Vector-like quarks confronting Cabibbo angle anomalies

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B. B. and Z. Berezhiani JHEP 10, 079 (2021), 2103.05549

B. B., R. Beradze and Z. Berezhiani, Eur. Phys. J. C 80, no.2, 149 (2020), 1906.02714

The Standard Model

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim (3,2)_{1/6} \quad u_R \sim (3,1)_{2/3} \quad d_R \sim (3,1)_{-1/3} \quad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim (1,2)_{-1/2} \quad e_R \sim (1,1)_{-1}$$

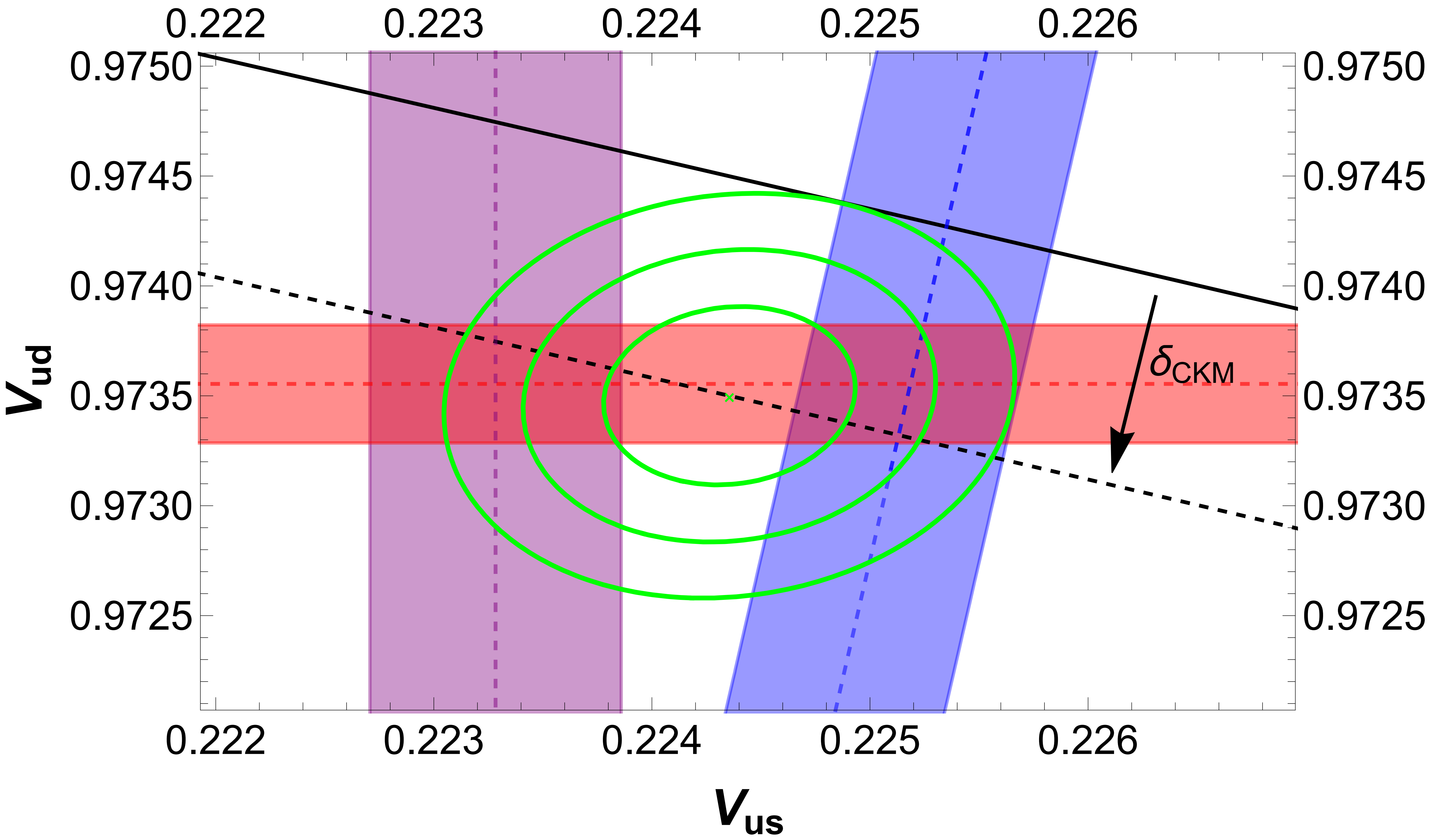
- Three fermion families in identical representations of gauge symmetry, chiral in EW part; a single Higgs doublet

$$-\mathcal{L}_{\text{Yuk}} = Y_u^{ij}\overline{q_{Li}}\tilde{\phi}u_{Rj} + Y_d^{ij}\overline{q_{Li}}\phi d_{Rj} + Y_e^{ij}\overline{\ell_{Li}}\phi e_{Rj} + \text{h.c.}$$

- Yukawa matrices are not diagonal in weak basis: $m^{(u,d,e)ij} = Y^{ij}_{u,d,e} v_w \,,\qquad \mathbf{m}^{u,d,e}_{\text{diag}} = V_L^{(u,d,e)\dagger}\mathbf{m}^{(u,d,e)}V_R^{(u,d,e)}$
- Quark charged currents in mass basis: $\frac{g}{\sqrt{2}}\overline{\begin{pmatrix} u & c & t \end{pmatrix}}_L \gamma^\mu V_L^{(u)\dagger}V_L^{(d)} \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^\dagger_\mu$
- $V_{\text{CKM}} = V_L^{(u)\dagger}V_L^{(d)} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$. **$\mathbf{V}_{\text{CKM}}$ is unitary.** $\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1}$
- Same transformation diagonalizes Yukawa and mass matrices.
- Neutral currents: $\frac{g}{\cos\theta_W}\left(g_L^f \overline{f_L}\gamma^\mu f_L + g_R^f \overline{f_R}\gamma^\mu f_R\right) Z_\mu \,,\qquad g^f_{L,R} = T_3(f_{L,R}) - Q(f)\sin^2\theta_W$
- Yukawa couplings and photon/Z couplings (for unitarity of $V_L^{(u)}$ and $V_L^{(d)}$) are diagonal in mass basis.
- **NO flavour changing neutral currents** at tree level.

Cabibbo angle anomalies

$3\,\sigma$ away from unitarity



C: $|V_{ud}| = 0.97355(27)$ β decays $(\Delta_R^V = 0.02467(22))$

A: $|V_{us}| = 0.22328(58)$ semileptonic $K \to \pi \ell \nu$ decays

$f_+(0) = 0.9698(17)$ FLAG21

B: $\frac{|V_{us}|}{|V_{ud}|} = 0.23130(49)$ ratio of leptonic $K\mu 2/\pi\mu 2$

$f_K/f_\pi = 1.1932(21)$ FLAG21

Unitarity

- CKM unitarity problem: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta_{\rm CKM} \;\to\; \delta_{\rm CKM} \approx 2 \times 10^{-3}$
- Discrepancy between determinations A and B.

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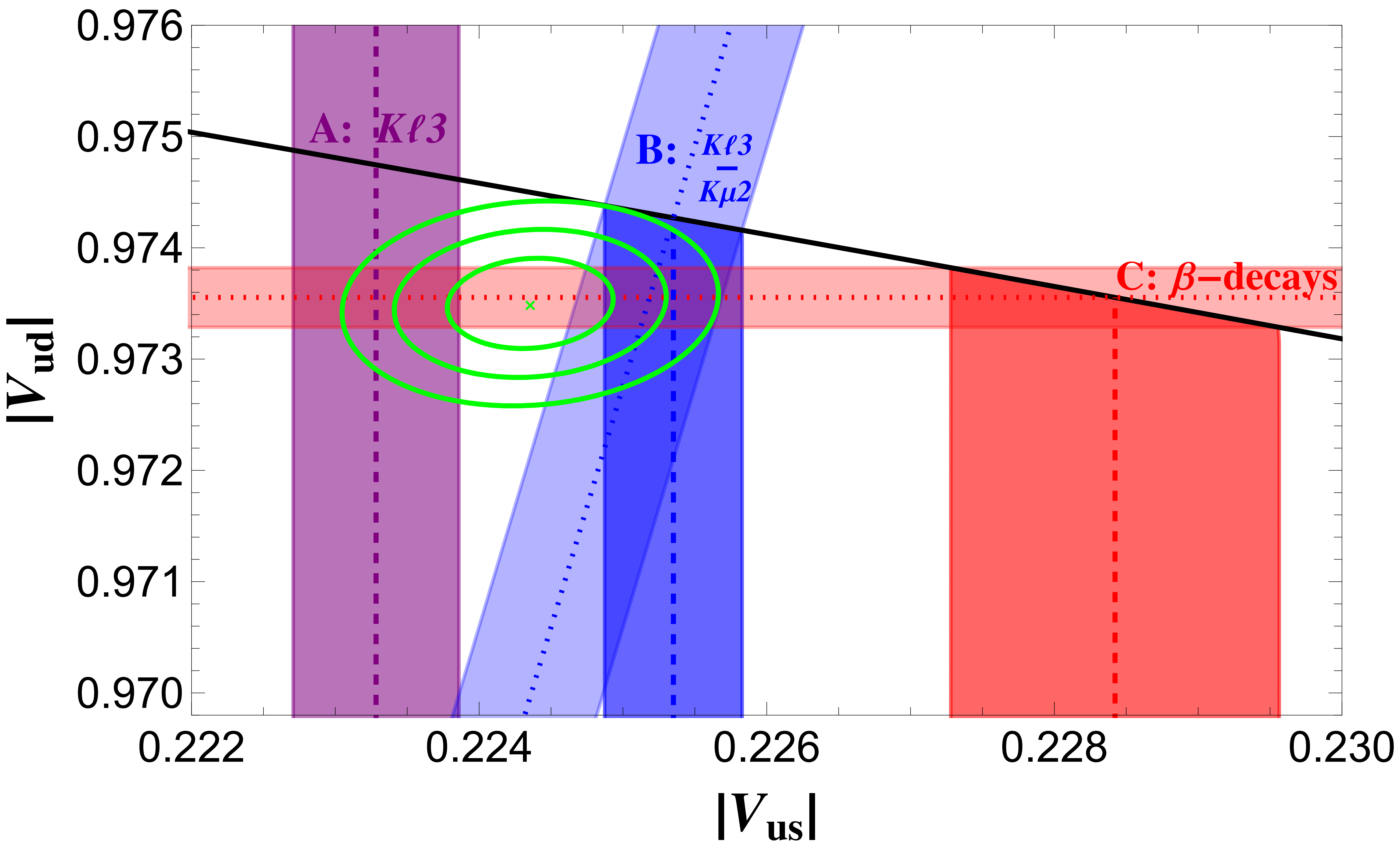

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— Unitarity

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- Discrepancy between determinations A and B.

Cabibbo angle anomalies

A: $|V_{us}| = 0.22328(58)$

B: $\frac{|V_{us}|}{|V_{ud}|} = 0.23130(52)$

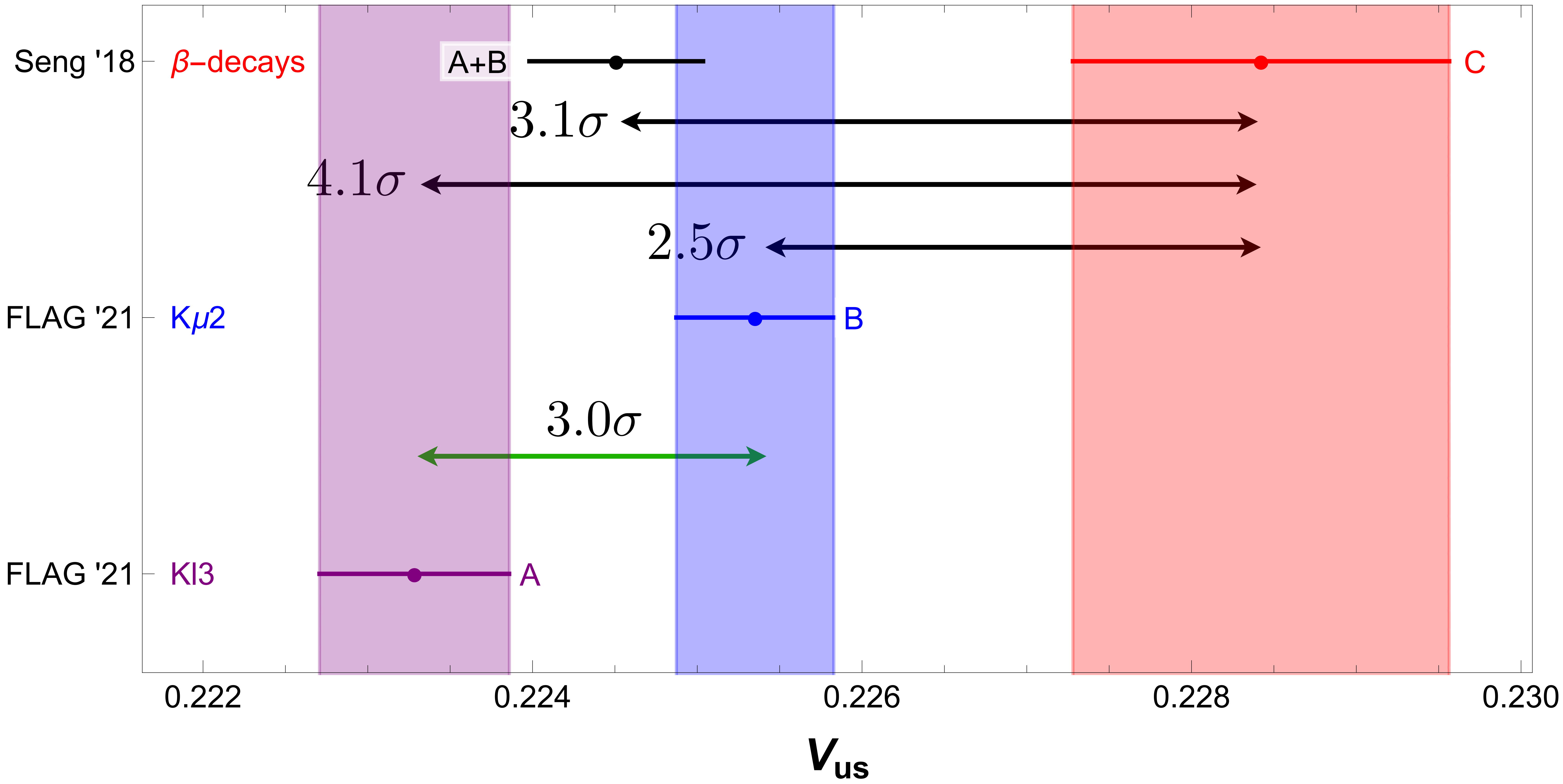
C: $|V_{ud}| = 0.97355(27)$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$|V_{us}|_{\mathrm{A}} = 0.22328(58)$

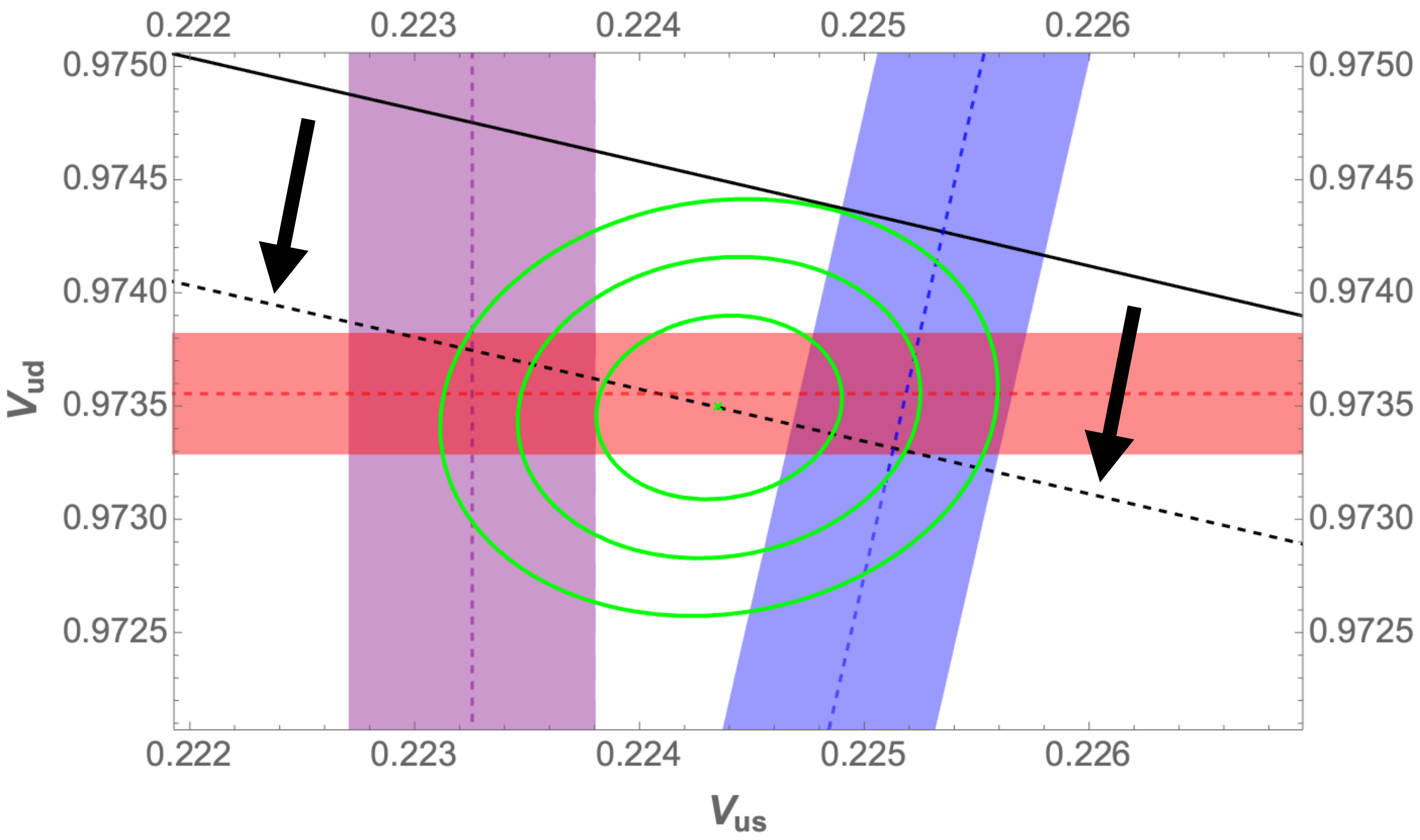
$|V_{us}|_{\mathrm{B}} = 0.22535(48)$

$|V_{us}|_{\mathrm{C}} = 0.2284(11)$



New quark-mixing

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{ub'}|^2 \,, \qquad |V_{ub'}| \approx 0.044 \qquad (|V_{ub'}| \gg |V_{ub}|)$$



4th sequential family is excluded by SM precision tests, LHC mass limits, Higgs production via gluon fusion and its 2γ decay. However additional vector-like fermions can be introduced.

Vector-like quarks

- LH and RH in the same representation of the SM
- predicted in some extensions of the SM
- Their mass terms are not protected by SM gauge symmetries
- Are they a solution for anomalies in the first row of CKM?
- Fermion species in the same representation as the standard quarks and leptons

- Extra down-type weak singlet
- Extra up-type weak singlet
- Extra weak doublet (Y=1/6)

Down-type weak singlet

Down-type vector-like quark $D_{L,R}$, with left and right components both $SU(2)$ singlets, mixing with SM quarks:

$$... + h_{Dj}\,\overline{q_{Lj}}\phi D_R + M_D \overline{D_L} D_R + \text{h.c.}$$

- $\overline{d_{Li}}\mathbf{m}^{(d)}_{ij} d_{Rj} + \text{h.c.} = (\overline{d_{L1}}, \overline{d_{L2}}, \overline{d_{L3}}, \overline{D_L}) \left(\begin{array}{ccc|c} & & & h_d v_w \\ & \mathbf{y}^{(d)}_{3\times 3} v_w & & h_s v_w \\ & & & h_b v_w \\ \hline 0 & 0 & 0 & M \end{array}\right) \begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ D_R \end{pmatrix} + \text{h.c.}$

- $V_L^{(d)\dagger} \mathbf{m}^{(d)} V_R^{(d)} = \mathbf{m}^{(d)}_{\text{diag}} = \text{diag}(m_d, m_s, m_b, M_{b'})$

$$\begin{pmatrix} d_{L1} \\ d_{L2} \\ d_{L3} \\ D_L \end{pmatrix} = V_L^{(d)} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix}, \quad V_L^{(d)} = \begin{pmatrix} V_{L1d} & V_{L1s} & V_{L1b} & V_{L1b'} \\ V_{L2d} & V_{L2s} & V_{L2b} & V_{L2b'} \\ V_{L3d} & V_{L3s} & V_{L3b} & V_{L3b'} \\ V_{LDd} & V_{LDs} & V_{LDb} & V_{LDb'} \end{pmatrix} \approx \begin{pmatrix} & & & 0 \\ & V^{(d)}_{3\times 3} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & \frac{h_d v_w}{M_D} \\ 0 & 1 & 0 & \frac{h_s v_w}{M_D} \\ 0 & 0 & 1 & \frac{h_b v_w}{M_D} \\ -\frac{h_d^* v_w}{M_D} & -\frac{h_s^* v_w}{M_D} & -\frac{h_b^* v_w}{M_D} & 1 \end{pmatrix}$$

$$|V_{LD\alpha}| \approx |h_\alpha| v_w / M_D \qquad s^d_{Ri} \approx \frac{y_i |h_i| v_w^2}{M_D^2}$$

Down-type weak singlet

- Charged weak interactions:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_{L1}} & \overline{u_{L2}} & \overline{u_{L3}} \end{pmatrix} \gamma^\mu \begin{pmatrix} d_{L1} \\ d_{L2} \\ d_{L3} \end{pmatrix} W_\mu^+ + \text{h.c.} = \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} \end{pmatrix} \gamma^\mu \tilde{V}_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix} W_\mu^+ + \text{h.c.}$$

- $\tilde{V}_{\text{CKM}}$ is a $\mathbf{3 \times 4}$ **matrix**
- $\tilde{V}_{\text{CKM}}^\dagger \tilde{V}_{\text{CKM}} \neq \mathbf{1}$, $\tilde{V}_{\text{CKM}} \tilde{V}_{\text{CKM}}^\dagger = \mathbf{1}$: $|\mathbf{V_{ud}}|^2 + |\mathbf{V_{us}}|^2 + |\mathbf{V_{ub}}|^2 = \mathbf{1} - |\mathbf{V_{ub'}}|^2$

$$\tilde{V}_{CKM} = V_L^{(u)\dagger} \tilde{V}_L^{(d)} = \tilde{V}_L^{(d)} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} & V_{ub'} \\ V_{cd} & V_{cs} & V_{cb} & V_{cb'} \\ V_{td} & V_{ts} & V_{tb} & V_{tb'} \end{pmatrix}$$

$\tilde{V}_L^{(d)}$

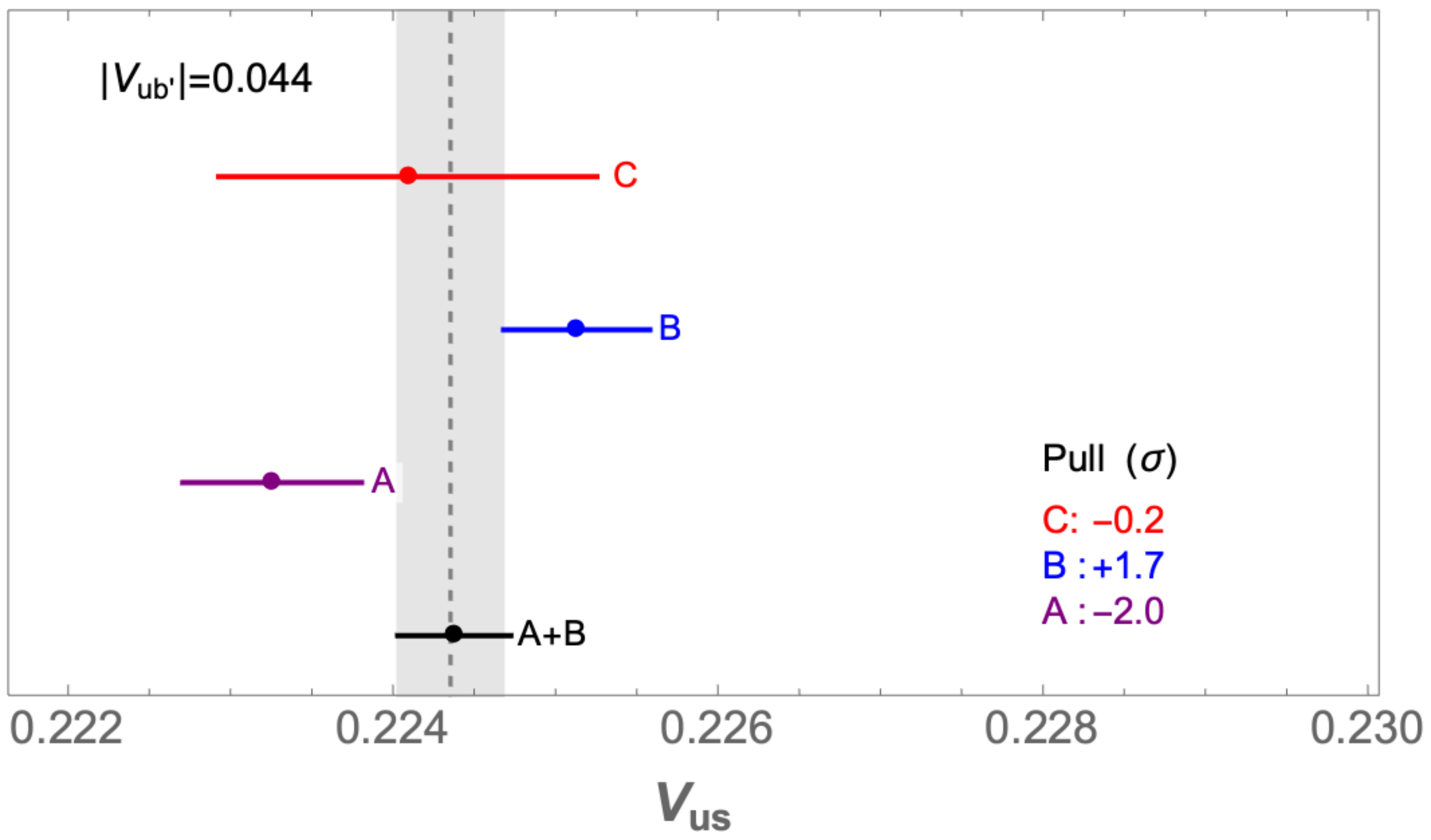
$$\begin{pmatrix} d_{L1} \\ d_{L2} \\ d_{L3} \\ D_L \end{pmatrix} = \begin{pmatrix} V_{L1d} & V_{L1s} & V_{L1b} & V_{L1b'} \\ V_{L2d} & V_{L2s} & V_{L2b} & V_{L2b'} \\ V_{L3d} & V_{L3s} & V_{L3b} & V_{L3b'} \\ V_{LDd} & V_{LDs} & V_{LDb} & V_{LDb'} \end{pmatrix} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix}$$

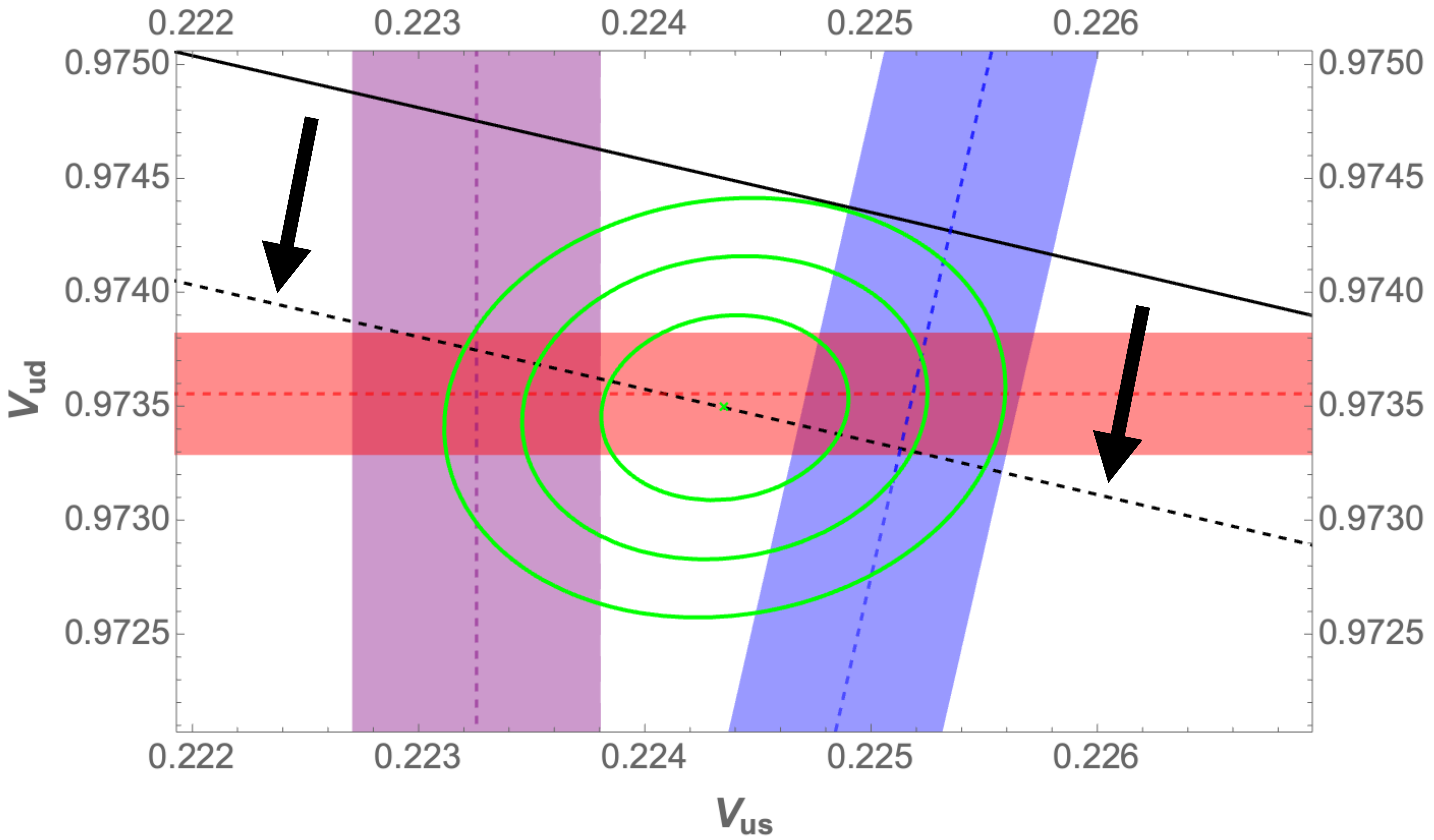
$$|V_{ub'}| \approx |V_{LDd}| \approx |h_d| v_w / M_D$$

Vector-like weak singlets

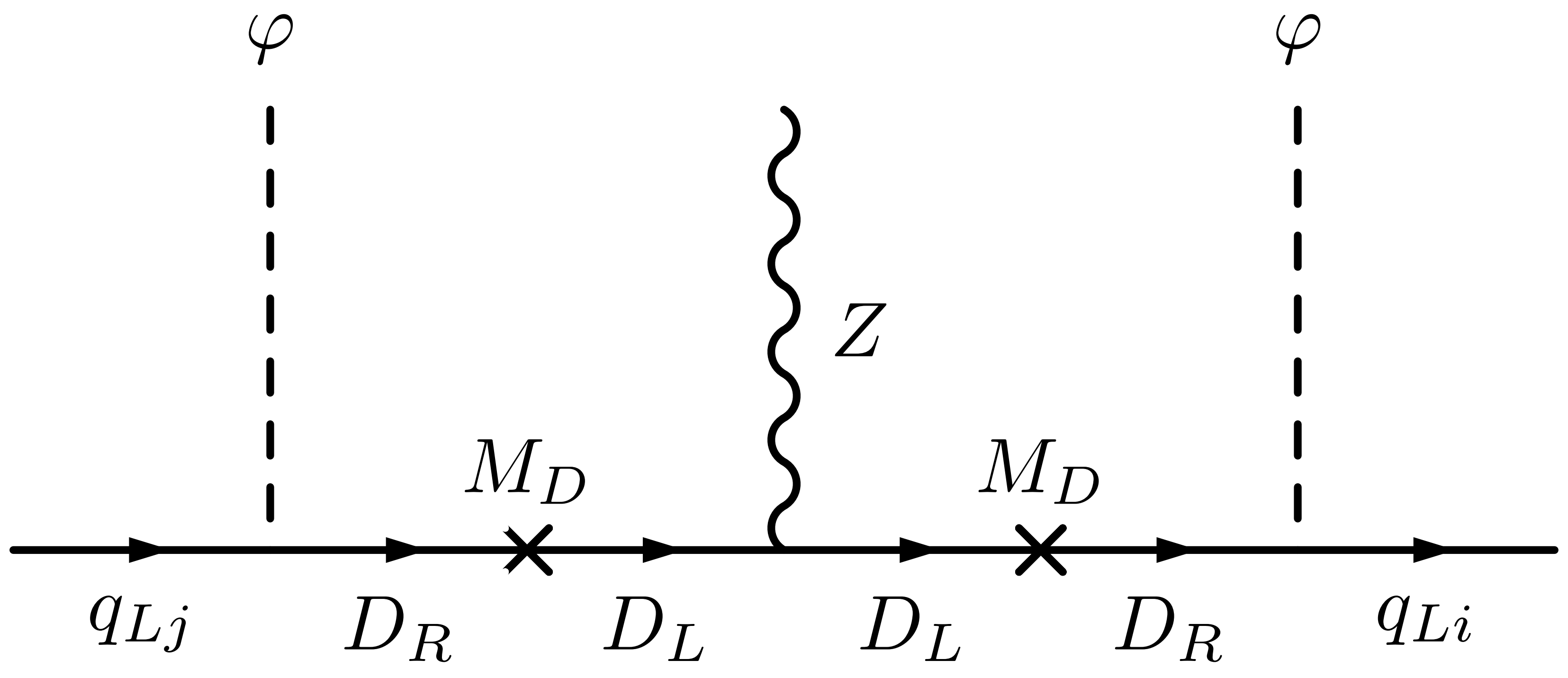
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{ub'}|^2 \,, \qquad |V_{ub'}| \approx 0.044 \qquad (|V_{ub'}| \gg |V_{ub}|)$$

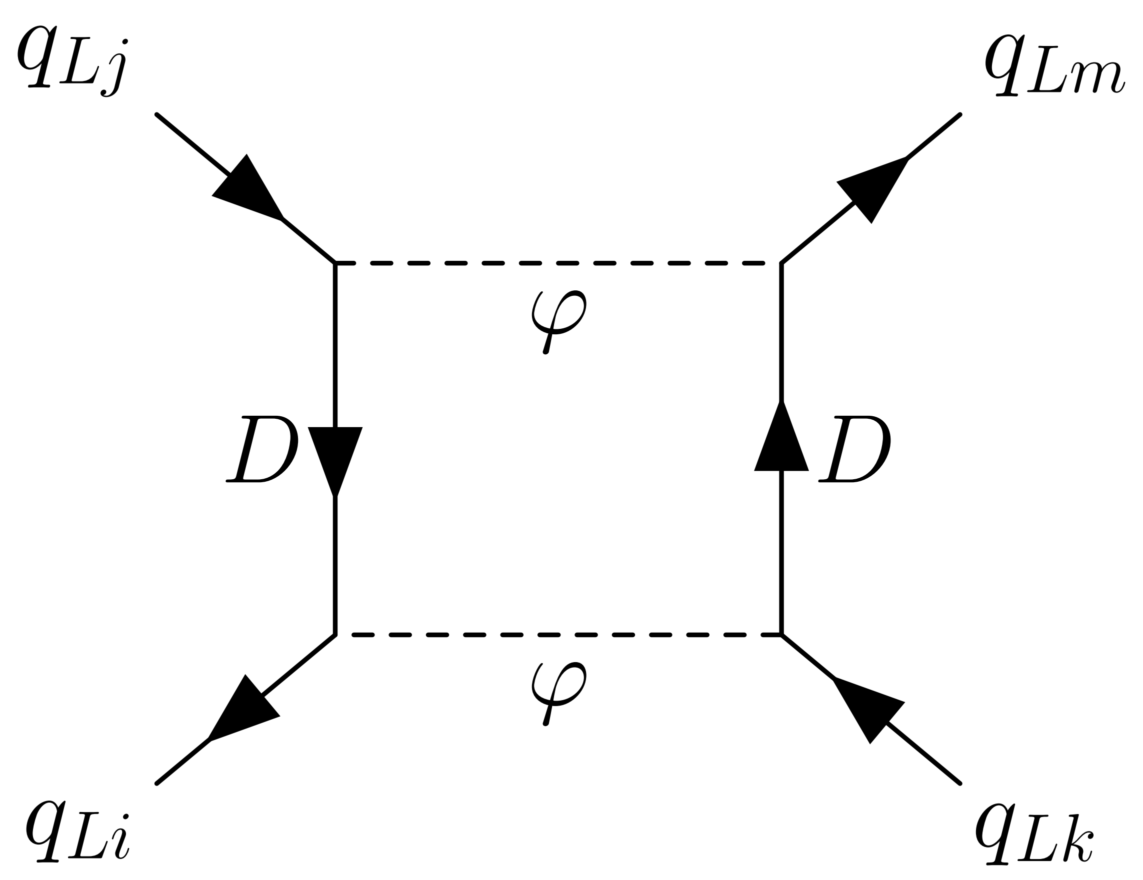
- Extra down-type weak singlets
- Extra up-type weak singlets



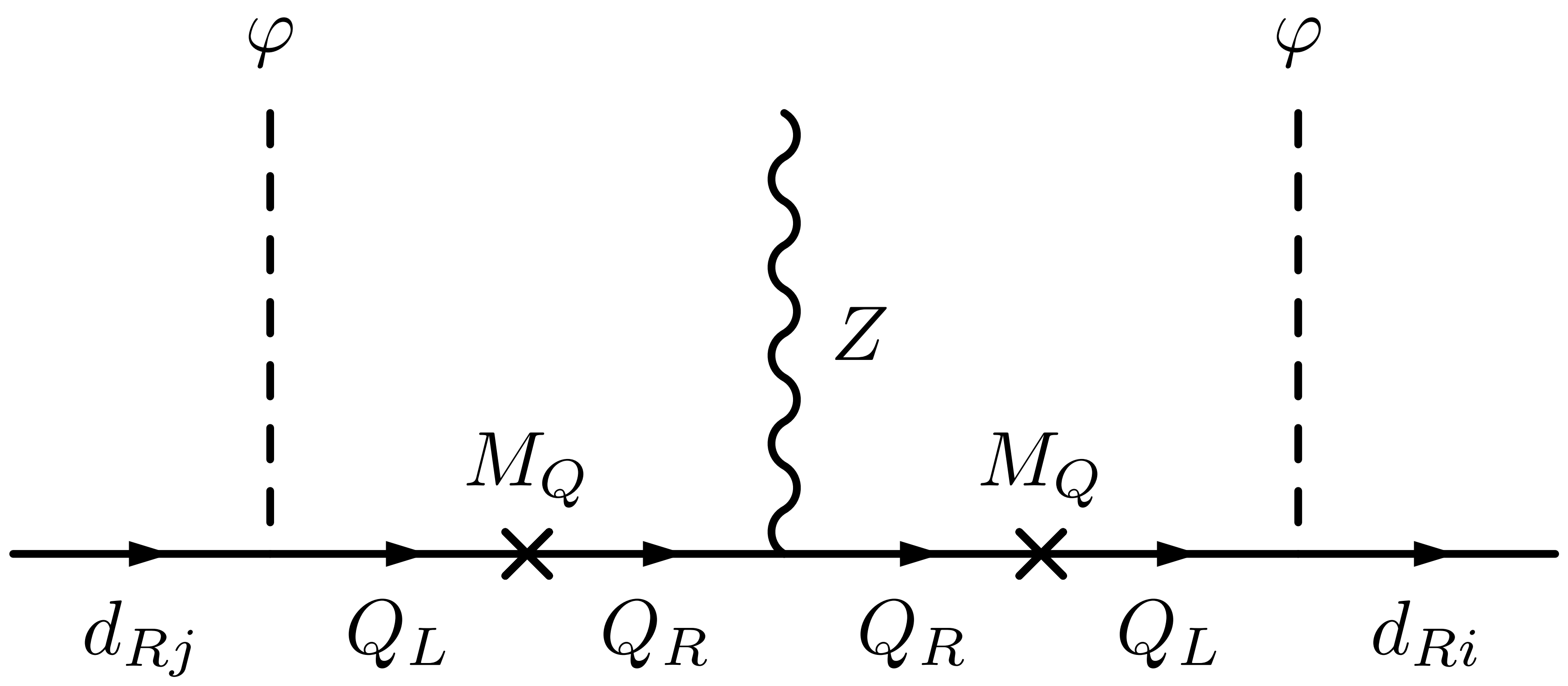


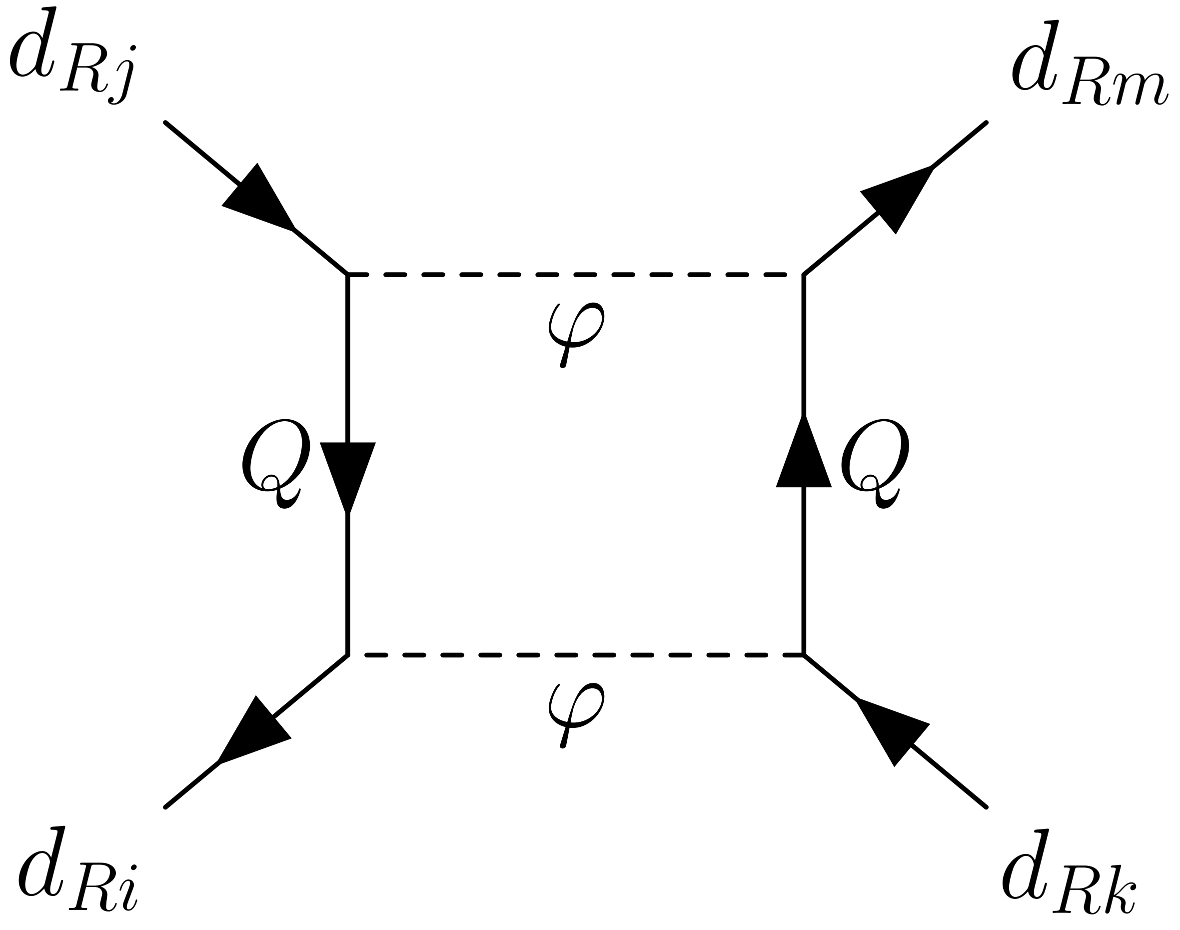
Flavour changing

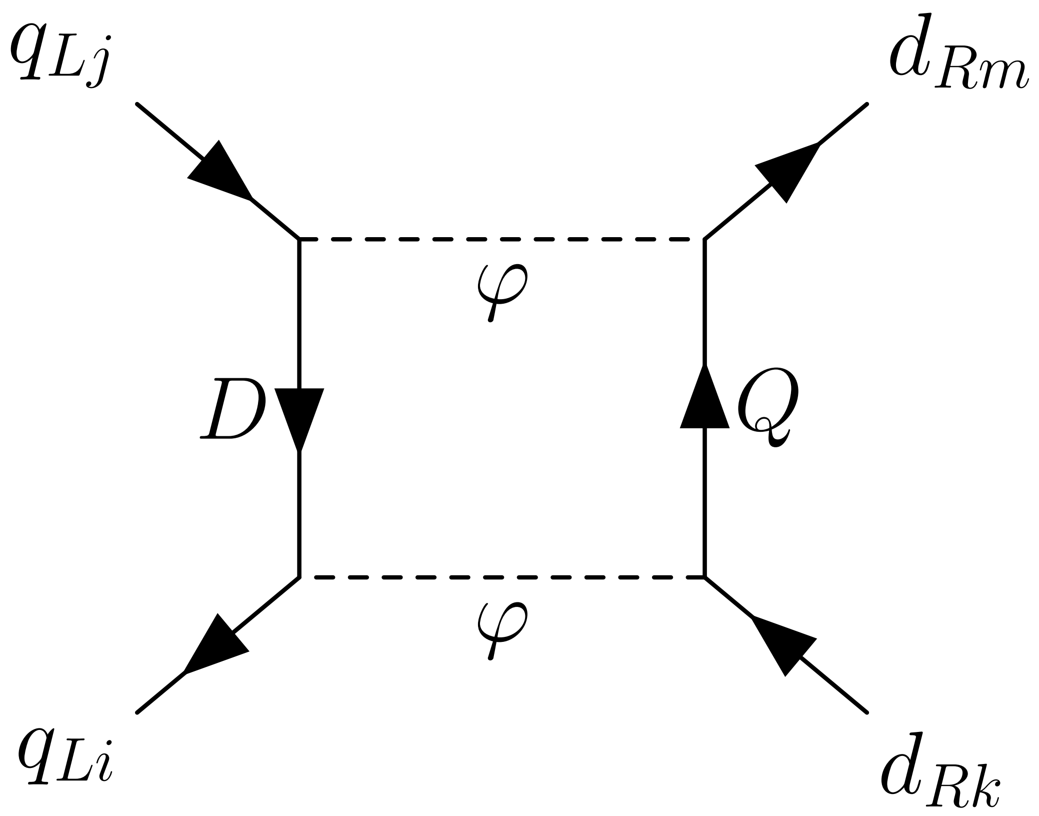




(similarly for up)







Down-type weak singlet

- Weak neutral currents:

$$-\frac{1}{2}\frac{g}{\cos\theta_W}\begin{pmatrix}\overline{d_L} & \overline{s_L} & \overline{b_L} & \overline{b'_L}\end{pmatrix}\gamma^\mu V_L^{(d)\dagger}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&0\end{pmatrix}V_L^{(d)}\begin{pmatrix}d_L\\s_L\\b_L\\b'_L\end{pmatrix}Z_\mu$$

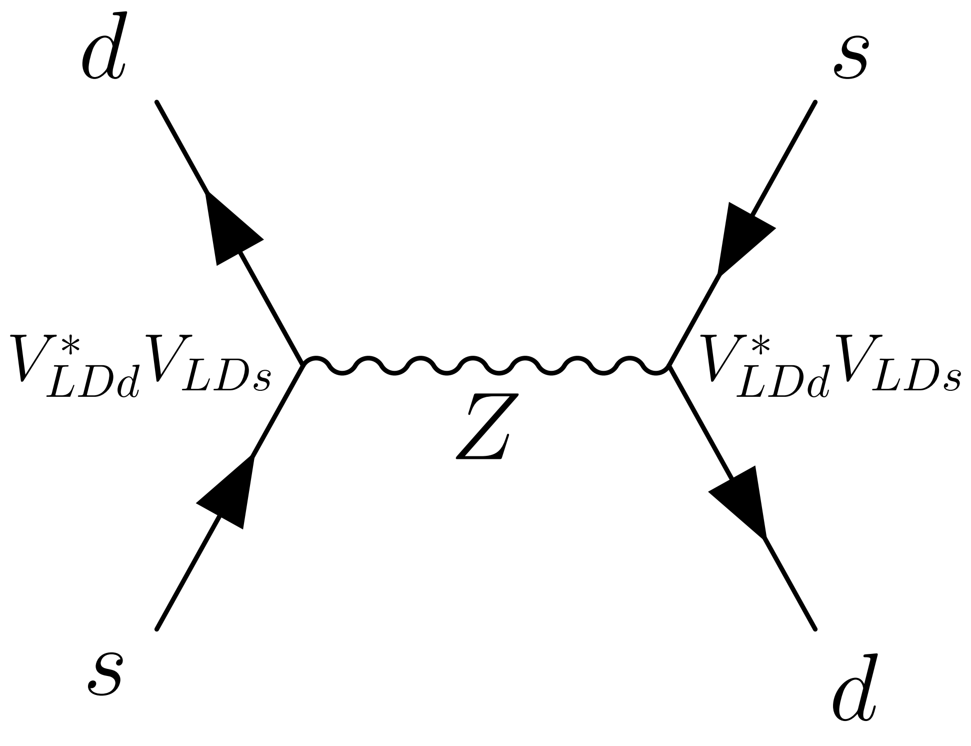
$$(\overline{d_L},\overline{s_L},\overline{b_L},\overline{b'_L})V_L^{(d)\dagger}\begin{pmatrix} & Y^{(d)}_{3\times3} & & h_d\\ & & & h_s\\ & & & h_b\\ 0&0&0&0\end{pmatrix}V_R^{(d)}\begin{pmatrix}d_R\\s_R\\b_R\\b'_R\end{pmatrix}\frac{H^0}{\sqrt{2}} \approx (\overline{d_L},\overline{s_L},\overline{b_L},\overline{b'_L})V_L^{(d)\dagger}\begin{pmatrix}y_d&0&0&h_d\\0&y_s&0&h_s\\0&0&y_b&h_b\\0&0&0&0\end{pmatrix}\begin{pmatrix}d_R\\s_R\\b_R\\b'_R\end{pmatrix}\frac{H^0}{\sqrt{2}}$$

- $\mathcal{L}_{\rm nc} = \frac{g}{\cos\theta_W}Z_\mu\left[\boxed{T_3(f_{L,R})} - Q(f)\sin^2\theta_W\right]\overline{f_{L,R}}\gamma^\mu f_{L,R}$ (not the same quantum numbers)
- Yukawa and mass matrices are not diagonalized by the same transformation;
- Tree and loop level **flavour changing** couplings with the Higgs boson and with Z-boson.
- Also **flavour diagonal** processes change.
- $\Gamma(Z\to \text{had}) - \Gamma(Z\to\text{had})_{\rm SM} \approx \frac{G_F M_Z^3}{\sqrt{2}\pi}\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)\left(|V_{LDd}|^2+|V_{LDs}|^2+|V_{LDb}|^2\right) < 0$

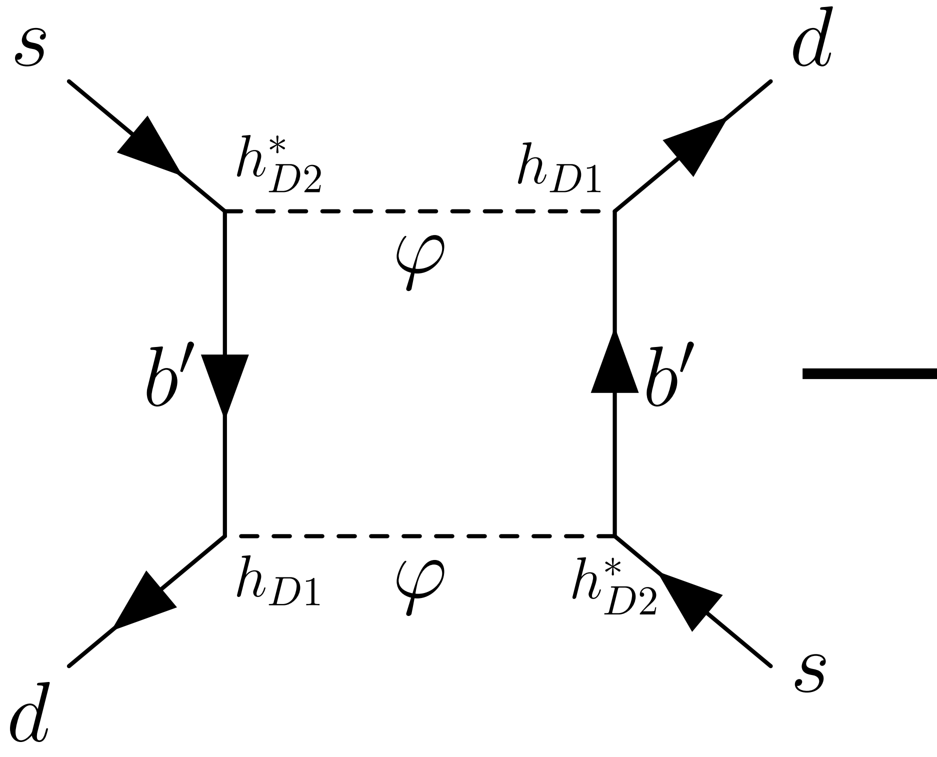
$|V_{LDd}|^2+|V_{LDs}|^2+|V_{LDb}|^2 < 2.5\times10^{-3}$

Down-type weak singlet

Neutral K mesons mixing (neutral currents):



$\propto (V^*_{LDd}V_{LDs})^2$

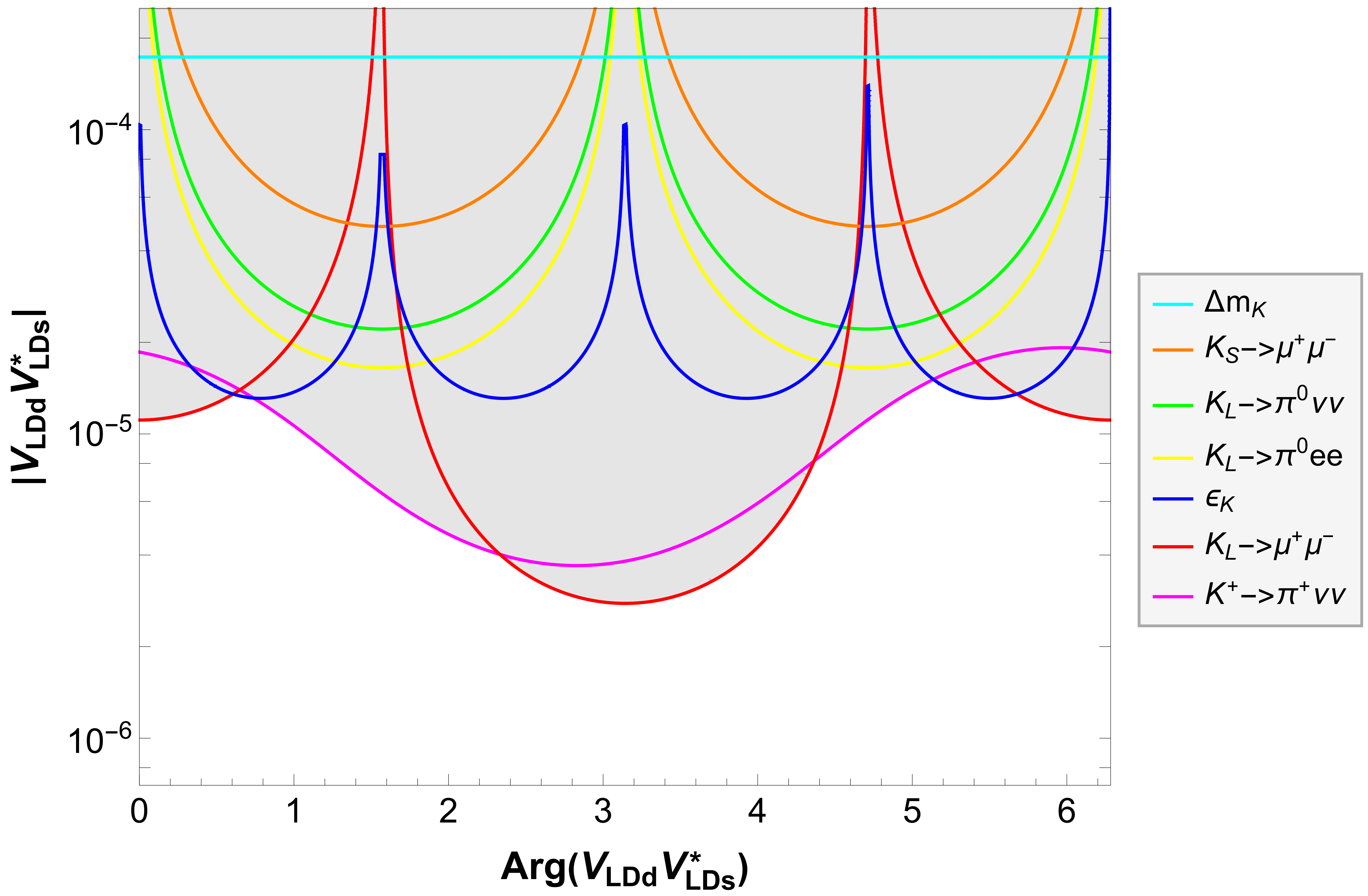


$\propto (V^*_{LDd}V_{LDs})^2 M^2_{b'}$

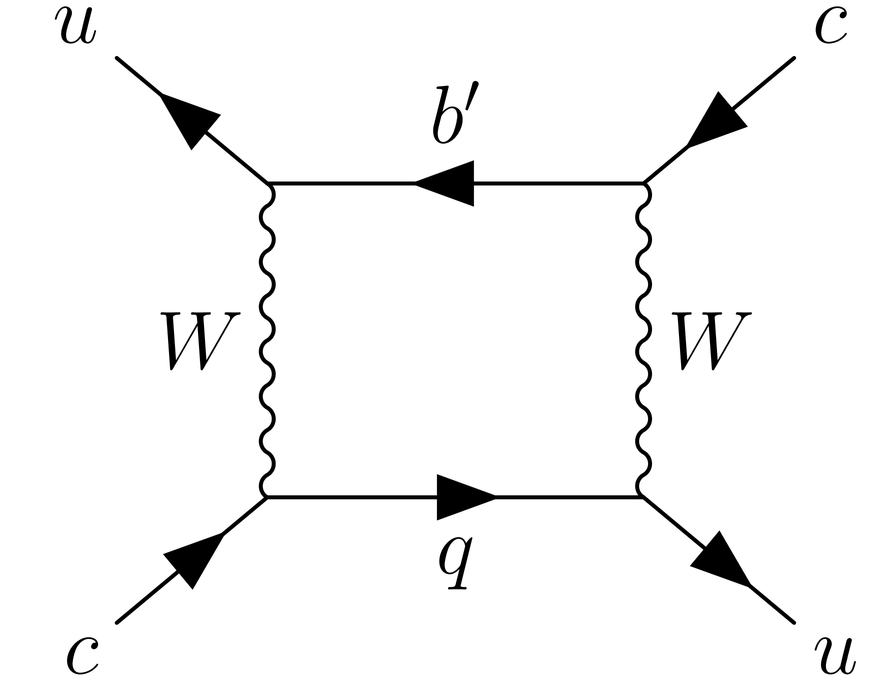
→ Relevant for $M_{b'} > 3$ TeV

$V_{LD\alpha} \approx -h^*_\alpha v_w / M_{b'}$

$M_{b'} = 1$ TeV

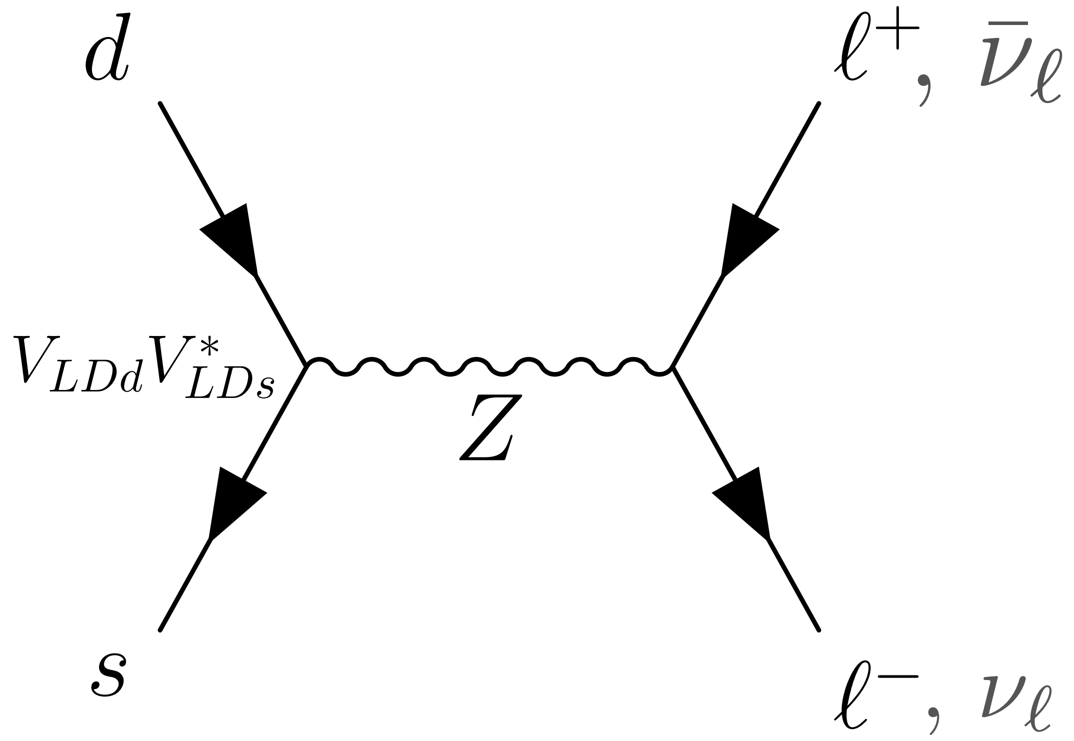


Neutral D mesons mixing (charge currents):



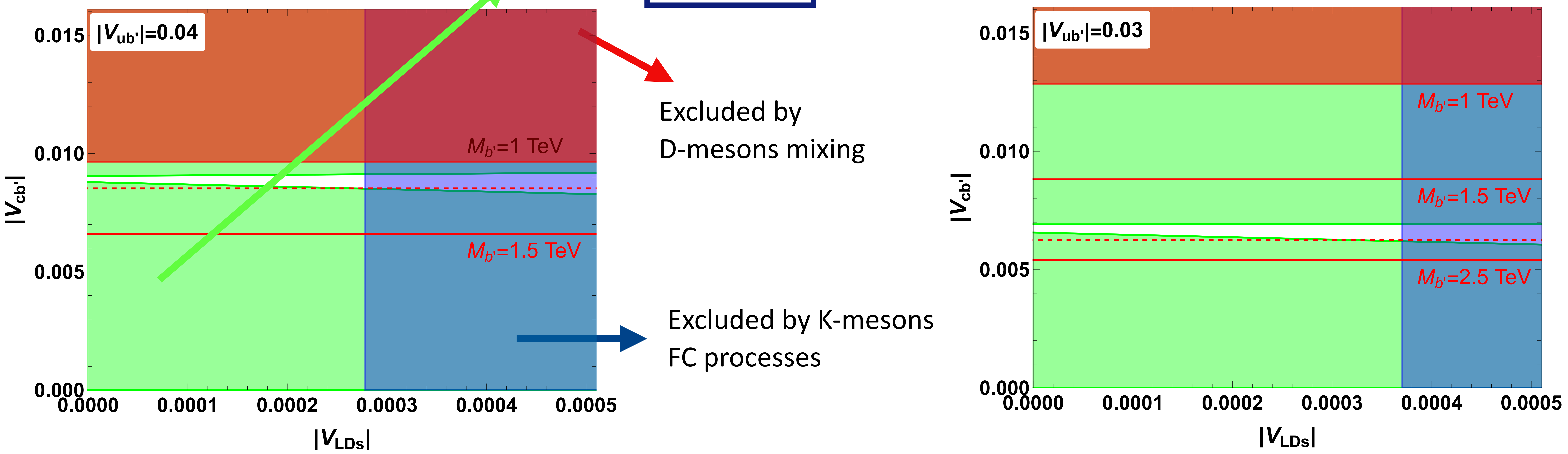
$\propto (V_{ub'}V^*_{cb'})^2 M^2_{b'}$

Kaon decays (nc):



Down-type weak singlet

$|V_{ub'}| \approx |V_{LDd}| \approx |h_d| v_w / M_D$ $\qquad$ $|V_{cb'}| = |-V^*_{LDd} V_{cd} - V^*_{LDs} V_{cs} - V^*_{LDb} V_{cb}|$



- Mass of the extra quark cannot exceed **few TeV**, also in the most conservative case.
- $|V_{ub'}| < 0.042$ with $M_{b'} = 1$ TeV (D^0-mesons mixing).
- $|V_{ub'}| < 0.050$ from Z boson decay rate.
- $|V_{cb'}| \lesssim 10^{-2}$. From B-mesons physics: $|V_{tb'}| = |V_{LDb}| = |h_b| v_w / M_D \lesssim 6 \cdot 10^{-3}$.
- Yukawa couplings h_s, h_b should be respectively 50 and 4 times smaller than h_d.

Up-type weak singlet

Vector-like up-type quark $U_{L,R}$ with left and right components both $SU(2)$ singlets, mixing with SM quarks:

$$\ldots + h_{Uj}\,\overline{q_{Lj}}\tilde{\phi}\,U_R + M_{t'}\overline{U_L}U_R + \text{h.c.}$$

$$\mathcal{L}_{\text{cc}} = \frac{g}{\sqrt{2}}\begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} & \overline{t'_L} \end{pmatrix}\gamma^\mu \tilde{V}_{\text{CKM}}\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W^+_\mu + \text{h.c.}$$

- $\tilde{V}_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{t'd} & V_{t's} & V_{t'b} \end{pmatrix} = \tilde{V}_L^{(u)\dagger} V_L^{(d)}$;
- $\tilde{V}_L^{(u)}$ is the 3×4 submatrix of $V_L^{(u)}$ without the last row, $\tilde{V}_{CKM}$ is 4×3 matrix.
- $\tilde{V}_L^{(u)}$ and $\tilde{V}_{\text{CKM}}$ are not unitary $\tilde{V}_{\text{CKM}}\tilde{V}^\dagger_{\text{CKM}} = \tilde{V}_L^{(u)\dagger}\tilde{V}_L^{(u)} \neq \mathbf{1}$.

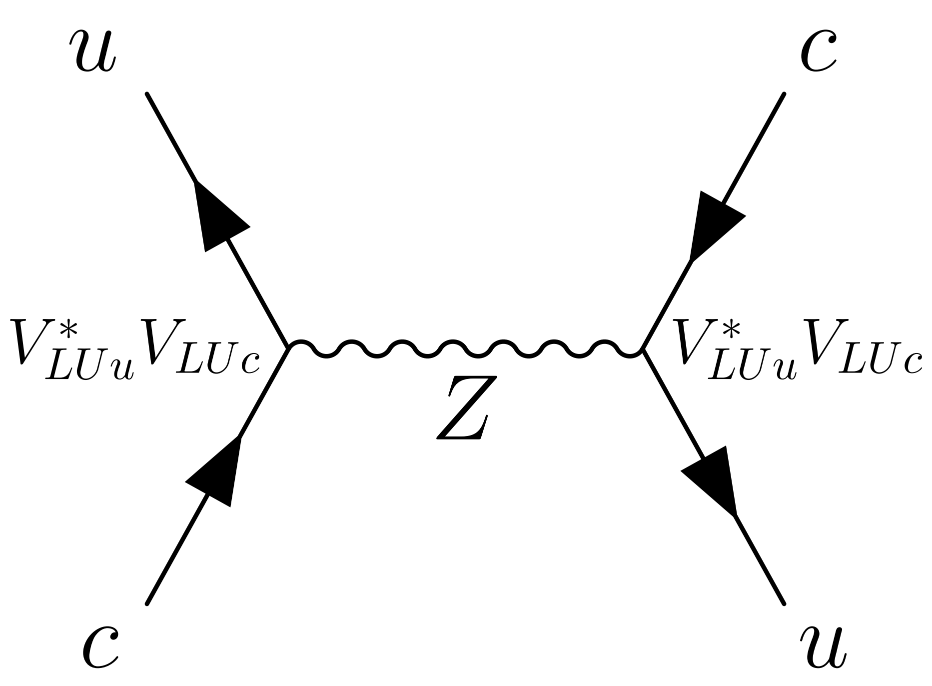
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{LUu}|^2\,, \qquad |V_{LUu}| \approx |h_u| v_w / M_{t'} \approx 0.044$$

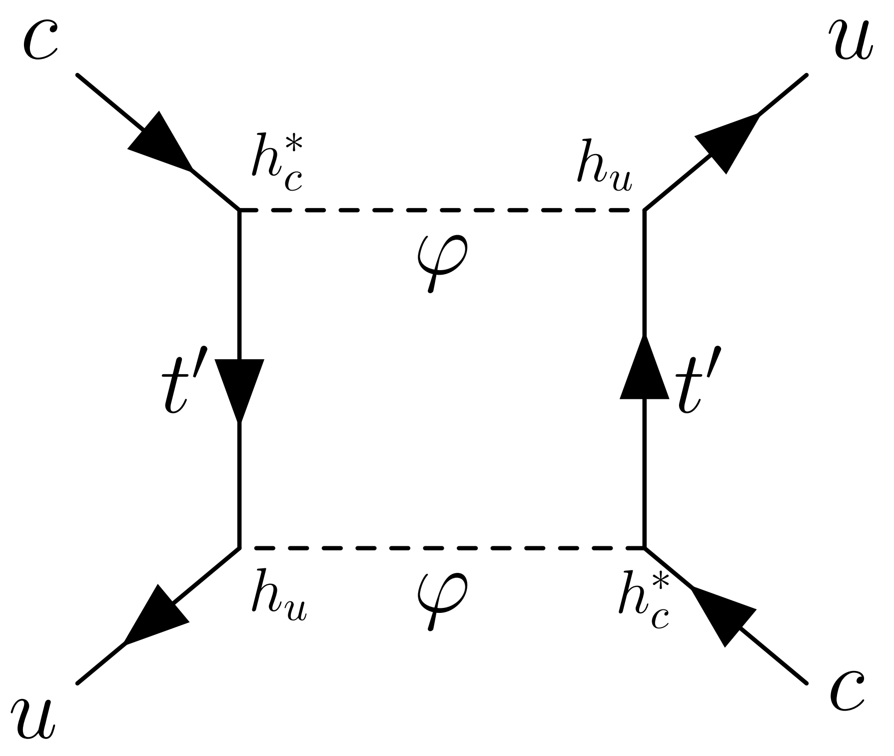
Up-type isosinglet

- The non-unitarity of $\tilde{V}_L^{(u)}$ gives flavour changing couplings of quarks with Z boson and Higgs.

$$\frac{g}{\cos\theta_W}\left[\frac{1}{2}\begin{pmatrix}\overline{u_L} & \overline{c_L} & \overline{t_L} & \overline{t'_L}\end{pmatrix}\gamma^\mu \tilde{V}_L^{(u)\dagger}\tilde{V}_L^{(u)}\begin{pmatrix}u\\c\\t\\t'\end{pmatrix}_L - \frac{2}{3}\sin^2\theta_W\left(\overline{\mathbf{u}_L}\gamma^\mu \mathbf{u}_L + \overline{\mathbf{u}_R}\gamma^\mu \mathbf{u}_R\right)\right] Z_\mu$$

$$(\overline{u_L}, \overline{c_L}, \overline{t_L}, \overline{t'_L}) V_L^{(u)\dagger}\begin{pmatrix} & & & h_u\\ & Y^{(u)}_{3\times 3} & & h_c\\ & & & h_t\\ 0 & 0 & 0 & 0\end{pmatrix} V_R^{(u)}\begin{pmatrix}u_R\\c_R\\t_R\\t'_R\end{pmatrix}\frac{H^0}{\sqrt{2}} + \text{h.c.}$$





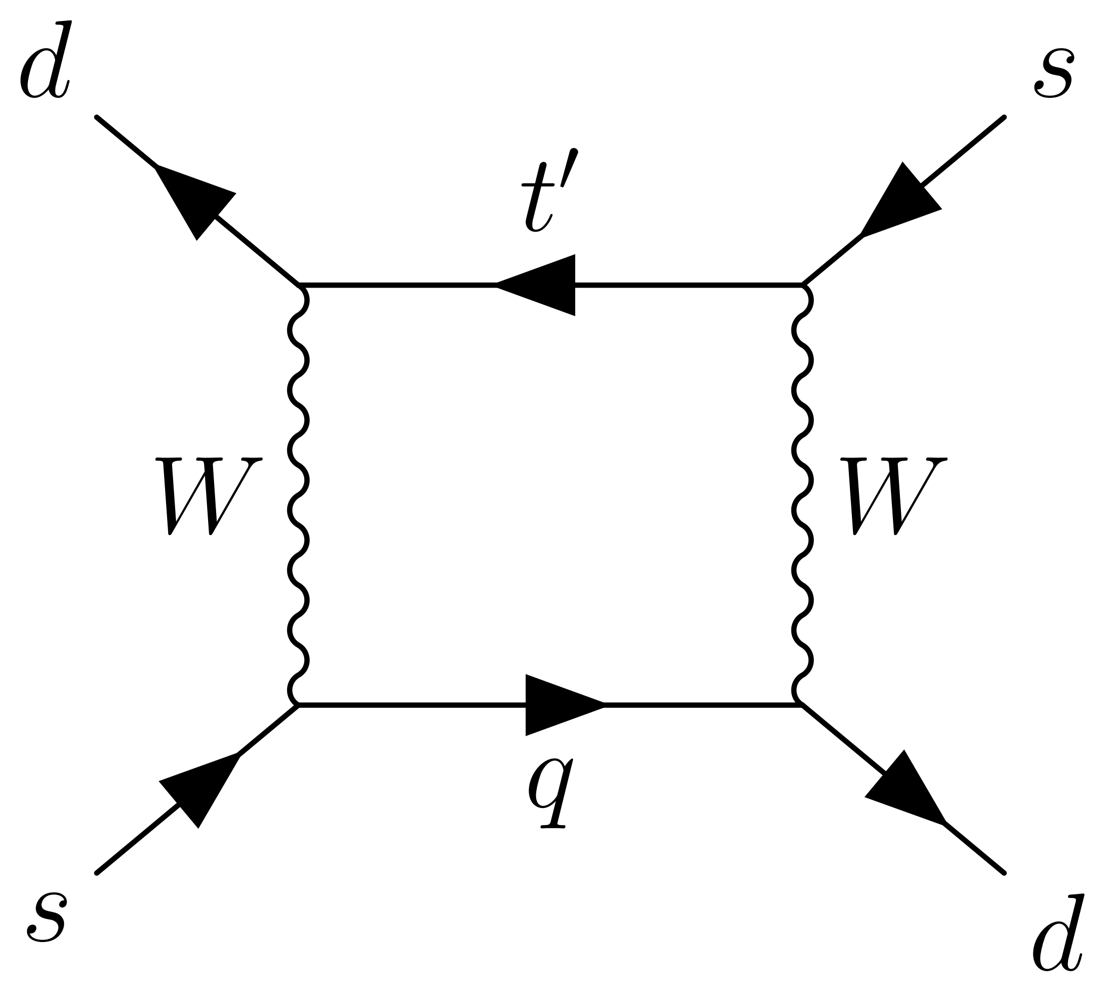
$$\propto (V^*_{LUu}V_{LUc})^2 M_{b'}^2\left[1 + \left(\frac{M_{t'}}{3.1\,\text{TeV}}\right)^2\right]$$

Up-type isosinglet

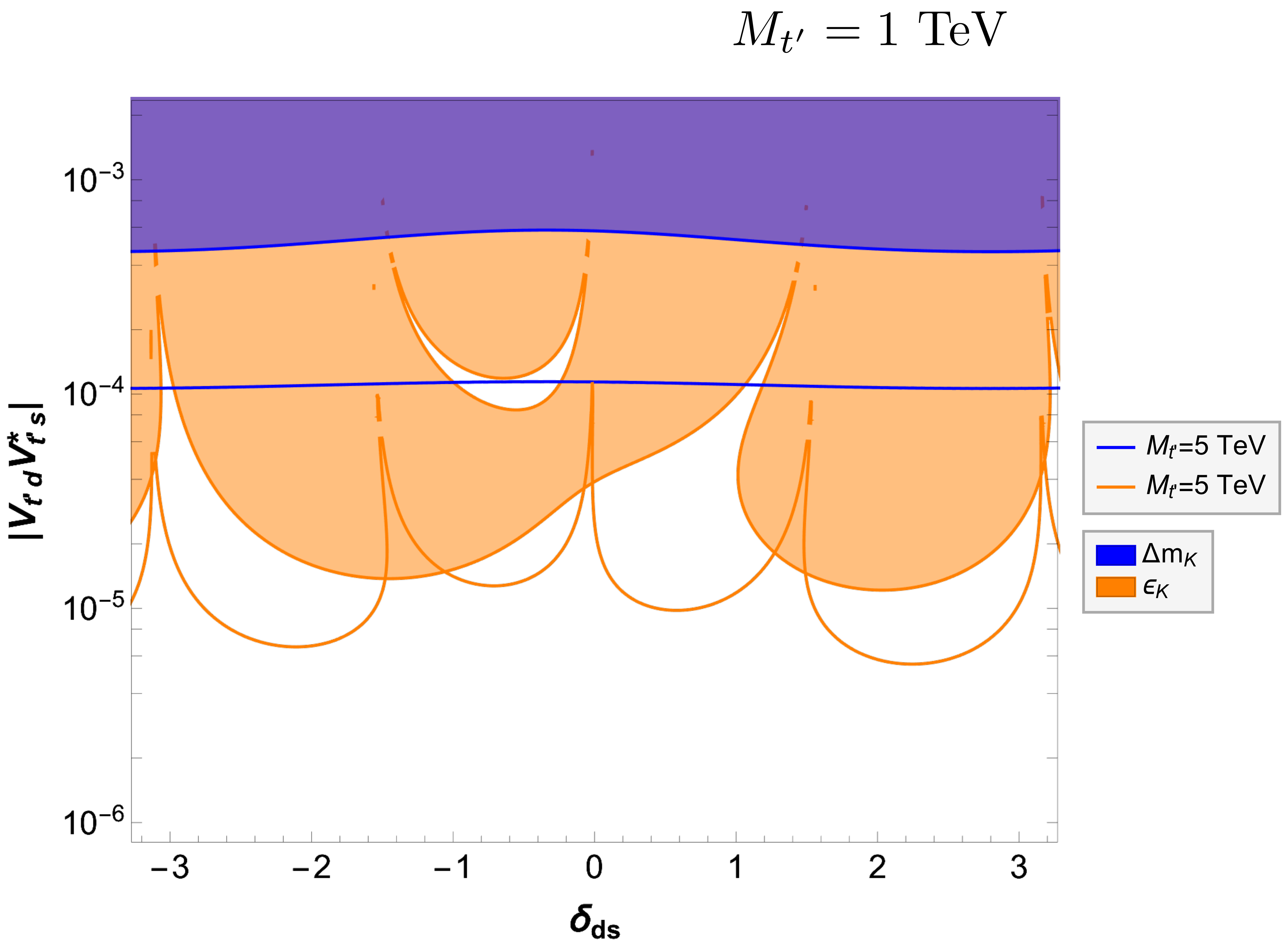
Constraints from K mesons mixing:

$$V_{t'd} = -V_{LUu} = h_u^* v_w / M_{t'}$$

$$V_{t's} = -V_{LUu}V_{us} - V_{LUc}V_{cs} - V_{LUt}V_{ts}$$



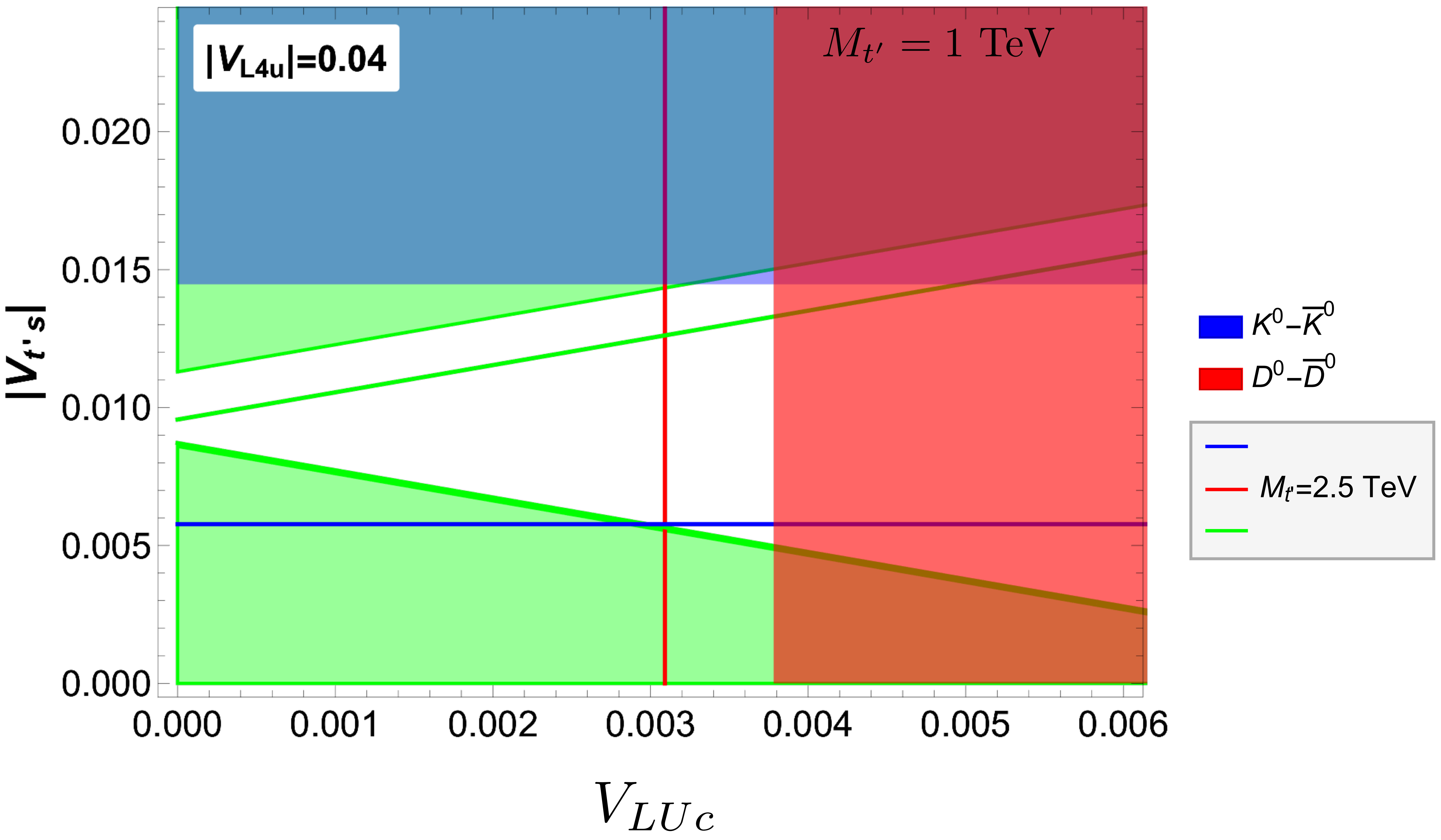
$$\sim (V_{t'd}V_{t's}^*)^2 M_{t'}^2 + \ldots$$

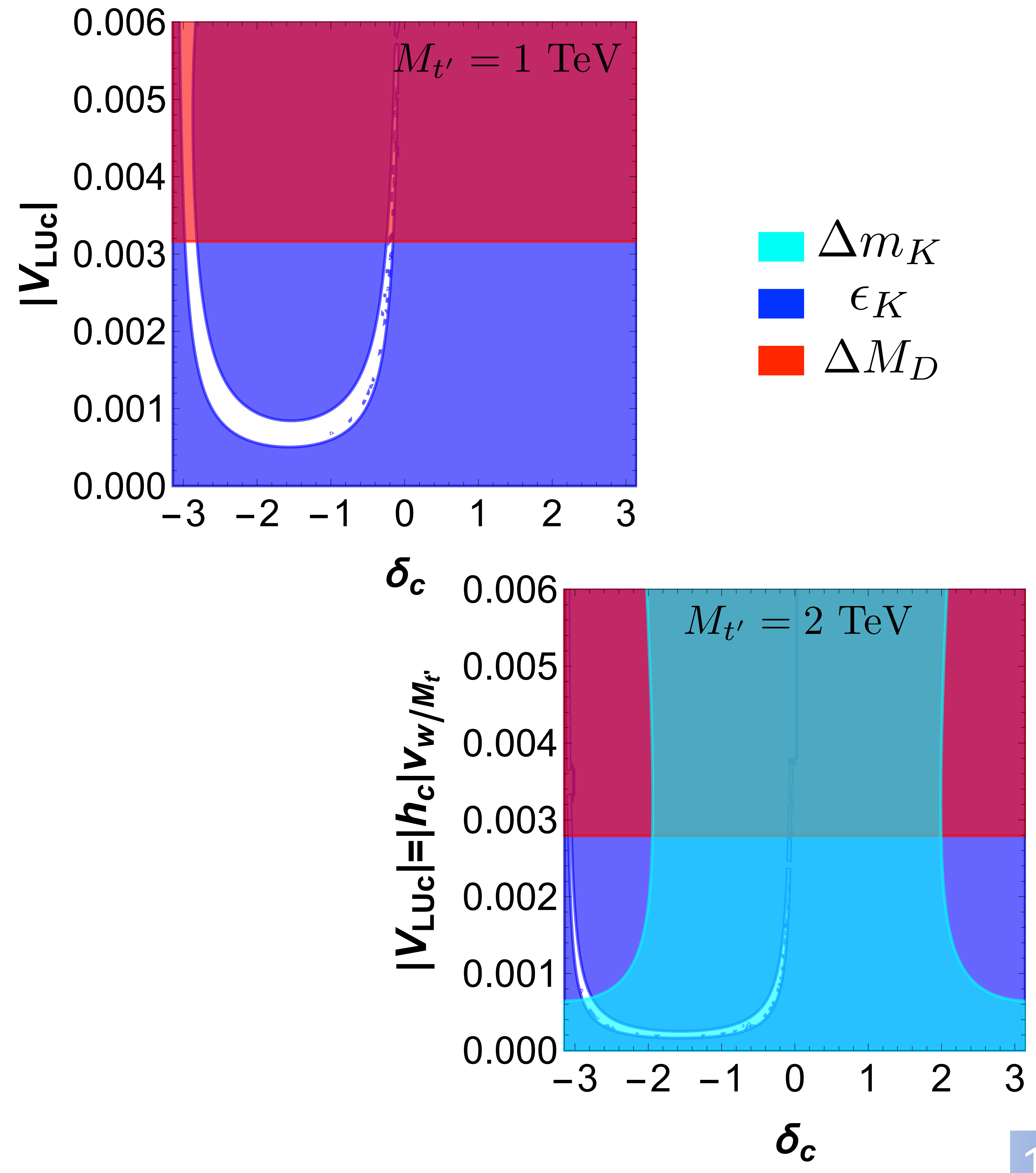


Up-type weak singlet

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{LUu}|^2 \,, \qquad |V_{t'd}| \approx |V_{LUu}| \approx |h_u| v_w / M_{t'} \approx 0.044$$

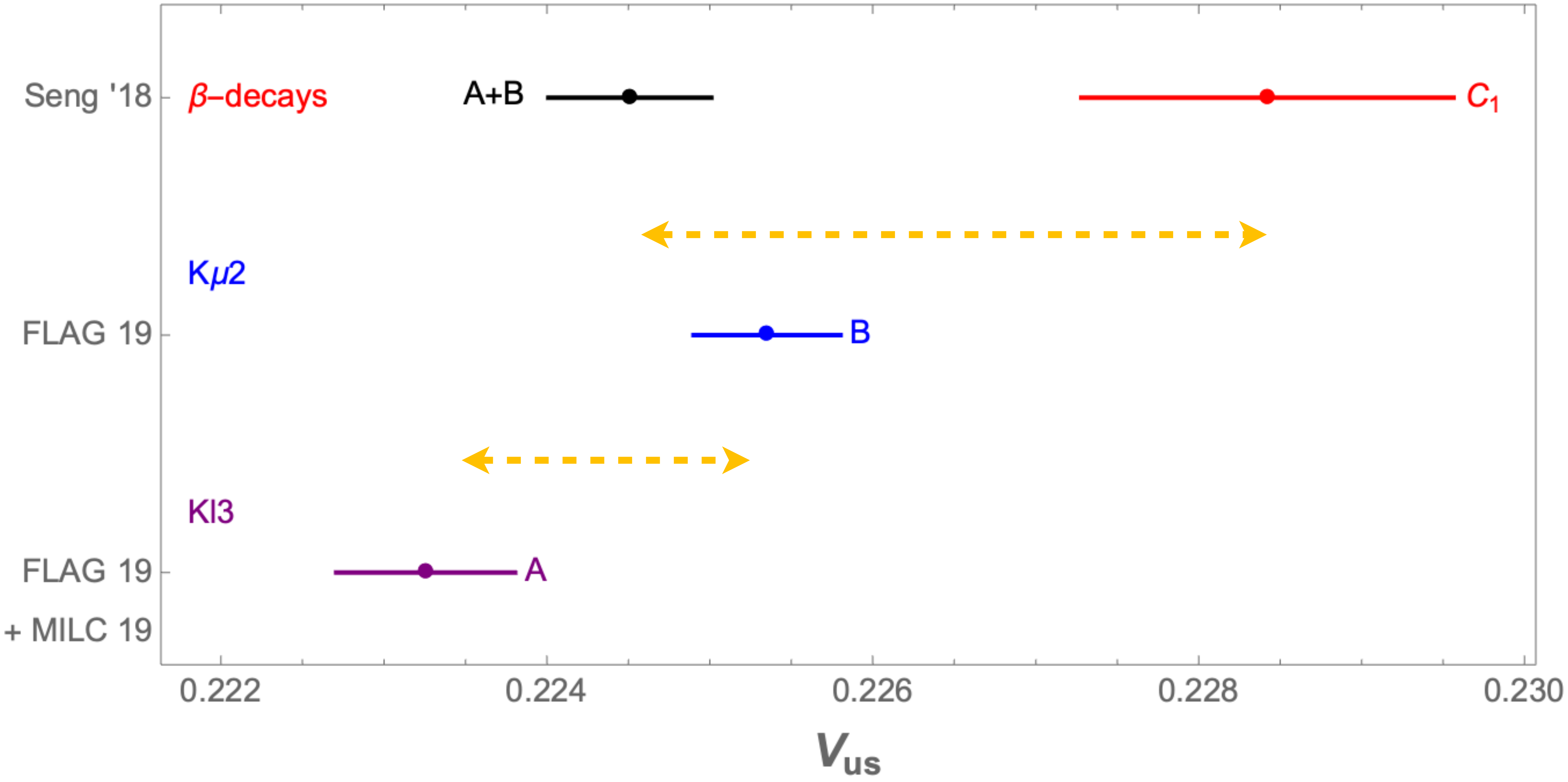
$$|V_{t's}| = |V_{t'd}V_{us} - V_{LUc}V_{cs} + V_{t'b}V_{ts}|$$





Vector-like weak doublet

- Vectorlike extra $SU(2)$-doublet: $q_{4\,L,R} = \begin{pmatrix} u_4 \\ d_4 \end{pmatrix}_{L,R}$



trying to explain both the gaps…

Vector-like weak doublet

- Vectorlike extra $SU(2)$-doublet: $q_{4\,L,R} = \begin{pmatrix} u_4 \\ d_4 \end{pmatrix}_{L,R}$ $\qquad \ldots + h_{uj}\overline{q_{L4}}\tilde{\varphi}u_{Rj} + h_{d\alpha}\overline{q_{L4}}\varphi d_{R\alpha} + M_q\overline{q_{L4}}q_{R4} + \text{h.c.}$

$$\overline{d_{Li}}\mathbf{m}^{(d)}_{ij}d_{Rj} + \text{h.c.} = \begin{pmatrix} \overline{d_{L1}} & \overline{d_{L2}} & \overline{d_{L3}} & \overline{d_{L4}} \end{pmatrix} \begin{pmatrix} & \mathbf{y}^{(d)}_{3\times3}v_w & & \begin{matrix}0\\0\\0\end{matrix} \\ h_d v_w & h_s v_w & h_b v_w & M_q \end{pmatrix} \begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ d_{R4} \end{pmatrix} + \text{h.c.}$$

$$V_L^{(d)\dagger}\mathbf{m}^{(d)}V_R^{(d)} = \mathbf{m}^{(d)}_{\text{diag}}\,, \quad V_L^{(u)\dagger}\mathbf{m}^{(u)}V_R^{(u)} = \mathbf{m}^{(u)}_{\text{diag}}$$

- $V^{(d,u)}_{L,R}$ are unitary 4×4 matrices: $\begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ d_{R4} \end{pmatrix} = V_R^{(d)} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \end{pmatrix}, \quad \begin{pmatrix} u_{R1} \\ u_{R2} \\ u_{R3} \\ u_{R4} \end{pmatrix} = V_R^{(u)} \begin{pmatrix} u_R \\ c_R \\ t_R \\ t'_R \end{pmatrix}$

$$V_{dR} = \begin{pmatrix} V_{R\,1d} & V_{R\,1s} & V_{R\,1b} & V_{R\,1b'} \\ V_{R\,2d} & V_{R\,2s} & V_{R\,2b} & V_{R\,2b'} \\ V_{R\,3d} & V_{R\,3s} & V_{R\,3b} & V_{R\,3b'} \\ V_{R\,4d} & V_{R\,4s} & V_{R\,4b} & V_{R\,4b'} \end{pmatrix} \approx \begin{pmatrix} 1 & 0 & 0 & \frac{h_d^* v_w}{M_Q} \\ 0 & 1 & 0 & \frac{h_s^* v_w}{M_Q} \\ 0 & 0 & 1 & \frac{h_b^* v_w}{M_Q} \\ -\frac{h_d v_w}{M_Q} & -\frac{h_s v_w}{M_Q} & -\frac{h_b v_w}{M_Q} & 1 \end{pmatrix} + O(h_i^* h_j v_w^2/M_Q^2)$$

$$V_{R4\alpha} \approx -h_\alpha v_w/M_q$$

$$s^{u,d}_{Li} \approx \frac{y_i|h_i|v_w^2}{M_q^2}$$

Vector-like weak doublet

- The charged-current Lagrangian is:

$$\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} \sum_{i=1}^{4} (\overline{u_{Li}} \gamma^\mu d_{Li}) W_\mu^+ + \frac{g}{\sqrt{2}} \overline{u_{R4}} \gamma^\mu d_{R4} W_\mu + \text{h.c.} =$$

$$= \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_L} & \overline{c_L} & \overline{t_L} & \overline{t'_L} \end{pmatrix} \gamma^\mu V_{\text{CKM,L}} \begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix} W_\mu^+ + \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u_R} & \overline{c_R} & \overline{t_R} & \overline{t'_R} \end{pmatrix} \gamma^\mu V_{\text{R}} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \end{pmatrix} W_\mu^+ + \text{h.c.}$$

- $V_{\text{CKM,L}} = V_L^{(u)\dagger} V_L^{(d)}$ is a 4×4 unitary matrix.
- Weak **charged RH currents** with mixing $\mathbf{V}_{\text{R}}$.

$$V_{\text{CKM,R}} = V_R^{(u)\dagger} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} V_R^{(d)} = \begin{pmatrix} V^*_{R\,4u} V_{R\,4d} & V^*_{R\,4u} V_{R\,4s} & V^*_{R\,4u} V_{R\,4b} & V^*_{R\,4u} V_{R\,4b'} \\ V^*_{R\,4c} V_{R\,4d} & V^*_{R\,4c} V_{R\,4s} & V^*_{R\,4c} V_{R\,4b} & V^*_{R\,4c} V_{R\,4b'} \\ V^*_{R\,4t} V_{R\,4d} & V^*_{R\,4t} V_{R\,4s} & V^*_{R\,4t} V_{R\,4b} & V^*_{R\,4t} V_{R\,4b'} \\ V^*_{R\,4t'} V_{R\,4d} & V^*_{R\,4t'} V_{R\,4s} & V^*_{R\,4t'} V_{R\,4b} & V^*_{R\,4t'} V_{R\,4b'} \end{pmatrix}$$

$$V_{R4\alpha} \approx -h_\alpha v_w / M_q$$

Vector-like weak doublet

- Couplings of u-quark with down quarks (first row of SM CKM matrix) become:

$$\frac{g}{\sqrt{2}}(\overline{u_L}\gamma^\mu V_{L\,ud} d_L + \overline{u_R} V_{R\,ud}\gamma^\mu d_R)\, W_\mu + \frac{g}{\sqrt{2}}(\overline{u_L}\gamma^\mu V_{L\,us} s_L + \overline{u_R} V_{R\,us}\gamma^\mu s_R)\, W_\mu + \text{h.c.}$$

- Then, in this scenario, we are determining vector and axial couplings:

$$\begin{aligned} \text{semileptonic K decay} \quad A &: \qquad |V_{L\,us} + V_{R\,us}| = 0.22326(55) \\ \text{leptonic K decay} \quad B &: \qquad \frac{|V_{L\,us} - V_{R\,us}|}{|V_{L\,ud} - V_{R\,ud}|} = 0.23130(49) \\ \text{superallowed beta decays} \quad C &: \qquad |V_{L\,ud} + V_{R\,ud}| = 0.97355(27) \end{aligned}$$

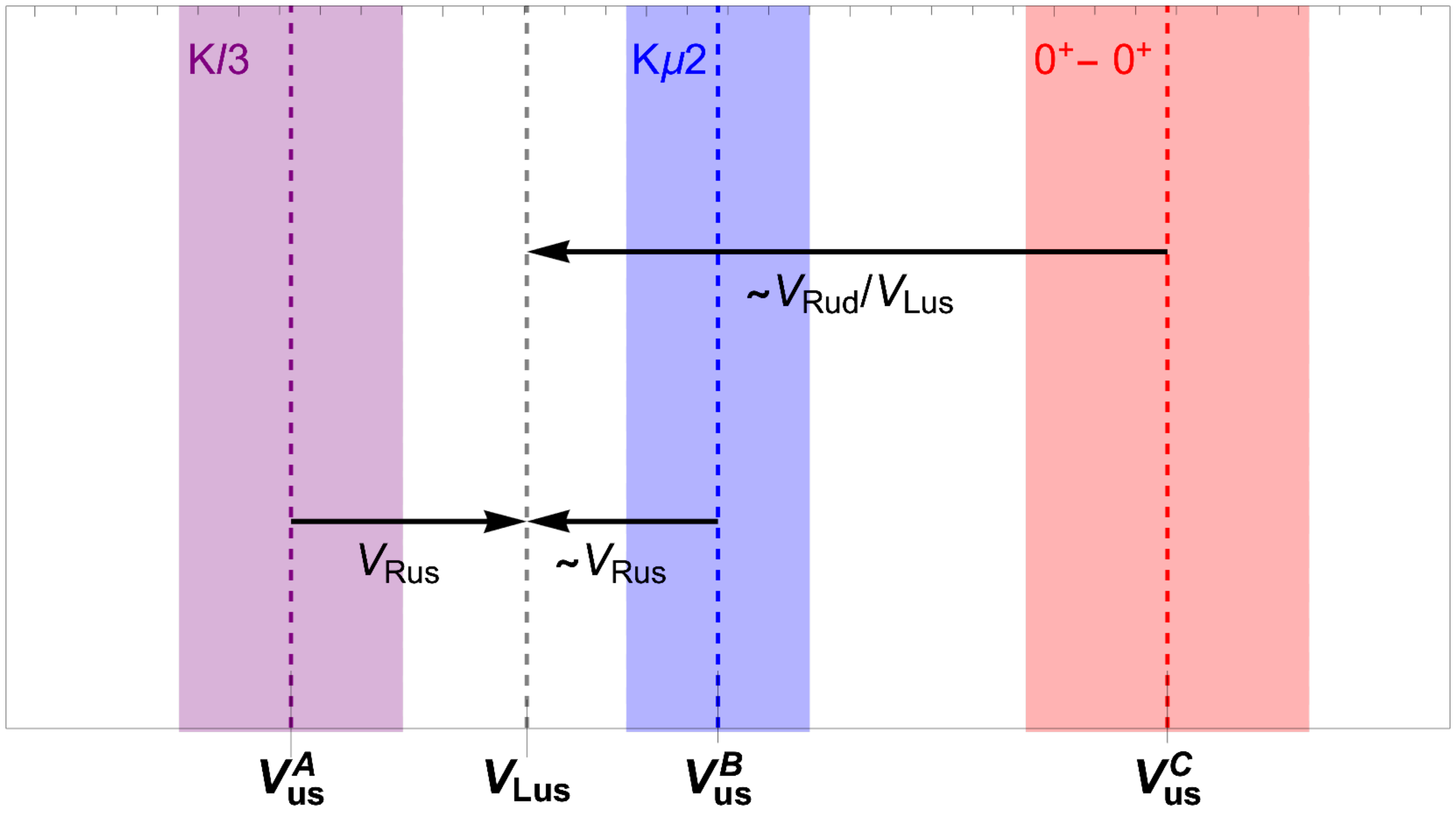
- $|V_{Lud}|^2 + |V_{Lus}|^2 + |V_{Lub}|^2 = 1 - |V_{Lub'}|^2 = 1$

Vector-like weak doublet

- Needed values:

$$V_{Rus} = V^*_{R4u} V_{R4s} = -1.17(37) \times 10^{-3} \qquad V_{Rud} = V^*_{R4u} V_{R4d} = -0.87(27) \times 10^{-3}$$

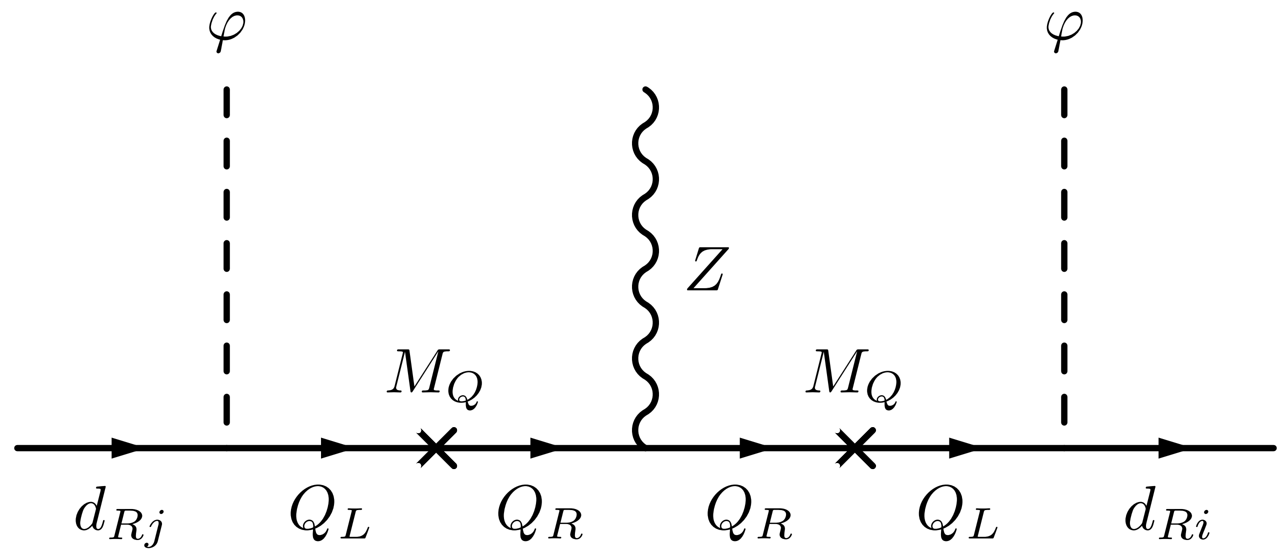
with $V_{Lus} = 0.22443(35)$

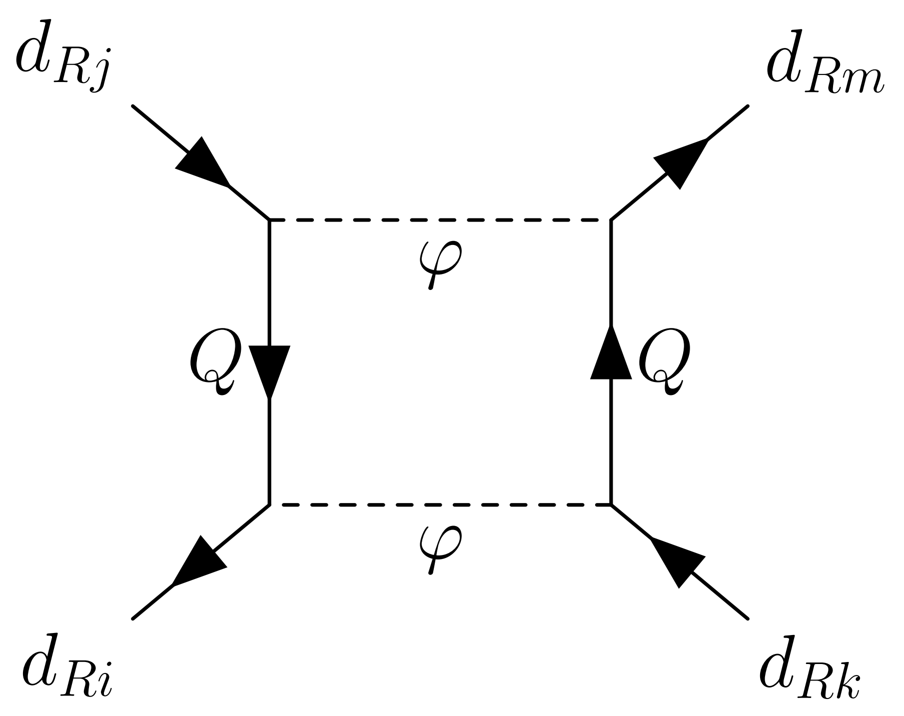


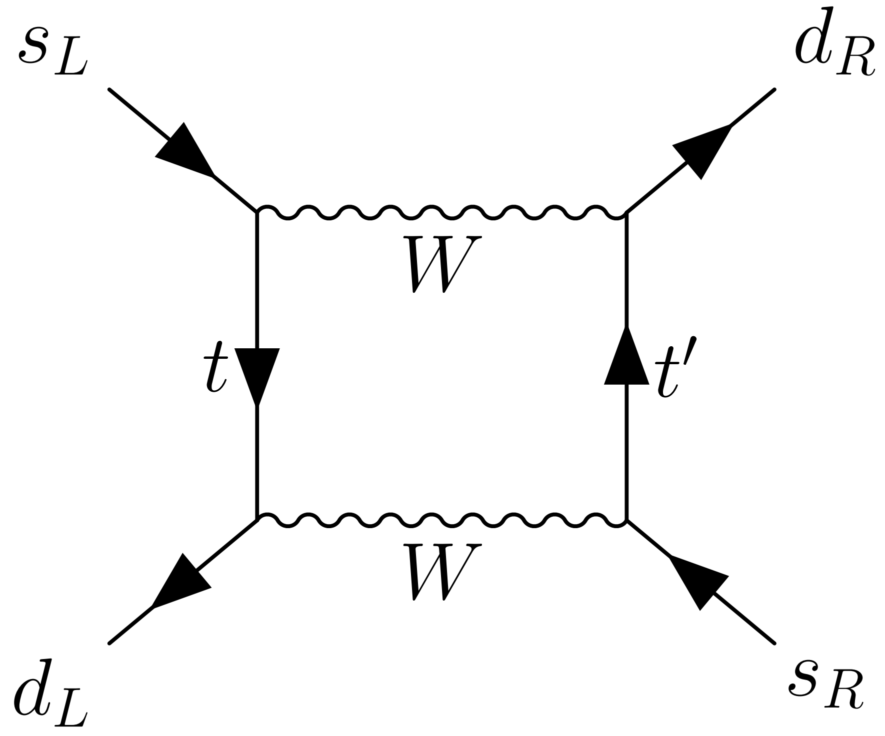
Vector-like weak doublet

- However also in this scenario constraints from FCNC and Z-boson physics and EW observables.
- For example, new Z couplings at tree level

$$\mathcal{L}_{\rm fcnc} = \frac{1}{2}\frac{g}{\cos\theta_W} Z^\mu \left(\overline{u_R}\,\overline{c_R}\,\overline{t_R}\,\overline{t'_R}\right)\gamma^\mu V_R^{(u)\dagger}\mathrm{diag}(0,0,0,1)V_R^{(u)}\begin{pmatrix}u_R\\ c_R\\ t_R\\ t'_R\end{pmatrix} - \frac{1}{2}\frac{g}{\cos\theta_W} Z^\mu \left(\overline{d_R}\,\overline{s_R}\,\overline{b_R}\,\overline{b'_R}\right)\gamma^\mu V_R^{(d)\dagger}\mathrm{diag}(0,0,0,1)V_R^{(d)}\begin{pmatrix}d_R\\ s_R\\ b_R\\ b'_R\end{pmatrix}$$







$$\propto (V^*_{R4d}V_{R4s})^2 M_{b'}^2\left[1+\left(\frac{M_q}{2.2\,\mathrm{TeV}}\right)^2\right] \quad (\text{in } \Delta m_K)$$

Vector-like weak doublet

However, also in this scenario there are constraints from FCNC and Z-boson physics and EW observables.

$$V_{Rus} = V^*_{R4u} V_{R4s} = -1.17(37) \times 10^{-3}\,, \qquad V_{Rud} = V^*_{R4u} V_{R4d} = -0.87(27) \times 10^{-3}$$

↓

$$|V_{R4u}|^2 V^*_{R4s} V_{R4d} \approx 1.0 \times 10^{-6}$$

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kaon flavor changing processes: $|V_{R4d} V_{R4s}^*| < 1.0 \times 10^{-5}$

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↓

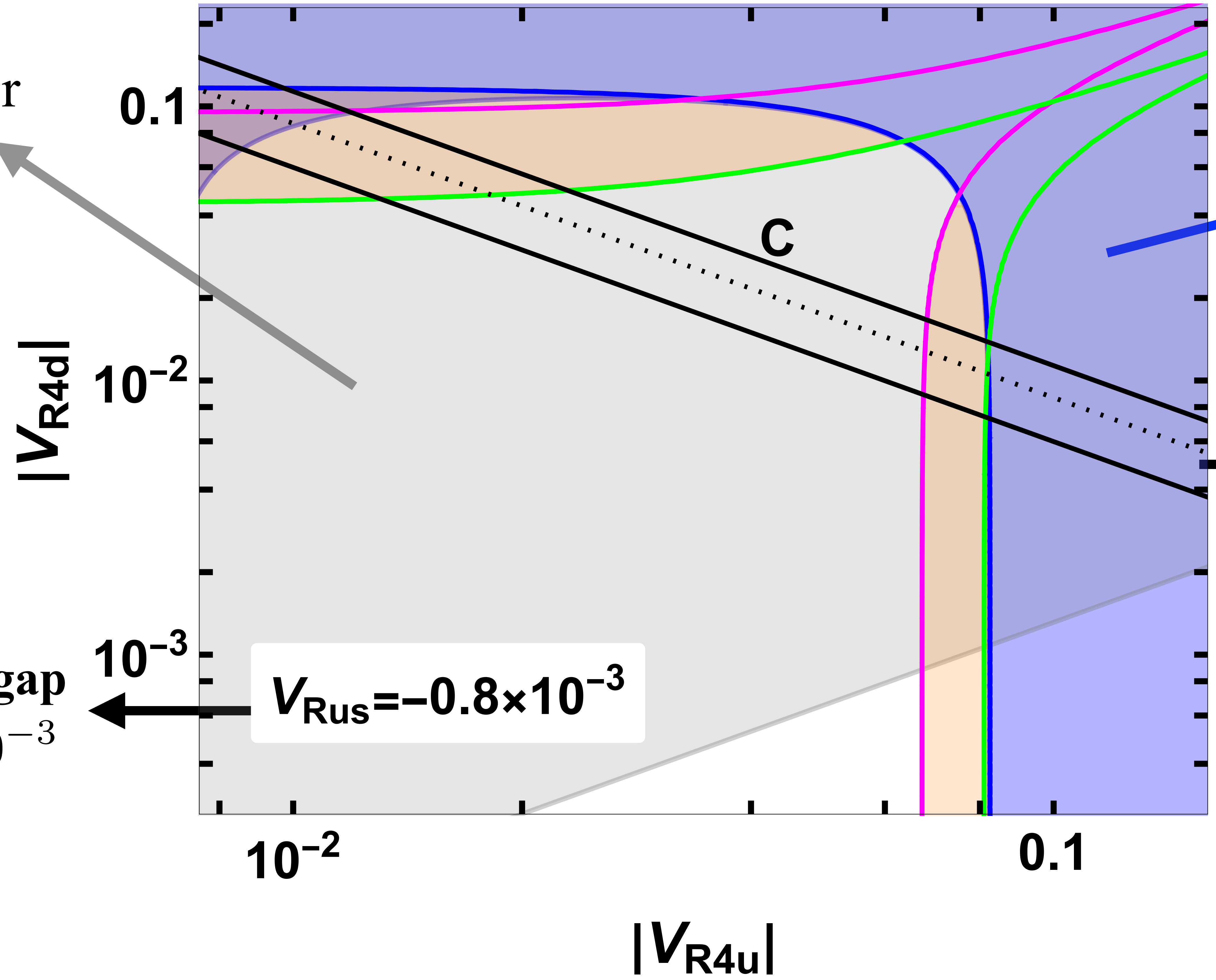
$|V_{R4u}| \gtrsim 0.3$ → excluded by Z decay into hadrons

Vector-like weak doublet

However, also in this scenario FCNC at tree level $\quad V_{R4\alpha} \approx -h_\alpha v_w / M_q$

$$V_{Rus} = V^*_{R4u} V_{R4s} = -1.17(37) \times 10^{-3}\,, \qquad V_{Rud} = V^*_{R4u} V_{R4d} = -0.87(27) \times 10^{-3}$$

Excluded by kaon flavor changing processes: $|V_{R4d} V^*_{R4s}| < 1.0 \times 10^{-5}$



Excluded by low energies EW quantities and Z physics (Z decay into hadrons,..)

needed value to explain the gap $|V^*_{R4u} V_{R4d}| = 0.87(27) \times 10^{-3}$

needed value to explain the gap $|V^*_{R4u} V_{R4s}| = 1.17(37) \times 10^{-3}$

Possible extensions

- Two weak doublets or one isodoublet with isosinglet (up or down type or both as a ”complete vector-like family”) (or one isodoublet and family symmetries...) can alleviate FC phenomena and explain all discrepancies.

Possible extensions

- Two weak doublets or one isodoublet with isosinglet (up or down type or both as a "complete vector-like family") (or one isodoublet and family symmetries...) can alleviate FC phenomena and explain all discrepancies.
- Two or more doublets: $\sum_{i=1}^{5}\sum_{j=1}^{3}\left[y^u_{ij}\tilde{\varphi}\overline{q_{Li}}u_{Rj} + y^d_{ij}\varphi\overline{q_{Li}}d_{Rj}\right] + M_4\overline{q_{L4}}q_{R4} + M_5\overline{q_{L5}}q_{R5} + \text{h.c.}$

$$\begin{pmatrix} \overline{d_{L1}} & \overline{d_{L2}} & \overline{d_{L3}} & \overline{d_{L4}} & \overline{d_{L5}} \end{pmatrix} \begin{pmatrix} & & & 0 & 0 \\ & \mathbf{y}^{(d)}_{3\times 3}\, v_w & & 0 & 0 \\ & & & 0 & 0 \\ y^d_{41} v_w & 0 & 0 & M_4 & 0 \\ 0 & y^d_{52} v_w & 0 & 0 & M_5 \end{pmatrix} \begin{pmatrix} d_{R1} \\ d_{R2} \\ d_{R3} \\ d_{R4} \\ d_{R5} \end{pmatrix}$$

$$\begin{pmatrix} \overline{u_{L1}} & \overline{u_{L2}} & \overline{u_{L3}} & \overline{u_{L4}} & \overline{u_{L5}} \end{pmatrix} \begin{pmatrix} & & & 0 & 0 \\ & \mathbf{y}^{(u)}_{3\times 3}\, v_w & & 0 & 0 \\ & & & 0 & 0 \\ y^u_{41} v_w & 0 & 0 & M_4 & 0 \\ y^u_{51} v_w & 0 & 0 & 0 & M_5 \end{pmatrix} \begin{pmatrix} u_{R1} \\ u_{R2} \\ u_{R3} \\ u_{R4} \\ u_{R5} \end{pmatrix}$$

$$V^*_{R\,5u}V_{R\,5s} = -1.17(37)\times 10^{-3} \qquad V^*_{R\,4u}V_{R\,4d} = -0.87(27)\times 10^{-3}$$

- Z decay: $|V_{R4u}|^2 + |V_{R5u}|^2 + 0.50(|V_{R4d}|^2 + |V_{R5s}|^2) < 6.8\times 10^{-3}$

Heavy vector-like fermions

General Lagrangian includes the mixed Yukawa terms:

$$\mathcal{L}_{\text{Yuk}}^{\text{mix}} = h_U \tilde{\varphi}\, \overline{q_L} U_R + h_D \varphi\, \overline{q_L} D_R + h_{\mathcal{U}} \tilde{\varphi}\, \overline{Q_L} u_R + h_{\mathcal{D}} \varphi\, \overline{Q_L} d_R \; + \; \text{h.c.}$$

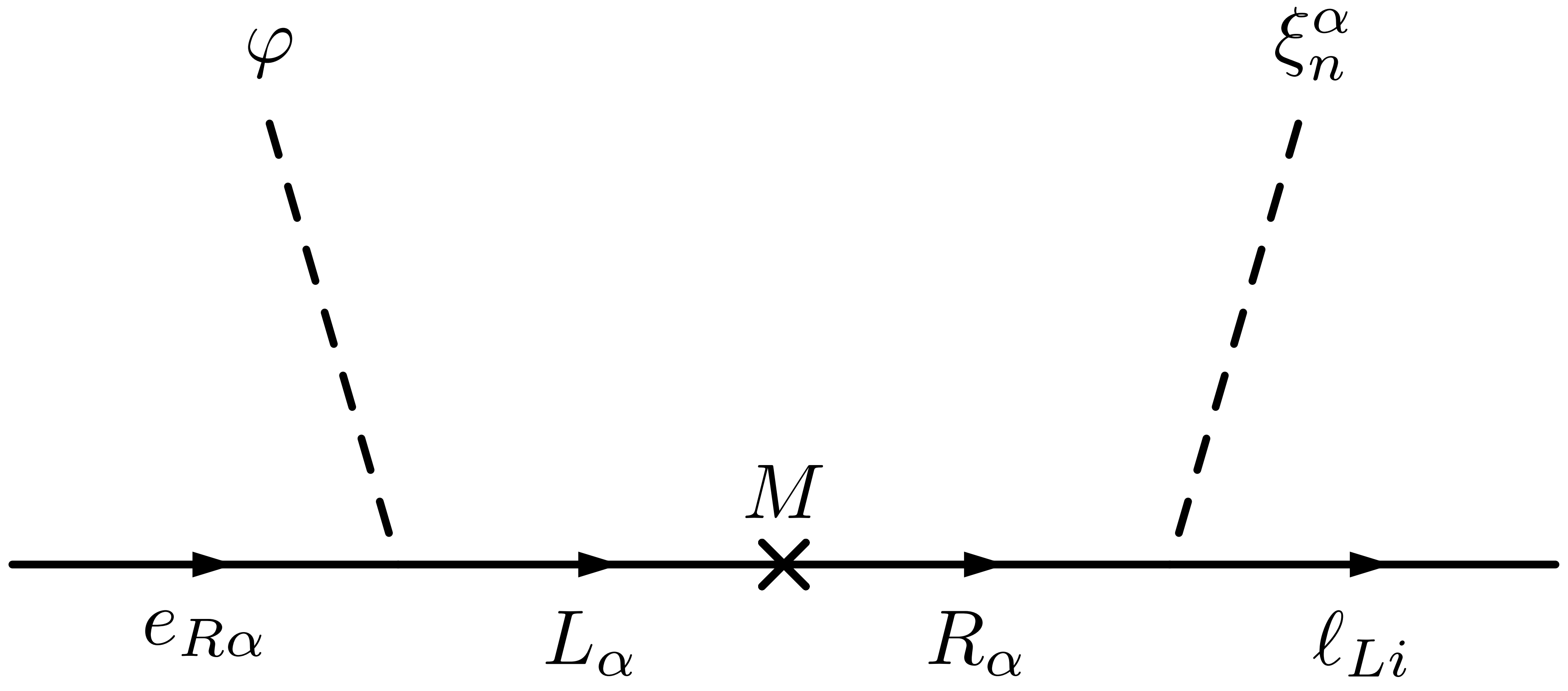
and the mass terms

$$\mathcal{L}_{\text{mass}} = M_U \overline{U_L} U_R + M_D \overline{D_L} D_R + M_Q \overline{Q_L} Q_R + \mu_u \overline{U_L} u_R + \mu_d \overline{D_L} d_R + \mu_q \overline{q_L} Q_R \; + \; \text{h.c.}$$

- Symmetries may forbid the direct Yukawa terms but allow the mixed ones.
- Mass terms μ and M can be originated from some physical scales.
- SM Yukawa terms for normal fermions will be induced after integrating out the heavy states.
  With mixing mass terms $\mu_{u,d,q}$ smaller than $M_{U,D,Q}$:
  $Y_u \simeq h_U M_U^{-1} \mu_u + \mu_q M_Q^{-1} h_{\mathcal{U}}, \quad Y_d \simeq h_D M_D^{-1} \mu_d + \mu_q M_Q^{-1} h_{\mathcal{D}}$
- Non-zero quark masses are induced via the mixings with the extra vector-like species.
- Predictive model building for fermion masses and mixings.

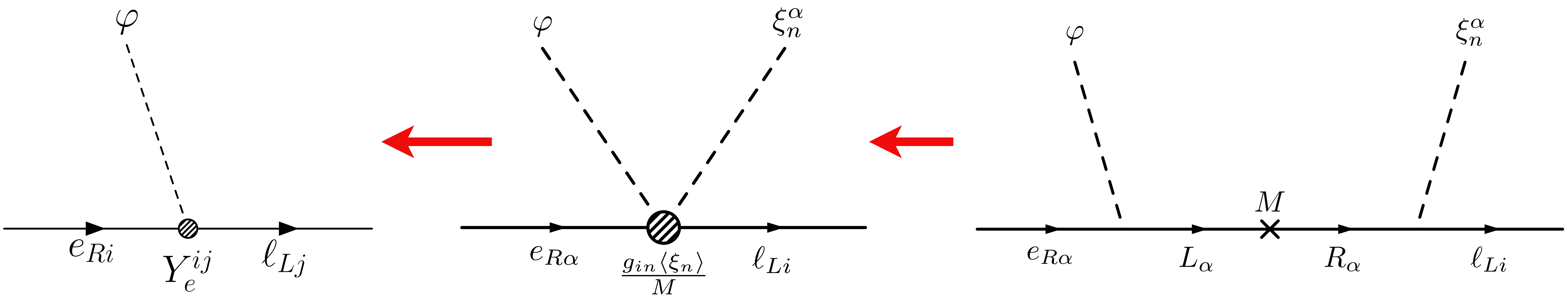
Heavy vector-like fermions

"Universal" seesaw mechanism for fermion masses



Heavy vector-like fermions

"Universal" seesaw mechanism for fermion masses



$Y_e^{ij}\varphi\overline{l_{Lj}}e_{Ri}$

$\frac{g_{in}\xi_n^\alpha}{M_L}\varphi\overline{\ell_{Li}}e_{R\alpha}$

Family symmetries

- In the limit of vanishing Yukawa couplings, $Y_f \to 0$, the SM acquires maximal global symmetry:
$$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$$

- Fermions transform as triplets, Yukawa interactions break the $SU(3)^5$ symmetry.

- Gauge the symmetry $SU(3)^5$ (the $U(1)$ factors remain as global symmetries).

- Fermions cannot get mass if the symmetry is unbroken: LH and RH particles transform in different representations.

- Yukawa couplings are induced by non-zero VEVs of scalars. The fermion mass hierarchy can be related to the breaking pattern.

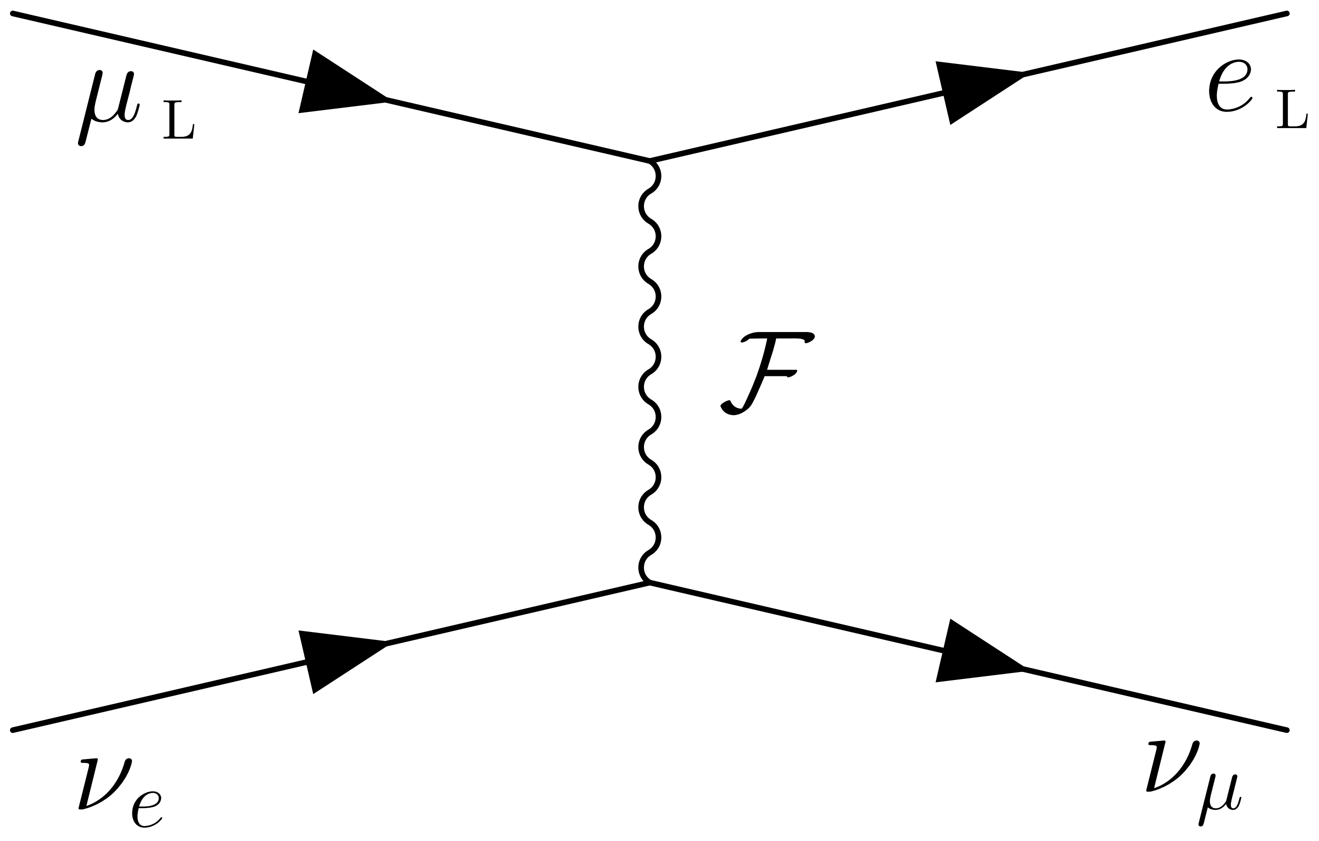
$$\frac{y_{ij}\bar{\xi}_j^\gamma \eta_{i\alpha}}{M_L^2}\varphi\overline{\ell_{L\alpha}}e_{Ri} + \frac{h_{ij}\bar{\eta}_i^\alpha \bar{\eta}_j^\beta}{M_\nu^3}\varphi\varphi\ell_{L\alpha}^T C\ell_\beta + \text{h.c.}$$

- Three $SU(3)_e$ triplets $\xi_i \sim 3_e$
- Three $SU(3)_\ell$ triplets $\eta_i \sim 3_\ell$

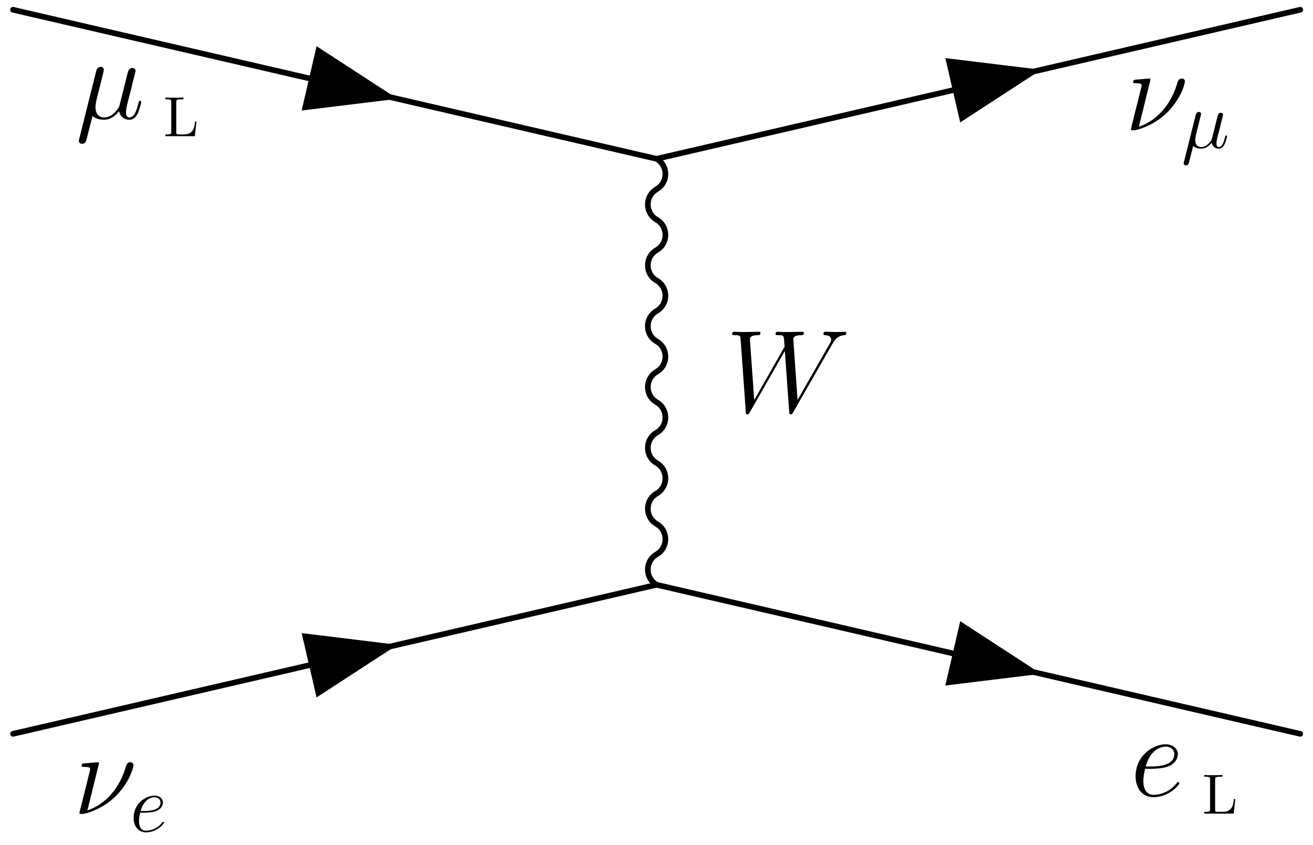
Solution #2

- Suppose the existence of **flavor changing bosons**.

$$-\frac{4G_{\mathcal{F}}}{\sqrt{2}}(\overline{e_L}\gamma^{\alpha}\mu_L)(\overline{\nu_\mu}\gamma_{\alpha}\nu_e)$$



$$-\frac{4G_{\mathrm{F}}}{\sqrt{2}}(\overline{\nu_\mu}\gamma^{\alpha}\mu_L)(\overline{e_L}\gamma_{\alpha}\nu_e)$$



- Horizontal interactions have positive interference with SM;
- After Fierz transformation, the sum of the diagrams gives the operator:

$$-\frac{4G_{\mu}}{\sqrt{2}}(\overline{\nu_\mu}\gamma^{\alpha}\mu_L)(\overline{e_L}\gamma_{\alpha}\nu_e) \qquad G_\mu = G_F + G_{\mathcal{F}} = G_F(1+\delta_\mu) \qquad \boxed{G_\mu \neq G_F}$$

Flavour bosons for CKM

- Different $\mathbf{G}_\mu = \mathbf{G_F} + \mathbf{G}_\mathcal{F} = \mathbf{G_F}(\mathbf{1} + \delta_\mu) = 1 + \frac{v_\mathrm{w}^2}{v_\mathcal{F}^2}$
- The values of V_us, V_ud (and corresponding errorbars) should be rescaled:

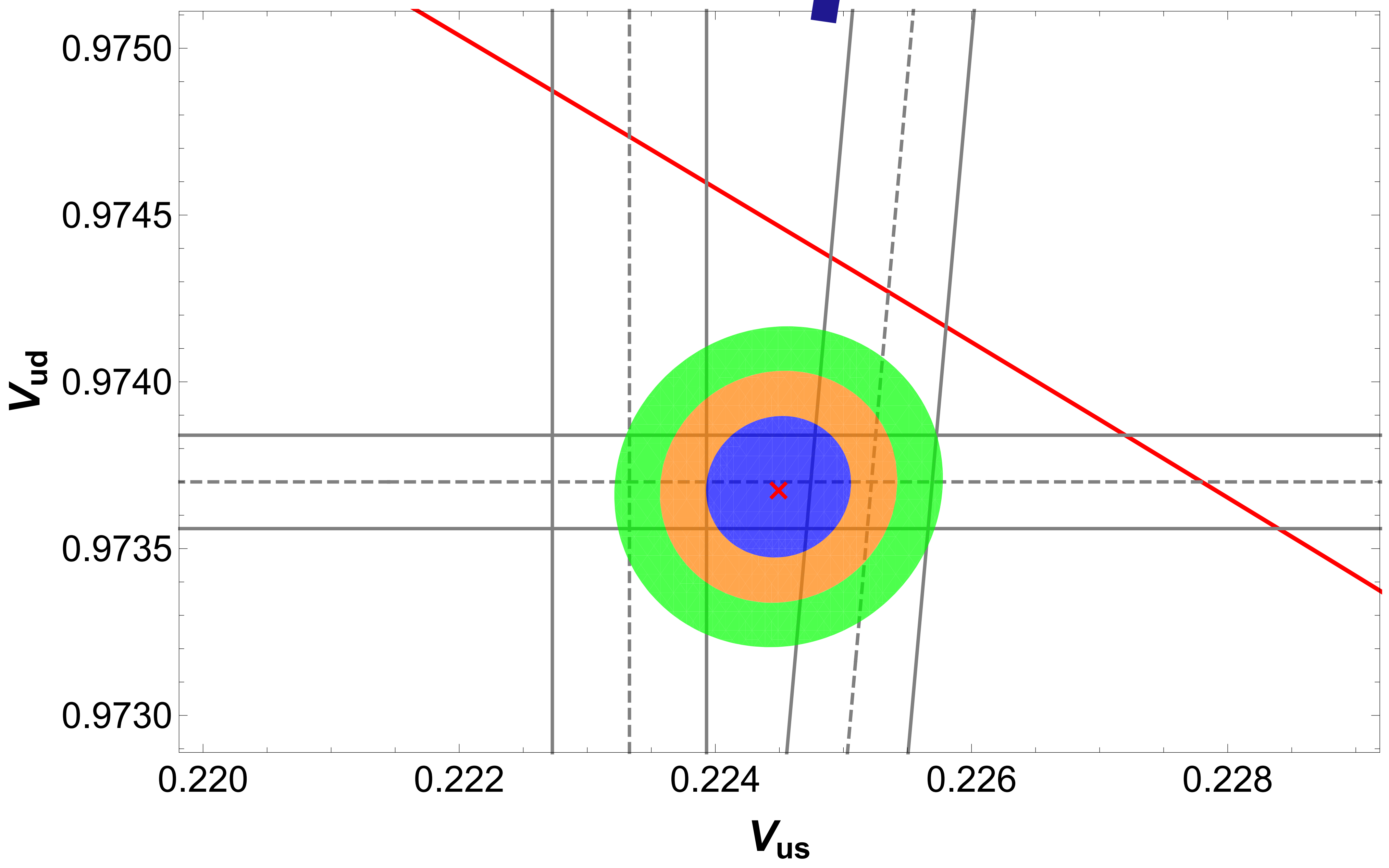
$$|V_\mathrm{us}| = 0.22333(60) \times (1 + \delta_\mu)\,, \qquad |V_\mathrm{ud}| = 0.97370(14) \times (1 + \delta_\mu)$$

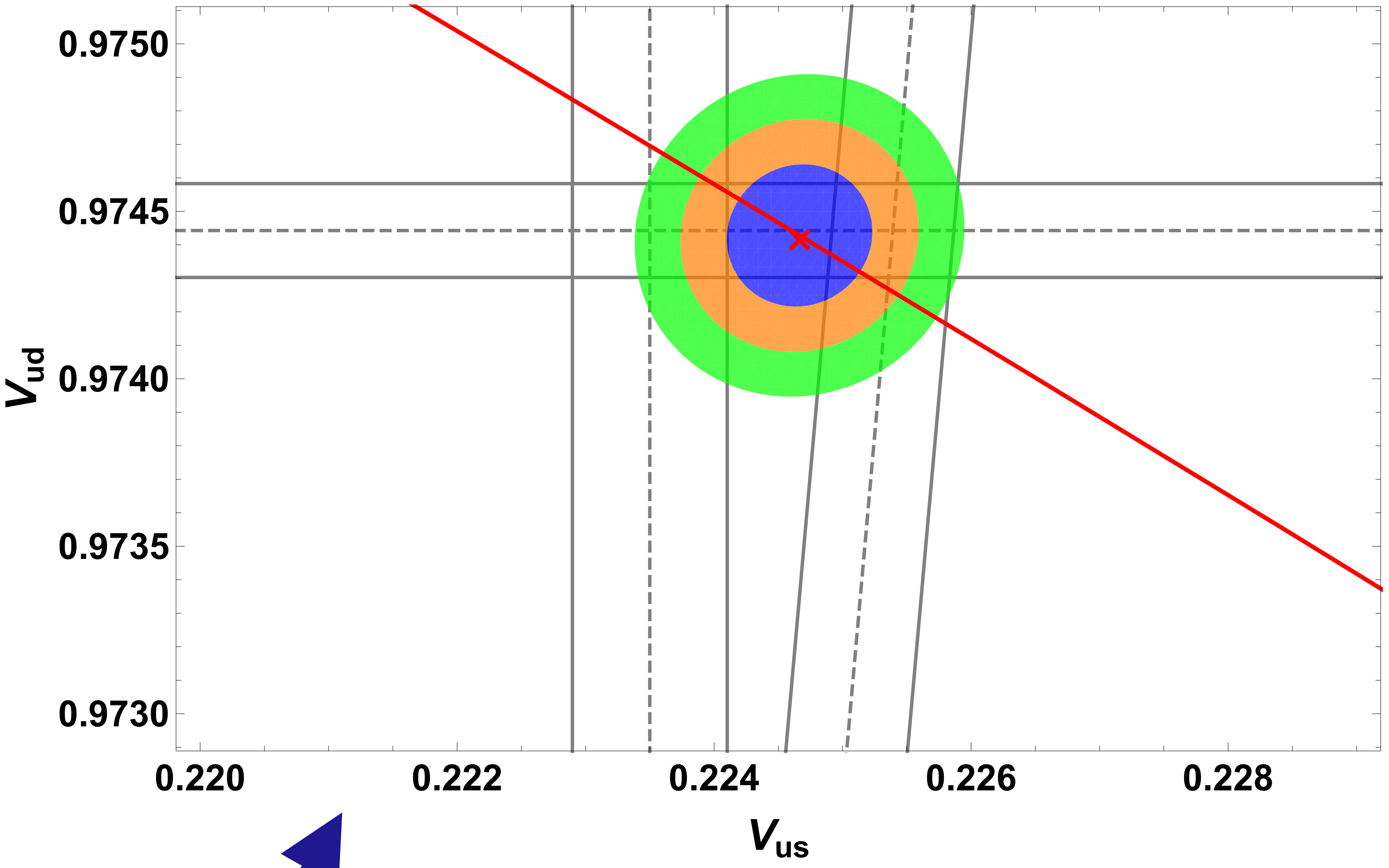
  while the ratio is not affected.
- Unitarity recovered: $\left(\frac{G_F}{G_\mu}\right)^2 (|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2) = 1 - \frac{2G_\mathcal{F}}{G_\mathrm{F}}$
- CKM unitarity is explained with $\delta_\mu \sim \mathbf{7.6 \times 10^{-4}}$, or

$$v_\mathcal{F} = 6\text{-}7\ \mathrm{TeV}$$

Flavour bosons for CKM

$$\delta_\mu = 7.6 \cdot 10^{-4}$$
$$v_{\mathcal{F}} = 6.3\,\text{TeV}$$





How light flavour can changing gauge bosons be?

FCNC in left-handed sector

$$\mathcal{L}_{\rm eff}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\, \gamma_\mu \frac{\lambda_a}{x_a}\, \boldsymbol{e}_L \right)^2$$

$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2}$ nothing $\qquad w_2^2 + w_1^2 = v_{\mathcal{F}}^2 \qquad (\delta_\mu = \frac{v_w^2}{v_{\mathcal{F}}^2} \simeq 7 \cdot 10^{-4})$

- Masses of gauge bosons $M_{\ell 1,2}^2 = \frac{g^2}{2}(w_2^2 + w_1^2) = \frac{g^2}{2} v_{\mathcal{F}}^2$.
- If $w_3 = w_2 = w_1$ (e. g. symmetry between ηs), gauge bosons have equal masses, $\lambda_a \to V^\dagger \lambda_a V$ is simply a basis redetermination of the Gell-Mann matrices. From Fierz identities for λ matrices:

$$\mathcal{L}_{eff} = -\frac{1}{4v_\ell^2}(\overline{\mathbf{e}_{\rm L}}\, \lambda^a\, \gamma^\mu \mathbf{e}_{\rm L})(\overline{\mathbf{e}_{\rm L}}\, \lambda^a\, \gamma_\mu \mathbf{e}_{\rm L}) = -\frac{1}{3v_\ell^2}(\overline{\mathbf{e}_{\rm L}}\, \mathbb{I}\, \gamma_\mu \mathbf{e}_{\rm L})^2$$

  No FCNC, the global $SO(8)_\ell$ symmetry acts as a custodial symmetry.

- In the general case, FC ($\mu \to 3e$, $\tau \to 3\mu$,...) under control.
- $\mathbf{v}_{\mathcal{F}} \simeq \mathbf{6}$ **TeV** is not contradicting experimental constraints.

Cabibbo angle and m_W

- New horizontal gauge bosonsgo to the opposite direction in m_W shift. Some other step is needed, like mixing with some extra gauge bosons like Z' at the TeV scale or perhaps also with the flavor gauge bosons or a scalar triplet.
- Down-type vector-like singlet cannot explain Cabibbo unitarity problem and m_W.
- Up-type vector-like singlet cannot explain Cabibbo unitarity problem and m_W.
- Vector-like doublet can explain Cabibbo unitarity problem and m_W.

(B.B., C.A. Manzari and S. Trifinopoulos arxiv:2211.XXXXX)

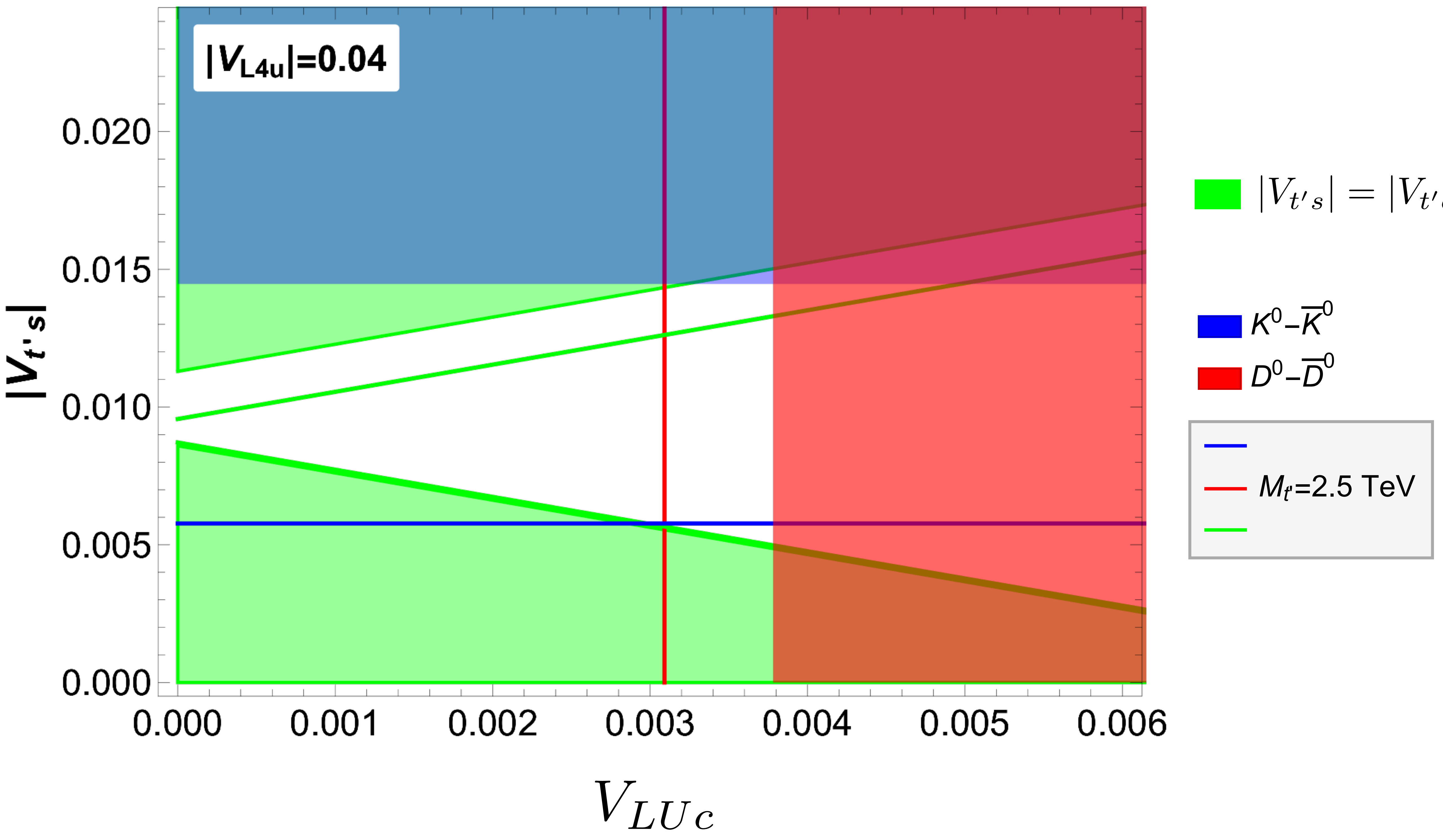
Conclusions

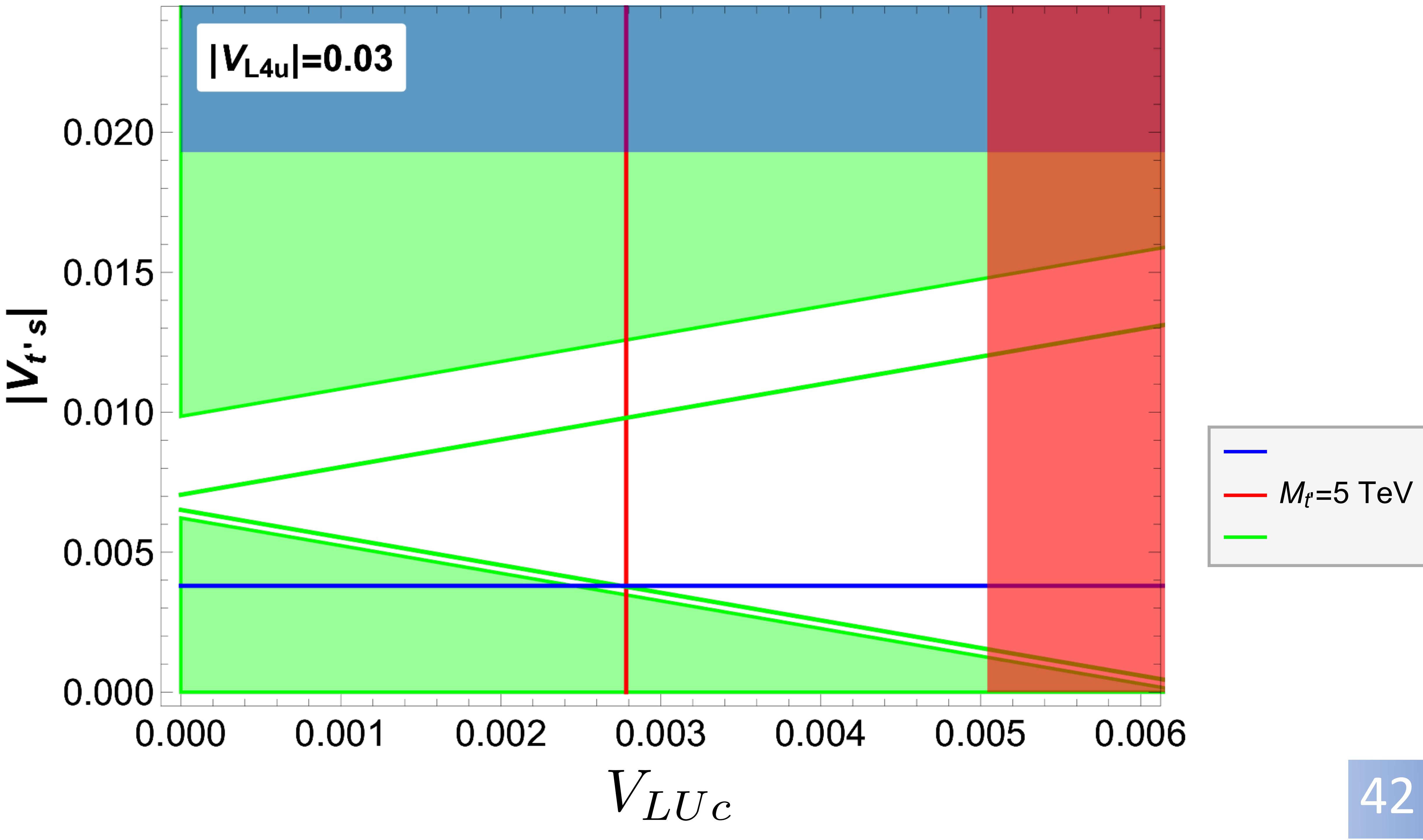
- There is **tension** between independent determinations of the **CKM matrix** elements of the first row.
- Extra vector-like quarks can be possible explanation for CKM anomalies. A quite large mixing with SM fermions is needed to restore unitarity.
- Their mass should be no more than few **TeV,** since experimental constraints on flavor changing phenomena become more stringent with larger masses.
- Only one type of extra multiplet cannot entirely explain all the discrepancies, and some their combination is required, e.g. two species of isodoublet, or one isodoublet and one (up or down type) isosinglet.
- These scenarios are **testable** with future experiments (Z boson decay, mass few TeV).
- A new effective operator in positive interference with the SM muon decay as the one generated by flavour changing gauge bosons can solve CKM unitarity problem. **G_F can be different** from muon decay constant without contradicting experimental data.
- Natural explanation for masses and mixings of fermions from the spontaneous breaking pattern of the symmetry, with generalized seesaw mechanism (integrating out heavy vector-like fermions).
- CKM unitarity is restored with a breaking scale of the symmetry (and gauge boson mass) of few TeV. Flavour gauge bosons can be as light as **TeV** without contradicting experimental constraints .

Backup

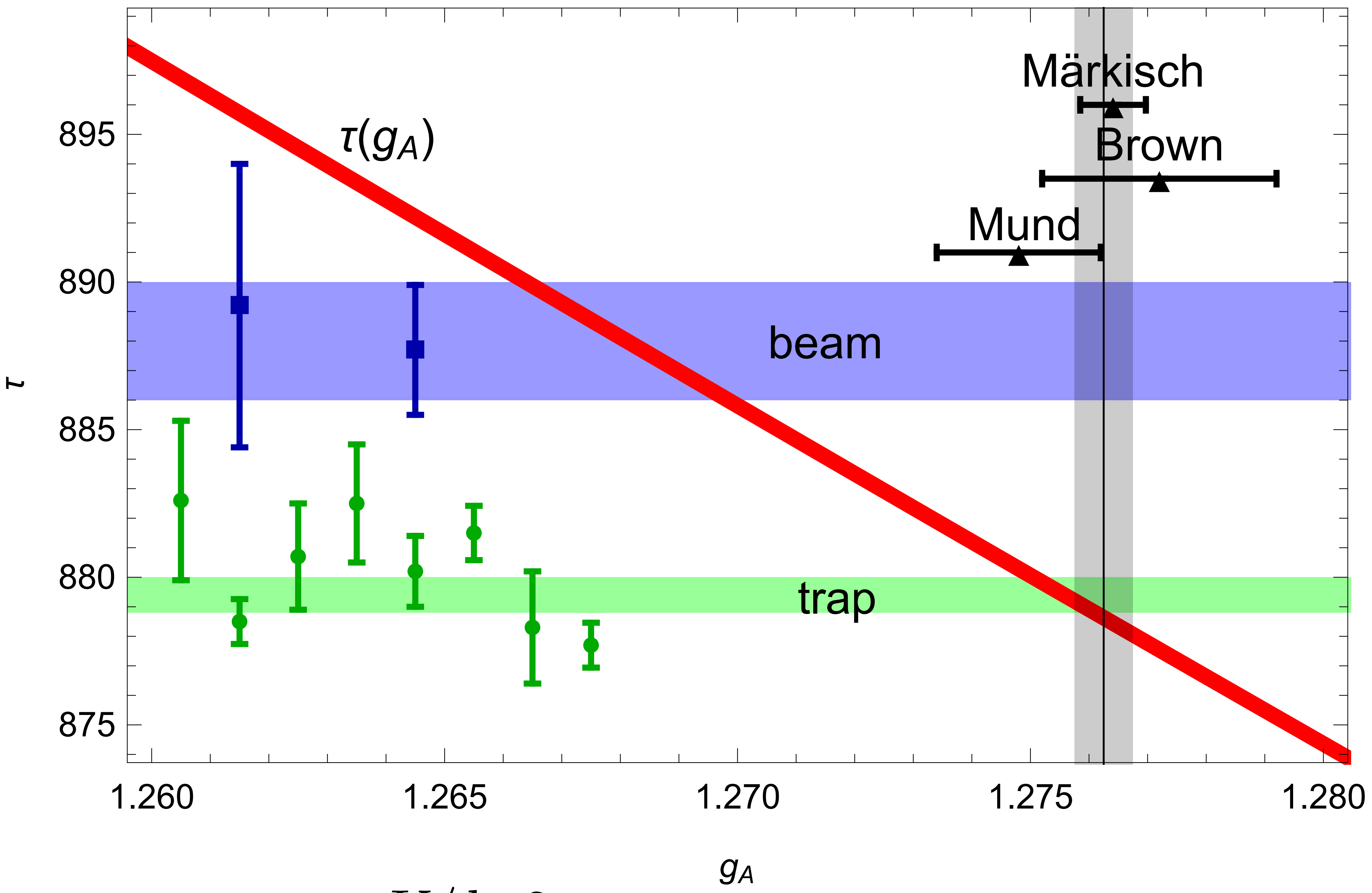
Up-type weak singlets

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - |V_{LUu}|^2 \,, \qquad |V_{LUu}| \approx |h_u| v_w / M_{t'} \approx 0.044$$





CKM and neutron lifetime problem



$$|V_{ud}|^2 = \frac{K/\ln 2}{G_F^2 \mathcal{F}_n \tau_n (1+3g_A^2)(1+\Delta_R^V)} = \frac{5024.46(30)\,\mathrm{s}}{\tau_n (1+3g_A^2)(1+\Delta_R^V)}$$

- $\mathcal{F}_n = f_n(1+\delta'_R)$ f-value corrected by LD QED correction.
- $g_A = 1.27625(50)$ axial current coupling from β-asymmetry.
- $\tau_{\mathrm{beam}} = 888.0(2.0)$ s (4.4σ away from SM prediction)
- $\tau_{\mathrm{trap}} = 879.4(6)$ s
- τ_{trap} & $\Delta_R^V = 0.02454(27)$ & $g_A \to |V_{ud}| = 0.97333(47)$

$$G_V^2 = \frac{K/\ln 2}{\mathcal{F}_n \tau_n (1+3g_A^2)(1+\Delta_R^V)}$$

$$G_V^2 = \frac{K}{2\mathcal{F}t(1+\Delta_R^V)}$$

→

$$\tau_n = \frac{2\mathcal{F}t}{\ln 2 \mathcal{F}_n (1+3g_A^2)} = \frac{5172.0(1.1)\,\mathrm{s}}{(1+3g_A^2)}$$

- G_V and Δ_R^V cancel out even in BSM $G_V \neq G_F|V_{ud}|$, $g_A = -G_A/G_V$
- new Δ_R^V calculations have no influence on τ_n determination.
- $g_A = 1.27625(50) \to \tau_n^{\mathrm{SM}} = 878.7(6)$ s $\approx \tau_{\mathrm{trap}}$

Vector-like weak doublet

- However also in this scenario flavour changing neutral currents appear at tree level.

$$\mathcal{L}_{\rm fcnc} = \frac{1}{2}\frac{g}{\cos\theta_W} Z^\mu \begin{pmatrix} \overline{u_R} & \overline{c_R} & \overline{t_R} & \overline{t'_R} \end{pmatrix} \gamma^\mu V_R^{(u)\dagger} {\rm diag}(0,0,0,1) V_R^{(u)} \begin{pmatrix} u_R \\ c_R \\ t_R \\ t'_R \end{pmatrix} + $$

$$- \frac{1}{2}\frac{g}{\cos\theta_W} Z^\mu \begin{pmatrix} \overline{d_R} & \overline{s_R} & \overline{b_R} & \overline{b'_R} \end{pmatrix} \gamma^\mu V_R^{(d)\dagger} {\rm diag}(0,0,0,1) V_R^{(d)} \begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \end{pmatrix}$$

where

$$V_R^{(u)\dagger}{\rm diag}(0,0,0,1)V_R^{(u)} = \begin{pmatrix} |V_{R\,4u}|^2 & V^*_{R\,4u}V_{R\,4c} & V^*_{R\,4u}V_{R\,4t} & V^*_{R\,4u}V_{R\,4t'} \\ V^*_{R\,4c}V_{R\,4u} & |V_{R\,4c}|^2 & V^*_{R\,4c}V_{R\,4t} & V^*_{R\,4c}V_{R\,4t'} \\ V^*_{R\,4t}V_{R\,4u} & V^*_{R\,4t}V_{R\,4c} & |V_{R\,4t}|^2 & V^*_{R\,4t}V_{R\,4t'} \\ V^*_{R\,4t'}V_{R\,4u} & V^*_{R\,4t'}V_{R\,4c} & V^*_{R\,4t'}V_{R\,4t} & |V_{R\,4t'}|^2 \end{pmatrix}$$

$$V_R^{(d)\dagger}{\rm diag}(0,0,0,1)V_R^{(d)} = \begin{pmatrix} |V_{R\,4d}|^2 & V^*_{R\,4d}V_{R\,4s} & V^*_{R\,4d}V_{R\,4b} & V^*_{R\,4d}V_{R\,4b'} \\ V^*_{R\,4s}V_{R\,4d} & |V_{R\,4s}|^2 & V^*_{R\,4s}V_{R\,4b} & V^*_{R\,4s}V_{R\,4b'} \\ V^*_{R\,4b}V_{R\,4d} & V^*_{R\,4b}V_{R\,4s} & |V_{R\,4b}|^2 & V^*_{R\,4b}V_{R\,4b'} \\ V^*_{R\,4b'}V_{R\,4d} & V^*_{R\,4b'}V_{R\,4s} & V^*_{R\,4b'}V_{R\,4b} & |V_{R\,4b'}|^2 \end{pmatrix}$$

Down-type isosinglet

- Weak neutral currents:

$$\mathcal{L}_{\rm nc} = \frac{g}{\cos\theta_W}\left[-\frac{1}{2}\begin{pmatrix} \overline{d_L} & \overline{s_L} & \overline{b_L} & \overline{b'_L}\end{pmatrix}\gamma^\mu \tilde{V}_L^{(d)\dagger}\tilde{V}_L^{(d)}\begin{pmatrix} d \\ s \\ b \\ b'\end{pmatrix} + \frac{1}{3}\sin^2\theta_W\left(\overline{\mathbf{d}_L}\gamma^\mu \mathbf{d}_L + \overline{\mathbf{d}_R}\gamma^\mu \mathbf{d}_R\right)\right] Z_\mu$$

- The non-unitarity of $\tilde{V}_L^{(d)}$ is at the origin of **tree level flavour changing** couplings with Z boson (and Higgs boson), determined by the matrix:

$$\tilde{V}_L^{(d)\dagger}\tilde{V}_L^{(d)} = \begin{pmatrix} 1-|V_{4d}|^2 & -V_{4d}^*V_{4s} & -V_{4d}^*V_{4b} & -V_{4d}^*V_{4b'} \\ -V_{4s}^*V_{4d} & 1-|V_{4s}|^2 & -V_{4s}^*V_{4b} & -V_{4s}^*V_{4b'} \\ -V_{4b}^*V_{4d} & -V_{4b}^*V_{4s} & 1-|V_{4b}|^2 & -V_{4b}^*V_{4b'} \\ -V_{4b'}^*V_{4d} & -V_{4b'}^*V_{4s} & -V_{4b'}^*V_{4b} & |V_{1b'}|^2+|V_{2b'}|^2+|V_{3b'}|^2 \end{pmatrix}_L$$

- If the 2nd and 3rd families are not mixed with the fourth, $V_{L4s} = V_{L4b} = 0$, there are not FCNC at tree level between the first three families.

Down-type weak singlet

- Weak neutral currents: $\mathcal{L}_{\rm nc} = \frac{g}{\cos\theta_W} Z_\mu \left[\boxed{T_3(f_{L,R})} - Q(f)\sin^2\theta_W \right] \overline{f_{L,R}}\gamma^\mu f_{L,R}$

(not the same quantum numbers)

- Yukawa and mass matrices are not diagonalized by the same transformation;
- Tree and loop level **flavour changing** couplings with the Higgs boson and with Z-boson.

$$-\frac{1}{2}\frac{g}{\cos\theta_W}\begin{pmatrix} \overline{d_L} & \overline{s_L} & \overline{b_L} & \overline{b'_L} \end{pmatrix}\gamma^\mu V_L^{(d)\dagger}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} V_L^{(d)}\begin{pmatrix} d_L \\ s_L \\ b_L \\ b'_L \end{pmatrix} Z_\mu$$

$$(\overline{d_L},\overline{s_L},\overline{b_L},\overline{b'_L})V_L^{(d)\dagger}\begin{pmatrix} & & & h_d \\ & Y^{(d)}_{3\times 3} & & h_s \\ & & & h_b \\ 0 & 0 & 0 & 0 \end{pmatrix} V_R^{(d)}\begin{pmatrix} d_R \\ s_R \\ b_R \\ b'_R \end{pmatrix}\frac{H^0}{\sqrt{2}} + \text{h.c.}$$

- Also flavour diagonal processes change.
- $\Gamma(Z\to \text{had}) - \Gamma(Z\to\text{had})_{\rm SM} \approx \frac{G_F M_Z^3}{\sqrt{2}\pi}\left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)\left(|V_{LDd}|^2+|V_{LDs}|^2+|V_{LDb}|^2\right) < 0$

$|V_{LDd}|^2+|V_{LDs}|^2+|V_{LDb}|^2 < 2.5\times 10^{-3}$

Gauge bosons in left-handed sector, $SU(3)_\ell$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing}$$

$$\frac{1}{2}\begin{pmatrix} \mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_1 - i\mathcal{F}_2 & \mathcal{F}_4 - i\mathcal{F}_5 \\ \mathcal{F}_1 + i\mathcal{F}_2 & -\mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_6 - i\mathcal{F}_7 \\ \mathcal{F}_4 + i\mathcal{F}_5 & \mathcal{F}_6 + i\mathcal{F}_7 & -\frac{2}{\sqrt{3}}\mathcal{F}_8 \end{pmatrix}$$

$$M^2_{\ell 4,5} = \frac{g^2}{2}(w_3^2 + w_1^2) \qquad M^2_{\ell 6,7} = \frac{g^2}{2}(w_3^2 + w_2^2) \qquad M^2_{\ell 38} = \frac{g^2}{2}\begin{pmatrix} w_2^2 + w_1^2 & \frac{1}{\sqrt{3}}(w_1^2 - w_2^2) \\ \frac{1}{\sqrt{3}}(w_1^2 - w_2^2) & \frac{1}{3}(4w_3^2 + w_1^2 + w_2^2) \end{pmatrix}$$

$$M^2_{\ell 1,2} = \frac{g^2}{2}(w_2^2 + w_1^2) = \frac{g^2}{2}v_{\mathcal{F}}^2$$

New interactions in left-handed sector

$$\mathcal{L}_{\rm eff}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L} \gamma_\mu \frac{\lambda_a}{x_a} \boldsymbol{e}_L \right)^2$$

Charged leptons flavour conserving interactions,
lepton flavour violating interactions

$$\mathcal{L}_{\rm eff}^{e\nu} = -\frac{2G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L} \gamma^\mu \frac{\lambda_a}{x_a} \boldsymbol{e}_L \right) \left(\overline{\boldsymbol{\nu}_L} \gamma_\mu \frac{\lambda_a}{x_a} \boldsymbol{\nu}_L \right)$$

muon decay, tau decays,
non-standard neutrino interactions with leptons

$$\mathcal{L}_{\rm eff}^{\nu\nu} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{\nu}_L} \gamma_\mu \frac{\lambda_a}{x_a} \boldsymbol{\nu}_L \right)^2$$

Non-standard interactions between neutrinos

New interactions in left-handed sector

$$\mathcal{L}_{\text{eff}}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L} \, \gamma_\mu \frac{\lambda_a}{x_a} \, \boldsymbol{e}_L \right)^2$$

Flavour conserving interactions
(compositeness limits):

$$v_{\mathcal{F}} > 3 \text{ TeV}$$

FCNC in left-handed sector

$$\mathcal{L}^{ee}_{\text{eff}} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\, \gamma_\mu \frac{\lambda_a}{x_a}\, \boldsymbol{e}_L \right)^2$$

If $u_3 = u_2 = u_1$ (e. g. symmetry between ηs) then

- Gauge bosons have equal masses and do not mix.
- $\lambda_a \to V^\dagger \lambda_a V$ is simply a basis redetermination of the Gell-Mann matrices
- From Fierz identities for λ matrices:

$$\mathcal{L}_{eff} = -\frac{1}{4v_\ell^2}(\overline{\mathbf{e}_\mathrm{L}}\,\lambda^a\,\gamma^\mu \mathbf{e}_\mathrm{L})(\overline{\mathbf{e}_\mathrm{L}}\,\lambda^a\,\gamma_\mu \mathbf{e}_\mathrm{L}) = -\frac{1}{3v_\ell^2}(\overline{\mathbf{e}_\mathrm{L}}\,\mathbb{I}\,\gamma_\mu \mathbf{e}_\mathrm{L})^2$$

- **no FCNC**, the global $SO(8)_\ell$ symmetry acts as a custodial symmetry.

FCNC in left-handed sector

$$\mathcal{L}_{\text{eff}}^{ee} = -\frac{G_{\mathcal{F}}}{\sqrt{2}} \sum_{a=1}^{8} \left(\overline{\boldsymbol{e}_L}\, \gamma_\mu \frac{\lambda_a}{x_a}\, \boldsymbol{e}_L \right)^2$$

- In general case e.g. $\mu \to 3e$ decay: $(\delta_\mu = \frac{v_w^2}{v_{\mathcal{F}}^2} \simeq 7 \cdot 10^{-4})$

$$\frac{\Gamma(\mu \to ee\bar{e})}{\Gamma(\mu \to e\nu_\mu \bar{\nu}_e)} \simeq \frac{1}{8} \left(C(r)|U_{3e}^* U_{3\mu}|\right)^2 \delta_\mu^2$$

$r = 2u_3^2/v_\ell^2$, $|C(r)| < 1$. $|U_{3\mu}|$ and $|U_{3e}|$ can be as large as $\sin\theta_C = V_{us}$.

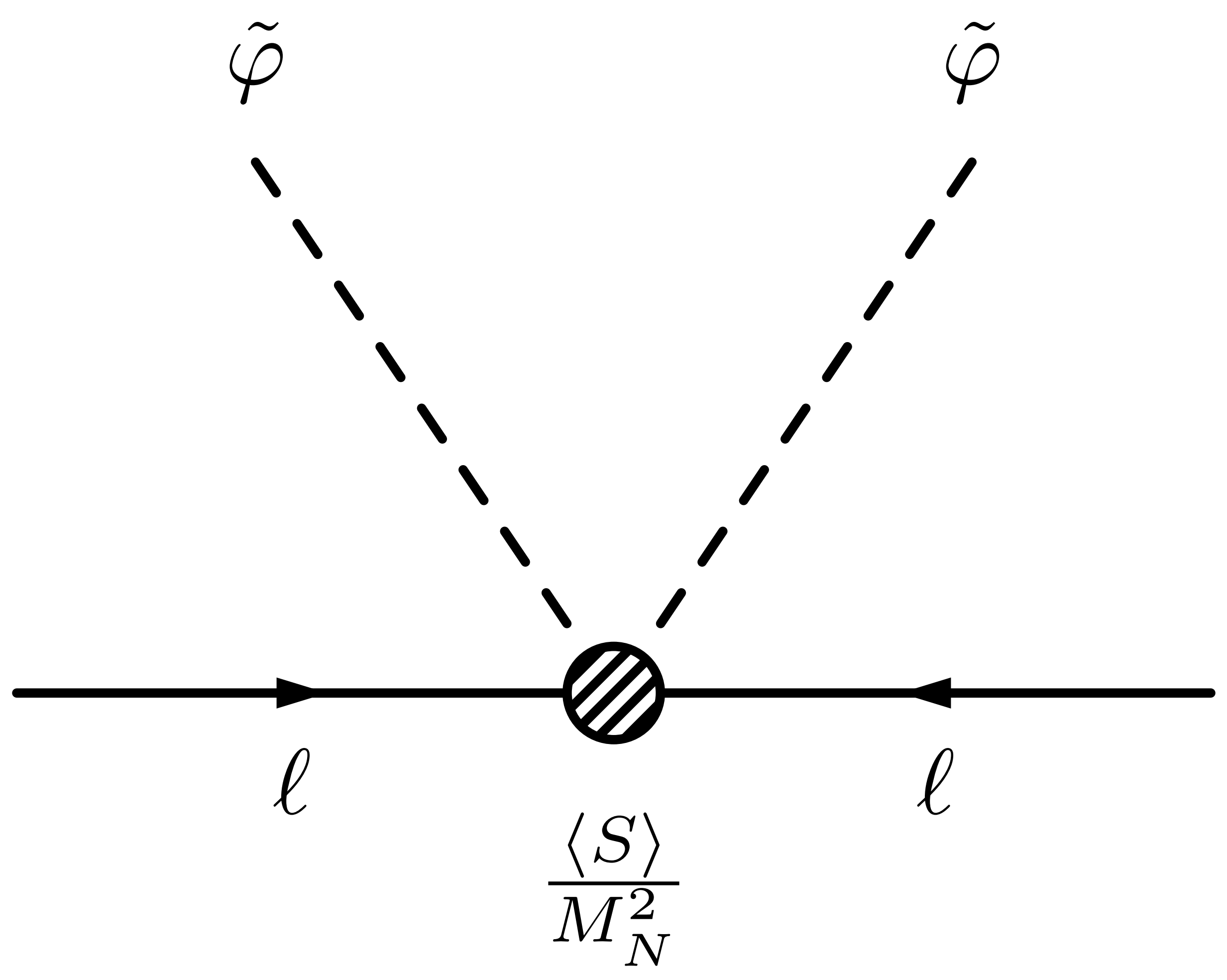
- The experimental limits on other LFV effects as $\tau \to 3\mu$ are much weaker.
- $\mathbf{v}_\ell \simeq$ **6 TeV** is not contradicting experimental constraints.
- For $r = 1$ all LFV effects are vanishing owing to custodial symmetry.

Effective operators for fermion masses

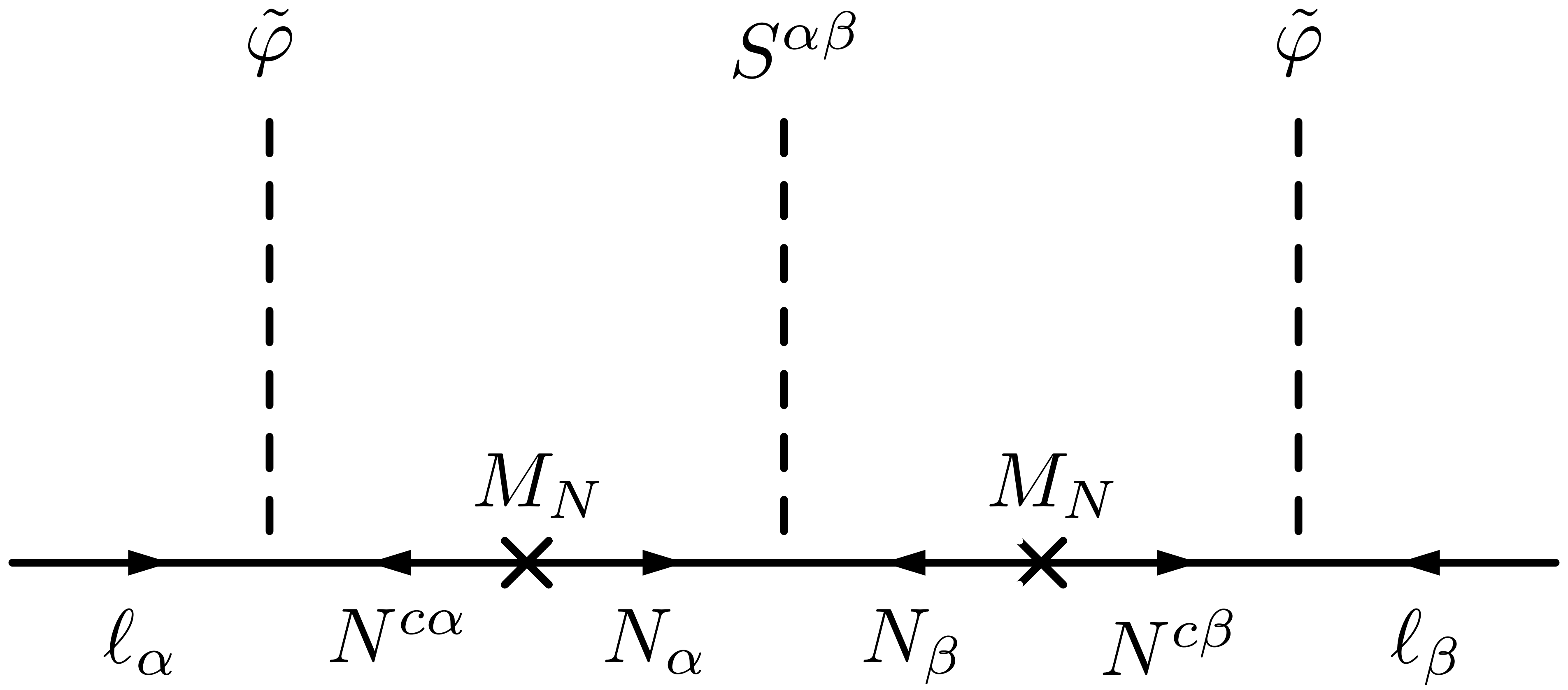
$$U(3)_\ell \times U(3)_e \times U(3)_q \times U(3)_u \times U(3)_d$$

$$\ell_L \sim 3_\ell\,, \qquad e_R \sim 3_e\,, \qquad q_L \sim 3_q \qquad u_R \sim 3_u\,, \qquad d_R \sim 3_d$$

- Fermions cannot get mass if the symmetry is unbroken.
- Yukawa couplings are induced by non-zero VEVs of scalars.



$$S_{\alpha\beta} \sim 6_l$$

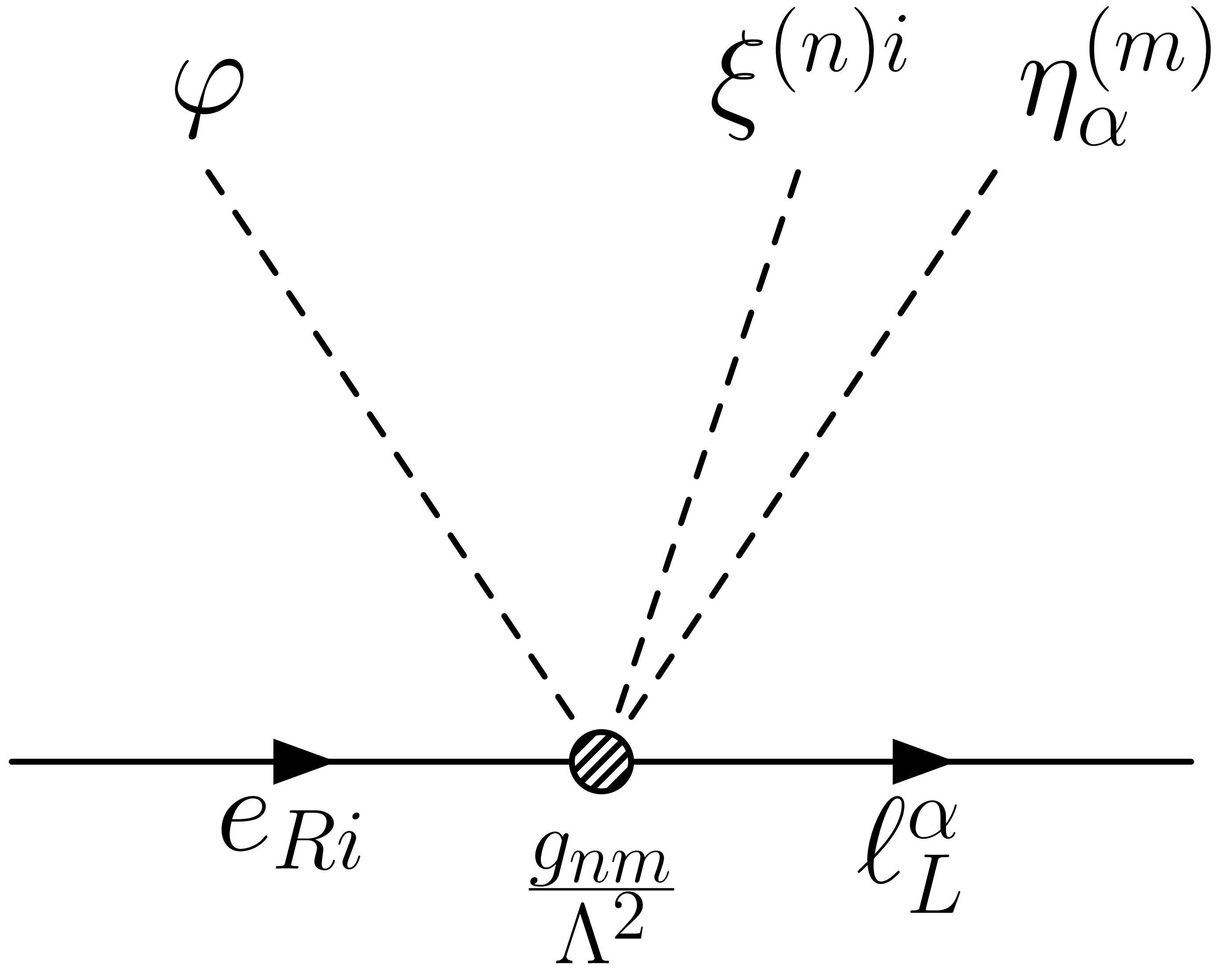


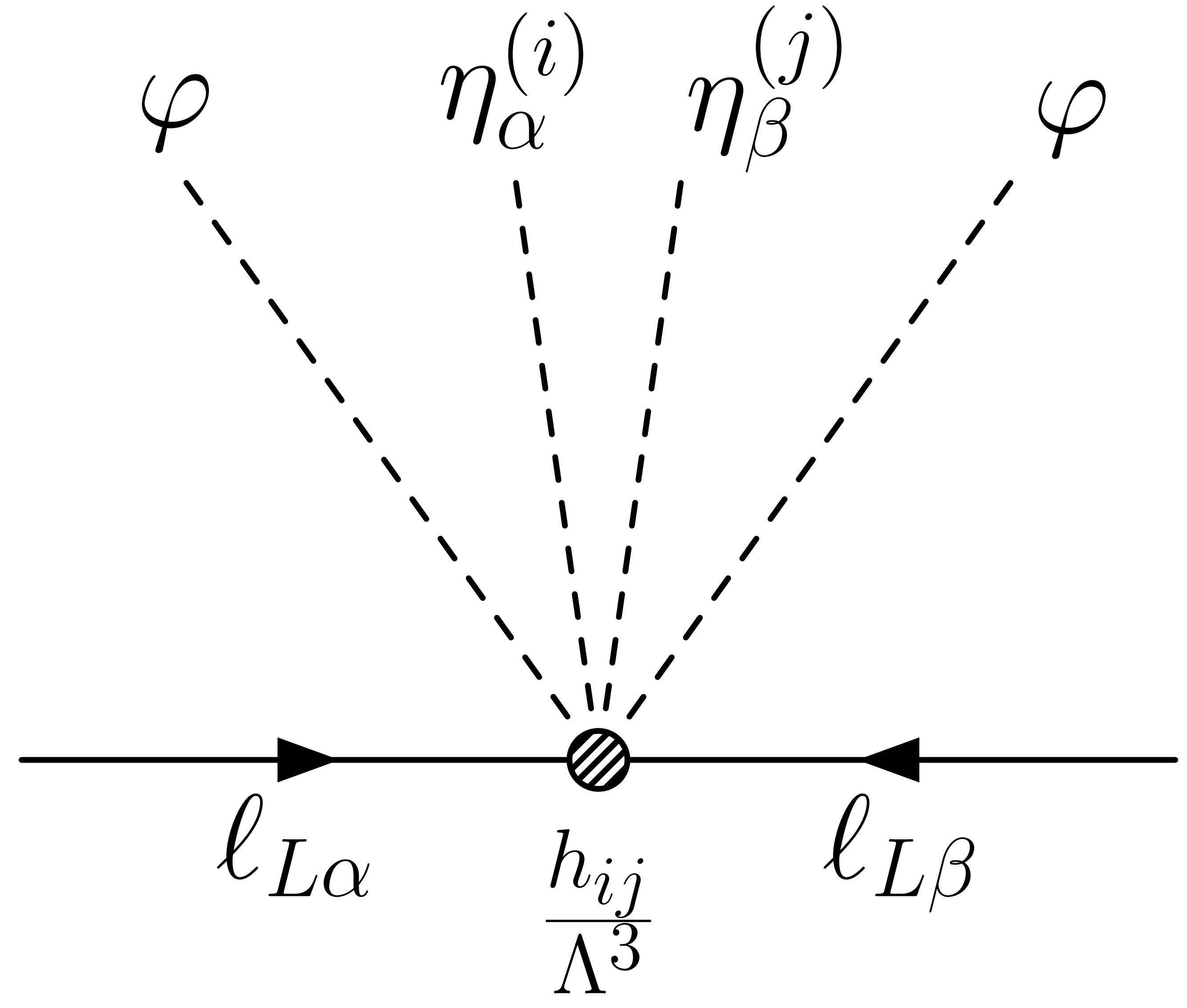
Leptons and family symmetry

$$SU(3)_\ell \times SU(3)_e$$

- Three $SU(3)_e$ triplets $\xi_i \sim 3_e$
- Three $SU(3)_\ell$ triplets $\eta_i \sim 3_\ell$

$$\frac{y_{ij}\bar{\xi}_j^{\gamma}\eta_{i\alpha}}{M_L^2}\varphi\overline{\ell_{L\alpha}}e_{Ri} + \frac{h_{ij}\bar{\eta}_i^{\alpha}\bar{\eta}_j^{\beta}}{M_\nu^3}\varphi\varphi\ell_{L\alpha}^T C\ell_\beta + h.c.$$





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$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing}$$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing}$$

$$\langle\eta_1\rangle = \begin{pmatrix} w_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle\eta_2\rangle = \begin{pmatrix} 0 \\ w_2 \\ 0 \end{pmatrix}, \quad \langle\eta_3\rangle = \begin{pmatrix} 0 \\ 0 \\ w_3 \end{pmatrix}$$

$$m_\nu^{ij} = \frac{h_{ij}w_iw_jv_w^2}{M_\nu^3}, \qquad U_{\text{PMNS}}$$

Leptons and family symmetry

$$SU(3)_\ell \times SU(3)_e$$

- Three $SU(3)_e$ triplets $\xi_i \sim 3_e$
- Three $SU(3)_\ell$ triplets $\eta_i \sim 3_\ell$

$$\frac{y_{ij}\bar{\xi}_j^\gamma \eta_{i\alpha}}{M_L^2}\varphi\overline{\ell_{L\alpha}}e_{Ri} + \frac{h_{ij}\bar{\eta}_i^\alpha \bar{\eta}_j^\beta}{M_\nu^3}\varphi\varphi\ell_{L\alpha}^T C\ell_\beta + h.c.$$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing}$$

$$SU(3)_\ell \xrightarrow{w_3} SU(2)_\ell \xrightarrow{w_2} \text{nothing}$$

$$\langle\xi_1\rangle = \begin{pmatrix} v_1 \\ 0 \\ 0 \end{pmatrix}, \quad \langle\xi_2\rangle = \begin{pmatrix} 0 \\ v_2 \\ 0 \end{pmatrix}, \quad \langle\xi_3\rangle = \begin{pmatrix} 0 \\ 0 \\ v_3 \end{pmatrix}$$

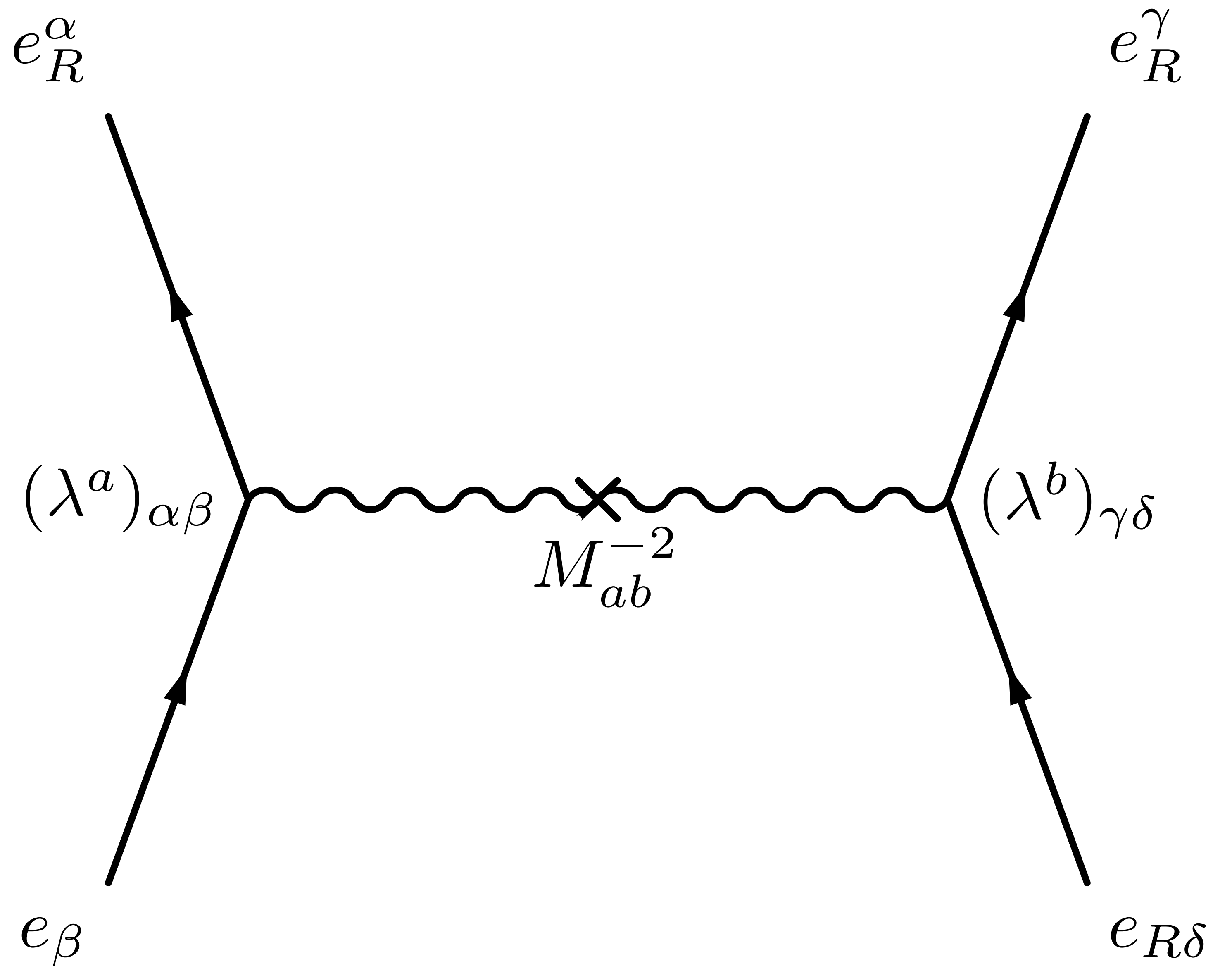
$$v_1 : v_2 : v_3 = \tilde{\epsilon}\epsilon : \epsilon : 1 \longrightarrow Y_e = y_\tau \begin{pmatrix} \sim\tilde{\epsilon}\epsilon & \sim\epsilon & \sim 1 \\ \sim\tilde{\epsilon}\epsilon & \sim\epsilon & \sim 1 \\ \sim\tilde{\epsilon}\epsilon & \sim\epsilon & \sim 1 \end{pmatrix} \longrightarrow m_e\,, m_\mu\,, m_\tau$$

$$Y_e^{ij} = \frac{y_{ij}w_i v_j}{M_L^2}$$

Gauge bosons & RH charged leptons

- Generically, FCNC:

$$SU(3)_e$$



$$e_{Ri} = V_{R1e}^{(e)} e_R + V_{R1\mu}^{(e)} \mu_R + V_{R1\tau}^{(e)} \tau_R$$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \quad m_\tau : m_\mu : m_e \approx v_3 : v_2 : v_1$$

$$v_1 : v_2 : v_3 = \tilde{\epsilon}\epsilon : \epsilon : 1$$

$$-\frac{1}{4v_2^2} \sum_{a=1}^{3} (\overline{\mathbf{e}_R}\, \lambda_a \gamma^\mu \mathbf{e}_R)^2 =$$

$$= -\frac{1}{4v_2^2} \left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix} \gamma_\mu V_R^{(e)\dagger} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V_R^{(e)} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \right]^2$$

Gauge bosons, $SU(3)_e$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_1 : v_2 : v_3 = \tilde{\epsilon}\epsilon : \epsilon : 1$$

$$\frac{1}{2}\begin{pmatrix} \mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_1 - i\mathcal{F}_2 & \mathcal{F}_4 - i\mathcal{F}_5 \\ \mathcal{F}_1 + i\mathcal{F}_2 & -\mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_6 - i\mathcal{F}_7 \\ \mathcal{F}_4 + i\mathcal{F}_5 & \mathcal{F}_6 + i\mathcal{F}_7 & -\frac{2}{\sqrt{3}}\mathcal{F}_8 \end{pmatrix}$$

$$M^2_{4,5,6,7} = \frac{g^2 v_3^2}{2} \qquad M^2_{1,2} = \frac{g^2 v_2^2}{2} \qquad M^2_{38} = \frac{g^2}{2}\begin{pmatrix} v_2^2 & -\frac{1}{\sqrt{3}}v_2^2 \\ -\frac{1}{\sqrt{3}}v_2^2 & \frac{1}{3}(4v_3^2 + v_2^2) \end{pmatrix}$$

Gauge bosons, $SU(3)_e$

$$SU(3)_e \xrightarrow{v_3} SU(2)_e \xrightarrow{v_2} \text{nothing} \qquad v_1 : v_2 : v_3 = \tilde{\epsilon}\epsilon : \epsilon : 1$$

$$\frac{1}{2}\begin{pmatrix} \mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_1 - i\mathcal{F}_2 & \mathcal{F}_4 - i\mathcal{F}_5 \\ \mathcal{F}_1 + i\mathcal{F}_2 & -\mathcal{F}_3 + \frac{1}{\sqrt{3}}\mathcal{F}_8 & \mathcal{F}_6 - i\mathcal{F}_7 \\ \mathcal{F}_4 + i\mathcal{F}_5 & \mathcal{F}_6 + i\mathcal{F}_7 & -\frac{2}{\sqrt{3}}\mathcal{F}_8 \end{pmatrix}$$

$$M^2_{4,5,6,7} = \frac{g^2 v_3^2}{2} \qquad M^2_{1,2} = \frac{g^2 v_2^2}{2} \qquad M^2_{38} = \frac{g^2}{2}\begin{pmatrix} v_2^2 & -\frac{1}{\sqrt{3}}v_2^2 \\ -\frac{1}{\sqrt{3}}v_2^2 & \frac{1}{3}(4v_3^2 + v_2^2) \end{pmatrix}$$

$$v_2 \gtrsim ?$$

Flavour changing neutral currents

In $\mathbf{SU(2)_e}$ gauge symmetry limit ($v_3 \gg v_2$):

- $SU(2)_e$ gauge bosons have equal masses;
- **no FCNC** for CUSTODIAL SYMMETRY, no matter if two families are mixed:

$$-\frac{1}{v_2^2}\sum_{a=1}^{3}(J_a^\mu)^2 = -\frac{1}{4v_2^2}\sum_{a=1}^{3}(\overline{\mathbf{e}_R}\,\lambda_a\gamma^\mu\mathbf{e}_R)^2 =$$

$$= -\frac{1}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix}\gamma_\mu V_R^{(e)\dagger}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V_R^{(e)}\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2$$

- no mixing with 3rd family $\rightarrow$ NO FCNC.
- Constraints on masses are proportional to violation of custodial symmetry (ϵ, $\tilde{\epsilon}$).

Flavour changing neutral currents

- Constraints on masses are proportional to violation of custodial symmetry:

$$\mathcal{L}_{\rm fcnc} = -\frac{1}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix} \gamma_\mu V_R^{(e)\dagger} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} V_R^{(e)} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2 + \\ - \frac{\epsilon^2}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix} \gamma_\mu V_R^{(e)\dagger} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} V_R^{(e)} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2 =$$

$$-\frac{1}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix} \gamma_\mu \begin{pmatrix} 1 & \sim\tilde{\epsilon}\epsilon^2 & \sim\tilde{\epsilon}\epsilon \\ \sim\tilde{\epsilon}\epsilon^2 & 1 & \sim\epsilon \\ \sim\tilde{\epsilon}\epsilon & \sim\epsilon & \sim\epsilon^2 \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2 + \\ - \frac{\epsilon^2}{4v_2^2}\left[\begin{pmatrix} \overline{e_R} & \overline{\mu_R} & \overline{\tau_R} \end{pmatrix} \gamma_\mu \begin{pmatrix} 1 & \sim\tilde{\epsilon} & \sim\tilde{\epsilon}\epsilon \\ \sim\tilde{\epsilon} & \sim\epsilon^2 & \sim\epsilon \\ \sim\tilde{\epsilon}\epsilon & \sim\epsilon & 1 \end{pmatrix} \begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix}\right]^2$$

Experimental constraint

- **Flavour conserving operators** constrained by compositeness limits:

$$-\frac{1}{4v_2^2}\left(\overline{e_R}\gamma^\nu e_R\right)^2 - \frac{1}{2v_2^2}\left(\overline{e_R}\gamma^\nu e_R\right)\left(\overline{\mu_R}\gamma_\nu \mu_R\right) \qquad \boxed{v_2 > 2\ \text{TeV}} \quad \to v_3 > 10\ \text{TeV}$$

- From FCNC:

LFV mode	Exp. $\Gamma_i/\Gamma_\mu(\Gamma_\tau)$	Main contribution to $\frac{\Gamma_i}{\Gamma_{\mu/\tau}}$	Predicted value of $\frac{\Gamma_i}{\Gamma_{\mu/\tau}}$
$\mu \to eee$	$< 1.0 \cdot 10^{-12}$	$\frac{1}{8}\left(\frac{v_{\text{EW}}}{v_2}\right)^4 \left\lvert V_{3e}^* V_{3\mu} + V_{2e}^* V_{2\mu}\epsilon^2\right\rvert^2$	$\leq 1.1 \cdot 10^{-13}\left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^4 \tilde{\epsilon}_{20}^2$
$\tau^- \to \mu^- e^+ e^-$	$< 1.8 \cdot 10^{-8}$	$\frac{1}{4}\left(\frac{v_{\text{EW}}}{v_2}\right)^4 \left\lvert V_{3\mu}^* V_{3\tau}\right\rvert^2 \frac{\Gamma_{\text{w}}}{\Gamma_\tau}$	$= 6.2 \cdot 10^{-9}\left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^2$
$\tau \to \mu\mu\mu$	$< 2.1 \cdot 10^{-8}$	$\frac{1}{8}\left(\frac{v_{\text{EW}}}{v_2}\right)^4 \left\lvert V_{3\mu}^* V_{3\tau}\right\rvert^2 \frac{\Gamma_{\text{w}}}{\Gamma_\tau}$	$= 3.1 \cdot 10^{-9}\left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^2$
$\mu \to e\gamma$	$< 4.2 \cdot 10^{-13}$	$\frac{3\alpha}{2\pi}\left(\frac{v_{\text{EW}}}{v_2}\right)^4 \left\lvert V_{3e}^* V_{3\mu}\right\rvert^2$	$= 3.1 \cdot 10^{-15}\left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^4 \tilde{\epsilon}_{20}^2$
$\tau \to \mu\gamma$	$< 4.4 \cdot 10^{-8}$	$\frac{3\alpha}{2\pi}\left(\frac{v_{\text{EW}}}{v_2}\right)^4 \left\lvert V_{3\mu}^* V_{3\tau}\right\rvert^2 \frac{\Gamma_{\text{w}}}{\Gamma_\tau}$	$= 8.7 \cdot 10^{-11}\left(\frac{2\,\text{TeV}}{v_2}\right)^4 \epsilon_{20}^2$

Quarks and family symmetries

$U(3)_q \times U(3)_d \times U(3)_u$ with gauge factors $SU(3)_q \times SU(3)_d \times SU(3)_u$

- Quark masses:

$$\frac{y^d_{ij}}{M_d^2}\,\eta^q_{i\alpha}\overline{\xi}^{d\gamma}_j\,\phi\,\overline{q_{L\alpha}}d_{R\gamma} + \frac{y^u_{ij}}{M_u^2}\,\eta^q_{i\alpha}\overline{\xi}^{u\gamma}_j\,\tilde{\phi}\,\overline{q_{L\alpha}}u_{R\gamma} + \text{h.c.}$$

- mass hierarchy is related with hierarchies in breaking of $SU(3)_q \times SU(3)_d \times SU(3)_u$ gauge symmetry:

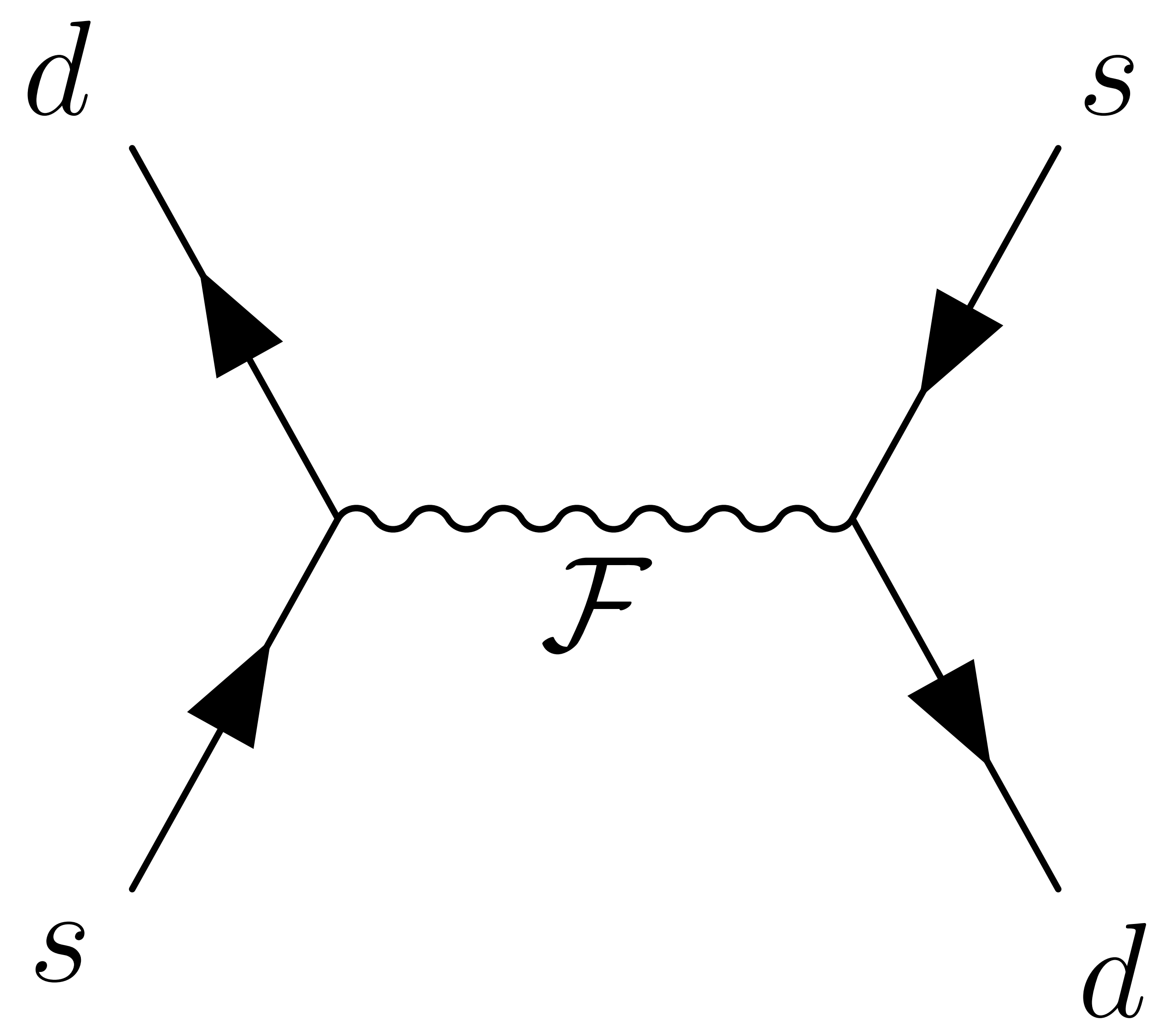
$$m_b : m_s : m_d = 1 : \epsilon_d\epsilon_q : \epsilon_d\tilde{\epsilon}_d\epsilon_q\tilde{\epsilon}_q$$
$$m_t : m_c : m_u = 1 : \epsilon_u\epsilon_q : \epsilon_u\tilde{\epsilon}_u\epsilon_q\tilde{\epsilon}_q$$

Quarks and family symmetries

- $K^0 - \bar{K}^0$ oscillation is induced by:

$$-\frac{1}{4v_{d2}^2}\left[(V_{3d}V_{3s}^*)^2 + \epsilon_d^2(V_{2d}V_{2s}^*)^2\right](\overline{s_R}\gamma^\mu d_R)^2$$

- Since $|V_{2d}V_{2s}^*| \sim \tilde{\epsilon}_d$, $|V_{3d}V_{3s}^*| \sim \epsilon_d^2\tilde{\epsilon}_d$, K^0 mixing is suppressed by $\epsilon_d^2\tilde{\epsilon}_d^2 \ll 1$.
- New contribution can be constrained to be less than the SM contribution. By taking $\epsilon_d\tilde{\epsilon}_d \sim 10^{-2}$, the mass scale $v_{d2} \sim 7\,\text{TeV}$ is compatible with the constraint from the neutral kaons mass difference.
- As regards the imaginary part contributing to ϵ_K, with the same choice $\epsilon_d\tilde{\epsilon}_d \sim 10^{-2}$, $v_{d2} \sim 7\,\text{TeV}$ is still allowed if the phase of $V_{2d}V_{2s}^*$ is $O(0.1)$.



VEVS

Can the hierarchy of the VEVs of ξs be natural? The generic potential is:

$$V(\xi) = \lambda_n(|\xi_n|^2 - \frac{\mu_n^2}{2\lambda_n})^2 + \lambda_{klnm}\xi_k^\dagger\xi_l\xi_n^\dagger\xi_m + (\mu\xi_1\xi_2\xi_3 + \text{h.c.})$$

- The dimensional constant μ can be arbitrarily small since if $\mu \to 0$ the Lagrangian acquires global $U(1)_e$ symmetry.
- $v_2/v_3 \sim m_\mu/m_\tau$ (one order of magnitude) can emerge from a natural fluctuation of mass terms μ_n^2 and coupling constants λ.
- Small v_1 is naturally obtained when the third flavon ξ_1 has positive mass squared. Then for $\mu \neq 0$ non-zero VEV $\langle\xi_1\rangle$ is induced:

$$v_1 = \frac{\mu v_2 v_3}{\mu_1^2}$$

- Taking μ small enough, say $\mu < v_2$, one can naturally get $v_1 \ll v_2$. **The hierarchy of the VEVs of ξs can be natural**.

Flavons and LFV

- Also flavons can mediate the LFV processes.
- Lepton Yukawa couplings with the flavon fields ξ_n:

$$h_{in}\,\xi_n^\alpha \overline{\ell_{Li}} e_{R\alpha} \qquad h_{in} = \frac{g_{in} v_{\mathrm{w}}}{M}$$

  which are generically flavor-changing.

- ξ_2, with mass $\mu_2 \sim v_2$, induces the effective operator:

$$-\frac{h_{32}h_{22}}{\mu_2^2}\,(\overline{\tau}\mu)\,(\overline{\mu}\mu)\,, \qquad \frac{h_{32}h_{22}}{\mu_2^2} \simeq \frac{m_\mu^2}{v_2^4}$$

  For $v_2 > 2$ TeV, the width of $\tau \to 3\mu$ decay induced by this operator is more than 12 orders of magnitude below the experimental limit.

- The width of $\mu \to 3e$ decay induced by analogous operator mediated by flavon ξ_1 is also suppressed by orders of magnitude.

Triangle anomalies

$$SU(3)_\ell \times SU(3)_e \times SU(3)_Q \times SU(3)_u \times SU(3)_d$$

$$\ell_L \sim 3_\ell,\ e_R \sim 3_e,\ Q_L \sim 3_Q,\ u_R \sim 3_u,\ d_R \sim 3_d$$

- In order to cancel $SU(3)^3$ anomalies for each triplet another triplet (SM singlet) with opposite chirality is needed.
- An interesting possibility is to introduce the mirror twins with opposite chirality and analogous representation of mirror SM gauge symmetry $SU(3)' \times SU(2)' \times U(1)'$:

$$\ell'_R \sim 3_\ell,\ e'_L \sim 3_e,\ Q'_R \sim 3_Q,\ u'_L \sim 3_u,\ d'_L \sim 3_d$$

- Couplings with flavons:

$$\frac{g_{in}\xi_n^\alpha}{M}(\phi\overline{\ell_{Li}}e_{R\alpha} + \phi'\overline{\ell'_{Ri}}e'_{L\alpha}) + h.c.$$

Triangle anomalies 2

- As an example, for $SU(3)_e$, mixed triangle anomaly $U(1) \times SU(3)_e^2$ must be cancelled. New leptons

$$\mathcal{E}_{L\alpha} \sim (1, -2, 3_e; X), \qquad \mathcal{E}_{Ri} \sim (1, -2, 1; X)$$

and for mirror parity

$$\mathcal{E}'_{R\alpha} \sim (1, -2', 3_e; X), \qquad \mathcal{E}'_{Li} \sim (1, -2', 1; X)$$

cancel the mixed triangle

$$U(1) \times SU(3)_e^2,\ U(1)_X \times SU(3)_e^2,\ U(1) \times U(1)_X^2,\ U(1)_X \times U(1)^2$$

- Masses from Yukawa couplings

$$y_{in} \xi_n^\alpha \overline{\mathcal{E}_{Ri}} \mathcal{E}_{L\alpha} + y_{in} \xi_n^\alpha \overline{\mathcal{E}_{Li}}' \mathcal{E}'_{R\alpha} + h.c.$$

- The lightest has mass $O(100)$ GeV. If $U(1)_X$ is unbroken, then it is stable. Current experimental lower limit on charged new leptons is 102.6 GeV.

Cosmological implications

- Mirror matter is a viable candidate for light dark matter dominantly consisting of mirror helium and hydrogen atoms.
- The flavor gauge bosons are messengers between the two sectors and so a portal for direct detection.
- $T'/T < 0.2 \div 0.3$ from CMB and large scale structures.
- Freeze-out temperature of horizontal interactions between the two sectors should not exceed $T_d \simeq (v_2/2)^{\frac{4}{3}} \times 130$ MeV. Or $v'_{\rm EW} \gg v_{\rm EW}$.
- For neutrinos

$$\frac{Y_\nu^{ij}}{\mathcal{M}}(\phi\phi l_{Li}^T C l_{Lj} + \phi'\phi' l_{Ri}'^T C l'_{Rj}) + \frac{\tilde{Y}_\nu^{ij}}{\mathcal{M}}\phi\phi' \overline{l_{Li}} l'_{Rj} + h.c.$$

the last operator gives COLEPTOGENESIS.