

Higher Orders and Jets at the LHC, MITP, Mainz, July 16, 2015

By now Soft-Collinear Effective Theory (SCET) is a textbook method to perform resummations of large logarithms in collider processes.

Many successful applications, but for hadron colliders mostly at the level of inclusive cross sections (threshold resummations).

Important to extent methods to processes with narrow jets in the final state. Difficulties

- Traditional jet definitions impose complicated phasespace constraints on high multiplicity final states
- Not all large logarithms are captured by standard softcollinear factorization ("non-global logarithms")

Dasgupta, Salam '01

Lecture Notes in Physics 896

Thomas Becher Alessandro Broggio Andrea Ferroglia

Introduction to Soft-Collinear Effective Theory





Size of corrections

For narrow jets, perturbative corrections are enhanced by Sudakov logarithms $L = \ln(M_J/E_J)$.

Correction to	L ~ 1	L ~ 1/a _s
NLO	α_s^2	$\alpha_s^n L^{2n}$!
LL		$\alpha_s^n L^n \sim 1$
NLL		$lpha_{S}$
NNLL		α_s^2

 $\exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \alpha_s^2 g_4(\alpha_s L) + \dots\right)$

Matching

For a description valid both at small and large jet mass, one can should match fixed-order and resummed results.

- State of the art is NLO + LL (parton shower)
- For uniform accuracy throughout phase space, we want NLO + NNLL.

Weak point of our current theoretical description of jet processes is poor parametric accuracy of resummation.

Outline

Will discuss three topics of increasing jettiness

- Automated NLO+NNLL resummation for jet-veto cross sections TB, Frederix, Neubert, Rothen '14 (JHEP)
- 2. NNLL resummation for hadron collider dijet event shapes TB, Garcia y Tormo '15 (JHEP) + ongoing with Jan Piclum
- 3. Jet Effective Theory: Towards resummation for conejet cross sections. Ongoing with Neubert, Rothen, Shao



Automated NNLL+NLO resummation for cross sections with a jet veto

TB, Frederix, Neubert, Rothen 1412.8408 (JHEP)

Higher-log resummations (in SCET or in QCD) are usually carried out analytically, on a case-to-case basis. (Notable exception: CAESAR Banfi, Salam, Zanderighi '04, ARES Banfi, McAslan, Monni, Zanderighi '14)

• Inefficient and error prone

In contrast, LO and NLO computations have been completely automated over the past years. These codes can be used as a basis to perform resummation:

- Large logarithms arise near Born-level kinematics. Can reweight LO events to achieve resummation.
- Can use NLO codes to compute ingredients for the resummation: hard function, jet and soft functions

Cross section with a jet veto

A veto on jets $p_T^{\rm jet} < p_T^{\rm veto} \approx 15 - 30 \, {\rm GeV}$ is used to suppress top background, in particular in processes involving W-bosons, e.g. in

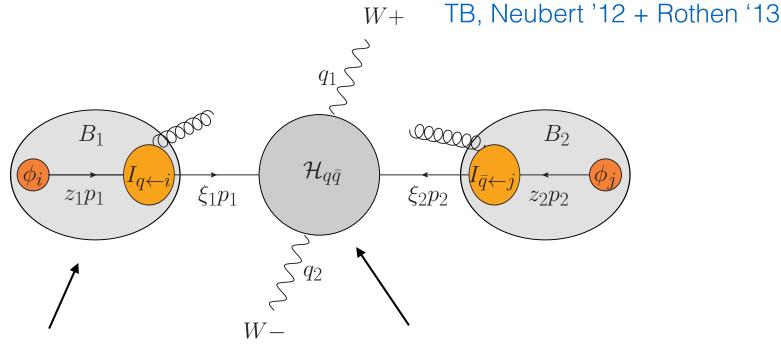
$$pp \rightarrow W^+ W^-$$
, $pp \rightarrow H \rightarrow W^+ W^-$, etc.

$$\rightarrow$$
 Large Sudakov logarithms $\alpha_s^n \ln^k \left(\frac{p_T^{\text{veto}}}{Q} \right)$

A lot of work on their resummation, both in QCD and SCET:

- Higgs: Banfi, Salam, Zanderighi '12; + Monni '12; TB Neubert '12 + Rothen '13; Tackmann, Walsh, Zuberi '12 + Stewart '13; Liu Petriello '13; + Boughezal, Tackmann and Walsh '14
- W+ W-: Jaiswal, Okui '14; Monni, Zanerighi '14; TB, Frederix, Neubert, Rothen '14

Factorization theorem for $\sigma(p_T^{veto})$



Beam functions *B*(*p*_T^{veto})

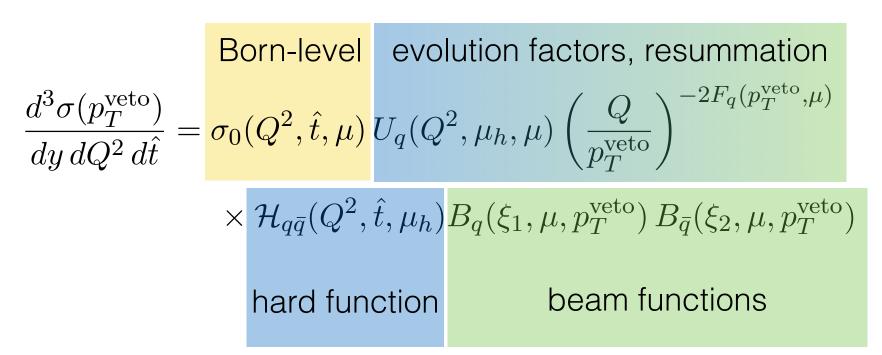
- real emission with veto.
 perturbative part ⊗ PDF
- process independent

Hard functions H(Q)

- virtual corrections, standard QCD loops
- process dependent

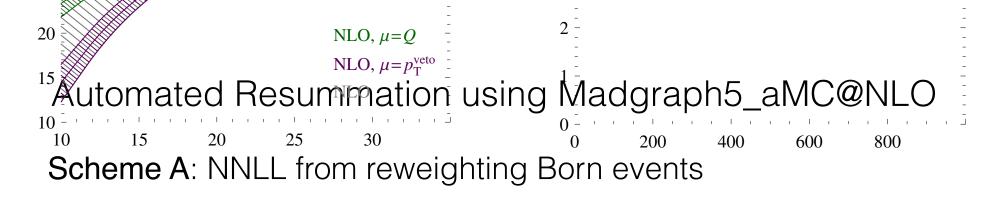
Born-level kinematics for small $p_{T^{veto}}$

Resummed cross section



"Born-level cross section" x "prefactor P(pTveto)"

 Can obtain resummed cross section by reweighting Born-level events with P(pTveto)



- Rescale each LO event weight with the ratio to the resummed cross section.
- Beam functions included via modified PDFs
 - Tabulate grid of values, use standard PDF interpolation
- One-loop hard function (only process dependent piece) computed using the MadGraph5_aMC@NLO code
- Additive matching to NLO fixed-order

 $\sigma_{\text{NNLL+NLO}} = \sigma_{\text{NNLL}}(\mu, \mu_h) + \left(\sigma_{\text{NLO}}(\mu_m) - \sigma_{\text{NNLL}}(\mu_m)\Big|_{\text{expanded to NLO}}\right)$

Automated Resummation using Madgraph5_aMC@NLO

Scheme B: NNLL+NLO with automated computation of the beam functions and matching corrections

• Define reduced cross section by dividing out hard function and evolution factors

Power-correction

$$d\tilde{\sigma}_{ij}(p_T^{\text{veto}}) = d\sigma_{ij}^0(Q^2, \hat{t}, \mu) \,\bar{B}_i(\xi_1, p_T^{\text{veto}}) \,\bar{B}_j(\xi_2, p_T^{\text{veto}}) + \Delta\tilde{\sigma}$$

- Reduced cross section is free of large log's. Compute it at NLO for $\mu \approx p_T^{veto}$ by running aMC@NLO in fixed-order mode
- multiply back evolution factor and hard function
- MadGraph5_aMC@NLO computes *both* hard and beam functions!
- Automatically includes multiplicative matching to NLO

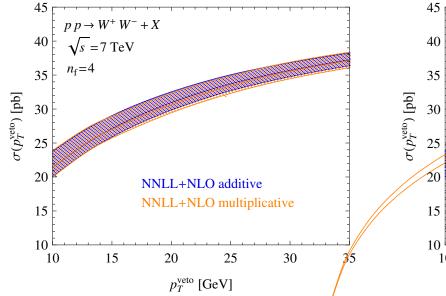
Comparison

Scheme A

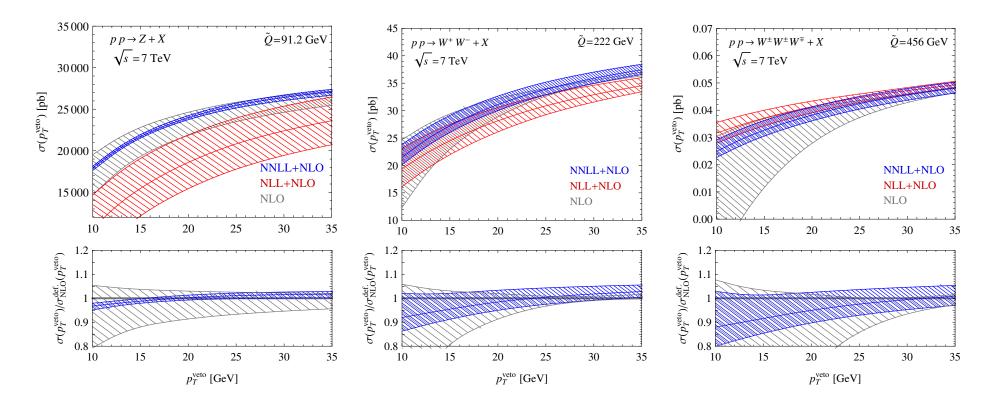
- Is easily extended to higher accuracy
- Can be applied to other processes
- Flexible, since it works with events (up to the NLO matching!)

Scheme B

- Resummation and NLO matching in one run
- Beam functions on the fly



Both will be included in version 2.3 of Madgraph5_aMC@NLO



- For NLO result we vary $p_{T}^{veto}/2 < \mu < 2Q$.
- NNLL+NLO is close to NLO at $\mu = Q$
- Matching corrections are small, grow linearly to 3% at $p_T^{\text{veto}}=80$ GeV. Can neglect matching at low p_T^{veto} .

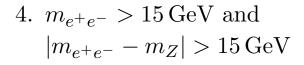
Decays and Cuts

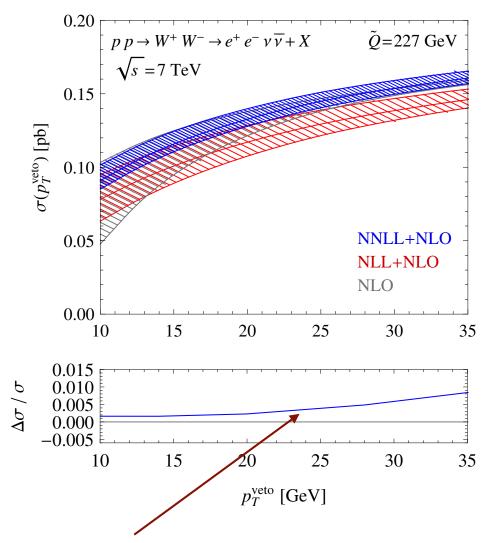
Important advantage:

Straightforward to include the decay of the vector bosons and cuts on the final state leptons.

E.g. cuts by ATLAS in *e*+*e*- channel

- 1. lepton $p_T > 20 \,\mathrm{GeV}$
- 2. leading lepton $p_T > 25 \,\mathrm{GeV}$
- 3. lepton pseudorapidity $\eta_e < 1.37$ or $1.52 < \eta_e < 2.47$



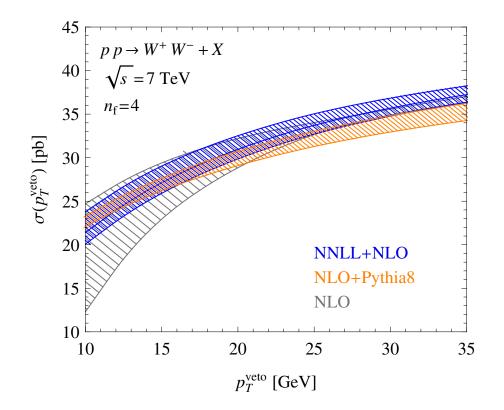


matching corrections remain small!

Comparison to matched PS

Observation: At higher values of p_T^{veto} the matched parton shower leads to lower results.

Unitarity of the shower, leads to compensation of changes at low transverse momentum.



Matched parton shower underestimates the jet-veto cross section

• In line with conclusions of Monni, Zanderighi '14

Extension to other observables

Since Sudakov logarithms always arise near Born-level kinematics, the same technique for automated resummation can also be used for more general observables.

Complications:

- Nontrivial color structure of the hard function. Need color information and imaginary part of amplitudes. Modified GoSam (Broggio + GoSam) can provide this information.
- NNLL needs **automated computations** of one-loop beam, jet, and soft functions, two-loop anomalous dimensions.
- Restriction to global observables



NNLL resummation for dijet event shapes at hadron colliders

TB, Xavier Garcia Tormo 1502.04136 (JHEP) and ongoing + Jan Piclum

Resummation for LHC processes

Many higher-log results for e^+e^- but, only for a handful of NNLL predictions for *differential* cross sections for hadron colliders

- Z/W/H transverse momentum spectra
- Z/W/H/WW/... cross sections with jet-veto
- Beam thrust
- 1-jettiness in *H* and *W* production

Not a single dijet observable! (Some threshold results.) Chien, Kelley, Schwartz, Zhu '10-'12

Canonical e^+e^- event shape: thrust 1.5 15 1.0 $\vec{p_i} \cdot$ $\frac{1}{\sigma} \frac{1}{T} \frac{1}{T}$ max $\vec{n}_{\scriptscriptstyle 0.10 \quad 0.15 \quad 0.20}$ $au = 1^{\frac{0.1}{1-T}} T$ 0.4 $\tau \approx 1/2$ $\tau \approx 0$

Precise measurement at LEP, theoretical predictions at N³LL+NNLO TB, Schwartz '08.

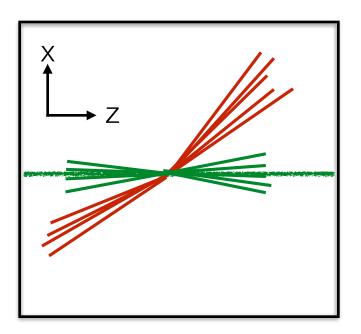
 $\alpha_s(m_Z) = 0.1135 \pm (0.0002)_{\text{expt}} \pm (0.0005)_{\text{hadr}} \pm (0.0009)_{\text{pert}}$

Abbate, Fickinger, Hoang, Mateu and Stewart '10

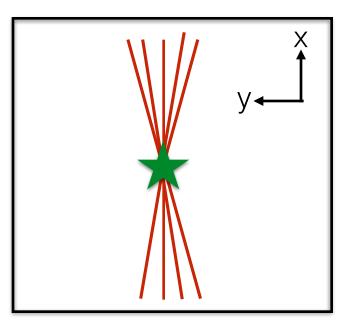
Hadron collider event shapes

- Each event has two jets down the beam pipe, no detector close to the beam.
- Natural to define event shapes in the transverse plane. (Alternative: *N*-jettiness Stewart, Tackmann, Waalewijn '10. Groups particles using multiple reference vectors.)

side view

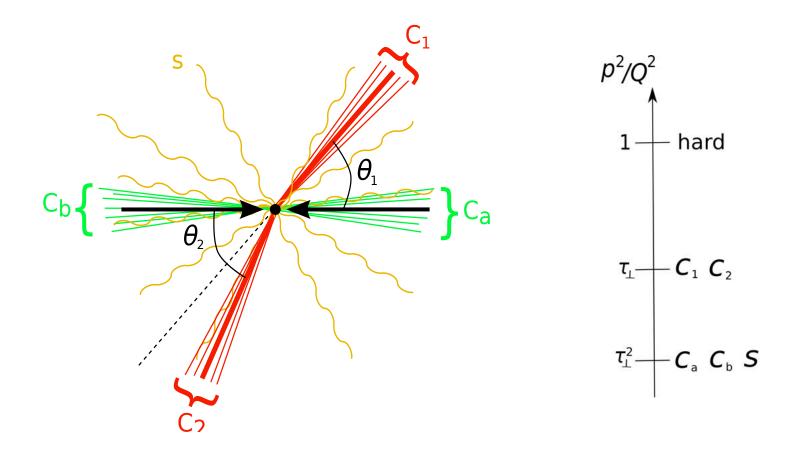


transverse plane



Hadron collider event shapes

- Going into the transverse plane, basically any e⁺e⁻ event shape can be turned into a hadron collider event shape.
- Large class of such observables was computed at NLL +NLO using automated CAESAR framework. Banfi, Salam, Zanderighi '04, '10
 - Ongoing work to extend this to NNLL ("ARES"), first results for e⁺e⁻ Banfi, McAslan, Monni and Zanderighi '14
- Transverse thrust has been measured both at the Tevatron and the LHC
- Have analyzed transverse thrust in SCET, as a first step towards a more general understanding of this class of event shapes.



Factorization involves several interesting aspects

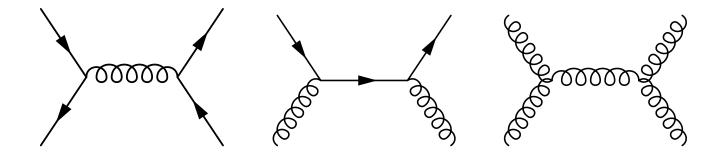
- Collinear fields with different virtuality: SCET_{I+II}
- Nontrivial color structure of hard and soft function
- Collinear anomaly (with color structure!)



Factorization theorem

 $d\sigma \sim H_{I,I} \mathcal{S}_{JI} \otimes J_1 \otimes J_2 \otimes \mathcal{B}_a \otimes \mathcal{B}_b$

- Beam functions \mathcal{B}_a , \mathcal{B}_b describe initial state radiation.
- Different partonic channels



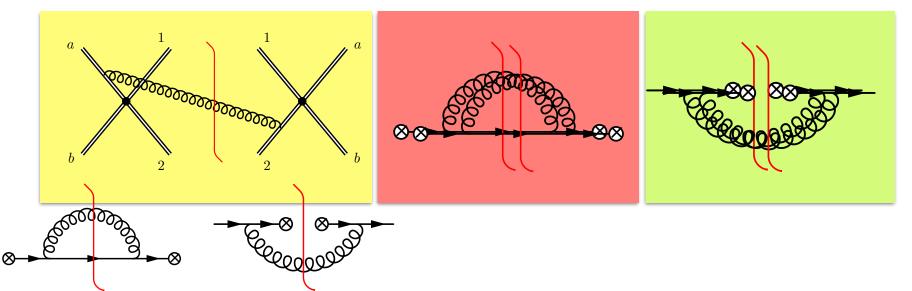
• nontrivial color structure in hard function H_{IJ} and soft functions S_{IJ} .

NNLL Resummation

Need

- One-loop hard, jet, soft, beam functions
- Two-loop anomalous dimensions for all these objects
- The two-loop anomaly exponent

Computed all one-loop ingredients in 1502.04136



At first sight, many two-loop computations seem necessary to achieve NNLL, but using

- RG invariance and universality
 - same jet functions in *p p* and *e*⁺*e*⁻ collisions
 - same beam func. in $pp \rightarrow 2$ jets and $pp \rightarrow e^+e^-$
- known results for two-loop hard anomalous dimensions
 Becher, Neubert '09, Casimir scaling of soft function

it turns out, everything is known except anomaly exponent F_{\perp} and jet anomalous dimension γ_{Jq} !

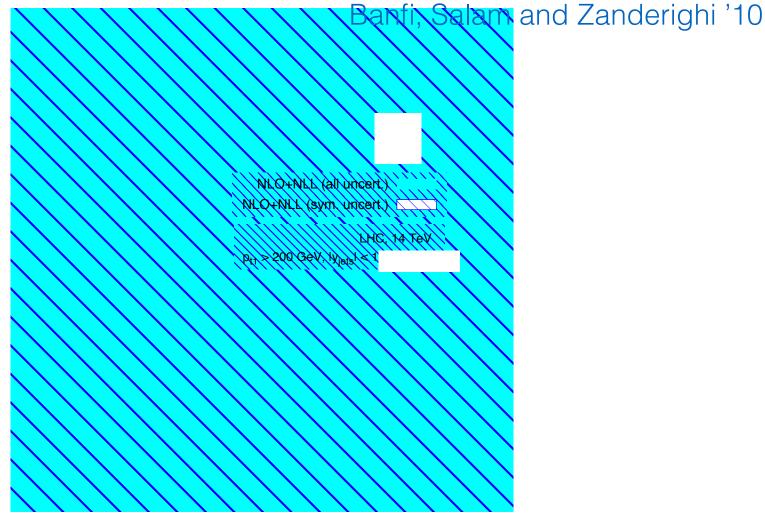
• Have determined both of these ingredients numerically. TB, Garcia-Tormo, Piclum, to appear.

NNLL

We now have *all* ingredients for full NNLL resummation. Implementation is work in progress

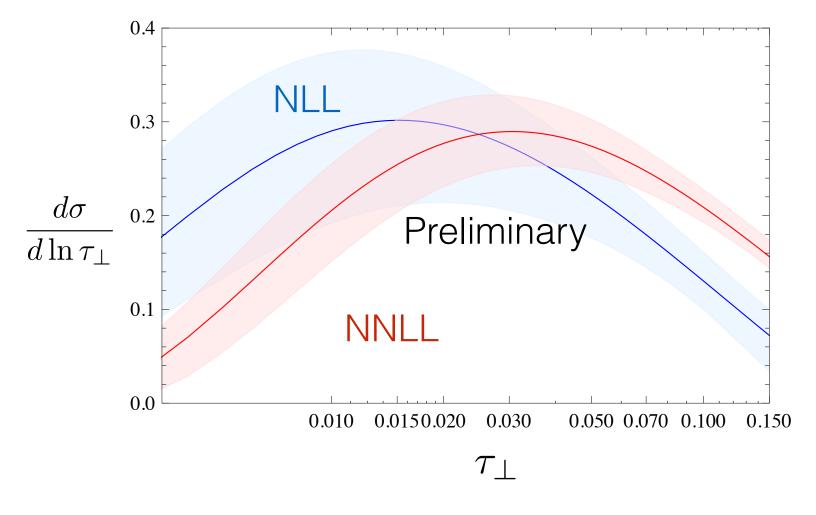
- Have coded up two-loop hard function matrices for the different channels Broggio, Ferroglia, Pecjak and Zhang 1409.5294, including RG evolution.
- Have beam function interpolations in PDF format, one-loop soft functions
- Find large perturbative corrections to jet, beam and soft functions and to their anomalous dimensions! This will translate into large corrections at NNLL.

NLL+NLO from CAESAR



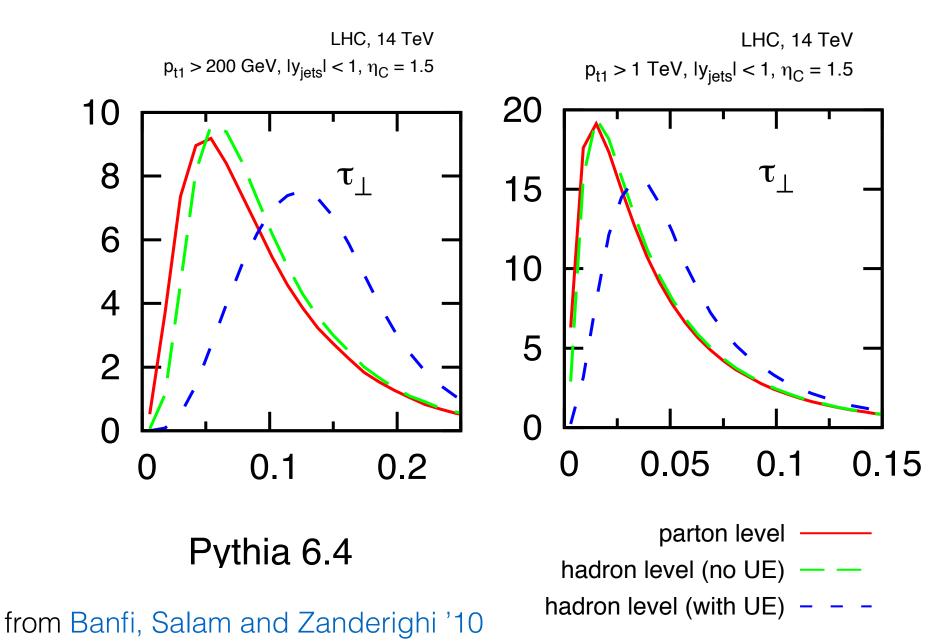
NNLL correction will be relatively large, but the basic shape stays the same.

Toy example: τ_{\perp} in e^+e^-



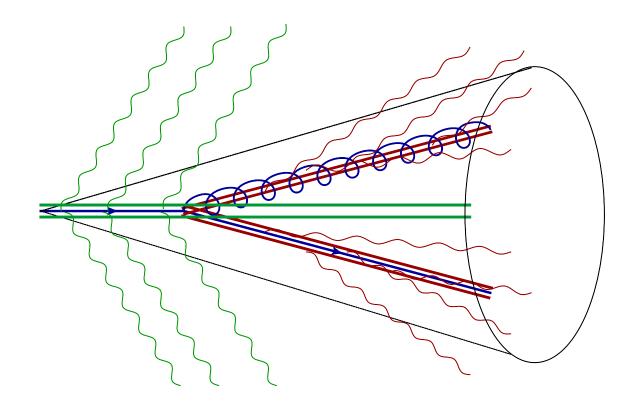
Pure resummation, no matching to NLO.

Underlying event



Glauber Gluons?

- From a theoretical perspective, UE modeling is quite unsatisfactory
 - True MPI is power suppressed!
 - Shouldn't we be able to model-independently describe O(1) effects in infrared safe observables?
- Glauber gluons [p^μ ~(λⁿ, λ^m, λ), m+n >2] could be the source of remnant interactions
 - Shown to be absent in DY Collins, Soper, Sterman, but could contribute to transverse thrust Gaunt '15.
 - Implementation in SCET is under way. Donoghue, Kamal El-Menoufi, Ovanesyan; Fleming; Rothstein and Stewart
 - Mao Zeng, last week: explicit example for the presence of factorization breaking in beam thrust (spin asymmetry for γγ scattering in toy model with scalar quarks)

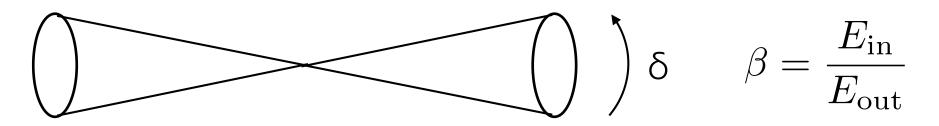


From SCET to Jet Effective Theory

TB, Neubert, Rothen, Shao, work in progress

Jet cross section in SCET

Cross sections for narrow cone jets (e.g. Sterman-Weinberg)



contains large logarithms $ln(\delta)$ and $ln(\beta)$.

Can compute such cross sections using standard SCET, but this does not translate into a resummation of all large logarithms:

 Non-global logarithms: soft function contains multiple scales and therefore large logarithms, independent of μ.

Dasgupta, Salam '01

Non-global logarithms in SCET

A number fixed-order computations for hemisphere soft functions

- Two-loop result for S(ω_L, ω_R). Kelley, Schwartz, Schabinger and Zhu '11; Hornig, Lee, Stewart, Walsh and Zuberi '11; Kelley; with jetcone Kelley, Schwartz, Schabinger and Zhu '11; von Manteuffel, Schabinger and Zhu '13
- Leading non-global log terms in S(ω_L, ω_R) up to 5 loops by solving BMS equation. Schwartz, Zhu '14

Recently, interesting framework for approximate resummation of such logs, based on resummation for observables with *n* soft subjets was proposed. Larkoski, Moult and Neill '15

• Seems to work numerically well in the considered example, but systematics of expansion in subjets unclear. Expansion parameter?

A systematic factorization of non-global observables is missing.

Cheung, Luke and Zuberi '09 have computed oneloop jet cross sections using SCET.

Result for the soft function for Sterman-Weinberg

$$\frac{1}{\sigma_0} \sigma_{\rm SW}^s = \frac{\alpha_s C_F}{2\pi} \left(\frac{4}{\epsilon} \ln \delta - 4 \ln^2 \delta + 8 \ln \delta \ln \frac{\mu}{\beta Q} - \frac{\pi^2}{3} \right)$$

multiple scales!

they use SCET with the following scaling:

$$(p_{+}, p_{-}, p_{\perp})$$

collinear: $p_{c} \sim Q(1, \delta^{2}, \delta)$
soft: $p_{s} \sim Q(\beta, \beta, \beta)$

The proper effective theory should completely separate the physics at different scales.

To achieve homogeneous scaling one must systematically expand away power suppressed contributions, also in the phase-space constraints: **method of regions**

As a result of the expansion

• Collinear fields are **always inside** the jet (they have generically large energies).

$$\theta(\beta Q - 2E_c) \longrightarrow \theta(-2E_c) = 0$$

• Soft fields are **always outside** jet (they have generically large angle).

Coft mode

To reproduce QCD when performing the expansion, we need additional region

$$(p_+, p_-, p_\perp)$$

coft: $p_t \sim \beta Q (1, \delta^2, \delta)$

This momentum mode is **simultaneously collinear and soft**

- Describes soft small angle radiation.
- Characteristic scale $\beta \delta Q$, much lower than soft scale!
- Can be be emitted both inside and outside of the jet.

One-loop result

hard
$$\Delta \sigma_{h} = \frac{\alpha_{s}C_{F}}{4\pi} \sigma_{0} \left(\frac{\mu}{Q}\right)^{2\epsilon} \left(-\frac{4}{\epsilon^{2}} - \frac{6}{\epsilon} + \frac{7\pi^{2}}{3} - 16\right)$$
(anti-)coll.
$$\Delta \sigma_{c+\bar{c}} = \frac{\alpha_{s}C_{F}}{4\pi} \sigma_{0} \left(\frac{\mu}{Q\delta}\right)^{2\epsilon} \left(\frac{4}{\epsilon^{2}} + \frac{6}{\epsilon} + c_{0}\right)$$
soft
$$\Delta \sigma_{s} = \frac{\alpha_{s}C_{F}}{4\pi} \sigma_{0} \left(\frac{\mu}{Q\beta}\right)^{2\epsilon} \left(\frac{4}{\epsilon^{2}} - \pi^{2}\right)$$
(anti-)coft
$$\Delta \sigma_{t+\bar{t}} = \frac{\alpha_{s}C_{F}}{4\pi} \sigma_{0} \left(\frac{\mu}{Q\delta\beta}\right)^{2\epsilon} \left(-\frac{4}{\epsilon^{2}} + \frac{\pi^{2}}{3}\right),$$

$$\Delta \sigma_{t+\bar{t}} = \frac{\alpha_{s}C_{F}}{4\pi} \sigma_{0} \left(\frac{\mu}{Q\delta\beta}\right)^{2\epsilon} \left(-\frac{4}{\epsilon^{2}} + \frac{\pi^{2}}{3}\right),$$

$$\Delta\sigma^{\text{tot}} = \frac{\alpha_s C_F}{4\pi} \sigma_0 \left(-16\ln\delta\ln\beta + 12\ln\delta + c_0 + \frac{5\pi^2}{3} - 16 \right)$$

Constant c_0 depends on definition of jet axis:

 $c_0 = -3\pi^2 + 26$, (Sterman-Weinberg) $c_0 = -5\pi^2/3 + 14 + 12\ln 2$ (thrust axis)

Higher-order structure?

Can now construct effective field theory with collinear, soft and coft degrees of freedom and then analyze factorization of cross section.

Simplest guess would be factorization theorem

$\sigma(\beta,\delta) \stackrel{?}{=}$	$\sigma_0 H(Q,\mu)$	$[J(Q\delta,\mu)]^2$	$S(Qeta,\mu)\otimes$	$U(Qeta\delta,\mu)$
	hard	(anti-)coll.	soft	(anti-)coft

It turns out that there is a much richer structure due to nontrivial interplay of coft and collinear fields!

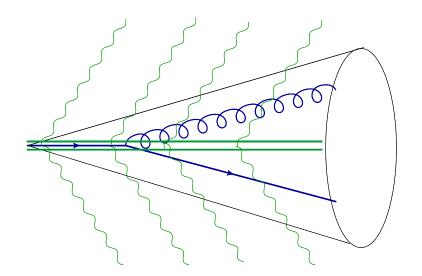
Factorization of the soft function

The soft function for cone jets factorizes as

$$S_{\text{full}}(Q\beta, \delta, \mu) = \int_{0}^{\beta} d\beta' \frac{S_{DY}(Q\beta - Q\beta', \mu)}{\text{soft contribution}} \frac{U(Q\delta\beta', \mu)}{(\text{anti-)coft}}$$

- Verified this explicitly at the 2-loop level. Two-loop S_{full} can be derived from results for the thrust conejet soft function. Manteuffel, Schabinger and Zhu '13
- Can resum large logs in S_{full} using RG.

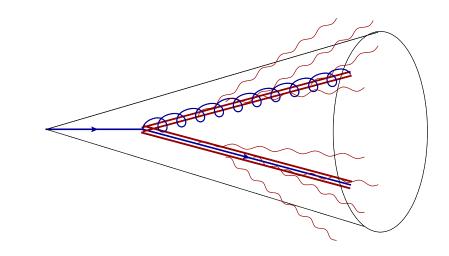
Soft-collinear factorization



Large angle soft radiation sees total charge of collinear radiation inside jet.

• Soft emissions described by single Wilson line.

Coft-collinear factorization



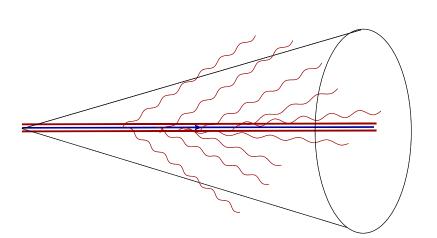
Small angle coft radiation resolves individual collinear particles.

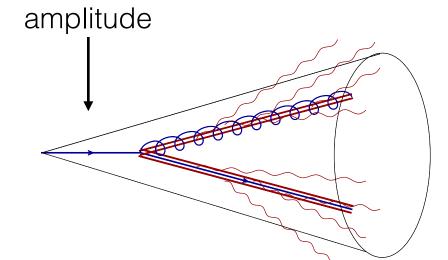
- Coft Wilson line for each final state collinear particle!
- Multi-Wilson-line structure of operators

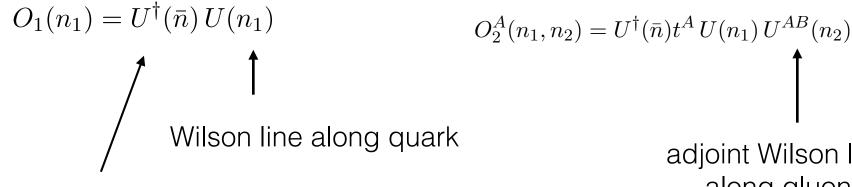
Verified by expanding $\gamma^* \rightarrow \overline{q}qgg$ amplitude in all regions.

Coft operator structure

collinear splitting







adjoint Wilson line along gluon

Wilson line along other jet

 n_i are light-like reference vectors along collinear partons

Operator matrix elements

Coft matrix element for collinear qg final state

$$\mathcal{A}_{2}(\delta\beta Q, \theta_{1}, \theta_{2}) = \sum_{\substack{X_{\text{coft}} \\ \text{phase-space} \\ \text{constraints}}} \mathcal{M}_{\text{coft}}(\{p_{i}\}) \langle 0|(O_{2}^{\dagger})_{ab}^{A}|X_{\text{coft}}\rangle \langle X_{\text{coft}}|(O_{2})_{ba}^{A}|0\rangle \\ O_{2}^{A}(n_{1}, n_{2}) = U^{\dagger}(\bar{n})t^{A}U(n_{1})U^{AB}(n_{2})$$

$$\theta_1 = \frac{1}{\delta^2} \frac{n \cdot n_1}{\bar{n} \cdot n_1} \quad \theta_2 = \frac{1}{\delta^2} \frac{n \cdot n_2}{\bar{n} \cdot n_2}$$

interesting similiarities to color density matrix by Simon Caron-Huot

gets integrated against collinear matrix element:

$$\int d\theta_1 d\theta_2 \, J_2(Q\delta, \theta_1, \theta_2) \, \mathcal{A}_2(\delta\beta Q, \theta_1, \theta_2)$$

collinear matrix element splitting function

Two-loop check

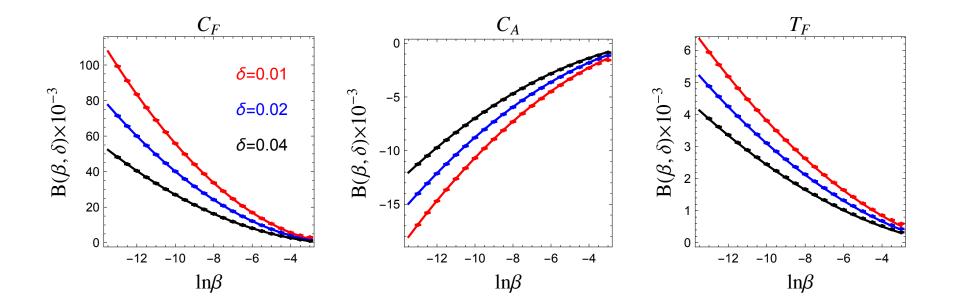
Have determined/computed all two-loop ingredients, except for purely collinear contribution 2-loop terms.

Checks:

- Cancellation of divergences involves nontrivial interplay of different regions, determines divergences and therefore all logarithms in collinear part
- Can then compare numerically to fixed-order results from Event2 at small β and δ , extract missing collinear constants numerically

$$\begin{aligned} &\frac{\sigma(\beta,\delta)}{\sigma_0} = 1 + \frac{\alpha_s}{2\pi} A(\beta,\delta) + \left(\frac{\alpha_s}{2\pi}\right)^2 B(\beta,\delta) \\ B(\beta,\delta) &= C_F^2 \left[\left(32\ln^2\beta + 48\ln\beta - \frac{16\pi^2}{3} + 18 \right) \ln^2\delta + \ln\beta \left(10\zeta_3 - 2 - 12\ln^2 2 + 4\ln 2 \right) \right. \\ &+ \ln\delta \left((8 - 48\ln 2)\ln\beta - 24\zeta_3 + 2\pi^2 + \frac{9}{2} - 36\ln 2 \right) + c_F^F \right] \\ &+ C_F C_A \left[\left(\frac{44\ln\beta}{3} + 11 \right) \ln^2\delta - \frac{2}{3}\pi^2 \ln^2\beta + \ln\beta \left(-4\zeta_3 + \frac{8}{3} - \frac{31\pi^2}{18} - 6\ln^2 2 - 4\ln 2 \right) \right. \\ &+ \ln\delta \left(\frac{44\ln^2\beta}{3} + \left(\frac{4\pi^2}{3} - \frac{268}{9} \right) \ln\beta + 12\zeta_3 - \frac{57}{2} - 22\ln 2 \right) + c_F^A \right] \\ &+ C_F T_F n_f \left[\left(-\frac{16\ln\beta}{3} - 4 \right) \ln^2\delta + \left(-\frac{16}{3}\ln^2\beta + \frac{80\ln\beta}{9} + 10 + 8\ln 2 \right) \ln\delta \right. \\ &+ \left. \left(\frac{4\pi^2}{9} - \frac{4}{3} \right) \ln\beta + c_2^f \right], \end{aligned}$$

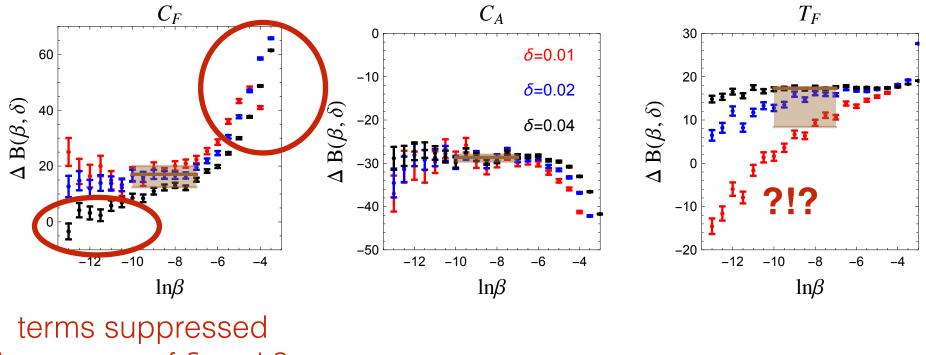
All logarithmic dependence is known, but three unknown constants c_2^{f} , c_2^{A} , c_2^{F}



Solid lines: analytic prediction

Points: Event 2 numerical result (billions of events in quadruple precision)

Two-loop constants are set to zero in these plots, but difference is not visible because logarithmic terms are huge.



by powers of δ and β

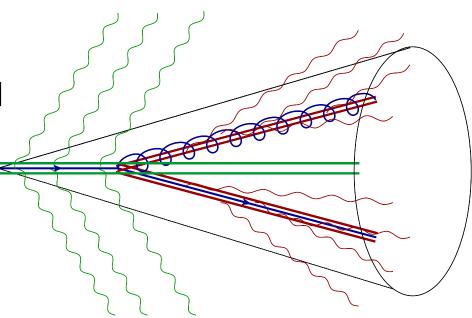
For δ and β both small, difference to full result must be missing two-loop constants

• Works, except for *n_f* terms. Problems for this color structure, also for other observables including thrust.

Jet Effective Theory

Understand

- the relevant scales and degrees of freedom
- the (complicated!) structure of the operators

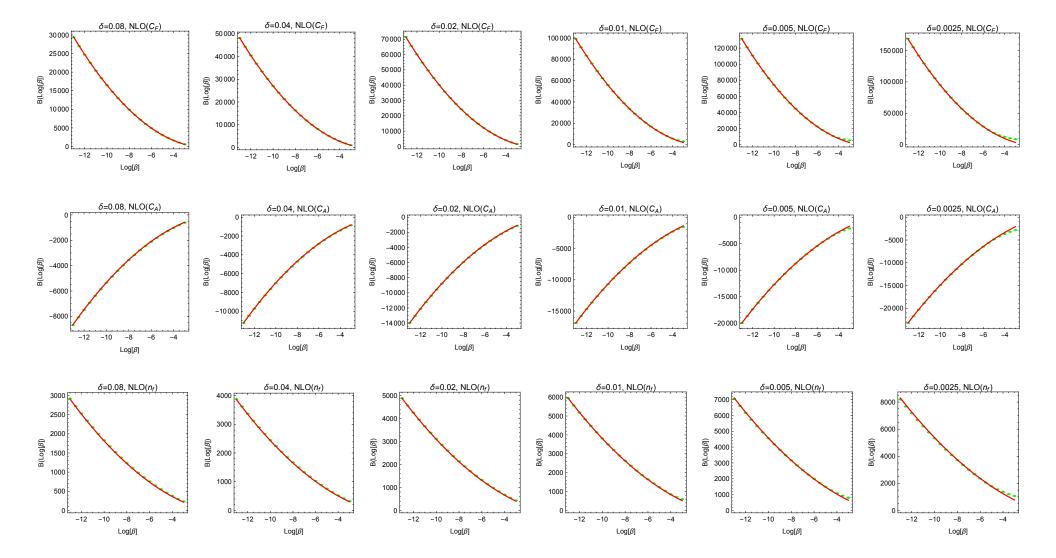


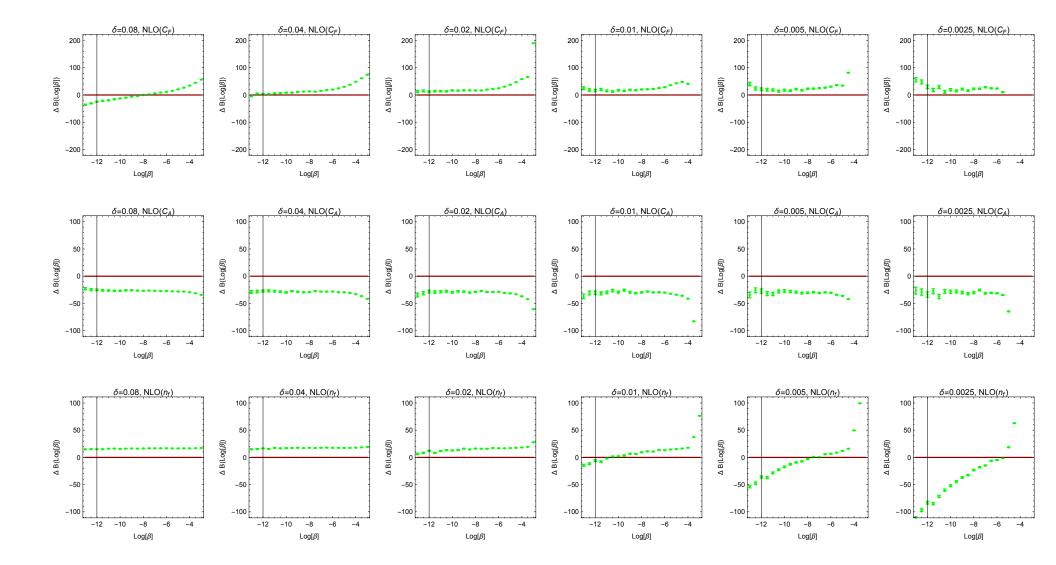
Important first step, but does not immediately translate into resummation. Next step

 Study renormalization and RG evolution in the effective theory!

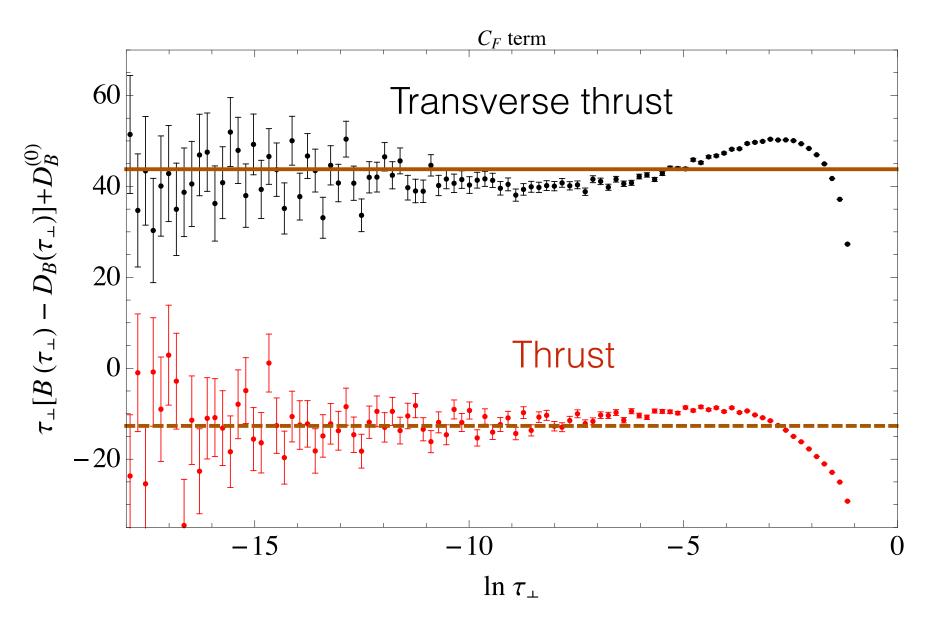
Summary

- Automated NNLL resummation for jet-veto cross sections
 - First example of an automated SCET resummation
 - Other observables can be resummed using the same technique
- NNLL resummation for transverse thrust
 - Interesting factorization theorem: SCET_I+SCET_{II}, rapidity divergences with nontrivial color structure, ...
 - Role of Glauber gluons? UE?
- Jet Effective Theory
 - New 'coft' mode to describe soft small angle radiation
 - Coft radiation resolves individual collinear final-state particles: leads to multi-Wilson-line structure of coft operators



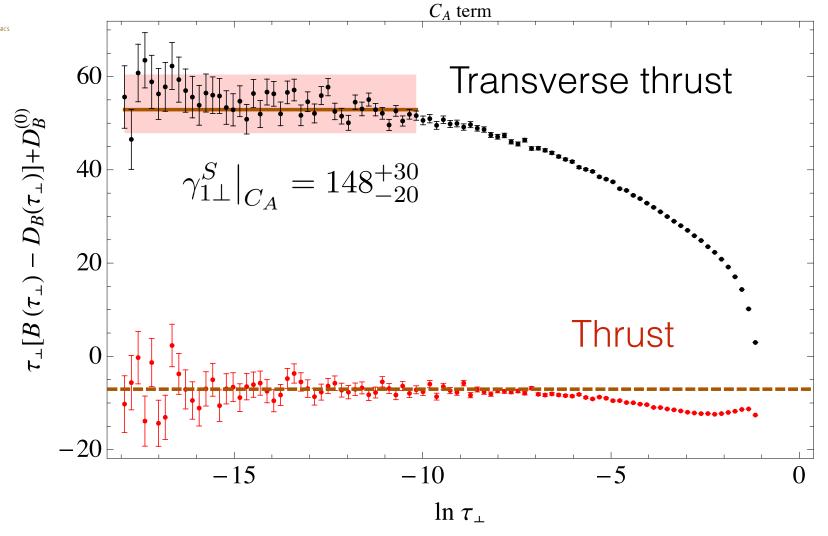


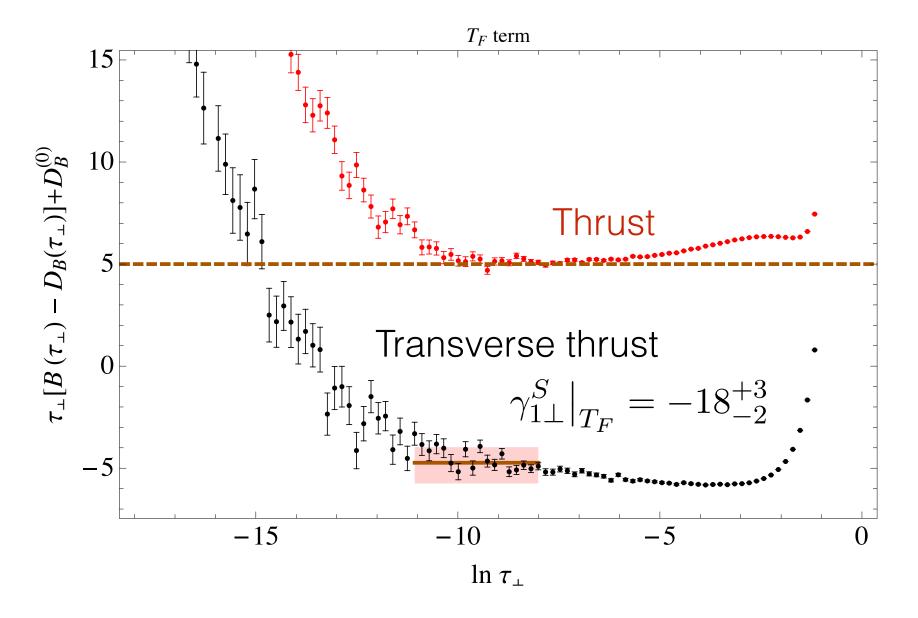
$O(\alpha_s^2)$ in e^+e^- from Event 2 vs. expansion of resummed result





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Numerical problem in Event2 code.

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