MITP workshop – Electroweak Precision Physics from Beta Decays to the Z Pole

Hadronic matrix elements from V_{ud} and V_{us} extractions from lattice QCD

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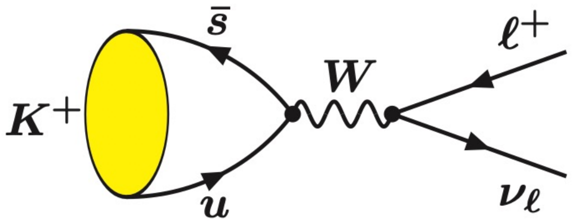
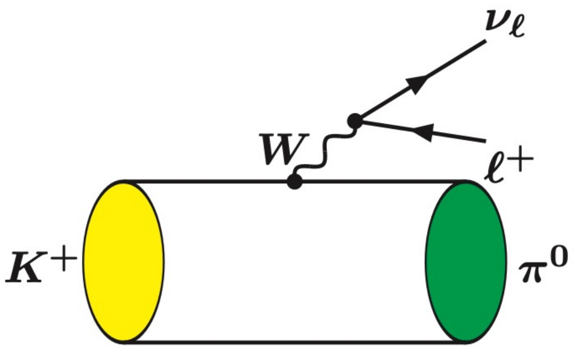
Lattice QCD and electroweak interactions

Role played by Lattice QCD: calculate the hadronic matrix elements

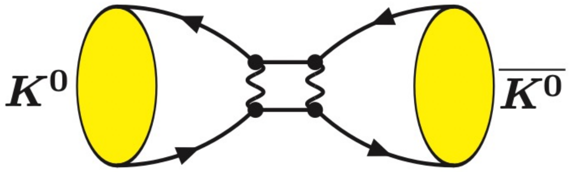
➢ Lattice QCD is powerful for "standard" hadronic matrix elements with



$K_{\ell 2} \rightarrow f_K$



$K_{\ell 3} \rightarrow f_+(0)$



K^0-$\overline{K^0}$ mixing $\rightarrow B_K$

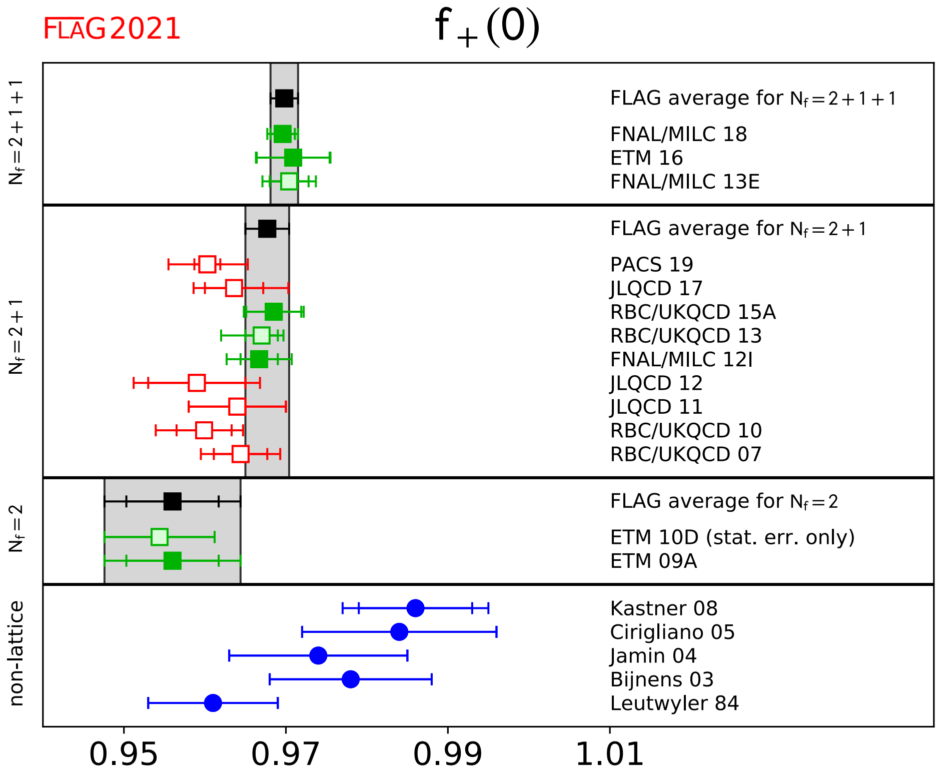
- Single local operator insertion
- Typically 2-point or 3-point correlation function
- No isospin breaking effects involved yet

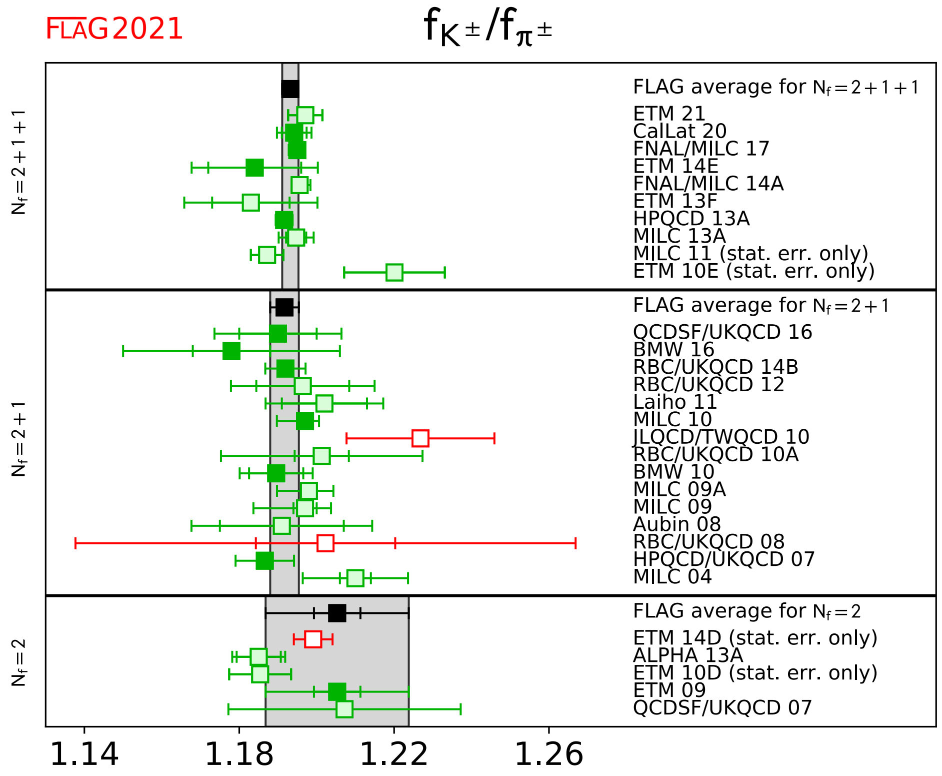
Precision era for lattice QCD

Flavor Lattice Averaging Group (FLAG) average, based on FLAG review 2021

$$f_+^{K\pi}(0) = 0.9698(17) \quad \Rightarrow \quad 0.18\% \text{ error}$$

$$f_{K^\pm}/f_{\pi^\pm} = 1.1932(21) \quad \Rightarrow \quad 0.18\% \text{ error}$$





High precision requires new directions to open in lattice QCD

FLAG average 2021

Error $< 1\%$

	N_f	FLAG average	Frac. Err.
f_K/f_π	$2+1+1$	1.1932(21)	0.18%
$f_+(0)$	$2+1+1$	0.9698(17)	0.18%
f_D	$2+1+1$	212.0(7) MeV	0.33%
f_{D_s}	$2+1+1$	249.9(5) MeV	0.20%
f_{D_s}/f_D	$2+1+1$	1.1783(16)	0.13%
f_B	$2+1+1$	190.0(1.3) MeV	0.68%
f_{B_s}	$2+1+1$	230.3(1.3) MeV	0.56%
f_{B_s}/f_B	$2+1+1$	1.209(5)	0.41%

Radiative corrections become important!

Error $< 5\%$

	N_f	FLAG average	Frac. Err.
$\hat{B}_K$	$2+1$	0.7625(97)	1.3%
$f_+^{D\pi}(0)$	$2+1$	0.666(29)	4.4%
$f_+^{DK}(0)$	$2+1$	0.747(19)	2.5%
$\hat{B}_{B_s}$	$2+1$	1.35(6)	4.4%
B_{B_s}/B_{B_d}	$2+1$	1.032(38)	3.7%
$\cdots$			

Time to go beyond local matrix elements and three-point functions

Go beyond local hadronic matrix elements - challenges

- Computational demanding
  - Three-point function

$$\langle H_f(x_f) O(0) H_i^\dagger(x_i)\rangle \quad \Rightarrow \quad \int d^3\vec{x}_i \int d^3\vec{x}_f \quad \Rightarrow \quad \sum_{\vec{x}_i}\sum_{\vec{x}_f} \sim L^6$$

  - Four-point function

$$\langle H_f(x_f) O_1(x) O_2(0) H_i^\dagger(x_i)\rangle\rangle \quad \Rightarrow \quad \int d^3\vec{x}_i \int d^3\vec{x}_f \int d^3\vec{x} \quad \Rightarrow \quad \sum_{\vec{x}_i}\sum_{\vec{x}_f}\sum_{\vec{x}} \sim L^9$$

  with $L = 24, 32, 48, 64, 96, \cdots$

- Complicated intermediate states

$$\langle H_f|O_1(x)O_2(0)|H_i\rangle = \sum_n \langle H_f|O_1(x)|n\rangle\langle n|O_2(0)|H_i\rangle$$

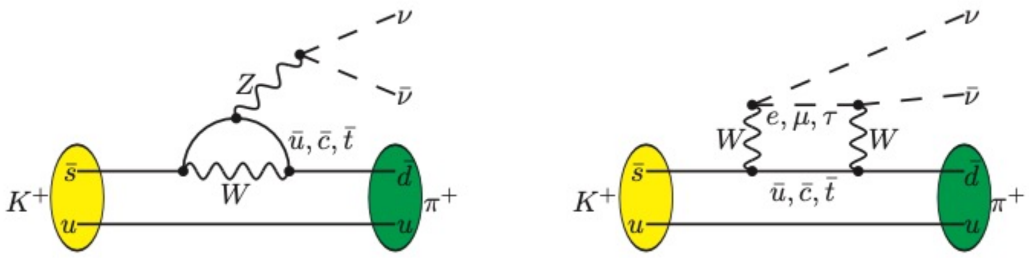
  - In Euclidean space, exponentially-growing unphysical effects from $|n\rangle$
  - Power-law FV effects if $|n\rangle$ given by low-lying multi-hadron states

- Short-distance divergence in $O_1(x)O_2(0)$ when $x \to 0$
  - Additional renormalization is required

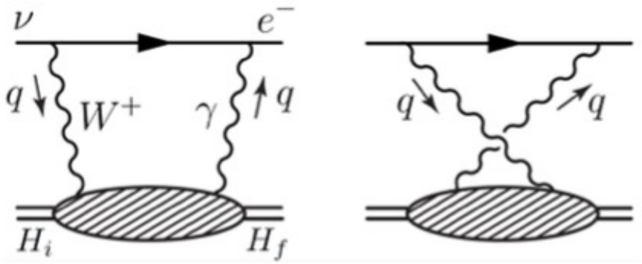
Go beyond local hadronic matrix elements - opportunities

Opportunities in flavor physics

- Rare decays, e.g. $\mathrm{Br}[K^+ \to \pi^+ \nu\bar{\nu}] = 1.73^{+1.15}_{-1.05} \times 10^{-10}$



- Electroweak radative corrections to hadronic decays
  $\Rightarrow$ superallowed nuclear β decay half-life time with precision 10^{-6}

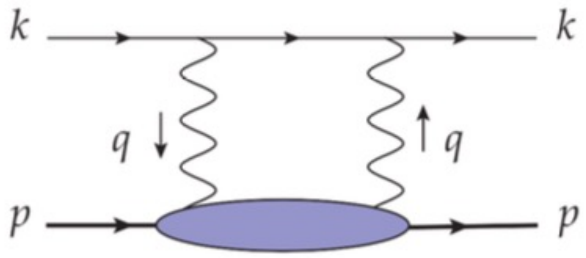


- Proton's weak charge $Q_W^p = 1 - 4\sin^2\theta_W$
  $\Rightarrow$ 0.3% measurement of $\sin^2\theta_W$ by Q-weak at JLab
  - Parity-violating e-p scattering, $\Box^V_{\gamma Z}$ contribution

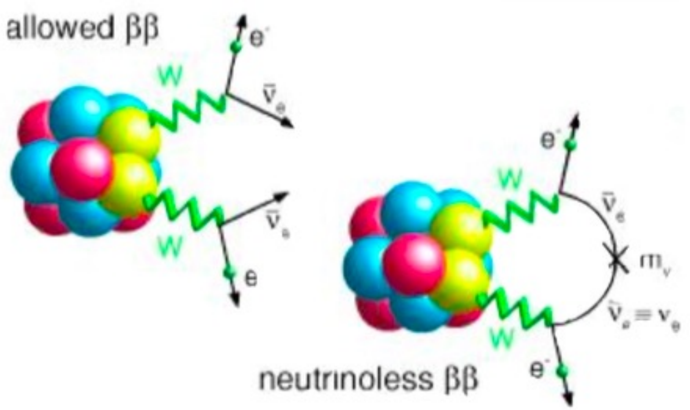
Go beyond local hadronic matrix elements - opportunities

Opportunities in nuclear physics

- Muonic hydrogen spectrum $\rightarrow$ proton charge radius $r_p = 0.84087(39)$ fm
  $\Rightarrow$ 10 times more accurate than e-p scattering



- Neutrioless double beta decays



- Hadron electromagentic polarizability

Our recent work on long-distance electroweak processes

- QCD+QED methodology & Pion mass splitting

  [XF, L. Jin, PRD100 (2019) 094509]

  [XF, L. Jin, M. Riberdy, PRL128 (2022) 052003]

- Rare kaon decays $K^+ \to \pi^+ \nu \bar{\nu}$

  [Z. Bai, XF, N. Christ, et.al. PRL118 (2017) 252001]

- Electroweak box contribution to $\pi_{\ell 3}$ and $K_{\ell 3}$ decay

  [XF, M. Gorchtein, L. Jin, P. Ma, C. Seng, PRL124 (2020) 192002]

  [P. Ma, XF, M. Gorchtein, L. Jin, C. Seng, PRD103 (2021) 114503]

- Neutrinoless double beta decays

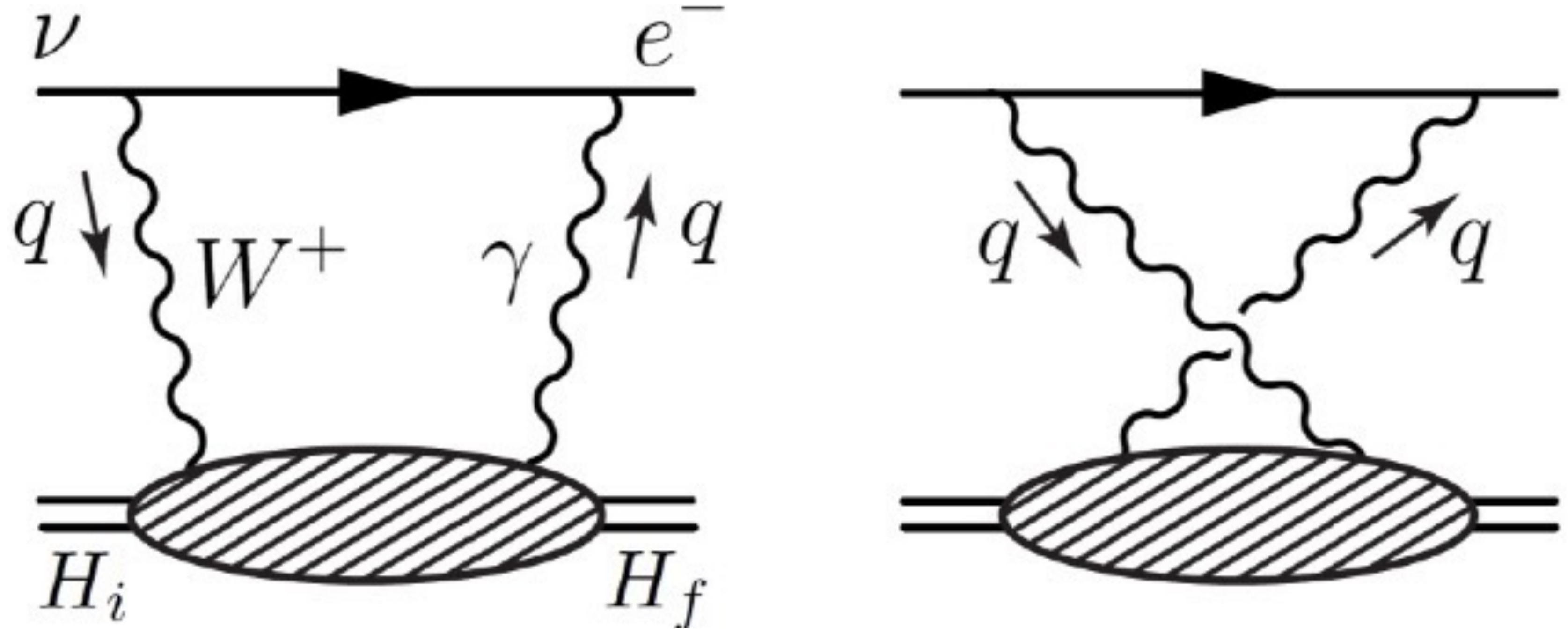
  [XF, L. Jin, X. Tuo, S. Xia, PRL122 (2019) 022001]

  [X. Tuo, XF, L. Jin, PRD100 (2019) 094511]

- Two-photon exchange contribution to μH spectroscopy

  [Y. Fu, XF, L. Jin, C. Lu, PRL128 (2022) 172002]

Electroweak box diagram



CKM unitarity – a constraint from Standard Model

First-row CKM unitarity

$$\Delta_{\rm CKM} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0$$

PDG 2019 $\Rightarrow$ PDG 2020

	PDG 2019	PDG 2020
$\lvert V_{ud}\rvert$	0.97420(21)	0.97370(14)
$\lvert V_{us}\rvert$	0.2243(5)	0.2245(8)
$\lvert V_{ub}\rvert$	0.00394(36)	0.00382(24)
$\Delta_{\rm CKM}$	−0.00061(47)	−0.00149(45)

- Main update from $|V_{ud}|$ $\Rightarrow$ 3.3 σ deviation from CKM unitarity
- $|V_{ud}|$ is from superallowed $0^+ \to 0^+$ nuclear beta decay
  - Pure vector transitions at leading order
  - Uncertainty is dominated by electroweak radiative correction

[J. Hardy, I. Towner (2015)]

CKM unitarity – a constraint from Standard Model

First-row CKM unitarity

$$\Delta_{\mathrm{CKM}} = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = 0$$

PDG 2019 $\Rightarrow$ PDG 2020 $\Rightarrow$ PDG 2022

	PDG 2019	PDG 2020	PDG 2022
$\|V_{ud}\|$	0.97420(21)	0.97370(14)	0.97373(31)
$\|V_{us}\|$	0.2243(5)	0.2245(8)	0.2243(8)
$\|V_{ub}\|$	0.00394(36)	0.00382(24)	0.00382(20)
Δ_{CKM}	−0.00061(47)	−0.00149(45)	−0.00152(70)

- Recent update from $|V_{ud}| \Rightarrow 2.2\,\sigma$ deviation from CKM unitarity
- The uncertainty is about twice larger than that in PDG 2020
- A more conservative estimate of the nuclear structure uncertainties is taken

[M. Gorchtein, PRL123 (2019) 042503]

V_{ud} from different measurements

- Superallowed β decays $|V_{ud}|=0.9737(3)$
  - $0^+ \to 0^+$ nuclear beta decays, which are pure vector transition at leading order
  - Estimate of nuclear structure uncertainties is important
- Neutron β decays $|V_{ud}|=0.9737(9)$
  - Free from nuclear structure uncertainties
  - Nuclear-structure independent radiative correction (RC) is same as superallowed nuclear β decay

$$\frac{1}{\tau_n} = \frac{G_\mu^2 |V_{ud}|^2}{2\pi^3} m_e^5 \left(1 + 3g_A^2\right)\left(1 + \Delta_R^V\right) f \quad \Rightarrow \quad |V_{ud}|^2 = \frac{5024.7\ \mathrm{s}}{\tau_n \left(1 + 3g_A^2\right)\left(1 + \Delta_R^V\right)}$$

$$|V_{ud}| = 0.9737(3)_{\tau_n}(8)_{g_A}(1)_{\mathrm{RC}}$$

- τ_n Neutron life time, measured by Ultra Cold Neutron Experiment, UCNτ+
- g_A Neutron axial charge, measured by UCNA+ [See the talk from Andre Walker-loud]
- Δ_R^V Radiative correction (RC)

New experiments under going, comparable precision as superallowed β decay in the near future

V_{ud} from different measurements

- Superallowed β decays $|V_{ud}|=0.9737(3)$
  - $0^+ \rightarrow 0^+$ nuclear beta decays, which are pure vector transition at leading order
  - Estimate of nuclear structure uncertainties is important
- Neutron β decays $|V_{ud}|=0.9737(9)$
  - Free from nuclear structure uncertainties
  - Nuclear-structure independent radiative correction (RC) is same as superallowed nuclear β decay
- Pion semileptonic β decays $|V_{ud}|=0.9739(29)$
  - More difficult to measure pion decays
  - Theoretically simpler, especially for lattice QCD

◆ Summary

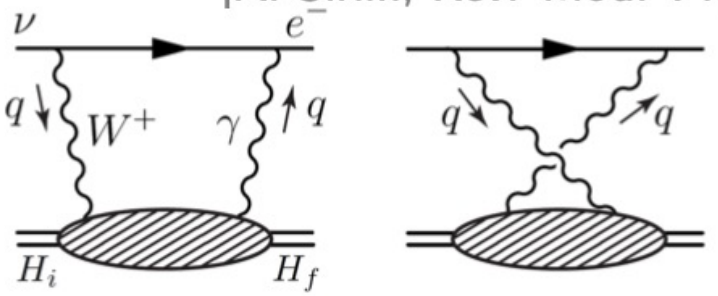
- To extract V_{ud} from superallowed decay or neutron β decay with controlled uncertainties at ~2×10^{-4} level ⇒ RC is important

Use pion decays to set up the lattice QCD calculation strategy

Axial γW-box diagram

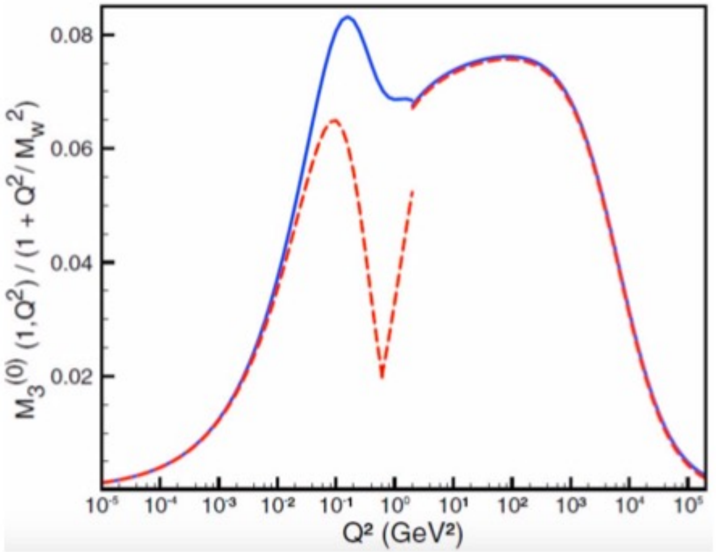
Based on current algebra, only axial γW-box diagram sensitive to hadronic scale

[A. Sirlin, Rev. Mod. Phys. 07 (1978) 573]



$$T_{\mu\nu}^{VA} = \frac{1}{2}\int d^4x\, e^{iqx}\langle H_f(p)|\,T\left[J_\mu^{em}(x)J_\nu^{W,A}(0)\right]|H_i(p)\rangle$$

Re-evaluation of the γW-box diagram



$|V_{ud}| = 0.97420(18)_{\mathrm{RC}}(10)_{\mathcal{F}t}$

Using VMD model

[Marciano & Sirlin, PRL 2006]

$\Downarrow$

$|V_{ud}| = 0.97370(10)_{\mathrm{RC}}(10)_{\mathcal{F}t}$

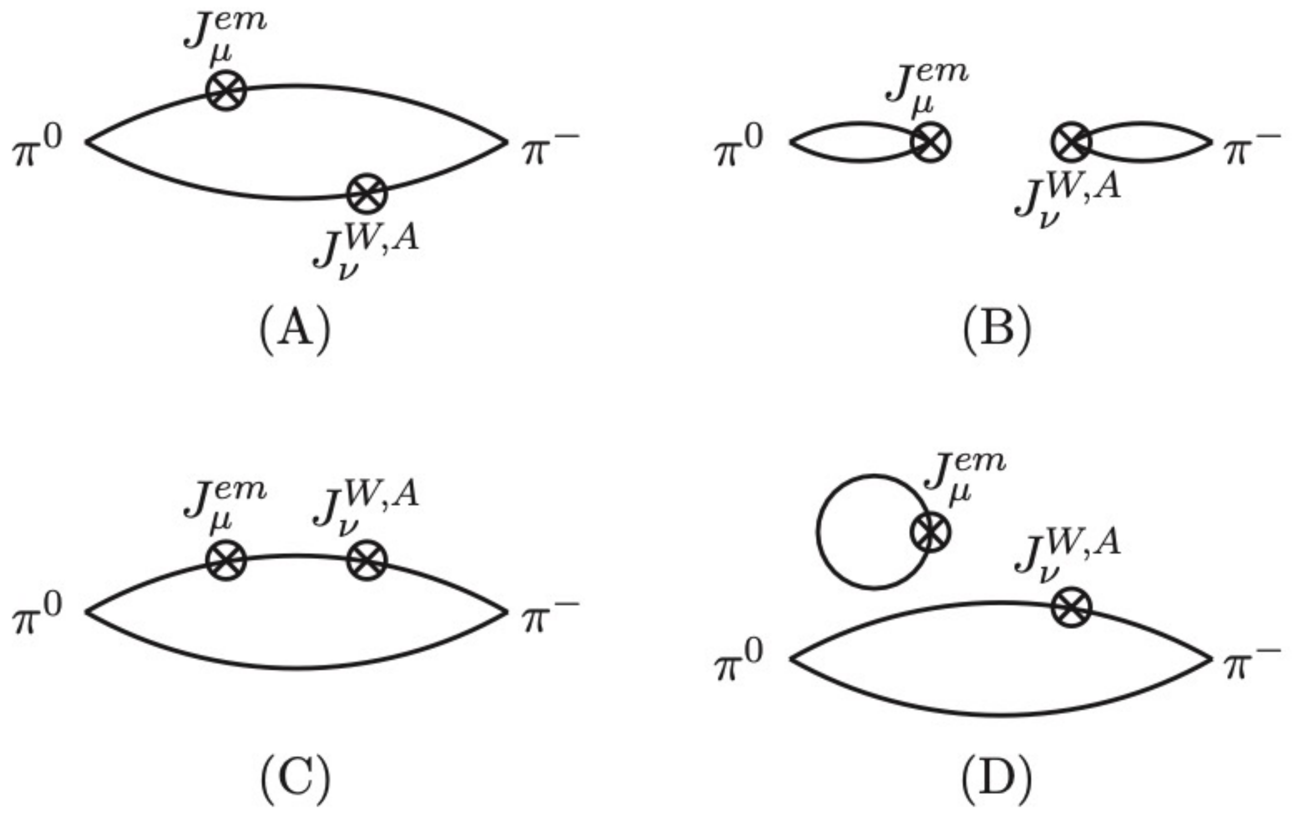
Using dispsive & data-driven analysis

[Seng et.al. PRL 2018]

It is the reason that caused 3.3σ deviation from CKM unitarity based on PDG 2020

Quark contractions for the γW-box diagram

$$\mathcal{H}^{VA}_{\mu\nu}(x) = \langle \pi^0(p) | T\left[J^{em}_\mu(x) J^{W,A}_\nu(0)\right] | \pi^-(p) \rangle$$



- Coulomb gauge fixed wall source is used for the pion interpolating field
- $J^{W,A}_\nu(0)$ is treated as a source and $J^{em}_\mu(x)$ is a sink
- Calculate $\mathcal{H}^{VA}_{\mu\nu}(x)$ as a function of x

Five gauge ensembles at physical pion mass

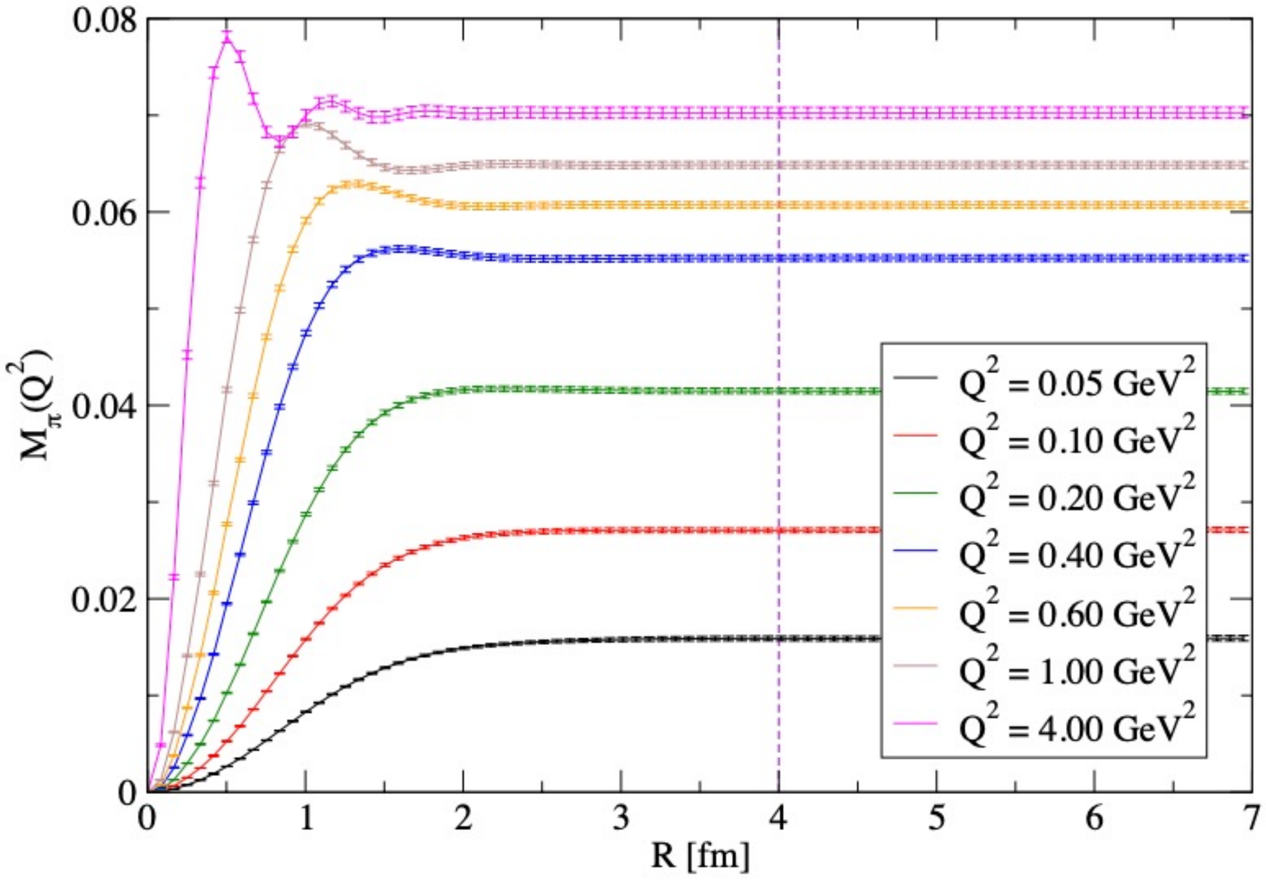
ensemble	M_π/MeV	$L^3\times T$	a/fm	L·a/fm	N_{conf}	N_r	$N_{conf}\times N_r$
24D	141.2(4)	$24^3\times64$	0.1944	4.665	46	1024	47104
32D	141.4(3)	$32^3\times64$	0.1944	6.221	32	2048	65536
32D-fine	143.0(3)	$32^3\times64$	0.1432	4.582	71	1024	72704
48I	135.5(4)	$48^3\times96$	0.1140	5.474	28	1024	28672
64I	135.3(2)	$64^3\times128$	0.0836	5.353	62	1024	63488

- Gauge ensembles generated by RBC-UKQCD Collaborations using 2+1 flavor domain wall fermion
- 24D, 32D, 32D-fine use Iwasaki+DSDR action; while 48I, 64I use Iwasaki gauge action

Lattice results for the hadronic functions

Construct the Lorentz scalar function $M_\pi(Q^2)$ from $\mathcal{H}^{VA}_{\mu\nu}(x)$

$$M_\pi(Q^2) = -\frac{1}{6\sqrt{2}}\frac{\sqrt{Q^2}}{m_\pi}\int d^4x\,\omega(Q,x)\epsilon_{\mu\nu\alpha0}x_\alpha\mathcal{H}^{VA}_{\mu\nu}(x)$$

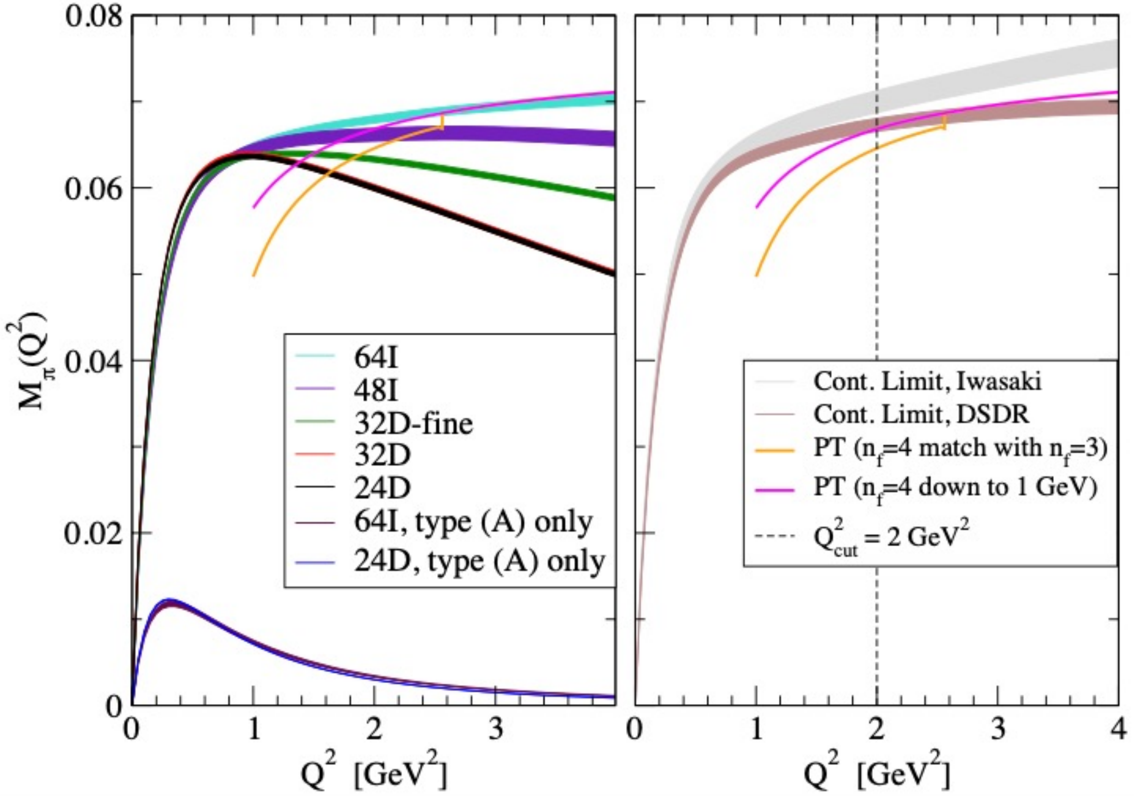


Combine lattice results with pQCD

Radiative correction requires the momentum integral from $0 < Q^2 < \infty$

$$\Box_{\gamma W}^{VA} = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}M_\pi(Q^2)$$

- Lattice data used for low-Q^2 region
- OPE and perturbative Wilson coefficients used for high-Q^2 region



Error analysis

Use the momentum scale Q^2_{cut} to separate the LD and SD contributions

$$\Box^{VA}_{\gamma W} = \begin{cases} 2.816(9)_{\mathrm{stat}}(24)_{\mathrm{PT}}(18)_{\mathrm{a}}(3)_{\mathrm{FV}} \times 10^{-3} & \text{using } Q^2_{\mathrm{cut}} = 1\ \mathrm{GeV}^2 \\ 2.830(11)_{\mathrm{stat}}(9)_{\mathrm{PT}}(24)_{\mathrm{a}}(3)_{\mathrm{FV}} \times 10^{-3} & \text{using } Q^2_{\mathrm{cut}} = 2\ \mathrm{GeV}^2 \\ 2.835(12)_{\mathrm{stat}}(5)_{\mathrm{PT}}(30)_{\mathrm{a}}(3)_{\mathrm{FV}} \times 10^{-3} & \text{using } Q^2_{\mathrm{cut}} = 3\ \mathrm{GeV}^2 \end{cases}$$

- When Q^2_{cut} increase, the lattice artifacts become larger

- When Q^2_{cut} decrease, systematic effects in pQCD become larger

- For 1 $\mathrm{GeV}^2 \le Q^2_{\mathrm{cut}} \le 3\ \mathrm{GeV}^2$, all results are consistent within uncertainties

Pion semileptonic β decay

Decay width measured by PIBETA experiment

$$\Gamma_{\pi\ell 3} = \frac{G_F^2 |V_{ud}|^2 m_\pi^5 |f_+^\pi(0)|^2}{64\pi^3}(1+\delta)I_\pi$$

- ChPT [Cirigliano et.al. (2002), Czarnecki, Marciano, Sirlin (2019)]

$$\delta = 0.0334(10)_{\mathrm{LEC}}(3)_{\mathrm{HO}}$$

- Sirlin's parametrization [A. Sirlin, Rev. Mod. Phys. 07 (1978) 573]

$$\begin{aligned}\delta &= \frac{\alpha_e}{2\pi}\left[\bar{g} + 3\ln\frac{m_Z}{m_p} + \ln\frac{M_Z}{M_W} + \tilde{a}_g\right] + \delta_{\mathrm{HO}}^{\mathrm{QED}} + 2\Box_{\gamma W}^{VA} \\ &= 0.0332(1)_{\gamma W}(3)_{\mathrm{HO}}\end{aligned}$$

where $\frac{\alpha_e}{2\pi}\bar{g} = 1.051\times 10^{-2}$, $\frac{\alpha_e}{2\pi}\tilde{a}_g = -9.6\times 10^{-5}$, $\delta_{\mathrm{HO}}^{\mathrm{QED}} = 0.0010(3)$

- Hadronic uncertainty reduced by a factor of 10, which results in

$$|V_{ud}| = 0.9739(28)_{\mathrm{exp}}(5)_{\mathrm{th}} \quad\Rightarrow\quad |V_{ud}| = 0.9739(28)_{\mathrm{exp}}(1)_{\mathrm{th}}$$

[XF, Gorchtein, Jin, Ma, Seng, PRL124 (2020) 192002]

First time to calculate γW box diagram $\Rightarrow$ method set up for nucleon decay

Status of V_{us}

- Semileptonic decays
  - Traditionally from $K_L^0 \to \pi e \nu$ decays to avoid isospin breaking effects
  - Latest experiments justify comparison between different decay modes
  - Global average from [FlaviaNet Working Group, EPJC, 2010]

$$K_L^0 \to \pi e \nu, \quad K_L^0 \to \pi \mu \nu, \quad K^\pm \to \pi^0 e^\pm \nu, \quad K^\pm \to \pi^0 \mu^\pm \nu, \quad K_S^0 \to \pi e \nu$$

  leads to $|V_{us}| f_+(0) = 0.21635(38)$
  - Using FLAG $N_f = 2+1+1$ lattice input of $f_+(0) = 0.9698(17) \Rightarrow$

$$|V_{us}| = 0.2231(4)_{\mathrm{exp+RCs}}(4)_{\mathrm{lat}}$$

- Leptonic decays
  - Exp. measurements of $K \to \mu\nu(\gamma)$ and $\pi \to \mu\nu(\gamma)$ e.g. from KLOE
  - Using FLAG $N_f = 2+1+1$ lattice input of $f_K/f_\pi = 1.1932(21) \Rightarrow$

$$|V_{us}| = 0.2252(5)_{\mathrm{lat}}$$

- $|V_{us}|$ from semileptonic and leptonic decays differs by 2.8σ

Important to include QED and QCD isospin violations in the lattice calculations

Basic idea

C. Seng, XF, M. Gorchtein, L. Jin, U.-G. Meißner, JHEP10 (2020) 179

Combine the lattice calculation with ChPT

- Use ChPT to determine EM correction

$$\delta_{\mathrm{em}}^{K^\pm} = 2e^2\left[-\frac{8}{3}X_1 - \frac{1}{2}\tilde{X}_6^{\mathrm{phys}}(M_\rho) - 2K_3^r(M_\rho) + K_4^r(M_\rho) + \frac{2}{3}K_5^r(M_\rho) + \frac{2}{3}K_6^r(M_\rho)\right]$$

$$\delta_{\mathrm{em}}^{K^0} = 2e^2\left[\frac{4}{3}X_1 - \frac{1}{2}\tilde{X}_6^{\mathrm{phys}}(M_\rho)\right] + \cdots$$

K^0 decays are much simpler, but still require LECs X_1 and $\tilde{X}_6^{\mathrm{phys}}(M_\rho)$

- Lattice QCD can provides LECs in principle
  - but needs to calculate all the diagrams, not only just γW diagram!
- Fortunately, these LECs are independent of quark mass

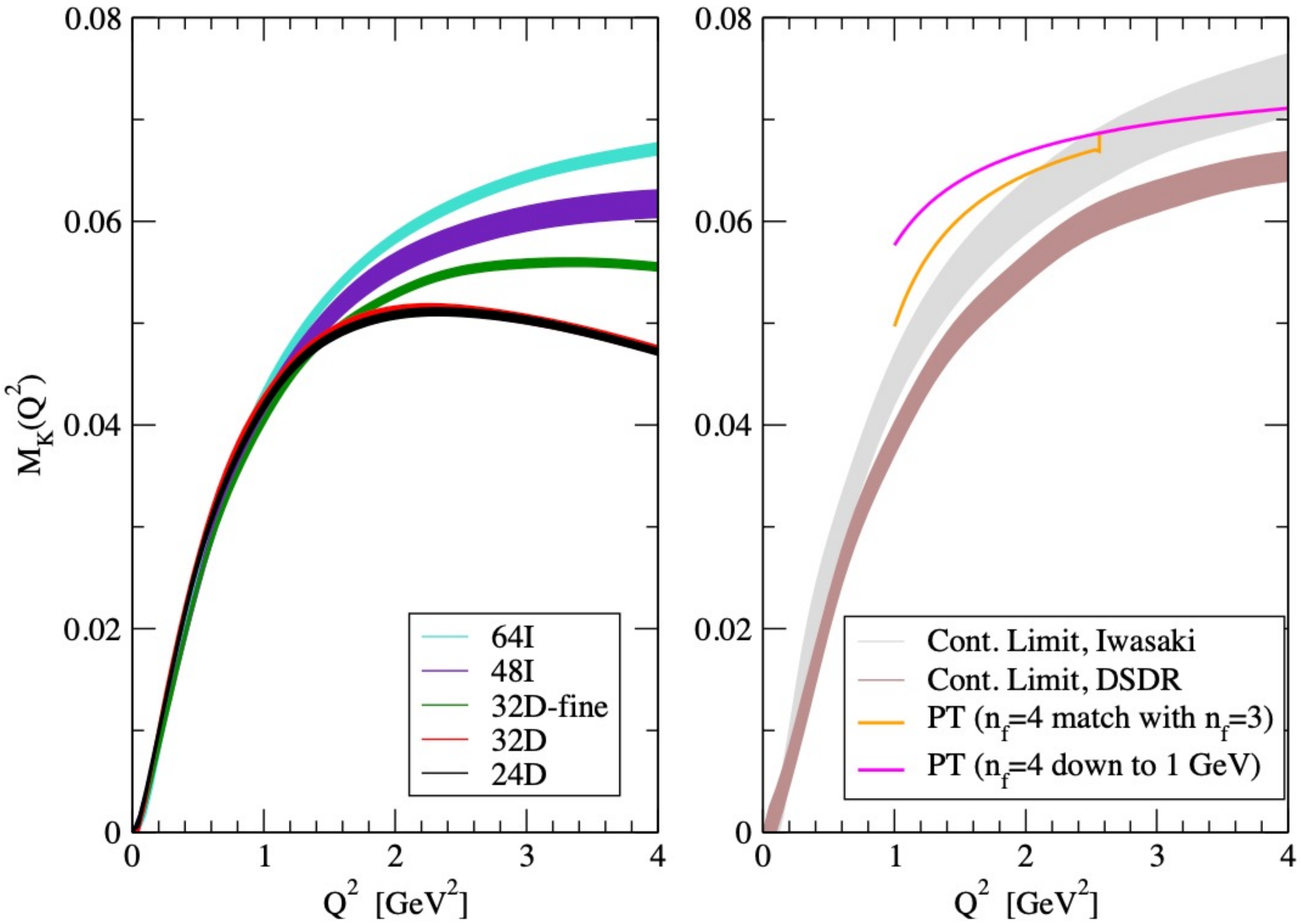
$K \to \pi\ell\nu$ at physical kinematics vs in flavor $SU(3)$ limit

Solution: Lattice calculation of EM correction to $K_{\ell 3}$ decay in flavor $SU(3)$ limit

Axial γW-box diagram contribution to $K^0 \to \pi^+$ decays

$$\square_{\gamma W}^{VA}\big|_H = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}M_H(Q^2)$$

Calculation is performed in the flavor $SU(3)$ limit with $m_K = m_\pi$



Lattice results

[P. Ma, XF, M. Gorchtein, L. Jin, C. Seng, PRD103 (2021) 114503]

- After combining the lattice data and PT results, we have

$$\Box_{\gamma W}^{VA}\big|_{K^0} = \begin{cases} 2.460(18)_{\rm stat}(42)_{\rm PT}(22)_a(1)_{\rm FV} \times 10^{-3} & Q^2_{\rm cut} = 1\ \mathrm{GeV}^2 \\ 2.443(20)_{\rm stat}(15)_{\rm PT}(36)_a(1)_{\rm FV} \times 10^{-3} & Q^2_{\rm cut} = 2\ \mathrm{GeV}^2 \\ 2.433(22)_{\rm stat}(7)_{\rm PT}(45)_a(1)_{\rm FV} \times 10^{-3} & Q^2_{\rm cut} = 3\ \mathrm{GeV}^2 \end{cases}$$

- The relation between box contribution and the LECs is given by

$$-\frac{8}{3}X_1 + \bar{X}_6^{\rm phys}(M_\rho) = -\frac{1}{2\pi\alpha}\left(\Box_{\gamma W}^{VA}\big|_{K^0} - \frac{\alpha}{8\pi}\ln\frac{M_W^2}{M_\rho^2}\right) + \frac{1}{8\pi^2}\left(\frac{5}{4} - \tilde{a}_g\right)$$

- This results in

$$-\frac{8}{3}X_1 + \tilde{X}_6^{\rm phys} = 0.0197(10)$$

- ChPT quoted the minimal resonance model as input

$$X_1 = -3.7(3.7) \times 10^{-3} \quad \text{and} \quad \tilde{X}_6^{\rm phys} = 10.4(10.4) \times 10^{-3}$$

$$-\frac{8}{3}X_1 + \tilde{X}_6^{\rm phys} = 0.0203(143)$$

Consistent between lattice and ChPT, but error from lattice is much smaller

Determination of LECs

- Combine the $SU(3)$ K^0 decay

$$-\frac{8}{3}X_1 + \tilde{X}_6^{\mathrm{phys}} = 0.0197(10) \quad \text{for } K^0 \to \pi^+$$

with semileptonic pion decay

$$\frac{4}{3}X_1 + \tilde{X}_6^{\mathrm{phys}} = 0.0110(6) \quad \text{for } \pi^- \to \pi^0$$

- We have

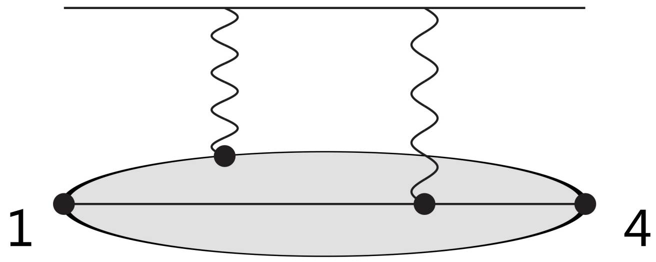
$$X_1 = -2.2(4) \times 10^{-3}, \quad \tilde{X}_6^{\mathrm{phys}} = 13.9(7) \times 10^{-3}$$

- This is comparable with the minimal resonance model

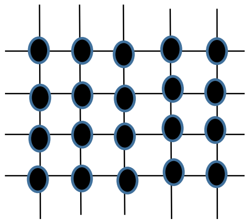
$10^3\, X_1$	$10^3\, X_2^r$	$10^3\, X_3^r$	$10^3\, \tilde{X}_6^{eff}$	$10^3 (X_6^{eff})_{\alpha_s}$	$10^3\, X_6^{eff}$
-3.7	3.6	5.00	10.4	3.0	-231.5

Move to nucleon sector

- Hadronic part from a typical 4-point function

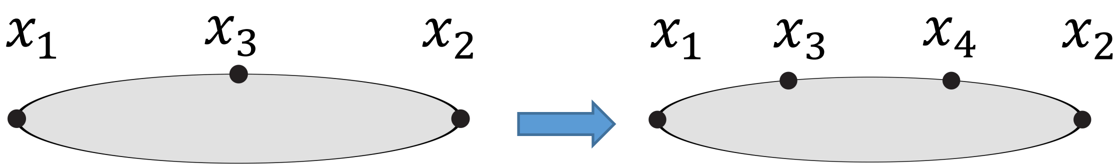


- Perform the volume summation for each point



$$\sum_{x \in L^3}$$

- From 3-point to 4-point function

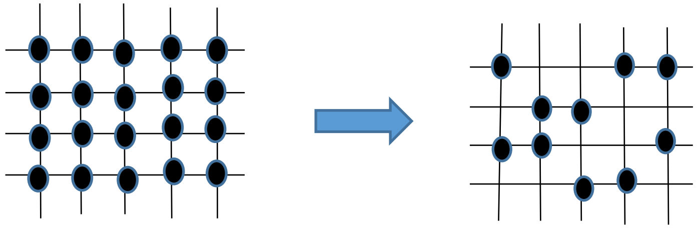


3-point: L^6 summation 4-point: L^9 summation

Increasing each point, computational cost increases by 10^5-10^6 times!
Cannot be solved by increasing resources …

Solution : Field sparsening method

【Y. Li, S. Xia, X. Feng, L. Jin, PRD 103 (2021) 014514】



- Less summation points may lead to lower precision
- It is not the case because of high correlation in lattice data

1000 times less points yields similar precision

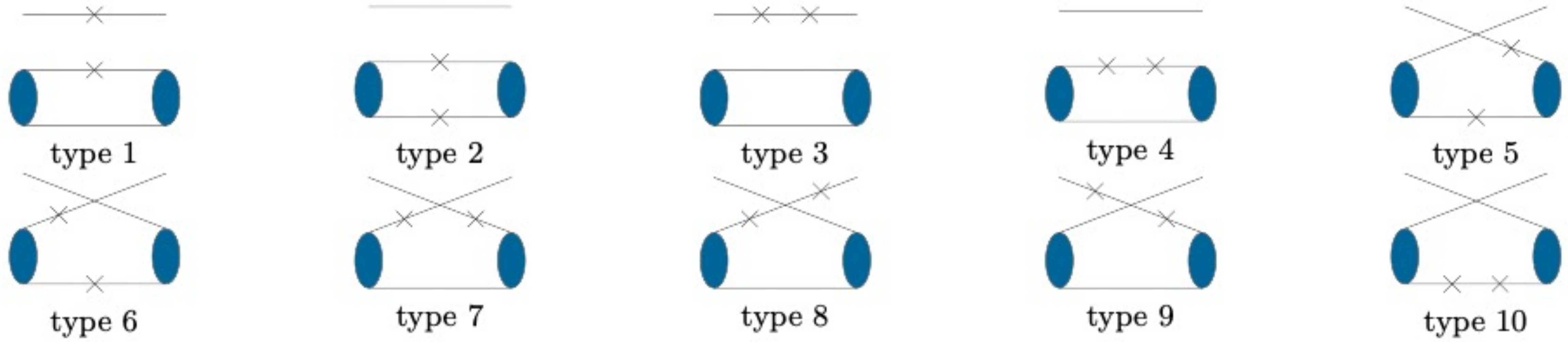
- Use both meson & baryon system to confirm universality

Utilize field sparsening method

- Reduce the computational cost by a factor of 1000 with almost no loss of precision!

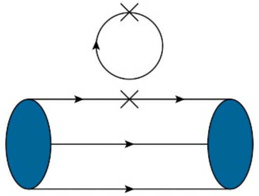
Computation of 4-point function

- Complicated quark field contraction for nucleon 4-point function – 10 types of connected diagrams



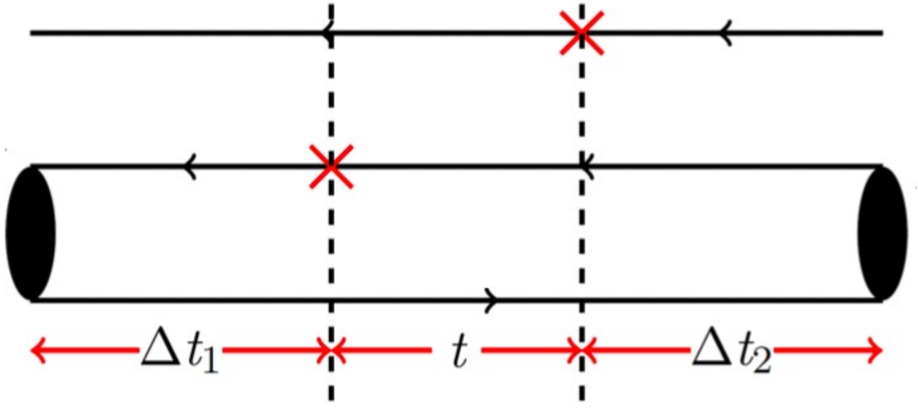
- Nucleon γW-box diagram requires 8 types of contractions (only type 2 & 4 do not contribute)

- Disconnected diagram is currently neglected as it vanishes in the flavor SU(3) limit



Lattice results

ensemble	M_π/MeV	$L^3\times T$	a/fm	L·a/fm	N_{conf}
24D	141.2(4)	$24^3\times64$	0.1944	4.665	111

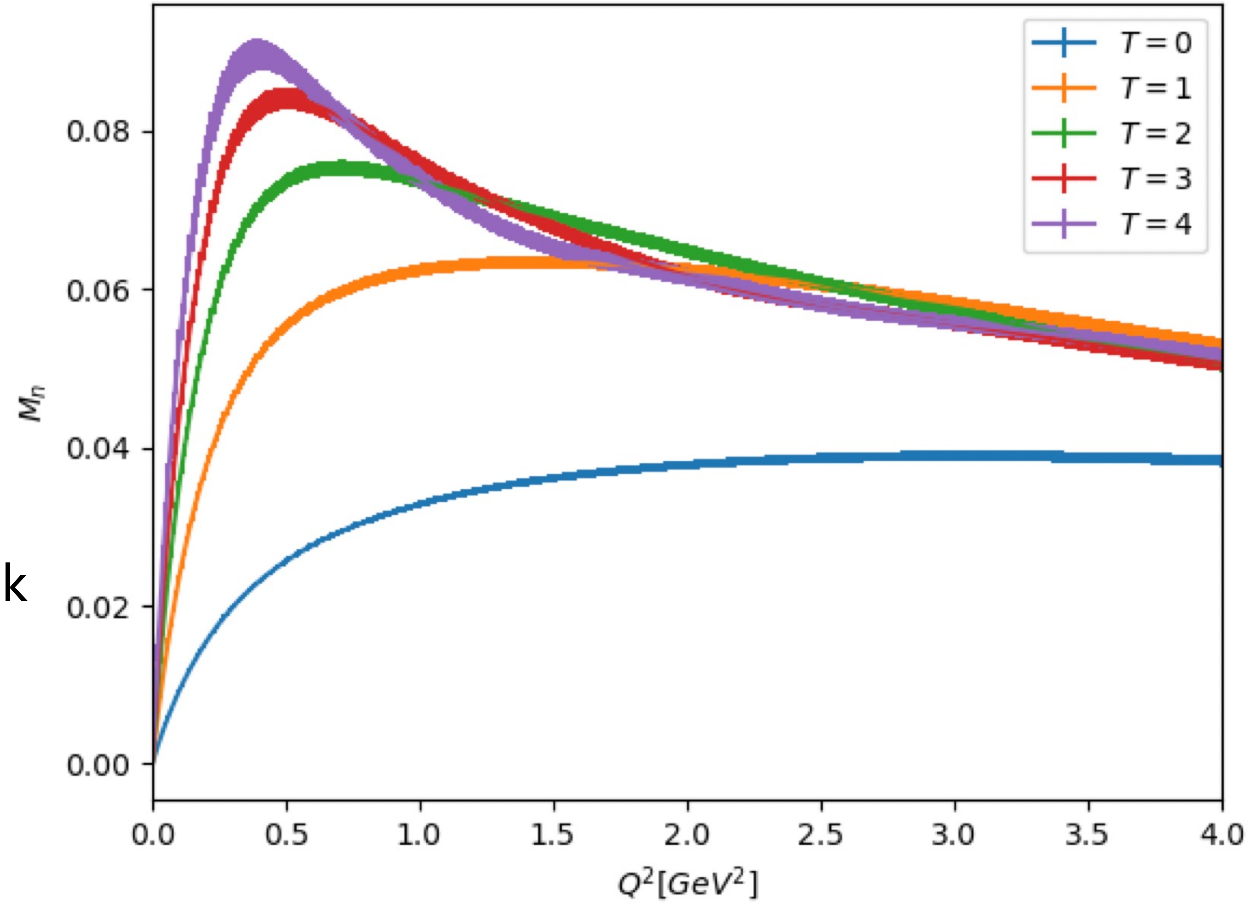


$\Delta t_1 = \Delta t_2 = 0.4$ fm

for ground state dominance in source and sink

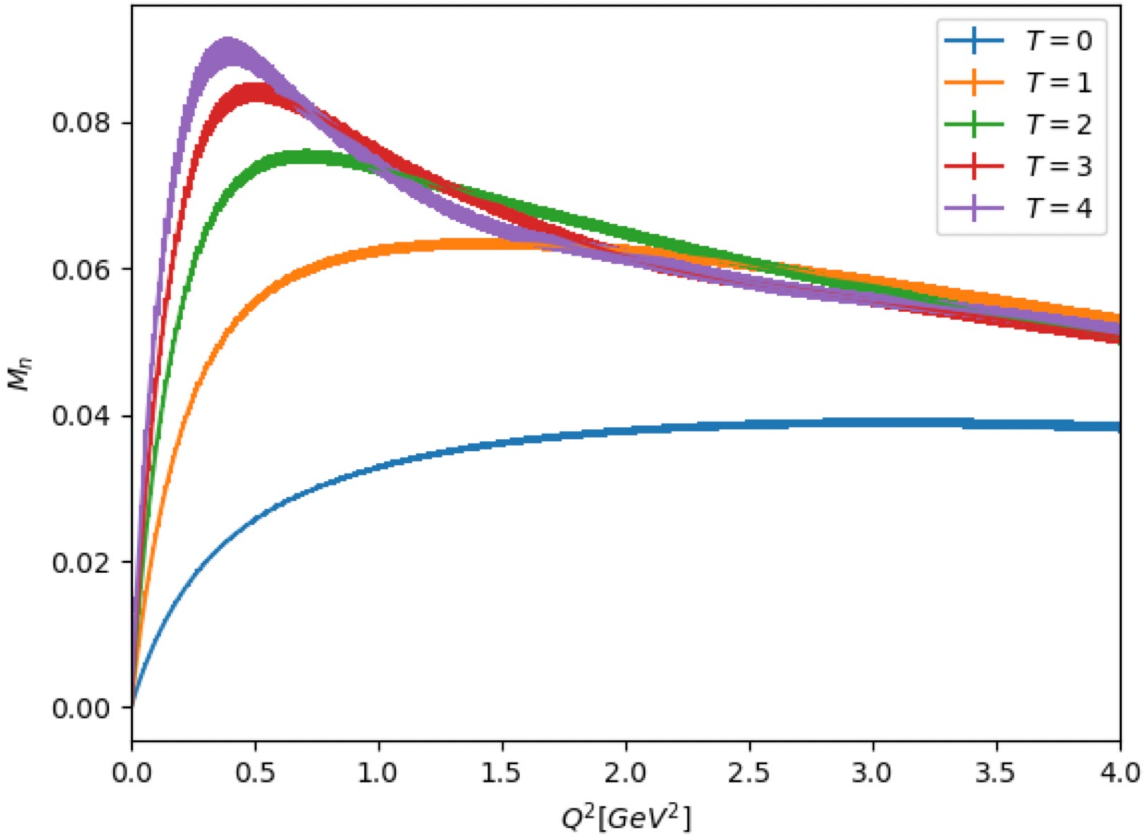
Sum over t with a temporal truncation T

needs large T for saturation

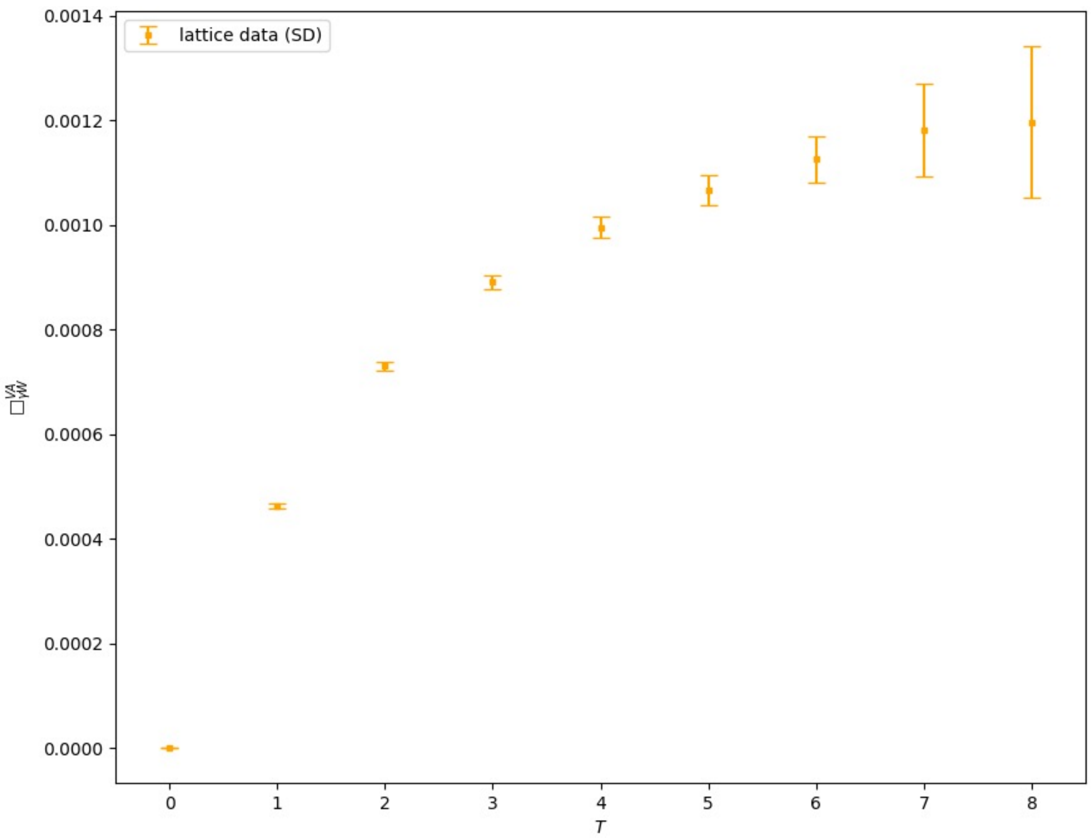


With momentum integral

$$\square_{\gamma W}^{VA} = \frac{3\alpha_e}{2\pi}\int_0^{Q^2_{cut}} \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}M_n(Q^2), \quad Q^2_{cut} = 2\ \mathrm{GeV}^2$$



Unintegrated data with Q^2 dependence



Integrated data with temporal truncation T

Question: Whether T=8 is safe for temporal truncation?

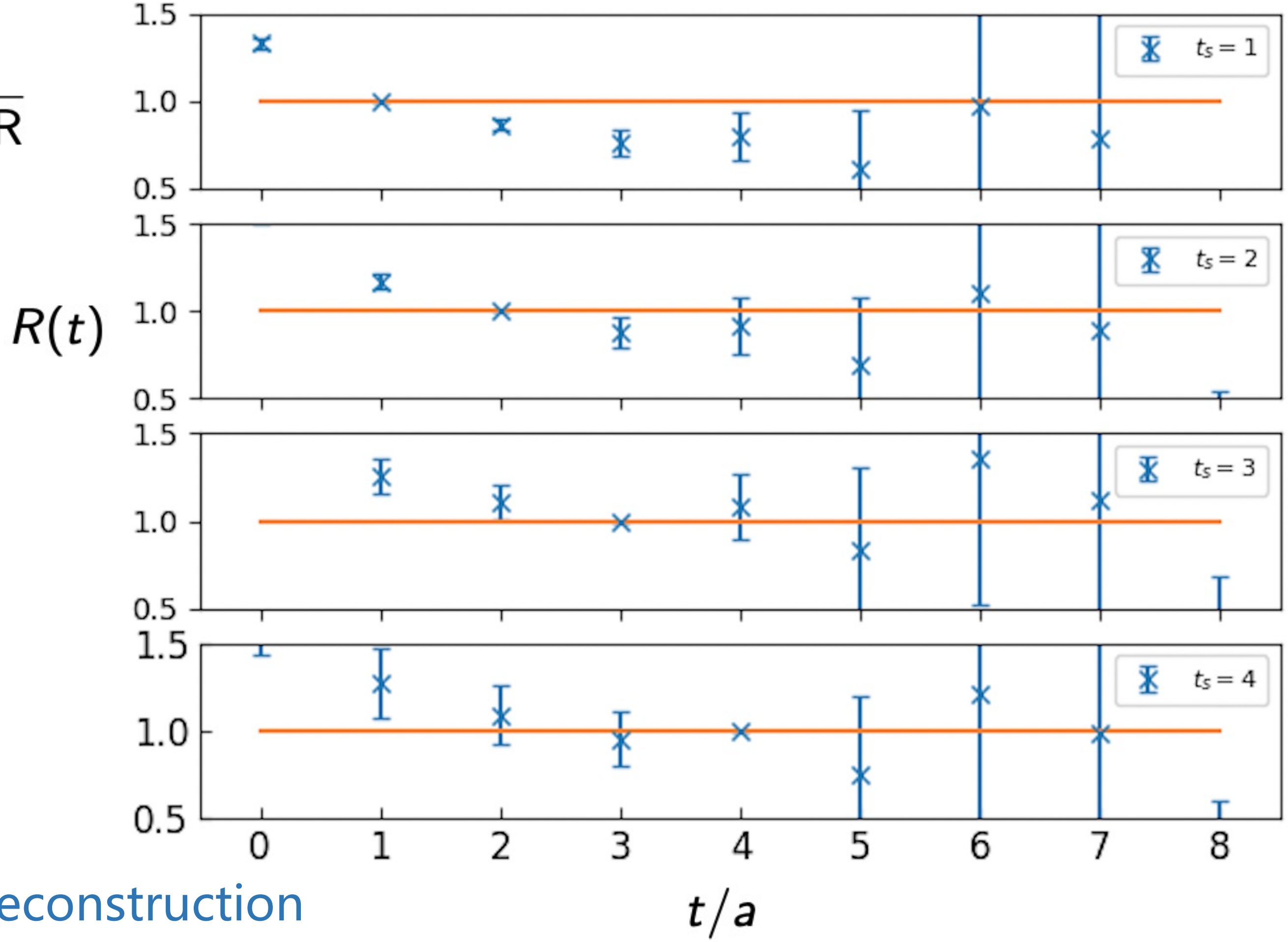
Examine the ground intermediate state dominance

Infinite-volume reconstruction method: [XF, L. Jin, PRD100 (2019) 094509]

Use $\mathcal{H}_{\mu\nu}(\vec{x}, t = t_s)$ to reconstruct the ground state contribution $\mathcal{H}^{GS}_{\mu\nu}(\vec{x}, t)$

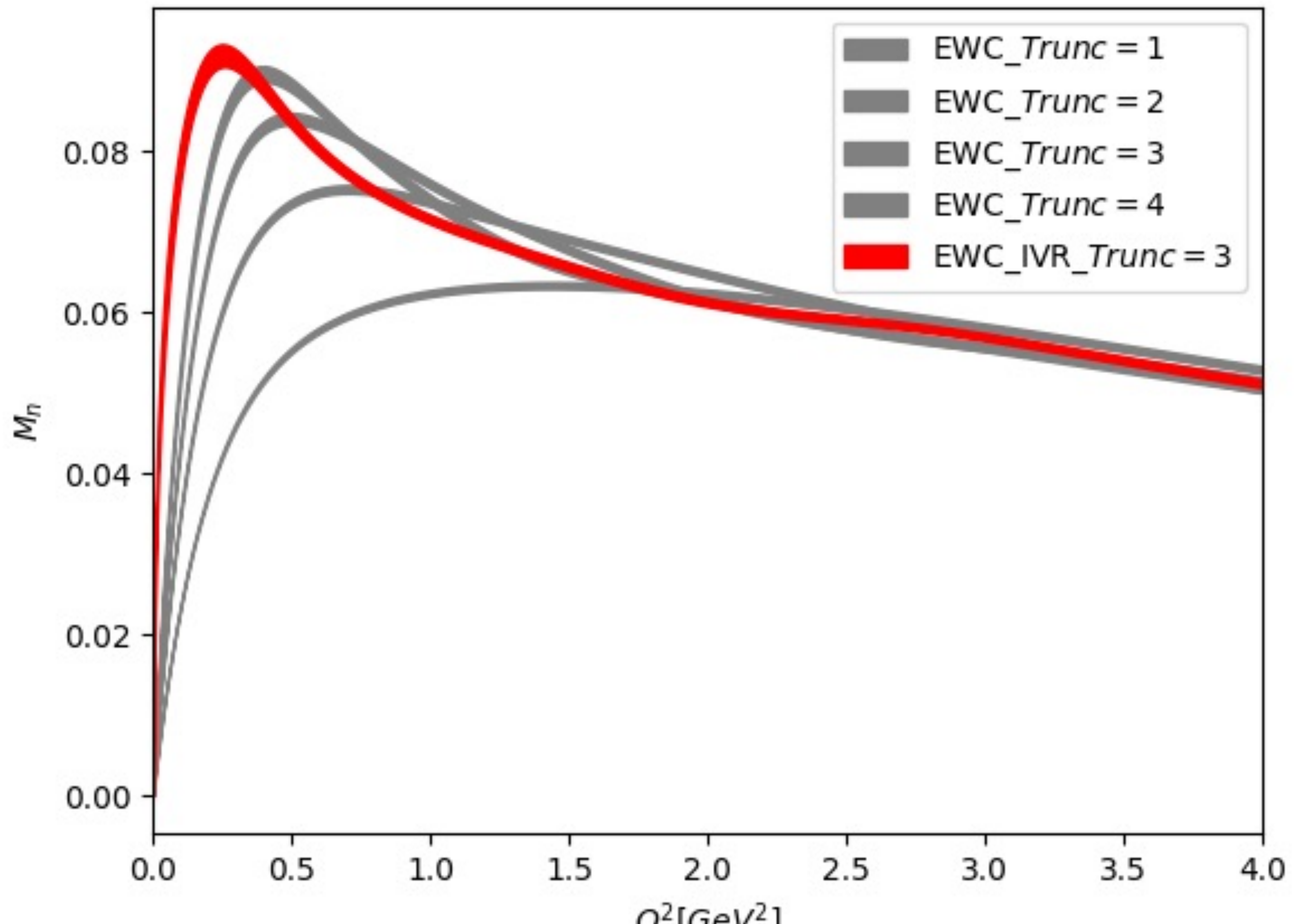
Construct a ratio to examine the ground-state dominance

$$R(t) = \frac{\text{Lattice data}}{\text{GS contribution from IVR}}$$

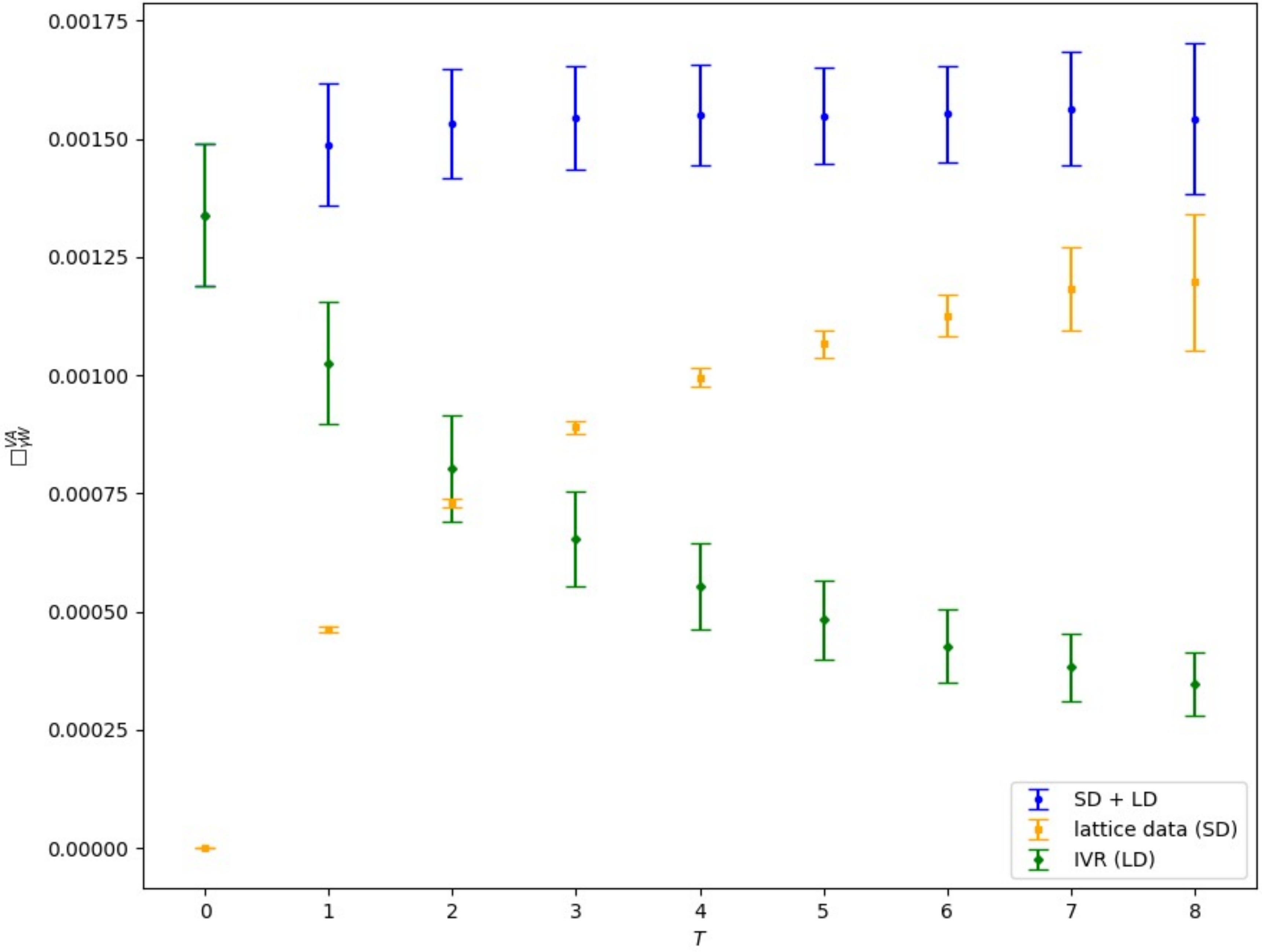


Confirm t_s=3a is a safe choice for reconstruction

Results from IVR



Temporal truncation effects are resolved



Conclusion

- Axial γW-box contribution to $\pi_{\ell 3}$ decay

$$\Box_{\gamma W}^{VA} = 2.830(11)_{\mathrm{stat}}(26)_{\mathrm{sys}} \times 10^{-3}$$

- $K_{\ell 3}$ decay: lattice QCD interplay with EFT

$$X_1 = -2.2(4) \times 10^{-3}, \quad \tilde{X}_6^{\mathrm{phys}} = 13.9(7) \times 10^{-3}$$

- Move towards nucleon beta decay

Target: Provide hadronic matrix elements from first-principles for the accurate determination of CKM matrix element