# GW Signal of Inflation Triggered Phase transition

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Work in collaboration with Haipeng An, KunFeng Lyu, and Siyi Zhou, 2009.12381, 2201.05171

Probing NP with GW workshop. MITP. August 5, 2022

# Why inflation triggered phase transition?

# The excursion of the inflaton

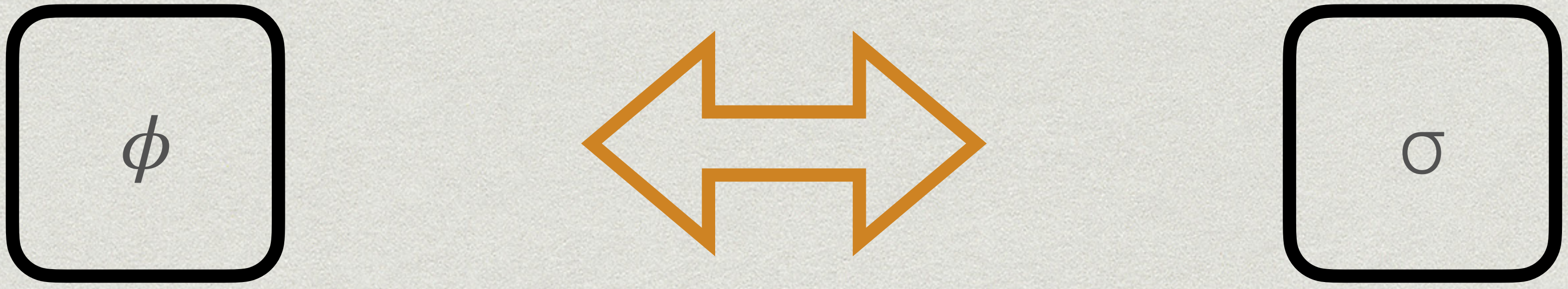
$$\Delta\phi \sim N_{\mathrm{efold}}\sqrt{\epsilon}M_{\mathrm{Planck}}$$

Large excursion of the inflaton field plausible, even if we restrict ourselves to the case where $\boldsymbol{\Delta\phi} < M_{\mathrm{Planck}}$

This is the case even for a small part of inflation with $N_{\mathrm{efold}} \approx O(1)$

Any physics/observable effect?

# Example: Inflaton + spectator




Inflaton sector
Single field slow roll
Approx. shift symmetry…

Spectator, less energy,
not driving spacetime evolution

Suppose the coupling is weak, suppressed by some high scale M, such as $M \approx M_{Planck}$

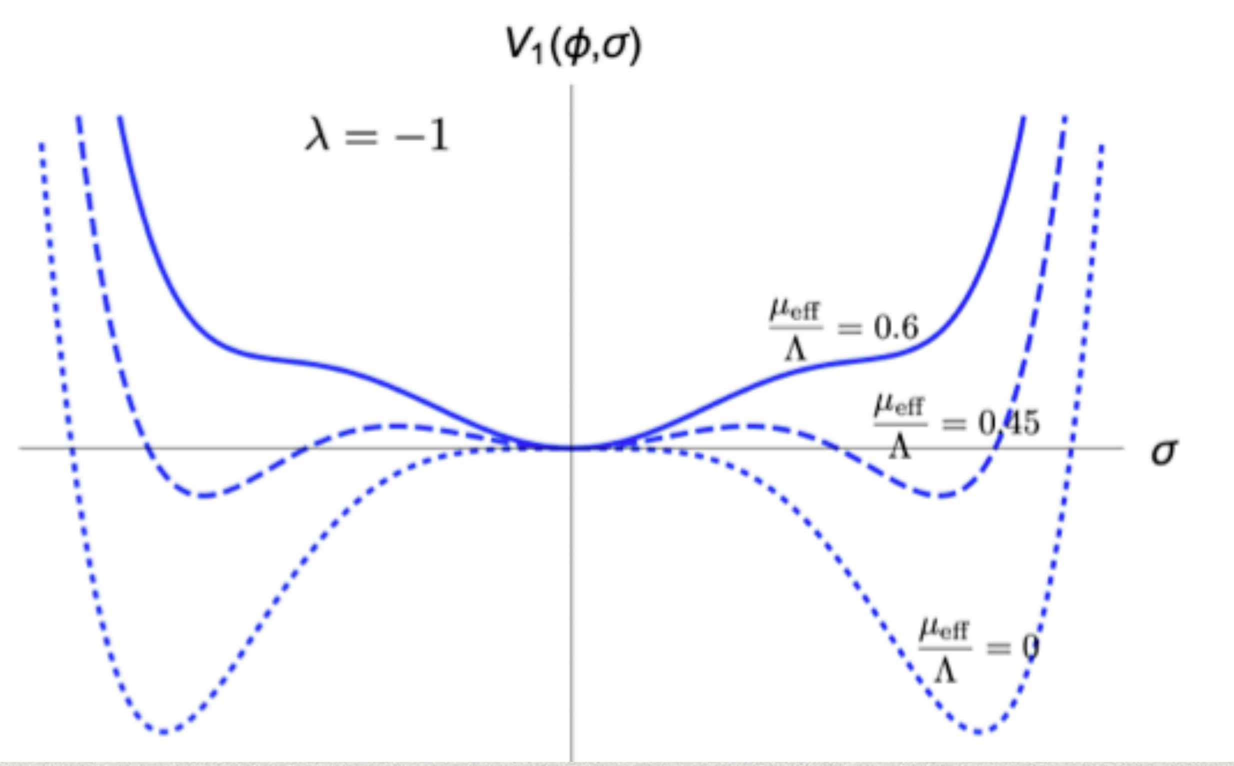
For example: $f\left(\frac{\phi}{M}\right) m_\sigma^2 \sigma^2, \quad g\left(\frac{\phi}{M}\right) \lambda \sigma^4,$ etc.

Field excursion of inflaton, $\boldsymbol{\Delta\phi} \sim M$, can change the mass and couplings in the spectator sector, leading to interesting dynamics.

# For example: 1st order PT

$$V(\phi,\sigma) = -\frac{1}{2}\mu_{\rm eff}^2\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6 + V_{\rm inf}(\phi), \quad \mu_{\rm eff}^2 = -\left(m_\sigma^2 - c^2\phi^2\right)$$

$$c^2 \sim \frac{m_\sigma^2}{M^2} \ll 1$$




Rolling inflaton → (1st order) phase transition in the spectator sector

# 1st order phase transition

Bubble nucleation rate: $$\frac{\Gamma}{V} \simeq m_\sigma^4 e^{-S_4}$$

$m_\sigma$: typical scale in the spectator sector

Efficient phase transition:

$$\int_{-\infty}^{t} dt' \frac{\Gamma}{V} \frac{1}{H^3} \simeq O(1) \;\rightarrow\; S_4 \sim \log\left(\frac{\phi H}{\dot{\phi}} \frac{m_\sigma^4}{H^4}\right) \sim \log\left(\frac{\phi}{\epsilon^{1/2} M_{\rm Pl}} \frac{m_\sigma^4}{H^4}\right)$$

Phase transition is 1st order ($S_4 \gg 1$).

Also, assume spectator sector does not dominate energy density.

$$H^4 \ll m_\sigma^4 \ll 3 M_{\rm Pl}^2 H^2$$

# 1st order phase transition

Phase transition completed with O(1) of Hubble volume in new phase (so that they will collide and produce GW)

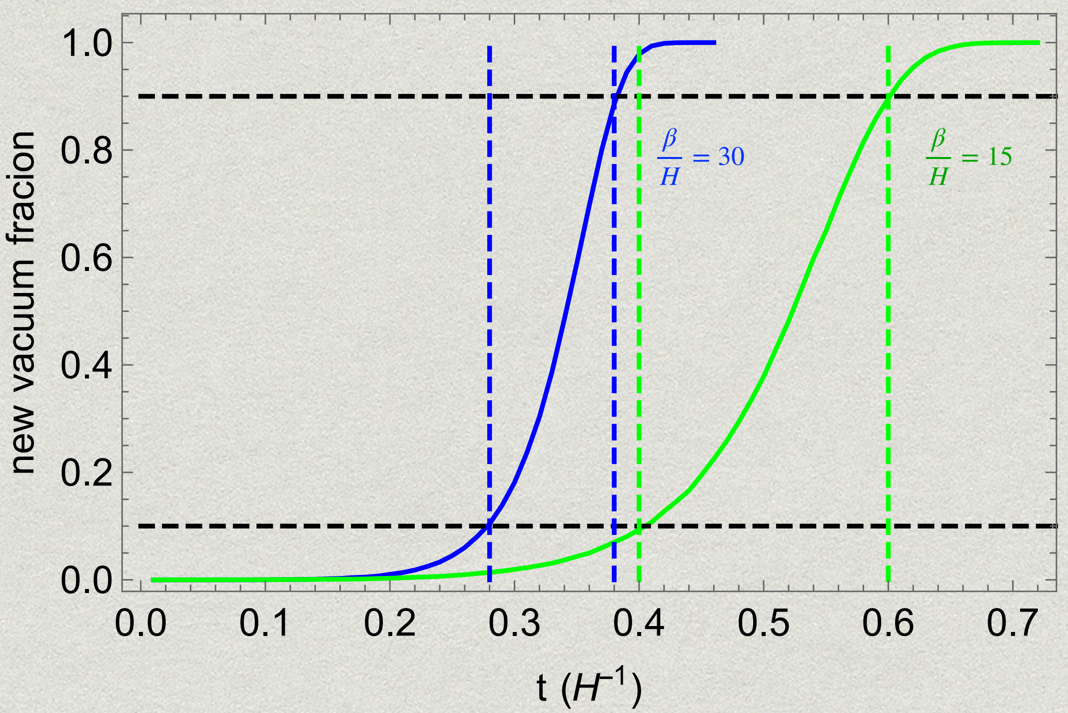
Guth and Weinberg, 83'

$$S_4(t) \simeq S(t_*) + \beta(t_* - t) + \ldots$$

$$r_{\text{bubble}}^{-1} \simeq \beta = \left| \frac{dS_4}{dt} \right|$$

$$\beta^4 \ll m_\sigma^4 \ll 3M_{\text{Pl}}^2 H^2$$

$$\beta \sim (10 - 100) \times H$$

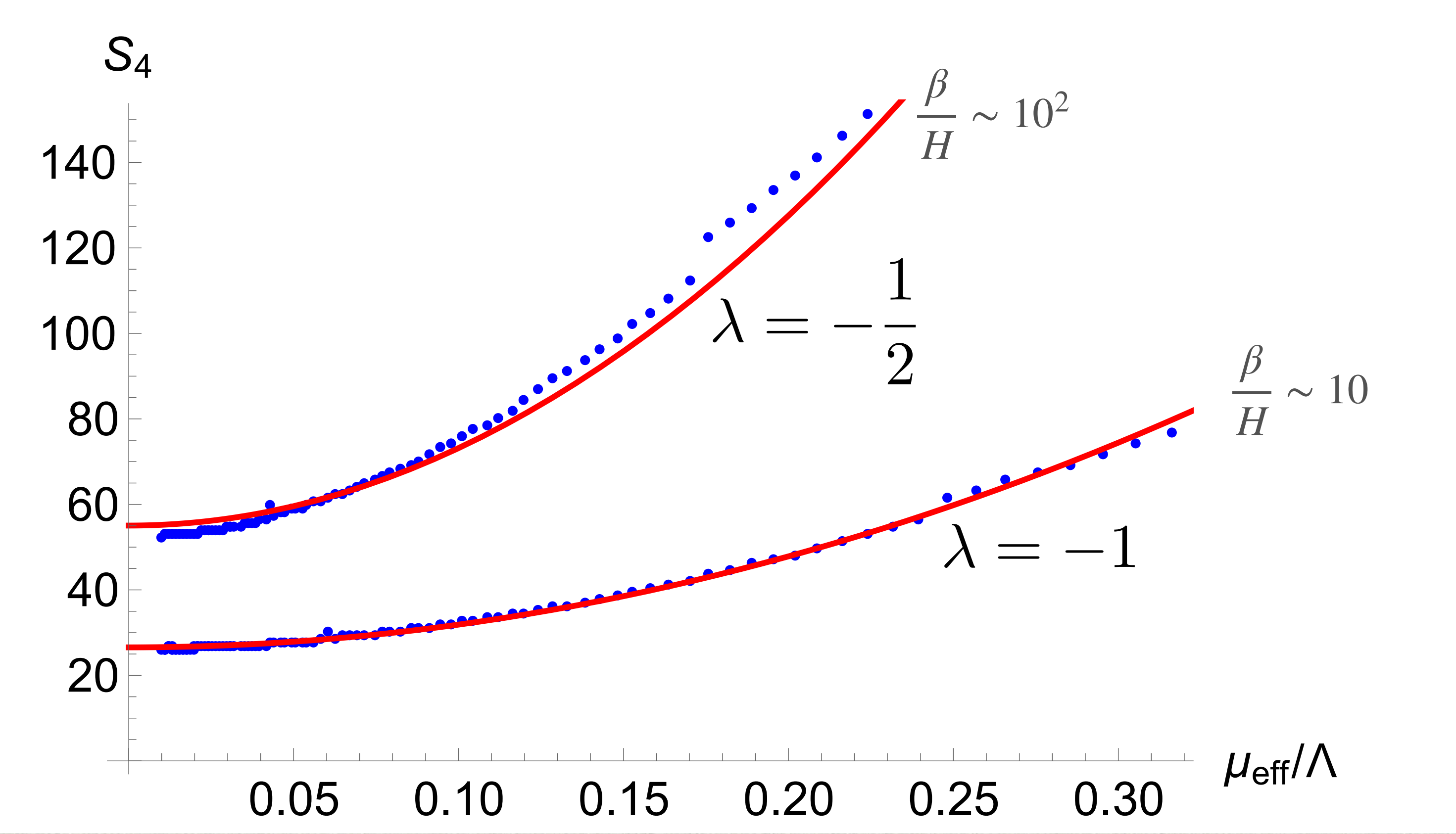



For GW signal in the opposite limit: J. Barir, M. Geller, C. Sun, T. Volansky, 2203.00693

# In our toy model:

$$\frac{\beta}{H} = \left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right|(2\epsilon)^{1/2}\times\frac{M_{\rm Pl}}{\phi\left(1-m_\sigma^2/(c^2\phi^2)\right)}$$

$$\left|\frac{dS_4}{d\log\mu_{\rm eff}^2}\right| \sim O(1)$$



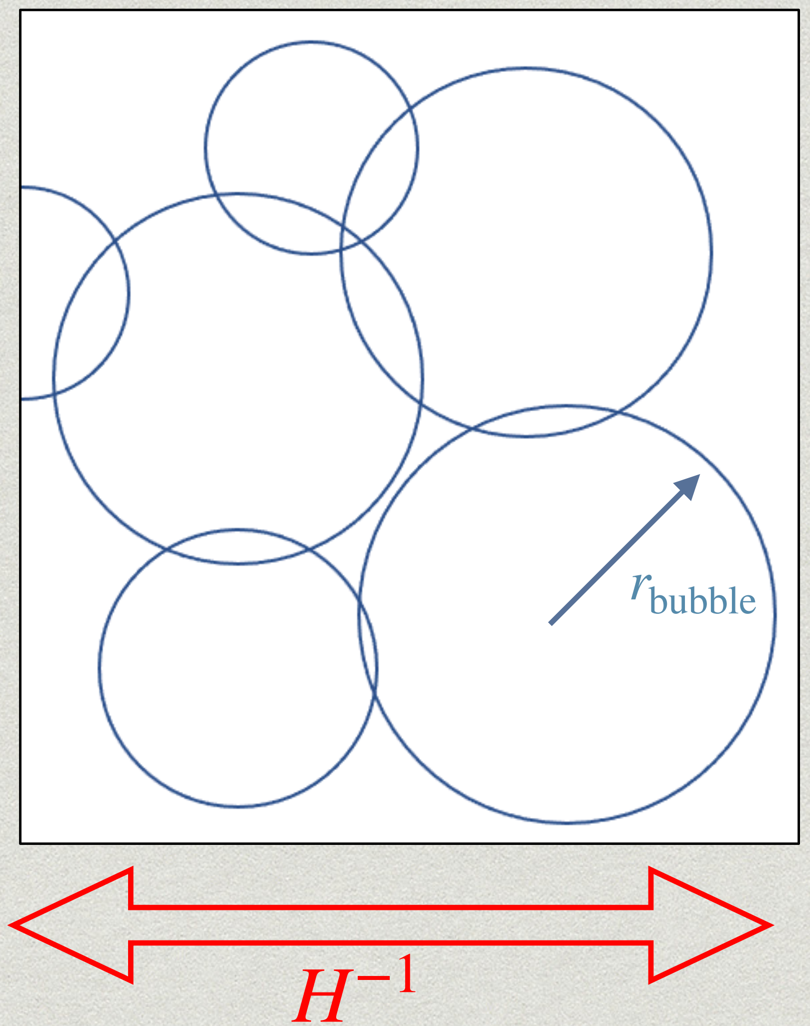

$$r_{\rm bubble}^{-1} \simeq \beta = \left|\frac{dS_4}{dt}\right|$$

CosmoTransitions

# 1st order phase transition

Phase transition is 1st order, and spectator sector does not dominate energy density:

$S_4(t) \simeq S(t_*) + \beta(t_* - t) + \dots$ $\qquad \beta^4 \ll m_\sigma^4 \ll 3M_{\rm Pl}^2 H^2$



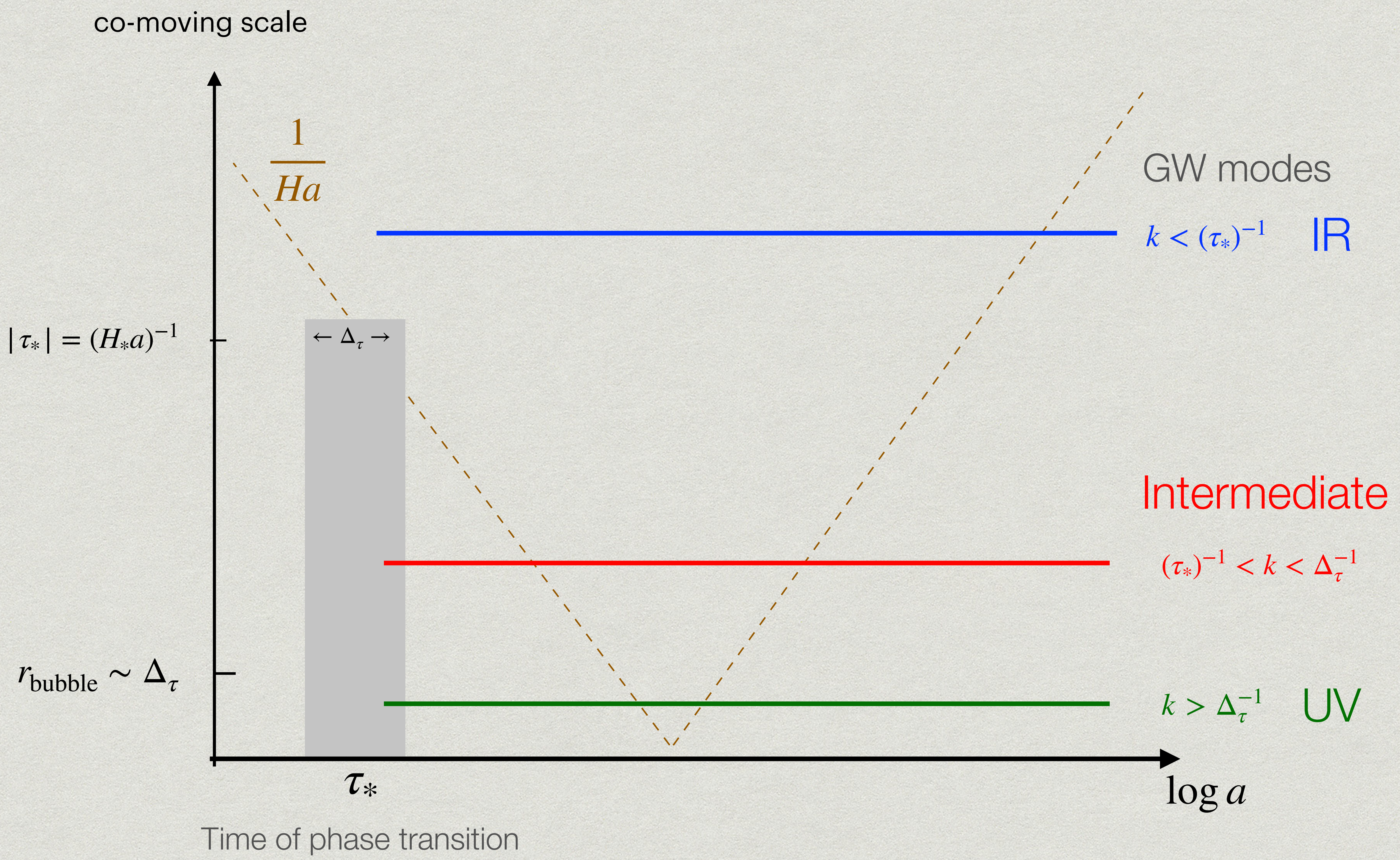

$$\beta^{-1} \sim r_{\rm bubble} \ll H^{-1}$$

$$t_{\rm bubble\ collision} \sim r_{\rm bubble} \ll H^{-1}$$

An instantaneous source of GW.

# Properties of GW signal

# GW in three regimes



# GW from instantaneous source

$$h'' + \frac{2a'}{a}h' + k^2 h = 16\pi G_{\rm N} a^3 T_{ij}$$

Instantaneous source: $T_{ij} \simeq T a^{-3}(\tau_*)\delta(\tau - \tau_*)$

Before the end of inflation:

$$h = 16\pi G_{\rm N}(-H\tau)\left[\frac{\sin k(\tau-\tau_*)}{k} + \left(\frac{1}{k^2\tau} - \frac{1}{k^2\tau_*}\right)\cos k(\tau-\tau_*) + \frac{1}{k^3\tau\tau_*}\sin k(\tau-\tau_*)\right]$$

Assume radiation domination after reheating (for now):

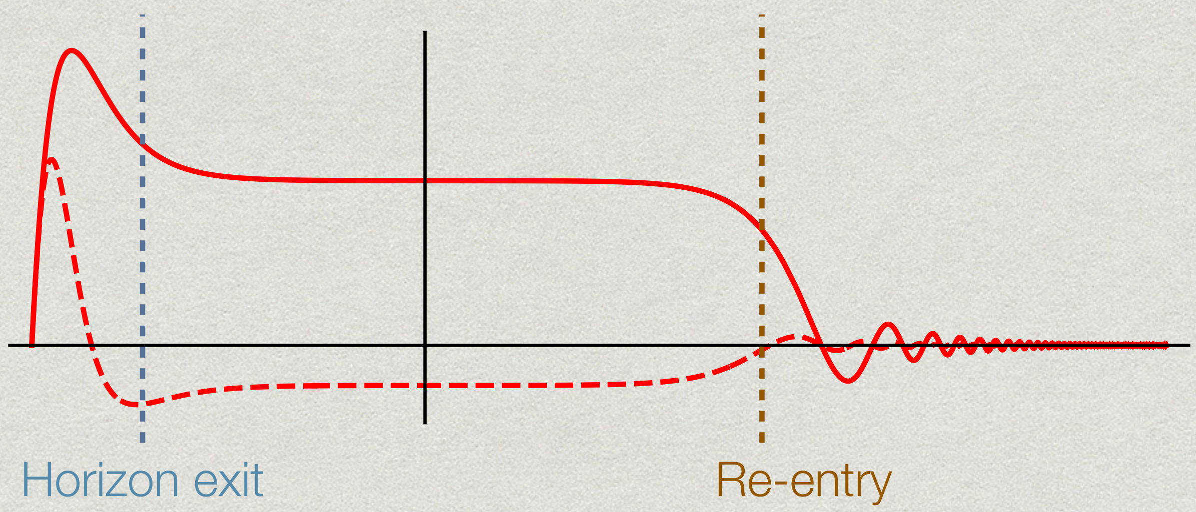
$$h \propto \frac{\sin k\tau}{k\tau}$$

# GW signal

$$h''_{ij} + \frac{2a'}{a} h'_{ij} - \nabla^2 h_{ij} = 16\pi G_N a^2 \sigma_{ij}$$

Intermediate

$\tau_*^{-1} < k < \Delta_\tau^{-1}$




During inflation:

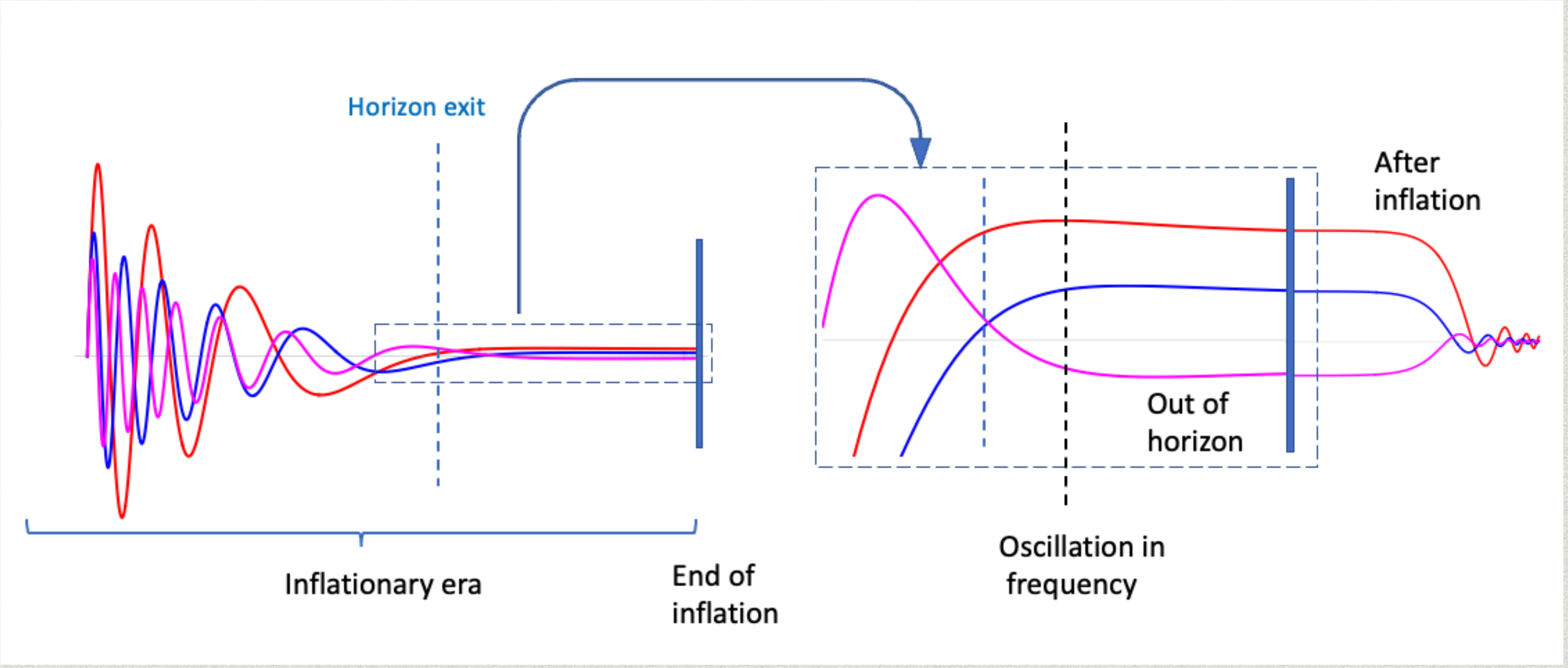
Mode starts inside horizon, oscillates till horizon exit.

➡ Amplitude depends on k.

➡ Leads to oscillatory pattern in frequency. $h \propto \frac{\cos(k\tau_*)}{k^2}$

# Oscillations

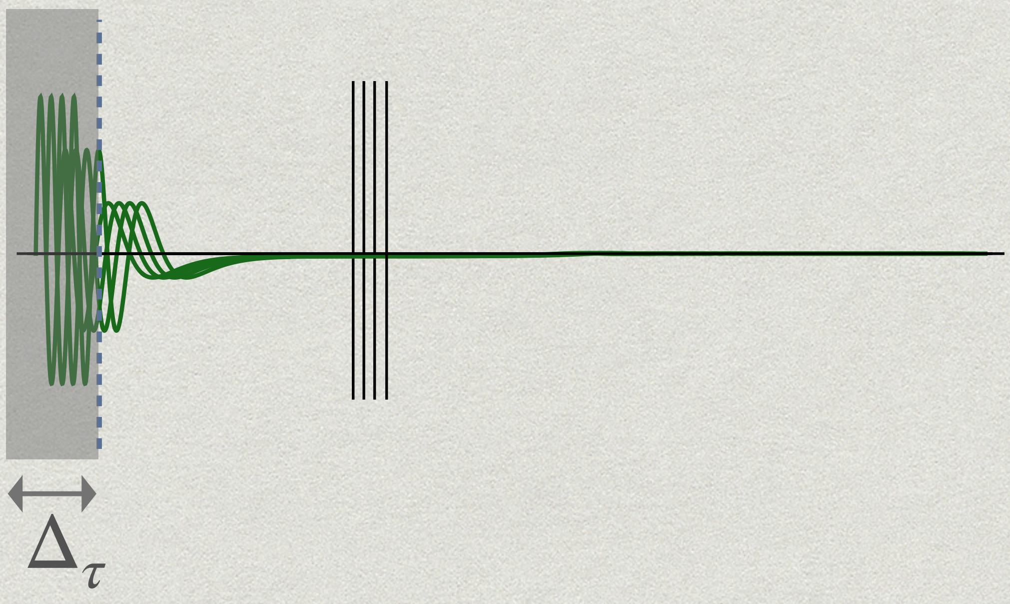
$\tau_*^{-1} < k < \Delta_\tau^{-1}$



# GW signal

$$h''_{ij} + \frac{2a'}{a}h'_{ij} - \nabla^2 h_{ij} = 16\pi G_N a^2 \sigma_{ij}$$

UV: $k > \Delta_\tau^{-1}$




Time scale of bubble collision $\approx \Delta_\tau$ .

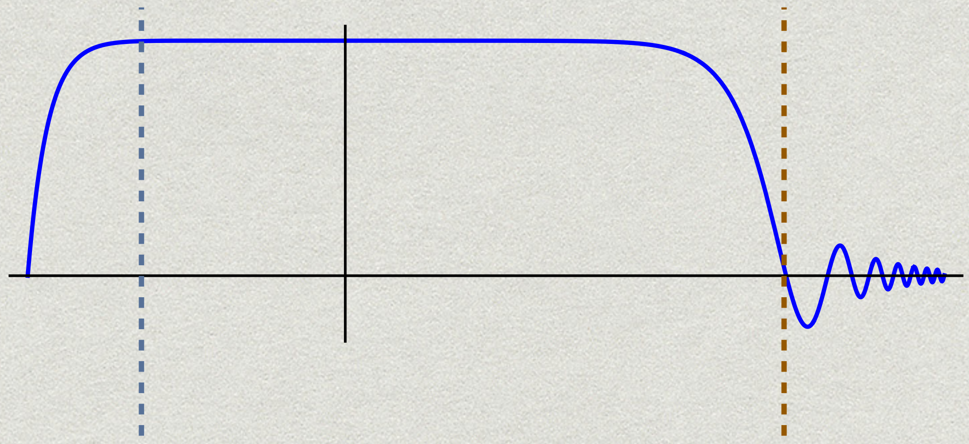
Oscillation pattern in frequency smeared out in this regime.

Spectrum depends on details of the source.

# GW signal

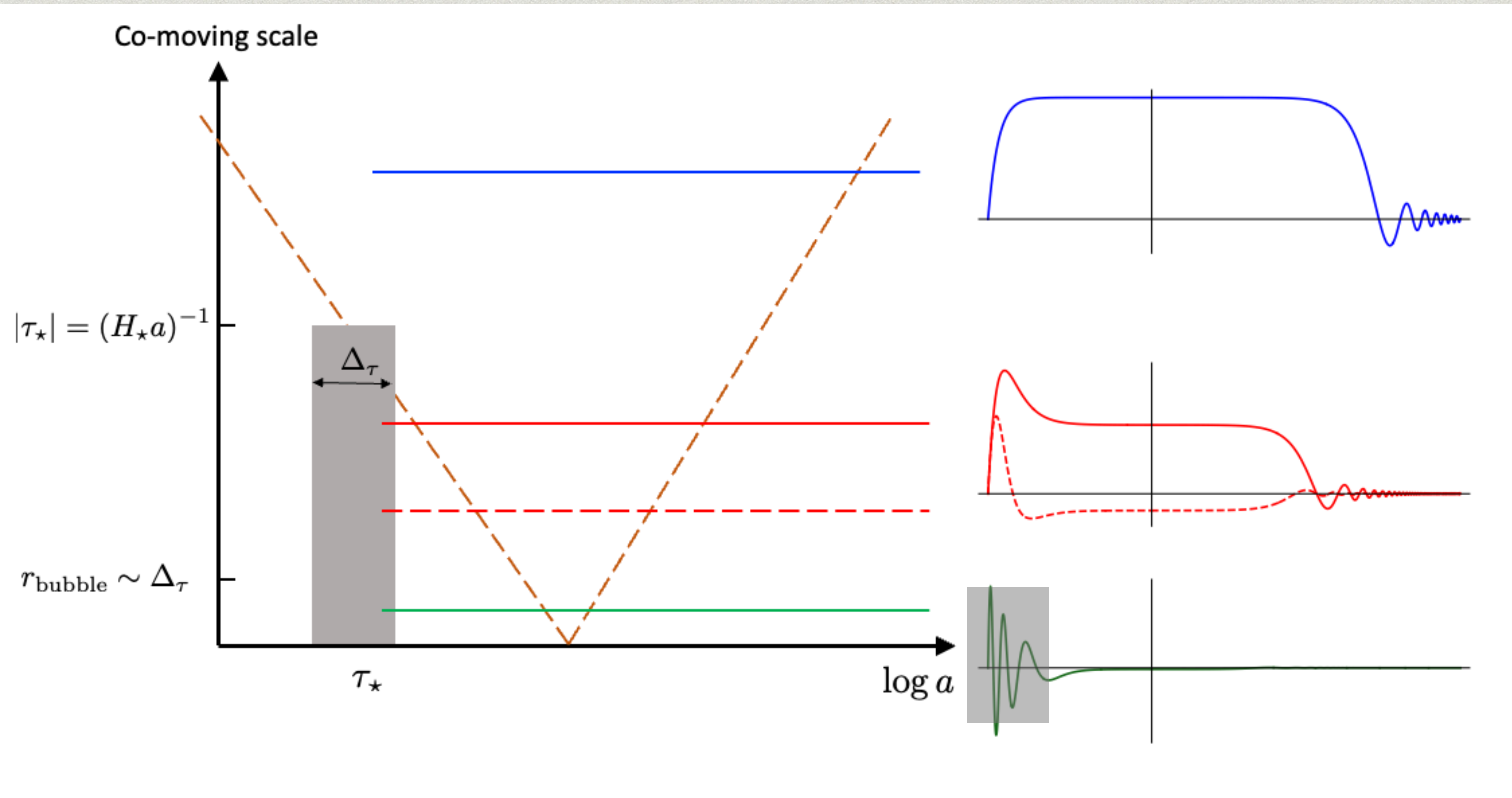
$$h''_{ij} + \frac{2a'}{a}h'_{ij} - \nabla^2 h_{ij} = 16\pi G_N a^2 \sigma_{ij}$$

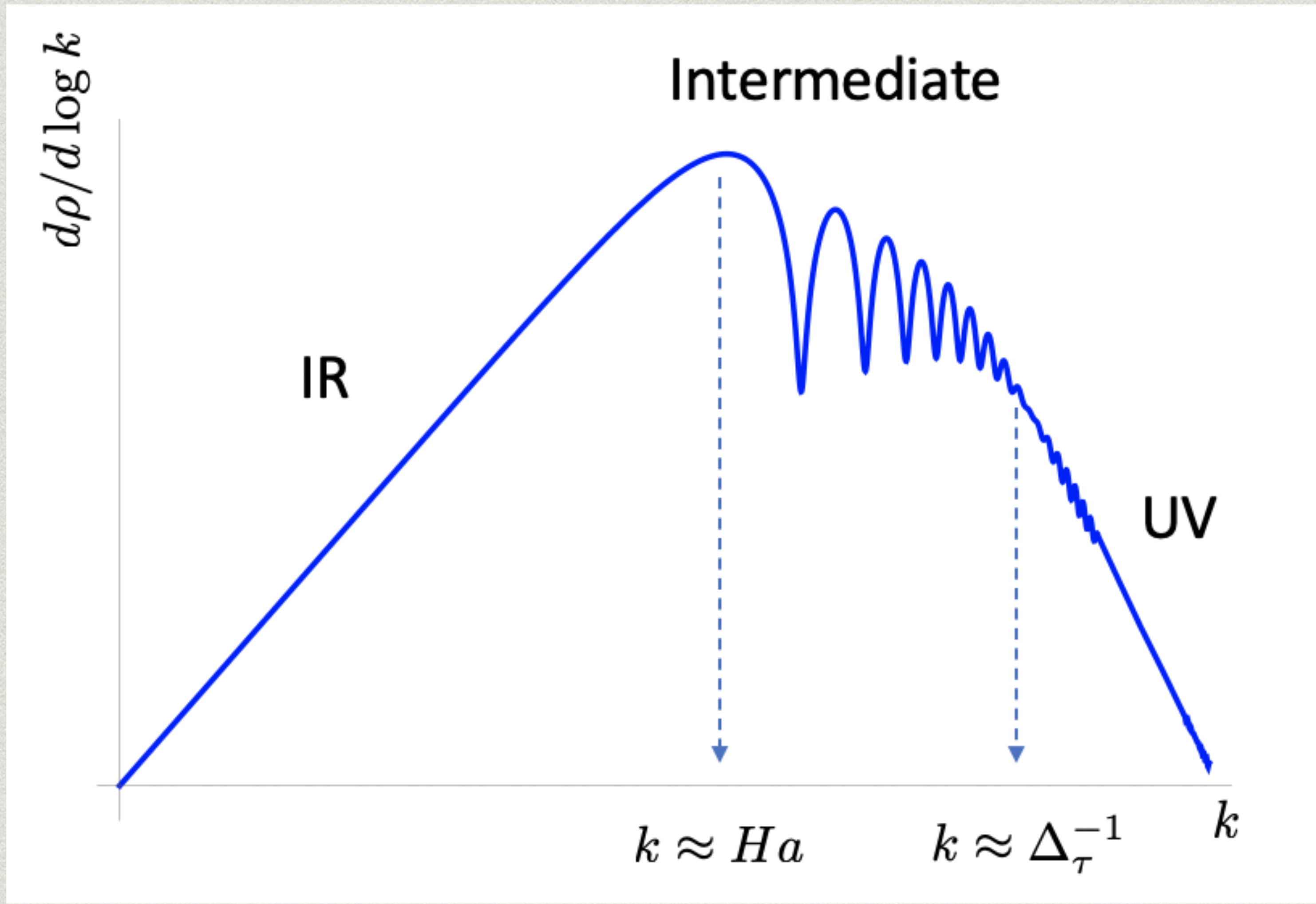
IR: $k < \tau_*$



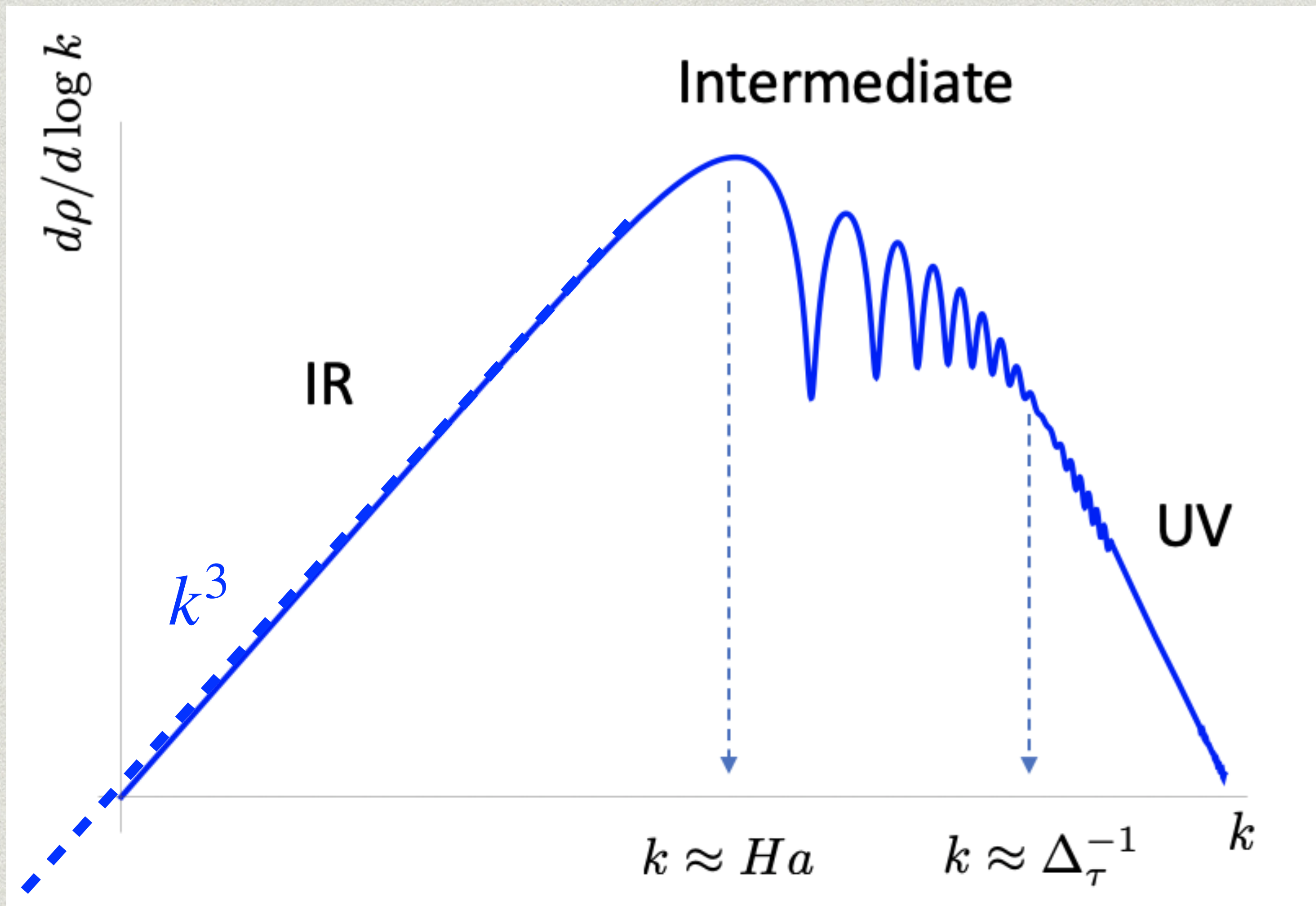
Mode outside horizon at the time of phase transition

No oscillation. Can treat the GW as if it is from a point source.


Co-moving scale
$|\tau_\star| = (H_\star a)^{-1}$
$\Delta_\tau$
$r_{\rm bubble} \sim \Delta_\tau$
$\tau_\star$
$\log a$

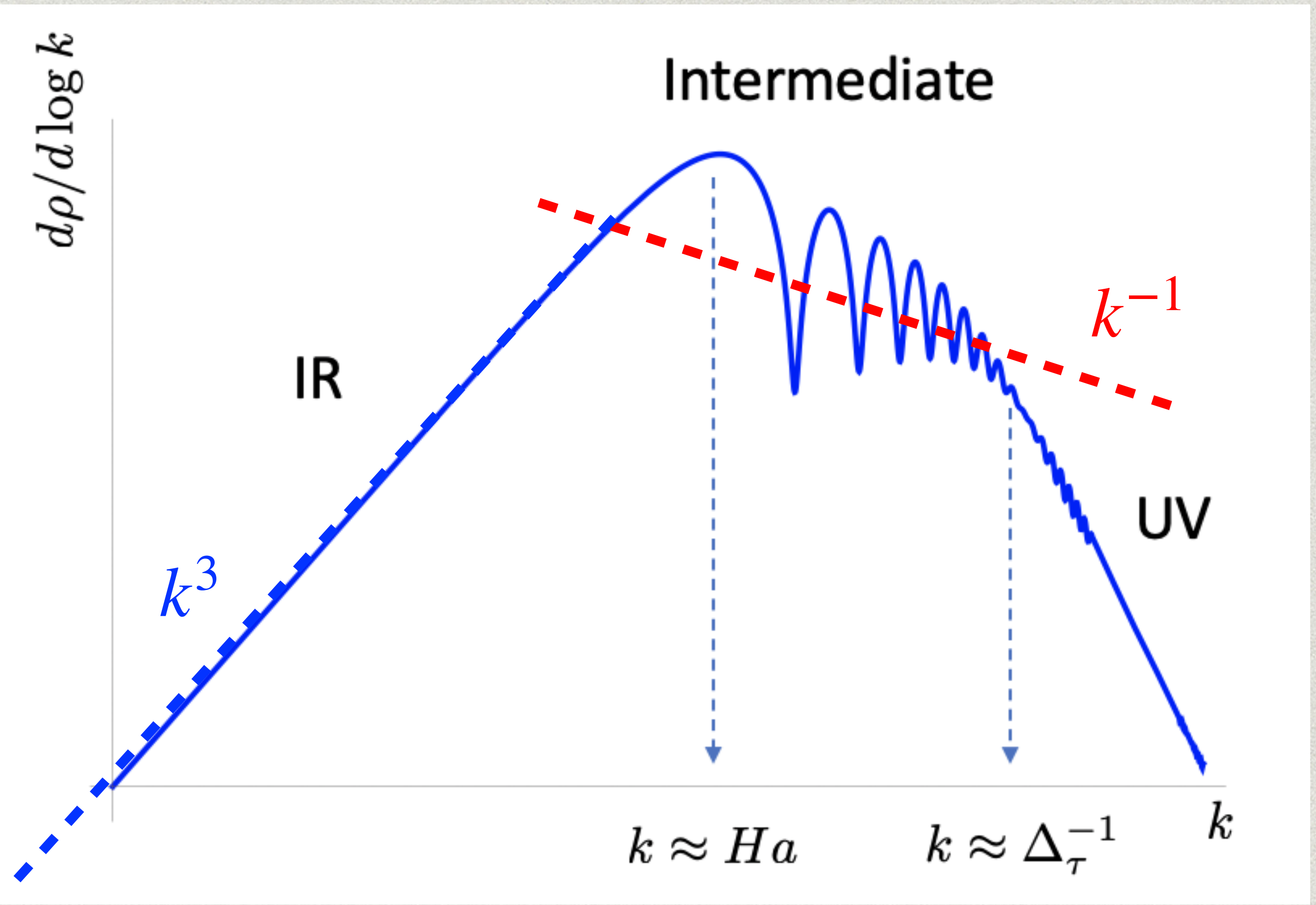



$$\frac{d\rho_{\rm GW}}{d\log k} \propto k^3 \langle (h')^2 \rangle$$




$$\frac{d\rho_{\rm GW}}{d\log k} \propto k^3 \langle (h')^2 \rangle$$

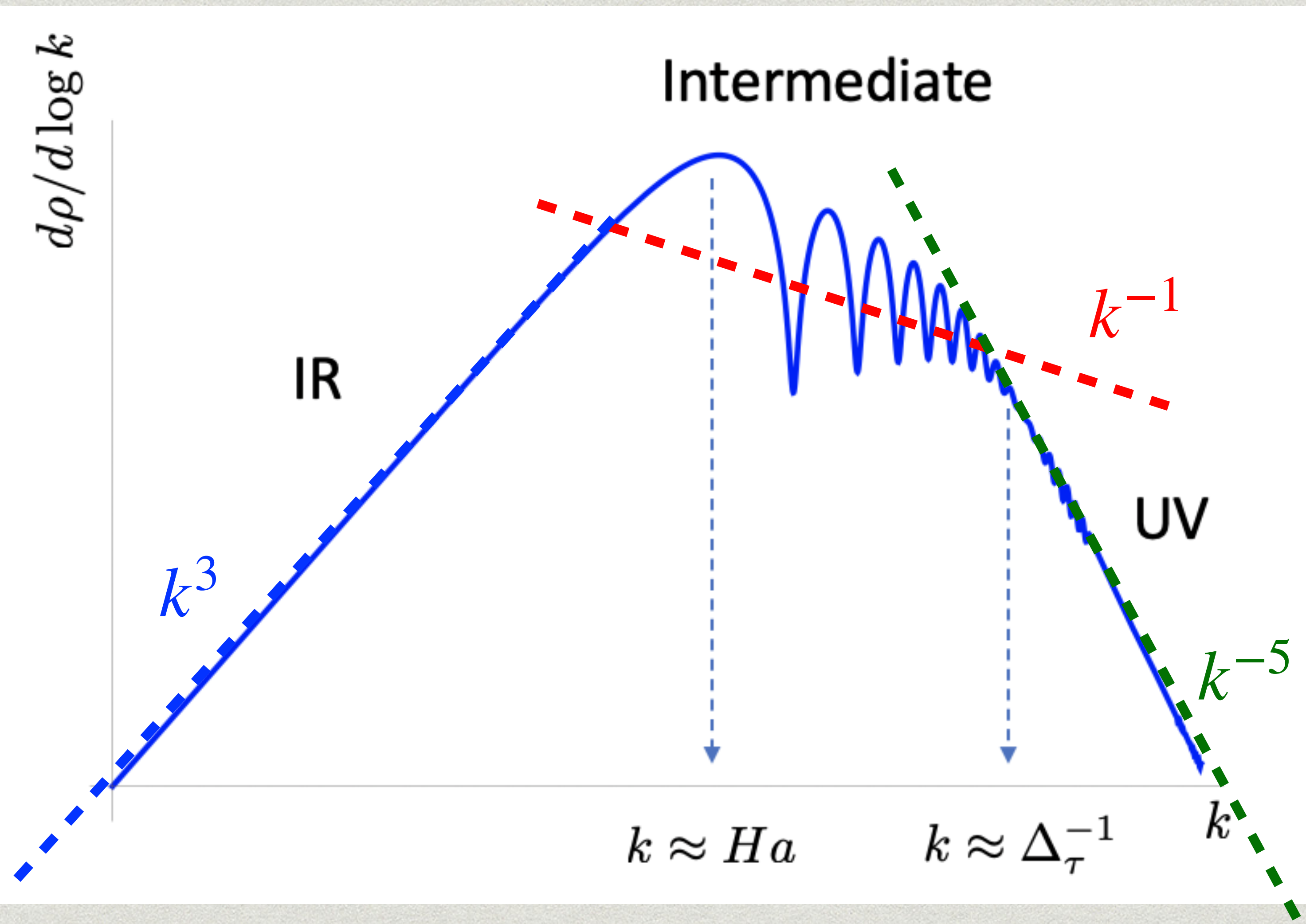
IR: $$h = \text{constant} \times \frac{\sin k\tau}{k\tau}$$




$$\frac{d\rho_{\rm GW}}{d\log k} \propto k^3 \langle (h')^2 \rangle$$

IR: $h = \text{constant} \times \dfrac{\sin k\tau}{k\tau}$

Intermediate: $h \propto \dfrac{\cos(k\tau_*)}{k^2} \dfrac{\sin k\tau}{k\tau}$




$$\frac{d\rho_{\rm GW}}{d\log k} \propto k^3 \langle (h')^2 \rangle$$

IR: $h = \text{constant} \times \dfrac{\sin k\tau}{k\tau}$

Intermediate: $h \propto \dfrac{\cos(k\tau_*)}{k^2} \dfrac{\sin k\tau}{k\tau}$

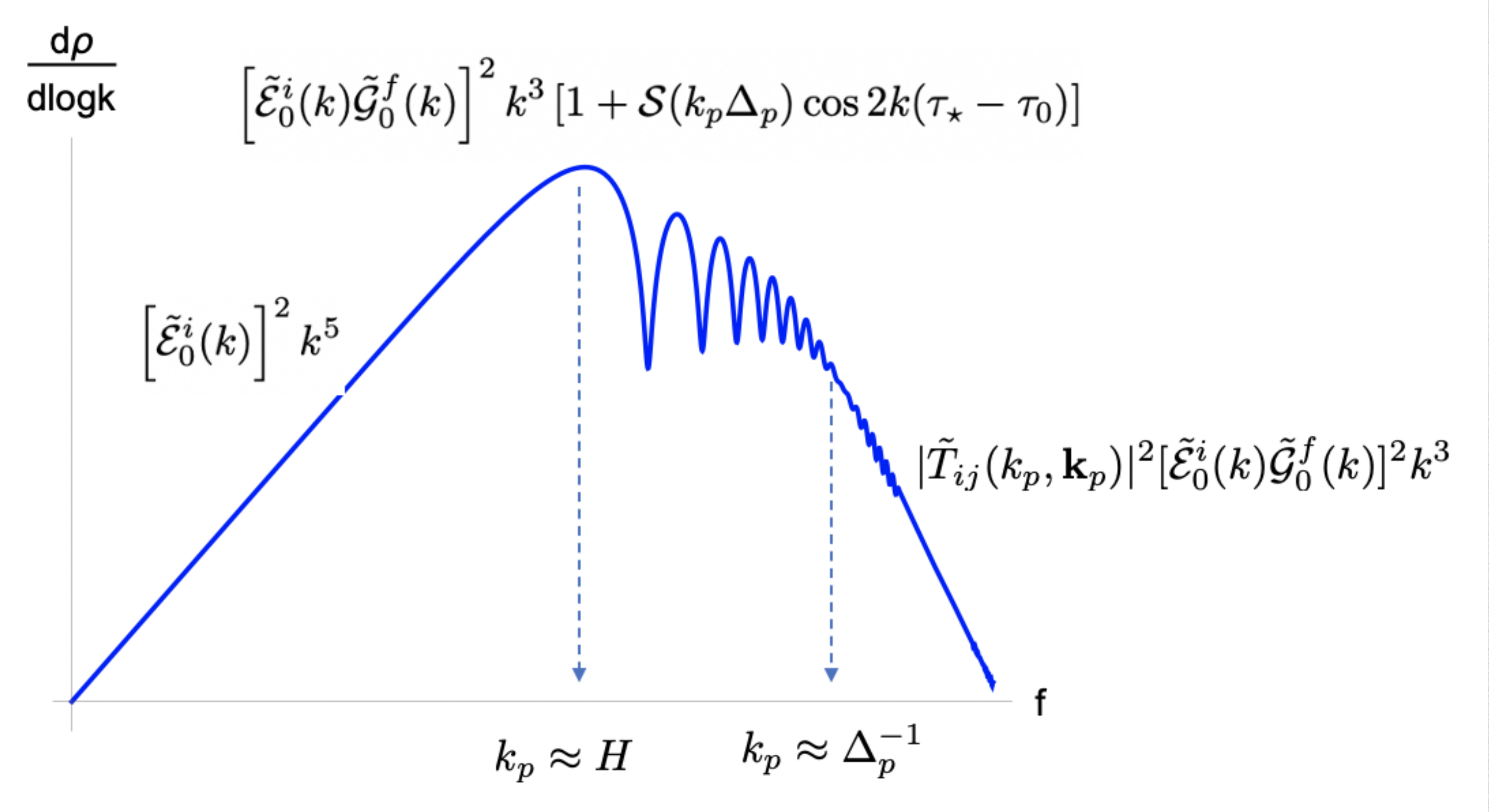
UV: $\dfrac{d\rho_{\rm GW}}{d\log k} \propto k^{-5}$

Details of source matters, determined by numerical simulation

Huber, Konstantin, 0806.1828

Cutting, Hindmarsh, Wier, 1802.05712

# More generally



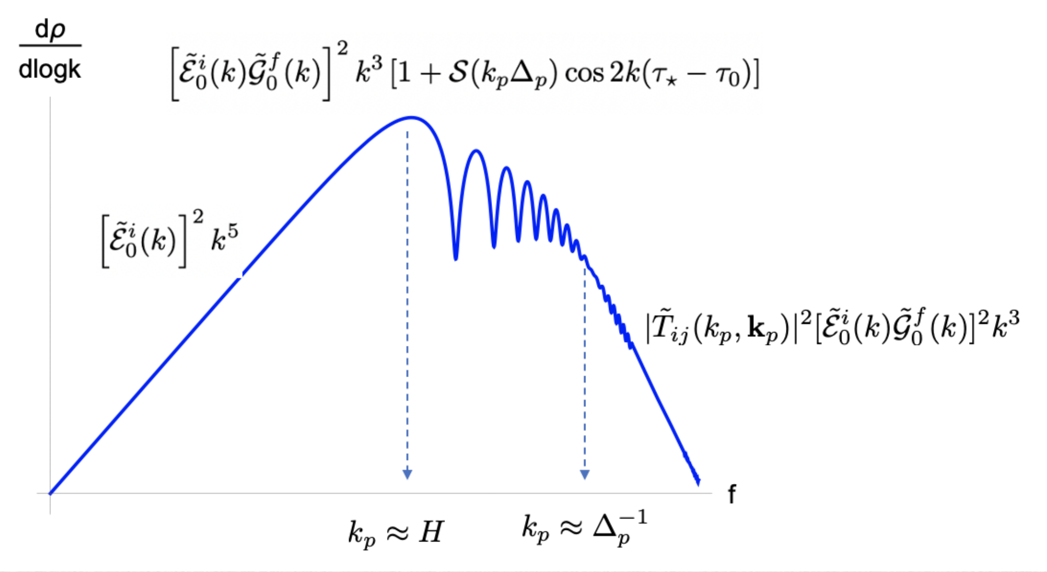

$$\frac{d\rho_{\rm GW}^{\rm osc}}{d\log k} = \frac{2G_N|\tilde{T}_{ij}(0,0)|^2}{\pi V a^4(\tau)a^2(\tau_\star)}\left\{\left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2 k^3\left[1+\mathcal{S}(k\Delta_\tau)\cos 2k(\tau_\star-\tau_0)\right]\right\}$$

Depending on Time evolution

Smearing for $k\boldsymbol{\Delta_\tau} \gg 1$

Oscillation

# Dependence on later evolution




$$\frac{d\rho_{\rm GW}^{\rm osc}}{d\log k} = \frac{2G_N|\tilde{T}_{ij}(0,0)|^2}{\pi V a^4(\tau)a^2(\tau_\star)}\left\{\left[\tilde{\mathcal{E}}_0^i(k)\tilde{\mathcal{G}}_0^f(k)\right]^2 k^3\left[1+\mathcal{S}(k\Delta_\tau)\cos 2k(\tau_\star-\tau_0)\right]\right\}$$

$\tilde{\mathcal{G}}_0^f(k)$ Depends on the evolution of the background spacetime during inflation

$\tilde{\mathcal{E}}_0^i(k)$ Depends on the evolution of the background spacetime after inflation

Alternative scenarios can change the shape of the GW signal!

Can be sensitive to era after the CMB mode exit the horizon and before BBN

# Scenarios of inflation and its aftermath

Scenarios of inflation

Parameterized by $p$

Quasi de Sitter: $a(\tau) = -\frac{1}{H\tau}$

Power law: $a(t) = a_0(t/t_0)^p$

Lucchin and Matarrese, 1985

$p \to \infty$, quasi de Sitter

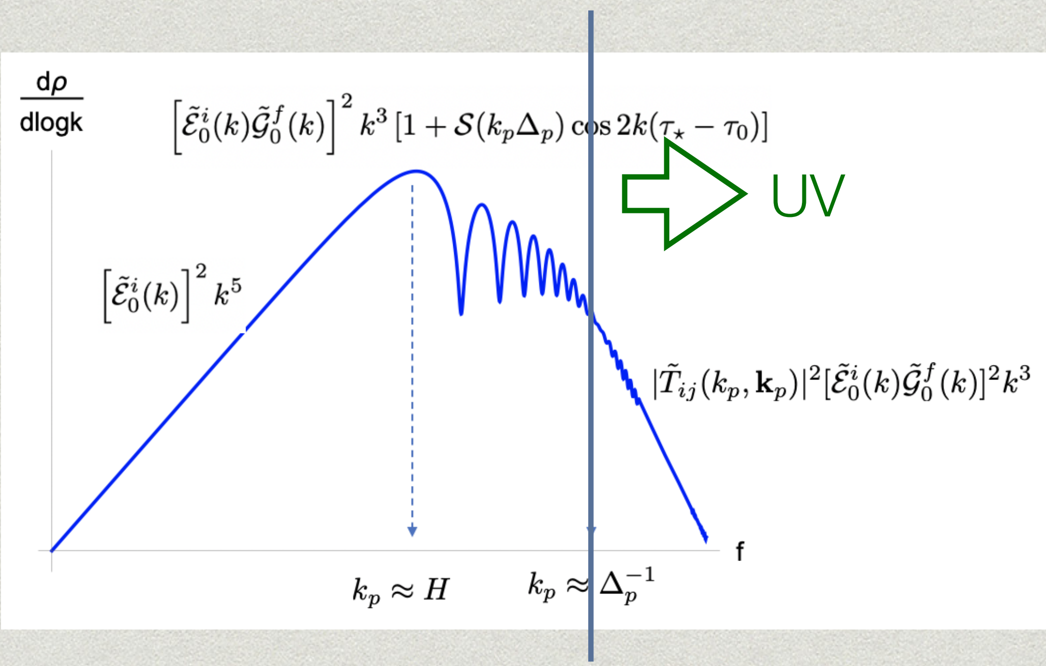
Scenarios after inflation:

Parameterized by $\tilde{p}$

$$a(t) \sim t^{\tilde{p}}$$

| | $w$ | $\rho(a)$ | $\tilde{p}$ | $\tilde{\alpha}$ |
|---|---|---|---|---|
| MD | 0 | $a^{-3}$ | 2/3 | -3/2 |
| RD | 1/3 | $a^{-4}$ | 1/2 | -1/2 |
| $\Lambda$ | -1 | $a^{0}$ | $\infty$ | 3/2 |
| Cosmic string | -1/3 | $a^{-2}$ | 1 | $\infty$ |
| Domain wall | -2/3 | $a^{-1}$ | 2 | 5/2 |
| kination | 1 | $a^{-6}$ | 1/3 | 0 |

# Impact on spectrum



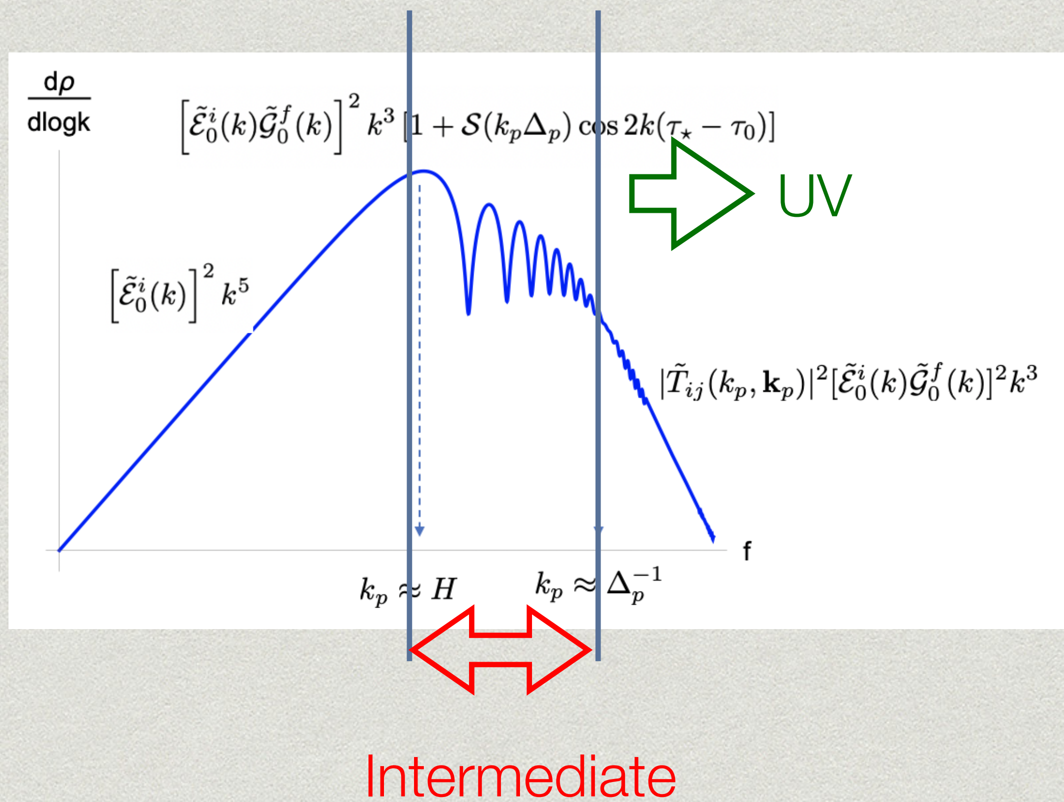

UV

Scenarios after inflation →

Inflationary scenarios ↓

| | RD | MD | $t^{\tilde{p}}$ |
|---|---|---|---|
| dS | $k^{-5}$ | $k^{-7}$ | $k^{-3+2\frac{\tilde{p}}{\tilde{p}-1}}$ |
| $t^p$ | $k^{-3+2\frac{p}{1-p}}$ | $k^{-5+2\frac{p}{1-p}}$ | $k^{-1+2\left(\frac{p}{1-p}+\frac{\tilde{p}}{\tilde{p}-1}\right)}$ |

# Impact on spectrum




## UV

Scenarios after inflation →

Inflationary scenarios ↓

| | RD | MD | $t^{\tilde{p}}$ |
|---|---|---|---|
| dS | $k^{-5}$ | $k^{-7}$ | $k^{-3+2\frac{\tilde{p}}{\tilde{p}-1}}$ |
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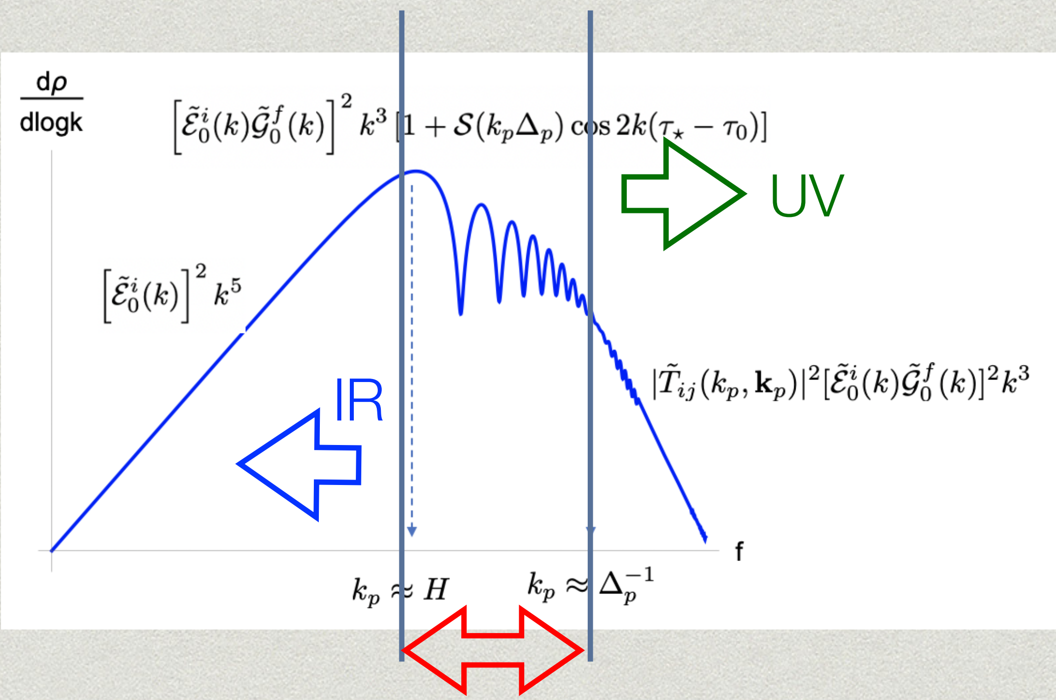
## Intermediate

Scenarios after inflation →

Inflationary scenarios ↓

| | RD | MD | $t^{\tilde{p}}$ |
|---|---|---|---|
| dS | $k^{-1}$ | $k^{-3}$ | $k^{1+2\frac{\tilde{p}}{\tilde{p}-1}}$ |
| $t^p$ | $k^{1+2\frac{p}{1-p}}$ | $k^{-1+2\frac{p}{1-p}}$ | $k^{3+2\left(\frac{p}{1-p}+\frac{\tilde{p}}{\tilde{p}-1}\right)}$ |

# Impact on spectrum




Intermediate

## UV

Scenarios after inflation →

Inflationary scenarios ↓

| | RD | MD | $t^{\tilde{p}}$ |
|---|---|---|---|
| dS | $k^{-5}$ | $k^{-7}$ | $k^{-3+2\frac{\tilde{p}}{\tilde{p}-1}}$ |
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## Intermediate

Scenarios after inflation →

Inflationary scenarios ↓

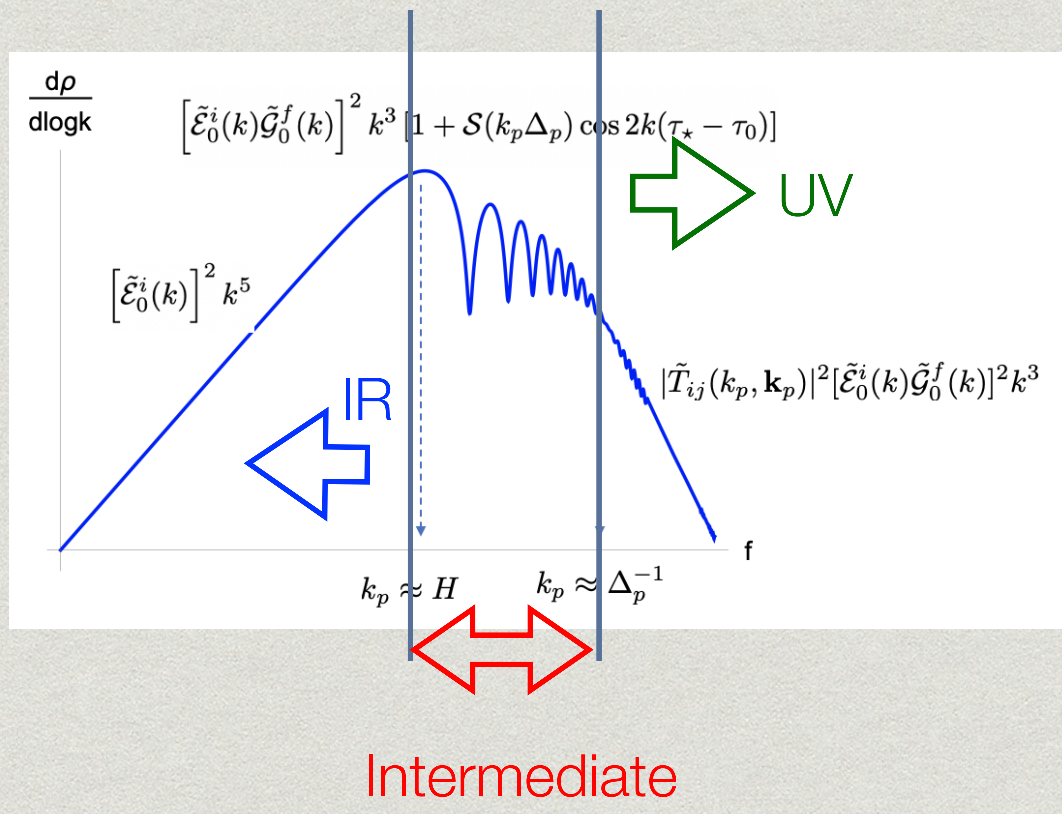
| | RD | MD | $t^{\tilde{p}}$ |
|---|---|---|---|
| dS | $k^{-1}$ | $k^{-3}$ | $k^{1+2\frac{\tilde{p}}{\tilde{p}-1}}$ |
| $t^p$ | $k^{1+2\frac{p}{1-p}}$ | $k^{-1+2\frac{p}{1-p}}$ | $k^{3+2\left(\frac{p}{1-p}+\frac{\tilde{p}}{\tilde{p}-1}\right)}$ |

## IR

Scenarios after inflation →

Inflationary scenarios ↓

| | RD | MD | $t^{\tilde{p}}$ |
|---|---|---|---|
| dS | $k^3$ | $k^1$ | $k^{5+2\frac{\tilde{p}}{\tilde{p}-1}}$ |
| $t^p$ | $k^3$ | $k^1$ | $k^{5+2\frac{\tilde{p}}{\tilde{p}-1}}$ |




## UV

Scenarios after inflation →

Inflationary scenarios ↓

| | RD | MD | $t^{\tilde{p}}$ |
|---|---|---|---|
| dS | $k^{-5}$ | $k^{-7}$ | $k^{-3+2\frac{\tilde{p}}{\tilde{p}-1}}$ |
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## Intermediate

Scenarios after inflation →

Inflationary scenarios ↓

| | RD | MD | $t^{\tilde{p}}$ |
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## IR

Scenarios after inflation →

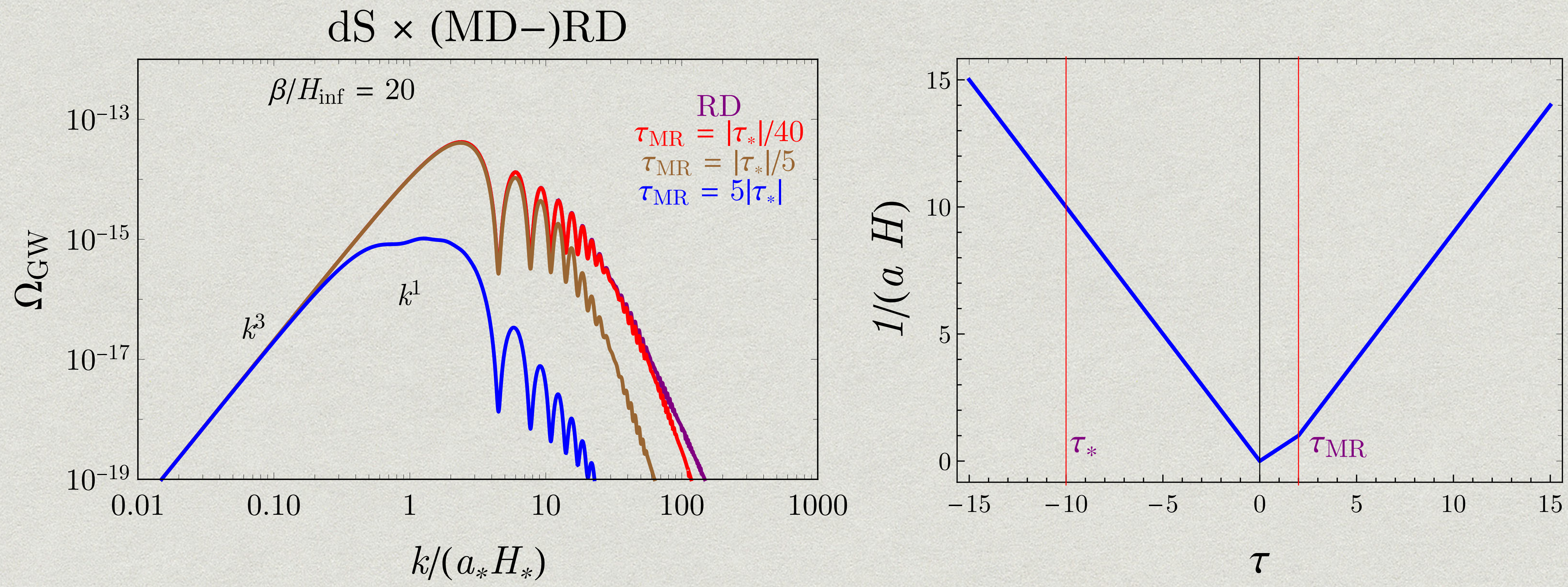
Inflationary scenarios ↓

| | RD | MD | $t^{\tilde{p}}$ |
|---|---|---|---|
| dS | $k^3$ | $k^1$ | $k^{5+2\frac{\tilde{p}}{\tilde{p}-1}}$ |
| $t^p$ | $k^3$ | $k^1$ | $k^{5+2\frac{\tilde{p}}{\tilde{p}-1}}$ |

Ideally, we will

1. Observe GW.
2. Observe Oscillation → Instantaneous source during inflation (1st order PT)
3. The spectral shape can tell us the evolution after PT.

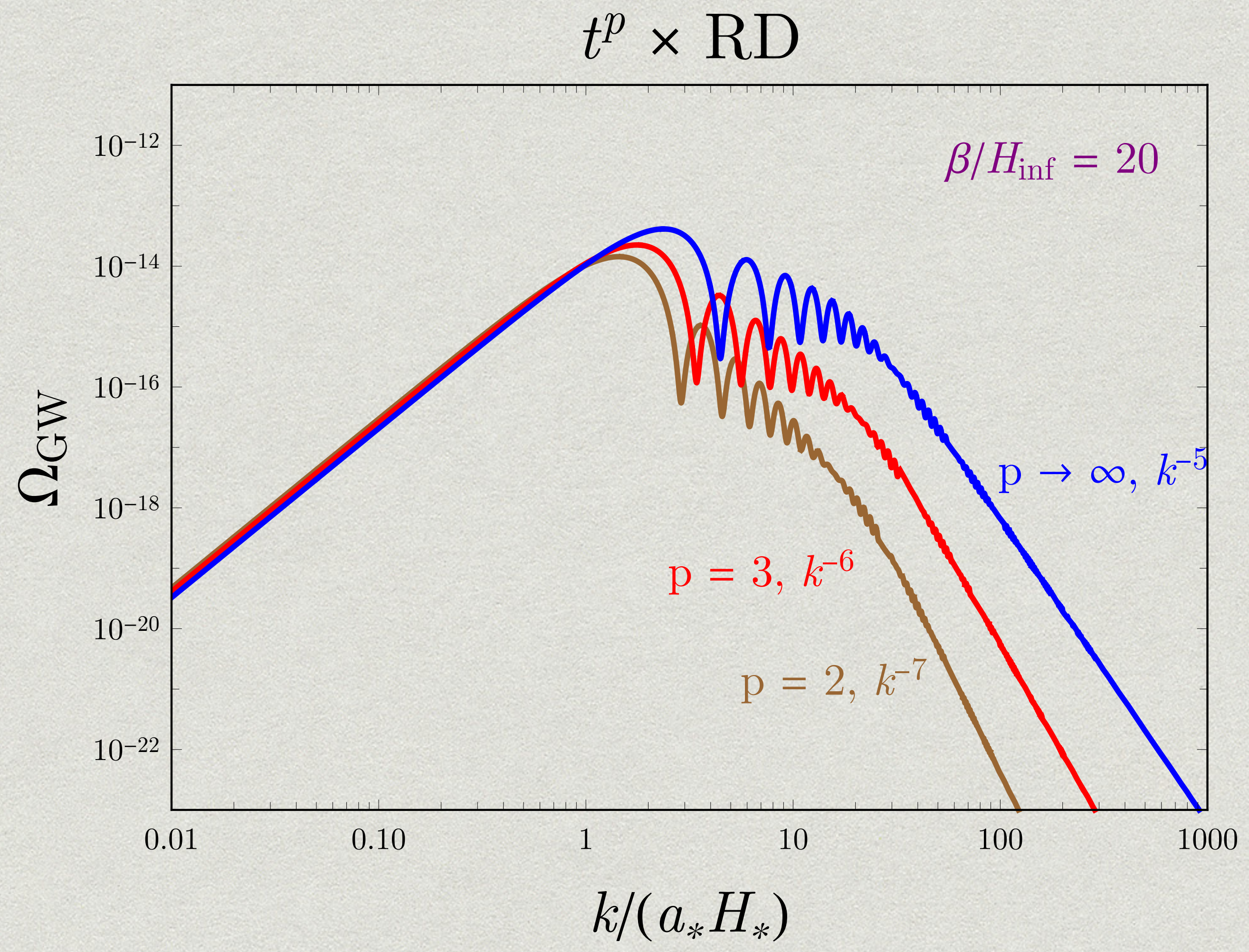
# Comparing scenarios




Scenarios after reheating.

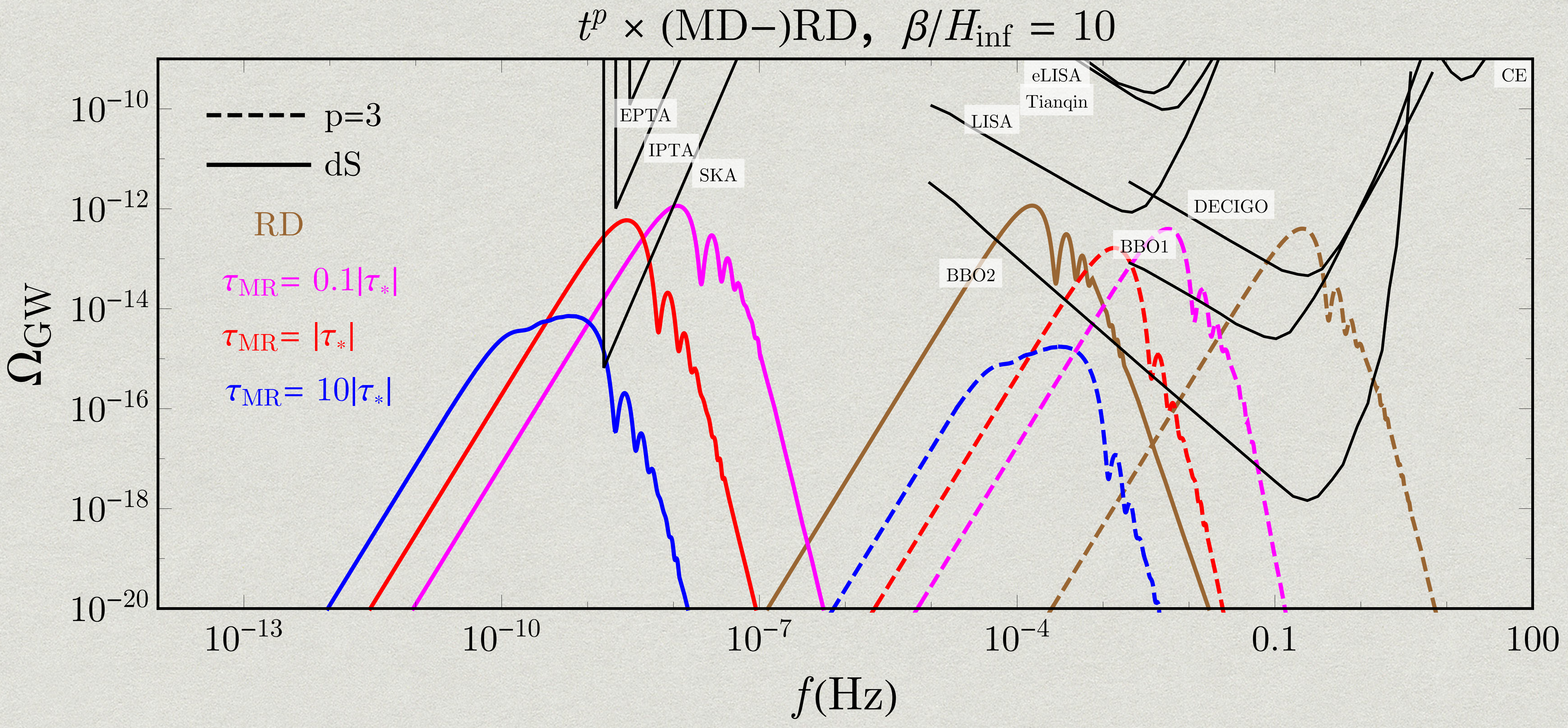
$\tau_{\rm MR}$ = MD-RD transition

# Comparing scenarios




Different inflationary scenarios.
→ different slope in UV part.

Haipeng An, KunFeng Lyu, Siyi Zhou, and LTW 2009.12381, 2201.05171




$$\Omega_{\rm GW}^{\rm max} \sim \Omega_R \times \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf\star}}\right)^2 \times \left(\frac{H_\star}{\beta}\right)^5 \tilde{\Delta} \times F(H_\star/H_r, a_\star/a_r, \cdots)$$

$$\approx 10^{-13} \times \left(\frac{\Delta\rho_{\rm vac}/\rho_{\rm inf\star}}{0.1}\right)^2 \times \left(\frac{H_\star/\beta}{0.1}\right)^5$$

# Conclusions

- Cosmological observations can reveal new dynamics in the inflationary era.
- Potentially large inflaton excursion can trigger new dynamics in a spectator sector.
  - Can trigger 1st order phase transition → GW.
    - GW Can probe an era invisible from other observables, such as CMB/LSS or BBN.