V_{us} from kaon decays

Matthew Moulson
INFN Frascati

in collaboration with **V. Cirigliano, A. Crivellin, M. Hoferichter**

Ref: V. Cirigliano et al., **arXiv:2208.11707** [hep-ph]

Electroweak Precision Physics from Beta Decays to the Z Pole
ELECTRO 2022 - Mainz Institute for Theoretical Physics
Johannes Gutenberg University, 28 October 2022

V_{us} from $K_{\ell 3}$: Déjà vu all over again?

→ **2002** (2004 PDG)	**Old $K_{\ell 3}$ data give $\Delta_{CKM} = 1 - \lvert V_{ud}\rvert^2 - \lvert V_{us}\rvert^2 = 0.0035(15)$** A 2.3$\sigma$ hint of unitarity violation?
2003	**BNL 865 measures BR($K^+ \to \pi^0 e^+ \nu$) = 5.13(10)%** Value for V_{us} consistent with unitarity
2004-2008 (mostly)	**Many new measurements from KTeV, ISTRA+, KLOE, NA48** • **BRs, lifetimes, form-factor slopes** • **Much higher statistics** than older measurements • Importance of **radiative corrections** • Proper reporting of **correlations** between measurements
2008-present	**Much progress on hadronic constants from lattice QCD** Value of V_{us} used in precision tests of the Standard Model
2018-present	**New evaluation of radiative corrections for V_{ud}** ~ 3σ evidence of unitarity violation in first row ~ 2σ tension between results for V_{us} from $K_{\ell 3}$ and $K_{\mu 2}$

Determination of V_{us} from $K_{\ell 3}$ data

$$\Gamma(K_{\ell 3(\gamma)}) = \frac{C_K^2 G_F^2 m_K^5}{192\pi^3} S_{\rm EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \times I_{K\ell}(\lambda_{K\ell}) \left(1 + 2\Delta_K^{SU(2)} + 2\Delta_{K\ell}^{\rm EM}\right)$$

with $K \in \{K^+, K^0\}$; $\ell \in \{e, \mu\}$, and:

C_K^2 1/2 for K^+, 1 for K^0

$S_{\rm EW}$ Universal SD EW correction (1.0232)

Inputs from experiment:

$\Gamma(K_{\ell 3(\gamma)})$ Rates with well-determined treatment of radiative decays:

- Branching ratios
- Kaon lifetimes

$I_{K\ell}(\{\lambda\}_{K\ell})$ Integral of form factor over phase space: λs parameterize evolution in t

- K_{e3}: Only λ_+ (or λ_+', λ_+'')
- $K_{\mu 3}$: Need λ_+ and λ_0

Inputs from theory:

$f_+^{K^0\pi^-}(0)$ Hadronic matrix element (form factor) at zero momentum transfer ($t = 0$)

$\Delta_K^{SU(2)}$ Form-factor correction for $SU(2)$ breaking

$\Delta_{K\ell}^{\rm EM}$ Form-factor correction for long-distance EM effects

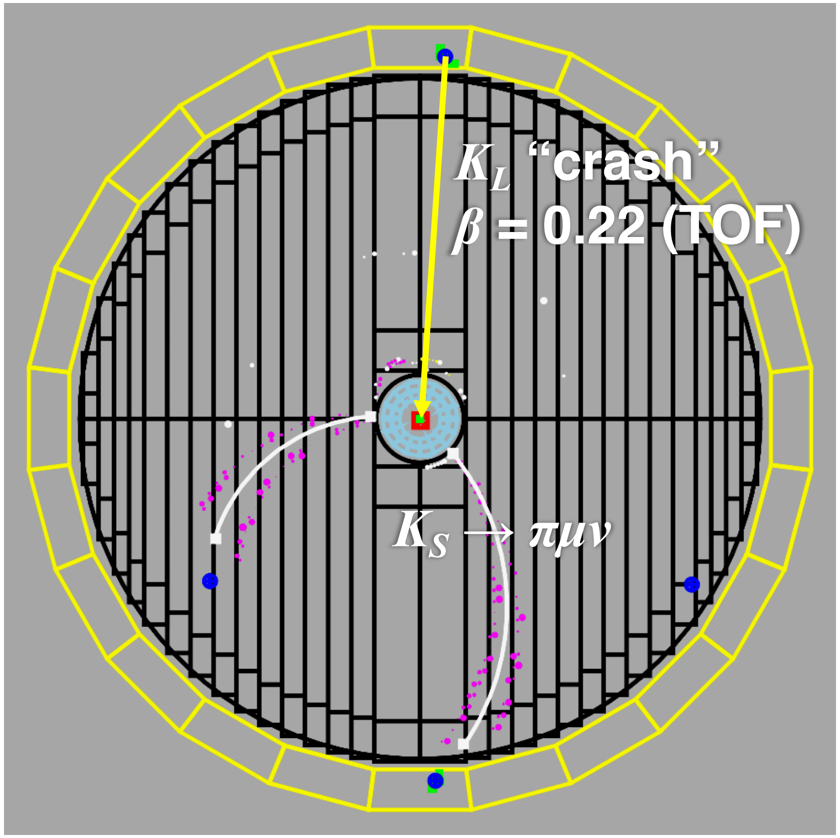
Modern experimental data for V_{us} from $K_{\ell 3}$

Experiment	Measurement	Year
BNL865	**BR($K^+ \to \pi^0{}_D e^+ \nu$)/BR($K^+ \to \pi^0{}_D X^+$)**	**2003**
KTeV	**$\tau(K_S)$**	**2003**
	BR(K_{Le3}), BR($K_{L\mu3}$), $\lambda_+(K_{Le3})$, $\lambda_{+,0}(K_{L\mu3})$	**2004**
ISTRA+	**$\lambda_+(K^-{}_{e3})$, $\lambda_{+,0}(K^-{}_{e3})$**	**2004**
KLOE	**$\tau(K_L)$**	**2005**
	BR(K_{Le3}), BR($K_{L\mu3}$), BR(K_{Se3}), $\lambda_+(K_{Le3})$	**2006**
	$\lambda_{+,0}(K_{L\mu3})$	**2007**
	$\tau(K^\pm)$, BR(K_{Le3}), BR($K_{L\mu3}$)	**2008**
NA48	**$\tau(K_S)$**	**2002**
	BR(K_{Le3}/2 tracks), $\lambda_+(K_{Le3})$	**2004**
	$\Gamma(K_{Se3}/K_{Le3})$, $\lambda_{+,0}(K_{L\mu3})$	**2007**
NA48/2	**BR($K^+{}_{e3}/\pi^+\pi^0$), BR($K+{}_{\mu3}/\pi^+\pi^0$)**	**2007**

Above data set used for 2010 FlaviaNet review (fits, averages, etc.)

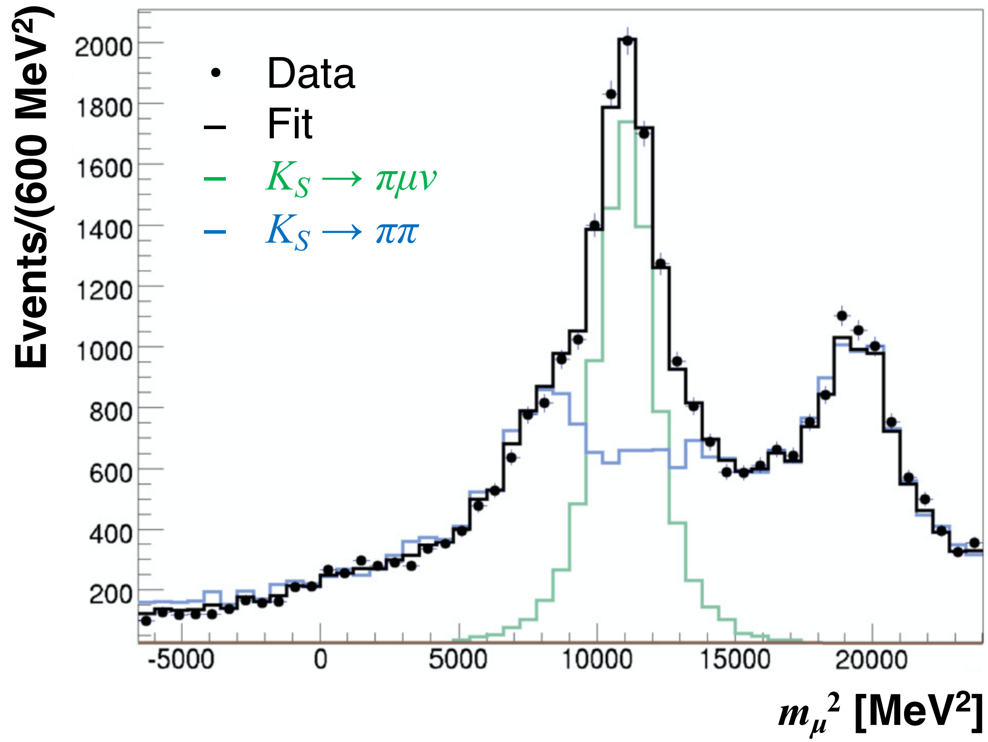
BR($K_S \to \pi\mu\nu$) from KLOE-2

K_S from $\phi \to K_L K_S$ tagged by K_L interaction in calorimeter barrel



Preselection with kinematic BDT and time-of-flight $\pi\mu$ assignment

Fit to $m_\mu^2 = (E_{KS} - E_\pi - p_{\text{miss}})^2 - \mathbf{p}_\mu^2$



KLOE-2
PLB 804 (2020)

BR($K_S \to \pi\mu\nu$) = (4.56 ± 0.20) × 10^{-4}
First measurement of this BR

Dominant systematic from $\pi\mu$ PID by TOF

BR($K_S \to \pi e \nu$) from KLOE-2

K_S lifetime known very precisely: Precision on V_{us} limited only by BR($K_S \to \pi e \nu$)

KLOE-2
PLB 636 (2006)

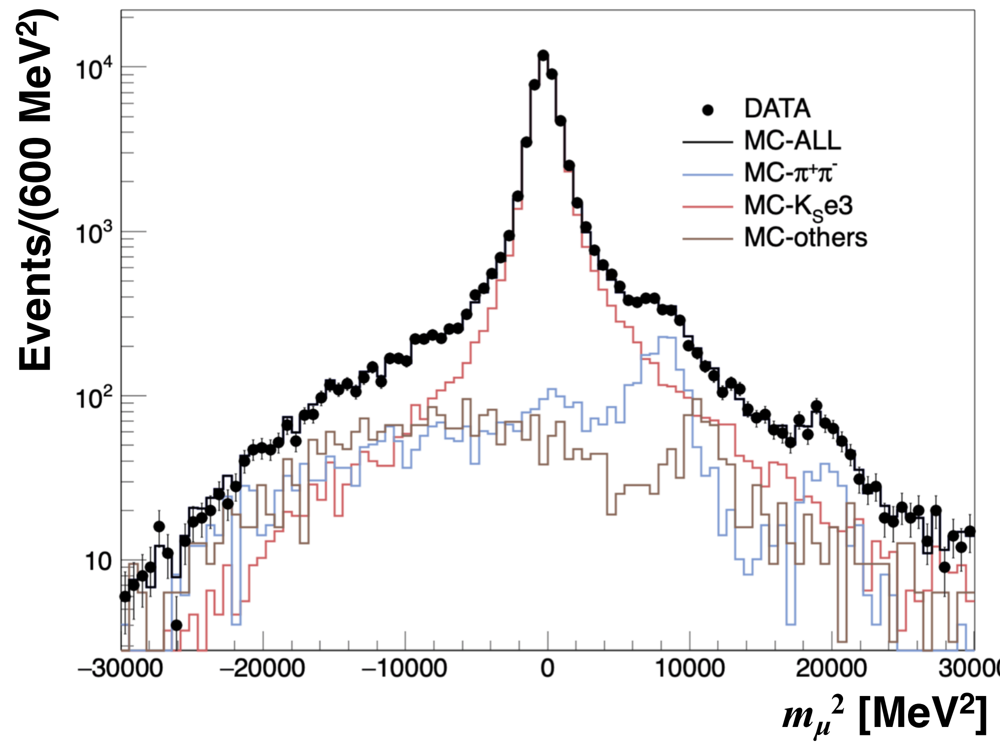
BR($K_S \to \pi e \nu$) = $(7.05 \pm 0.09) \times 10^{-4}$
Previous best measurement, 0.4 fb^{-1}: 1.3% uncertainty

K_S from $\phi \to K_L K_S$ tagged by K_L interaction in calorimeter barrel

Preselection with kinematic BDT and time-of-flight πe assignment

Fit to $m_e^2 = (E_{KS} - E_\pi - p_{\text{miss}})^2 - \mathbf{p}_e^2$

Reconstruction systematics: MC validation with $K_L \to \pi e \nu$ control sample



KLOE-2
2208.04872

BR($K_S \to \pi e \nu$) = $(7.153 \pm 0.037 \pm 0.043) \times 10^{-4}$
0.4 + 1.6 fb^{-1}: 0.8% uncertainty

Fit to K_S rate data (2022)

7 input measurements:

- **KLOE '06** BR $\pi^0\pi^0/\pi^+\pi^-$
- **NA48** $\Gamma(K_S \to \pi e\nu)/\Gamma(K_L \to \pi e\nu)$, τ_S
- **KLOE '11** τ_S
- **KTeV '11** τ_S
- **KLOE-2 '22** BR $\pi e\nu/\pi^+\pi^-$ **New!**
- **KLOE-2 '20** BR $\pi\mu\nu/\pi^+\pi^-$

2 possible constraints:

- **Σ BR = 1**
- **BR(K_{e3})/BR($K_{\mu3}$) = 0.6640(17)**
  From ratio of phase-space integrals from current fit to dispersive $K_{\ell3}$ form factor parameters

Only sum constraint used for fit

Parameter	Value
BR($\pi^+\pi^-(\gamma)$)	69.20(5)%
BR($\pi^0\pi^0$)	30.69(5)%
BR(K_{e3})	7.15(6) × 10^{-4}
BR($K_{\mu3}$)	4.56(20) × 10^{-4}
τ_S	89.58(4) ns

χ^2/ndf = 0.36/3 (Prob = 95%)

Little correlation for K_{e3} $K_{\mu3}$ from fit

10-20% correlations with $\pi^0\pi^0/\pi^+\pi^-$

Input measurements essentially unchanged

Fit to K_L rate data (2010)

21 input measurements:

- **5 KTeV** ratios
- **NA48** BR(K_{e3}/2 track)
- **4 KLOE** BRs
  with dependence on τ_L
- **KLOE**, **NA48** BR($\pi^+\pi^-/K_{\ell3}$)
- **KLOE**, **NA48** BR($\gamma\gamma/3\pi^0$)
- **BR($2\pi^0/\pi^+\pi^-$) from K_S fit, Re ε'/ε**
- **KLOE** τ_L from $3\pi^0$
- **Vosburgh '72** τ_L
- **KTeV** BR($\pi^+\pi^-\gamma/\pi^+\pi^-(\gamma)$)
- **E731, 2 KTeV** BR($\pi^+\pi^-\gamma_{\mathrm{DE}}/\pi^+\pi^-\gamma$)

1 constraint: Σ BR = 1

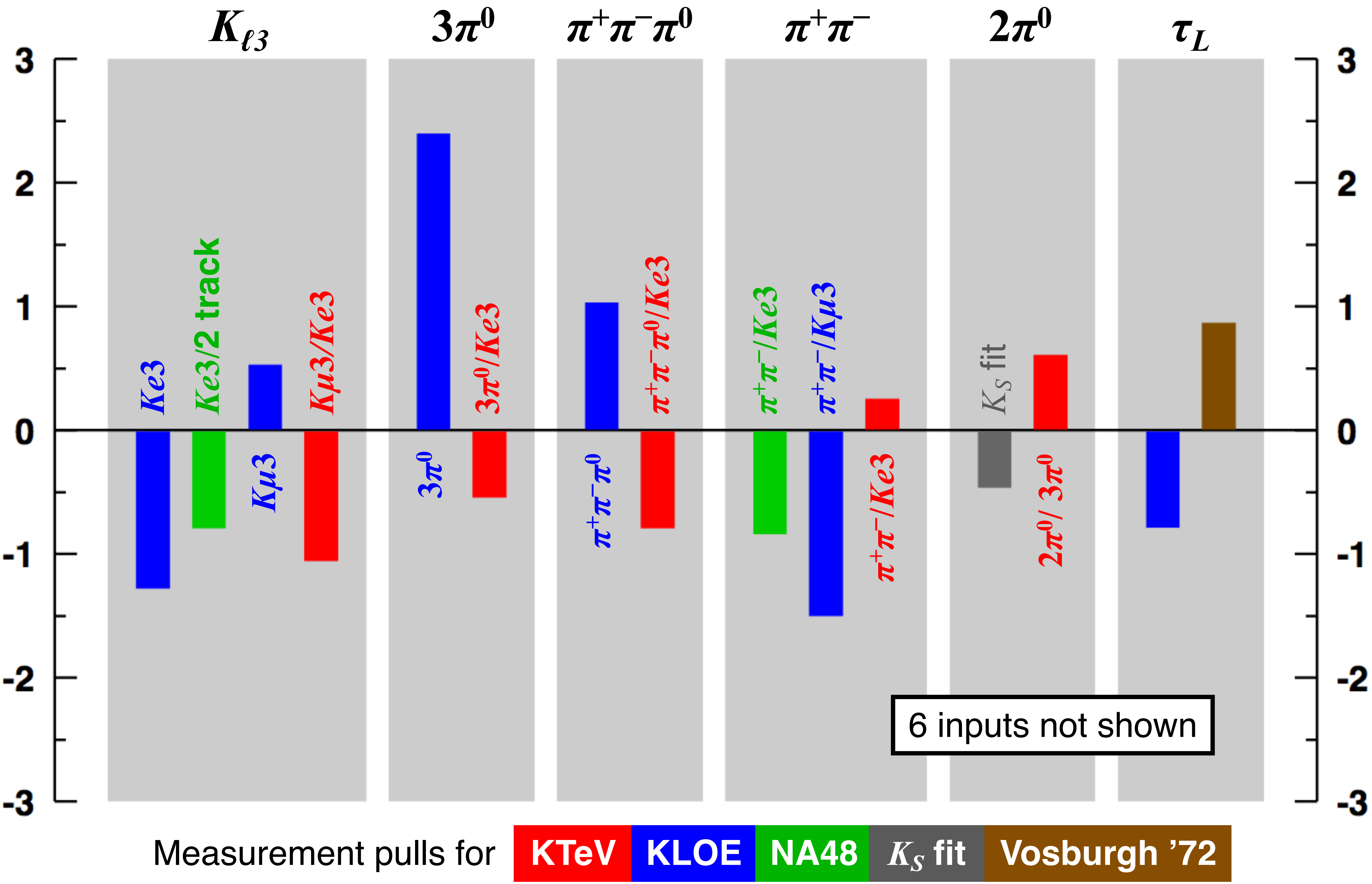
Parameter	Value	S
BR(K_{e3})	0.4056(9)	1.3
BR($K_{\mu3}$)	0.2704(10)	1.5
BR($3\pi^0$)	0.1952(9)	1.2
BR($\pi^+\pi^-\pi^0$)	0.1254(6)	1.3
BR($\pi^+\pi^-(\gamma_{\mathrm{IB}})$)	1.967(7) × 10^{-3}	1.1
BR($\pi^+\pi^-\gamma$)	4.15(9) × 10^{-5}	1.6
BR($\pi^+\pi^-\gamma_{\mathrm{DE}}$)	2.84(8) × 10^{-5}	1.3
BR($2\pi^0$)	8.65(4) × 10^{-4}	1.4
BR($\gamma\gamma$)	5.47(4) × 10^{-4}	1.1
τ_L	51.16(21) ns	1.1

χ^2/ndf = 19.8/12 (Prob = 7.0%)

Essentially same result as 2010 fit

Current PDG (since '09): **37.4/17** (0.30%)

Comparison: K_L fit result vs. input data



Fit to $K^\pm$ rate data (2014)

17 input measurements:

- **3 old τ values in PDG**
- **KLOE τ**
- **KLOE BR $\mu\nu$, $\pi\pi^0$**
- **KLOE BR K_{e3}, $K_{\mu3}$**
  with dependence on τ
- **NA48/2 BR $K_{e3}/\pi\pi^0$, $K_{\mu3}/\pi\pi^0$**
- **E865 BR K_{e3}/KDal**
- **3 old BR $\pi\pi^0/\mu\nu$**
- **KEK-246 $K_{\mu3}/K_{e3}$**
- **KLOE BR $\pi\pi\pi$, $\pi\pi^0\pi^0$**
- (Bisi '65 BR $\pi\pi^0\pi^0/\pi\pi\pi$ removed)

1 constraint: Σ BR = 1

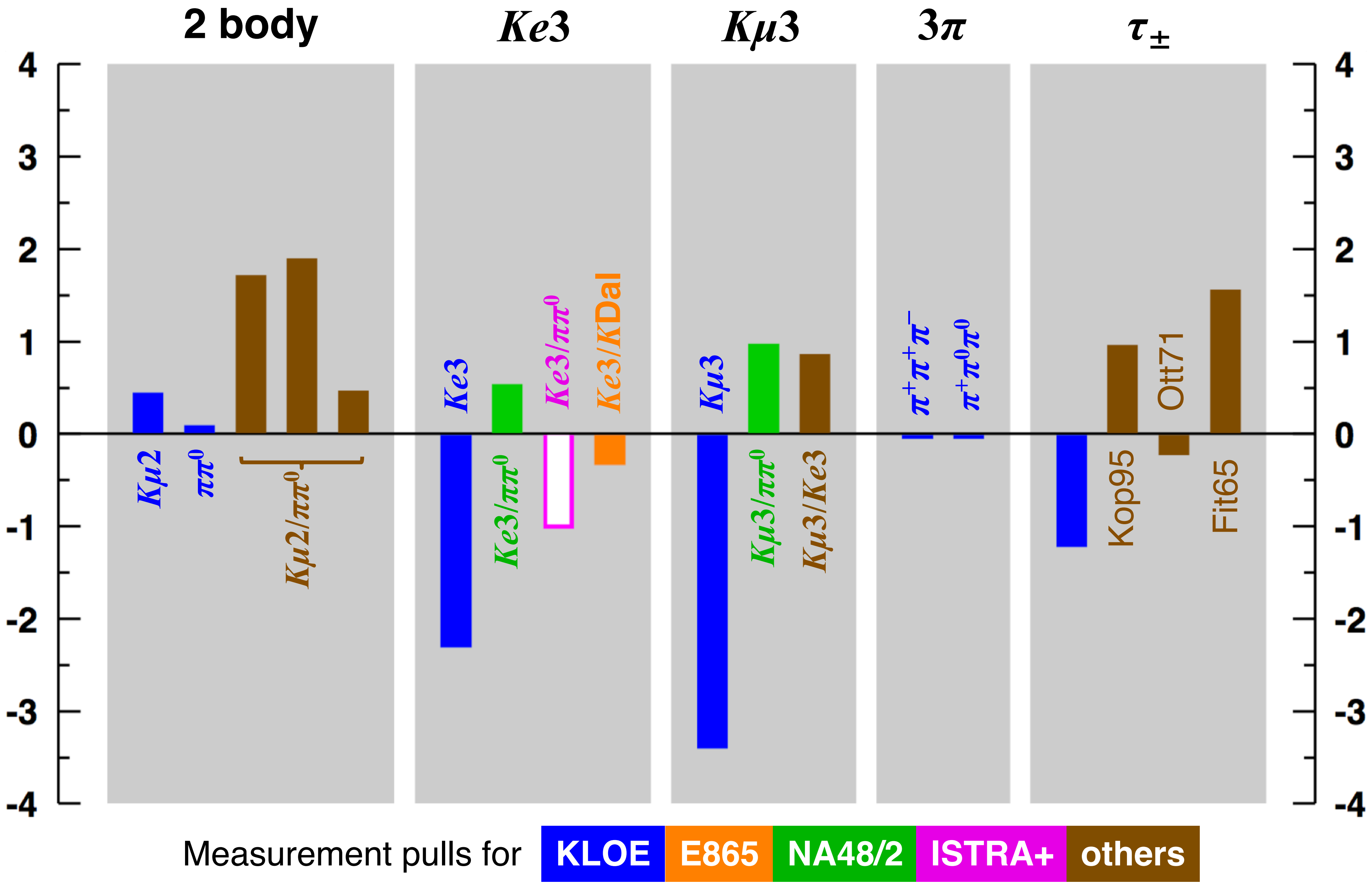
Much more selective than PDG fit
PDG '16: 35 inputs, 8 parameters

Parameter	Value	S
BR($\mu\nu$)	63.58(11)%	1.1
BR($\pi\pi^0$)	20.64(7)%	1.1
BR($\pi\pi\pi$)	5.56(4)%	1.0
BR(K_{e3})	5.088(27)%	1.2
BR($K_{\mu3}$)	3.366(30)%	1.9
BR($\pi\pi^0\pi^0$)	1.764(25)%	1.0
$\tau_\pm$	12.384(15) ns	1.2

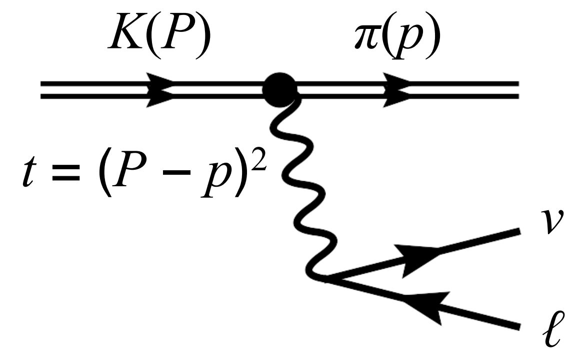
χ^2/ndf = 25.5/11 (Prob = 0.78%)
compare PDG '16: 53/28 (0.26%)

With **ISTRA+ '14 BR($K^-_{e3}/\pi^-\pi^0$)**
- **BR(K_{e3}) = 5.083(27)%**
- Negligible changes in other parameters, fit quality

Comparison: $K^{\pm}$ fit result vs. input data



$K_{\ell 3}$ form factors



Hadronic matrix element:

$$\langle \pi | J_\alpha | K \rangle = f(0) \times \left[\tilde{f}_+(t)(P+p)_\alpha + \tilde{f}_-(t)(P-p)_\alpha \right]$$

K_{e3} decays: Only **vector form factor:** $\tilde{f}_+(t)$

$K_{\mu 3}$ decays: Also need **scalar form factor:** $\tilde{f}_0(t) = \tilde{f}_+ + \tilde{f}_- \dfrac{t}{m_K^2 - m_\pi^2}$

For V_{us}, need integral over phase space of squared matrix element:

Parameterize form factors and fit distributions in t (or related variables)

Parameterizations based on systematic expansions

Taylor expansion:

$$\tilde{f}_{+,0}(t) = 1 + \lambda_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right)$$

$$\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right) + \lambda''_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right)^2$$

Notes:

Many parameters: λ_+', λ_+'', λ_0', λ_0''

Large correlations, unstable fits

Higher-order terms ignored

$K_{\ell 3}$ form-factor parameterizations

Parameterizations incorporating physical constraints

Pole dominance:

$$\tilde{f}_{+,0}(t) = \frac{M_{V,S}^2}{M_{V,S}^2 - t}$$

Notes:

What does M_S correspond to?

Dispersion relations:

$$\tilde{f}_{+}(t) = \exp\left[\frac{t}{m_\pi^2}(\Lambda_+ - H(t))\right]$$

$$\tilde{f}_{0}(t) = \exp\left[\frac{t}{m_K^2 - m_\pi^2}(\ln C - G(t))\right]$$

Notes:

Allows tests of ChPT & low-energy dynamics

$H(t)$, $G(t)$ evaluated from $K\pi$ scattering data and given as polynomials

Bernard et al., PRD 80 (2009)

Uncertainties from representations $H(t)$, $G(t)$ of $K\pi$ phase-shift data contribute to fit results for Λ_+, $\ln C$

– Small compared to other uncertainties for single measurements (so far)

2010 FlaviaNet analysis used average of FF parameters from dispersive fits

– Parameterization uncertainties beginning to dominate averages for Λ_+, $\ln C$

$K_{\ell 3}$ form factor data

Form-factor parameter measurements in FlaviaNet 2010 fit:

K_L: **KTeV, KLOE, NA48** (K_{e3} only)

K^-: **ISTRA+**

Even if not in the original publications, all experiments have:

- Obtained results for Taylor, pole, and dispersive parameterizations
- Supplied parameter correlation coefficients

Recent measurements

NA48/2
JHEP 1810 (2018)

2.3×10^6 $K^{\pm}_{\mu 3}$
4.4×10^6 $K^{\pm}_{e3}$

Updates 2012 preliminary

K^+ and K^- simultaneously acquired in dedicated minimum-bias run

Taylor, pole, and dispersive fits with complete investigation of systematics

OKA
JETPL 107 (2018)

5.25×10^6 K^{+}_{e3}

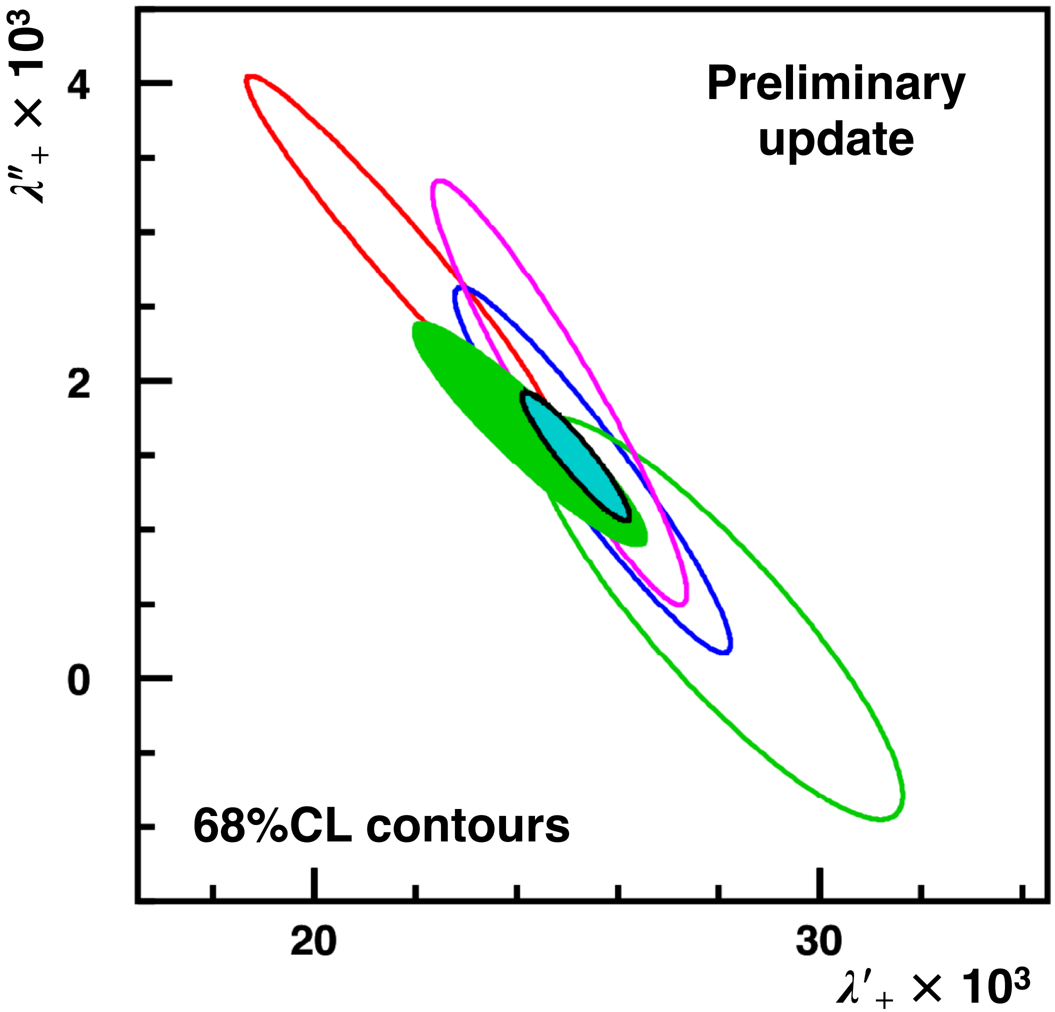
Extraordinarily high precision claimed, esp. for λ_+', λ_+''

Rudimentary discussion of systematics

Not yet included in updated K_{e3} fit

Fit to K_{e3} form-factor slopes

Slopes from **KTeV** **KLOE** **ISTRA+** **NA48** **NA48/2** **Current**



Slope parameters × 10^3

λ'_+ = **25.17 ± 0.70**

λ''_+ = **1.49 ± 0.29**

$\rho(\lambda'_+, \lambda''_+)$ = −0.929

χ^2/ndf = 6.4/10 (61%)

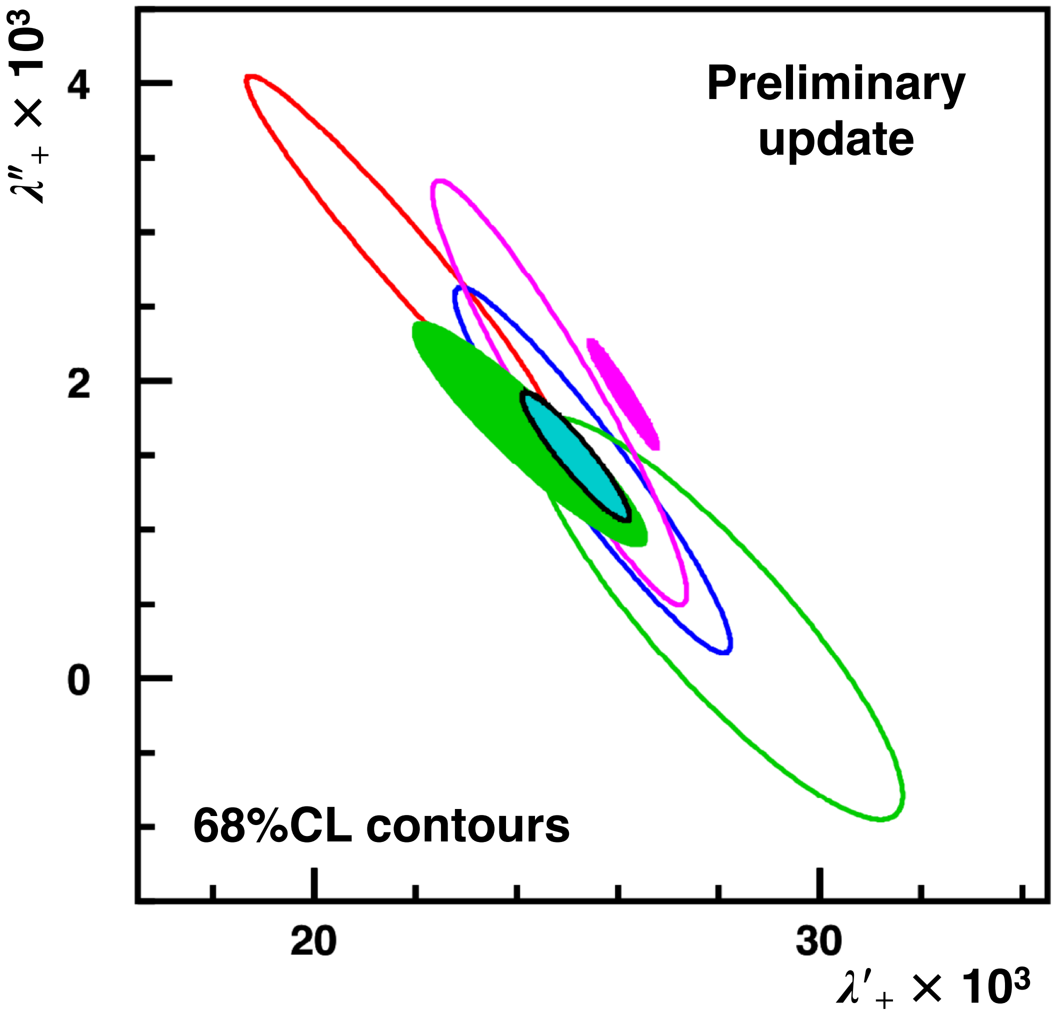
Excellent compatibility

$I(K^0_{e3})$ = 0.15458(19)

$I(K^+_{e3})$ = 0.15895(20)

Fit to K_{e3} form-factor slopes

Slopes from KTeV KLOE ISTRA+ NA48 NA48/2 Current



Slope parameters × 10³

λ'_+ = **25.17 ± 0.70**

λ''_+ = **1.49 ± 0.29**

$\rho(\lambda'_+, \lambda''_+)$ = −0.929

χ^2/ndf = 6.4/10 (61%)

OKA
JETPL 107 (2018)

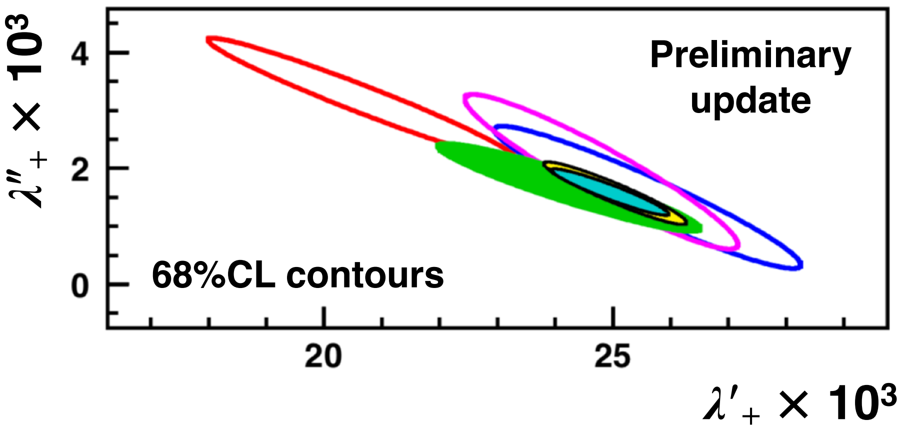
Not included in fit

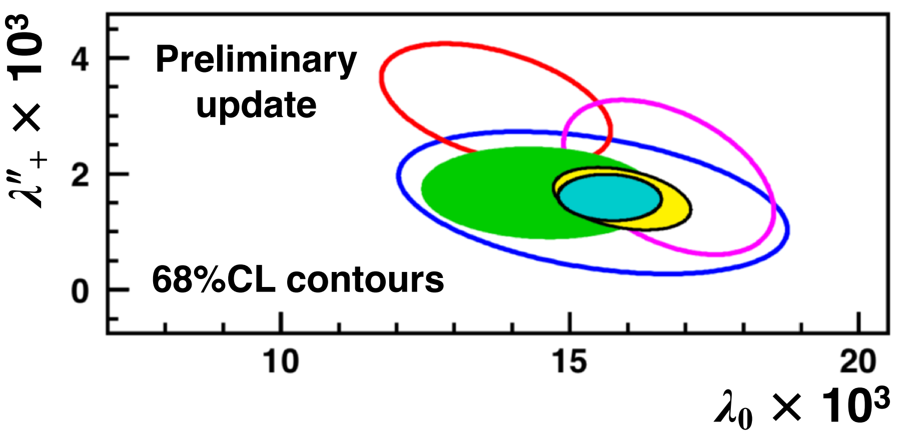
- Stated as preliminary
- If included: χ^2/ndf → 45/10 ($P \sim 10^{-6}$)

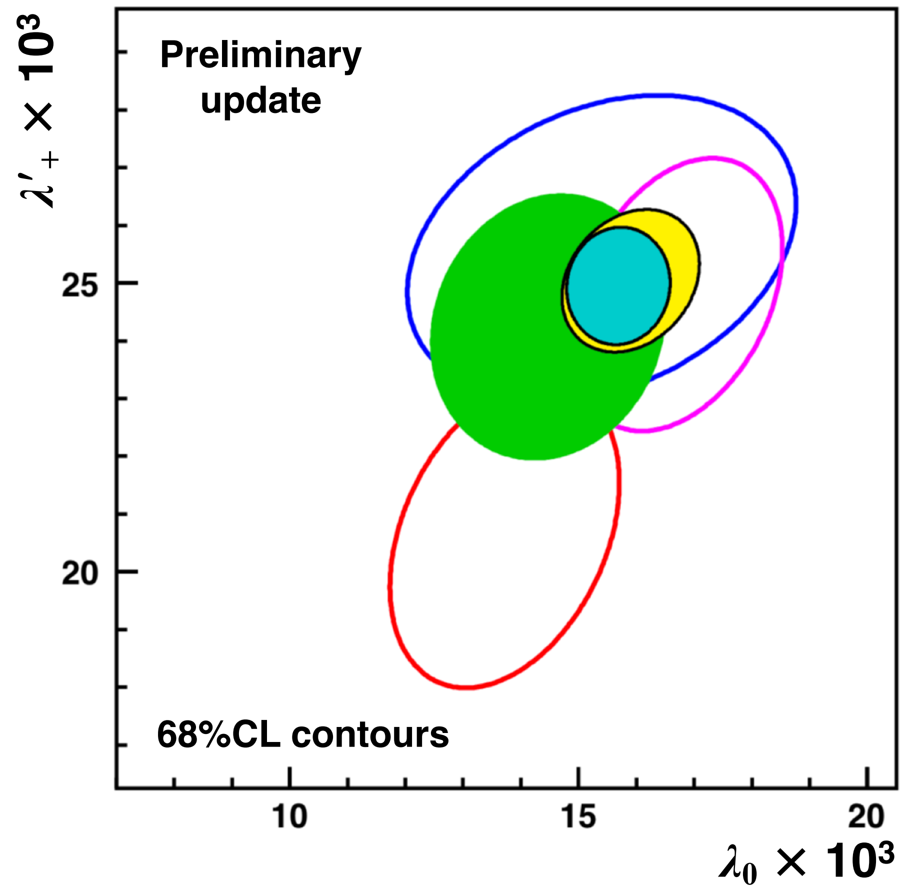
Fits to $K_{e3} + K_{\mu 3}$ form-factor slopes

KTeV | KLOE | ISTRA+ | NA48/2 — 2010 fit | Current

NA48 K_{e3} data included in fits but not shown







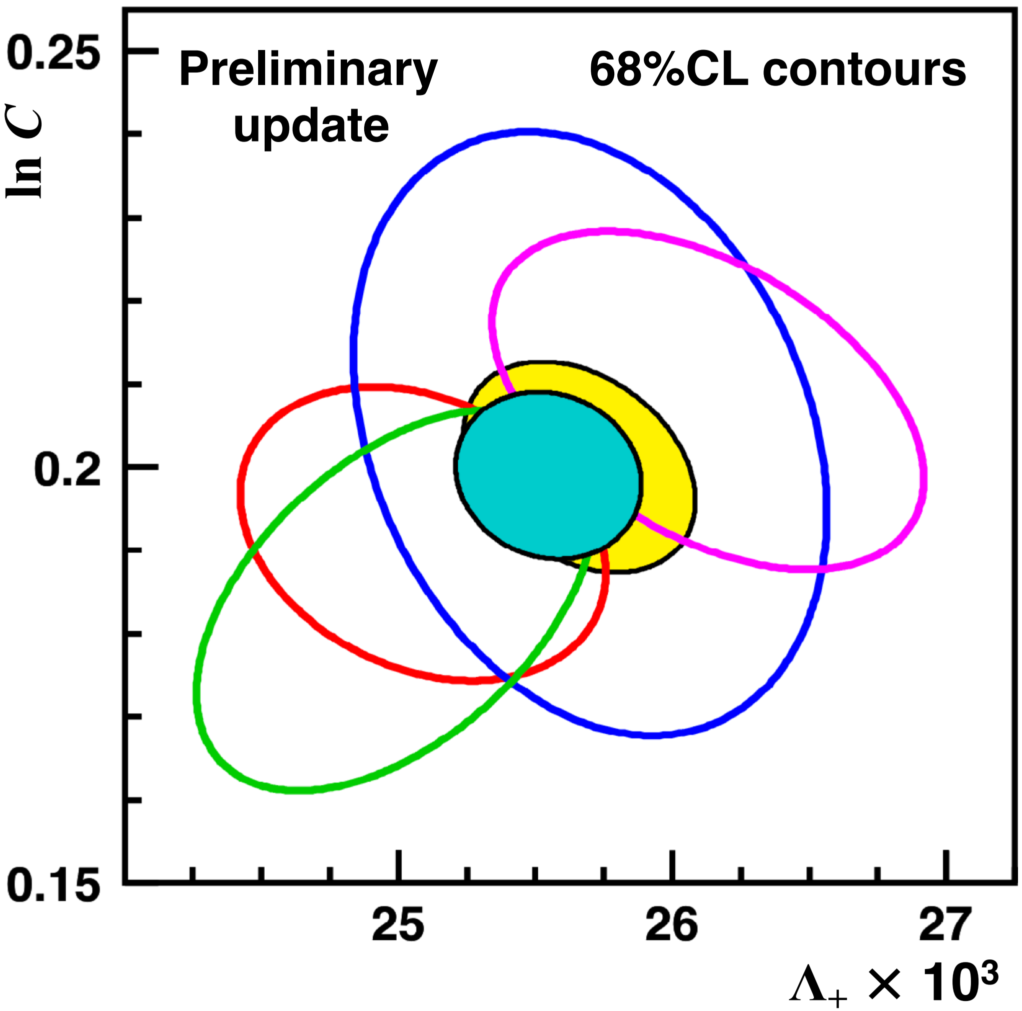
2010: $\chi^2 = 12.1/8$ ($P = 14.5\%$) | Update: $\chi^2 = 13.4/11$ ($P = 26.8\%$)

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2**

NA48 K_{e3} data included in fits but not shown

2010 fit **Current**



$\Lambda_+ \times 10^3$ = **25.55 ± 0.38**

$\ln C$ = **0.1992(78)**

$\rho(\Lambda_+, \ln C)$ = −0.110

χ^2/ndf = 7.5/7 (38%)

Integrals

Mode	Update	2010
$K^0{}_{e3}$	**0.15470(15)**	0.15476(18)
$K^+{}_{e3}$	**0.15915(15)**	0.15922(18)
$K^0{}_{\mu 3}$	**0.10247(15)**	0.10253(16)
$K^+{}_{\mu 3}$	**0.10553(16)**	0.10559(17)

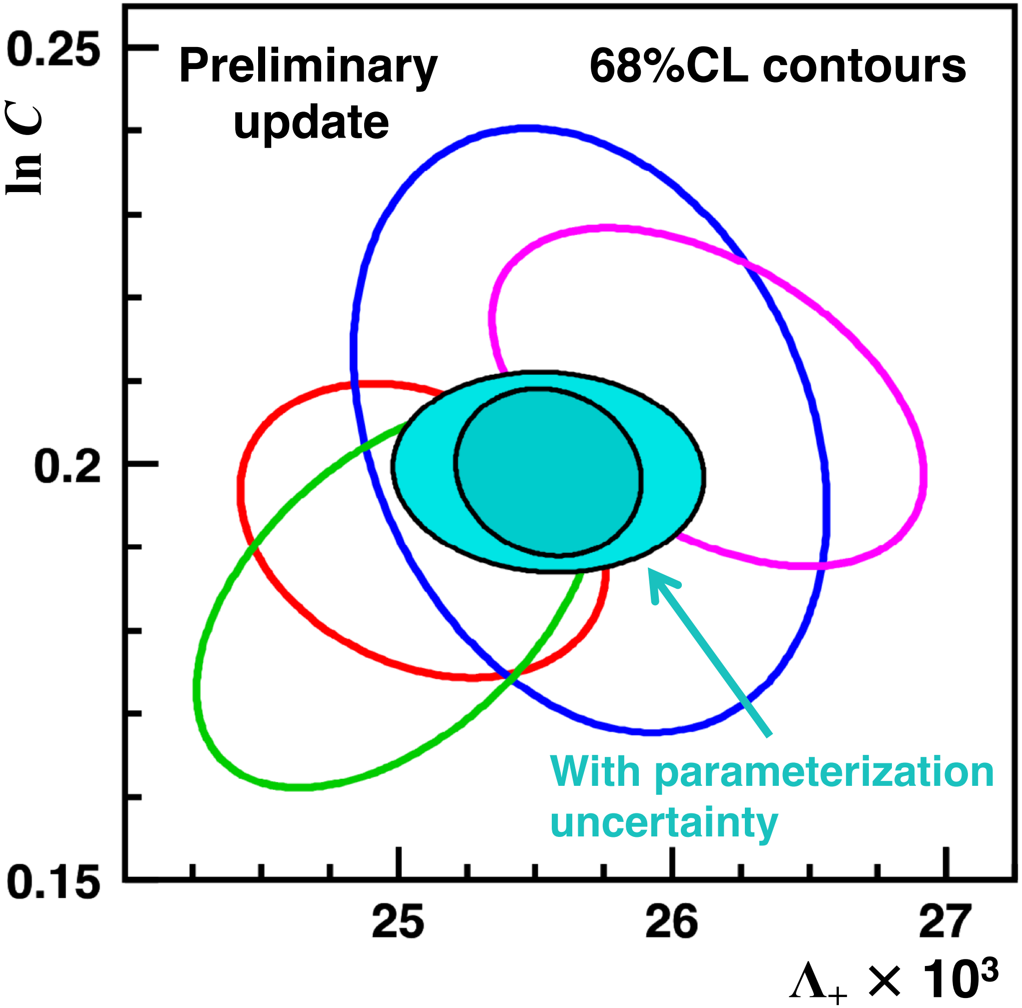
Only tiny changes in central values

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from KTeV | KLOE | ISTRA+ | NA48/2

NA48 K_{e3} data included in fits but not shown

2010 fit | Current



$\Lambda_+ \times 10^3 = 25.55 \pm 0.38$

$\ln C = 0.1992(78)$

$\rho(\Lambda_+, \ln C) = -0.110$

$\chi^2/\text{ndf} = 7.5/7\ (38\%)$

Fit results include common uncertainty from $H(t)$, $G(t)$:

$\sigma_{\text{param}}(\Lambda_+) = 0.3 \times 10^{-3}$

$\sigma_{\text{param}}(\ln C) = 0.0040$

KTeV, Bernard et al. ’09

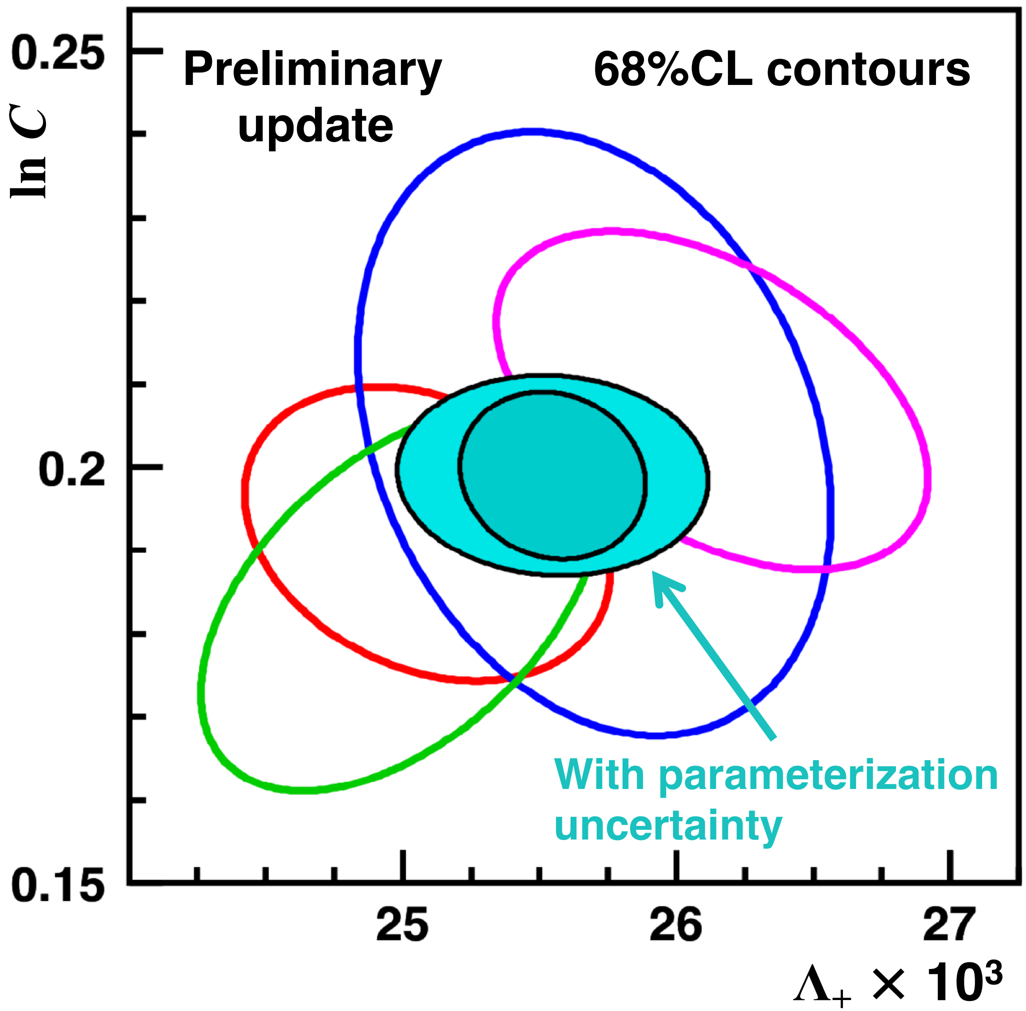
Confidence ellipses shown **without** common uncertainty

(except as indicated)

Dispersive parameters for $K_{\ell 3}$ form factors

$K_{\ell 3}$ avgs from **KTeV** **KLOE** **ISTRA+** **NA48/2** **2010 fit** **Current**

NA48 K_{e3} data included in fits but not shown



$\Lambda_+ \times 10^3$	=	**25.55 ± 0.38**
$\ln C$	=	**0.1992(78)**
$\rho(\Lambda_+, \ln C)$	=	−0.110
χ^2/ndf	=	7.5/7 (38%)

Fit results include common uncertainty from $H(t)$, $G(t)$

Without common uncertainty:

$\sigma(\Lambda_+)$	**(0.38 → 0.22) × 10^{-3}**
$\sigma(\ln C)$	**0.0078 → 0.0067**
$\sigma(K_{e3}$ int)	**0.10% → 0.09%**
$\sigma(K_{\mu 3}$ int)	**0.15% → 0.11%**

Long-distance EM corrections

Mode-dependent corrections $\Delta^{EM}_{K\ell}$ to phase-space integrals $I_{K\ell}$ from EM-induced Dalitz plot modifications

- Values depend on acceptance for events with additional real photon(s)
- All recent measurements assumed fully inclusive

FlaviaNet analysis and updates used Cirigliano et al. '08

- Comprehensive analysis at fixed order e^2p^2

Seng et al.
JHEP 07 (2022)

Calculation of complete EW RC using hybrid current algebra and ChPT with resummation of largest terms to all chiral orders

- Reduced uncertainties at O(e^2p^4)
- Lattice evaluation of QCD contributions to γW box diagrams
- Conventional value of S_{EW} subtracted from results for use with standard formula for V_{us}

	Cirigliano et al. '08	Seng et al. '21
$\Delta^{EM}(K^0_{e3})$ [%]	0.50 ± 0.11	**0.580 ± 0.016**
$\Delta^{EM}(K^+_{e3})$ [%]	0.05 ± 0.12	**0.105 ± 0.023**
$\Delta^{EM}(K^+_{\mu3})$ [%]	0.70 ± 0.11	**0.770 ± 0.019**
$\Delta^{EM}(K^0_{\mu3})$ [%]	0.01 ± 0.12	**0.025 ± 0.027**

$K_{\ell 3}$ data and lepton universality

For each state of kaon charge, evaluate:

$$r_{\mu e} = \frac{(R_{\mu e})_{\rm obs}}{(R_{\mu e})_{\rm SM}} = \frac{\Gamma_{\mu 3}}{\Gamma_{e3}} \cdot \frac{I_{e3}\,(1+\delta_{e3})}{I_{\mu 3}\,(1+\delta_{\mu 3})} = \frac{[|V_{us}|\,f_+(0)]^2_{\mu 3,\,\rm obs}}{[|V_{us}|\,f_+(0)]^2_{e3,\,\rm obs}} = \frac{g_\mu^2}{g_e^2}$$

Modes	2004 BRs*,†	Current
K_L	1.054(14)	**1.002(5)**
$K^\pm$	1.014(12)	**0.999(9)**
Avg	1.030(9)	**1.001(4)**

*Assuming current values for form-factor parameters and $\Delta^{\rm EM}$ †K_S not included

As statement on lepton universality

Compare to other precise tests:

$\pi \to \ell\nu$ **$(r_{\mu e})$ = 1.0020(19)**

PDG with PIENU ’15 result

$\tau \to \ell\nu\nu$ **$(r_{\mu e})$ = 1.0036(28)**

HFLAV May ’19 unofficial prelim.

As statement on calculation of $\Delta^{\rm EM}$

Confirmed at per-mil level

$SU(2)$-breaking correction

$$\Delta^{SU(2)} \equiv \frac{f_+(0)^{K^+\pi^0}}{f_+(0)^{K^0\pi^-}} - 1$$

Strong isospin breaking

Quark mass differences, η-π^0 mixing in $K^+\pi^0$ channel

$$= \frac{3}{4}\frac{1}{Q^2}\left[\frac{\overline{M}_K^2}{\overline{M}_\pi^2} + \frac{\chi_{p^4}}{2}\left(1+\frac{m_s}{\hat{m}}\right)\right] \qquad Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

$\chi_{p^4} = 0.252$
NLO in strong interaction
$O(e^2p^2)$ term $\varepsilon_{EM}^{(4)} \sim 10^{-6}$

Cirigliano et al., '02; Gasser & Leutwyler, '85

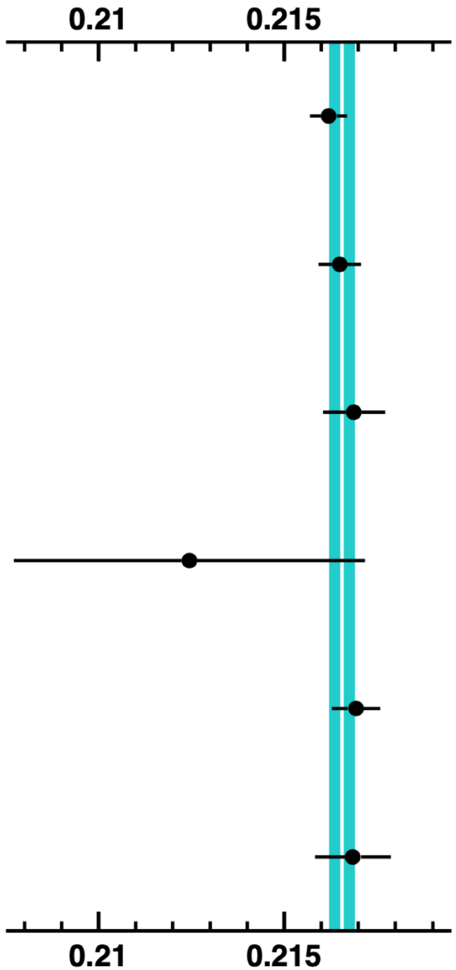
$=$ **+2.52(11)%** **Calculated using:**

$Q = 22.5(5)$
$m_s/m = 27.23(10)$
] FLAG '21, $N_f = 2+1+1$ avg.
Good agreement with ChPT
Cf. Colangelo et al., $Q = 22.1(7)$ from $\eta \to 3\pi$

$M_K = 494.2(3)$
$M_\pi = 134.8(3)$
] Isospin-limit meson masses from FLAG '17
No difference if $M_K = 494.58$, $M_\pi = m_\pi$

Test by evaluating V_{us} from $K^\pm$ and K^0 data with **no** corrections:
Equality of V_{us} values would require $\Delta^{SU(2)}$ **= 2.76(33)%**

$|V_{us}| f_+(0)$ from world data: 2022 update

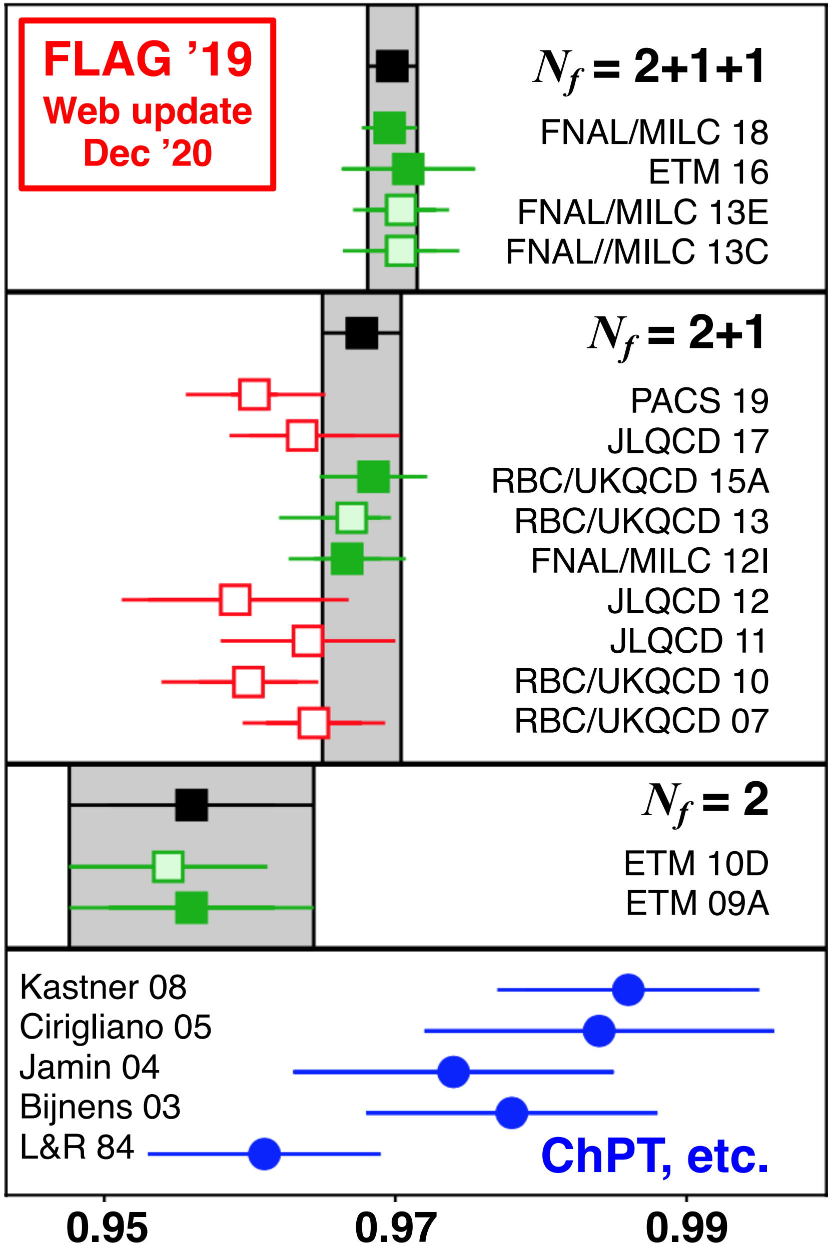


Approx. contrib. to % err from:

		% err	BR	τ	Δ	Int
$K_L e3$	**0.2162(5)**	**0.23**	0.09	0.20	0.02	0.05
$K_L \mu 3$	**0.2165(6)**	**0.26**	0.15	0.18	0.02	0.07
$K_S e3$	**0.2169(8)**	**0.39**	0.38	0.02	0.02	0.05
$K_S \mu 3$	**0.2125(47)**	**2.2**	2.2	0.02	0.02	0.08
$K^\pm e3$	**0.2169(6)**	**0.30**	0.27	0.06	0.11	0.05
$K^\pm \mu 3$	**0.2168(10)**	**0.47**	0.45	0.06	0.11	0.08

Average: $|V_{us}| f_+(0)$ = 0.21656(35) **χ^2/ndf = 1.89/5 (86%)**

Evaluations of $f_+(0)$



FLAG '21 averages:

N_f = 2+1+1 $\quad f_+(0) = 0.9698(17)$

Uncorrelated average of:

FNAL/MILC 18: HISQ, 5sp, $m_\pi \to$ 135 MeV, new ensembles added to FNAL/MILC 13E

ETM 16: TwMW, 3sp, $m_\pi \to$ 210 MeV, full q^2 dependence of f_+, f_0

N_f = 2+1 $\quad f_+(0) = 0.9677(27)$

Uncorrelated average of:

FNAL/MILC 12I: HISQ, $m_\pi \sim$ 300 MeV

RBC/UKQCD 15A: DWF, $m_\pi \to$ 139 MeV

JLQCD 17 not included because only single lattice spacing used

ChPT $\quad f_+(0) = 0.970(8)$

Ecker 15, Chiral Dynamics 15:
Calculation from Bijnens 03,
with new LECs from Bijnens, Ecker 14

Evaluations of $f_+(0)$

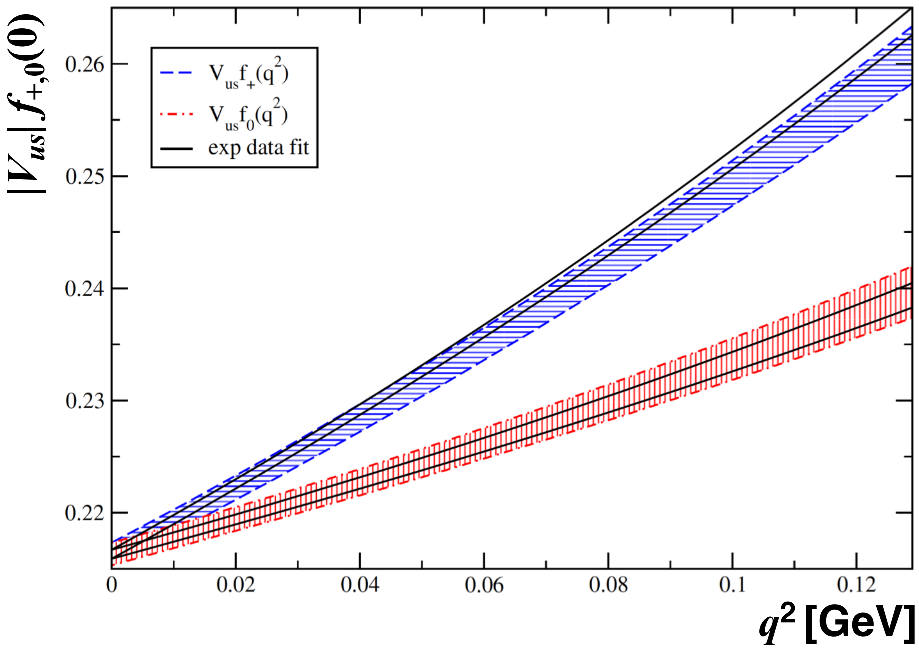
ETM
PRD 93 (2016)

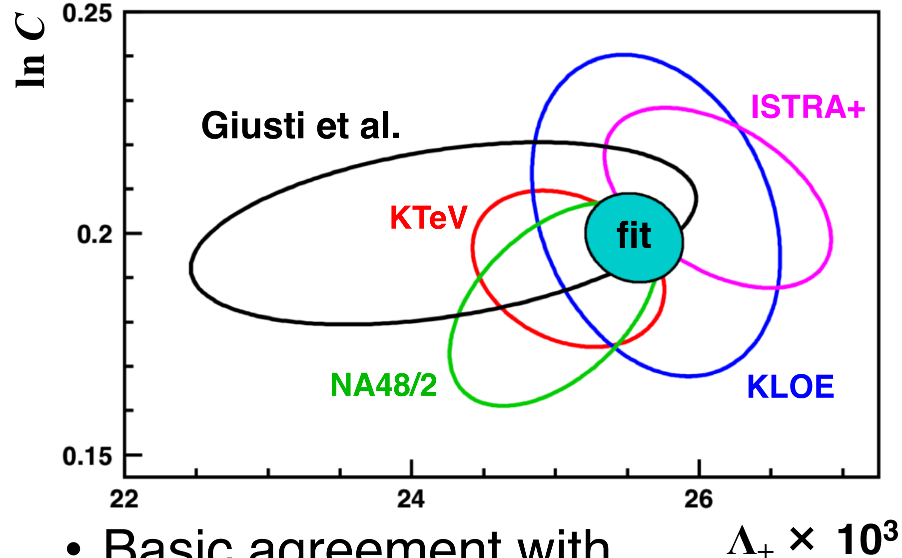
N_f **= 2+1+1**

$f_+(0) = 0.9709(44)_{st}(9)_{sy}(11)_{ext}$

Full q^2 dependence of f_+, f_0

See also:
PACS PRD 101 (2020)
ETM PRD 105 (2022)





Fit synthetic data points with dispersive parameterization

Λ_+ **= 24.22(1.16) × 10^{-3}**

ln C = 0.1998(138)

$\rho(\Lambda_+, f_+(0)) = -0.228$

$\rho(\ln C, f_+(0)) = -0.719$

$\rho(\Lambda_+, \ln C) = +0.376$

- Basic agreement with experimental results
- Confirms basic correctness of lattice calculations for $f_+(0)$
- In the near future FF parameters will be obtained on lattice?

V_{us}/V_{ud} and $K_{\ell 2}$ decays

$$\frac{|V_{us}|}{|V_{ud}|}\frac{f_K}{f_\pi} = \left(\frac{\Gamma_{K_{\mu 2(\gamma)}} m_{\pi^\pm}}{\Gamma_{\pi_{\mu 2(\gamma)}} m_{K^\pm}}\right)^{1/2} \frac{1 - m_\mu^2/m_{\pi^\pm}^2}{1 - m_\mu^2/m_{K^\pm}^2}\left(1 - \frac{1}{2}\delta_{\mathrm{EM}} - \frac{1}{2}\delta_{SU(2)}\right)$$

Inputs from experiment:

From $K^\pm$ BR fit:

BR($K^\pm_{\mu 2(\gamma)}$) = 0.6358(11)

$\tau_{K\pm}$ = 12.384(15) ns

From PDG:

BR($\pi^\pm_{\mu 2(\gamma)}$) = 0.9999

$\tau_{\pi\pm}$ = 26.033(5) ns

Inputs from theory:

δ_{EM} Long-distance EM corrections

$\delta_{SU(2)}$ Strong isospin breaking

$f_K/f_\pi \rightarrow f_{K\pm}/f_{\pi\pm}$

f_K/f_π Ratio of decay constants

Cancellation of lattice-scale uncertainties from ratio

NB: Most lattice results already corrected for $SU(2)$-breaking: $f_{K\pm}/f_{\pi\pm}$

V_{us}/V_{ud} and $K_{\ell 2}$ decays

Giusti et al.
PRL 120 (2018)

First lattice calculation of EM corrections to P_{l2} decays

- Ensembles from ETM
- N_f = 2+1+1 Twisted-mass Wilson fermions

$\boldsymbol{\delta_{SU(2)} + \delta_{\text{EM}} = -0.0122(16)}$

- Uncertainty from quenched QED included (0.0006)

Compare to ChPT result from Cirigliano, Neufeld ’11:

$\boldsymbol{\delta_{SU(2)} + \delta_{\text{EM}} = -0.0112(21)}$

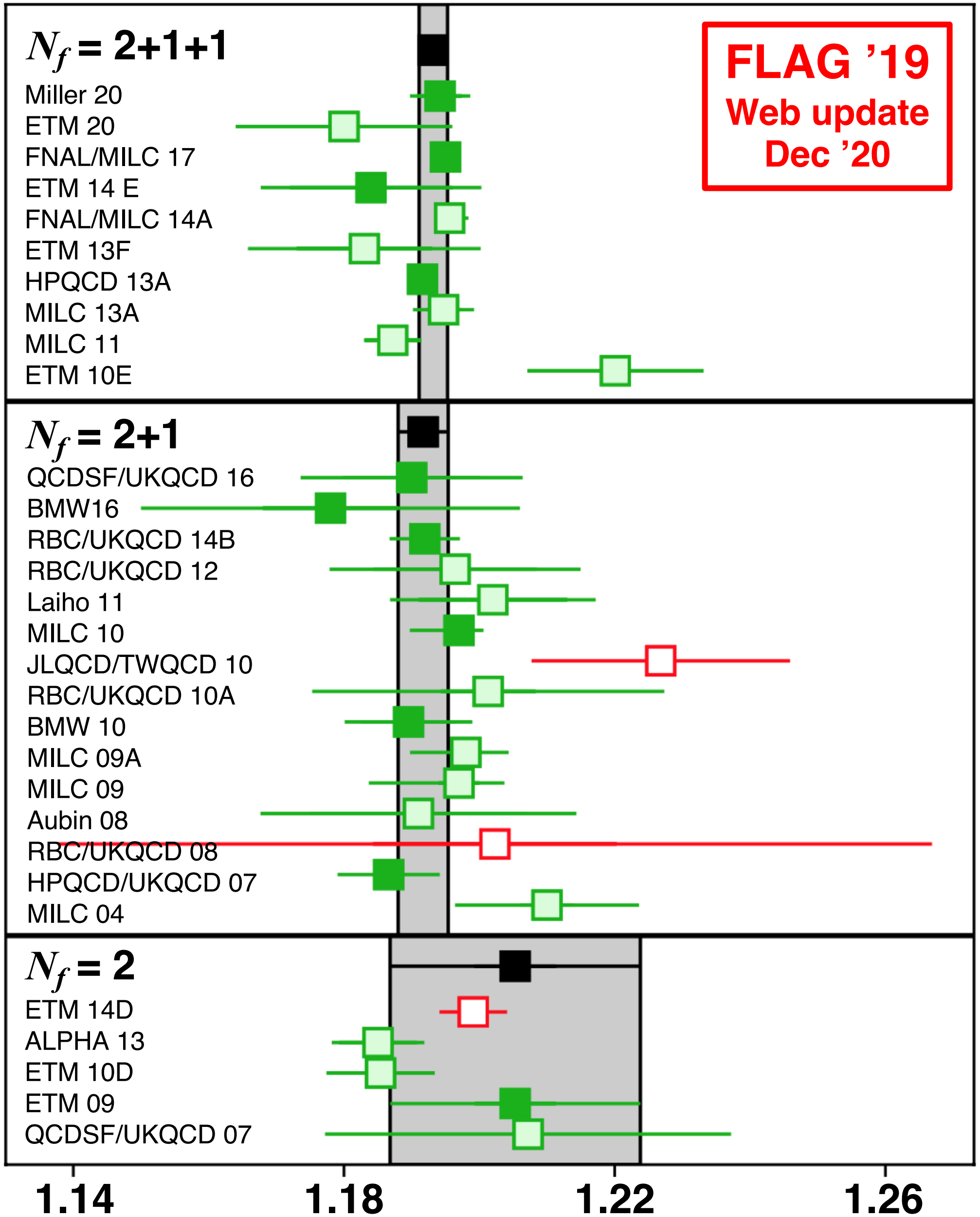
Di Carlo et al.
PRD 100 (2019)

Update, extended description, and systematics of Giusti et al.

$\boldsymbol{\delta_{SU(2)} + \delta_{\text{EM}} = -0.0126(14)}$

$\boldsymbol{|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(28)_{\text{BR}}(20)_{\text{corr}}}$

Lattice evaluations of f_K/f_π



N_f = 2+1+1 $f_{K\pm}/f_{\pi\pm}$ = 1.1932(19)

Miller 20
HISQ + DWF, 4sp, $m_\pi \to$ 130 MeV
Uses FNAL/MILC ensembles

FNAL/MILC 17
HISQ, 4sp, m_π phys
Updates MILC 13A, FNAL/MILC 14A

ETM 14E
TwM, 3sp, m_π = 210-450 MeV

HPQCD 13A
HISQ, 3sp, m_π phys
Uses FNAL/MILC ensembles

N_f = 2+1 $f_{K\pm}/f_{\pi\pm}$ = 1.1917(37)

QCDSF/UKQCD 16
Clover, 4sp, $m_\pi \to$ 220 MeV

BMW 16
Clover, 5sp, $m_\pi \to$ 139 MeV

RBC/UKQCD 14B
DWF, m_π = 139 MeV
f_K and f_π separately (isospin limit)

Lattice results for f_K/f_π

Recalculate FLAG averages for results without $SU(2)$-breaking
Isospin-limit results as reported in original papers

N_f = 2+1+1

f_K/f_π = 1.1978(22) S = 1.1

ETM 21 New!	**1.1995(44)(7)**

TM quarks, 3sp, $m_\pi \rightarrow$ physical
Not yet in FLAG '21 average!
Replaces ETM 14E in our average

Miller 20	1.1964(44)
FNAL/MILC17	$1.1980(^{+13}_{-19})$
HPQCD13A	1.1948(15)(18)

Average is problematic with correlations assumed by FLAG, dominated by FNAL/MILC17 (symmetrized)

Share ensembles
Partially correlated uncertainties using FLAG prescription

N_f = 2+1

f_K/f_π = 1.1946(34)*

QCDSF/UKQCD17	1.192(10)(13)
BMW16	1.182(10)(26)
RBC/UKQCD14B	1.1945(45)
BMW10	1.192(7)(6)
HPQCD/UKQCD07	1.198(2)(7)

* MILC10 omitted from average because unpublished

$|V_{ud}|$ from $0^+ \to 0^+$ nuclear β decays

For a given decay mode:

$$ft\,(1+\mathrm{RC}) = \frac{K}{2(V_{ud}G_F)^2(1+\Delta_R^V)}$$

f	statistical rate function
t	measured half life
K	constant
RC	mode-dependent radiative corr.
$\Delta_R{}^V$	inner (universal) radiative corr.

Hardy & Towner
1807.01146

$|V_{ud}|$ = 0.97420(21) $\Delta_R{}^V$ = 2.361(38)%
World $0^+ \to 0^+$ data set very robust: 14 transitions with compatible measurements at 0.1% precision or better
$\Delta_R{}^V$ from Marciano & Sirlin '06

Seng et al.
PRL 125 (2018)

$|V_{ud}|$ = 0.97370(14) $\Delta_R{}^V$ = 2.467(22)%
New calculation of γW-box contribution to $\Delta_R{}^V$ using dispersion relations and DIS structure functions

Czarnecki et al.
PRL 125 (2018)

$|V_{ud}|$ = 0.97389(18) $\Delta_R{}^V$ = 2.426(32)%
Improved use of Bjorken sum rule to constrain strong-interaction corrections to axial-vector component of the γW-box

Hardy & Towner
PRC 102 (2020)

$|V_{ud}|$ = 0.97373(31) $\Delta_R{}^V$ = 2.454(19)%
Use weighted average of above values for $\Delta_R{}^V$
23 new publications, some older measurements eliminated

Status of first-row unitarity

Cirigliano et al.
2208.11707

Evaluation of Δ_R^V and Δ_R^V:

- Hadronic scheme for resummation of infrared logs
- Non correlated average of contributions to γW box

V_{ud} from neutron decays uses current best measurements (not averages) for τ_n and $\lambda = g_A/g_V$

$|V_{ud}|$ = 0.97384(26)

Avg. of results from

$0^+ \rightarrow 0^+$ with $\Delta_R^V = 2.467(27)\%$

n decays with $\Delta_R = 3.983(27)\%$

$K_{\mu3}$

$f_+(0) = 0.9698(17)$

$N_f = 2+1+1$

$V_{us} = 0.22330(35)_{exp}(39)_{lat}(8)_{IB}$

$\Delta^{(1)}_{CKM} = -0.00176(16)_{exp+IB}(17)_{lat}(51)_{ud}$ $= -3.1\sigma$

$K_{\mu2}$

$f_K/f_\pi = 1.1978(22)$

$N_f = 2+1+1$

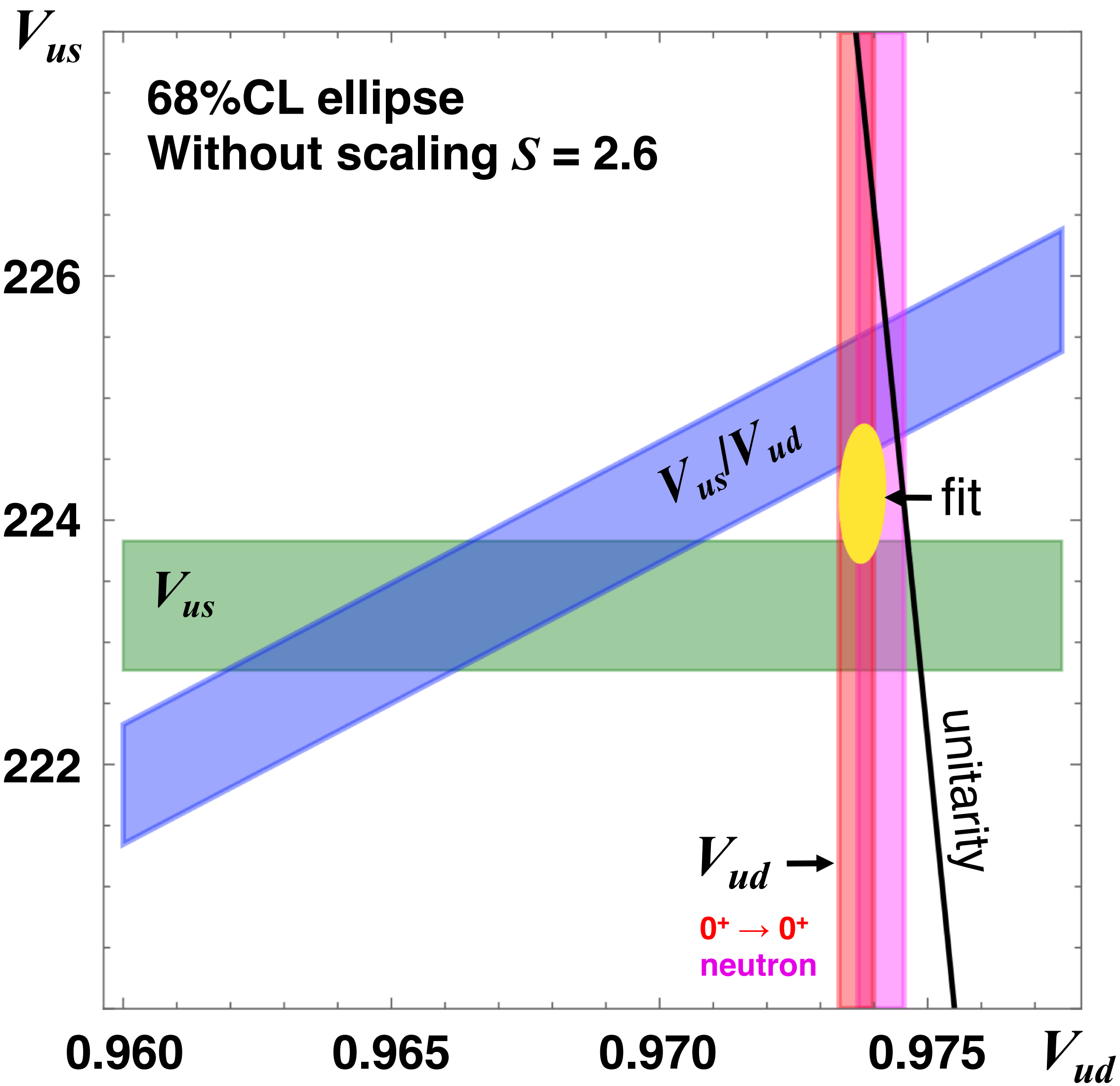
$V_{us}/V_{ud} = 0.23108(23)_{exp}(42)_{lat}(16)_{IB}$

$V_{us} = 0.22504(28)_{exp}(41)_{lat}(06)_{ud}$

$\Delta^{(2)}_{CKM} = -0.00098(13)_{exp}(19)_{lat}(53)_{ud}$ $= -1.8\sigma$

$\Delta V_{us}(K_{\mu3} - K_{\mu2}) = -0.0174(73) \quad -2.4\sigma$

Status of first-row unitarity



Fit results, no constraint

V_{ud} = **0.97378(26)**

V_{us} = **0.22422(36)**

χ^2/ndf = 6.4/2 (4.1%)

Δ_{CKM} = **−0.0018(6)**

−2.8σ

With scale factor S = 2.6

V_{ud} = **0.9737(8)**

V_{us} = **0.2242(10)**

Impact of hypothetical $K_{\mu3}/K_{\mu2}$ result

	current fit	$K_{\mu3}/K_{\mu2}$ BR at 0.5% central	$K_{\mu3}/K_{\mu2}$ BR at 0.5% $+2\sigma$	$K_{\mu3}/K_{\mu2}$ BR at 0.5% -2σ	$K_{\mu3}/K_{\mu2}$ BR at 0.2% central	$K_{\mu3}/K_{\mu2}$ BR at 0.2% $+2\sigma$	$K_{\mu3}/K_{\mu2}$ BR at 0.2% -2σ
χ^2/dof	25.5/11	25.5/12	31.8/12	32.1/12	25.5/12	35.6/12	35.9/12
p-value [%]	0.78	1.28	0.15	0.13	1.28	0.04	0.03
BR($\mu\nu$) [%]	63.58(11)	63.58(09)	63.44(10)	63.72(11)	63.58(08)	63.36(10)	63.80(11)
$S(\mu\nu)$	1.1	1.1	1.3	1.4	1.2	1.6	1.7
BR($\pi\pi^0$) [%]	20.64(7)	20.64(6)	20.73(7)	20.55(8)	20.64(6)	20.78(7)	20.50(10)
$S(\pi\pi^0)$	1.1	1.2	1.3	1.5	1.2	1.5	2.0
BR($\pi\pi\pi$) [%]				5.56(4)			
$S(\pi\pi\pi)$				1.0			
BR(K_{e3}) [%]	5.088(27)	5.088(24)	5.113(25)	5.061(31)	5.088(23)	5.128(24)	5.046(32)
$S(K_{e3})$	1.2	1.2	1.2	1.6	1.3	1.3	1.8
BR($K_{\mu3}$) [%]	3.366(30)	3.366(13)	3.394(16)	3.336(27)	3.366(7)	3.411(13)	3.320(18)
$S(K_{\mu3})$	1.9	1.2	1.5	2.6	1.1	2.2	3.1
BR($\pi\pi^0\pi^0$) [%]				1.764(25)			
$S(\pi\pi^0\pi^0)$				1.0			
$\tau_\pm$ [ns]	12.384(15)	12.384(15)	12.382(15)	12.385(15)	12.384(15)	12.381(15)	12.386(15)
$S(\tau_\pm)$				1.2			

Hypothetical $K_{\mu3}/K_{\mu2}$ measurement to 0.2% giving result $\pm2\sigma$ from current fit:

- Changes BR(K_{e3}) and BR($K_{\mu3}$) by $\pm1.5\sigma$
- Changes BR($K_{\mu2}$) by $\mp2\sigma$ (i.e. in opposite direction)

Impact of $K_{\mu 3}/K_{\mu 2}$ on unitarity tests

$\Delta^{(1)}_{\rm CKM} = \left|V^{\beta}_{ud}\right|^2 + \left|V^{K_{\ell 3}}_{us}\right|^2 - 1$ — V_{us} from $K_{\ell 3}$ + V_{ud} from β decays

$\Delta^{(2)}_{\rm CKM} = \left|V^{\beta}_{ud}\right|^2 + \left|V^{K_{\ell 2}/\pi_{\ell 2},\beta}_{us}\right|^2 - 1$ — V_{us}/V_{ud} from $K_{\mu 2}$ + V_{ud} from β decays

$\Delta^{(3)}_{\rm CKM} = \left|V^{K_{\ell 2}/\pi_{\ell 2},K_{\ell 3}}_{ud}\right|^2 + \left|V^{K_{\ell 3}}_{us}\right|^2 - 1$ — V_{us} from $K_{\ell 3}$ + V_{us}/V_{ud} from $K_{\mu 2}$

	current fit	$K_{\mu 3}/K_{\mu 2}$ BR at 0.5%			$K_{\mu 3}/K_{\mu 2}$ BR at 0.2%		
		central	$+2\sigma$	-2σ	central	$+2\sigma$	-2σ
$\left.\frac{V_{us}}{V_{ud}}\right\vert_{K_{\ell 2}/\pi_{\ell 2}}$	0.23108(51)	0.23108(50)	0.23085(51)	0.23133(51)	0.23108(49)	0.23071(51)	0.23147(52)
$V^{K_{\ell 3}}_{us}$	0.22330(53)	0.22337(51)	0.22360(52)	0.22309(54)	0.22342(49)	0.22386(52)	0.22287(52)
$\Delta^{(1)}_{\rm CKM}$	−0.00176(56)	−0.00173(55)	−0.00162(56)	−0.00185(56)	−0.00171(55)	−0.00151(56)	−0.00195(56)
	-3.1σ	-3.1σ	-2.9σ	-3.3σ	-3.1σ	-2.7σ	-3.5σ
$\Delta^{(2)}_{\rm CKM}$	−0.00098(58)	−0.00098(58)	−0.00108(58)	−0.00087(58)	−0.00098(58)	−0.00114(58)	−0.00081(58)
	-1.7σ	-1.7σ	-1.9σ	-1.5σ	-1.7σ	-2.0σ	-1.4σ
$\Delta^{(3)}_{\rm CKM}$	−0.0164(63)	−0.0157(60)	−0.0118(62)	−0.0202(63)	−0.0153(59)	−0.0083(62)	−0.0233(62)
	-2.6σ	-2.6σ	-1.9σ	-3.2σ	-2.6σ	-1.4σ	-3.8σ

- $\Delta^{(3)}_{\rm CKM}$ has no inputs from β decays
- Less sensitive as an absolute unitarity test but clearly shows impact of new measurements of V_{us}

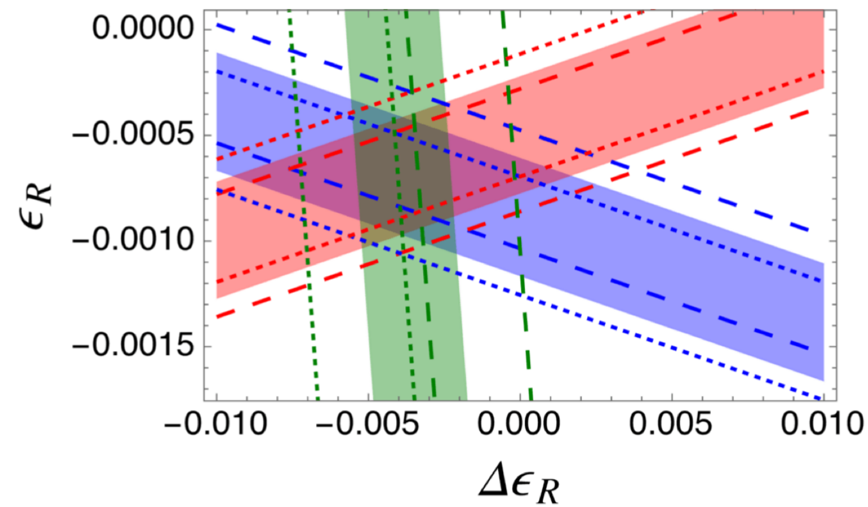
Constraints on right-handed currents

Cirigliano et al.
2208.11707

- In SM, W couples only to LH chiral fermion states
- New physics with couplings to RH currents could explain both unitarity deficit and $K_{\ell 3}$-$K_{\mu 2}$ difference
- Define ϵ_R = admixture of RH currents in non-strange sector
  $\epsilon_R + \Delta\epsilon_R$ = admixture of RH currents in strange sector

$$\Delta^{(1)}_{\rm CKM} = 2\epsilon_R + 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(2)}_{\rm CKM} = 2\epsilon_R - 2\Delta\epsilon_R V_{us}^2$$
$$\Delta^{(3)}_{\rm CKM} = 2\epsilon_R + 2\Delta\epsilon_R (2 - V_{us}^2)$$

$$r \equiv \left(\frac{1+\Delta^{(2)}_{\rm CKM}}{1+\Delta^{(3)}_{\rm CKM}}\right)^{1/2} = \frac{\left.\frac{V_{us}}{V_{ud}}\right|_{K_{\ell 2}/\pi_{\ell 2}}}{\frac{V_{us}^{K_{\ell 3}}}{V_{ud}^{\beta}}} = 1 - 2\Delta\epsilon_R$$



From current fit:

$\epsilon_R = -0.69(27) \times 10^{-3}$ (2.5σ)
$\Delta\epsilon_R = -3.9(1.6) \times 10^{-3}$ (2.4σ)

With hypothetical $K_{\mu 3}/K_{\mu 2}$ to 0.2%:

$K_{\mu 3}/K_{\mu 2} + 2\sigma \rightarrow \Delta\epsilon_R = -1.8(1.6) \times 10^{-3}$ (1.1σ)
$K_{\mu 3}/K_{\mu 2} - 2\sigma \rightarrow \Delta\epsilon_R = -5.7(1.6) \times 10^{-3}$ (3.5σ)

What can NA62 say by itself?

NA62 can make a precision measurement of $K_{\mu 3}/K_{\mu 2}$, with many systematics cancelling. What can this measurement alone tell us?

r is proportional to $(K_{\mu 3}/K_{\mu 2})^{-1/2}$:

$$r \equiv \left(\frac{1 + \Delta_{\rm CKM}^{(2)}}{1 + \Delta_{\rm CKM}^{(3)}} \right)^{1/2} = \frac{\left.\frac{V_{us}}{V_{ud}}\right|_{K_{\ell 2}/\pi_{\ell 2}}}{\frac{V_{us}^{K_{\ell 3}}}{V_{ud}^{\beta}}} = 1 - 2\Delta\epsilon_R$$

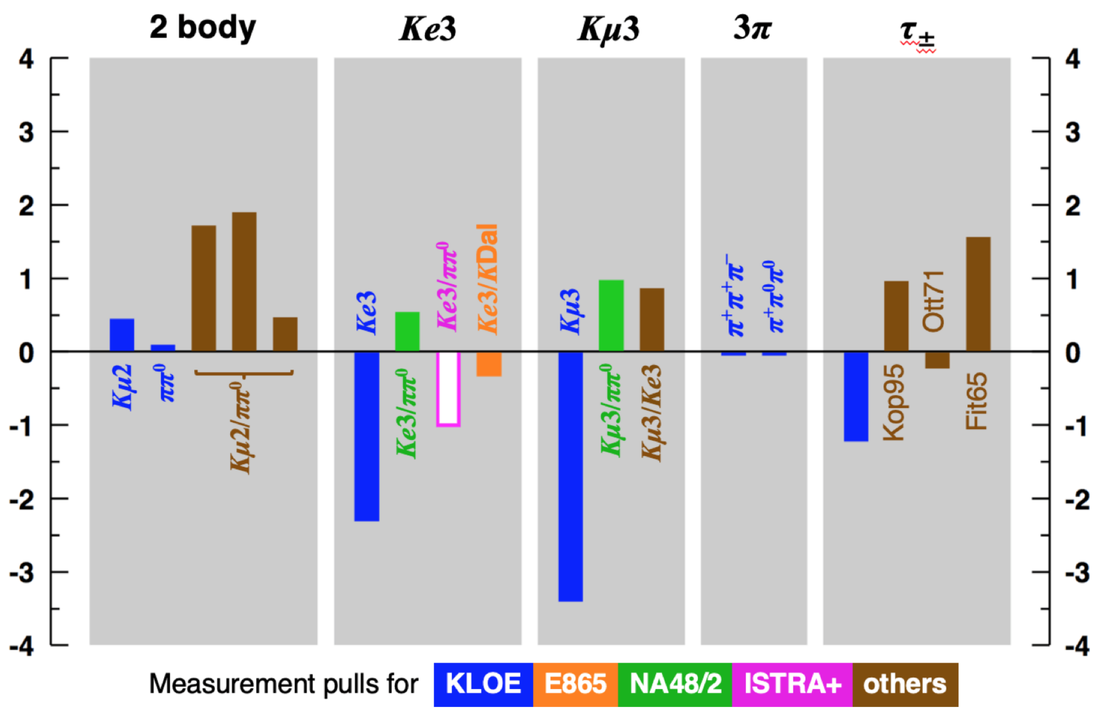
- Uses input from β decays, but provides a qualified statement about consistency of data set
- Search for right-handed currents

NA62 hypothetical $K_{\mu 3}/K_{\mu 2}$ to 0.5%:

Result	$\Delta\epsilon_R$		Remarks
Same as fit	–4.0(1.9)×10^{-3}	2.1σ	Almost same precision as result from world average
+ 1.5σ	–0.4(1.9)×10^{-3}	0.2σ	$K_{\mu 2}$, $K_{\mu 3}$, V_{ud} consistent: current tensions have experimental origin?
– 1.5σ	–7.6(1.9)×10^{-3}	4.0σ	Evidence for right-handed currents contributing to CKM non-unitarity

(Almost) Final thoughts

While a high priority, $K_{\mu3}/K_{\mu2}$ is not the only measurement that NA62 can make to help clarify the inconsistencies in the first row



Other important measurements

$K_{\mu2}/\pi\pi^0$	See if KLOE mmt *really* has 0.27% error
$K_{e3}/\pi\pi^0$, $K_{\mu3}/\pi\pi^0$	K_{e3}, $K_{\mu3}$ have large scale factors
$K_{e3}/K_{\mu3}$	Test of lepton universality

- Suite of redundant measurements for good control of systematics
- Best to measure with a single analysis framework to maximize cancellation of systematics
- Very important to report full covariance matrix for results

What can we learn from $\pi^+ \to \pi^0 e\nu$?

Czarnecki, Marciano, Sirlin, PRD 101 (2020)

$$\frac{\Gamma(K_L \to \pi e\nu(\gamma))}{\Gamma(\pi^+ \to \pi^0 e\nu(\gamma))} = \frac{1}{3}\left(\frac{m_{K^0}}{m_{\pi^+}}\right)^5 \left(\frac{V_{us} f_+^K(0)}{V_{ud} f_+^\pi(0)}\right)^2 \left(\frac{I_K}{I_\pi}\right)\left(\frac{1+\mathrm{RC}_K}{1+\mathrm{RC}_\pi}\right)$$

Ratio not sensitive to short-distance EW radiative corrections

- $(1 + \mathrm{RC}_K - \mathrm{RC}_\pi) = 1.000(2)_K(1)_\pi$
- Cancellation of S_{EW} and short-distance radiative corrections
- $\Delta_{\mathrm{EM}}{}^K$ (long-distance correction) fortuitously cancels when using K_{Le3}

Consider K_{Le3} mode as an example:

Most precise value of V_{us}; LD corrections known better than for other modes

$$\frac{V_{us} f_+^K(0)}{V_{ud} f_+^\pi(0)} = 0.22221(53)_{\Gamma(K)}(64)_{\Gamma(\pi)}(22)_{\mathrm{RC}}(12)_{\mathrm{int}} = \mathbf{0.22221(87)}$$

	K_{Le3}/π_{e3}*	$K_{\mu2}/K_{\pi2}$†	$K_{\ell3}$* & $0^+ \to 0^+$
V_{us}/V_{ud}	**$0.2291(9)_{\mathrm{exp}}(4)_{\mathrm{lat}}$**	0.2311(5)	$0.22930(54)_{us}(6)_{ud}$
diff with K_{Le3}/π_{e3}	–	**+1.7σ**	**+0.2σ**

*with $f_+{}^K(0) = 0.9698(17)$ and $f_+{}^\pi(0) = 1$ in $SU(2)$ limit

† with $f_K/f_\pi = 1.1978(22)$

V_{us} from kaons: Not over till it's over

Experimental results from kaons		With $|V_{ud}|(0^+ \to 0^+)$ and $N_f = 2+1+1$ lattice
$|V_{us}|\, f_+(0) = 0.21656(35)$	▶	$\Delta^{(1)}_{\mathrm{CKM}} = -0.00176(56) = -3.1\sigma$
$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(34)$	▶	$\Delta^{(2)}_{\mathrm{CKM}} = -0.00098(58) = -1.7\sigma$

$K_{\ell 2}$ result shows better agreement with unitarity than $K_{\ell 3}$ result when $|V_{ud}|$ obtained from $0^+ \to 0^+$ decays

New measurement of $K_{\mu 3}/K_{\mu 2}$ (e.g. from NA62) could be very helpful in distinguishing if origin of discrepancy is experimental

- Other measurements of main K BRs also very important!

Normalization of $\Gamma(K_{Le3})$ to $\Gamma(\pi_{e3})$ result agrees with $K_{\ell 3}$ results obtained with V_{ud} from $0^+ \to 0^+$ decays

- Discrepancy not in RC for $0^+ \to 0^+$?
- K_{Le3}/π_{e3} not sensitive to universal radiative corrections: is this a clue?

V_{us} from kaons: Not over till it's over

Experimental results from kaons	With $|V_{ud}|(0^+ \to 0^+)$ and $N_f = 2+1+1$ lattice
$|V_{us}| f_+(0) = 0.21656(35)$	$\Delta^{(1)}_{CKM} = -0.00176(56) = -3.1\sigma$
$|V_{us}/V_{ud}| \times f_K/f_\pi = 0.27679(34)$	$\Delta^{(2)}_{CKM} = -0.00098(58) = -1.7\sigma$

"It's hard to make predictions, especially about the future"

Attributed to Yogi Berra
Catcher, NY Yankees, 1946-1963

V_{us} from kaon decays

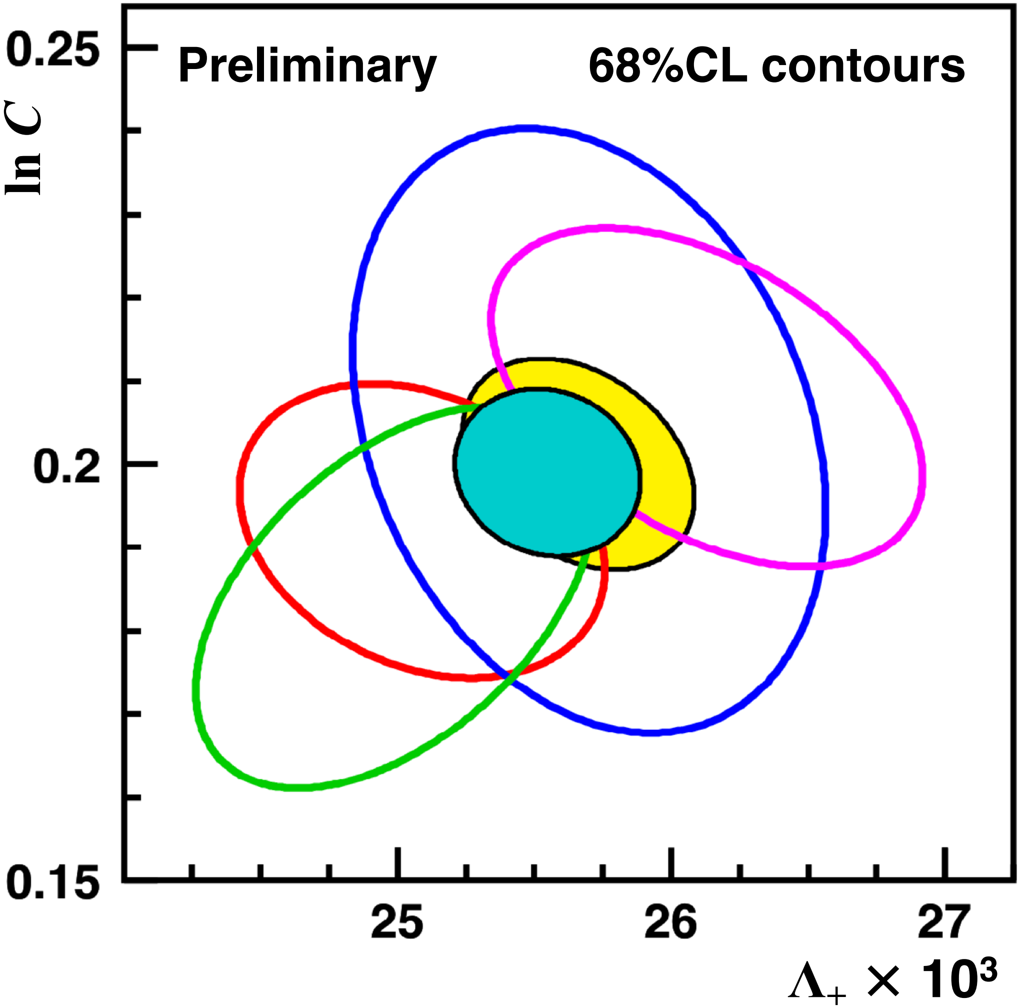
Additional information

Electroweak Precision Physics from Beta Decays to the Z Pole
ELECTRO 2022 - Mainz Institute for Theoretical Physics
Johannes Gutenberg University, 28 October 2022

Phase-space integrals 2021

Averages of form-factor parameters for dispersive parameterization Λ_+ and ln C

Integrals calculated from average values



$$\Lambda_+ \times 10^3 = 25.55 \pm 0.38$$
$$\ln C = 0.1992(78)$$
$$\rho(\Lambda_+, \ln C) = -0.110$$
$$\chi^2/\text{ndf} = 7.5/7\ (38\%)$$

Integrals	
$K^0{}_{e3}$	**0.15470(15)**
$K^+{}_{e3}$	**0.15915(15)**
$K^0{}_{\mu3}$	**0.10247(15)**
$K^+{}_{\mu3}$	**0.10553(16)**

Correlation matrix for integrals

$K^0{}_{e3}$	1	1	0.530	0.521
$K^+{}_{e3}$		1	0.530	0.521
$K^0{}_{\mu3}$			1	1
$K^+{}_{\mu3}$				1