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A Quantum-Mechanical Mechanism for Reducing the Cosmological Constant

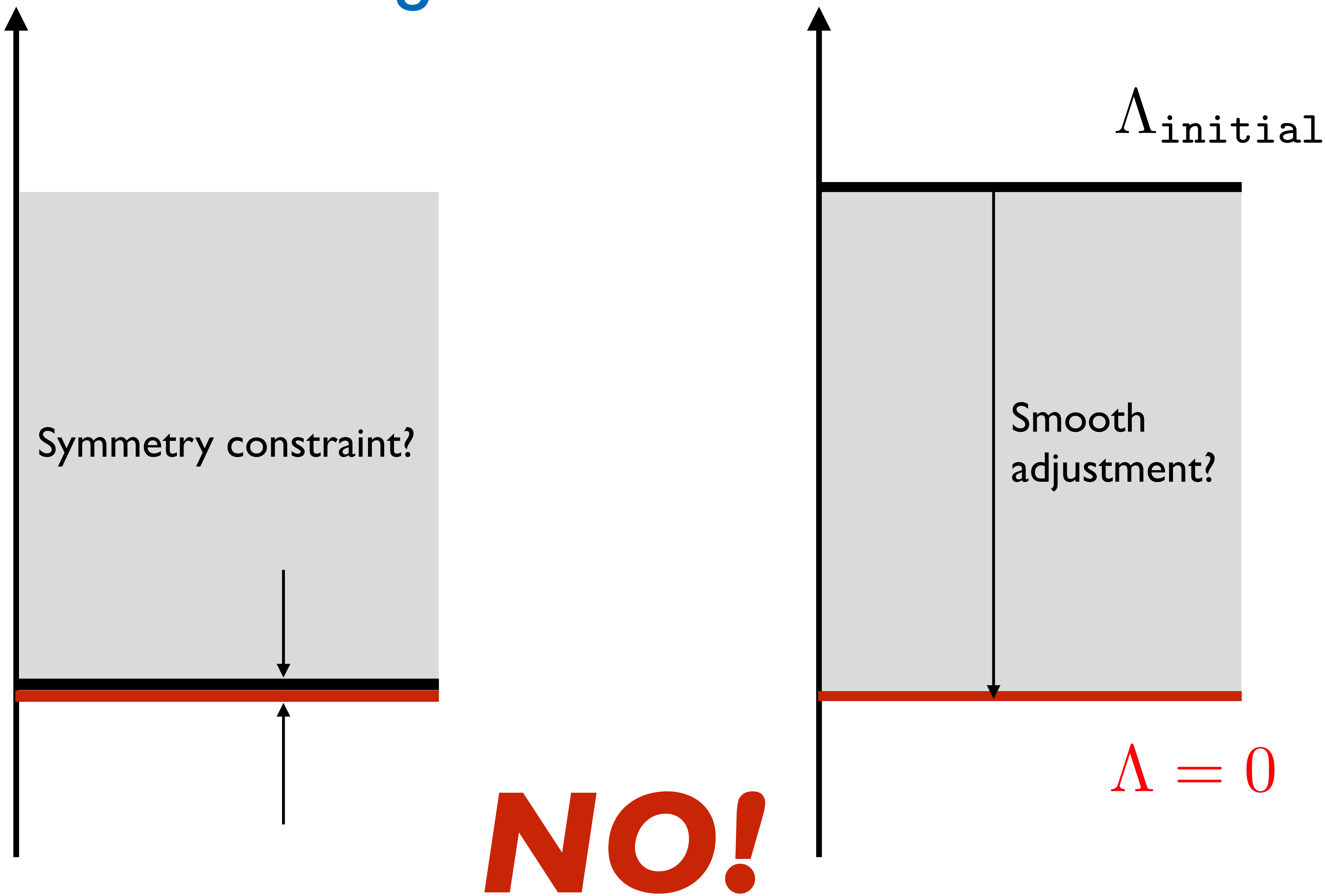
arXiv:2202.06977, arXiv:2202.08860, arXiv:2204.13124

PGW Workshop, MITP Mainz, August 2022 last paper with A. Westphal

Before we start...

- What does it take to 'solve' the cosmological constant problem???
- (by this I mean, to set it to zero; let's ignore this recent nuisance of current cosmic acceleration)

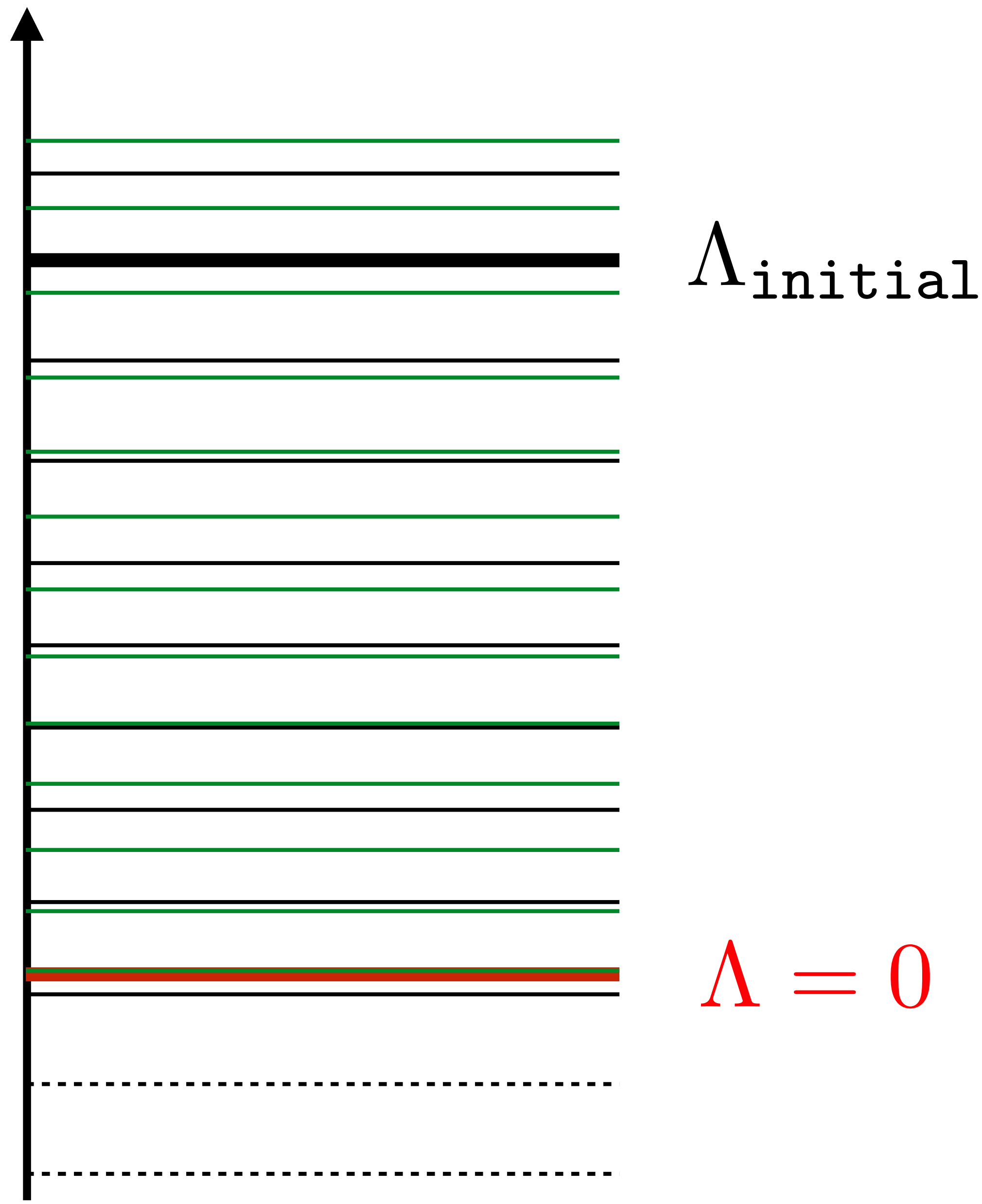
Cosmological Constant Problem



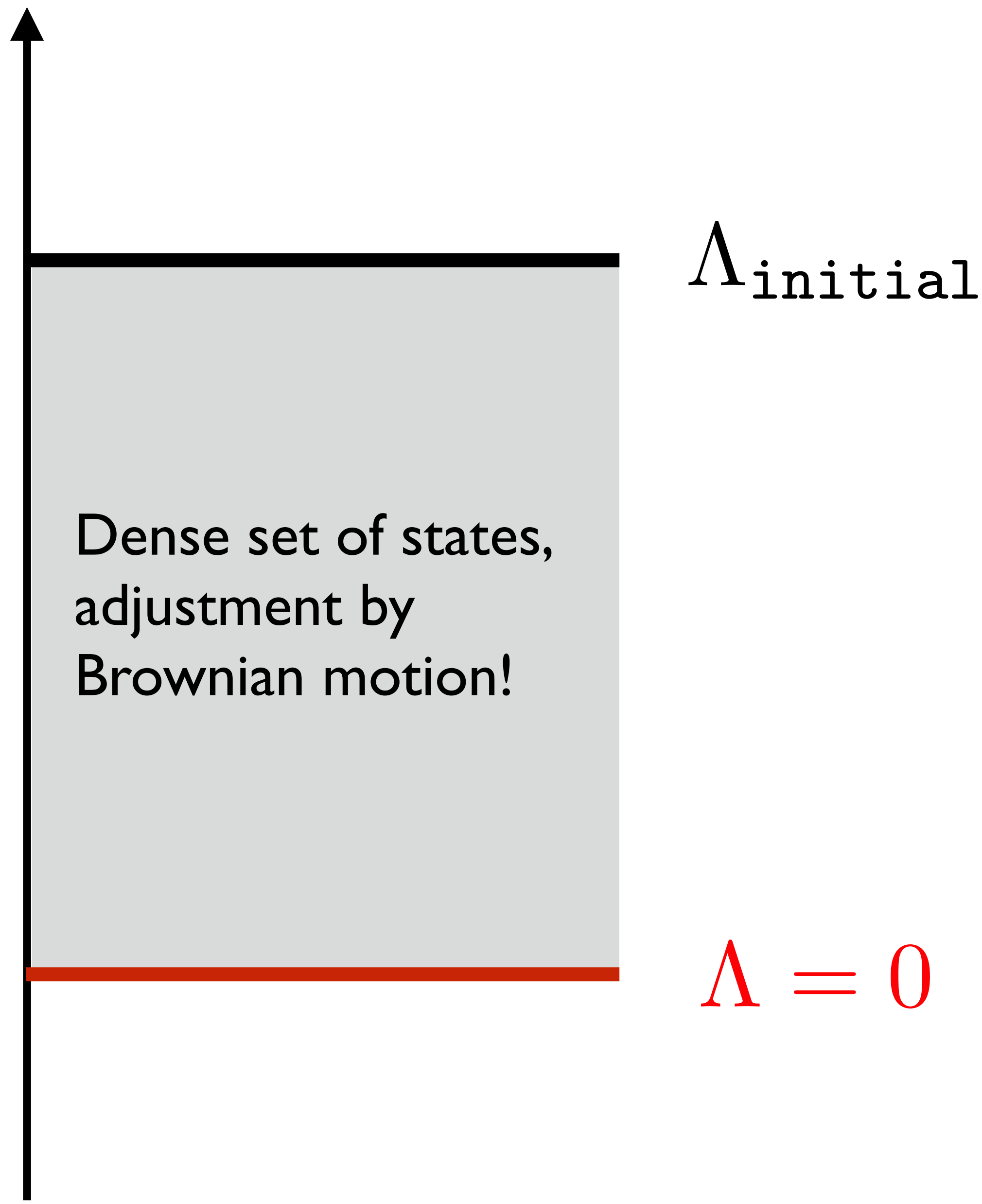
Stairway in Heaven

CC is unstable, it decays toward the attractor where dynamics stops.

As long as the last gap is wide enough we can fit the "real universe" inside it, all 60ish efolds of inflation, reheating, BBN, etc etc



Stairway in Heaven



Wisdom After the Fact…

- Cosmological constant problem is not a UV nor an IR problem; it is an ALL SCALES PROBLEM
- ANY hard particle at any mass threshold contributes m^4: quarks, tau, mu, e, neutrinos… and whatever else is in the universe - and none of it (except maybe neutrino's) gravitates
- Actually Pauli already knew this in '20s… we seem to have forgotten it Rugh & Zinkernagel; tale of Matvei Bronstein
- If we do not want to violate locality and causality (read: mess with QFT in a big bad way)… that leaves one option
- DECAY!

Hiding in Plain View…

- Gravity has latent discrete degrees of freedom which persist in the far IR
- \`Teasing' them out uncovers new symmetries; interpreting those as gauge symmetries requires introducing charges; fluxes sourced by those charges and their discharge realize a very simple, GR-only example of Landscape
- Let's see how this simple Landscape relaxes the cosmological constant to zero without deploying anthropics

Hidden Degrees of Freedom of GR

- Usual textbook statement: the measure in GR is unique.
- NOT TRUE!

$$\int d^4x \sqrt{g}\, \frac{M_{pl}^2}{2}\, R \to \int \mathcal{F}\, R \qquad\qquad \mathcal{F} = d\mathcal{A}$$

- This measure is perfectly valid: but this is BD-like: $\frac{\mathcal{F}}{\sqrt{g} d^4x} = \Phi$
- Project out the local fluctuations; add $-\frac{1}{4!}\int \mathcal{F}\,\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}\mathcal{G}_{\mu\nu\lambda\sigma}$ $\quad \mathcal{G} = d\mathcal{B}$
- Couple QFT minimally, and consider

$$S = \int \mathcal{F}\Big(R - \frac{1}{4!}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}\mathcal{G}_{\mu\nu\lambda\sigma}\Big) - \int d^4x \sqrt{g}\, \mathcal{L}_{\texttt{QFT}}$$

- Locally, this is JUST GR!!!

arXiv:2202.06977, 2202.08860

Proof

- Vary the action:

$$-\frac{2}{4!}\frac{\epsilon^{\rho\zeta\gamma\delta}}{\sqrt{g}}\mathcal{F}_{\rho\zeta\gamma\delta}\Big(R^{\mu}{}_{\nu}-\frac{\epsilon^{\alpha\beta\lambda\sigma}\mathcal{G}_{\alpha\beta\lambda\sigma}}{2\cdot 4!\sqrt{g}}\delta^{\mu}{}_{\nu}\Big)=T^{\mu}{}_{\nu}\,,$$

$$R-\frac{1}{4!}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}\mathcal{G}_{\mu\nu\lambda\sigma}=2\lambda\,,\qquad -\frac{1}{4!}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}\mathcal{F}_{\mu\nu\lambda\sigma}=\frac{\kappa^2}{2}\,.$$

- Here λ, κ^2 are integration constants due to the (rigid) gauge symmetries of $\mathcal{F} = d\mathcal{A}, \mathcal{G} = d\mathcal{B}$

- Now manipulate the eqs a bit: substitute bottom 2 into the top

$$\kappa^2\Big(R^{\mu}{}_{\nu}-\frac{1}{2}R\,\delta^{\mu}{}_{\nu}\Big)=-\kappa^2\lambda\,\delta^{\mu}{}_{\nu}+T^{\mu}{}_{\nu}$$

$$R-\frac{1}{4!}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}\mathcal{G}_{\mu\nu\lambda\sigma}=2\lambda\,,\qquad -\frac{1}{4!}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}\mathcal{F}_{\mu\nu\lambda\sigma}=\frac{\kappa^2}{2}\,.$$

- Locally this is just GR!!! BUT: a fascinating new thing happens
- Since λ, κ^2 are integration constants this is infinitely many GRs!

A Proto-Landscape

- Since λ, κ^2 are completely arbitrary - they are fluxes of 4-forms - the meta-theory has infinitely many versions of GR which behave as superselection sectors as long as the QFT parameters are fixed. These sectors, for now, do not mix.

- However, if there is a QFT phase transition, which changes QFT vacuum energy, then the superselection sectors can mix - transitioning into one another.

- To generalize the theory promote rigid to local gauge symmetries: add charges sourcing 4-forms; those are membranes.

$$
\begin{aligned}
S \;=\; & \int \mathcal{F}\Big(R - \frac{1}{4!}\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{g}}\mathcal{G}_{\mu\nu\lambda\sigma}\Big) - \int d^4x \sqrt{g}\,\mathcal{L}_{\mathtt{QFT}} + S_{\mathtt{boundary}} \\
& - \mathcal{T}_A \int d^3\xi \sqrt{\gamma}_A - \mathcal{Q}_A \int \mathcal{A} - \mathcal{T}_B \int d^3\xi \sqrt{\gamma}_B - \mathcal{Q}_B \int \mathcal{B}\,.
\end{aligned}
$$

- A landscape of couplings - which is spanned by fluxes

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Dual Description

- Replace 4-forms by their magnetic duals - analogous to E&M, $\vec{E} \to \vec{B}, \vec{B} \to -\vec{E}$
- The trick: use 1st order path integral & integrate out 4-forms

$$Z = \int \ldots [\mathcal{D}\mathcal{A}][\mathcal{D}\mathcal{B}][\mathcal{D}\mathcal{F}][\mathcal{D}\mathcal{G}][\mathcal{D}\mathcal{P}_A][\mathcal{D}\mathcal{P}_B]\, e^{iS(\mathcal{A},\mathcal{B},\mathcal{F},\mathcal{G},\ldots)+i\int \mathcal{P}_A(\mathcal{F}-d\mathcal{A})+i\int \mathcal{P}_B(\mathcal{G}-d\mathcal{B})}$$

- Answer: *arXiv:2202.06977, 2202.08860*

$$S = \int d^4x \Big\{ \sqrt{g}\Big(\frac{\kappa^2}{2}R - \kappa^2\lambda - \mathcal{L}_{\texttt{QFT}}\Big) - \frac{\lambda}{3}\epsilon^{\mu\nu\lambda\sigma}\partial_\mu \mathcal{A}_{\nu\lambda\sigma} - \frac{\kappa^2}{12}\epsilon^{\mu\nu\lambda\sigma}\partial_\mu \mathcal{B}_{\nu\lambda\sigma}\Big\}$$
$$+ \; S_{\texttt{boundary}} - \mathcal{T}_A \int d^3\xi \sqrt{\gamma}_A - \mathcal{Q}_A \int \mathcal{A} - \mathcal{T}_B \int d^3\xi \sqrt{\gamma}_B - \mathcal{Q}_B \int \mathcal{B}\,.$$

- We could have started with this action - it is technically simpler; note the topological sector

"stiff flux monodromy": arXiv:0811.1989

- For simplicity here I will `freeze' the Planck scale by decoupling B-membranes

QM of Fluxes and Membranes

- The idea: Euclideanize the action and consider semiclassical discharge processes by solving equations with membrane sources and cosmological constant alone, taking the bulk geometry to be locally maximally symmetric

- Construct instantons which change the geometry as sourced by membranes and compute the bounce actions which control instability rates

- Will find that $\lambda, \xcancel{\kappa}^2$ are not constant but change discretely, controlled by membrane charges and tensions

- Will focus on the differences relative to Brown-Teitelboim (BP).

- The bottomline: dS is unstable. It decays to Minkowski. (Almost) Flat space is the accumulation point - i.e. quantum attractor.

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Euclidean Field Eqs

- Bulk:

$$ds_E^2 = dr^2 + a^2(r)\, d\Omega_3 \qquad\qquad 3\kappa_{\mathtt{eff}}^2\Big(\big(\frac{a'}{a}\big)^2 - \frac{1}{a^2}\Big) = -\kappa^2\lambda = -\Lambda$$

- Membrane junction conditions:

$$\lambda_{out} - \lambda_{in} = \frac{1}{2}\mathcal{Q}_A \qquad\qquad \kappa_{out}^2 - \kappa_{in}^2 = \cancel{2\mathcal{Q}_B}$$

$$a_{out} = a_{in} \qquad\qquad \kappa_{\mathtt{eff}\ out}^2\frac{a'_{out}}{a} - \kappa_{\mathtt{eff}\ in}^2\frac{a'_{in}}{a} = -\frac{1}{2}\Big(\mathcal{T}_A + \cancel{\mathcal{T}_B}\Big)$$

- 3-form boundary terms all cancel out

- Bulk solutions: sections of (horo)spheres, we glue them together

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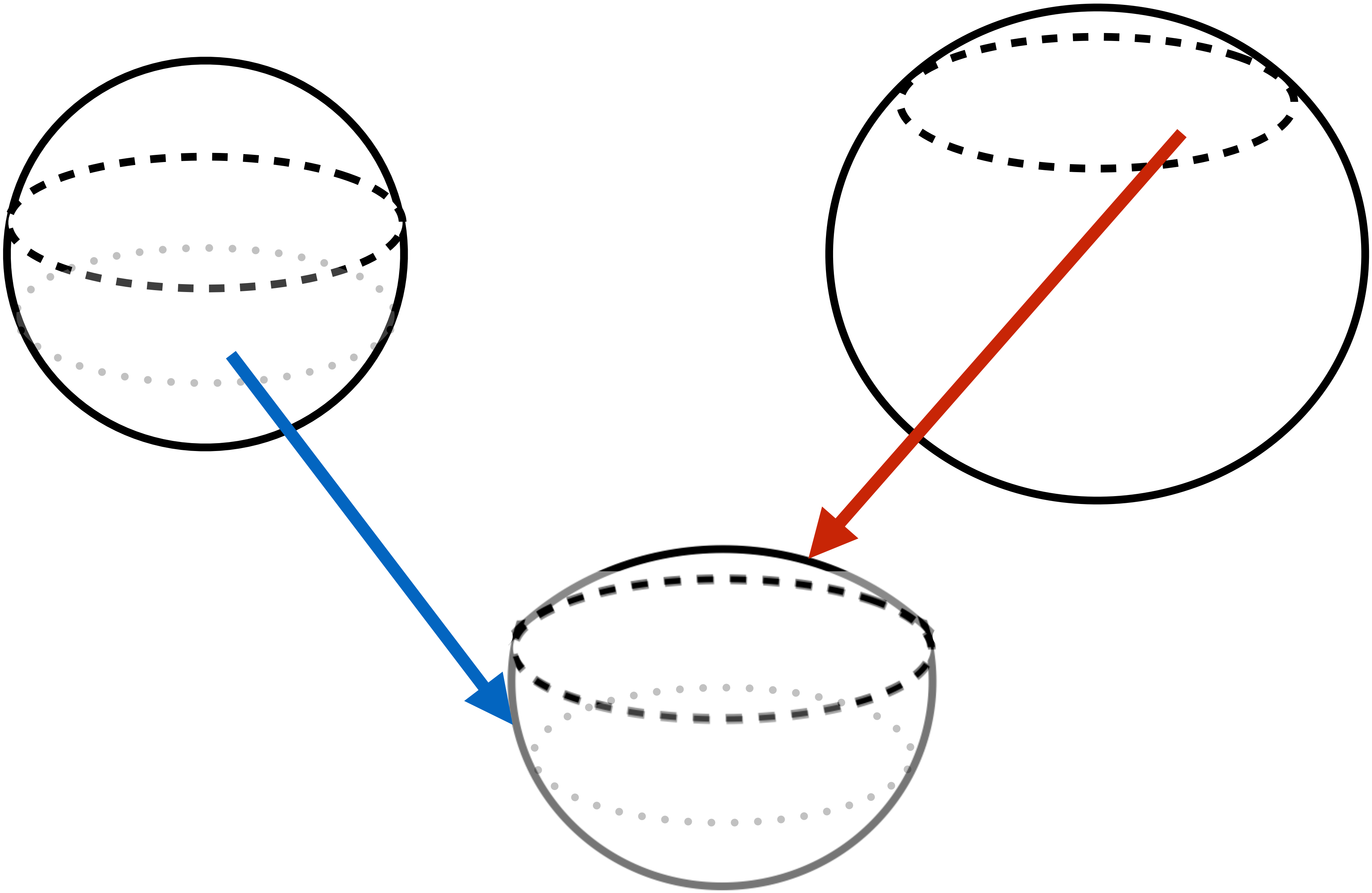
de Sitter Instanton Surgery



$\mathcal{T}_A, \mathcal{Q}_A \neq 0$

- Bulk sections:

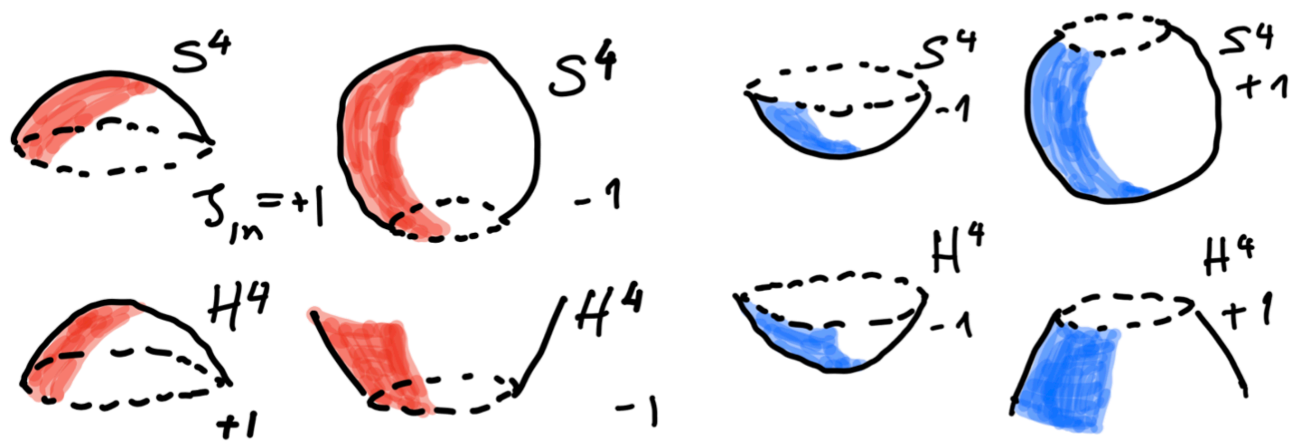


Figure 1: Spherical (S^4, top row) and horospherical (a.k.a. hyperbolic; H^4, bottom row) sections which are glued together to form instantons. Red ones are the interiors and the blue ones the exterior geometries of the instanton. The $\pm$ are the values of $\zeta_{in/out}$.

- Junction conditions: massaging the eqs, can rewrite them as

$$\zeta_{out}\sqrt{1-\frac{\Lambda_{out}a^2}{3\kappa_{\tt eff}^2}} = -\frac{\mathcal{T}_A}{4\kappa_{\tt eff}^2}\left(1-\frac{2\kappa_{\tt eff}^4\mathcal{Q}_A}{3\mathcal{T}_A^2}\right)a\,,$$

$$\zeta_{in}\sqrt{1-\frac{\Lambda_{in}a^2}{3\kappa_{\tt eff}^2}} = \frac{\mathcal{T}_A}{4\kappa_{\tt eff}^2}\left(1+\frac{2\kappa_{\tt eff}^4\mathcal{Q}_A}{3\mathcal{T}_A^2}\right)a\,.$$

A Crucial Feature of Junction Conditions

- The junction conditions are controlled by

$$\left(1 \mp \frac{2\kappa_{\tt eff}^4 \mathcal{Q}_A}{3\mathcal{T}_A^2}\right)$$

- This differs from BT (BP) in a crucial way: in BT the junction conditions depend on charge QUADRATICALLY - one power of charge and one power of background flux
- IN OUR CASE NOT SO - the instanton junction conditions do not care about the background value of the flux
- They only care about

$$\frac{2\kappa_{\tt eff}^4 \mathcal{Q}_A}{3\mathcal{T}_A^2} = q > 1 \quad or \quad < 1$$

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Instanton `Baedeker'

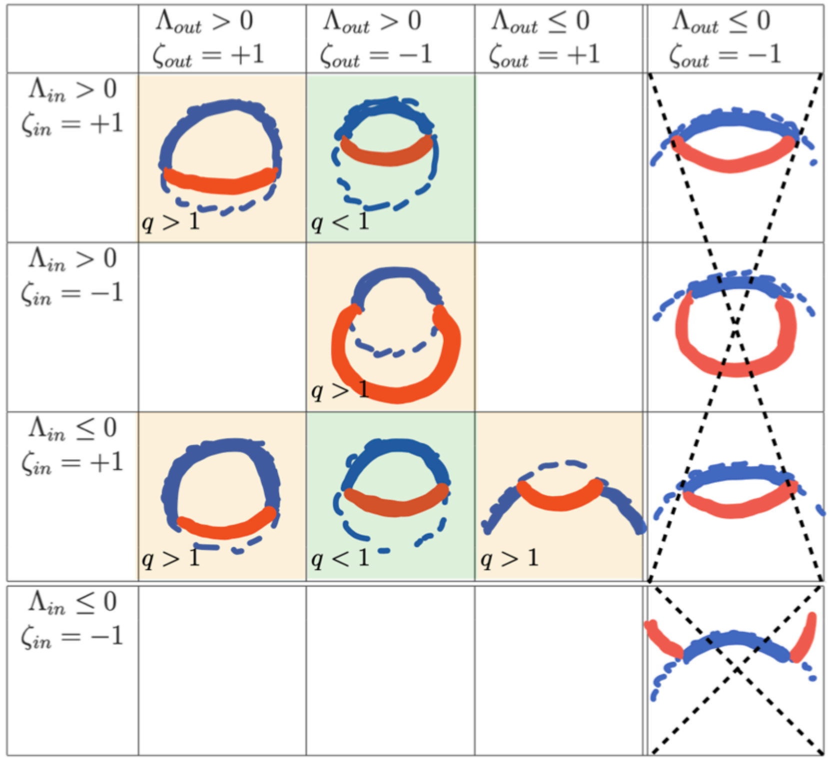
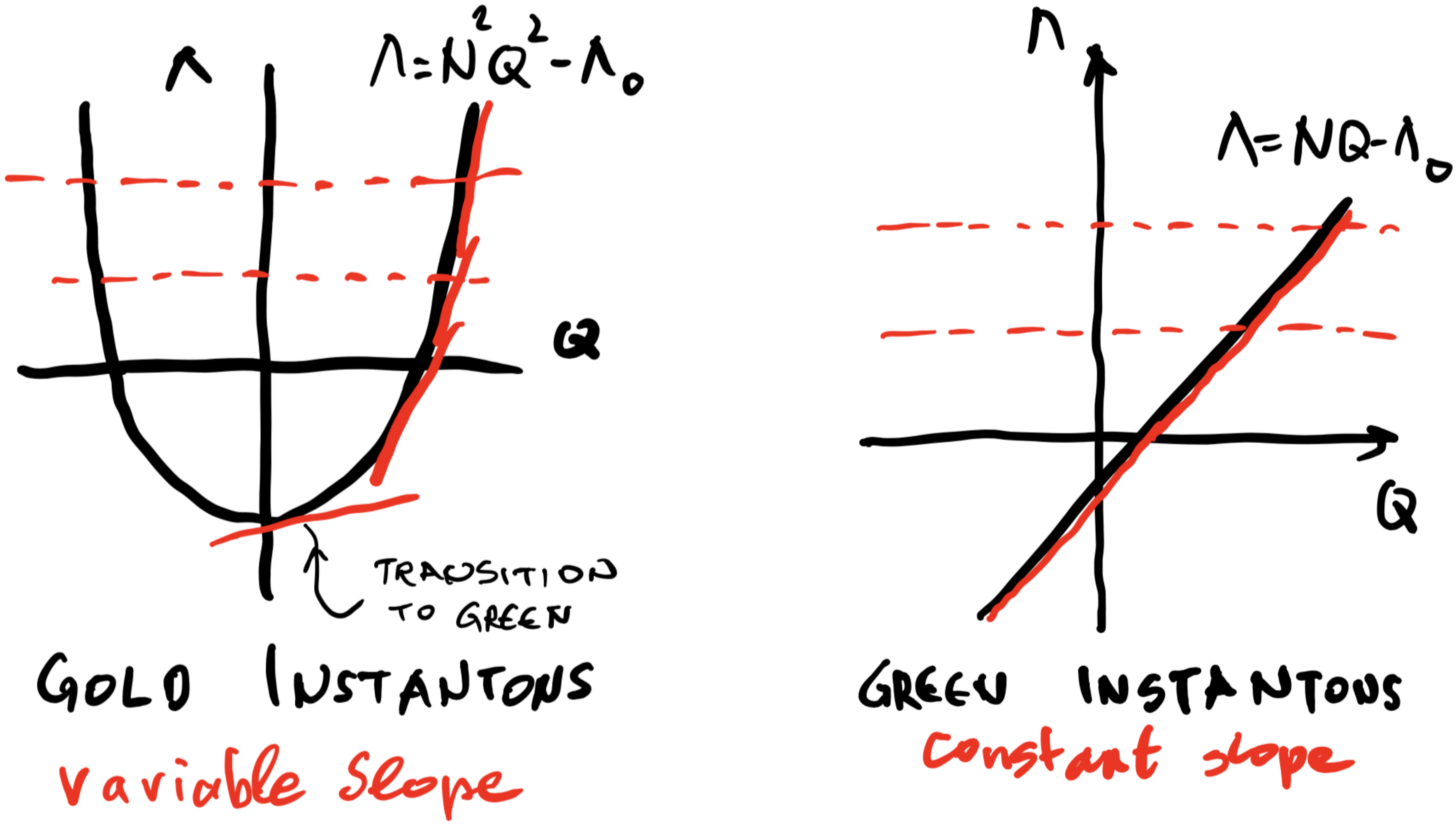


Figure 2: The instanton 'Baedeker'. The instantons fall into four types, divided by double lines in the table, and counted clockwise from the top corner [44]. The transitions corresponding to empty squares are ruled out kinematically by Eqs. (49), (50). The top nine are further split by $q = \frac{2\kappa_{\tt eff}^4 \mathcal{Q}_A}{3\mathcal{T}_A^2} < 1$ (pale green) or $q > 1$ (pale gold). We keep both since $\kappa_{\tt eff}^2$ *might* vary independently (we will suppress those variations later on). The "ogre"-like configurations in the right column which are crossed out are allowed kinematically, but are suppressed dynamically since their bounce action is huge and positive, $S_{\tt bounce} \gg 1$, diverging when Anti-de Sitter sections are non-compact (see the text).

Why is this happening???



Bounce Action and Decay Rate

- The rate and bounce action are defined by

$$\Gamma \sim e^{-S(\mathtt{bounce})} \qquad S(\mathtt{bounce}) = S(\mathtt{instanton}) - S(\mathtt{parent})$$

- The bounce action evaluated on the instanton is

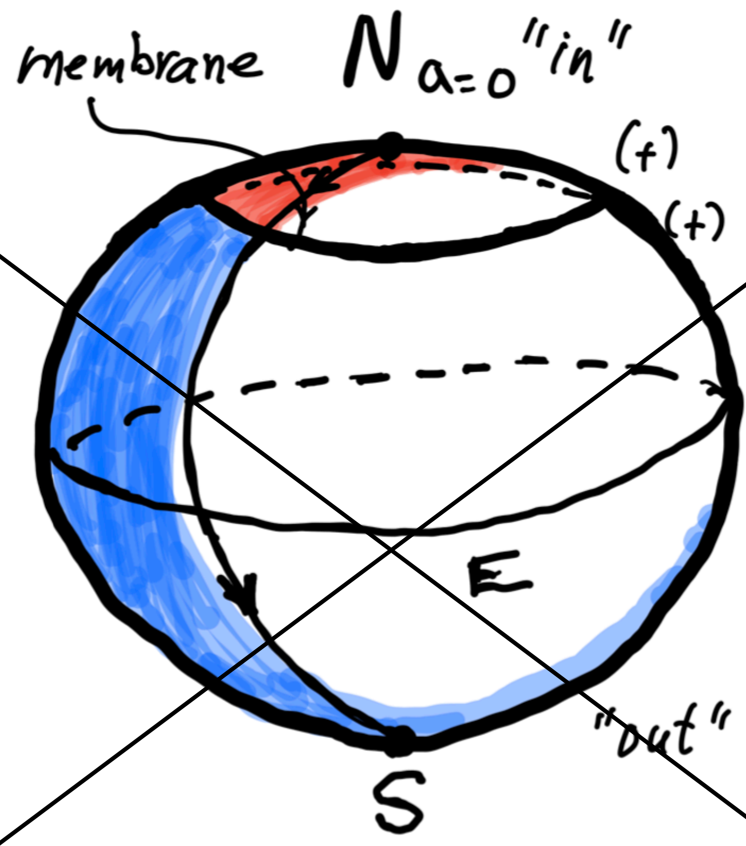
$$S(\mathtt{bounce}) = 2\pi^2 \Big\{ \Lambda_{out} \int_{North\ \ Pole}^{a} da \Big(\frac{a^3}{a'}\Big)_{out} - \Lambda_{in} \int_{North\ \ Pole}^{a} da \Big(\frac{a^3}{a'}\Big)_{in} \Big\} - \pi^2 a^3 \mathcal{T}_A$$

$$2\pi^2 \Lambda_{in/out} \int_{North\ \ Pole}^{a} da \Big(\frac{a^3}{a'}\Big) = 18\pi^2 \frac{\kappa_{\mathtt{eff}}^4}{\Lambda_{in/out}} \Big(\frac{2}{3} - \zeta_{in/out} \big(1 - \frac{\Lambda_{in/out} a^2}{3\kappa_{\mathtt{eff}}^2}\big)^{1/2} + \frac{\zeta_{in/out}}{3} \big(1 - \frac{\Lambda_{in/out} a^2}{3\kappa_{\mathtt{eff}}^2}\big)^{3/2} \Big)$$

- (After quite a bit of algebraic tedium)
- Using these formulas we can calculate the rate for any instanton from the `Baedeker'; in some cases it diverges (the corresponding entries are crossed out).
- These formulas are identical to Brown-Teitelboim (Coleman etc too), except that the differences in junction conditions produce different final answers.

An Example of a 'Gold' Instanton

$$\frac{2\kappa_{\texttt{eff}}^2 \kappa^2 \mathcal{Q}_A}{3\mathcal{T}_A^2} = q > 1$$

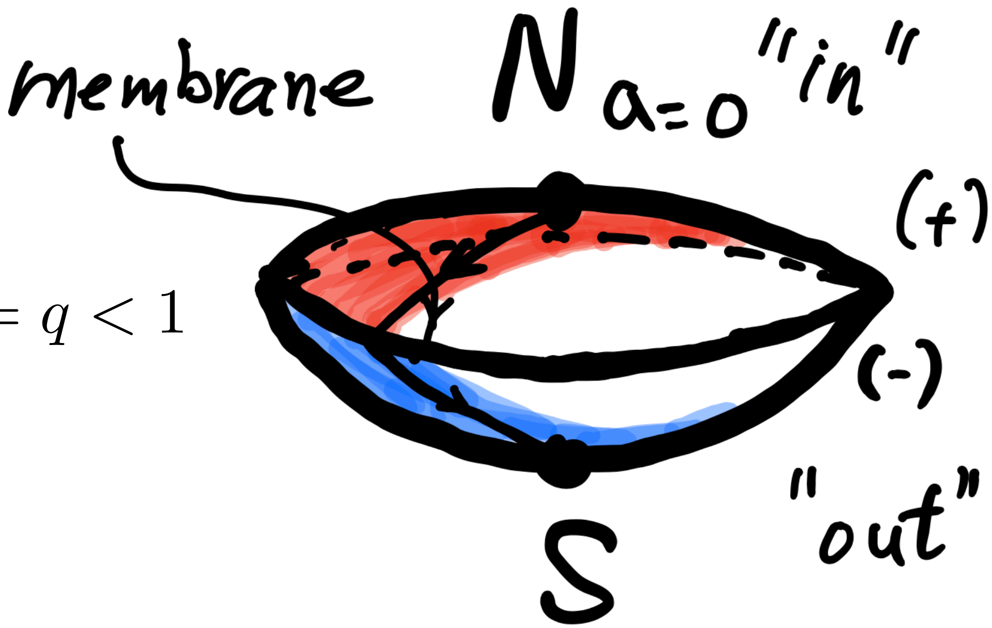


$$S_{\texttt{bounce}} \simeq \frac{27\pi^2}{2}\frac{\mathcal{T}_A^4}{(\Delta\Lambda)^3} \simeq 108\pi^2 \frac{\mathcal{T}_A^4}{\kappa^6 \mathcal{Q}_A^3} \qquad for \quad q > 1$$

This instanton mediates processes that lead to the decay rate which does not depend on the background cosmological constant and so it can overshoot Minkowski space. This instanton is unavoidable in the quadratic flux models like Brown-Teitelboim (BP).

An Example of a `Green' Instanton

$$\frac{2\kappa_{\texttt{eff}}^2 \kappa^2 \mathcal{Q}_A}{3\mathcal{T}_A^2} = q < 1$$



This instanton mediates processes where discharge rate DOES depend on the background cosmological constant. This selects Minkowski as the accumulation point of the evolution and makes (almost) Minkowski (quasi)stable.

$$S_{\texttt{bounce}} \simeq \frac{24\pi^2 \kappa_{\texttt{eff}}^4}{\Lambda_{out}} \left(1 - \frac{8}{3}\frac{\kappa_{\texttt{eff}}^2 \Lambda_{out}}{\mathcal{T}_A^2}\right) \quad for \quad q < 1$$

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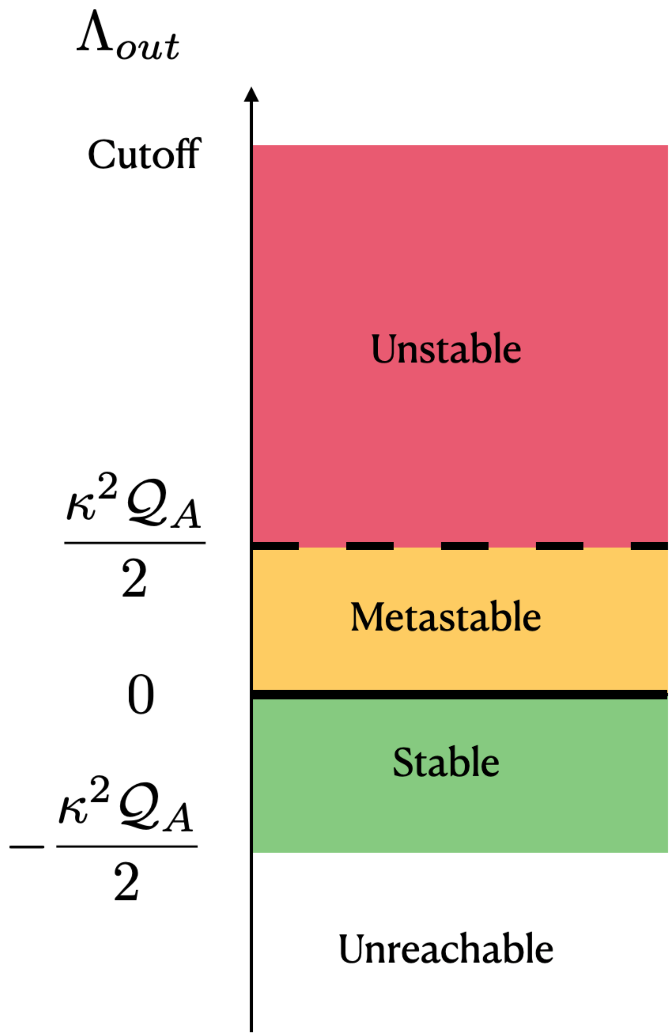
The Green Spectrum



Cosmological Constant: No Problem!

- Define the problem first

$$\Lambda_{\mathtt{total}} = \kappa^2_{\mathtt{eff}} \Big(\frac{\mathcal{M}^4_{\mathtt{UV}}}{\mathcal{M}^2} + \frac{V}{\mathcal{M}^2} + \lambda \Big) \, , \qquad \lambda = \lambda_0 + N \frac{\mathcal{Q}_A}{2} \, ,$$

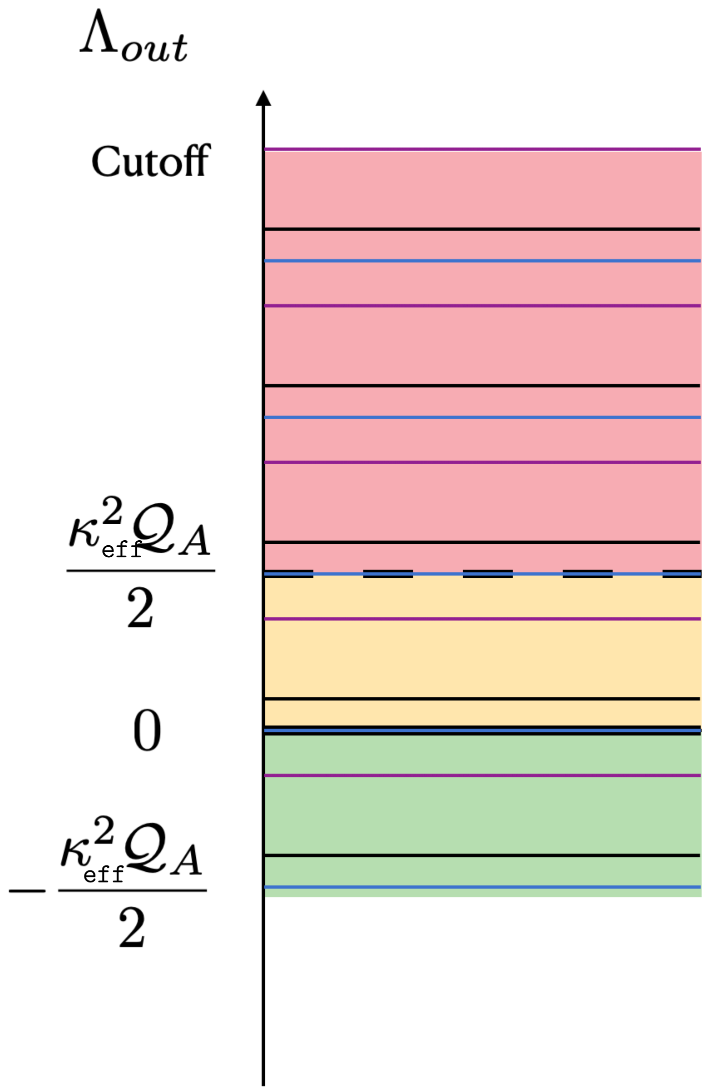
- So:

$$\Lambda_{\mathtt{total}} = \kappa^2_{\mathtt{eff}} \Big(\frac{\Lambda_0}{\mathcal{M}^2} + N \frac{\mathcal{Q}_A}{2} \Big) \, ,$$

- Thus the CC is unstable - BUT - to make it arbitrarily small eventually we must either take a tiny membrane charge or fine tune initial value

- This is the problem.

The Superselection Sectors in the Spectrum



Each color is a set of levels for a fixed superselection sector; they do not mix.

The Resolution: Add One More Charge

$$\mathcal{S} = S - \int d^4x\sqrt{g}\left(\kappa_{\mathtt{eff}}^2\hat{\lambda} + \frac{\hat{\lambda}}{3}\epsilon^{\mu\nu\lambda\sigma}\partial_\mu\hat{\mathcal{A}}_{\nu\lambda\sigma}\right) - \mathcal{T}_{\hat{A}}\int d^3\xi\sqrt{\gamma}_{\hat{A}} - \mathcal{Q}_{\hat{A}}\int\hat{\mathcal{A}}$$

Banks, Dine, Seiberg, 1991

$$\frac{\mathcal{Q}_{\hat{A}}}{\mathcal{Q}_A} = \omega \in \textbf{Irrational Numbers}$$

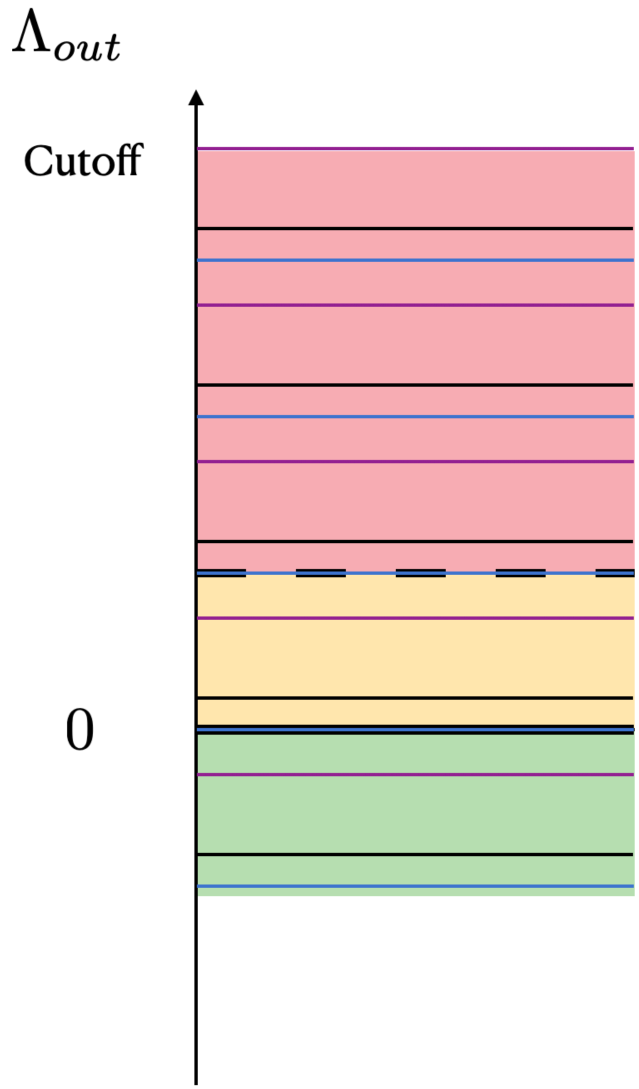
Other ways too!

- As a result: $\Lambda_{\mathtt{total}} = \kappa_{\mathtt{eff}}^2\left(\frac{\Lambda_0}{\mathcal{M}^2} + \frac{\mathcal{Q}_A}{2}(N + \hat{N}\omega)\right)$.
- Here, $N, \hat{N}$ are any pair of integers; since the ratio of charges is irrational, $N, \hat{N}$ exist such that CC is arbitrarily close to zero!
- The idea is this is achieved by a long sequence of membrane nucleations/ discharges where cc changes discretely from one to another, mediated by the `green' instantons, and continuing as long as CC is nonzero
- As CC approaches zero the nucleation rate becomes tiny since

$$S_{\mathtt{bounce}} \simeq \frac{24\pi^2\kappa_{\mathtt{eff}}^4}{\Lambda_{out}} \to \infty \quad \Rightarrow \quad \Gamma \to 0$$

arXiv:2204.13124

Fine Structure of the Spectrum



Now all the superselection sectors mix together because there are two discharge channels and the CC=0 is the accumulation point since it is the only stable state in the spectrum

Approximate Density of Occupied States

- The evolution by discrete emissions realizes the density of states of the cosmological constant advocated by Hawking and Baum in 1984,

$$Z = \int e^{-S_E} \simeq e^{-S_{classical}} = \begin{cases} e^{24\pi^2 \frac{\kappa_{\mathtt{eff}}^4}{\Lambda}} = e^{\frac{A_{\mathtt{horizon}}}{4G_N}}\,, & \Lambda > 0\,; \\ e^{\Lambda \int d^4x \sqrt{g}} = 1\,, & \Lambda = 0\,; \\ e^{-|\Lambda| \int d^4x \sqrt{g}} \to 0\,, & \Lambda < 0\,, \quad \text{noncompact}\,. \end{cases}$$

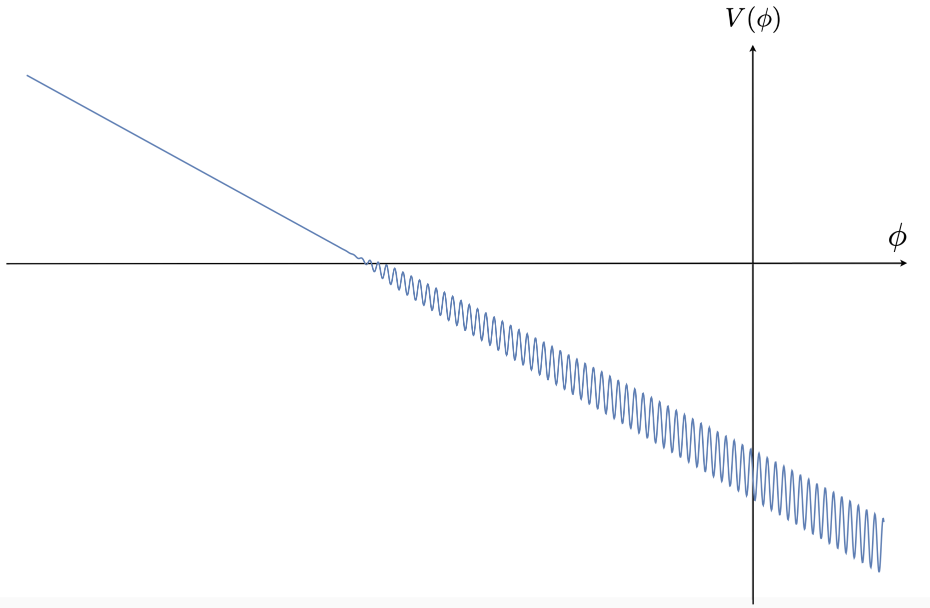
- This is in the leading order of the approximation which suffices here

- The conclusion is, that due to the irrational ratio of charges and the evolution controlled by `green instantons' since $q < 1$

$$\frac{\Lambda}{\kappa_{\mathtt{eff}}^4} \to 0 \qquad \text{without anthropics!!!}$$

Abbott?

- With wisdom after the fact, this reminds one of Abbott 1985



- Abbot relaxed CC using a linear potential, with small bumps near the terminal value required to stop overshooting.
- However since the bumps were negligible at large values of CC his evolution was completely classical - so the field always dominated the geometry and generated the empty universe problem!
- We evade this problem since evolution is quantum Brownian drift and the terminal value is the asymptotic attractor!!!

Inflation?

- Because the evolution is by discrete jumps and CC=0 is the "semi-classical attractor" it is possible to have the jumps finish before the last stage of inflation, like in BP. There may also be interruptions that could yield observational signatures.
- It can also happen that a universe `restarts' itself by up-jumps; eg evolution brings it close to zero CC, and then a jump to a large value occurs; the universe recycles itself. In classical limit, this requires NEC violations, but in QM it is perfectly reasonable

Dark Energy?

- In the framework with irrational charge ratio to have it be a CC we must fine tune since CC = 0 is the favored value. So what is it???
  - Transient quintessence?
  - A late stage phase transition?
  - The ratio of charges is actually a rational # but it is a fraction of two very large mutual primes - so a tiny value of CC exists but it is nonzero? (I kinda like this but can't quite calculate… (but, stay tuned))
  - Even tho CC not zero right now seems unlikely, maybe using a different measure (than Hawking's) it is more likely; eg some argument related to inflation?
  - … ?

Summary

- Properly understood, there is really infinitely many GRs and the one we use to describe the universe is an a posteriori fit. There is a single action for all of them (a.k.a. GRs span a landscape).
- Each specific choice of parameters looks like a superselection sector, but dynamics induced by charges (fundamental, or "emergent") mixes them up
- dS is unstable and decays to Minkowski - this is a good thing, since it can relax CC
- dS may be pretty long lived - a good thing too, inflation can work
- SM parameters may also be subject to such discrete variations, is there a connection?