The Sensitivity of Spin-Precession Axion Experiments+

arXiv: 22XX.XXXXX & 22XX.XXXXX
J.A. Dror, S. Gori, N.L. Rodd

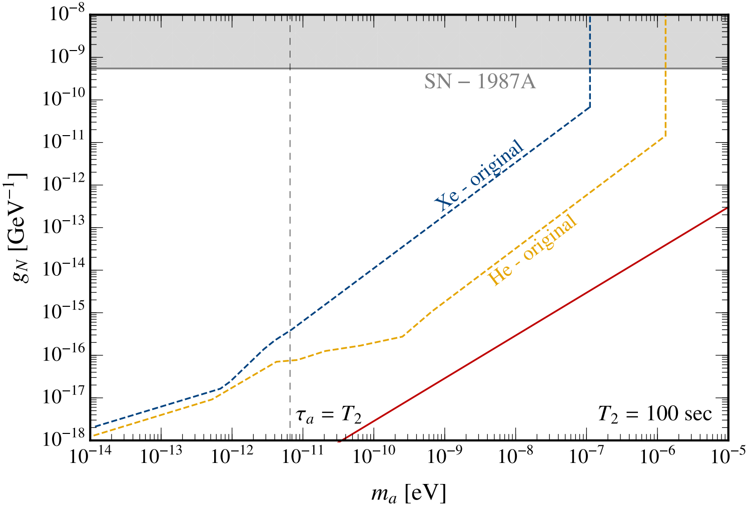
Jacob M. Leedom

Wavy DM Detection with QN, 19.08.2022

CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Spoiler Slide

[Graham, Rajendran,'13]



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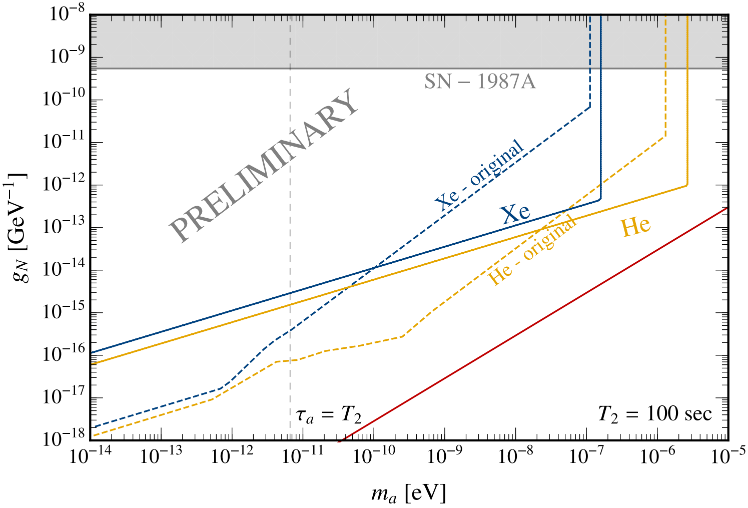

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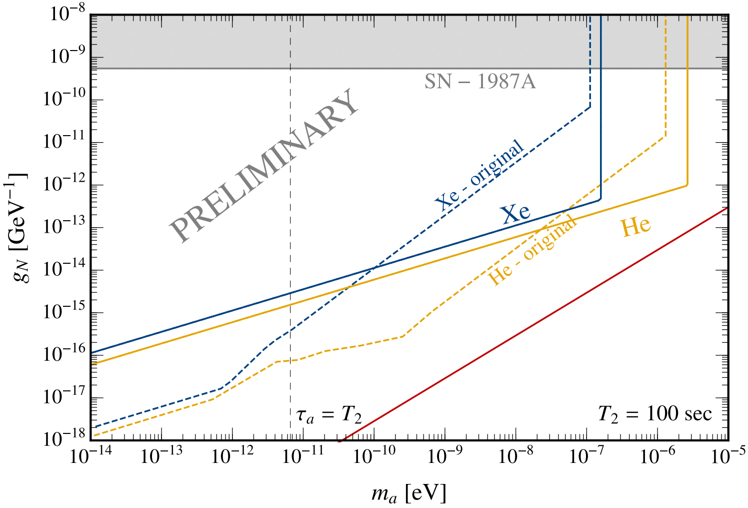

[Graham, Rajendran,'13]

$$B_{max} = 10, 20 \text{ T}$$

$$\bar{\gamma}_{Xe} = 2.436 \times 10^7 \text{ s}^{-1}\text{T}^{-1}$$
$$\gamma_{He} = 2.037 \times 10^8 \text{ s}^{-1}\text{T}^{-1}$$

$$\omega_{max}^{Xe} = \bar{\gamma}_{Xe} B_{max} \approx 1.60 \times 10^{-7} \text{ eV}$$
$$\omega_{max}^{He} = \bar{\gamma}_{He} B_{max} \approx 2.68 \times 10^{-6} \text{ eV}$$

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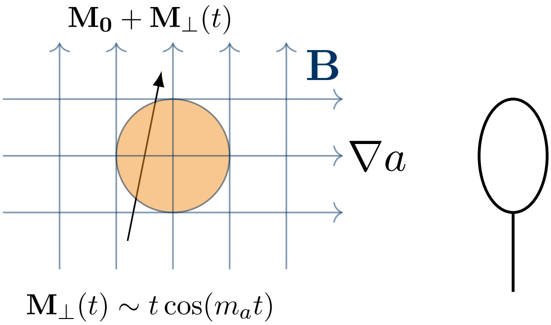
Axion-Induced NMR: Basics

[See Hendrik's talk from yesterday/last week]



Axion-Induced NMR: Bloch Equations

- One can start from Schrödinger equation for spins to find magnetization....or use the Bloch Equations:

$$\frac{d\mathbf{M}}{dt} = \mathbf{M} \times \gamma \mathbf{B} - \frac{M_x \hat{\mathbf{x}} + M_y \hat{\mathbf{y}}}{T_2} - \frac{(M_z - M_0)\hat{\mathbf{z}}}{T_1}$$

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> Perturbative expansion in g_N & decouple the equations

$$\ddot{M}_x + \frac{2}{T_2}\dot{M}_x + \omega_0^2 M_x = 2 g_N M_0 \omega_0 (\hat{x} \cdot \nabla a) - 2 g_N M_0 \frac{d}{dt}(\hat{y} \cdot \nabla a)$$

Axion-Induced NMR: General Solution

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- General Solution via Green's Function

$$G(t) = \frac{1}{\Omega} e^{-t/T_2} \sin[\Omega t] \qquad \Omega^2 = \omega_0^2 - T_2^{-2}$$

$$M_x(t) = C_1 e^{-t/T_2}\cos(\omega_0 t) + C_2 e^{-t/T_2}\sin(\omega_0 t) + \int_0^t G(t-s)F(s)ds$$

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- From here, can obtain DFT & PSD of the signal
- However, must model the dark matter source

Axion-Induced NMR: Resonance & Beats

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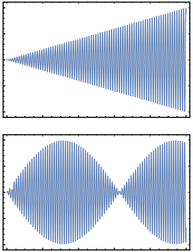
$T_2 \to \infty$

$\omega = \omega_0$

$\omega \approx \omega_0$



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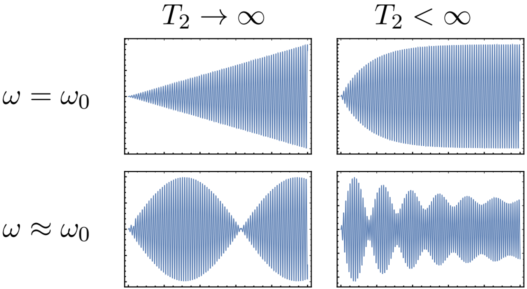
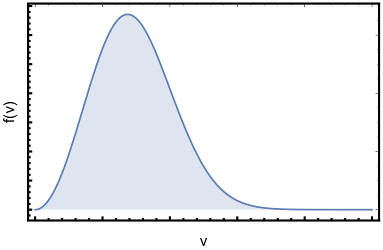


Modeling Wavy Dark Matter

> Wavy axion dark matter is not just a cosine



$$f_{SHM}(v) = \frac{v}{\sqrt{\pi} v_0 v_\odot} \exp\left[-\frac{(v+v_\odot)^2}{v_0^2}\right]\left(\exp\left[\frac{4 v v_\odot}{v_0^2}\right] - 1\right)$$

$$f_{SHM}(\mathbf{v}) = \frac{1}{\pi^{3/2} v_0^3} \exp\left[-(\mathbf{v}+\mathbf{v}_\odot)^2 / v_0^2\right]$$

Modeling Wavy Dark Matter: Plane Wave Model

$$a(t) = \sqrt{\frac{2\rho_a}{N_a \bar{\omega}}} \sum_{i=1}^{N_a} \frac{1}{\sqrt{\omega_i}} \cos[\omega_i t + \phi_i] \implies \nabla a(t) = \sqrt{\frac{2\rho_a}{N_a \bar{\omega}}} \sum_{i=1}^{N_a} \frac{\vec{k}_i}{\sqrt{\omega_i}} \sin[\omega_i t + \phi_i]$$

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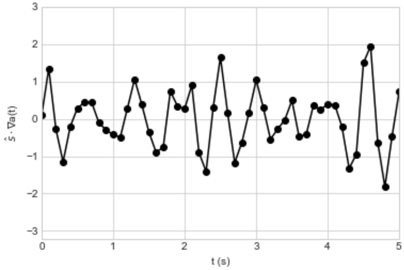

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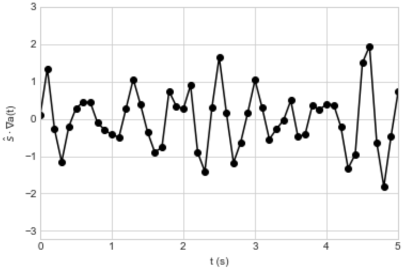

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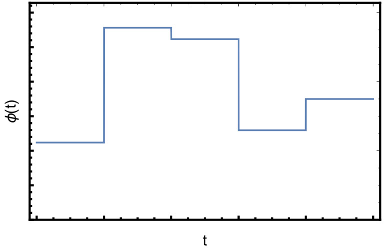

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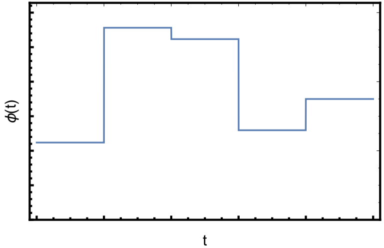

$$f_{SHM}(\vec{v}) \to \vec{v}_i$$

- Inside a coherence time patch, Bloch solution is simple. Only nontrivial aspect is stitching patches together.

The Original Argument

[Graham, Rajendran,'13]

[Budker,Graham, Ledbetter,Rajendran,Sushkov , '14]

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$$M_s \approx \sqrt{S_n}(\tau_a T)^{-1/4} \to g_n \propto m_a^{5/4}$$

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Claim: Both assumptions need to be revisited

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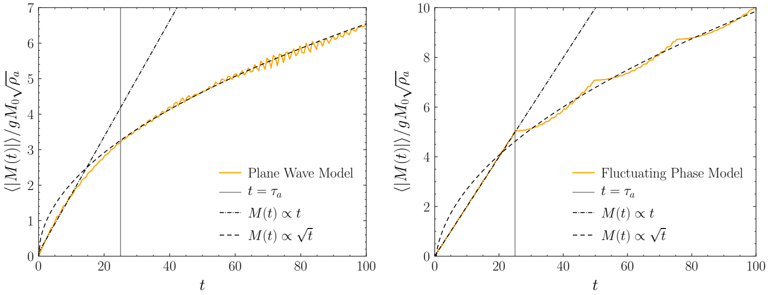

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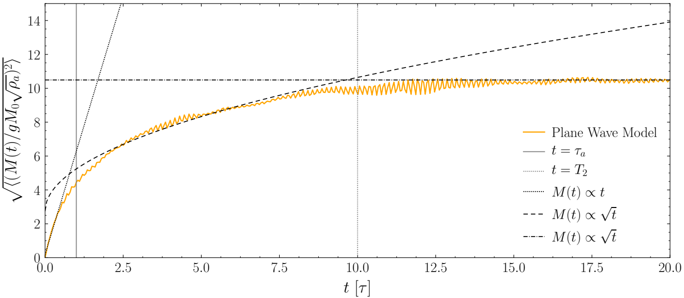

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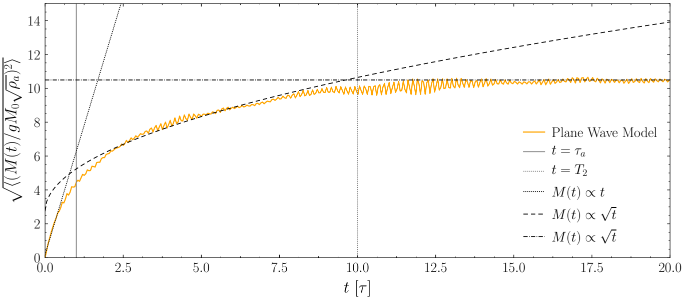

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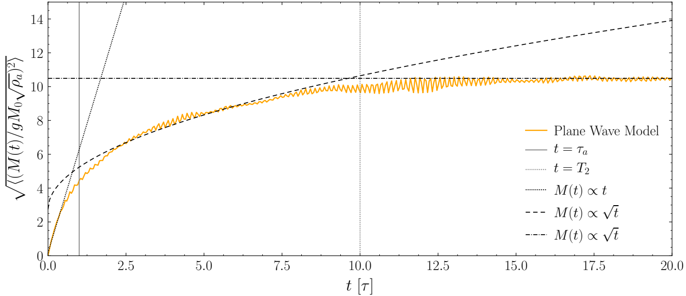

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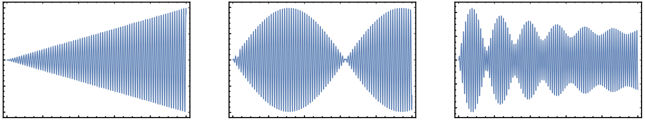
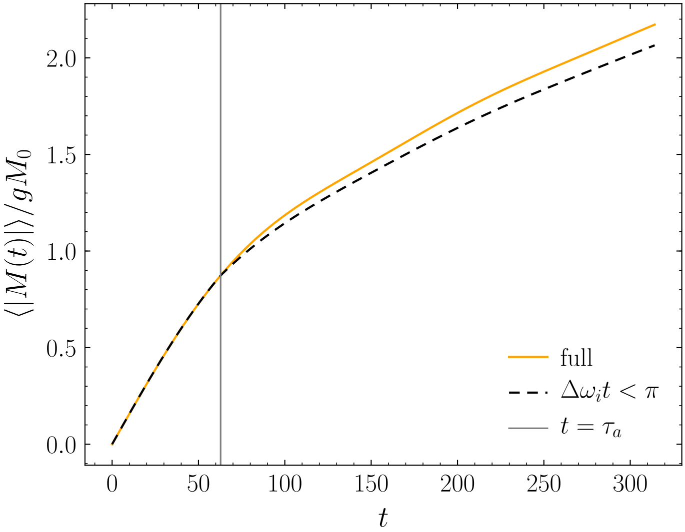


Axion-Induced NMR: Back of the Envelope



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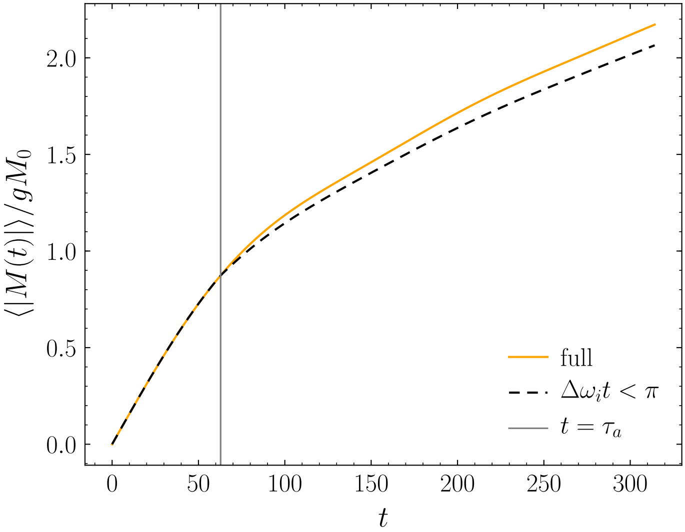

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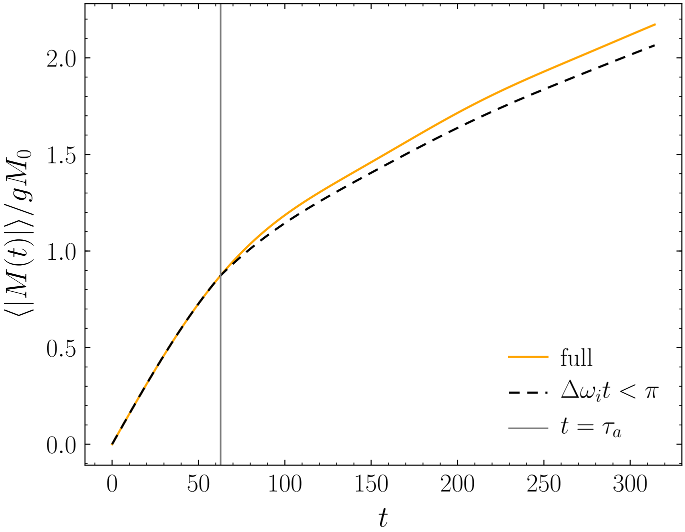

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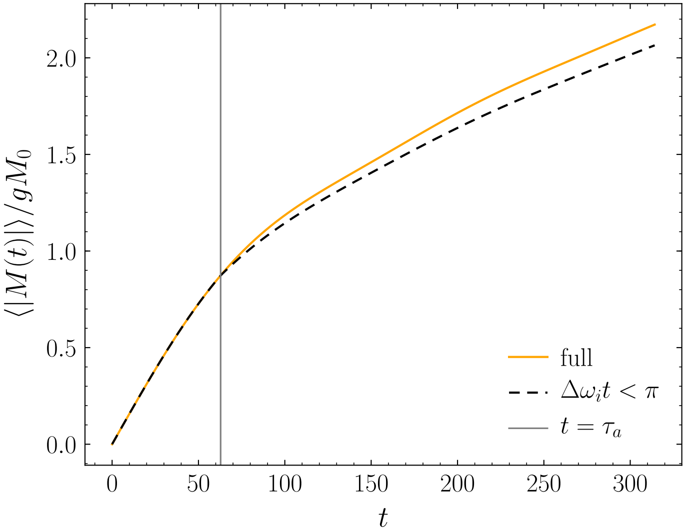

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- If $\Delta\omega_i t << 1$, axion wave is in **resonant regime**

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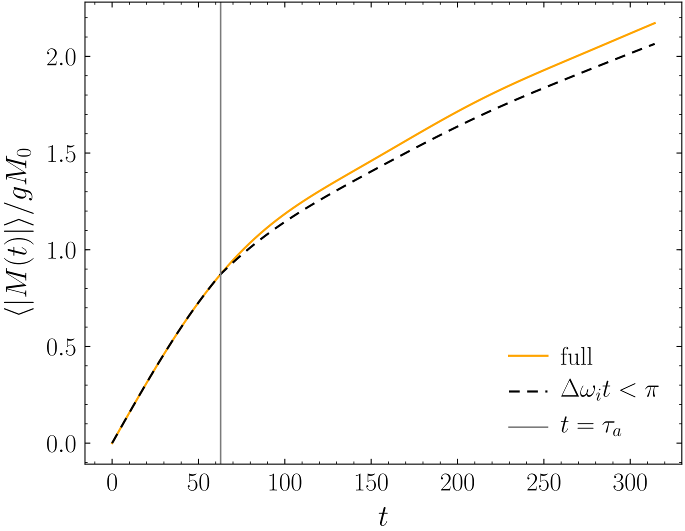

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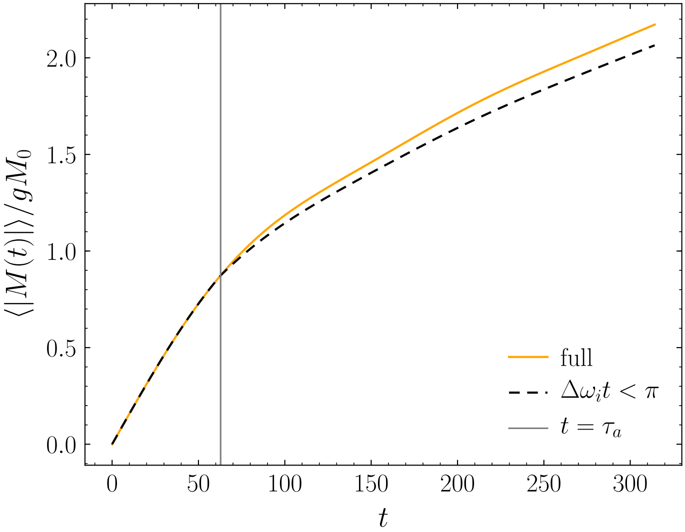
Axion-Induced NMR: Back of the Envelope



$$M_x(t) \sim \sum_{i=1}^{N_a} \int_0^t G(\omega_0|t-t') \cos(\omega_i t')$$

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$$\sqrt{\langle M^2 \rangle} \sim \sqrt{\frac{\tau_a}{t}} \times t \cos(m_a t)$$

Axion-Induced NMR: Master Equation

- Take Discrete Fourier Transform of Bloch equation:

$$M_x^{(k)} = \sum_{n=0}^{N-1} M_x(n\Delta t) e^{-2\pi i k/N} \qquad T = N\Delta t$$

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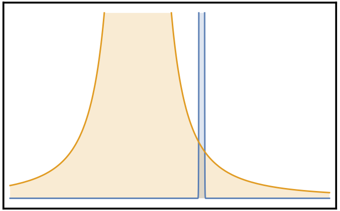

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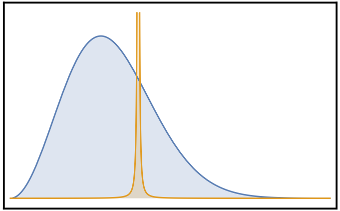

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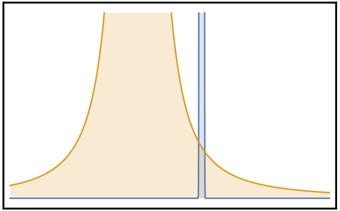

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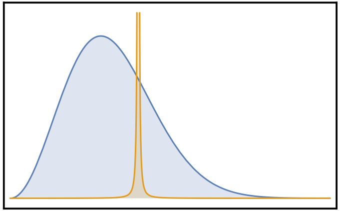

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In both regimes, the PSD is sharply peaked and power is primarily in a single bin

Master Equation & Analytics

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$$\langle|\hat{s}\cdot\nabla a(\omega)|^2\rangle = \frac{\pi\rho_{DM}}{m_a v_\omega}\int d^3v f(\vec{v})(\hat{s}\cdot\vec{v})\delta(|\vec{v}|-v_\omega) \qquad v_\omega = \sqrt{2\omega/m_a - 2}$$

$$= \frac{\rho_{DM}}{\pi^{1/2} m_a v_\omega v_0^3}\int d^3v e^{-(\vec{v}+\vec{v}_\odot)^2/v_0^2}(\hat{s}\cdot\vec{v})^2\delta(|\vec{v}|-v_\omega) \qquad \hat{s}\rightarrow\hat{v}_\odot$$

$$= \frac{\sqrt{\pi}\rho_{DM}}{m_a}e^{-v_\omega^2/v_0^2-1}[(1+2(v_\omega/v_0)^2)\sinh(2v_\omega/v_0) - 2(v_\omega/v_0)\cosh(2v_\omega/v_0)]$$

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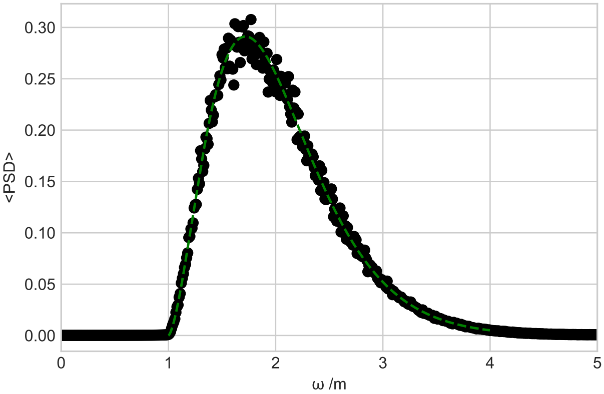


Axion DM Models & Master Equation

$$S_x(\omega) = \begin{cases} \dfrac{\rho_{DM} g_N^2 M_0^2 T_2^2}{m_a} \delta_{k,k_s} & \text{when } \tau_a < T_2 \\ \dfrac{\rho_{DM} g_N^2 M_0^2 v_{DM}^2 T T_2^2}{12} \delta_{k,k_s} & \text{when } T_2 < \tau_a \end{cases}$$

$$T = T_2 \times \max\left[1, \frac{\alpha \tau_a}{T_2}\right]$$

Noise

- Magnetometer Noise:

$$\lambda_M \sim 1.6 \times 10^5 \text{ eV}^{-1}$$

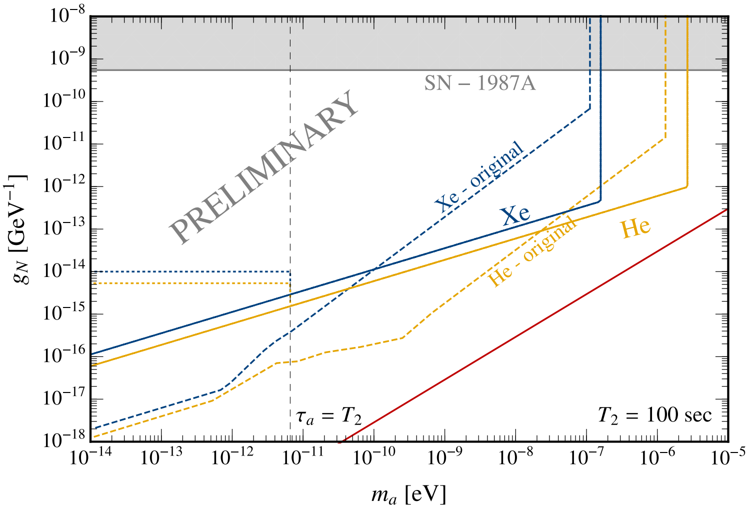
- Spin Projection Noise

$$\lambda_{SP}(\omega) = \frac{\gamma^2 n}{4\pi V T_2} \frac{1}{T_2^{-2} + (\omega_0 - \omega)^2}$$

[Braun, König,'06]
[Aybas+,'21]

- There is also thermal noise, but we assume the sample is cooled sufficiently that is it subdominant.
- These are fed into a likelihood framework & 95% limits are derived via the Asimov dataset

CASPEr for DM Conclusions



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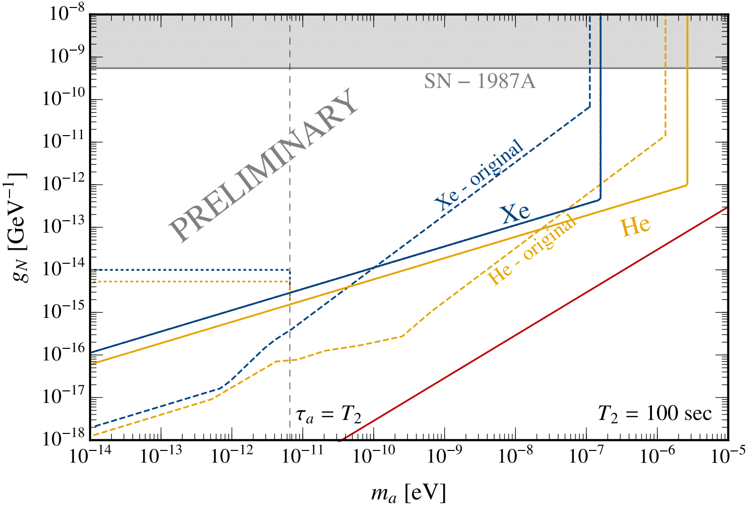

- Effects lead to enhanced sensitivity at higher masses
- Applicable to any experiment utilizing NMR or related techniques. In particular, also true for CASPEr-electric (will be in paper)
- Note: If SP noise can be mitigated, He contour approaches QCD axion line

The CaB, String Theory, & CASPEr

- The Cosmic axion Background - relic density of relativistic utlralight axions

[Dror,Murayama,Rodd, '21]

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$$m_a^2 \sim \frac{M_p^2 \Lambda^2 e^{-S}}{f_a^2}$$

[Hui, Ostriker, Tremaine, Witten, '16]

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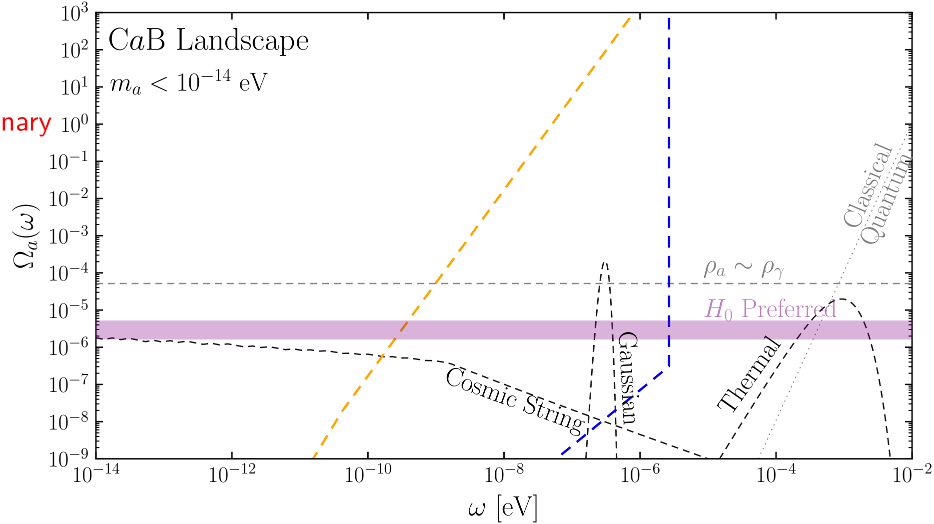
CASPEr as a probe of cosmology & quantum gravity!

The CaB, String Theory, & CASPEr

Very Preliminary



Thank you!

Contact

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