Determination of V_{ud} from superallowed β decays

- Master formula Hardy, Towner 2018

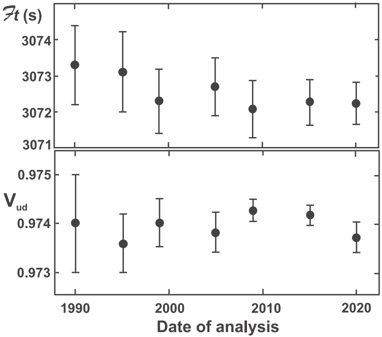
$$|V_{ud}|^2 = \frac{2984.432(3)\,\text{s}}{\mathcal{F}t(1+\Delta_R^V)}$$

with (universal) radiative corrections Δ_R^V

- Value of V_{ud} crucially depends on Δ_R^V:

Ref.	Δ_R^V
Marciano, Sirlin 2006	0.02361(38)
Seng, Gorchtein, Patel, Ramsey-Musolf 2018	0.02467(22)
Czarnecki, Marciano, Sirlin 2019	0.02426(32)
Seng, Feng, Gorchtein, Jin 2020	0.02477(24)
Hayen 2020	0.02473(27)
Shiells, Blunden, Melnitchouk 2021	0.02472(18)
Cirigliano, Crivellin, MH, Moulson 2022	0.02467(27)

- Similar correction Δ_R for neutron decay



Hardy, Towner 2020

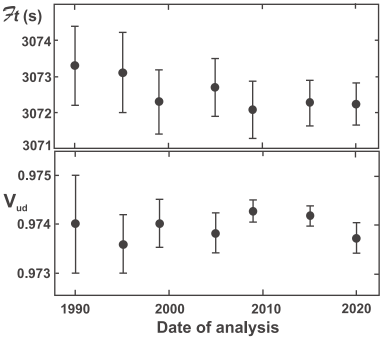
Determination of V_{ud} from superallowed β decays

- Further corrections
  - Isospin breaking Miller, Schwenk 2008, 2009, Condren, Miller 2022, Seng, Gorchtein 2022, Crawford, Miller 2022
  - Nuclear corrections Seng, Gorchtein, Ramsey-Musolf 2018, Gorchtein 2018
- Estimate from Gorchtein 2018 becomes dominant source of uncertainty

$$V_{ud}^{0^+\to 0^+} = 0.97367(11)_{\text{exp}}(13)_{\Delta_V^R}(27)_{\text{NS}}[32]_{\text{total}}$$

- Improvements from ab-initio nuclear structure? Martin, Stroberg, Holt, Leach 2021
- More on nuclear corrections later talk by J. Holt

$\hookrightarrow$ for now, can we converge on Δ_R, Δ_R^V?



Hardy, Towner 2020

Master formulae for universal radiative corrections

- Master formula for Δ_R Czarnecki et al. 2004

$$1+\Delta_R=\left[1+\frac{\alpha}{2\pi}\left(\bar{g}(E_m)-3\log\frac{m_p}{2E_m}\right)\right]\left[L(2E_m,m_p)+\frac{\alpha}{2\pi}\delta+2\Box_A^{\gamma W}\big|_{Q^2\leq Q_0^2}\right]$$
$$\times\left[\tilde{S}(m_p,M_Z)-\frac{\alpha}{2\pi}\log\frac{M_W}{M_Z}+2\Box_A^{\gamma W}\big|_{Q^2\geq Q_0^2}+\text{NLL}\right]$$

- Includes
  - Resummation of logs

$$L(2E_m,m_p)=\left(\frac{\hat{\alpha}(m_\mu)}{\hat{\alpha}(2E_m)}\right)^{9/4}\left(\frac{\hat{\alpha}(M_\pi)}{\hat{\alpha}(m_\mu)}\right)^{9/8}\left(\frac{\hat{\alpha}(M_K)}{\hat{\alpha}(M_\pi)}\right)^{1}\left(\frac{\hat{\alpha}(m_p)}{\hat{\alpha}(M_K)}\right)^{9/10}=1.02090$$

$$\tilde{S}(m_p,M_Z)=\left(\frac{\hat{\alpha}(m_c)}{\hat{\alpha}(m_p)}\right)^{9/16}\left(\frac{\hat{\alpha}(m_\tau)}{\hat{\alpha}(m_c)}\right)^{27/64}\left(\frac{\hat{\alpha}(m_b)}{\hat{\alpha}(m_\tau)}\right)^{27/76}\left(\frac{\hat{\alpha}(M_W)}{\hat{\alpha}(m_b)}\right)^{27/80}\left(\frac{\hat{\alpha}(M_Z)}{\hat{\alpha}(M_W)}\right)^{27/17}=1.01682$$

$\hookrightarrow$ adopt hadronic scheme for $L(2E_m,m_p)$, matching correction for $\hat{\alpha}$ at $\mu=m_p$

  - γW box diagram

$$\Box_A^{\gamma W}=\frac{\alpha}{2\pi}\int_0^\infty\frac{dQ^2}{Q^2}\frac{M_W^2}{M_W^2+Q^2}\int_0^1 dx\,F_3^{(0)}(x,Q^2)\frac{1+2r}{(1+r)^2}\qquad r=\sqrt{1+4m_p^2x^2/Q^2}$$

separated into different Q^2 regions

An attempt at unification

$\Box_A^{\gamma W} \times 10^{-3}$	Shiells et al. 2020	Seng et al. 2018	Seng et al. 2020	Czarnecki et al. 2019	Hayen 2020	our estimate
Elastic	1.05(4)	1.06(6)	1.06(6)	0.99(10)	1.06(6)	1.06(6)
Resonance	0.04(1)	0.05(1)	0.05(1)	–	–	0.04(1)
Regge	0.52(7)	0.51(8)	0.56(9)	0.38^*	0.46^*	0.49(11)
DIS ($Q^2 \geq Q_0^2$)	2.29(3)	2.26^*	2.26^*	2.24^*	2.32^*	2.28(4)

- Strategy:
  - Convert all numbers to $Q_0^2 = 2\,\text{GeV}^2$ (modified numbers carry *)
  - Running of α included in DIS, not as prefactor
  - Inelastic contributions from Czarnecki et al. 2019, Hayen 2020 booked as “Regge”

$\hookrightarrow$ looks pretty consistent, with main uncertainty from Regge region

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- Questions:
  - Can one do better to find a "consensus value"?
  - What would it take to improve the Regge region?

Prescription proposal

- Proximity to the quark mass scheme is desired to make contact with phenomenology
- **Prescription proposal** (both with $\alpha = 0$)

Pure QCD	*Iso-symmetric QCD*
$\hat{M}_{\pi^+} = 135.0 \text{ MeV}$	$\bar{M}_\pi = 135.0 \text{ MeV}$
$\hat{M}_{K^+} = 491.6 \text{ MeV}$	$\bar{M}_K = 494.6 \text{ MeV}$
$\hat{M}_{K^0} = 497.6 \text{ MeV}$	$\bar{M}_{D_s} = 1967 \text{ MeV}$
$\hat{M}_{D_s} = 1967 \text{ MeV}$	

- Scale setting with $\hat{M}_{\Omega^-} = \bar{M}_{\Omega^-} = 1672.45 \text{ MeV}$ possibly using a theory scale as proxy

A. Portelli, 5th plenary meeting of the Muon $g-2$ theory initiative at Higgs Centre

Isospin schemes in $(g-2)_\mu$

- Difference between different isospin schemes higher order in α, $\delta = m_u - m_d$

  $\hookrightarrow$ small as long as close to quark-mass scheme (BMWc vs. RBC/UKQCD)
- Cottingham formula

$$M_{K^\pm} = \big(494.58 - 3.05_\delta + 2.14_{e^2}\big)\,\text{MeV} \qquad M_{K^0} = \big(494.58 + 3.03_\delta\big)\,\text{MeV}$$

  $\hookrightarrow$ close to proposed lattice prescription
- Phenomenological estimates of isospin breaking in HVP largely consistent with lattice QCD
- Significant scheme dependence only likely if parts of the lattice calculation were replaced by phenomenology

Isospin schemes in $(g-2)_\mu$

	SD window		int window		LD window		full HVP	
	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$	$\mathcal{O}(e^2)$	$\mathcal{O}(\delta)$
$\pi^0\gamma$	0.16(0)	–	1.52(2)	–	2.70(4)	–	4.38(6)	–
$\eta\gamma$	0.05(0)	–	0.34(1)	–	0.31(1)	–	0.70(2)	–
ρ–ω mixing	–	0.05(0)	–	0.83(6)	–	2.79(11)	–	3.68(17)
FSR (2π)	0.11(0)	–	1.17(1)	–	3.14(3)	–	4.42(4)	–
M_{π^0} vs. $M_{\pi^\pm}$ (2π)	0.04(1)	–	$-0.09(7)$	–	$-7.62(14)$	–	$-7.67(22)$	–
FSR (K^+K^-)	0.07(0)	–	0.39(2)	–	0.29(2)	–	0.75(4)	–
kaon mass (K^+K^-)	$-0.29(1)$	0.44(2)	$-1.71(9)$	2.63(14)	$-1.24(6)$	1.91(10)	$-3.24(17)$	4.98(26)
kaon mass $(\bar{K}^0K^0)$	0.00(0)	$-0.41(2)$	$-0.01(0)$	$-2.44(12)$	$-0.01(0)$	$-1.78(9)$	$-0.02(0)$	$-4.62(23)$
total	0.14(1)	0.08(3)	1.61(12)	1.02(20)	$-2.44(16)$	2.92(17)	$-0.68(29)$	4.04(39)
BMWc 2020	–	–	$-0.09(6)$	0.52(4)	–	–	$-1.5(6)$	1.9(1.2)
JLM 2021	–	–	–	–	–	–	–	3.32(89)