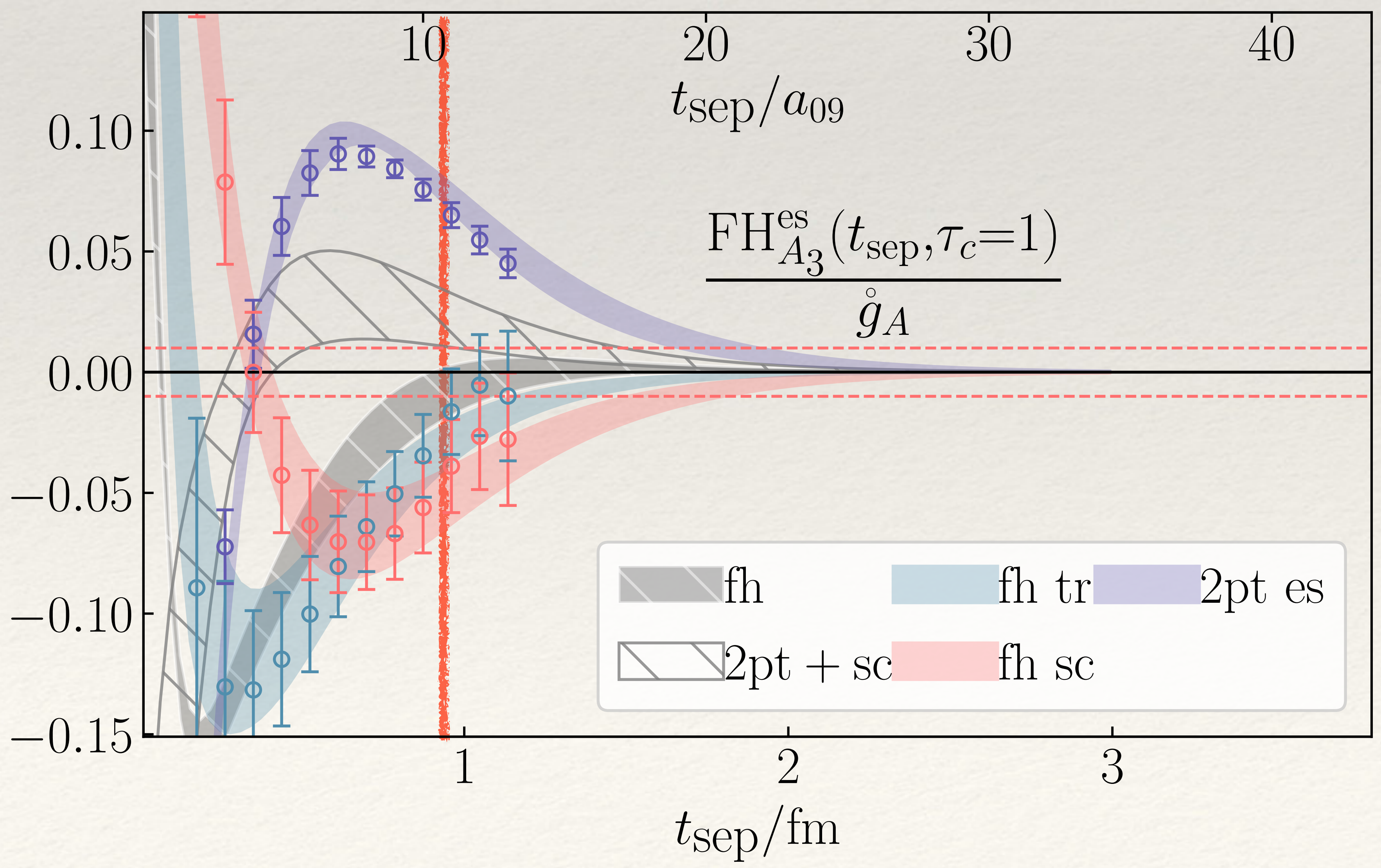
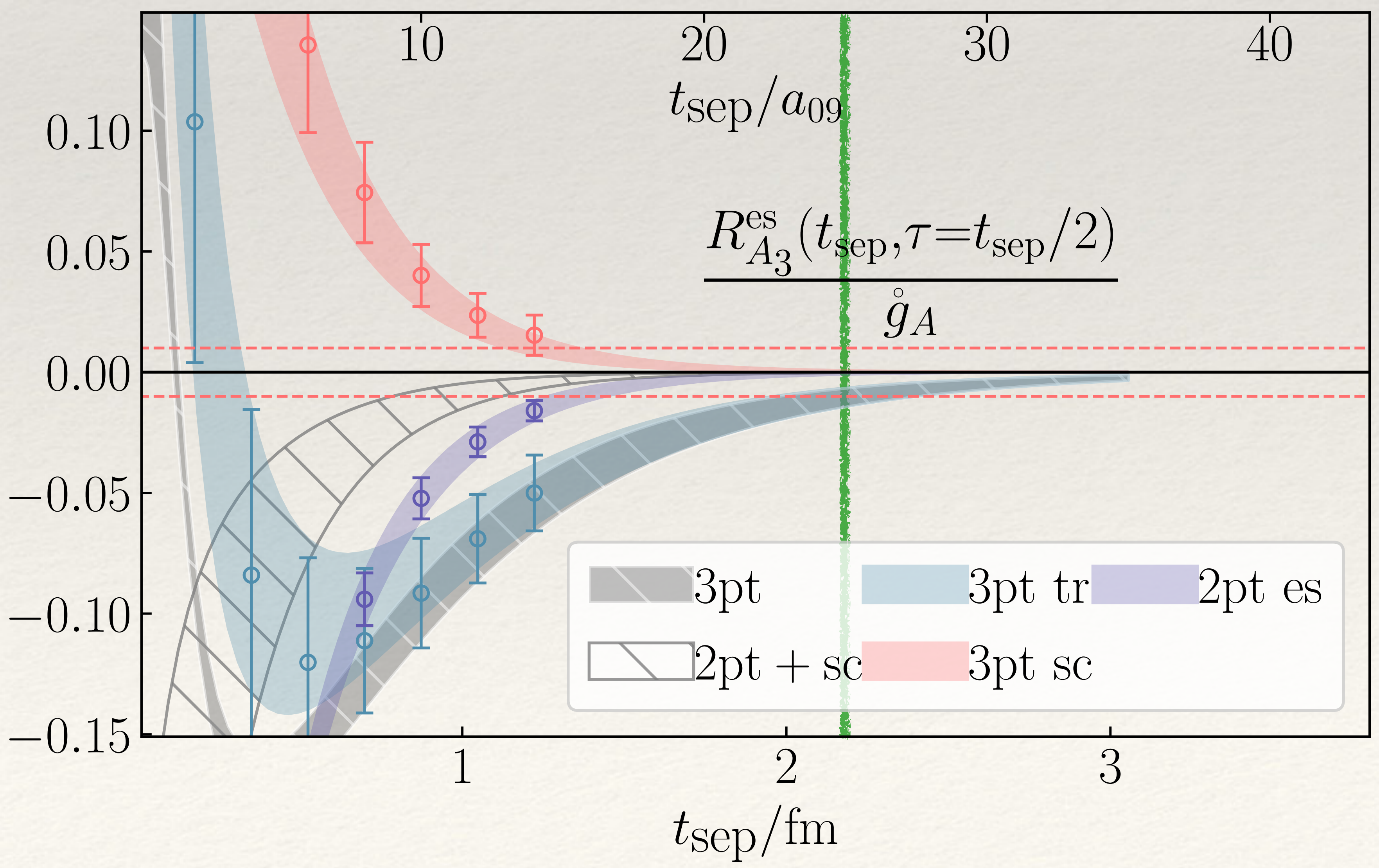
es(FH) $\lesssim$ 1% at $t_{sep} \gtrsim$ 1fm

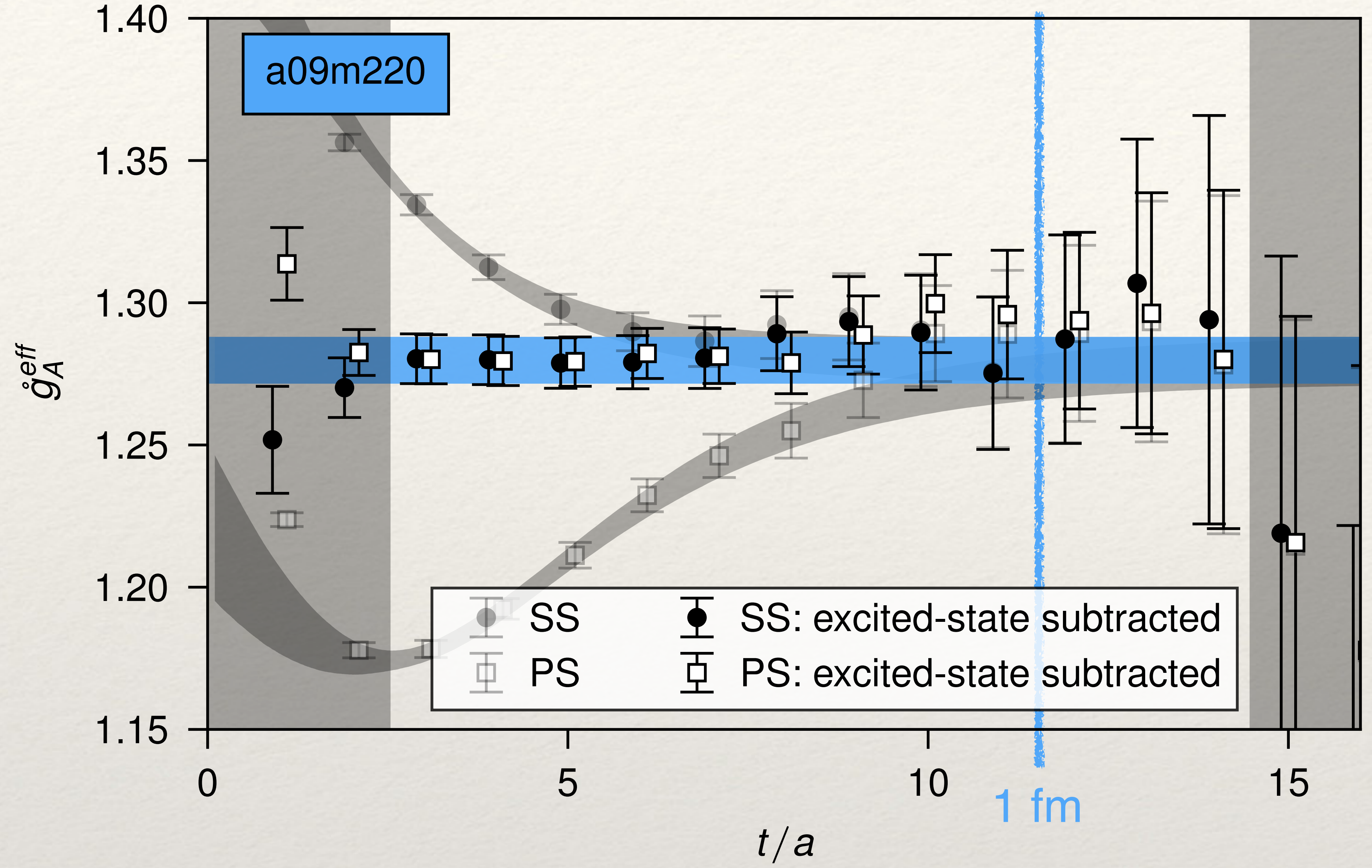


es(ratio) $\lesssim$ 1% at $t_{sep} \gtrsim$ 2fm

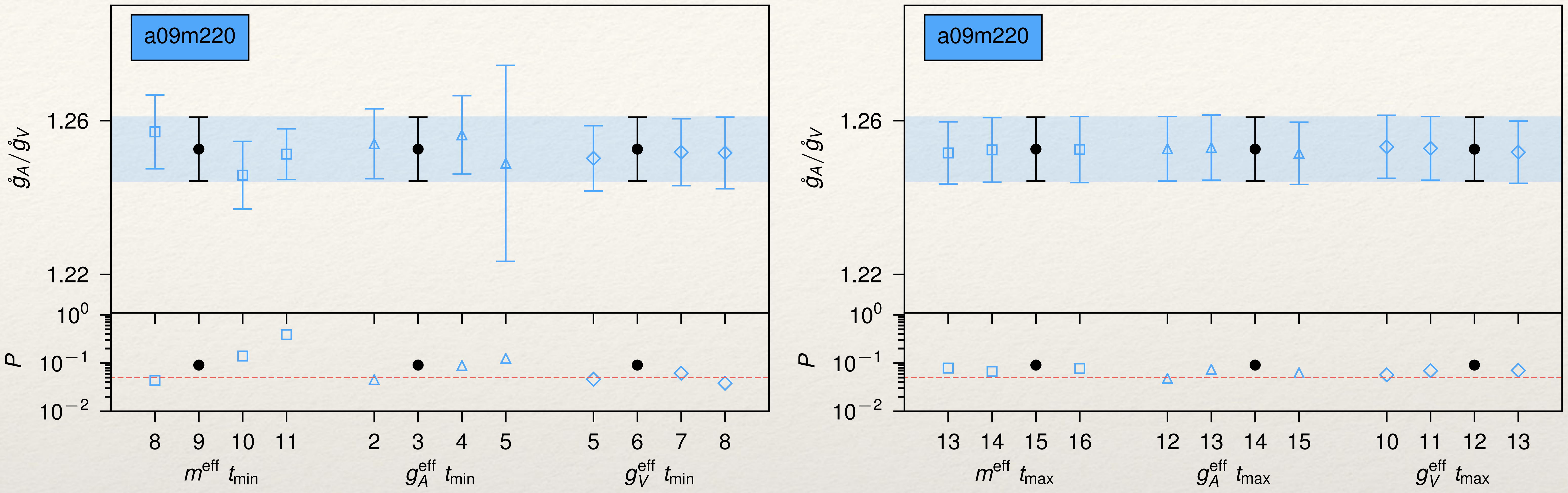


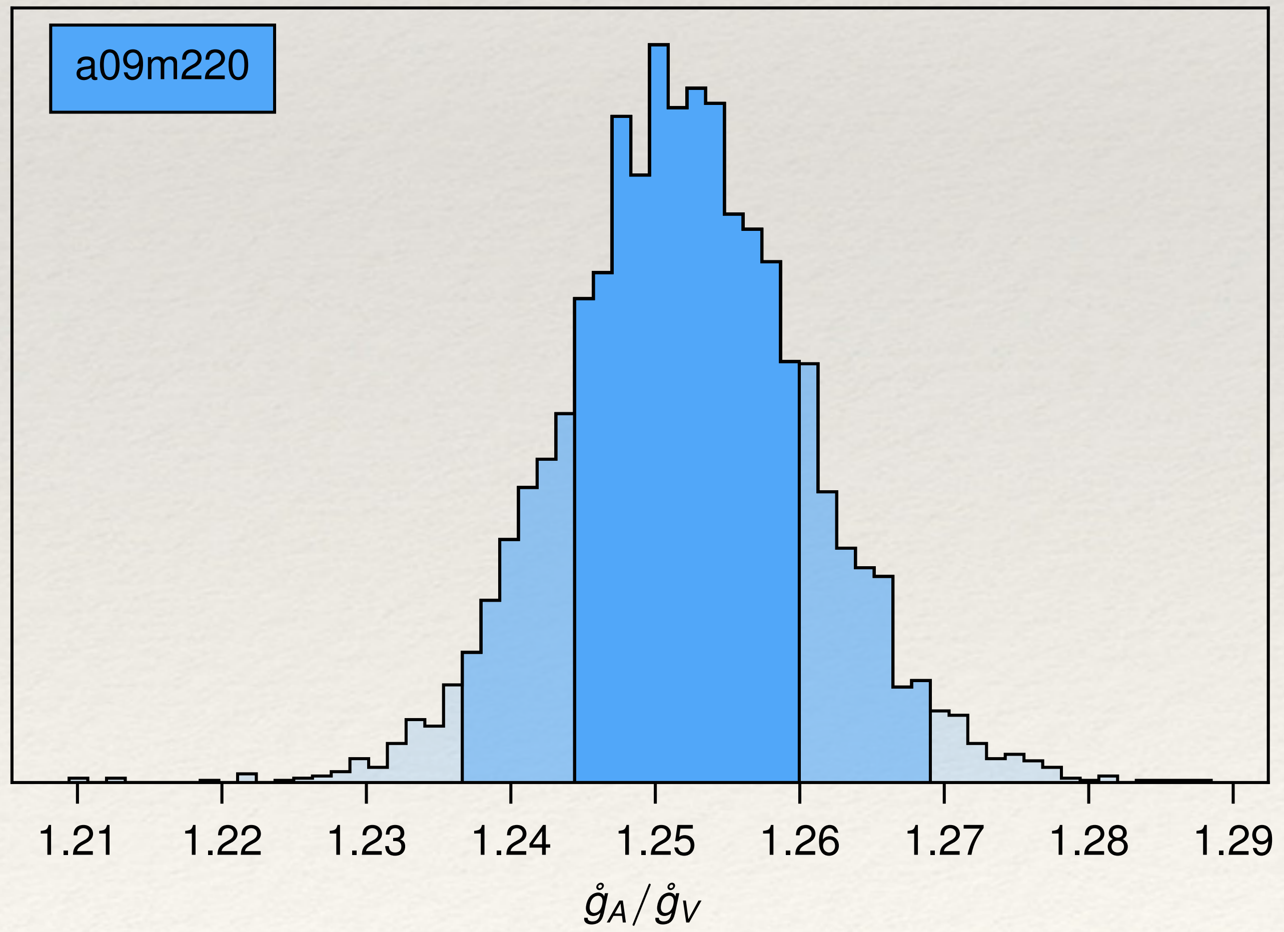
Chang et al, *A per-cent-level determination of g_A from QCD,* Nature 558 (2018) [1805.12130]

https://github.com/callat-qcd/project_gA — raw correlators and data analysis



Fit Stability





Gaussian Bootstrap (N_{bs}=5000) distribution

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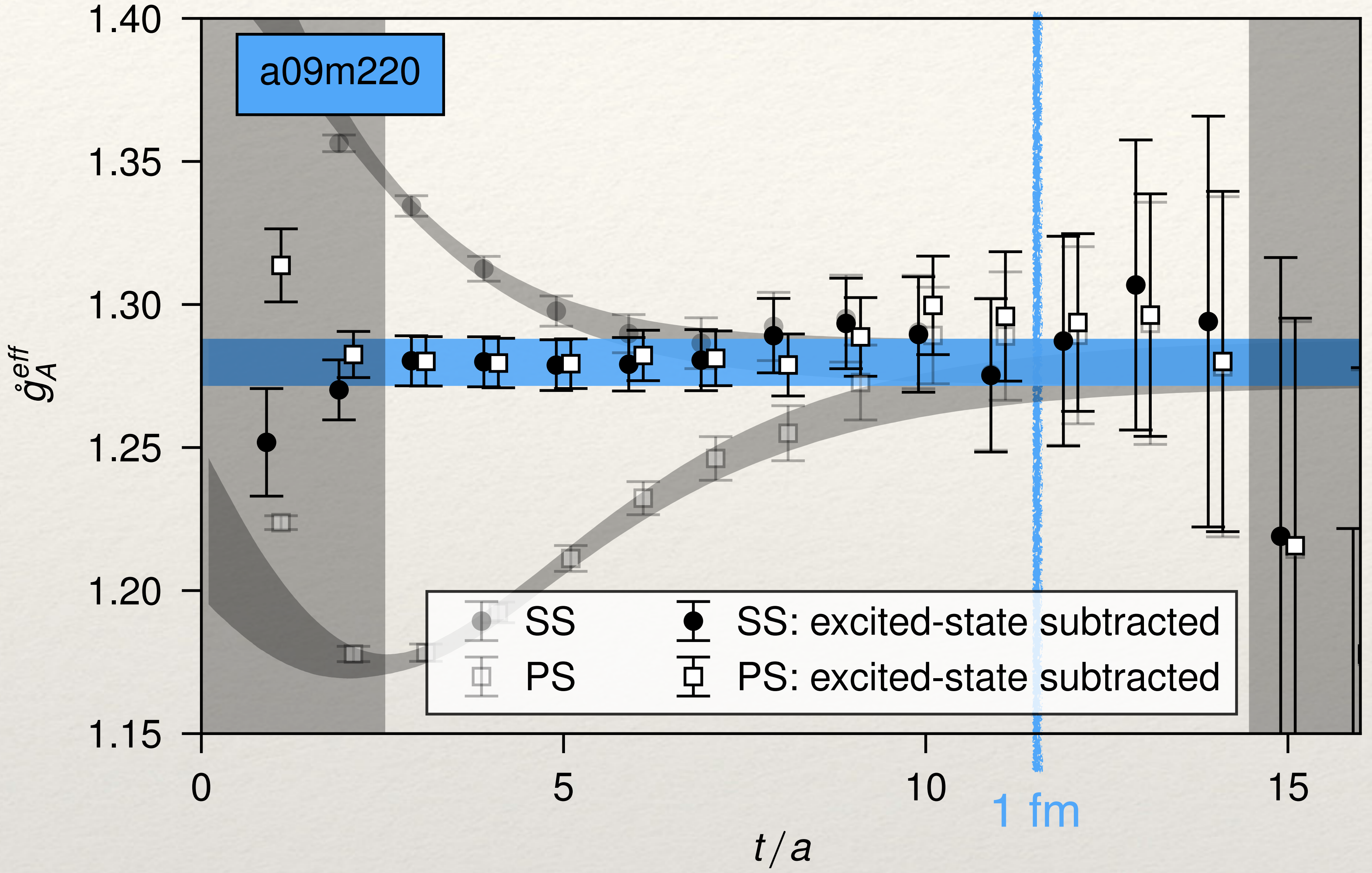

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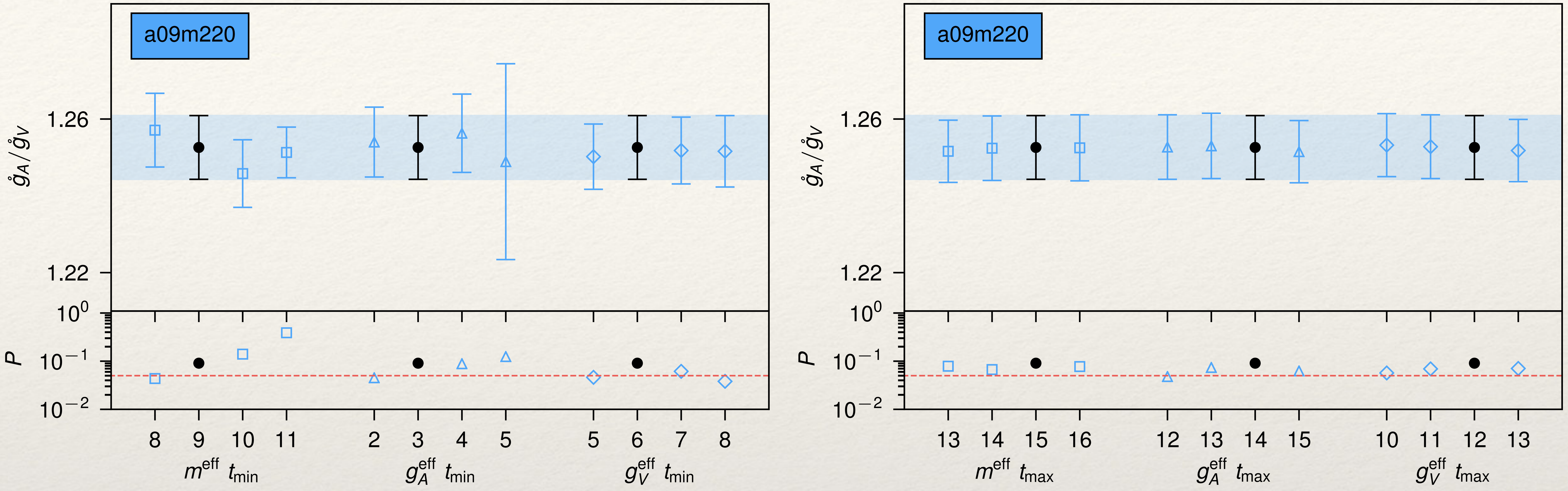
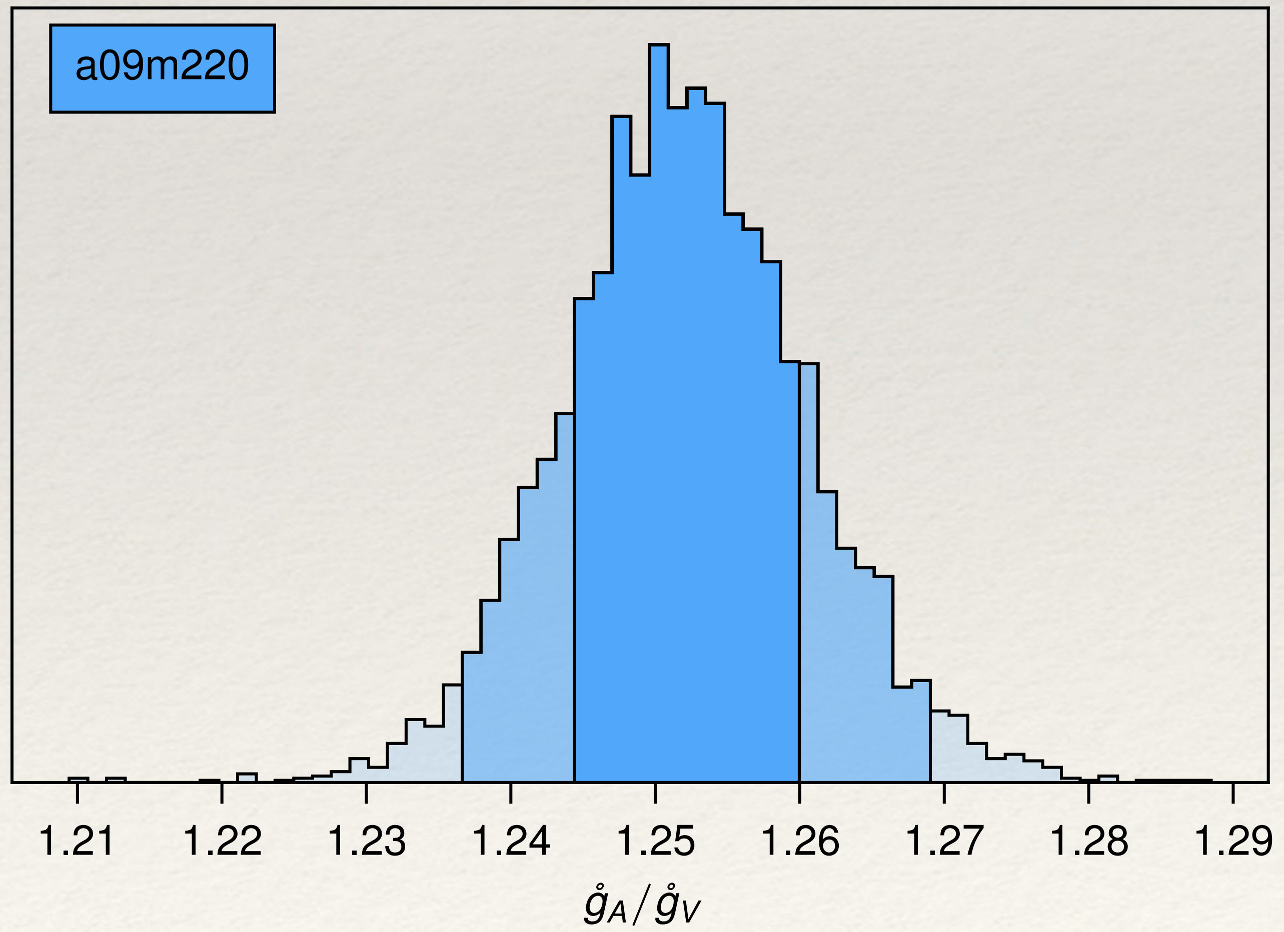


- blue band: ground state
- black/white data: correlator - excited states
- gray data/bands: full correlator data/fit



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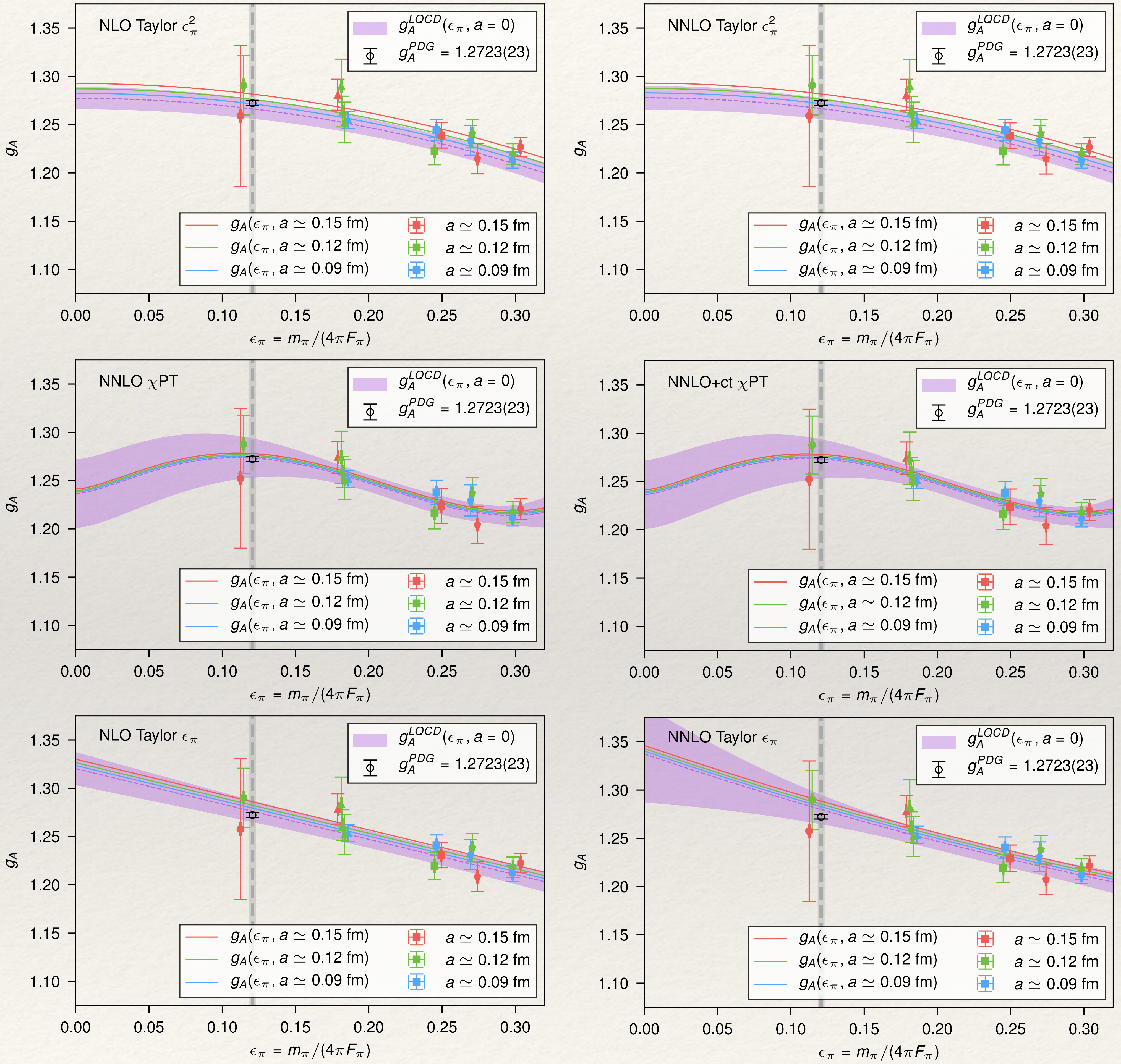
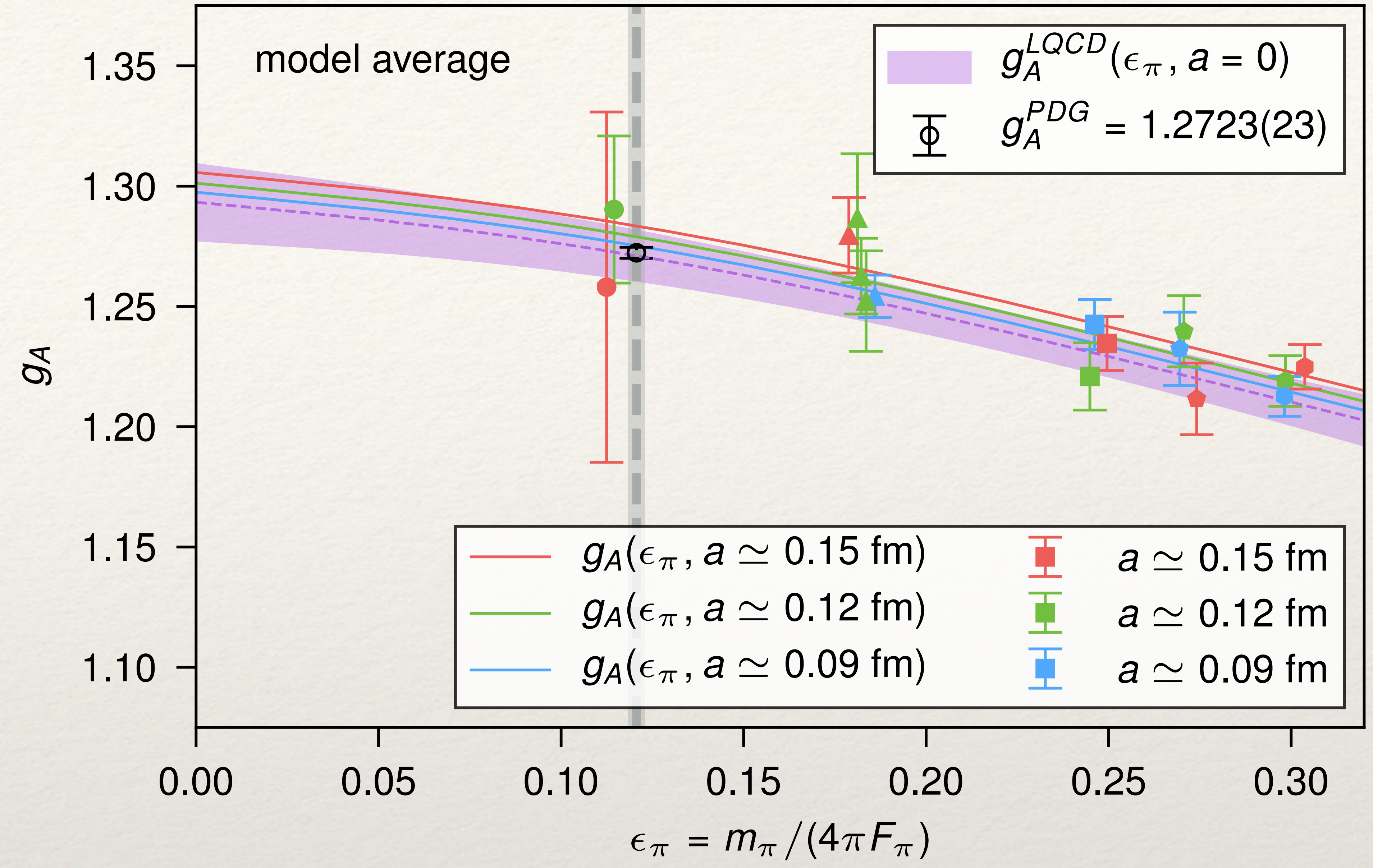
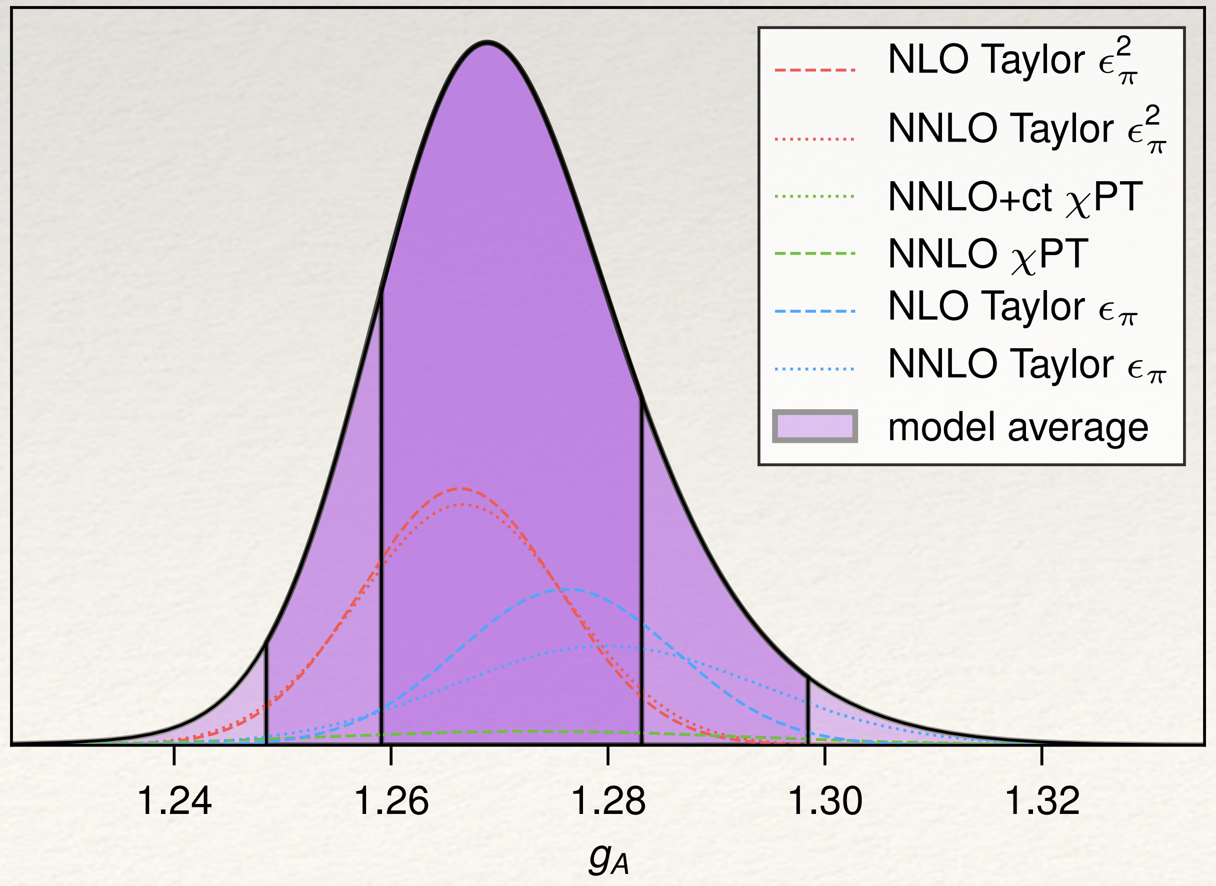
Use various “models” to extrapolate



Final result



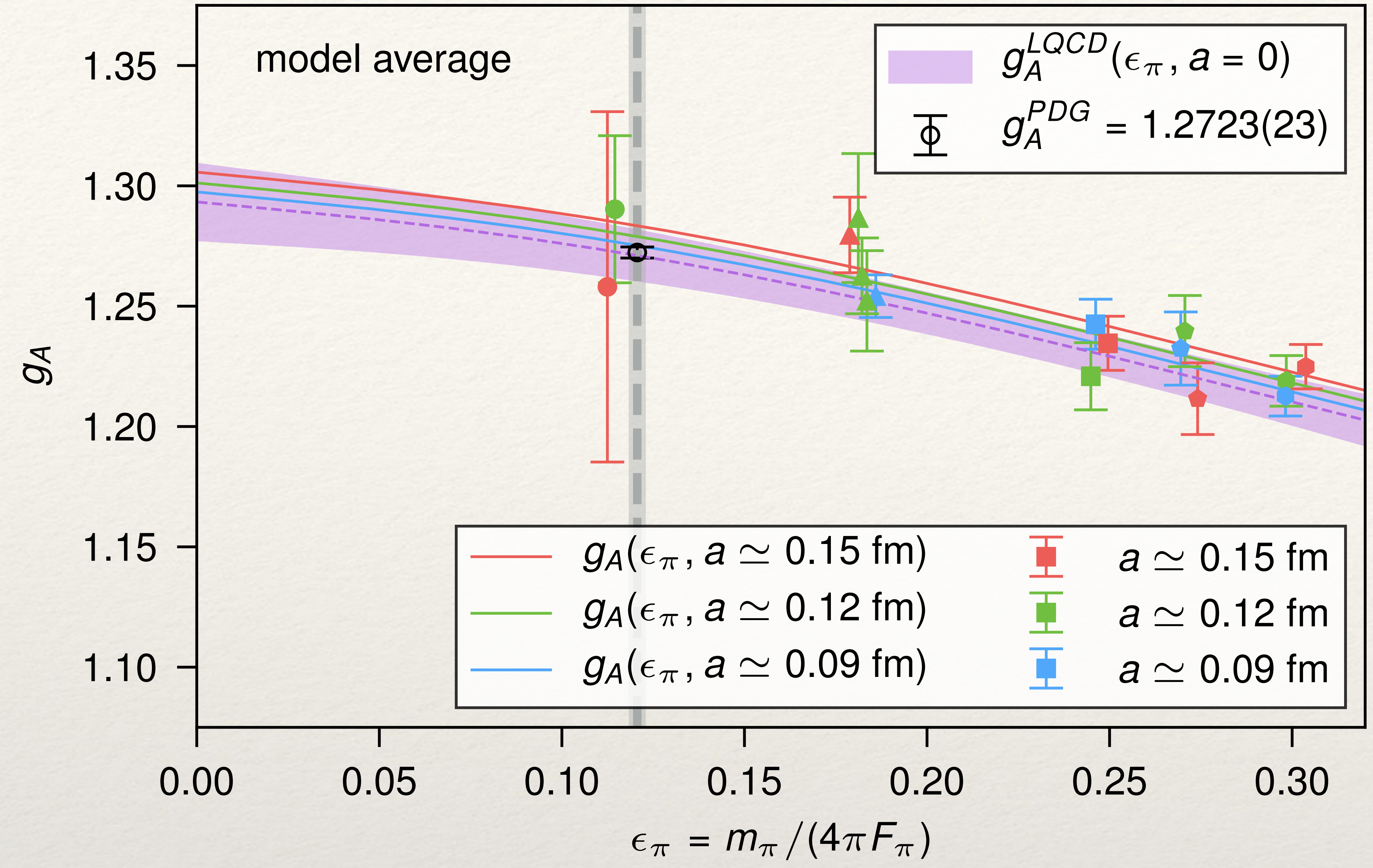
$$g_A^{\text{QCD}} = 1.2711(103)^s(39)^\chi(15)^a(19)^V(04)^I(55)^M$$



Fit	χ^2/dof	$\mathcal{L}(D\|M_k)$	$P(M_k\|D)$	$P(g_A\|M_k)$
NNLO χPT	0.727	22.734	0.033	1.273(19)
NNLO+ct χPT	0.726	22.729	0.033	1.273(19)
NLO Taylor ϵ_π^2	0.792	24.887	0.287	1.266(09)
NNLO Taylor ϵ_π^2	0.787	24.897	0.284	1.267(10)
NLO Taylor ϵ_π	0.700	24.855	0.191	1.276(10)
NNLO Taylor ϵ_π	0.674	24.848	0.172	1.280(14)
average				1.271(11)(06)

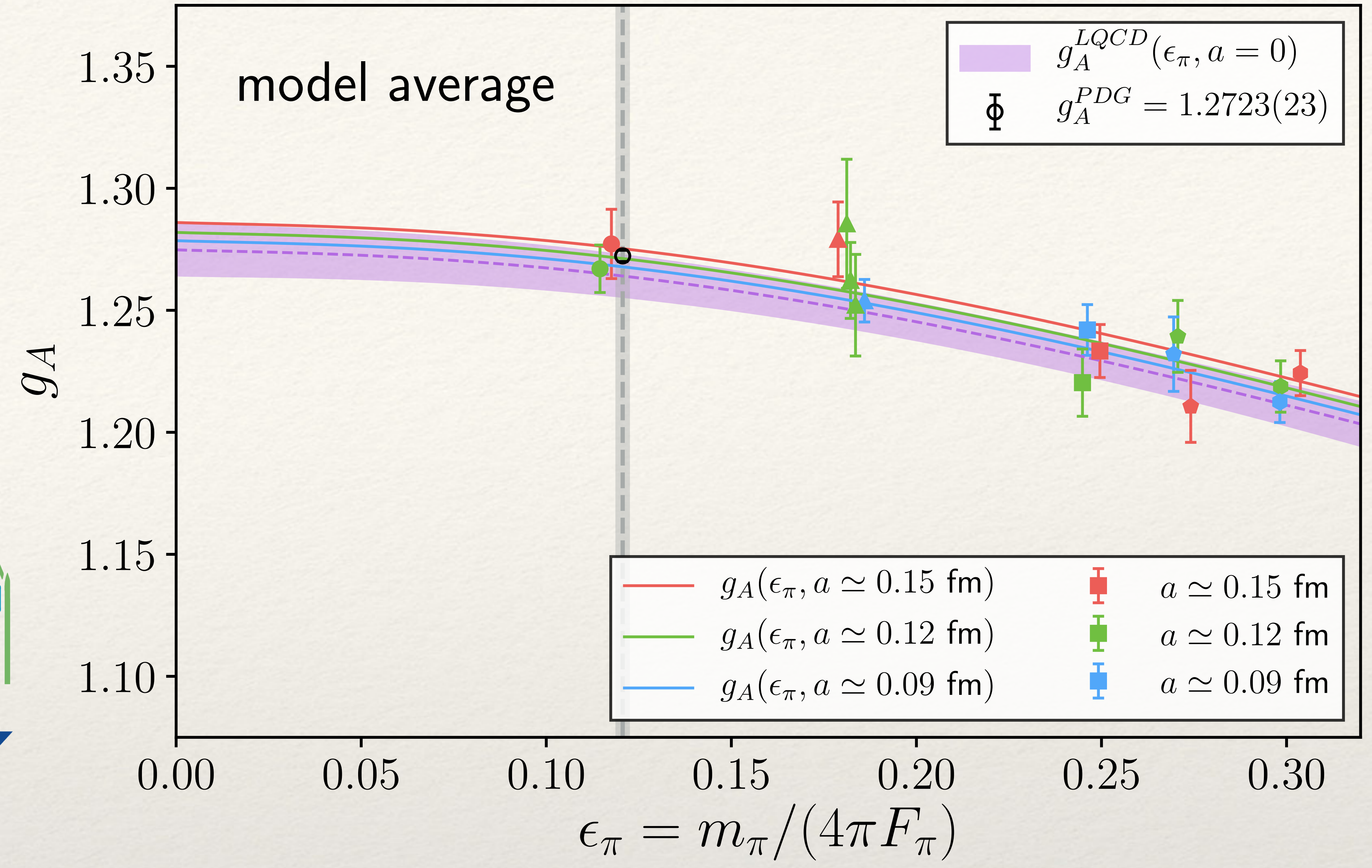
Nature 558 (2018) no. 7708, 91-94
Chang et al. [arXiv:1805.12130]

1 year on Titan (ORNL) + 2 years on GPU machines at LLNL



Sierra Early Science

PRELIMINARY



California Lattice Collaboration

- The a12m130 (48^3 x 64 x 20) with 3 sources cost as much as all other ensembles combined
  - 2.5 weekends on Sierra → 16 srcs
  - Now, 32 srcs (un-constrained, 3-state fit)
- We generated a new a15m135XL (48^3 x 64) ensemble (old a15m130 is 32^3 x 48)
  - $M_\pi L = 4.93$ (old $M_\pi L = 3.2$)
  - $L_5 = 24$, $N_{src} = 16$

$$g_A = 1.2711(125) \rightarrow 1.2641(93) \ \ [0.74\%]$$

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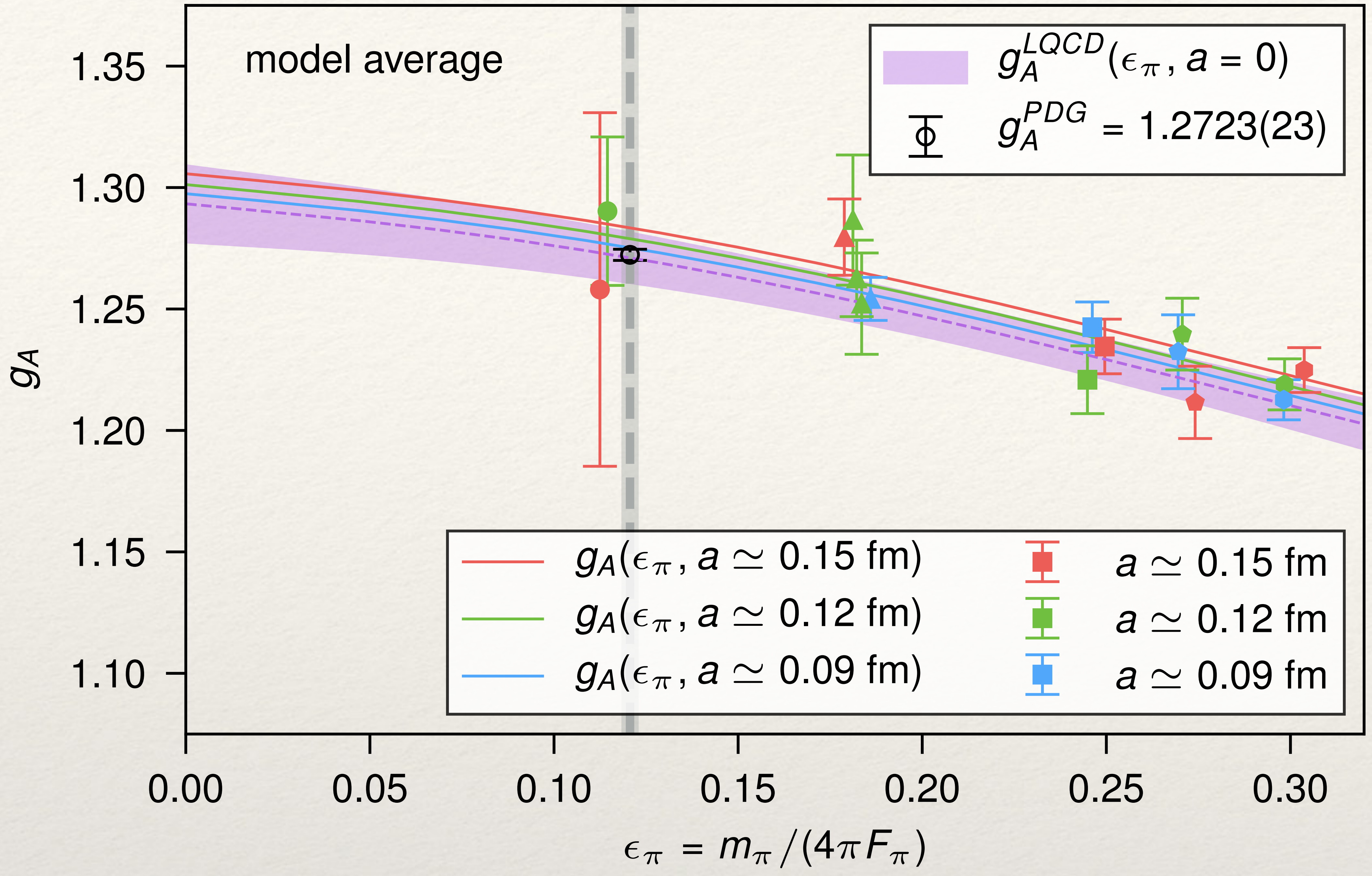

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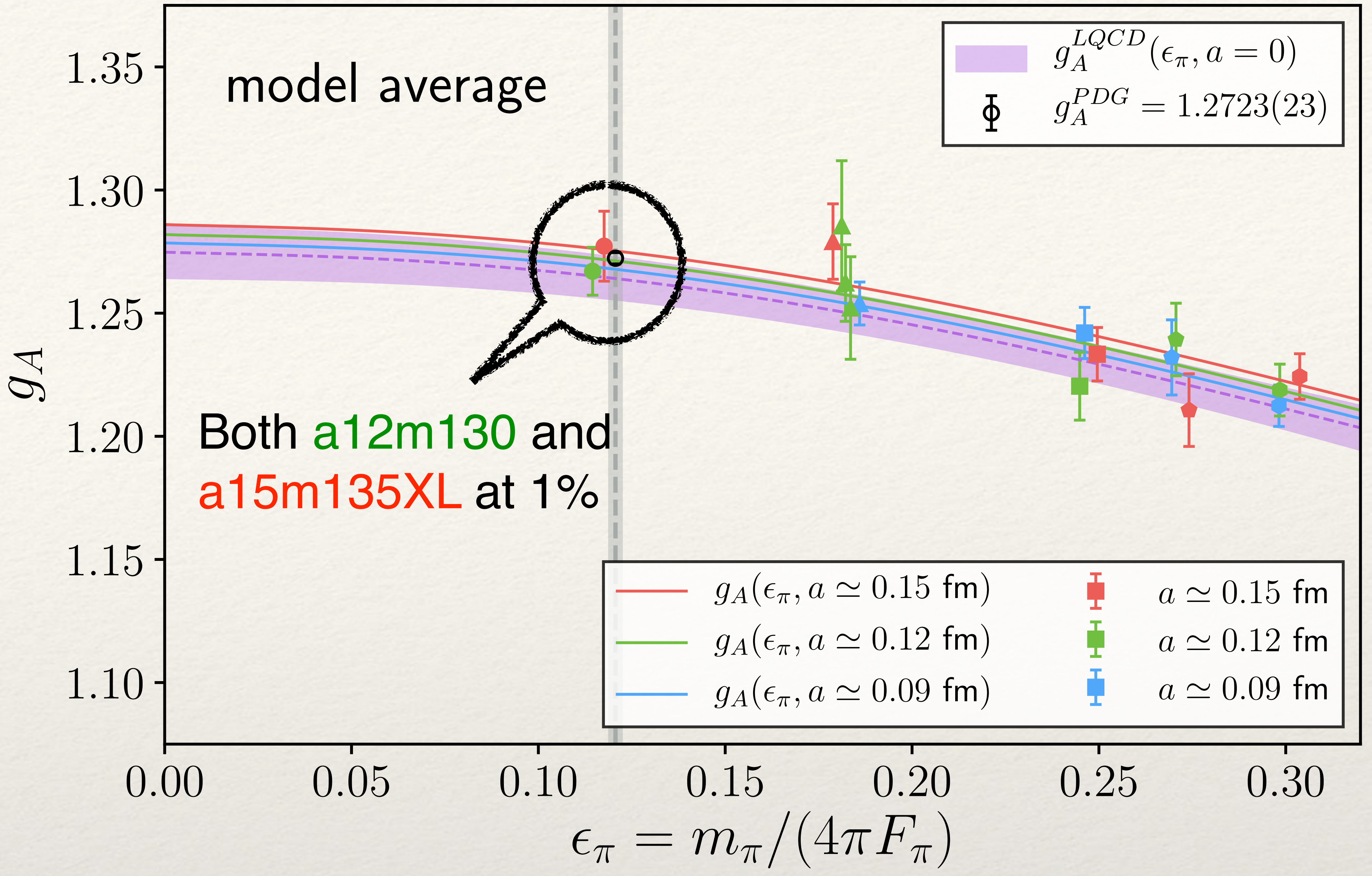

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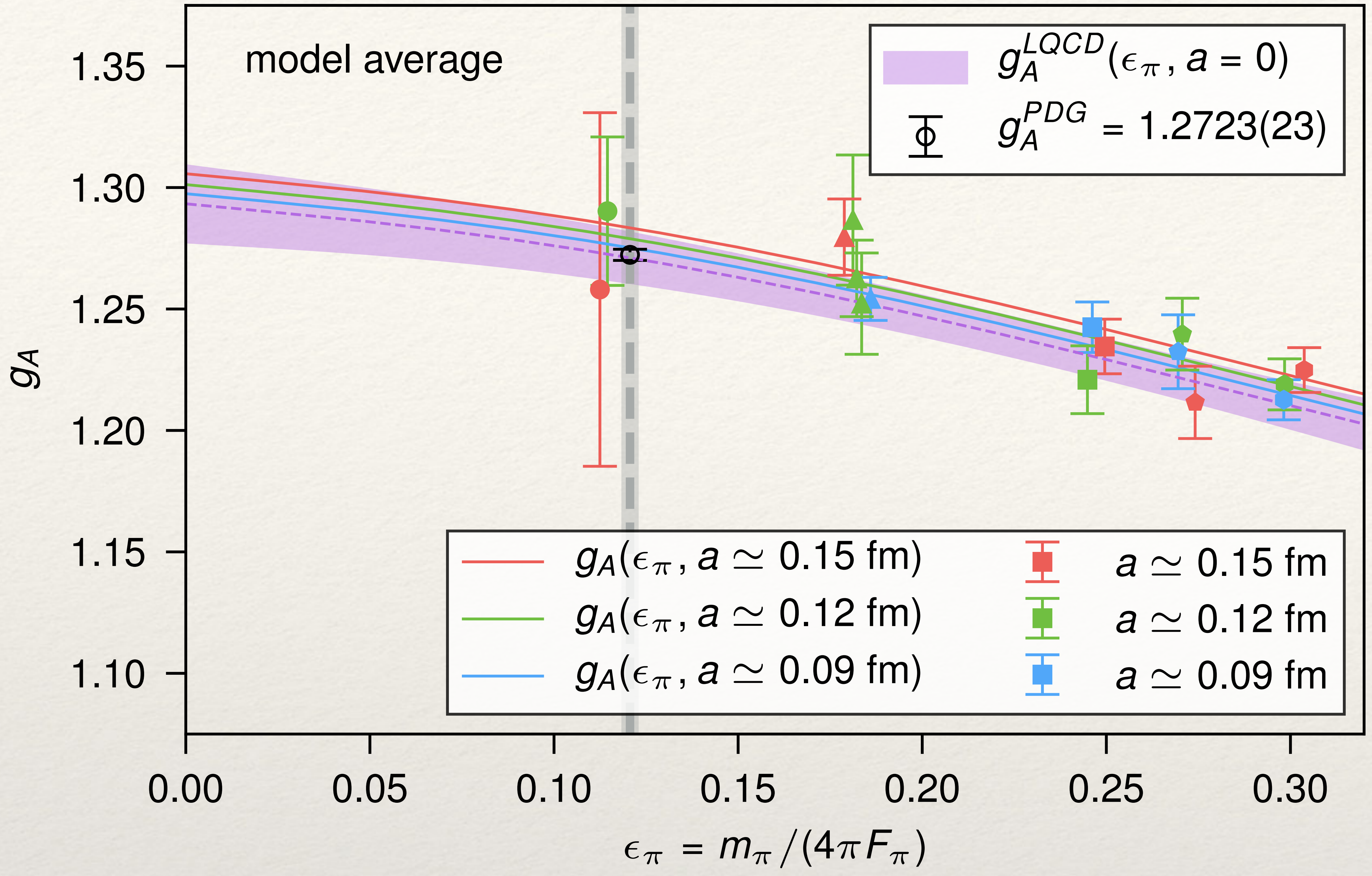

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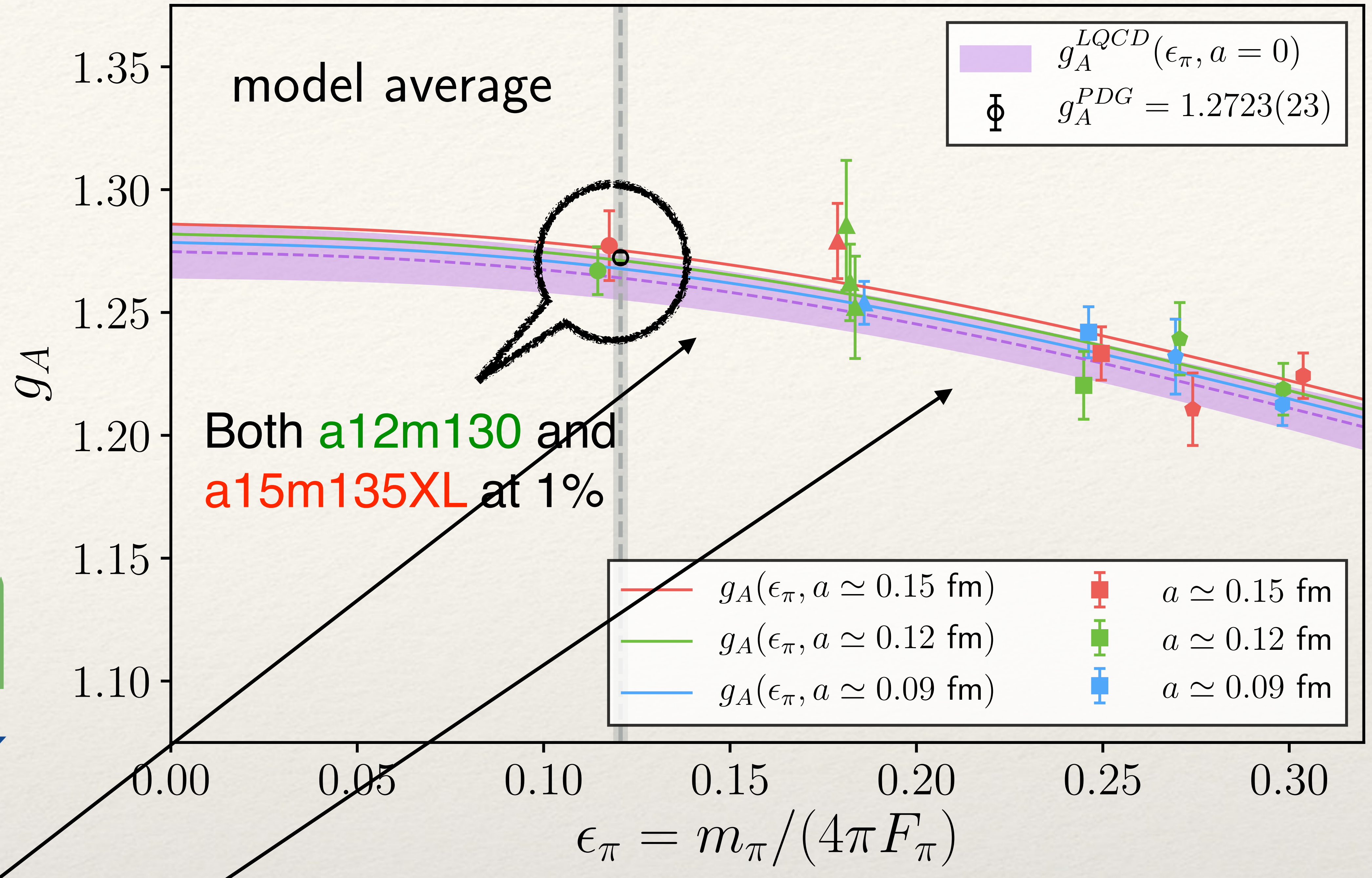

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$$g_A = 1.2711(125) \to 1.2641(93) \quad [0.74\%]$$

- We have 2 additional pion masses (180, 260) and a 4th finer lattice spacing, a≈0.06fm @ $M_\pi \approx 220$, 310 MeV
  - We anticipate improving g_A to ~0.5% — we need to address the radiative QED correction to make this useful

Radiative QED corrections to Neutron Beta Decay

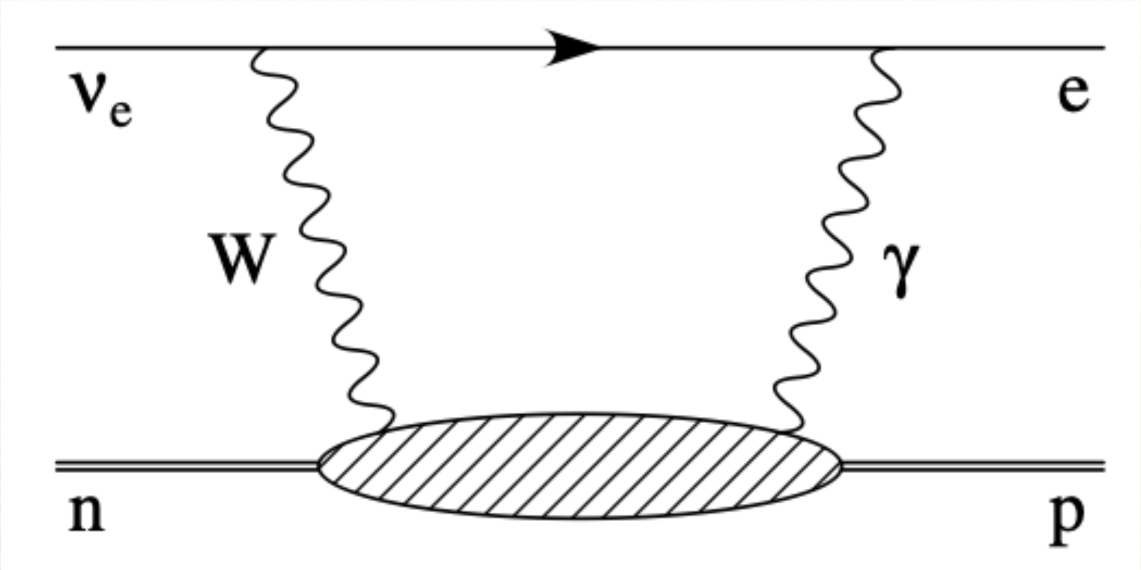
- Radiative QED corrections to beta decay are well understood
  Kinoshita & Sirlin, Phys. Rev. 113 (1959)
  Berman Phys. Rev. 112 (1958)
  Berman & Sirlin Annals Phys. 20 (1962)
  …
  Czarnecki, Marciano & Sirlin (2018)
  Seng, Gorchtein, Patel & Ramsey-Musolf, PRL 121 (2018) [1807.10197]
  Seng, Gorchtein & Ramsey-Musolf, PRD 100 (2019) [1812.03352]
  Gorchtein, PRL 123 (2019) [1812.04229]
  Czarnecki, Marciano & Sirlin, PRD 100 (2019) [1907.06737]
  Hayen & Young (2020) [2009.11364]
  Hayen, PRD 103 (2021) [2010.07262]
  Gorchtein, Seng, JHEP 53 (2021) [2106.09185]
  …
  Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

Pion-induced radiative corrections to neutron beta-decay

Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

- Previous determinations of radiative corrections utilize current algebra, dispersive methods, …

  they identify *vertex corrections* and the γW box corrections



- The vertex correction is proportional to the 3-point correlation function

$$\mathcal{D}_\gamma = \int \frac{d^4k}{k^2} \int d^4y e^{i\bar{q}y} \int d^4x e^{ikx} \times \langle p_f | T \left\{ \partial_\mu J_W^\mu(y) J_\gamma^\lambda(x) J_\lambda^\gamma(0) \right\} | p_i \rangle$$

  - At large momentum - the Operator Product Expansion can be used and $\mathcal{D}_\gamma^{\mathrm{OPE}} = 0$
  - For more general momenta, the integral is saturated (and approximated) by on-shell nucleons with elastic form factors, finding $\mathcal{D}_\gamma \approx 0$
  - We used low-energy Effective Field Theory (EFT) - χPT - to determine the *nucleon structure* corrections

Pion-induced radiative corrections to neutron beta-decay

Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

- Systematic, EFT treatment of neutron β-decay

$$\frac{d\Gamma}{dE_e d\Omega_e d\Omega_\nu} = \frac{(G_F V_{ud})^2}{(2\pi)^5}(1+3\lambda^2)w(E_e)$$

$$\times\left[1+\bar{a}(\lambda)\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu}+\bar{A}(\lambda)\frac{\vec{\sigma}_n\cdot\vec{p}_e}{E_e}+\ldots\right]$$

The parameters can be measured

If we want to connect them to Standard Model (SM) parameters
we need to start from a Lagrangian with parameters related to SM parameters

pion-less low-energy EFT

$$\lambda = \frac{g_A}{g_V}$$

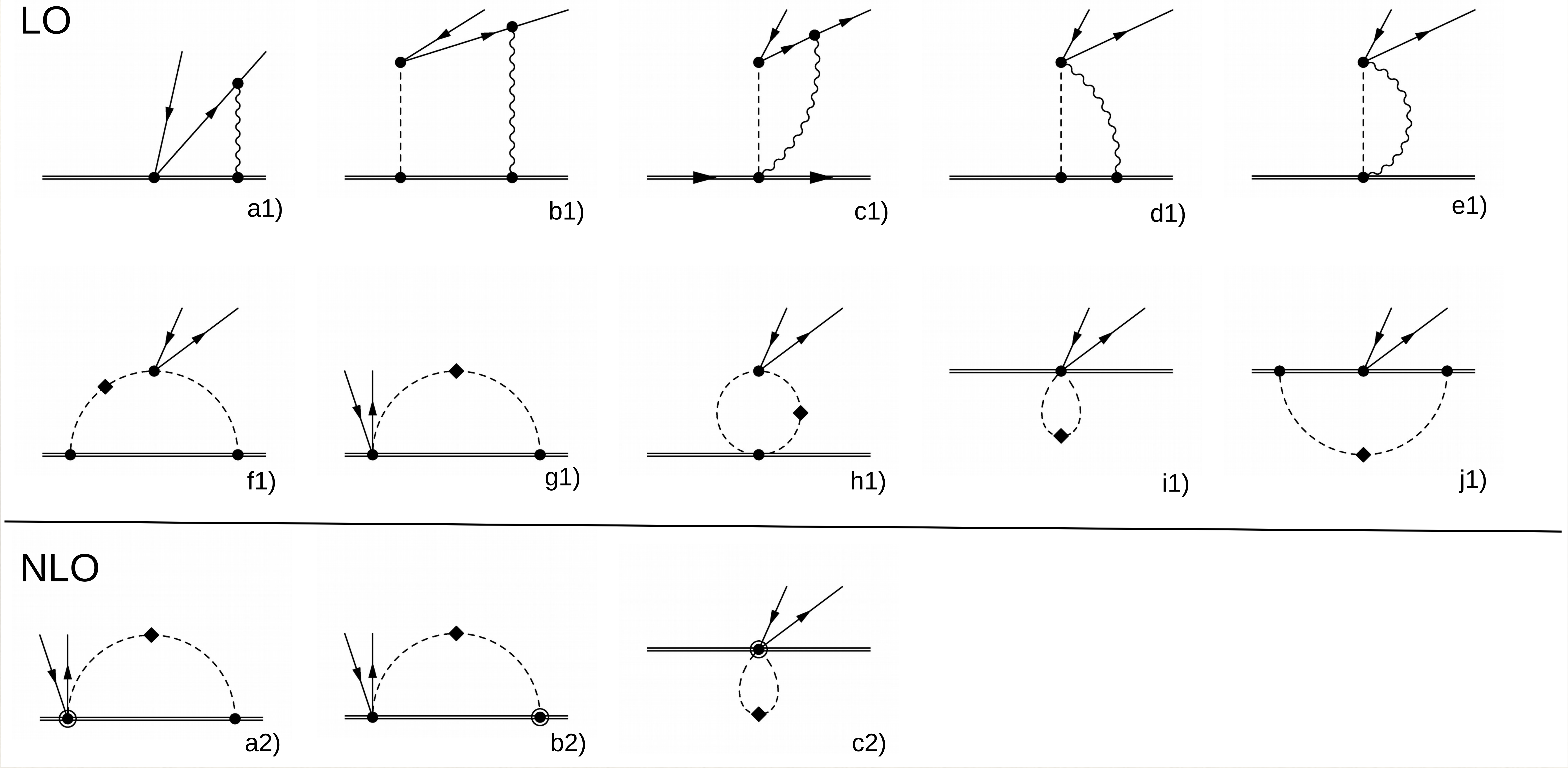
$$\begin{aligned}\mathcal{L}_{\not\pi} = -\sqrt{2}G_F V_{ud}\Big[&\bar{e}\gamma_\mu P_L \nu_e\Big(\bar{N}\left(g_V v_\mu - 2g_A S_\mu\right)\tau^+ N \\ &+\frac{i}{2m_N}\bar{N}(v^\mu v^\nu - g^{\mu\nu} - 2g_A v^\mu S^\nu)(\overleftarrow{\partial}-\overrightarrow{\partial})_\nu \tau^+ N\Big) \\ &+\frac{ic_T m_e}{m_N}\bar{N}\left(S^\mu v^\nu - S^\nu v^\mu\right)\tau^+ N\ \left(\bar{e}\sigma_{\mu\nu}P_L\nu\right) \\ &+\frac{i\mu_{\text{weak}}}{m_N}\bar{N}[S^\mu,S^\nu]\tau^+ N\,\partial_\nu\left(\bar{e}\gamma_\mu P_L\nu\right)\Big]+\ldots \qquad (2)\end{aligned}$$

Perform the calculation with SU(2) heavy-baryon χPT and match the results to this pion-less EFT whose parameters can be matched to experimentally measured quantities

Pion-induced radiative corrections to neutron beta-decay

Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

- Sub-set of O(50) diagrams



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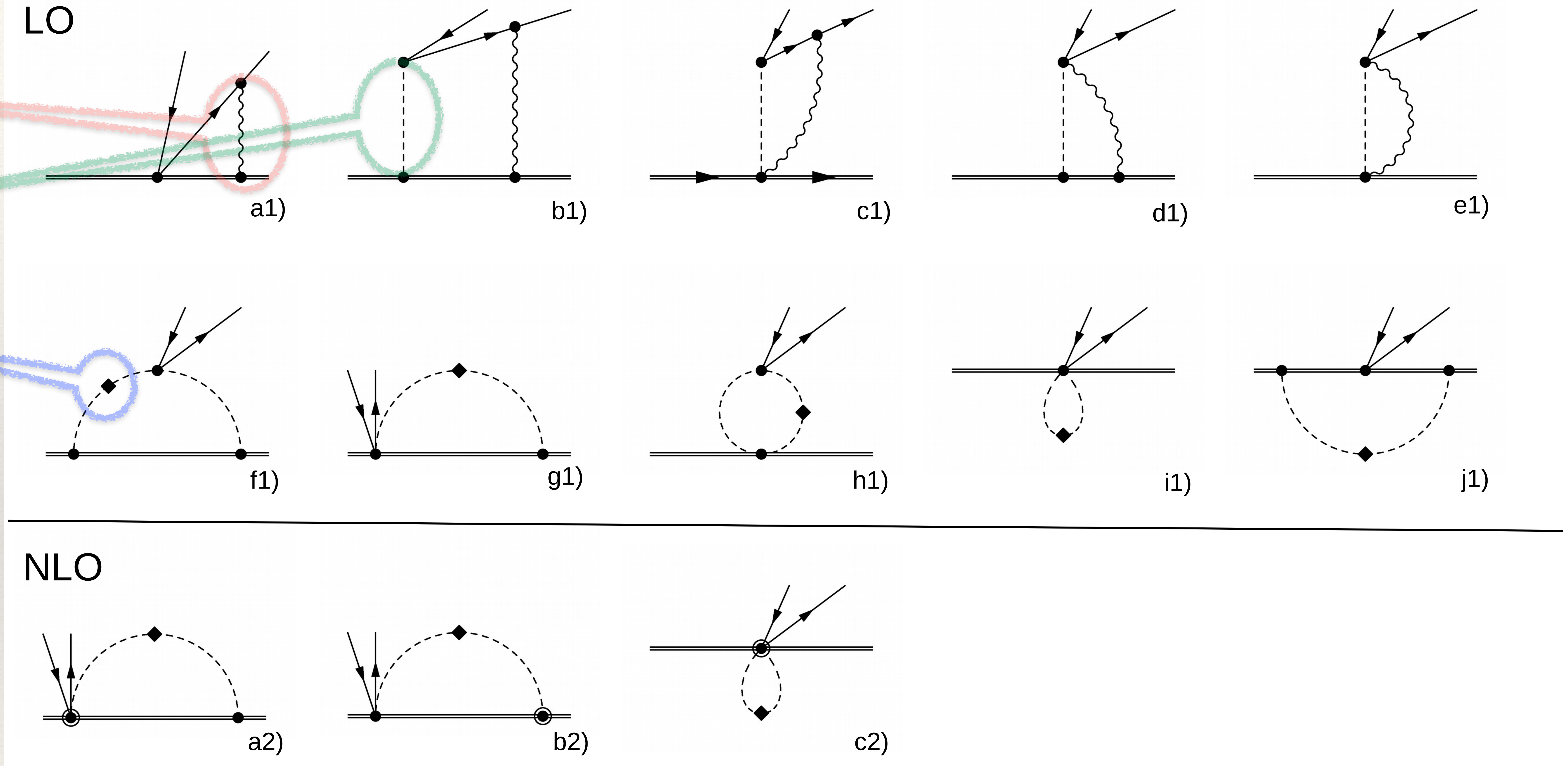
photons

pions

pion electromagnetic mass splitting

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 = 2e^2 F_\pi^2 Z_\pi$$



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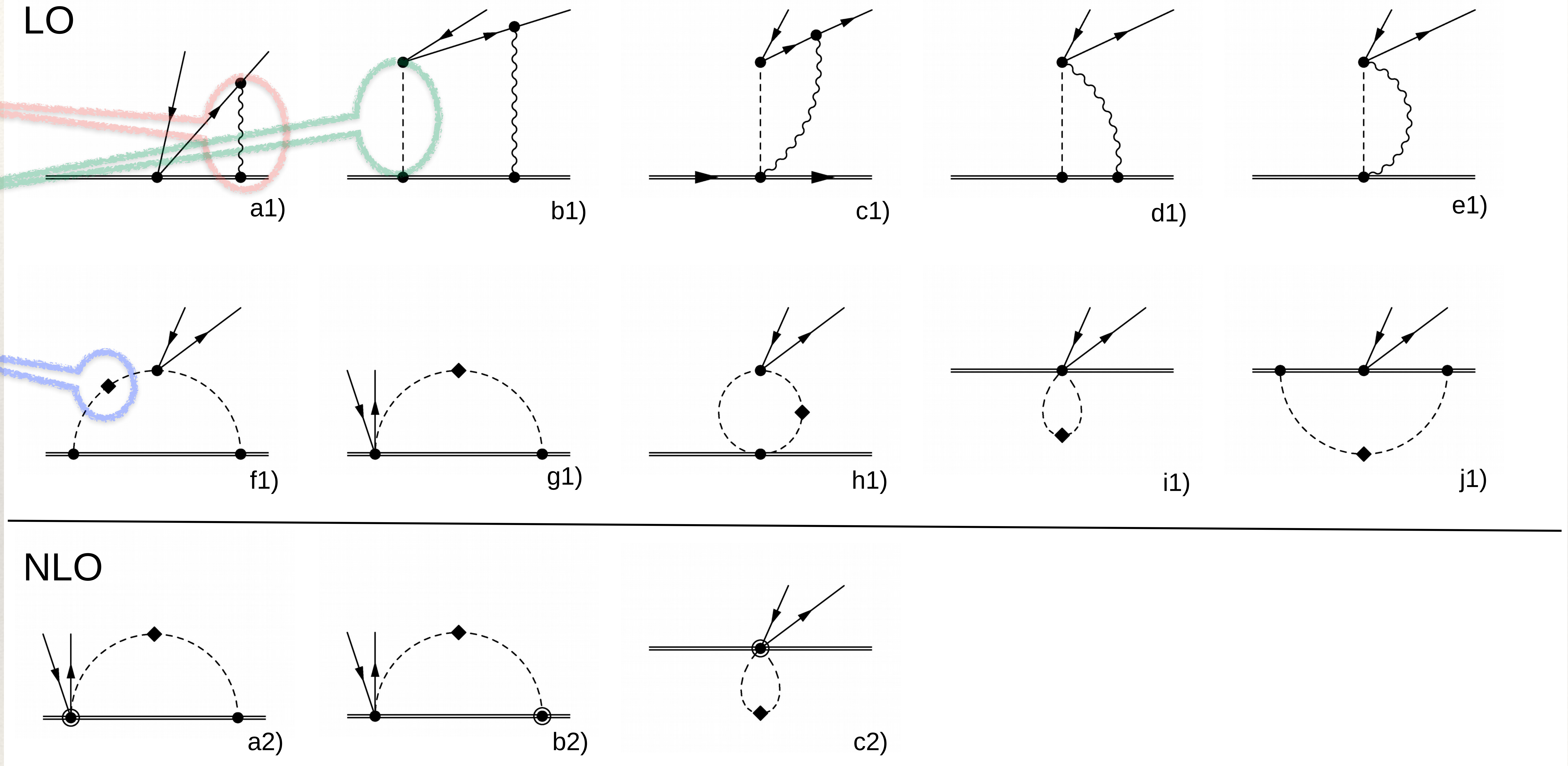

NOTE: at this order, we also include QED, m_d-m_u corrections to M_n-M_p

- iso-vector contributions to M_n-M_p vanish from symmetry constraints for τ^+ current
- iso-scalar contributions do not vanish - but the sum of all of them does vanish through NLO

Pion-induced radiative corrections to neutron beta-decay

Cirigliano, de Vries, Hayen, Mereghetti & Walker-Loud, PRL 129 (2022) [2202.10439]

- Matching $\lambda = g_A^{\mathrm{QCD}}\left(1+\delta_{\mathrm{RC}}^{(\lambda)} - 2\mathrm{Re}(\epsilon_R)\right)$ $\quad \delta_{\mathrm{RC}}^{(\lambda)} = \frac{\alpha}{2\pi}\left(\Delta_{A,\mathrm{em}}^{(0)} + \Delta_{A,\mathrm{em}}^{(1)} - \Delta_{V\mathrm{em}}^{(0)}\right)$

$$g_{V/A} = g_{V/A}^{(0)}\left[1+\sum_{n=2}^{\infty}\Delta_{V/A,\chi}^{(n)} + \frac{\alpha}{2\pi}\sum_{n=0}^{\infty}\Delta_{V/A,\mathrm{em}}^{(n)} + \left(\frac{m_u-m_d}{\Lambda_\chi}\right)^{n_{V/A}}\sum_{n=0}^{\infty}\Delta_{V/A,\delta m}^{(n)}\right]$$

$g_V^{(0)} = 1 \qquad \Delta_{\chi,\mathrm{em},\delta m}^{(n)} \sim O(\epsilon_\chi^n)$

$n_V = 2$ CVC

$n_A = 1$ explicit calculation:

$$\Delta_{A,\delta m}^{(0),(1)} = 0$$
$$\Delta_{V,\delta m}^{(0)} = 0$$

$$\Delta_{A,\mathrm{em}}^{(0)} = Z_\pi\left[\frac{1+3g_A^{(0)2}}{2}\left(\log\frac{\mu^2}{m_\pi^2}-1\right) - g_A^{(0)2}\right] + \hat{C}_A(\mu)$$

Low-Energy-Constants (LECs)

$$\Delta_{A,\mathrm{em}}^{(1)} = Z_\pi\, 4\pi m_\pi\left[c_4 - c_3 + \frac{3}{8m_N} + \frac{9}{16m_N}g_A^{(0)2}\right]$$

$C_A(\mu)$ - completely unknown

c_3 & c_4 are estimated from literature

Using Naive Dimensional Analysis (NDA) to estimate $C_A(\mu)$ and $c_{3,4}$ from the literature

$\delta_{\mathrm{RC}}^{(\lambda)} \in \{1.4,\ 2.6\}\cdot 10^{-2}$ an order of magnitude larger than previous estimates

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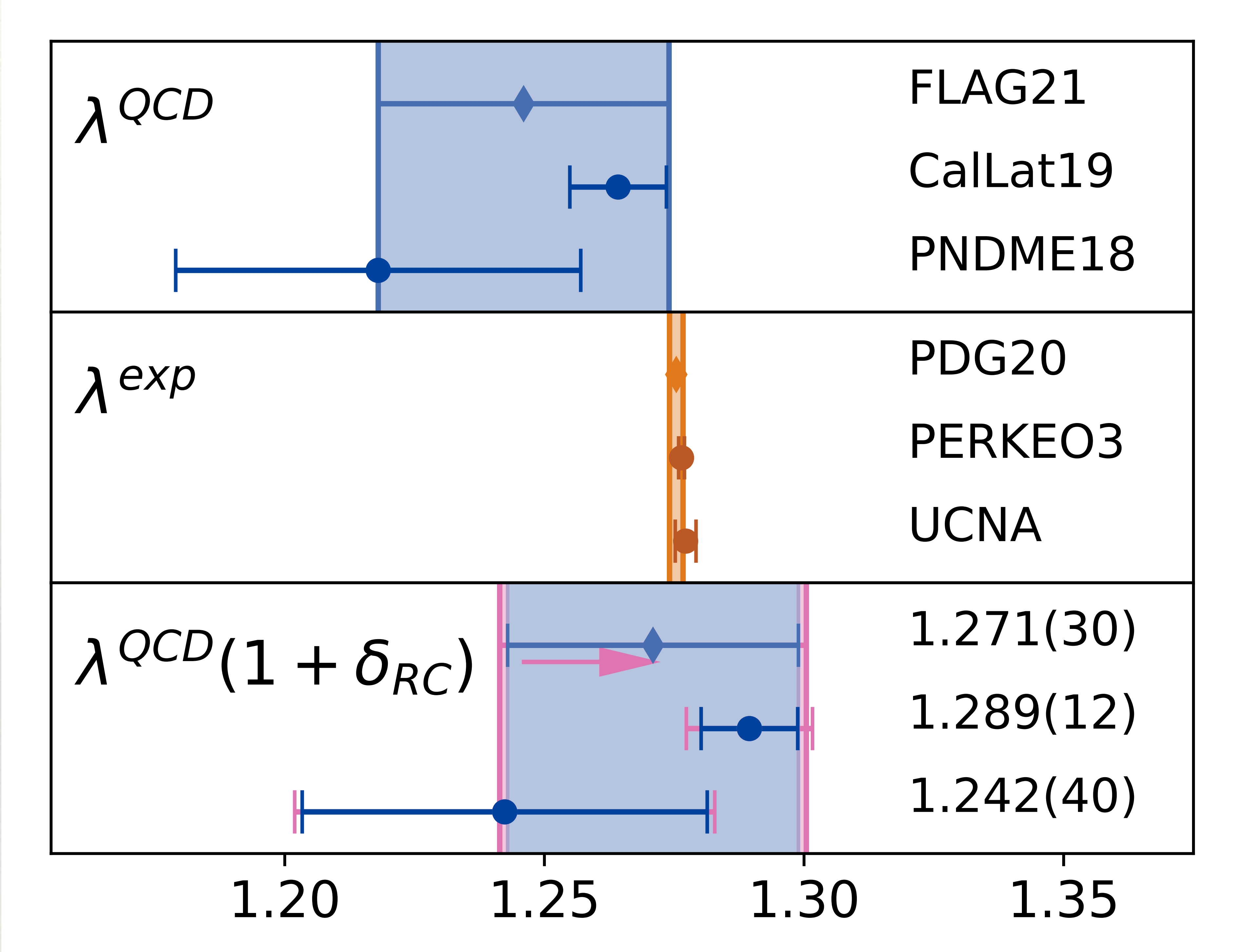
- Comments:
  - This does not affect neutron lifetime/g_A measurements - it enters both experiments in the same way
  - It impacts our ability to compare lattice QCD results to measured values of g_A and constrain BSM RH currents
  - We need lattice QCD + QED to determine the unknown LECs and compare with estimate

    (work in progress)

  - This has led us to wonder if there are similarly sized corrections in other matrix elements
    - Is the full $n \to pe\bar{\nu}$ under control?

      (is computing γ-W box sufficient?)
    - two-nucleon matrix elements, such as pp-fusion



Conclusions and Future Directions

- Lattice QCD is capable of delivering precise (sub-percent) values of isospin symmetric g_A
  - We are waiting for other groups to achieve the same level of precision as us - next best is 3%
  - In the meantime, we have 0.74%
  - We anticipate publishing at O(0.5%) next year
  - I believe LQCD can reach 0.1 - 0.2% for isospin symmetric g_A in the ~5-year time-scale

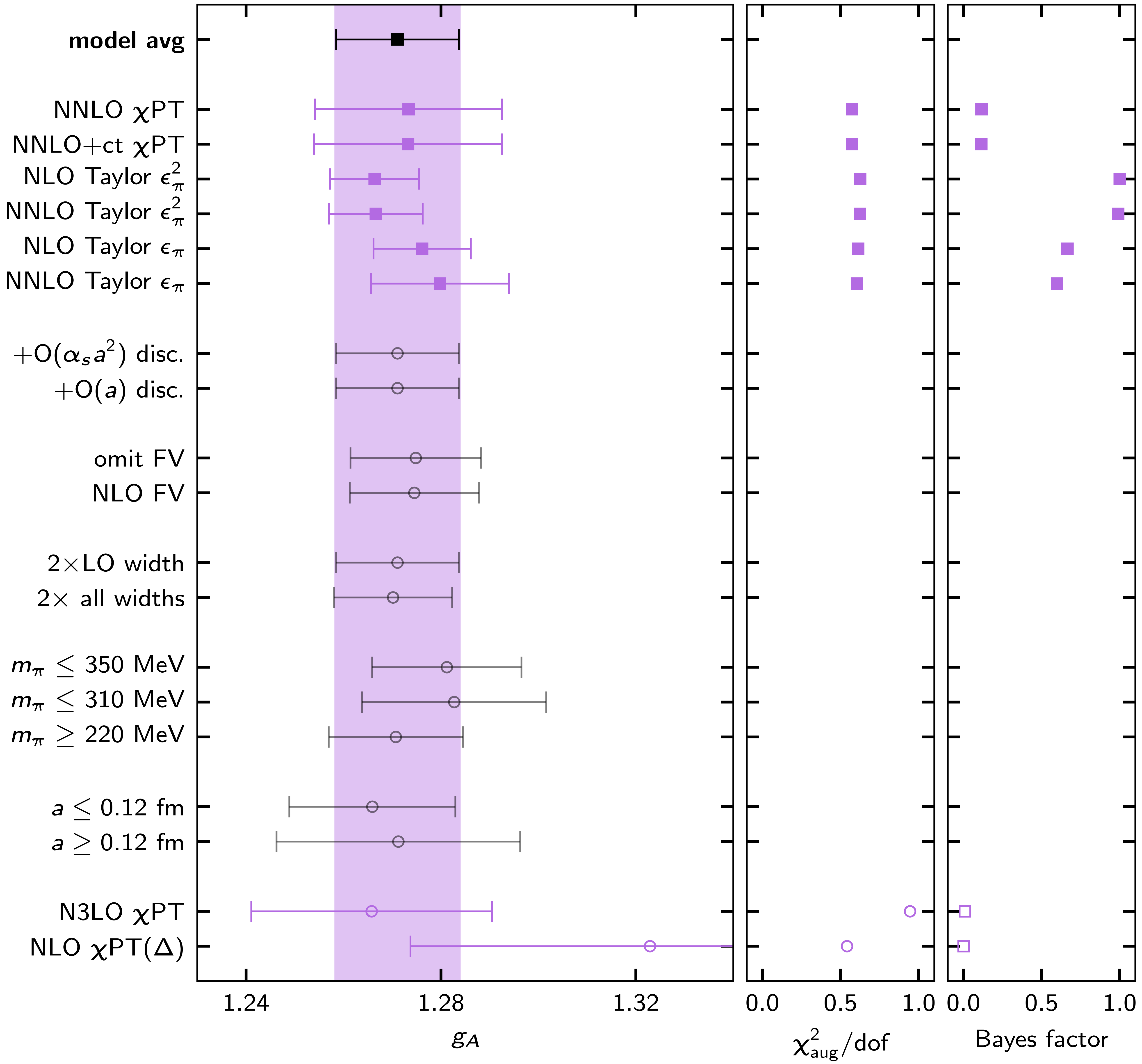
- To make use of this precision, we must determine the QED corrections to g_A
  - We believe this is possible by isolating the hadronic contributions
  - Requires an understanding of the renormalization — and a choice of gauge in XPT/LQCD
  - If the QED corrections are controllable - we can add/subtract them from LQCD/experiment to make use of the higher precision iso-symmetric result

- If the first-row CKM unitarity tension is at 4+ sigma
  - We will want a full lattice QCD+QED calculation of $n \to p e \bar{\nu}$
  - Feng, Gorchtein, Jin, Seng, et al are making very exciting progress in their γ-W box calculations

Thank You

Analysis Details

Stability of Extrapolation Analysis



Final result

6 fits included in the model average and their relative weights

discretization corrections

finite volume corrections

sensitivity to prior width

sensitivity to pion mass cuts

sensitivity to lattice spacing cuts

additional XPT analysis

Analysis Details

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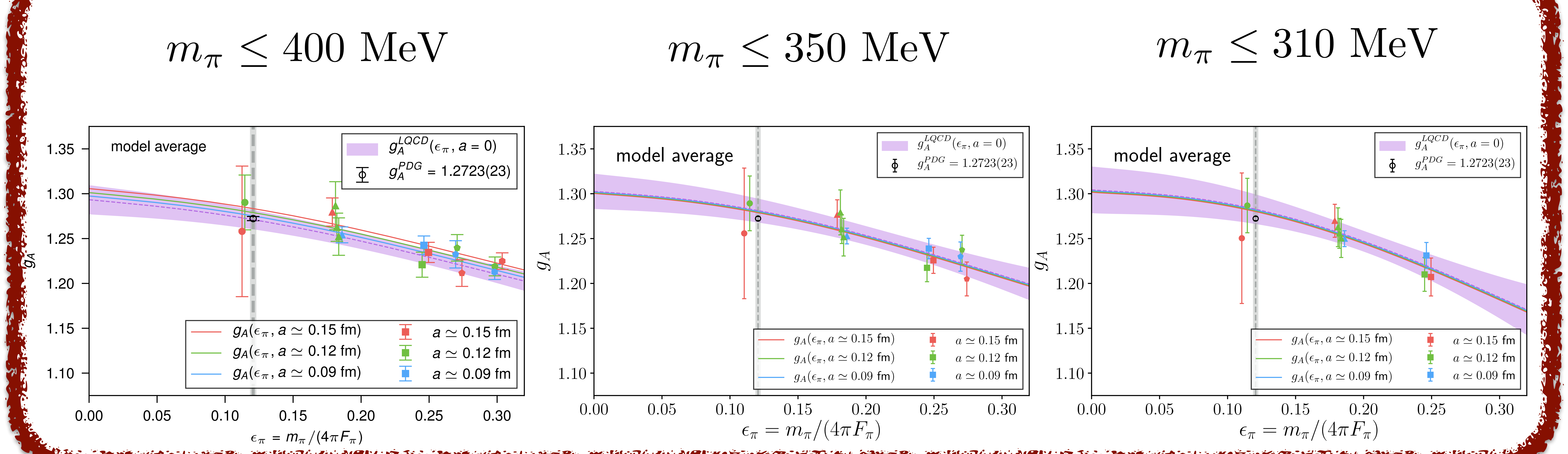

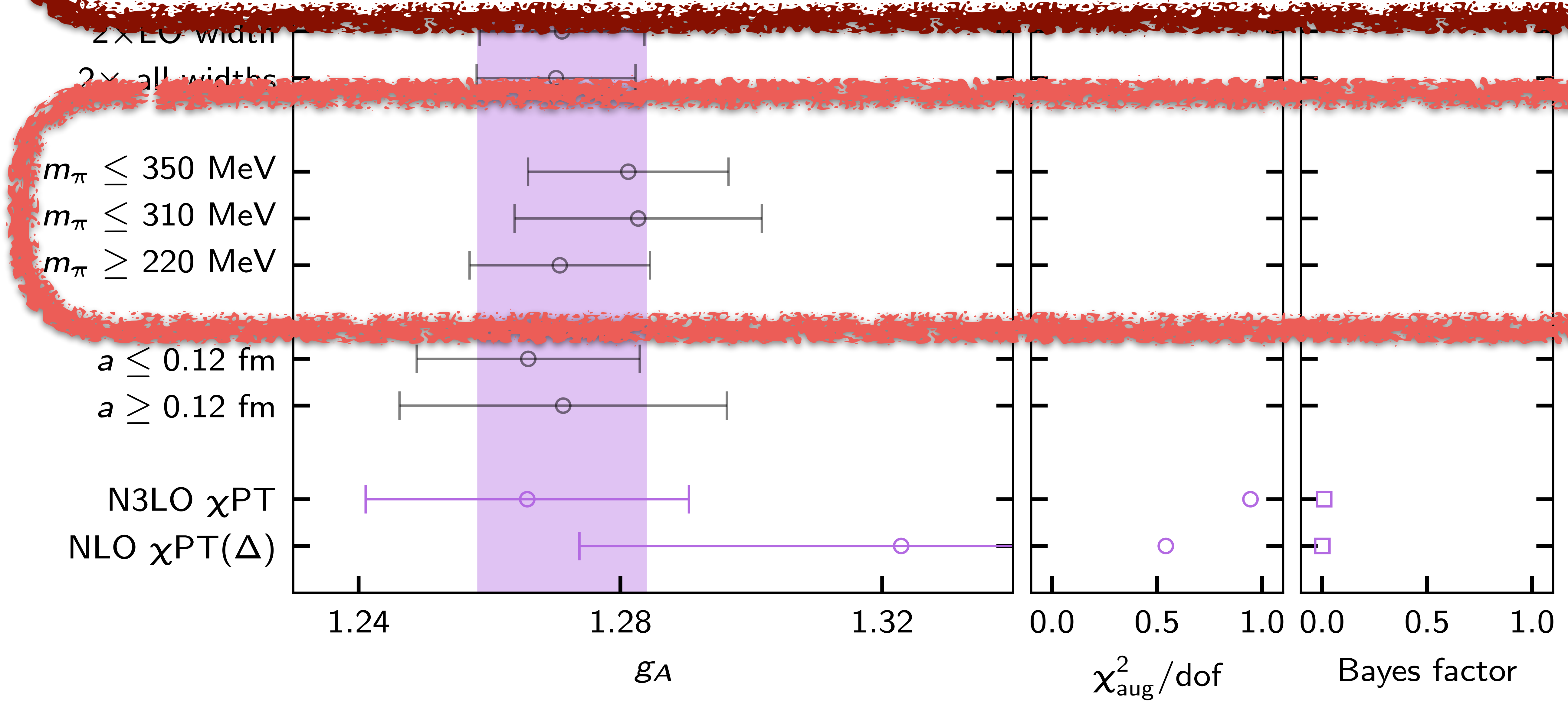

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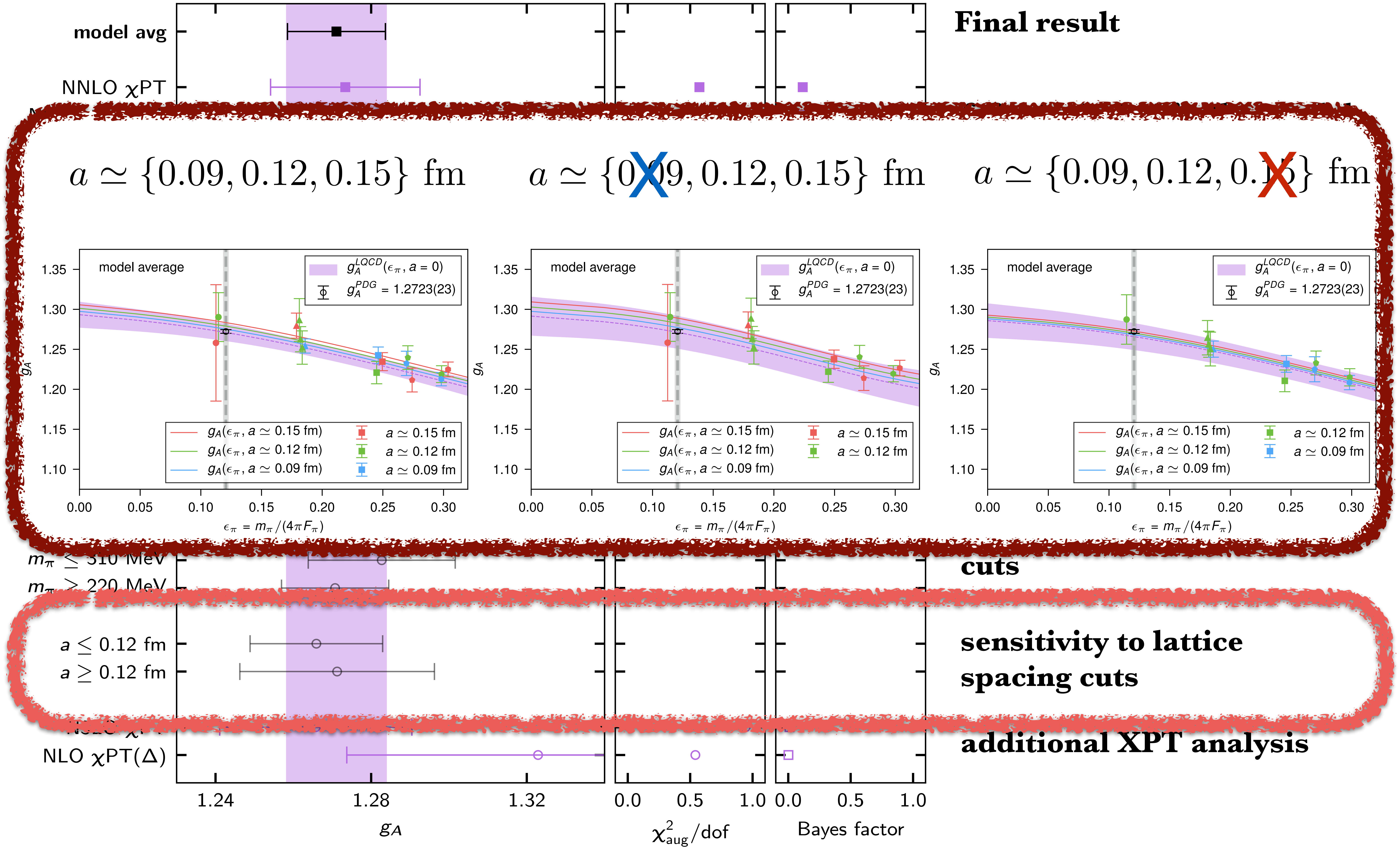
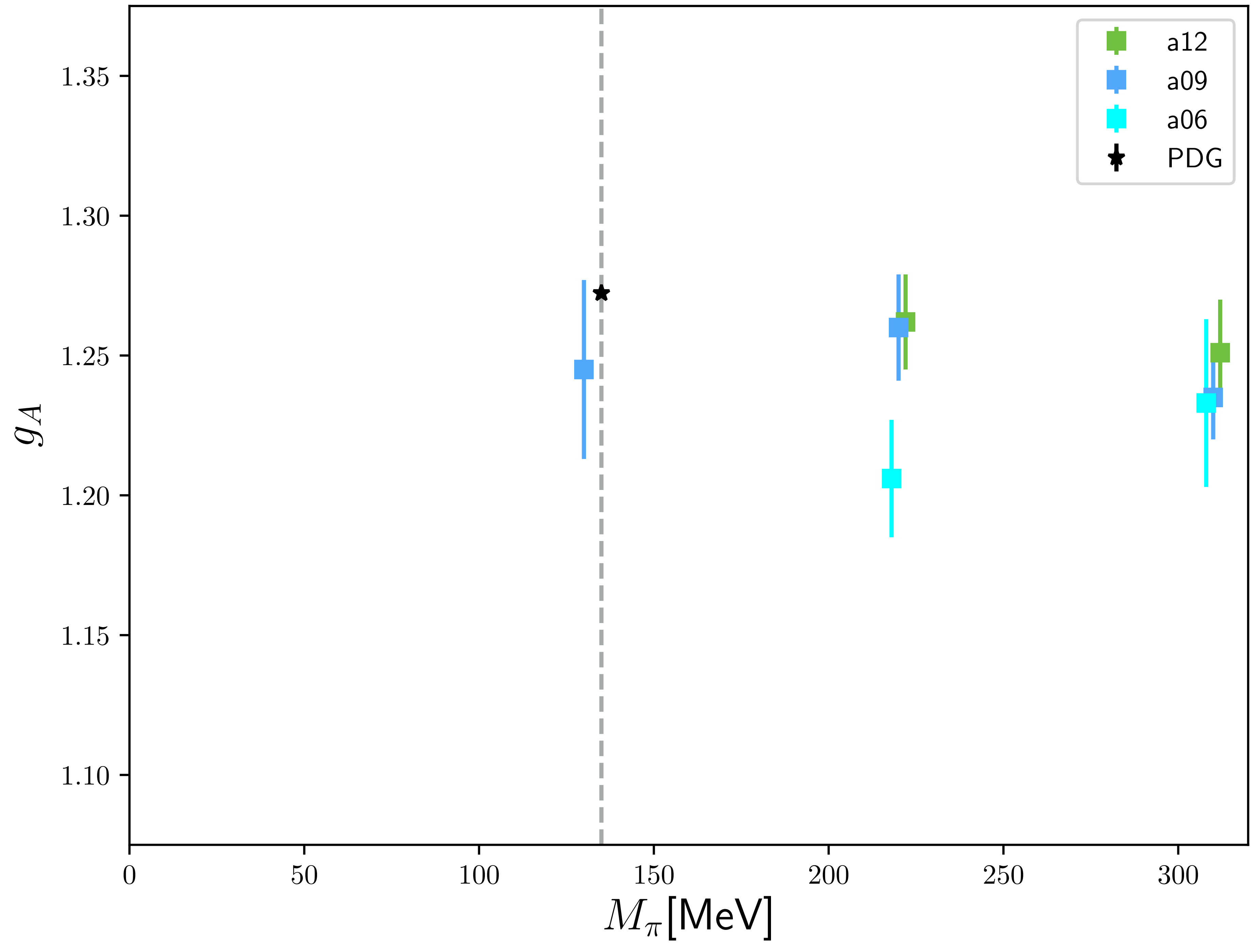


Comparison with other LQCD results

- *The difference in the continuum extrapolation is driven by the smaller estimates on all three fine $a \approx 0.06$fm ensembles*



arXiv:1606.07049

$g_A = 1.195(33)(22)$

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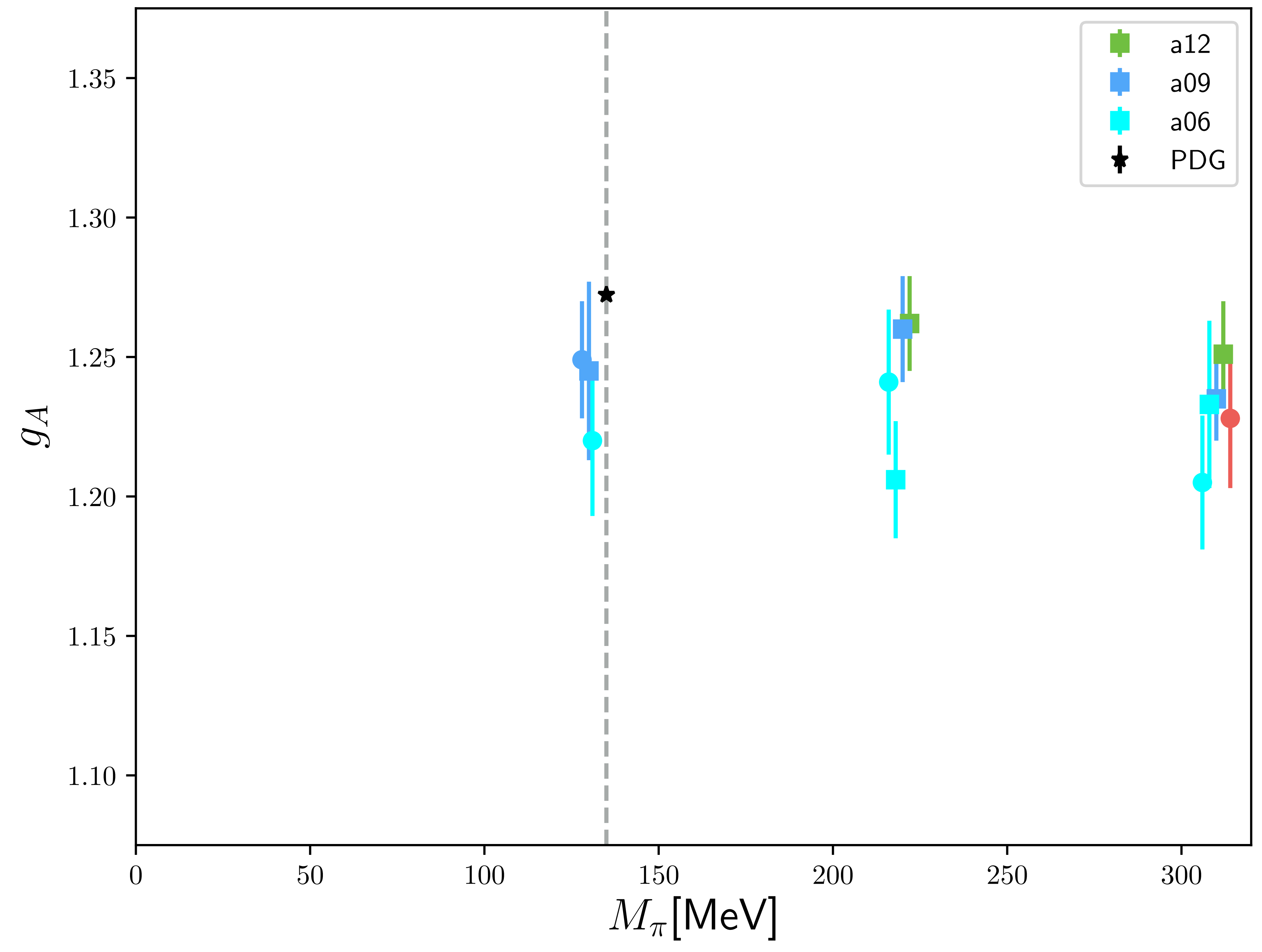

arXiv:1606.07049 $g_A = 1.195(33)(22)$

arXiv:1806.09006 $g_A = 1.218(25)(30)$

g_A[no a06] $= 1.245(42)(xx)$

- A change in quark smearing caused a ~2 sigma shift in the a06m220 point
  - The correlated difference will be ~5-sigma
  - suggests an underestimate of systematic uncertainties on this ensemble
- In conferences - it has been stated it is only the physical pion mass a06m135 point that causes the discrepancy - **note - this is the very endpoint result in an extrapolation**

Comparison with other LQCD results

- *The difference in the continuum extrapolation is driven by the smaller estimates on all three fine $a \approx 0.06$fm ensembles*
- "Surely, You're Joking, Mr. Feynman!"

  I went out and found the original article on the experiment that said the neutron-proton coupling is T, and I was shocked by something. I remembered reading that article once before (back in the days when I read every article in the Physical Review—it was small enough). And I remembered, when I saw this article again, looking at that curve and thinking, "That doesn't prove anything!"

  You see, it depended on one or two points at the very edge of the range of the data, and there's a principle that a point on the edge of the range of the data—the last point—isn't very good, because if it was, they'd have another point further along. And I had realized that the whole idea that neutron-proton coupling is T was based on the last point, which wasn't very good, and therefore it's not proved. I remember noticing that!

  And when I became interested in beta decay, directly, I read all these reports by the "beta-decay experts," which said it's T. I never looked at the original data; I only read those reports, like a dope. Had I been a good physicist, when I thought of the original idea back at the Rochester Conference I would have immediately looked up "how strong do we know it's T?"—that would have been the sensible thing to do. I would have recognized right away that I had already noticed it wasn't satisfactorily proved.

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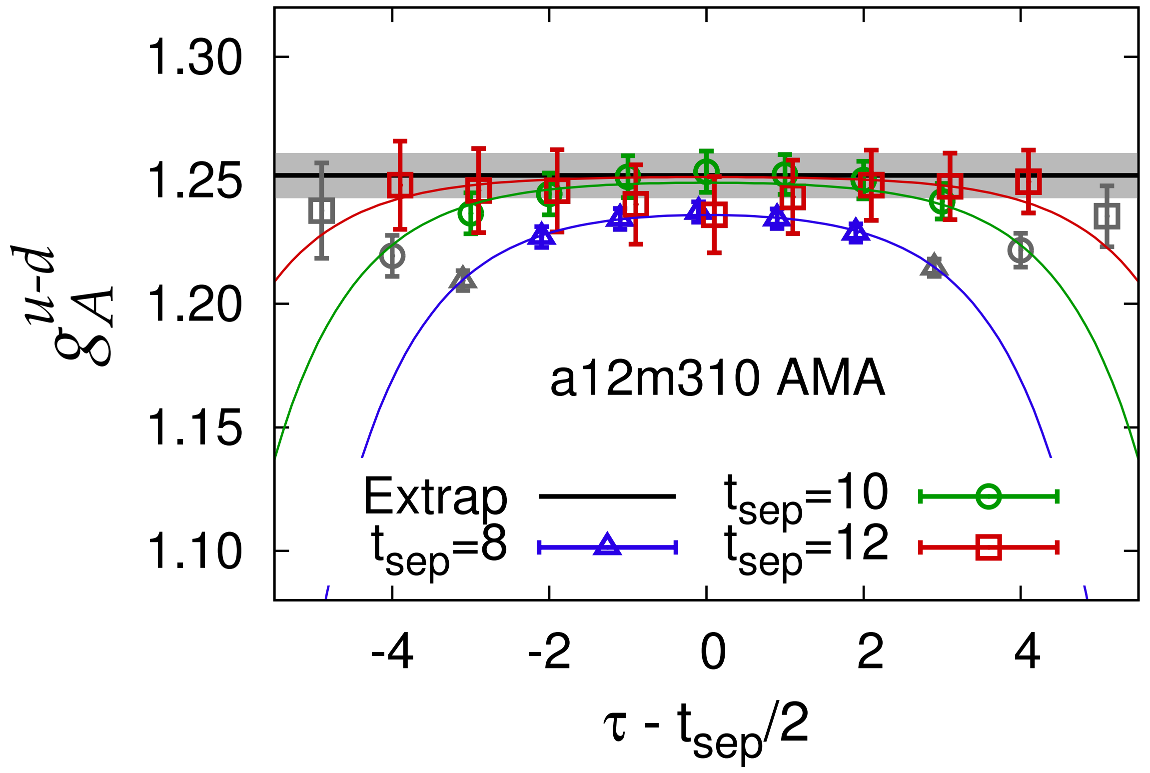
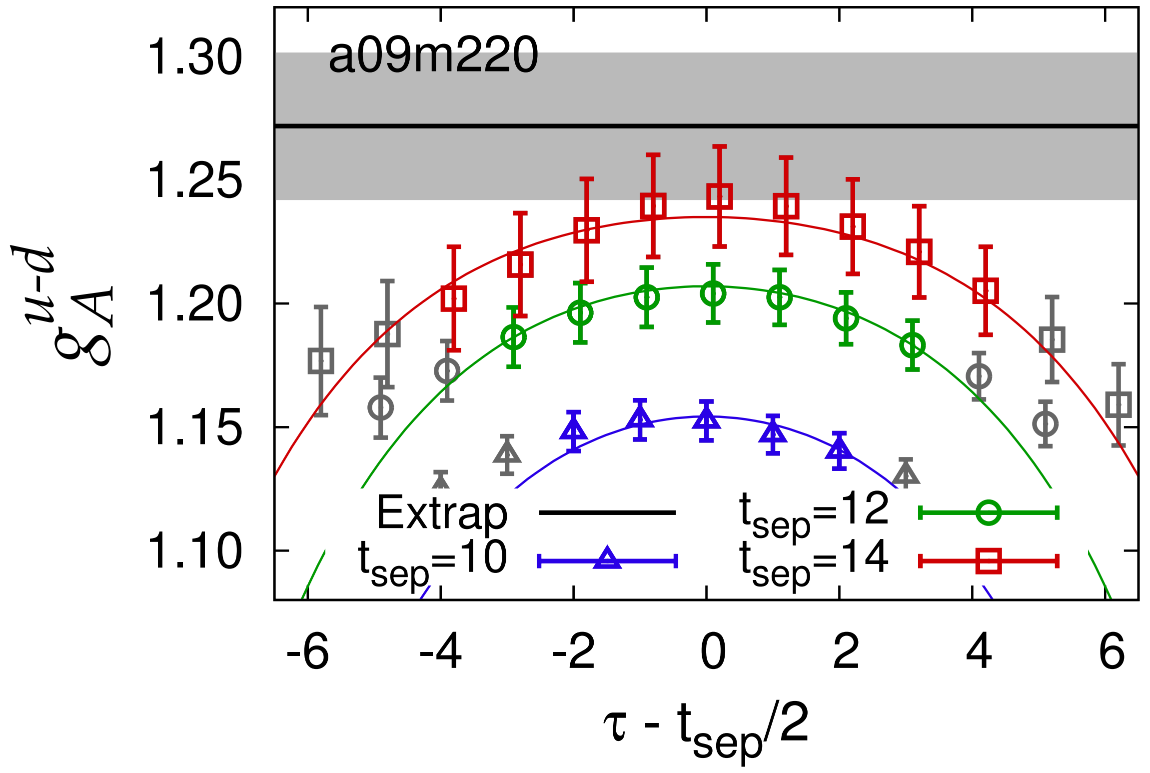


0.49 fm

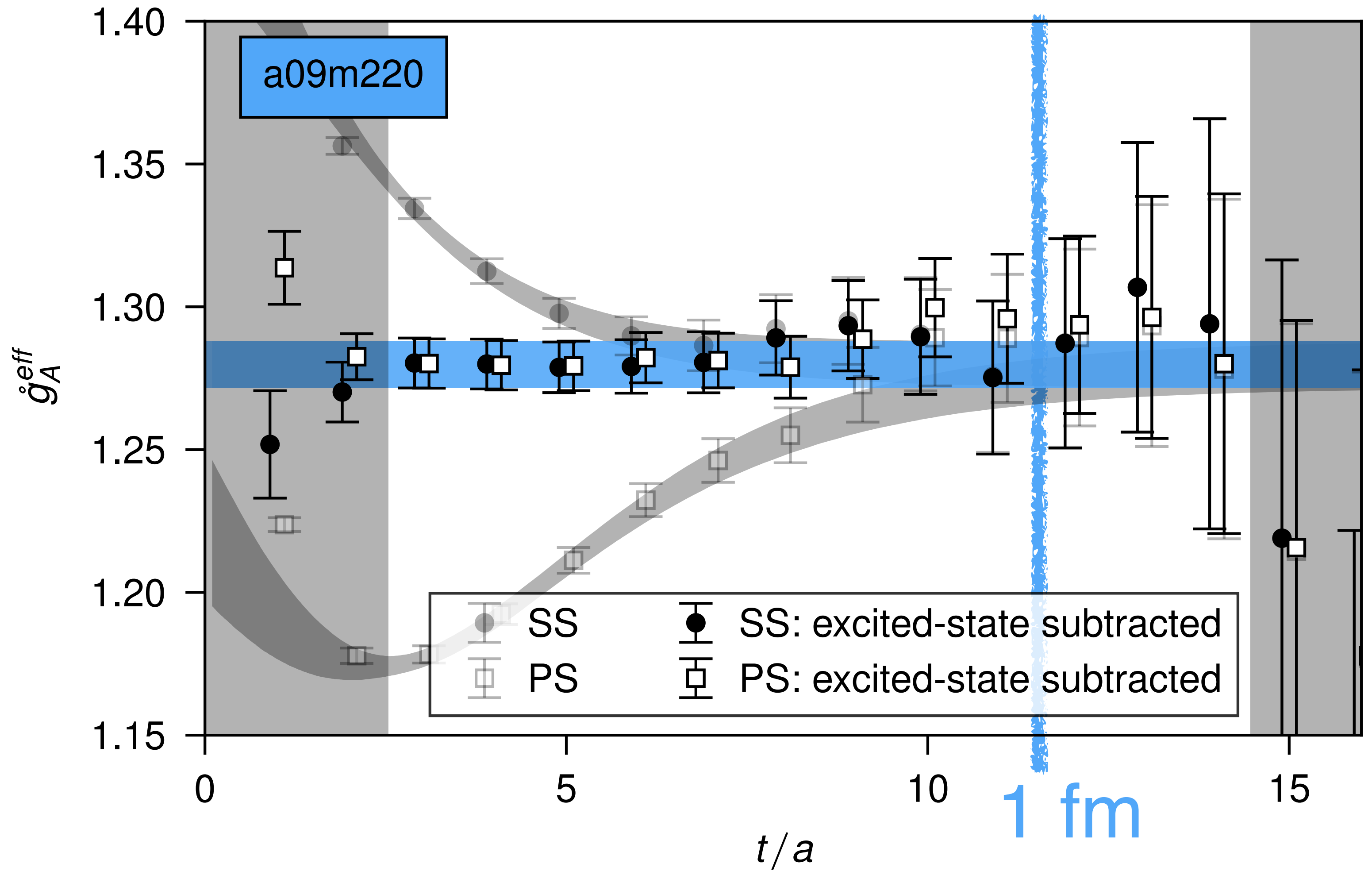
lever arm to extrapolate to infinite $\mathbf{t_{sep}}$

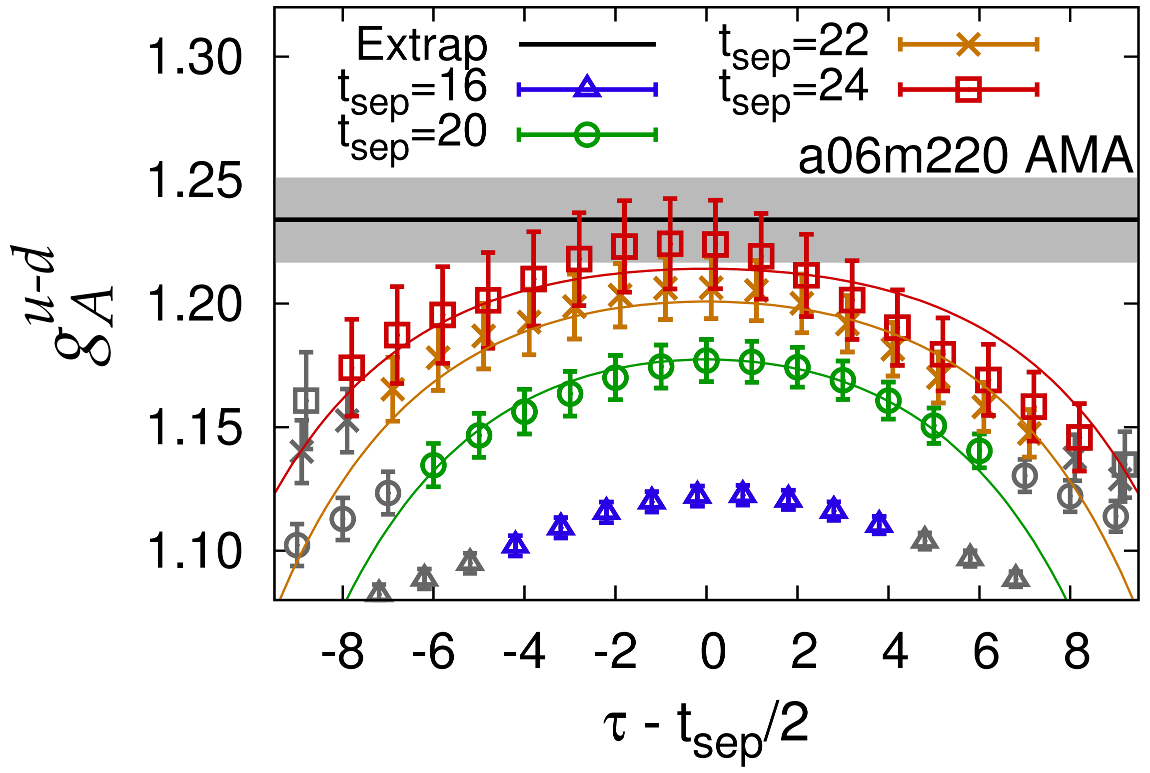
$$R_3 = g_\lambda + z_1 e^{-t_{sep}\Delta_{10}} + z_{10} e^{-(\tau - t_{sep}/2)\Delta_{10}} + \dots$$

from curvature at fixed $\mathbf{t_{sep}}$



0.35 fm





0.23 fm

Also, the a06 results, from time-slice to time-slice, are more correlated than the other ensembles, and therefore more susceptible to correlated fluctuations that will fool the eye (analysis)

Comparison with other LQCD results

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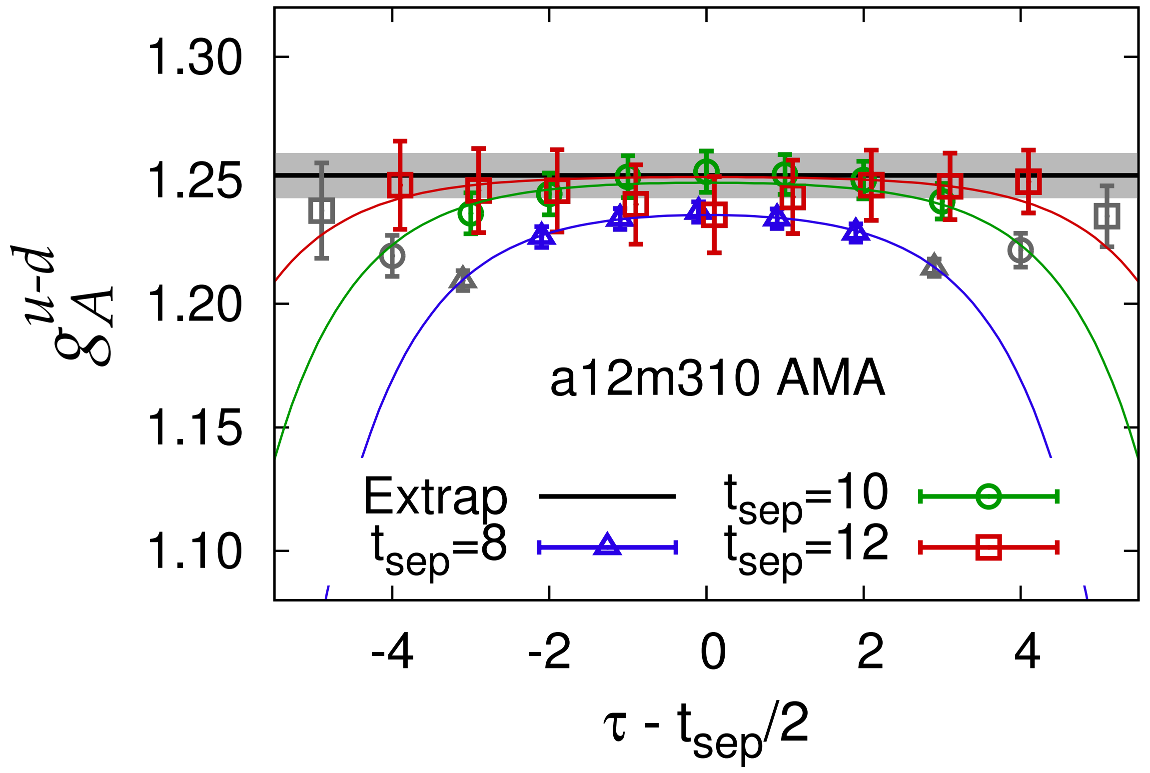

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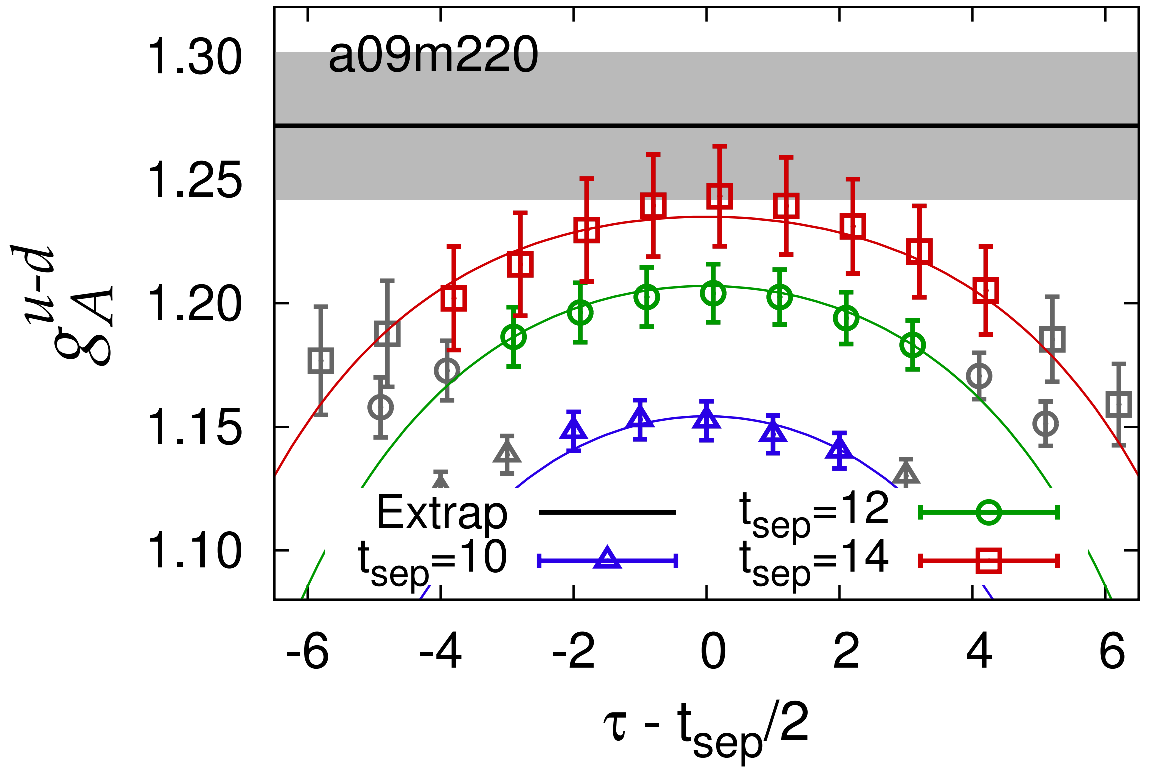

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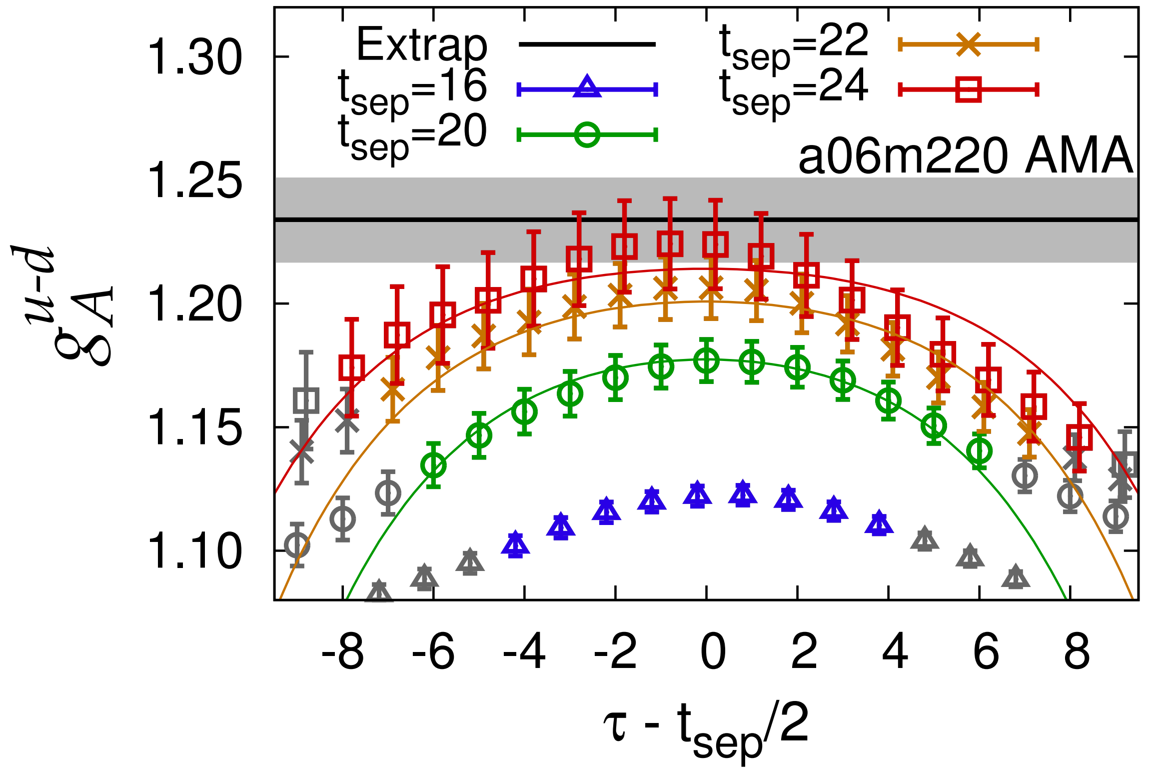

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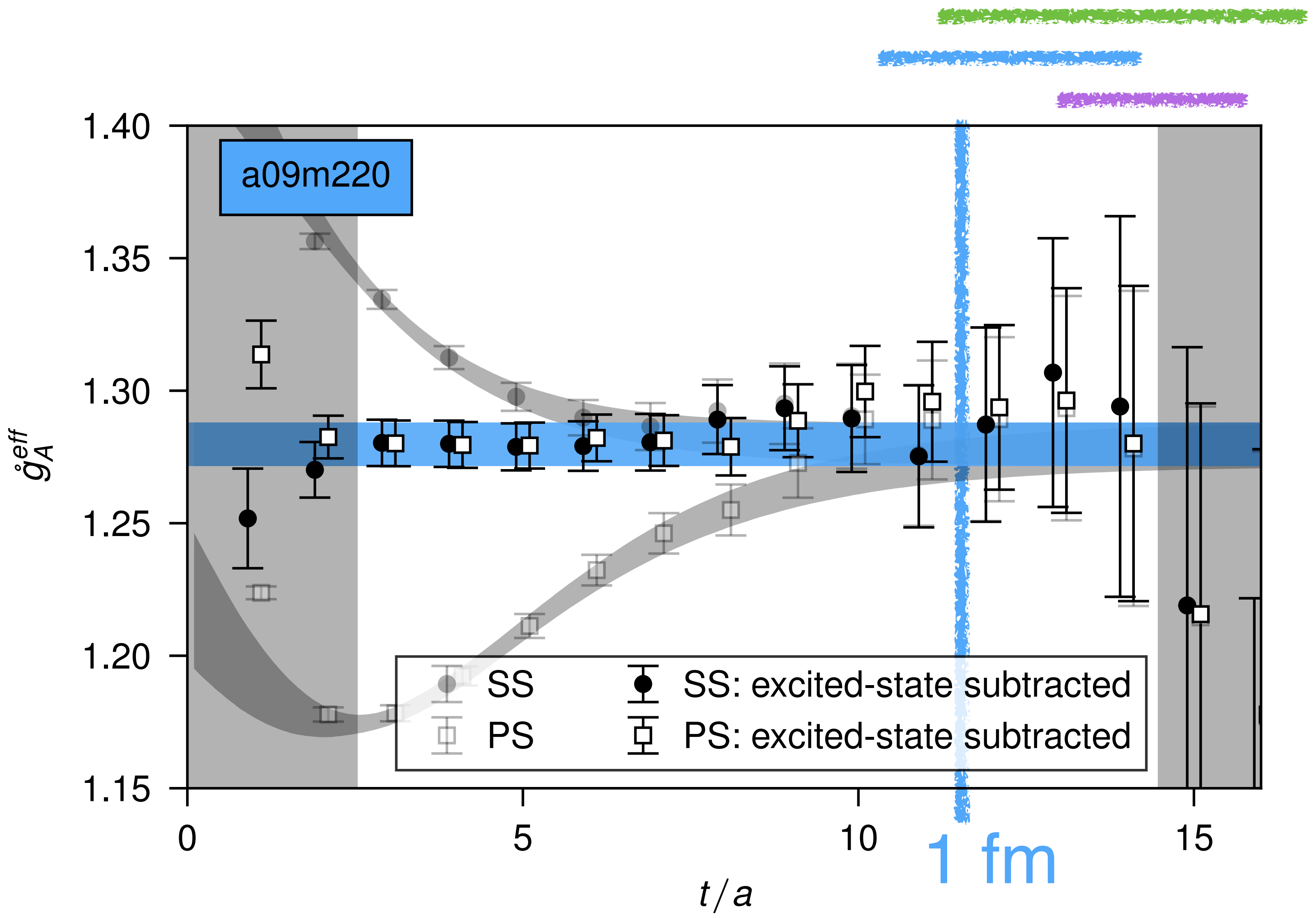

from curvature at fixed $\mathbf{t_{sep}}$

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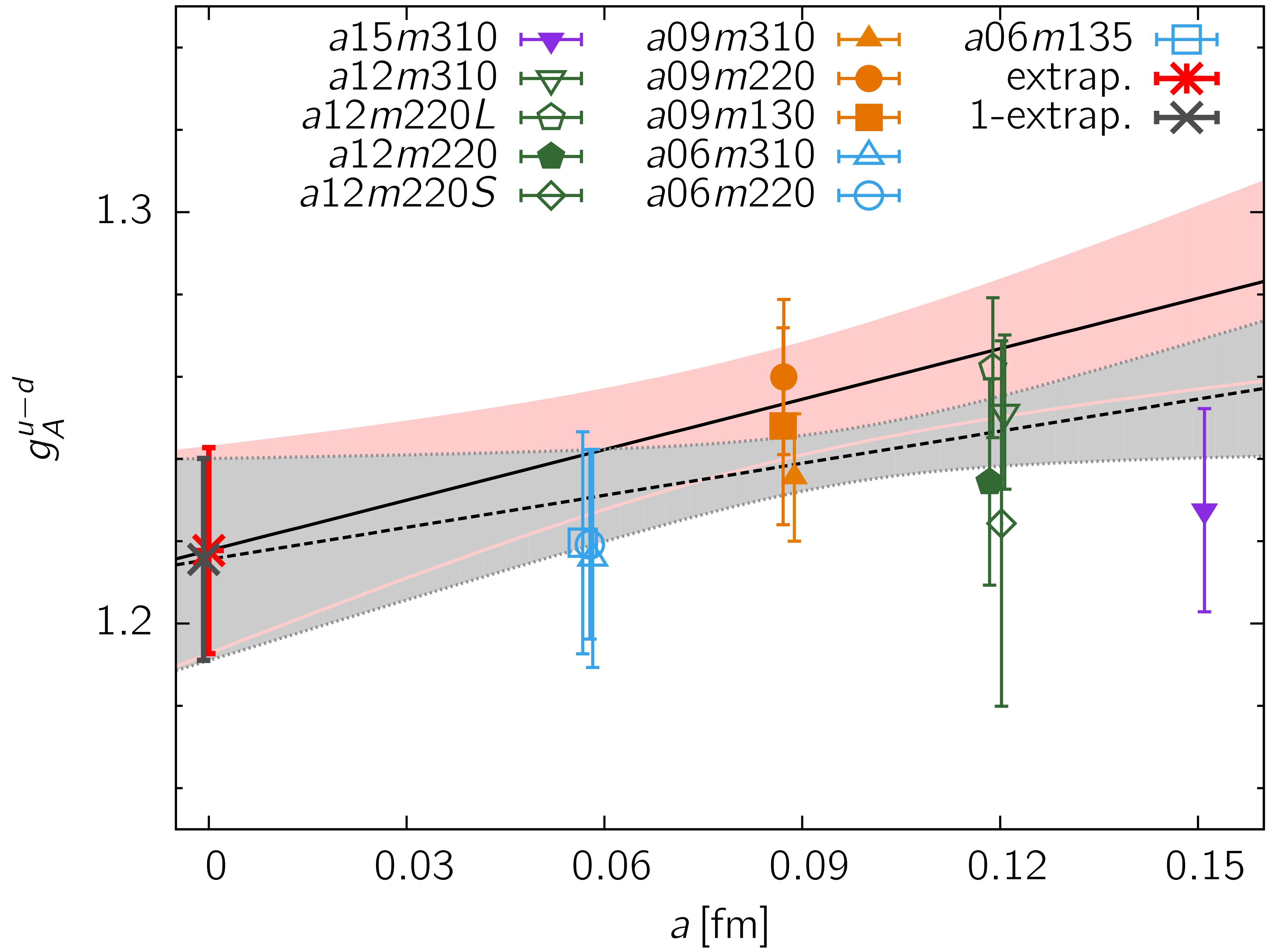
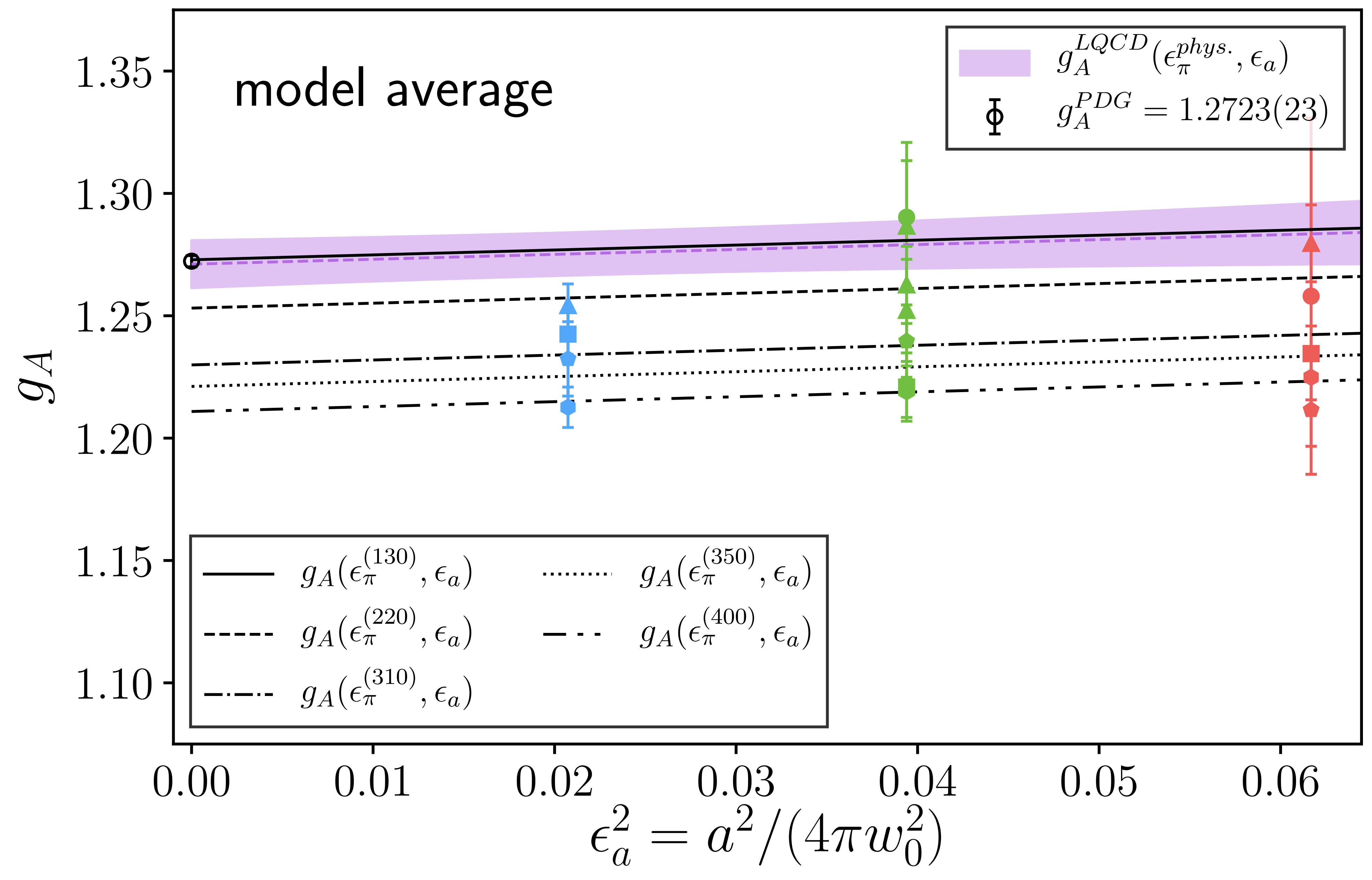
Comparison with other LQCD results

- Suppose the results in arXiv:1806.09006 are correct (not biased by a systematic uncertainty) What are the implications?



arXiv:1806.09006



arXiv:1805.12130

- **Either**
  - There is significant new physics in g_A
  - The continuum extrapolation would follow a dramatic curve

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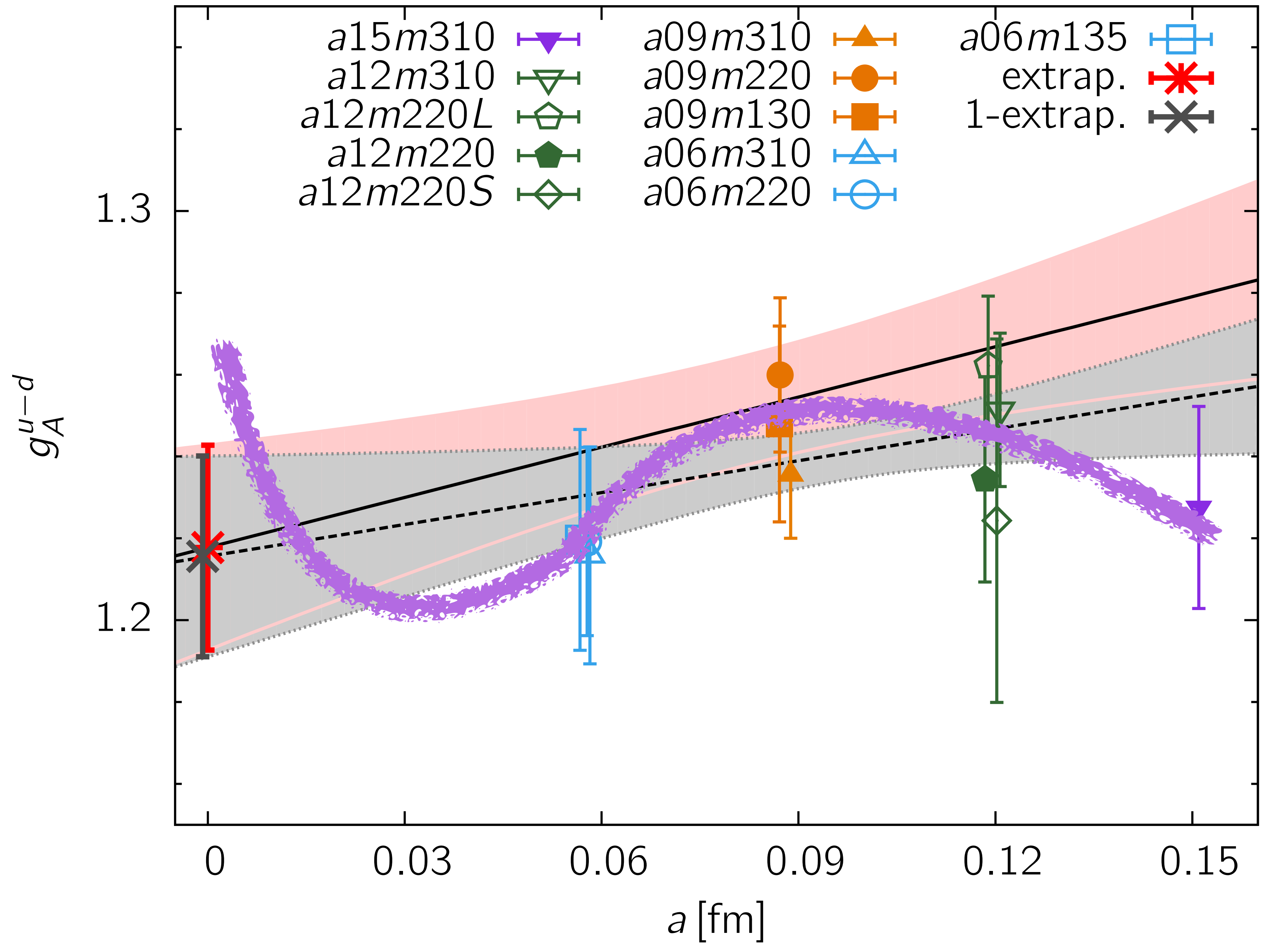

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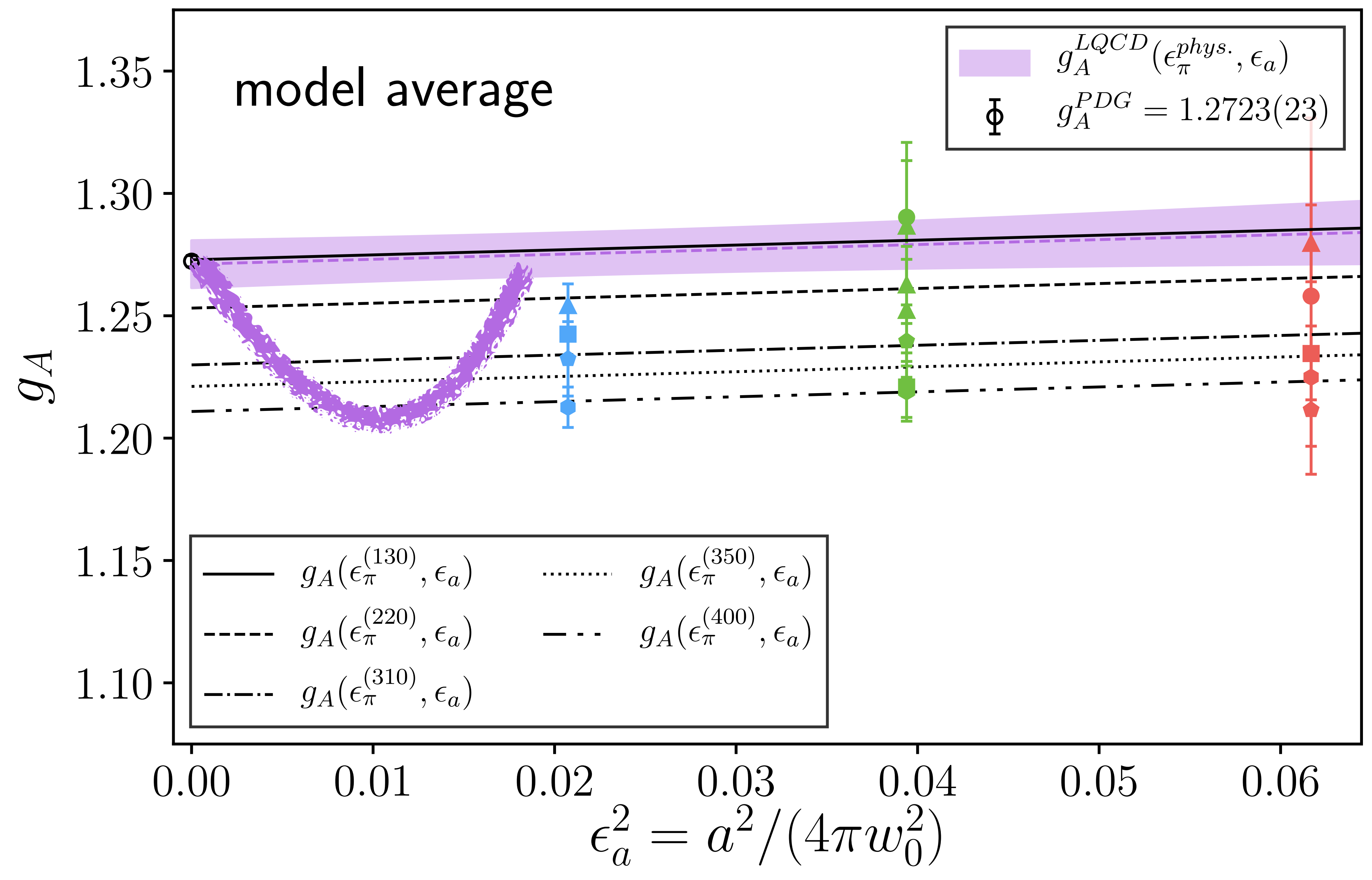

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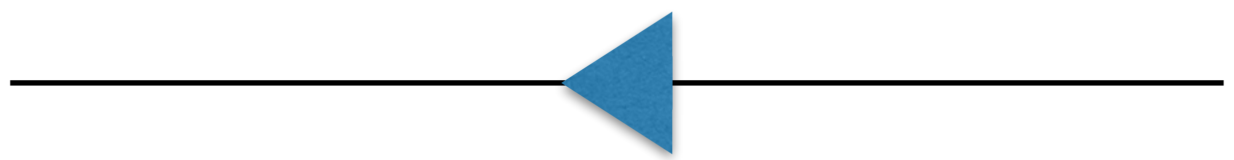
LQCD challenges for NP

Most difficult challenge: an exponentially bad signal-to-noise problem

Lepage, TASI 1989

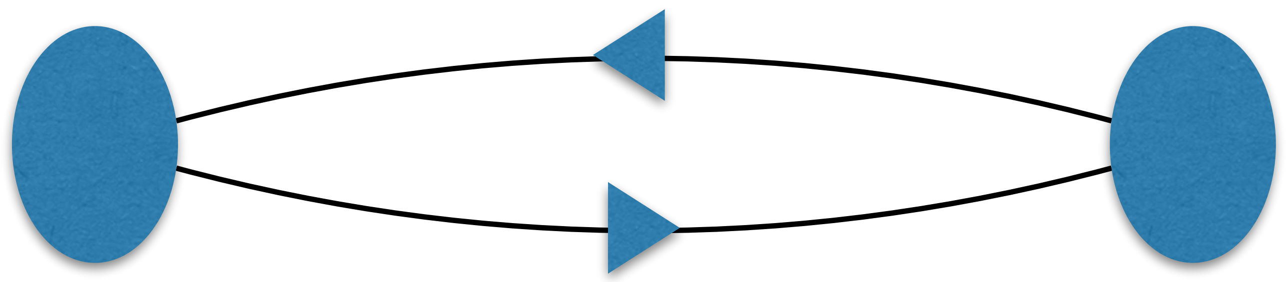


$$\sim e^{-\frac{1}{2}m_\pi t} + e^{-\frac{1}{3}m_N t} + \cdots$$

Each **quark propagator** carries information about pions and nucleons (conversations with David Kaplan)

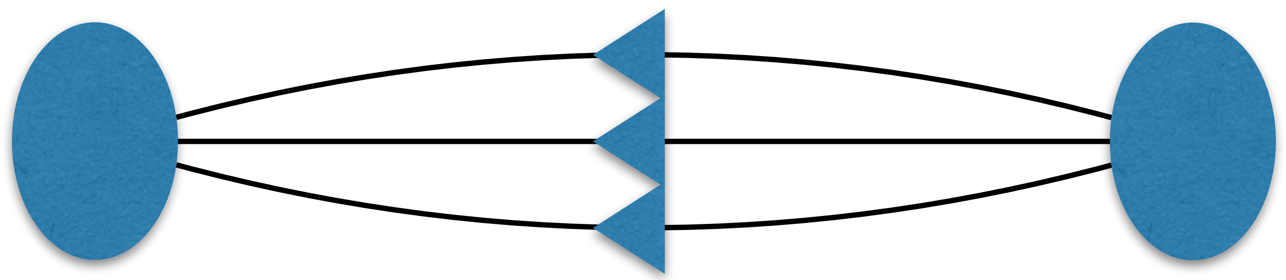
$$\lambda_\pi(t) \gg \lambda_N(t)$$

$$\lambda_i(t) \sim e^{-E_i t}$$



$$\bar{d}\gamma_5 u : C(t) = A_\pi e^{-m_\pi t} + \cdots$$

Large pion eigenvalues must cancel to expose small nucleon eigenvalues $e^{-m_N t} \ll e^{-m_\pi t}$



$$(u^T C\gamma_5 d)u : C(t) = A_N e^{-m_N t} + \cdots$$

$$\frac{\text{Signal}}{\text{Noise}} \sim \sqrt{N}\exp\left[-A\left(m_N - \frac{3}{2}m_\pi\right)t\right]$$

exponential noise
power-law statistics