Earth as a transducer for ultralight dark-matter detection

Saarik Kalia

based on arXiv:2106.00022, arXiv:2108.08852, arXiv:2112.09620

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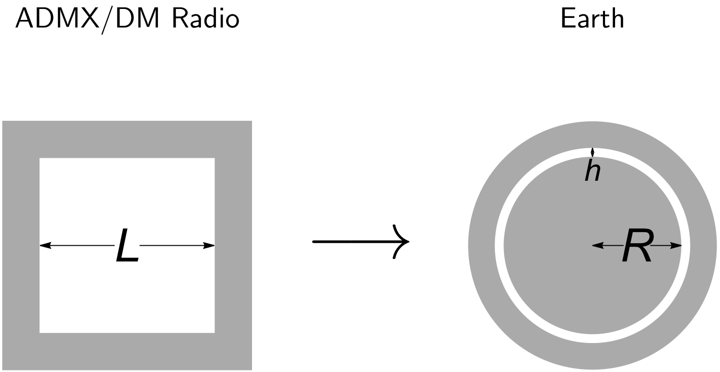
Searches for Wave-Like Dark Matter with Quantum Networks

August 18, 2022

Introduction

- Cavity/shielded experiments search for ultralight EM-coupled DM:
  - Dark-photon dark matter
  - Axionlike dark matter
- Signal scales with size of apparatus
- DPDM constraints below 10^{-14} eV (sub-Hz) all astrophysical
- We use the Earth as our apparatus/transducer!
- Ultralight DM $\longrightarrow$ magnetic field at Earth's surface

Introduction



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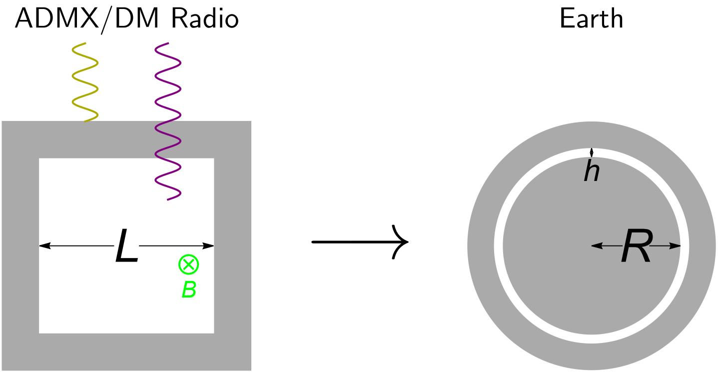

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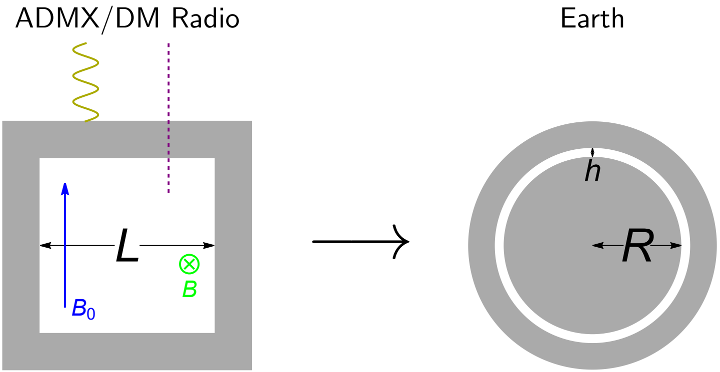
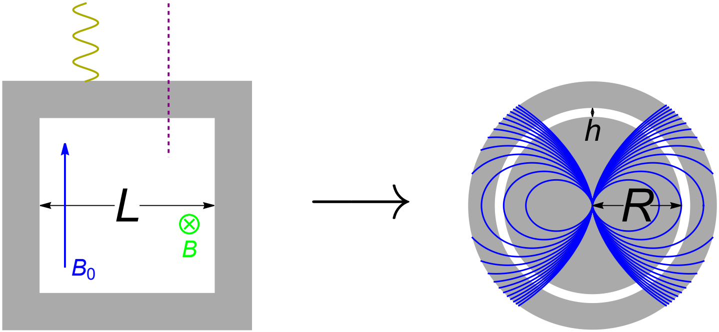


Introduction

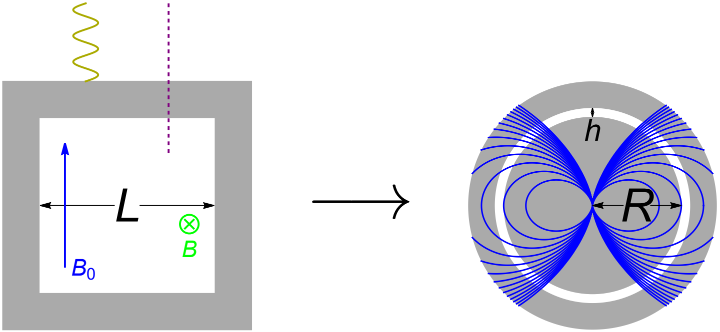
ADMX/DM Radio | Earth



Introduction

ADMX/DM Radio

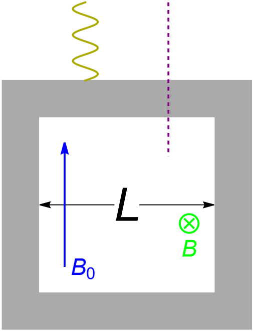
Earth



Scales with L

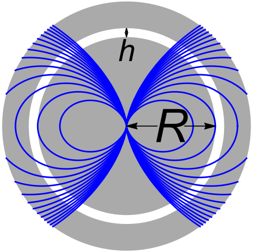
Introduction

ADMX/DM Radio



$\longrightarrow$

Earth



Scales with L

Scales with R, not h!

Outline

1. Effective Current Approach
2. Earth Signal
3. Analysis of SuperMAG Data

Kinetically Mixed Dark Photon

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}m_{A'}^2 A'_\mu A'^\mu + \varepsilon m_{A'}^2 A'^\mu A_\mu - J^\mu_{\mathrm{EM}} A_\mu$$

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- One massless and one massive (mass $m_{A'}$) propagation state

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- Only A couples to charges
  - Only A is affected (at leading order) by conductors
  - The observable fields are E and B (no contribution from E' and B')
- One massless and one massive (mass $m_{A'}$) propagation state
- A and A' are not propagation states in vacuum!
  - Mixing (and all observable effects) are proportional to $m_{A'}$
  - A and A' *are* propagation states in conductor $\to$ mixing at boundary

Dark-Photon Effective Current

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \frac{1}{2}m_{A'}^2 A'_\mu A'^\mu + \varepsilon m_{A'}^2 A'^\mu A_\mu - J^\mu_{\mathrm{EM}} A_\mu$$

- When A' is DM and $\varepsilon \ll 1$ (no backreaction), then $J^\mu_{\mathrm{eff}} = -\varepsilon m_{A'}^2 A'^\mu$.

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  - $J_{\mathrm{eff}}^0 = 0$
  - $\boldsymbol{J}_{\mathrm{eff}}$ constant in space (for dark photons)
  - Oscillates with frequency $\omega = m_{A'}$

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- Non-relativistic ($v = 0$)
  - $J_{\mathrm{eff}}^0 = 0$
  - $\boldsymbol{J}_{\mathrm{eff}}$ constant in space (for dark photons)
  - Oscillates with frequency $\omega = m_{A'}$
- Just a single-photon EM problem with a background current!

Axionlike Particle

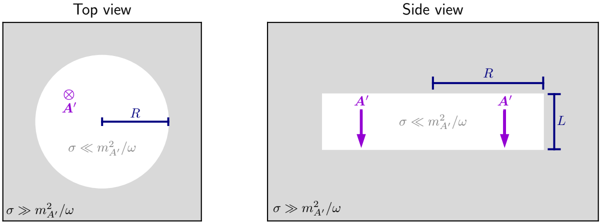
$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu a)^2 - \frac{1}{2}m_a^2 a^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{4}g_{a\gamma}aF_{\mu\nu}\tilde{F}^{\mu\nu}$$

- Allows axion to convert into photon in background $\boldsymbol{B}_0$
- In non-relativistic limit,

$$\nabla \times \boldsymbol{B} - \partial_t \boldsymbol{E} = -g_{a\gamma}(\partial_t a)\boldsymbol{B}$$

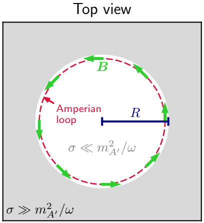
- Also behaves as $\boldsymbol{J}_{\mathrm{eff}} = ig_{a\gamma}m_a a\boldsymbol{B}_0$ (replace $\varepsilon m_{A'}\boldsymbol{A}' \to -ig_{a\gamma}a\boldsymbol{B}_0$)
- Note that direction set by $\boldsymbol{B}_0$ not by DM!

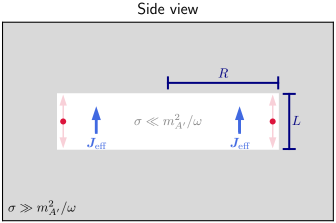
Toy Geometry: ADMX/DM Radio Cavity



Scales with transverse direction R,
not longitudinal direction L

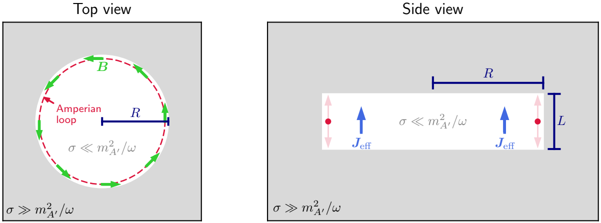
Ampère's Law Argument





$$BR \sim \oint \boldsymbol{B} \cdot d\boldsymbol{\ell} = \iint \boldsymbol{J}_{\text{eff}} \cdot d\boldsymbol{A} \sim \varepsilon m_{A'}^2 R^2 A'$$

Ampère's Law Argument



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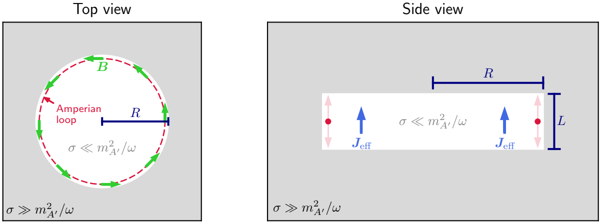
$$B_{A'} \sim \varepsilon m_{A'}^2 R A' \sim \varepsilon m_{A'} R \sqrt{\rho_{\text{DM}}}$$

Ampère's Law Argument



$$BR \sim \oint \boldsymbol{B} \cdot d\boldsymbol{\ell} = \iint \boldsymbol{J}_{\rm eff} \cdot d\boldsymbol{A} \sim \varepsilon m_{A'}^2 R^2 A'$$

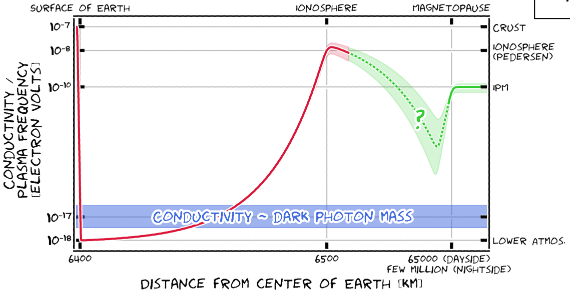
$$B_{A'} \sim \varepsilon m_{A'}^2 R A' \sim \varepsilon m_{A'} R \sqrt{\rho_{\rm DM}} \quad \left(B_a \sim g_{a\gamma} B_0 R \sqrt{\rho_{\rm DM}}\right)$$

What makes a good shield?

- Good conductor or plasma: $\sigma, \omega_p \gg m$
- Sufficiently thick: $h \gg \delta$
  - $\delta \sim \sqrt{\frac{2}{m\sigma}}$ for conductor
  - $\delta \sim \omega_p^{-1}$ for collisionless plasma

Earth Conductivity Profile

$m \sim 10^{-18}$ eV
$f \sim 10^{-4}$ Hz



	Core	Lower Atmosphere	Ionosphere $\sigma_{\parallel}$	Ionosphere $\sigma_{\perp}$	IPM
σ (ω_p) [eV]					
h [km]					
δ [km]					
Shield?					

Earth Conductivity Profile

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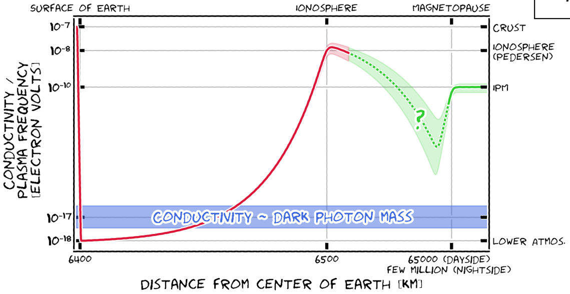

	Core	Lower Atmosphere	Ionosphere $\sigma_{\parallel}$	Ionosphere $\sigma_{\perp}$	IPM
σ (ω_p) [eV]	100				
h [km]	3000				
δ [km]	0.03				
Shield?	Yes				

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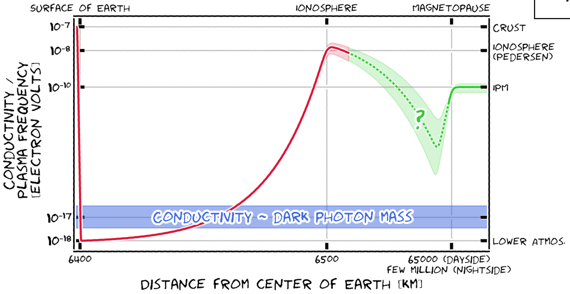

	Core	Lower Atmosphere	Ionosphere $\sigma_{\parallel}$	Ionosphere $\sigma_{\perp}$	IPM
σ (ω_p) [eV]	100	10^{-18}			
h [km]	3000	5			
δ [km]	0.03	10^8			
Shield?	Yes	No			

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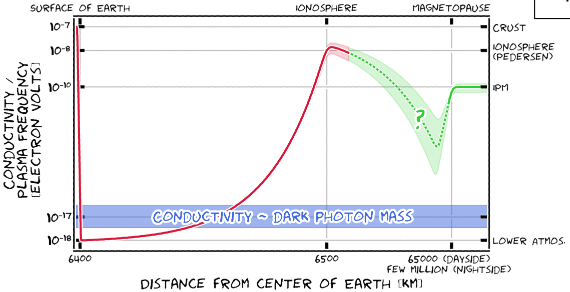

	Core	Lower Atmosphere	Ionosphere $\sigma_{\parallel}$	Ionosphere $\sigma_{\perp}$	IPM
σ (ω_p) [eV]	100	10^{-18}	10^{-2}	10^{-8}	
h [km]	3000	5	100		
δ [km]	0.03	10^{8}	2	1000	
Shield?	Yes	No	???		

Earth Conductivity Profile

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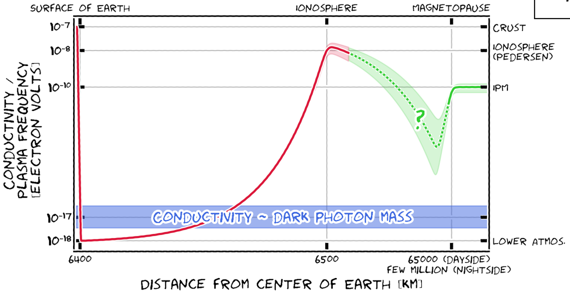

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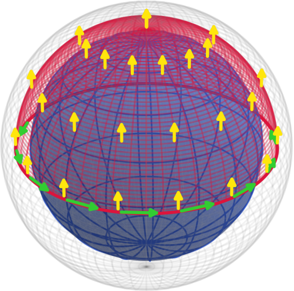
σ (ω_p) [eV]	100	10^{-18}	10^{-2}	10^{-8}	10^{-10}
h [km]	3000	5	100		3×10^5
δ [km]	0.03	10^8	2	1000	2
Shield?	Yes	No	???		Yes

Ampère's Law Argument (for Earth)



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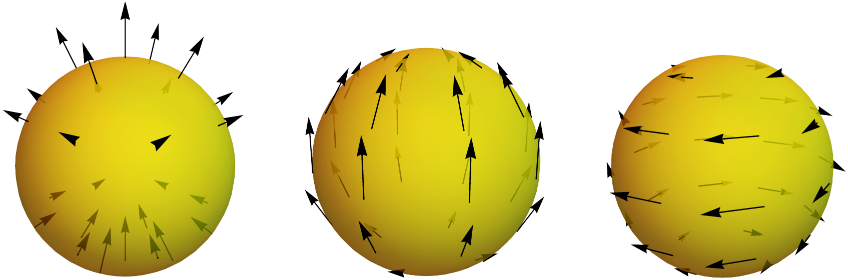
$$B \sim \varepsilon m_{A'}^2 R A' \sim \varepsilon m_{A'} R \sqrt{\rho_{\text{DM}}}$$

Vector Spherical Harmonics

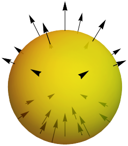
- Three types of vector spherical harmonics: $\boldsymbol{Y}_{\ell m}$, $\boldsymbol{\Psi}_{\ell m}$, $\boldsymbol{\Phi}_{\ell m}$
- Only $\ell = 1$ relevant for dark photon case
- Real and imaginary parts of $m = \pm 1$ oriented along x- and y-axes



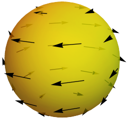
$\boldsymbol{Y}_{10}$ $\boldsymbol{\Psi}_{10}$ $\boldsymbol{\Phi}_{10}$

VSH Calculus

$$\nabla \times \left[r \cdot \boldsymbol{Y}_{10} \right] = - \boldsymbol{\Phi}_{10}$$

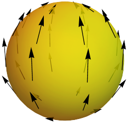


$\boldsymbol{Y}_{10}$

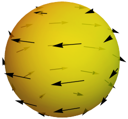


$\boldsymbol{\Phi}_{10}$

$$\nabla \times \left[r \cdot \boldsymbol{\Psi}_{10} \right] = 2 \cdot \boldsymbol{\Phi}_{10}$$

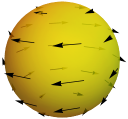


$\boldsymbol{\Psi}_{10}$



$\boldsymbol{\Phi}_{10}$

$$\nabla \times \left[r \cdot \boldsymbol{\Phi}_{10} \right] = -2 \cdot \left(\boldsymbol{Y}_{10} + \boldsymbol{\Psi}_{10} \right)$$



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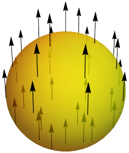

$\boldsymbol{Y}_{10} + \boldsymbol{\Psi}_{10}$

Dark Photon Signal

- Uniform $\boldsymbol{A}'$ sources

$$\boldsymbol{J}_{\text{eff}} = -\varepsilon m_{A'}^2 \boldsymbol{A}' \propto \varepsilon m_{A'}^2 \sum_{m=-1}^{1} A'_m \left(\boldsymbol{Y}_{1m} + \boldsymbol{\Psi}_{1m} \right) e^{-i m_{A'} t}$$

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- Since cavity is sub-wavelength, $\boldsymbol{E}$ is generically small, so

$$\nabla \times \boldsymbol{B} - \cancel{\partial_t \boldsymbol{E}} = \boldsymbol{J}_{\text{eff}}$$

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- Magnetic field at surface $r = R$ is

$$\boldsymbol{B} \propto \varepsilon m_{A'}^2 R \sum_{m=-1}^{1} A'_m \boldsymbol{\Phi}_{1m} e^{-i m_{A'} t} + \text{curl-free}$$

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- Curl-free part made of $\boldsymbol{Y}_{\ell m}$'s and $\boldsymbol{\Psi}_{\ell m}$'s
- $\boldsymbol{\Phi}_{1m}$ part independent of boundary conditions (to leading order)!

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- Magnetic field at surface $r = R$ is (in corotating coordinates)

$$\boldsymbol{B} \propto \varepsilon m_{A'}^2 R \sum_{m=-1}^{1} A'_m \boldsymbol{\Phi}_{1m} e^{-im_{A'} t + 2\pi i f_d m t} + \text{curl-free}$$

where $f_d = 1/(\text{sidereal day})$

- Curl-free part made of $\boldsymbol{Y}_{\ell m}$'s and $\boldsymbol{\Psi}_{\ell m}$'s
- $\boldsymbol{\Phi}_{1m}$ part independent of boundary conditions (to leading order)!

Axion Signal

- International Geomagnetic Reference Field model: $\boldsymbol{B}_0 = -\nabla V_0$ with

$$V_0 = \sum_{\ell,m} C_{\ell m} \frac{R^{\ell+2}}{r^{\ell+1}} Y_\ell^m,$$

where $C_{\ell m}$ provided at 5-year intervals

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$$V_0 = \sum_{\ell,m} C_{\ell m} \frac{R^{\ell+2}}{r^{\ell+1}} Y_\ell^m,$$

where $C_{\ell m}$ provided at 5-year intervals

- Effective current is

$$\boldsymbol{J}_{\text{eff}} = i g_{a\gamma} m_a a_0 \sum_{\ell,m} C_{\ell m} \left(\frac{R}{r}\right)^{\ell+2} \left[(\ell+1)\boldsymbol{Y}_{\ell m} - \boldsymbol{\Psi}_{\ell m}\right] e^{-i m_a t}$$

Axion Signal

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- Axion signal at surface is (in corotating coordinates)

$$\boldsymbol{B} = -i(g_{a\gamma}R)(m_a a_0) \sum_{\ell,m} \frac{C_{\ell m}}{\ell} \boldsymbol{\Phi}_{\ell m} e^{-i m_a t} + \text{curl-free}$$

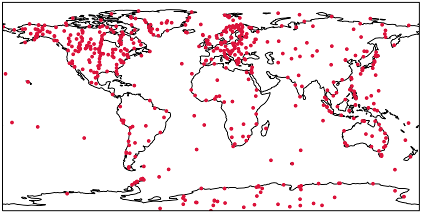
Signal Properties

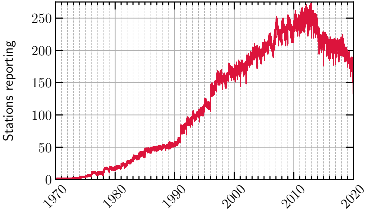
- Observable magnetic field at Earth's surface
- Large: scales with R not h
- Spatially coherent: global spatial pattern $\mathbf{\Phi}_{\ell m}$
- Temporally coherent: sharply peaked in frequency with $Q \sim 10^6$
- Robust: $\mathbf{\Phi}_{\ell m}$ component of signal is unaffected to leading order by boundary conditions!

Dark-Photon vs. Axion Signal

Dark Photon	Axion
Only uses $\ell = 1$ modes	Mostly (but not exactly) dipolar
Sidebands at $f = m_{A'}/2\pi \pm f_d$	Only at $f = m_a/2\pi$
Amplitude, phase, and orientation change between coherence times	Only amplitude and phase change between coherence times

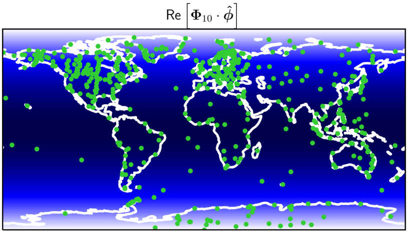
SuperMAG

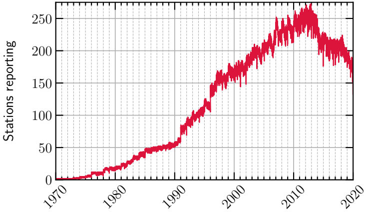




- Collaboration of over 500 ground-based magnetometers
- Data collected over 50 years
- 1-minute time resolution

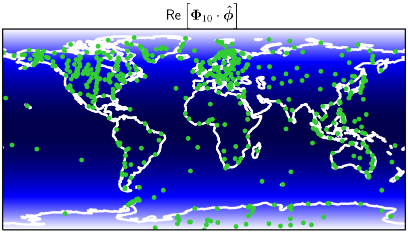
Analysis Difficulties

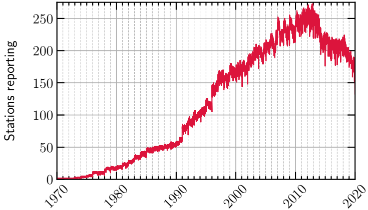




- What we'd like to do:
  - Project onto $\boldsymbol{\Phi}_{\ell m}$ modes
  - Fourier transform
  - Look for single-frequency peak

Analysis Difficulties

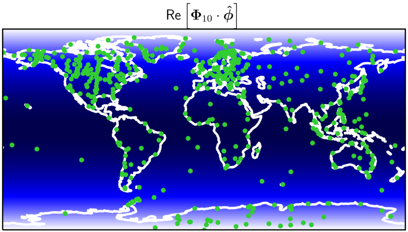


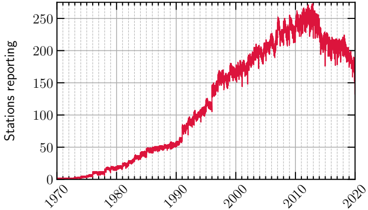


- What we'd like to do:
  - Project onto $\boldsymbol{\Phi}_{\ell m}$ modes
  - Fourier transform
  - Look for single-frequency peak

Noise variations/correlations

Analysis Difficulties





- What we'd like to do:
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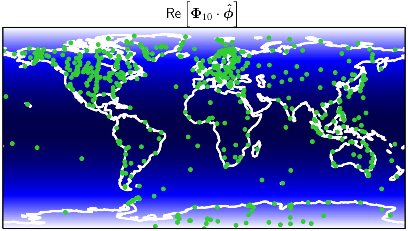
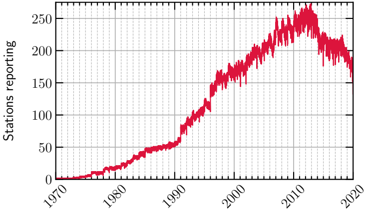
  - Fourier transform — Active stations highly variable
  - Look for single-frequency peak

Analysis Difficulties



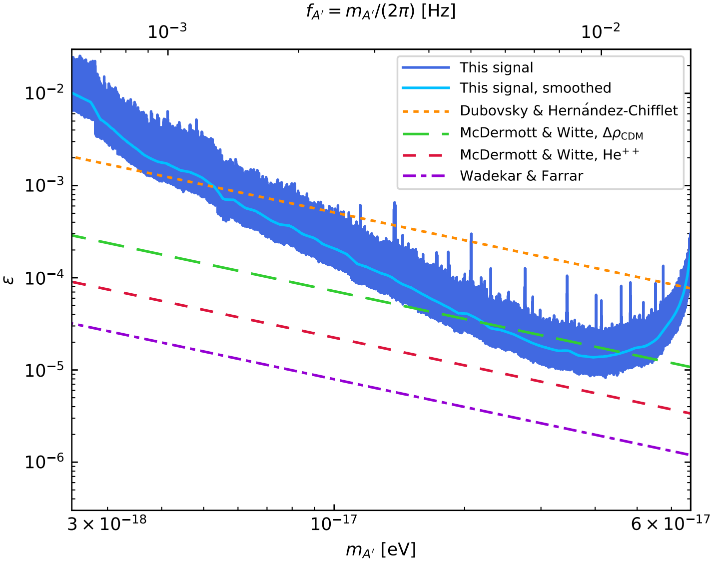


- What we'd like to do:
  - Project onto $\mathbf{\Phi}_{\ell m}$ modes — Noise variations/correlations
  - Fourier transform — Active stations highly variable
  - Look for single-frequency peak — Total time $>$ coherence time

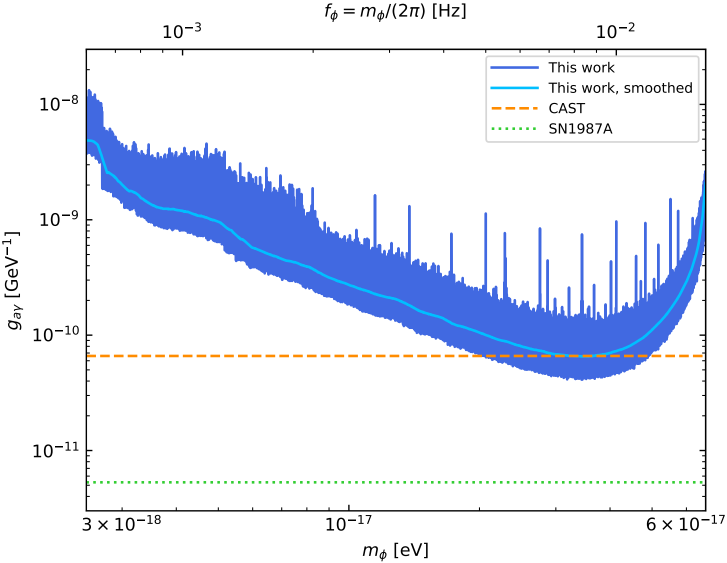
Analysis Overview

- Combine data from active stations into a few time series
- Weight by $\mathbf{\Phi}_{\ell m}$ modes *and* inverse noise
- Do same to signal
- Divide temporally into chunks of duration equal to coherence time
- Search each chunk coherently, then combine incoherently
- Utilize Bayesian framework to derive posterior for ε or $g_{a\gamma}$

Dark-Photon Constraint



Axion Constraint



Future Prospects

- Currently analyzing SuperMAG's 1-second resolution data
  - Will probe higher masses
  - If $1/f$ noise continues, then our bound scales better than others!
- SNIPE Hunt: Derek goes camping with a magnetometer
  - Take our own data to probe above 1 Hz
  - Won't have as much data, and may lose m scaling of signal
  - If noise is mostly man-made, may have much lower noise!

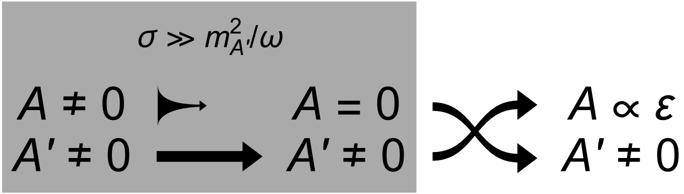
Summary

- We demonstrated a novel mechanism to probe ultralight dark matter using the Earth as a transducer.
- It utilizes the natural conductivity environment near the Earth.
- Our signal is not suppressed by the height of the atmosphere!
- It is highly spatially and temporally coherent, and robust to environmental details.
- We set competitive bounds on DPDM and axion parameter space.
- Stay tuned for 1-second analyses and SNIPE Hunt!

Mixing in Medium

- Consider (transverse) modes of frequency ω

	In vacuum		In good conductor ($\sigma \gg m_{A'}^2/\omega$)	
State	$A - \varepsilon A'$	$A' + \varepsilon A$	A	A'
Propagation	Massless	Mass $m_{A'}$	Damped	Mass $m_{A'}$



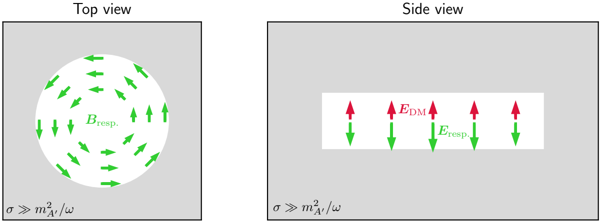
Solving the wave equation with a current

$$(\nabla^2 - \partial_t^2)\boldsymbol{E} = \partial_t \boldsymbol{J}_{\text{eff}}$$

$$\boldsymbol{E} = \boldsymbol{E}_{\text{DM}} + \boldsymbol{E}_{\text{response}}$$

$\boldsymbol{E}_{\text{DM}}$ (specific)	$\boldsymbol{E}_{\text{response}}$ (homogeneous)
$(\nabla^2 - \partial_t^2)\boldsymbol{E}_{\text{DM}} = \partial_t \boldsymbol{J}_{\text{eff}}$	$(\nabla^2 - \partial_t^2)\boldsymbol{E}_{\text{response}} = 0$
Field "sourced by" DM	Cavity response to cancel $\boldsymbol{E}_{\parallel}$ at boundary
Constant in space	(Slowly) varying with $k = m_{A'}$
$\boldsymbol{B}_{\text{DM}} = 0$	$\boldsymbol{B}_{\text{response}} \neq 0$

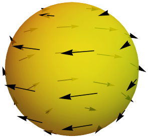
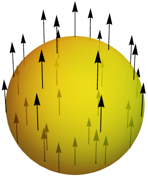
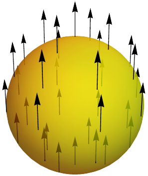
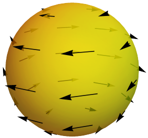
ADMX/DM Radio Solution



$$\boldsymbol{E} = \boldsymbol{E}_{\text{DM}} + \boldsymbol{E}_{\text{response}} \propto m_{A'}^2 (R^2 - r^2)$$

$$\boldsymbol{B} = -\frac{i}{m_{A'}} \nabla \times \boldsymbol{E} \propto m_{A'} r$$

Spherical Modes

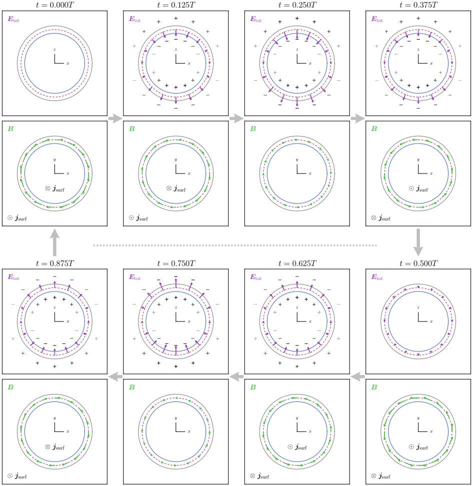
Transverse Electric (TE)	Transverse Magnetic (TM)
$\boldsymbol{E}_{\mathrm{TE}} \sim \boldsymbol{\Phi}_{\ell m}$ 	$\boldsymbol{E}_{\mathrm{TM}} \sim \boldsymbol{Y}_{\ell m} + \boldsymbol{\Psi}_{\ell m}$ 
$\boldsymbol{B}_{\mathrm{TE}} \sim \boldsymbol{Y}_{\ell m} + \boldsymbol{\Psi}_{\ell m}$ 	$\boldsymbol{B}_{\mathrm{TM}} \sim \boldsymbol{\Phi}_{\ell m}$ 

Full TM Modes

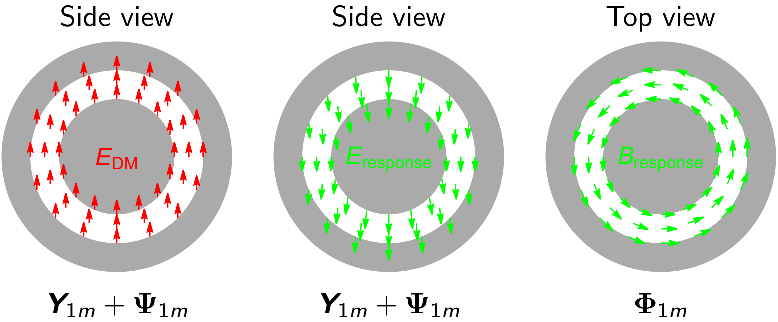
$$\boldsymbol{E}_{\mathrm{TM}} = \sum_{\ell m} \left(-\frac{\ell(\ell+1) g_{\ell m}(m_{A'} r)}{m_{A'} r} \boldsymbol{Y}_{\ell m} - \left(g'_{\ell m}(m_{A'} r) + \frac{g_{\ell m}(m_{A'} r)}{m_{A'} r} \right) \boldsymbol{\Psi}_{\ell m} \right) e^{-i m_{A'} t}$$

$$\boldsymbol{B}_{\mathrm{TM}} = -i \sum_{\ell m} g_{\ell m}(m_{A'} r) \boldsymbol{\Phi}_{\ell m} e^{-i m_{A'} t}$$

Earth Field Oscillations



Earth Signal



- B has particular $\boldsymbol{\Phi}_{1m}$ spatial pattern that we can search for!
  - If A' points in z-direction, then only $\boldsymbol{\Phi}_{10}$ mode
- Note that E_{response} (almost) cancels E_{DM} everywhere!
- E_{response} correct (to leading order) regardless of boundary conditions
- Can show $\boldsymbol{\Phi}_{1m}$ component of B correct (to LO) regardless of BCs!

Robustness to Boundary Conditions

- As long as geometry is sub-wavelength, our solution is the correct leading-order TM magnetic field, regardless of boundary conditions!

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NLO	✗	✗		
NNLO	?	?		
NNNLO	?	?		

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NLO	✗	✗	?	✓
NNLO	?	?	?	✗
NNNLO	?	?	?	?

- $\boldsymbol{B}_{\text{TM}}$ *higher* order than $\boldsymbol{E}_{\text{TM}}$, but $\boldsymbol{B}_{\text{TE}}$ *lower* order than $\boldsymbol{E}_{\text{TE}}$

- As long as our search projects onto Φ_{1m}, we can just look for $\boldsymbol{B}_{\text{TM}}$!

IGRF Model

- V_0 expansion described by $g_{\ell m}$ and $h_{\ell m}$ via

$$V_0 = \sum_{\ell=1}^{\infty} \sum_{m=0}^{\ell} \frac{R^{\ell+2}}{r^{\ell+1}} \left(g_{\ell m} \cos m\phi + h_{\ell m} \sin m\phi\right) P_\ell^m(\cos\theta),$$

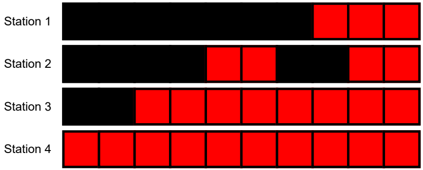
where Schmidt-normalized associated Legendre polynomials are

$$P_\ell^m(x) = \sqrt{(2-\delta_0^m)\frac{(\ell-m)!}{(\ell+m)!}} \left(1-x^2\right)^{m/2} \frac{d^m}{dx^m} P_\ell(x)$$

- $C_{\ell m}$ related to $g_{\ell m}$ and $h_{\ell m}$ by

$$C_{\ell m} = (-1)^m \sqrt{\frac{4\pi(2-\delta_0^m)}{2\ell+1}} \frac{g_{\ell m} - i h_{\ell m}}{2}$$

Time Series Construction



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- Weight by inverse noise and Φ_{1m} (different m's will be correlated)
- Do same for signal and just work with time series

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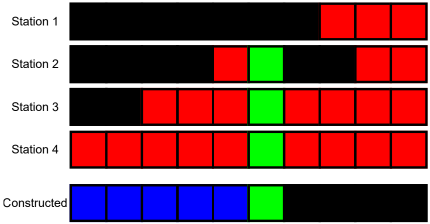

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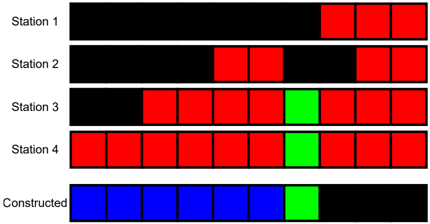

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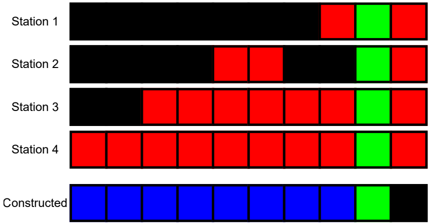

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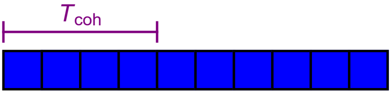
Time Series Partitioning



- Split time series into chunks of length T_{coh}
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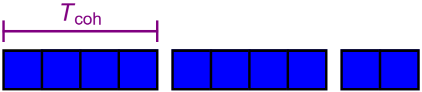

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Bayesian Analysis

- Analysis variables:

$$z_{ik} \sim \frac{\text{Data}}{\sqrt{\text{Noise}}} \qquad s_{ik} \sim \frac{\text{Signal}}{\sqrt{\text{Noise}}}$$

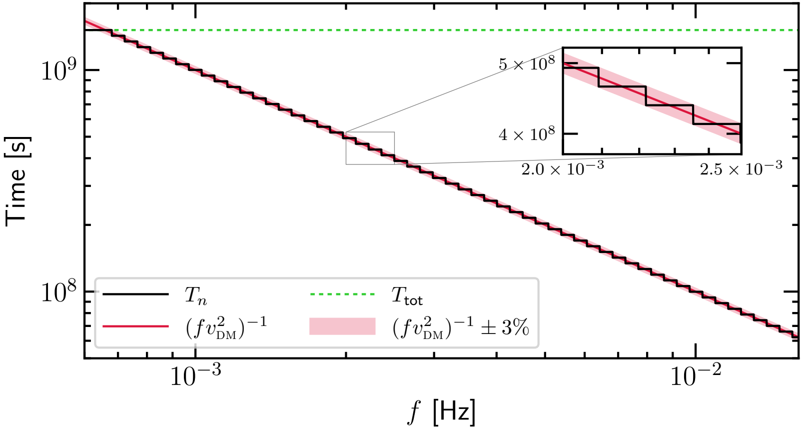
- Likelihood of data z_{ik} given coupling ε

$$\mathcal{L}(\{z_{ik}\}|\varepsilon) \propto \prod_{i,k} \frac{1}{3+\varepsilon^2 s_{ik}^2} \exp\left(-\frac{3|z_{ik}|^2}{3+\varepsilon^2 s_{ik}^2}\right)$$

- Definition of bound $\hat{\varepsilon}$ (using Jeffreys prior $p(\varepsilon)$)

$$\int_0^{\hat{\varepsilon}} d\varepsilon \; \mathcal{L}(\{z_{ik}\}|\varepsilon) \cdot p(\varepsilon) = 0.95$$

Coherence Time Approximation



Candidate Rejection

- Identified 30 DPDM signal candidates and 27 axion signal candidates by comparing $\sum_{i,k} |z_{ik}|^2$ to χ^2-distribution
- Tested candidates with resampling analysis
  - Reran analysis with 4 subsets of time and saw if z_{ik} consist with signal
  - Also with 4 subsets of stations
- All candidates in tension with tests $\longrightarrow$ no robust DM candidates