Model dependence of the GW spectrum of a first order phase transition

Probing New Physics with Gravitational Waves — MITP Workshop

01/08/2022

Jorinde van de Vis

Based on:

T. Tenkanen, JvdV		arXiv:2206.01130
F. Giese, T. Konstandin, K. Schmitz, JvdV	*JCAP* 01 (2021) 072	arXiv:2010.09744
F. Giese, T. Konstandin, JvdV	*JCAP* 07 (2020) 07, 057	arXiv:2004.06995

HELMHOLTZ

DESY.

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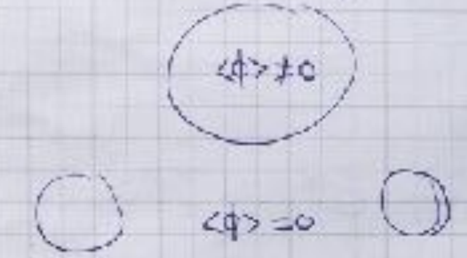
2004.06995 w/ Giese, Konstandin
2010.09744 w/ " & Schmitz
2206.01130 w/ Tenkanen

Context: cosmological first order phase transition



Release of vacuum energy ΔV drives bubble expansion and sources fluid motion, which sources GWs.

Can take place during Electroweak PT ~~→~~ (requires BSM) or in dark sector.

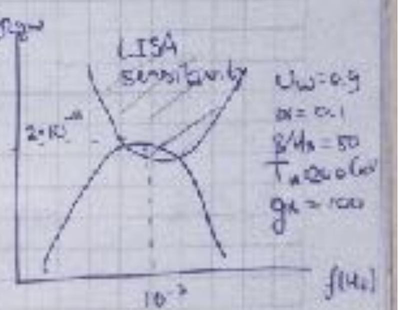
* How to compute the GW signal?

Hydrodynamic simulations of coupled fluid-scalar field system

Fit to simulations (LISA cosmo WG 2019) for sound waves

$$\frac{d\Omega_{gw,0}}{d\ln(f)} = 0.687\, F_{gw,0}\, K^2\, (H_* \tau_s)(H_* R_*)/c_s\, \tilde{\Omega}_{gw}\, C(f/f_0)$$

Relevant parameters:
α (PT strength)
β (PT duration)
v_w (wall velocity)
T_* (PT temperature)
$c_s (\sim 1/\sqrt{3})$ (Speed of sound)



K: kinetic energy fraction $\frac{\rho_{fluid}}{e(T_n)}$ – can be obtained from a single bubble. ②

Often-used procedure: Assume $c_s^2 = 1/3$ (radiation) and use a fit of K as a function of α, v_w (Espinosa Konstandin, No, Servant 2010) obtained in bag equation of state (pure radiation + T independent vacuum energy difference).

$\boxed{\text{Main question: can we improve this procedure?}}$

I Energy budget for $c_s^2 \neq 1/3$

II What are realistic values for c_s^2?

① Hydrodynamics of a single bubble

$$T^{\mu\nu} = w u^\mu u^\nu - g^{\mu\nu} p$$

$p = -V_{eff}$

$e = T\frac{\partial p}{\partial T} - p$

$w = e + p$

$c_s^2 = \frac{dp/dT}{de/dT}$

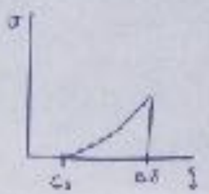
Fluid eqs follow from $\partial_\mu T^{\mu\nu} = 0$

- $$\frac{dv}{d\xi} = \frac{2v(1-v^2)}{\xi(1-v\xi)}\left(\frac{\mu(\xi,v)}{c_s^2} - 1\right)^{-1}$$

$$\mu = \frac{\xi - v}{1 - \xi v}$$

- $$\frac{dw}{d\xi} = w\left(1 + \frac{1}{c_s^2}\right)\gamma^2 \mu(\xi,v)\frac{dv}{d\xi}$$

$\xi = r/t$ v: fluid velocity e.g.



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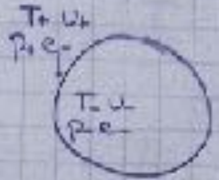
broken   symmetric

Matching: $\frac{v_+}{v_-} = \frac{e_b(T_-) + p_s(T_+)}{e_s(T_+) + p_b(T_-)}$, $v_+ v_- = \frac{p_s(T_+) - p_b(T_-)}{e_s(T_+) - e_b(T_-)}$



$w(\text{far away}) = w(T_n)$

↳ Energy budget: $K = \frac{3}{\xi_w^3 e_n} \int d\xi\, \xi^2 v^2 \gamma^2 w$.

We can solve the equations numerically for each EOS, but most of the model dependence can actually be captured in a few parameters.

Assuming $T_+ \simeq T_-$, the matching can be approximated as

$$\frac{v_+}{v_-} = \frac{v_+ v_- / c_{s,b}^2 - 1 + 3\alpha_{\bar\theta}}{v_+ v_- / c_{s,b}^2 - 1 + 3 v_+ v_- \alpha_{\bar\theta}} \quad \text{with} \quad \alpha_{\bar\theta} = \frac{D\bar\theta(T_+)}{3 w_+} \quad \bar\theta = e - \frac{p}{c_{s,b}^2}$$

$$DX = X_s(T_+) - X_b(T_+)$$

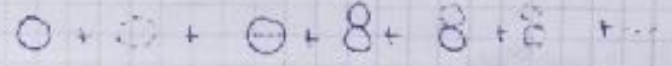
If we also assume $c_{s,b}$, $c_{s,s}$ constant, $\kappa \left(= \frac{4 e(T_n)}{D\bar\theta} K\right)$ becomes a function of v_w, $\alpha_{\bar\theta}$, $c_{s,b}$, $c_{s,s}$ alone.

2010.09744 provides a Python snippet which computes κ.

Upshot: in addition to $\alpha_{\bar\theta}$, β/H, v_w and T_*, the amplitude of the GW signal depends on (model-dependent) $c_{s,b}$ and $c_{s,s}$. The shape of the GW signal is likely also affected, but this requires new (hydrodynamic) simulations.

(II) What is c_s?

Finite-temperature corrections to V_{eff}:



Equilibrium thermodynamics can be formulated as $\mathbb{R}^3 \times S^1$, with $1/T$ the size of the compact dimension.

The fields can be expanded in Matsubara modes:

bosons $\phi(\vec{x}, \tau) = T \sum_{n=-\infty}^{\infty} \tilde\phi_n(\omega_n, \vec{x})\, e^{i\omega_n \tau}$, $\omega_n = 2\pi T n$

fermions $\psi(\vec{x}, \tau) = T \sum_{n=-\infty}^{\infty} \tilde\psi_n(\omega_n, \vec{x})\, e^{i\omega_n \tau}$, $\omega_n = 2\pi T (n + \frac{1}{2})$

One-loop thermal potential

$$V_T(h) = \frac{T^4}{2\pi^2}\sum_a n_a \int_0^\infty dx\, x^2 \log\left[1 \mp e^{-\sqrt{x^2+M_a^2/T^2}}\right]$$

high-T expansion

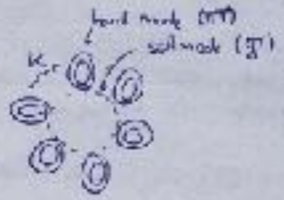
$$T^4\left(-\frac{\pi^2}{90} + \frac{1}{24}\frac{m^2}{T^2} - \frac{1}{4(4\pi)^2}\frac{m^4}{T^4}\log\frac{m^2}{a_b T^2} + \dots\right) \quad \text{bosons}$$

$$T^4\left(-\frac{7}{8}\frac{\pi^2}{90} - \frac{1}{48}\frac{m^2}{T^2} - \frac{1}{4(4\pi)^2}\frac{m^4}{T^4}\log\frac{m^2}{a_f T^2} + \dots\right) \quad \text{fermions}$$

Light particles all contribute to the pressure as T^4, so they push the speed of sound towards $c_s^2 \to 1/3$.

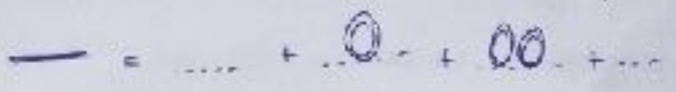
Beyond one-loop

Problems: • IR-divergent diagrams (daisies)



Solution: resummation of the propagator by the thermal mass

$$G_{full} = G_0 + G_0 \Pi G_0 + \dots = \frac{1}{p^2+m^2+\underbrace{\Pi}_{\text{thermal mass}}}$$



• RG scale dependence:

at zero T, the running of the couplings of the tree-level potential cancel the scale-dependence of the CW $m^4\log\frac{m^2}{\mu^2}$-term.

at finite-T, the leading order contribution to V_{eff} gets modified

$$V^{g^2}_{thermal} = \frac{1}{2}(m^2 + \overbrace{\Pi_T}^{O(g^2T^2)})v^2 + \frac{1}{4!}g^2 v^4$$

But the β-functions remain the same.

Explicitly, there is a left-over RG-scale dependence $\mu \frac{d}{d\mu}\left(\frac{\mu^2 T}{2\pi}\right)$,

corresponding to a theoretical uncertainty.

Both problems can be solved by constructing an EFT for the light DOFs (remember that all fermionic modes are heavy and all non-zero bosonic modes). (Ginsparg 1980, Appelquist Pisarski 1981)

The coefficients of the EFT are determined by matching of correlators - the effect of the hard modes is captured in the parameters of the EFT.

e.g. (4D) $k^2 + m_s^2 + \underset{\text{light}}{\Pi_s(k^2)} + \underset{\text{heavy}}{\overline{\Pi}(k^2)} = k^2 + m_3^2 + \underset{\text{light}}{\Pi_3(k^2)}$ (3D)

$\rightarrow$ m_3 in terms of $\overline{\Pi}$, $\overline{\Pi}'$.

In order to compute the speed of sound, we need to know the field independent part and the field-dependent part of the pressure. This is obtained via "matching of the unit operator", i.e.: vacuum bubbles.

We work in a theory with SM + real singlet with Z_2 symmetry, undergoing a two-step PT.

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{2}\mu_s^2 S^T S + \frac{1}{4}\lambda_s (S^T S)^2 + \frac{1}{2}\lambda_m S^T S H^\dagger H.$$

we assume the scaling $\lambda \sim \overset{\text{weak coupling}}{g^2}$

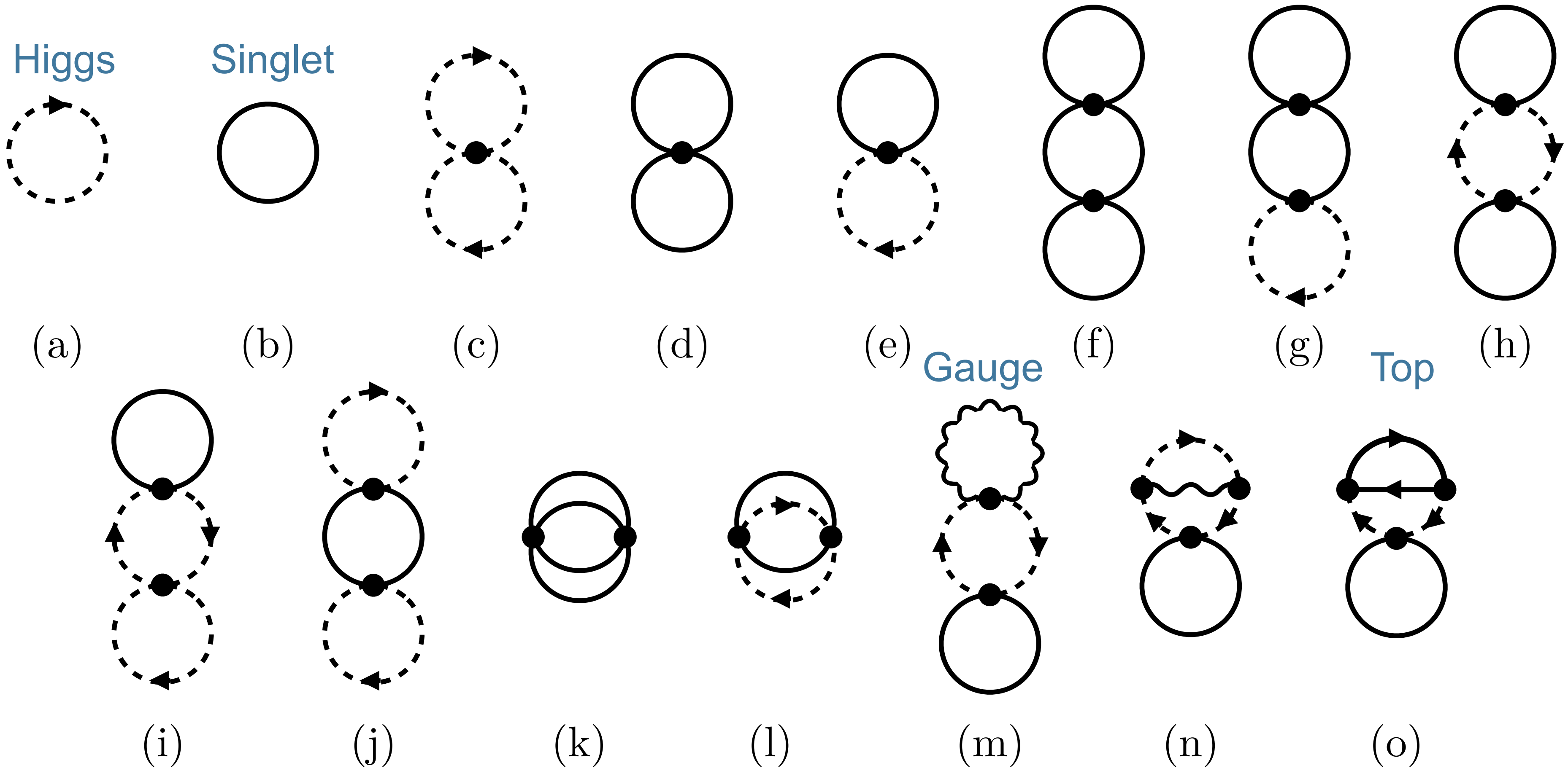
$\mu_s^2, \mu_h^2 \sim g^2 T^2$

and work to order g^6.

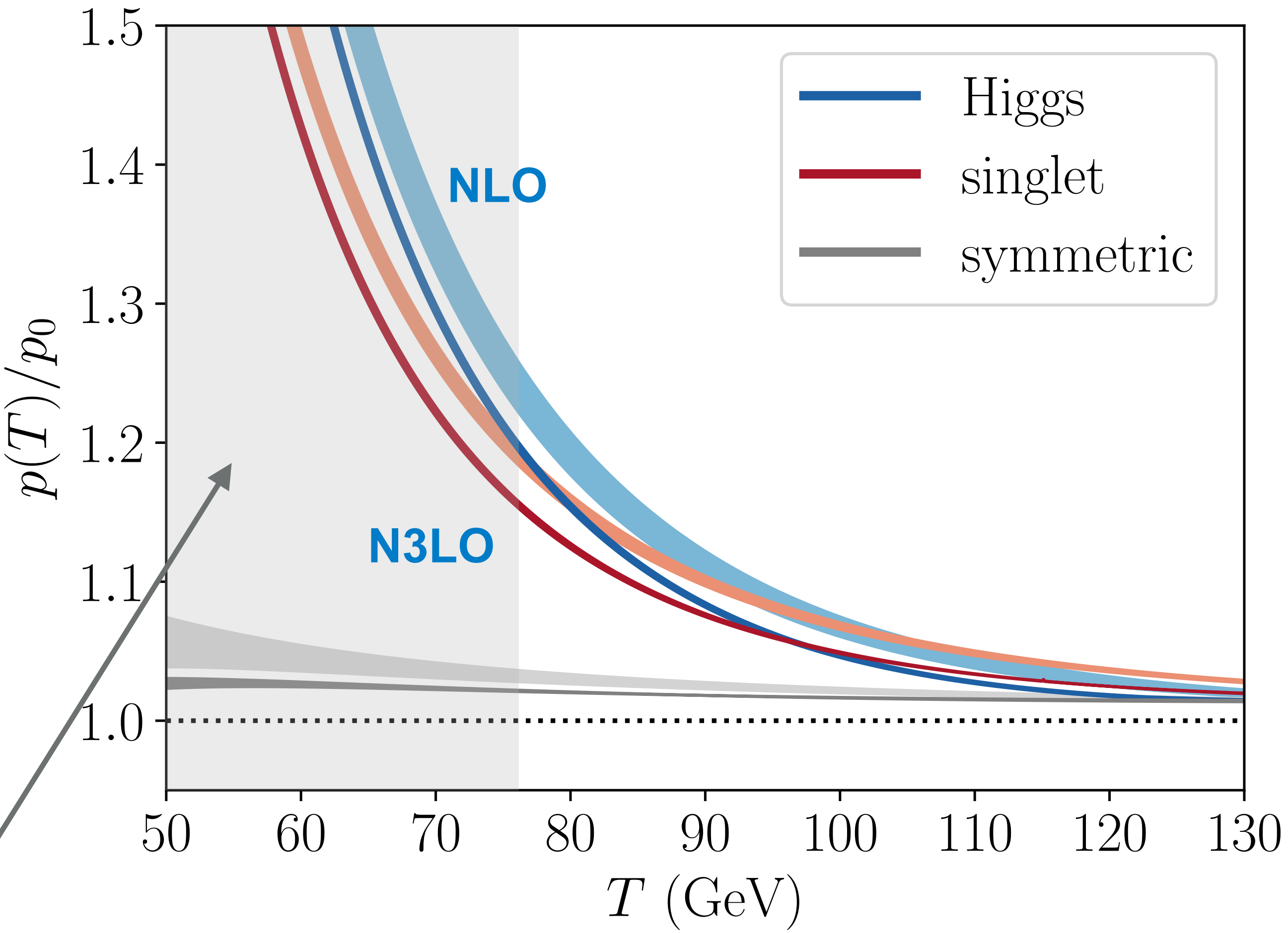
Computation of the pressure with g^4 accuracy (N3LO)

For SM part, see Gynther, Vepsalainen 2005&2006

- Matching of the unit operator - singlet contributions

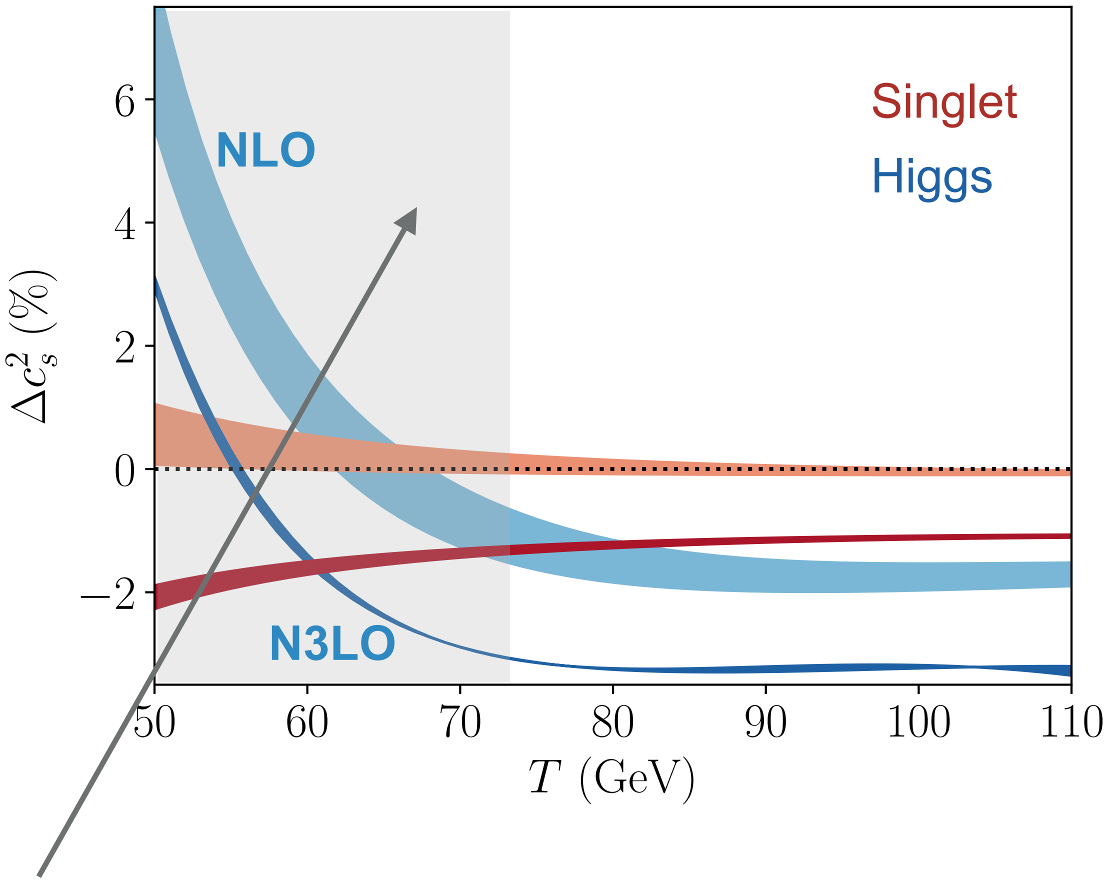


Deviation of the sound speed from $c_s^2 = 1/3$ for $N = 1$



Not so reliable, due to breakdown of high-T expansion

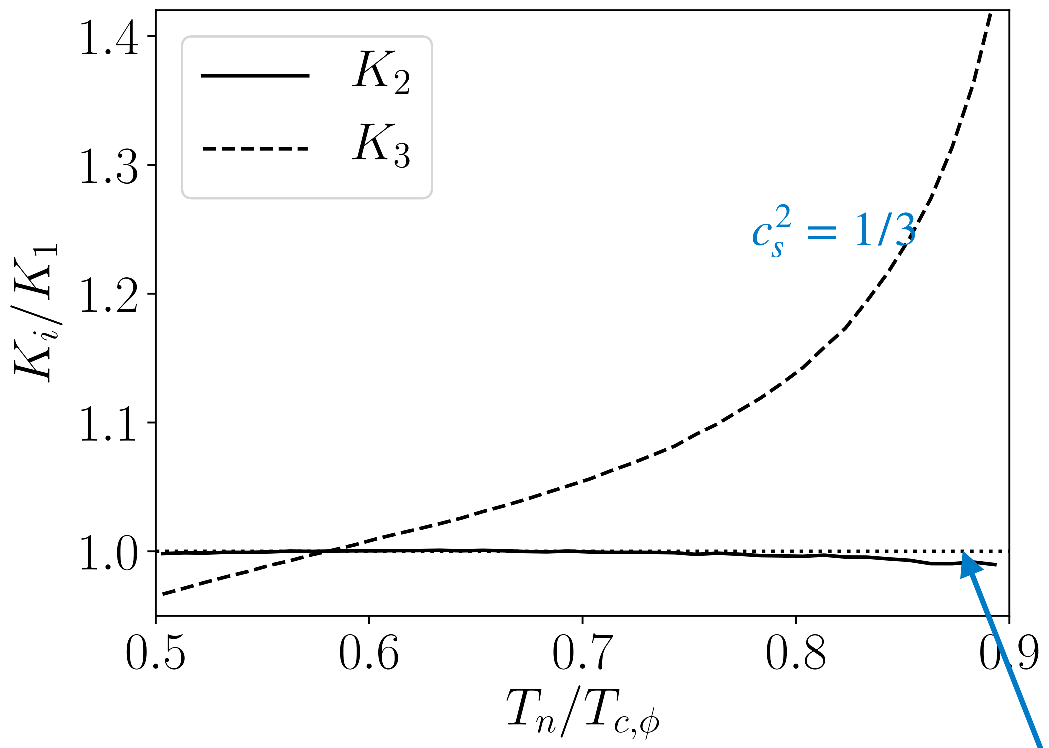
Deviation of the sound speed from $c_s^2 = 1/3$ for $N = 1$



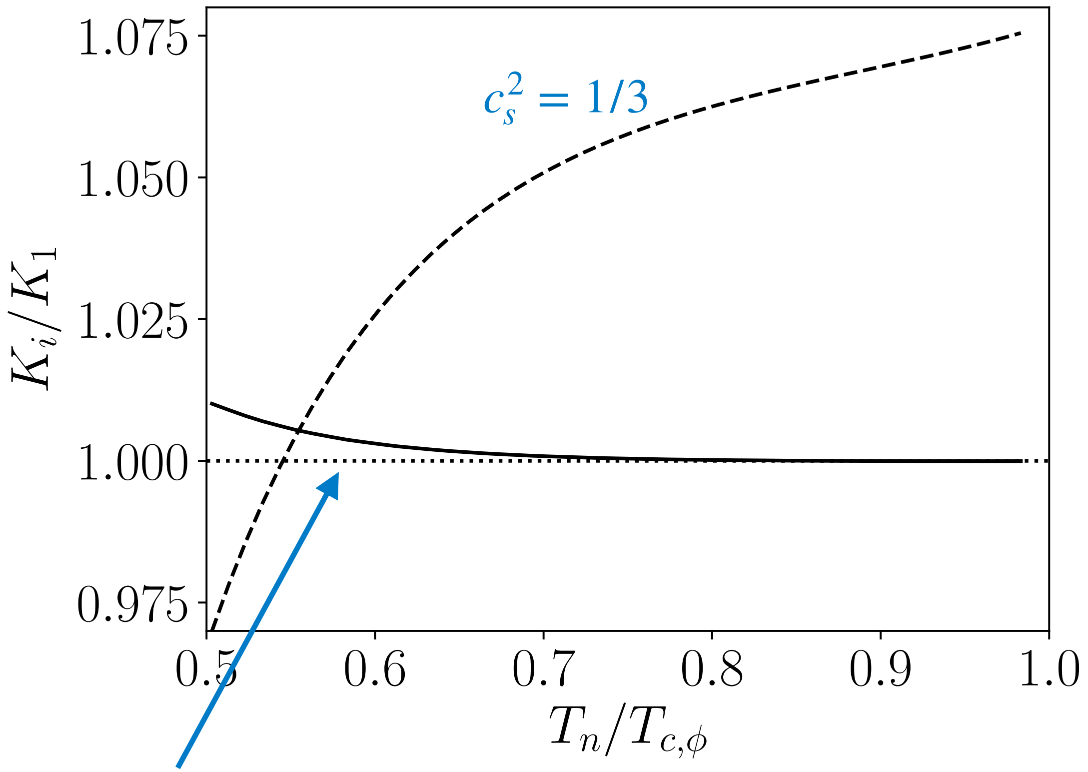
Not so reliable, due to breakdown of high-T expansion

Deviation in the kinetic energy fraction with $c_s^2 = 1/3$

Hybrid

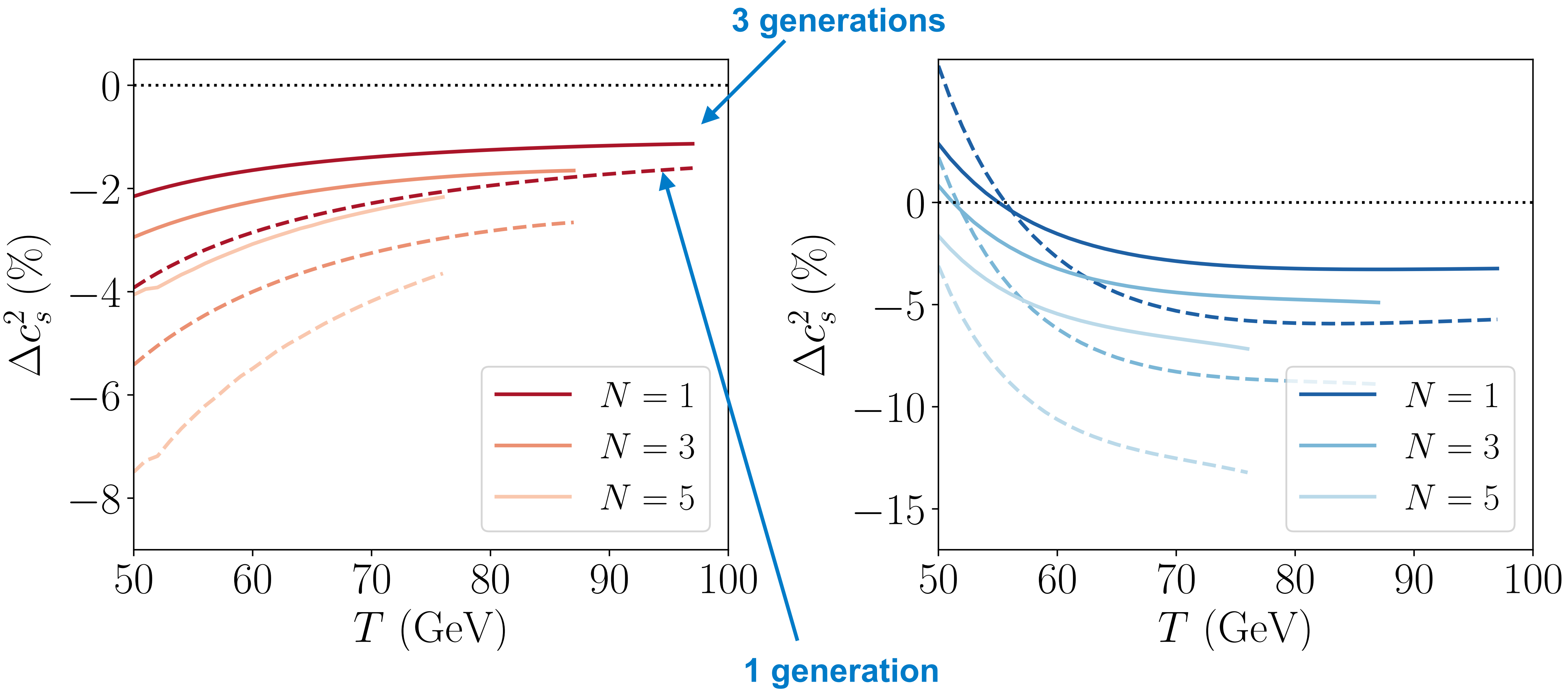


Detonation



Full numerical

Modified particle content: fewer fermions



- Up to order-of-magnitude suppression of GW signal

Back-up

Contributions at different orders in g

LO (ag^0): one-loop hard contributions to p_{sym}.

NLO (bg^2): tree-level terms in V_{eff} and two-loop hard pieces in p_{sym} .

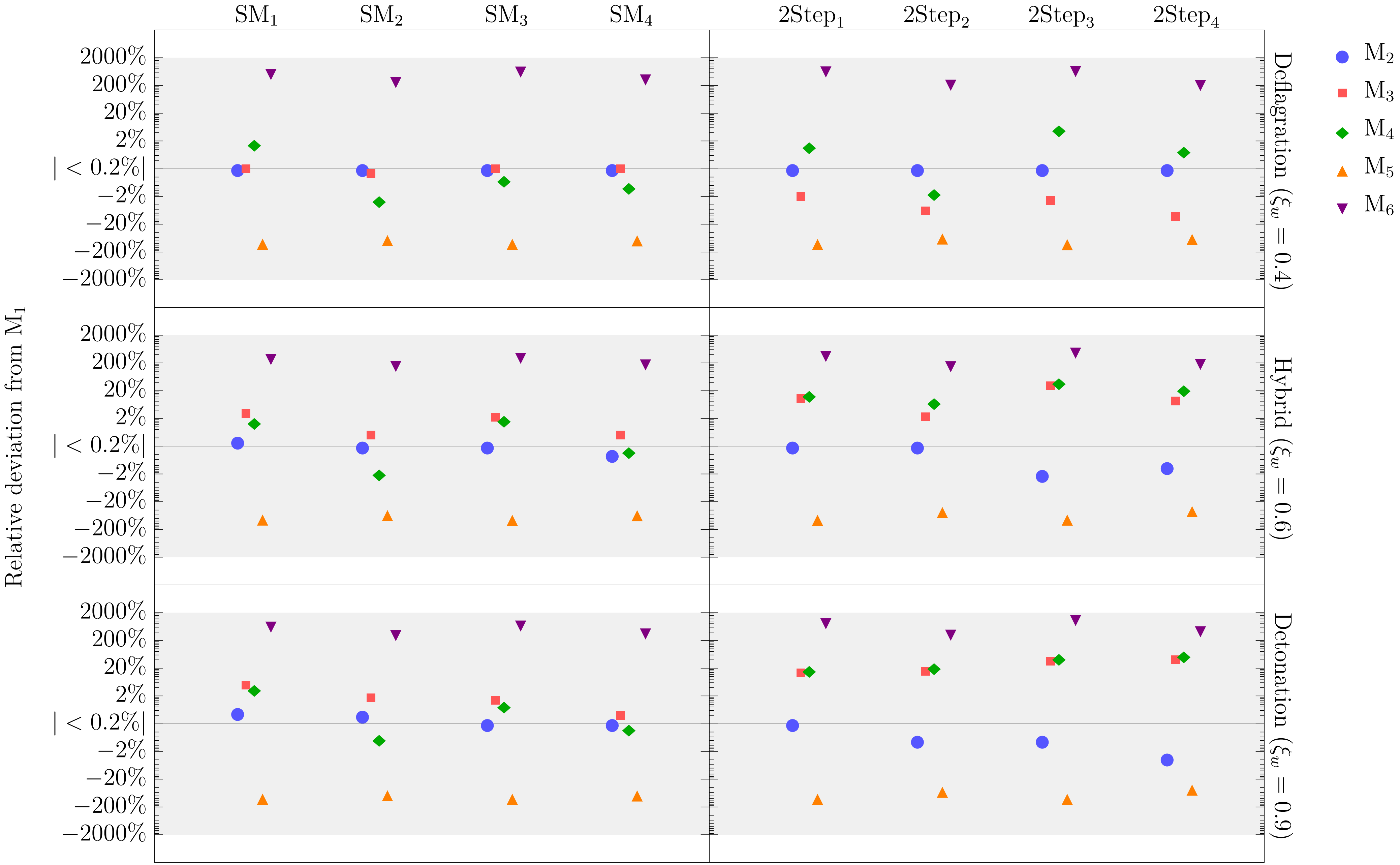
NNLO (cg^3): one-loop soft (ultrasoft) terms in p_{sym} (V_{eff}).

N^{3}LO (dg^4): two-loop (three-loop) soft (hard) pieces in p_{sym} and two-loop ultrasoft pieces in V_{eff}. Tree-level V_{eff} includes contributions at this order, via the resummed parameters in the EFT.

N^{4}LO (eg^5): three-loop soft (ultrasoft) contributions to p_{sym} (V_{eff}).

Six different methods to determine K

M1	Full numerical solution
M2	Assuming constant speed of sound and use Python snippet
M3&4	Mapping onto bag model via trace of energy momentum tensor
M5	Mapping onto bag model via pressure difference
M6	Mapping onto bag model via energy-density difference


SM1
SM2
SM3
SM4
2Step1
2Step2
2Step3
2Step4
M2
M3
M4
M5
M6
Deflagration ($\xi_w = 0.4$)
Hybrid ($\xi_w = 0.6$)
Detonation ($\xi_w = 0.9$)
Relative deviation from M1
2000%
200%
20%
2%
$|<0.2\%|$
−2%
−20%
−200%
−2000%