# Cosmology Prior to the BBN and its Impact on Gravitational Waves

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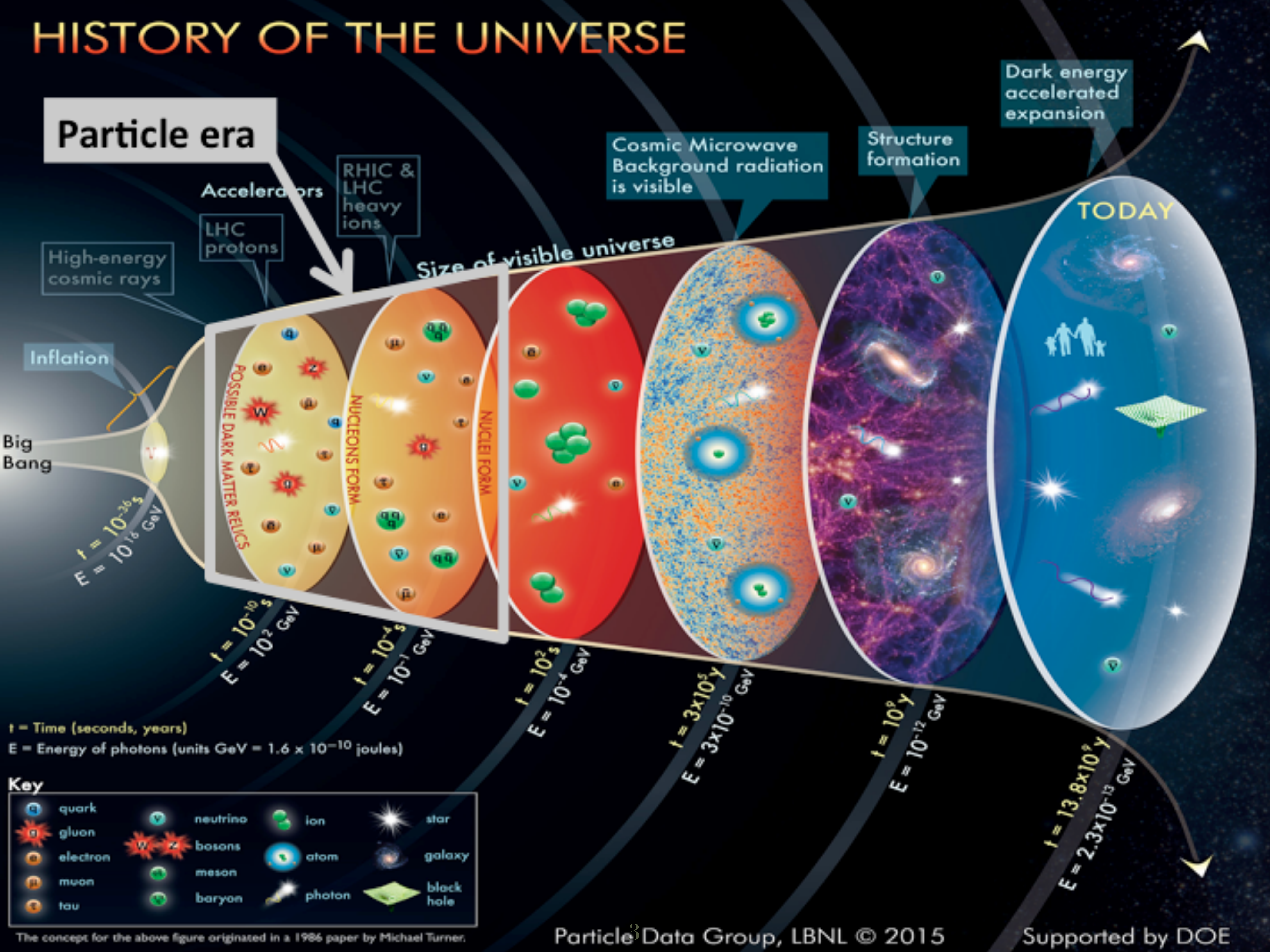
# Overview

- Primordial gravitational waves in standard cosmology
- Pion condensation in the early universe at nonvanishing lepton flavour asymmetry and its gravitational wave signatures
- Primordial gravitational waves in nonstandard cosmologies
- Primordial gravitational waves in modified cosmologies
- Thermal history of the early universe and the induced PGW
- Primordial black hole dominated era and QCD axion dark matter


HISTORY OF THE UNIVERSE
Particle era
Accelerators
RHIC & LHC heavy ions
LHC protons
High-energy cosmic rays
Inflation
Big Bang
Size of visible universe
Cosmic Microwave Background radiation is visible
Structure formation
Dark energy accelerated expansion
TODAY
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
E = 3x10^-10 GeV
t = 10^9 y
E = 10^-12 GeV
t = 13.8x10^9 y
E = 2.3x10^-13 GeV
t = Time (seconds, years)
E = Energy of photons (units GeV = 1.6 x 10^-10 joules)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
The concept for the above figure originated in a 1986 paper by Michael Turner.
Particle Data Group, LBNL © 2015
Supported by DOE

# Tensor Perturbation and Primordial Gravitational Waves
# Relic Density

Evolution equation for gravitational wave amplitude "h" (1st order):

$$h^{''}(k,\eta) + 2\mathcal{H}(\eta)h^{'}(k,\eta) + k^2 h(k,\eta) = 0\,, \quad \mathcal{H} = a'/a = aH$$

Wave number

Conformal time

Hubble rate:

$$H^2 = \frac{8\pi G}{3}\rho_{\rm tot}$$

Tensor perturbation polarisation modes:

Transfer function

$$h_\lambda(k,\eta) = h_\lambda^{\rm prim}(k)Y(\eta,k) = \frac{v(k,\eta)}{a(\eta)}$$

Tensor perturbation in the comoving frame:

$$v^{''}(k,\eta) + \left(k^2 - \frac{a^{''}}{a}\right)v(k,\eta) = 0$$



Starobinsky 1979 Guth 1980 Linde 1981 Maggiore 1999

PGW energy density:

$$\rho_{\rm GW}(\eta) = \frac{M_{\rm Pl}^2}{32\pi a(\eta)^2} \left\langle h'_{ij}(k,\mathbf{x}) h^{ij\prime}(k,\mathbf{x}) \right\rangle$$

$$\left\langle h'_{ij}(k,\mathbf{x}) h^{ij\prime}(k,\mathbf{x}) \right\rangle = \int \frac{dk}{k} \mathcal{P}_T(k,\eta)$$

Tensor power spectrum:

$$\mathcal{P}_T(k,\eta) = \frac{k^3}{\pi^2} \sum_\lambda \left\langle |h_\lambda(k,\eta)|^2 \right\rangle = \mathcal{P}_T^{\rm prim}(k) [Y(k,\eta)]^2$$

PGW relic density:

$$\Omega_{\rm GW}(k,\eta) = \frac{\mathcal{P}_T^{\rm prim}(k)}{12 a(\eta)^2 H(\eta)^2} [Y'(k,\eta)]^2$$

At horizon crossing:

$$[Y'(k,\eta)]^2 = k^2 [Y(k,\eta)]^2$$

In standard cosmology:

Assuming no phase transition or modified cosmology → Entropy conservation

Friedmann equation:

$$\frac{a''}{a} = \frac{4\pi G}{3} a^2 (\rho_{tot} - 3p_{tot})$$

Trace anomaly:

$$\frac{I(T)}{T^4} = \frac{\rho_{\rm tot} - 3p_{\rm tot}}{T^4} = T \frac{\partial}{\partial T} \left( \frac{p_{\rm tot}}{T^4} \right)_{\mu/T}$$

Horizon scale:

$$k = a(\eta_{\rm hc}) H(\eta_{\rm hc})$$

We do not consider the effect of neutrinos and photons damping on the PGW, since we are interested in temperatures above 10 MeV.

PGW relic density:

$$\Omega_{\rm GW}(k,\eta_0) \propto \Omega_{\rm GW}(k,\eta_{\rm hc}) \propto k^5 |v(k,\eta_{\rm hc})|^2$$

It gives roughly a flat spectrum

PGW relic density and the SM equation of state:

$$\Omega_{\rm GW}(k,\eta_0) \propto \rho_{\rm tot}(T_{hc}) s_{\rm tot}(T_{\rm hc})^{-4/3}$$

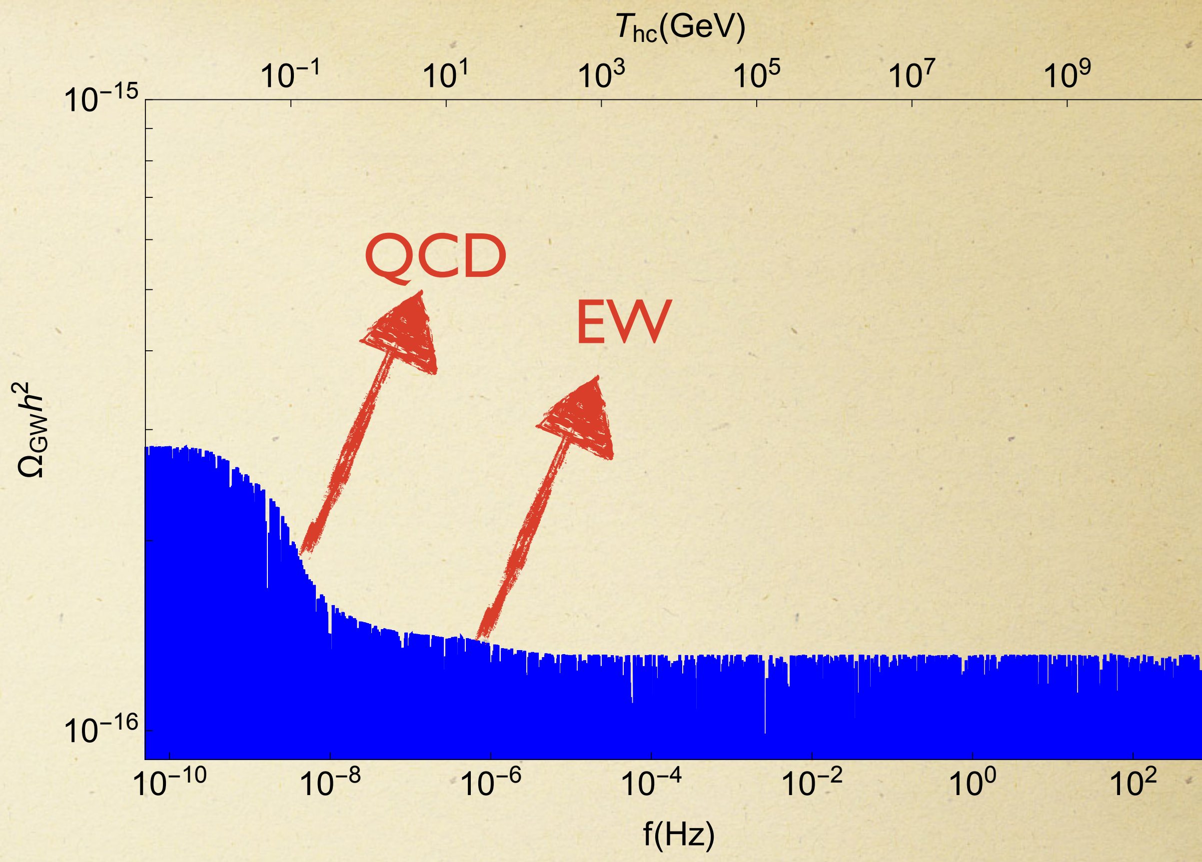
Any changes of degrees of freedom or equation of state in the early universe especially around cosmic transitions influence on the PGW spectrum. QCD affects the GW background in the frequency range of pulsar timing arrays, e.g. EPTA, SKA, etc.



Schwarz 1997 Watanabe & Komatsu, 2006 Saikawa & Shirai, 2018 F. H., J. Schaffner-Bielich, et al. arXiv:1904.01046

Scale independent tensor power spectrum and the scale of inflation:

$$\mathcal{P}_T(k) = \frac{2}{3\pi^2}\frac{V_{\rm inf}}{M_{Pl}^4}\ ,\ V_{\rm inf}^{1/4} = 1.5 \times 10^{16}\ GeV$$




Tensor power spectrum and its scale dependence:

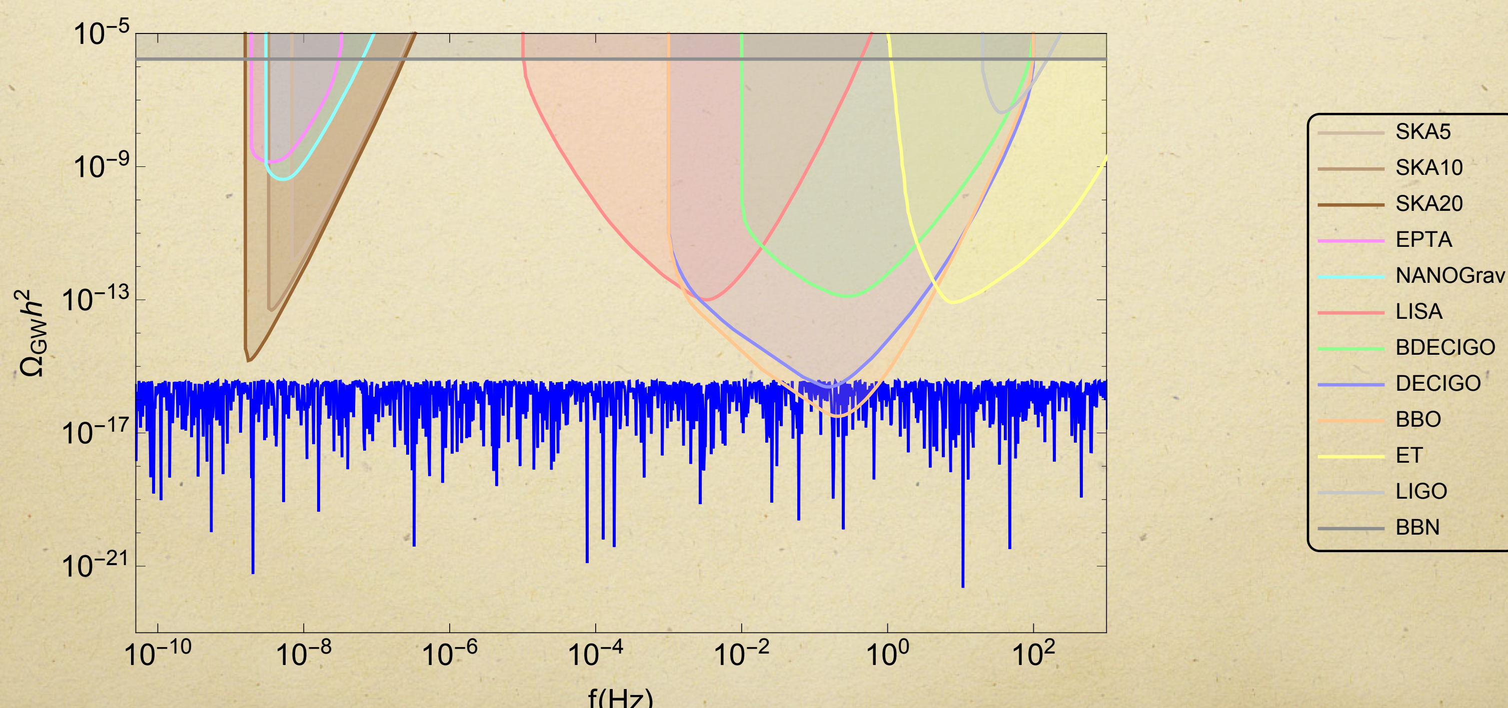
$$\mathcal{P}_T(k) = A_T \left(\frac{k}{\bar{k}}\right)^{n_T}$$

Tensor tilt

Pivot scale

Tensor to scalar perturbation ratio:

$$r \equiv \frac{A_T}{A_S}$$



# Pion Condensation in the Early Universe at Nonvanishing Lepton Flavour Asymmetry

- Using hadron resonance gas and the effective mass model of pions for lepton flavour asymmetry and pion condensation regime in the early universe.
- Lattice QCD at nonvanishing isospin chemical potential and nonzero temperature.
- Equation of state for cosmology: predictions for the spectrum of primordial gravitational waves and mass distribution of primordial black hole formation.

V.Vovchenko et al. 2009.02309

# Lepton Asymmetry in the Early Universe

Conservation of charge, lepton number, and baryon number in the early universe:

$$\frac{n_Q(T, \mu_B, \mu_Q, \{\mu_l\})}{s(T, \mu_B, \mu_Q, \{\mu_l\})} = 0$$

$$\frac{n_{L_\alpha}(T, \mu_Q, \{\mu_\alpha\})}{s(T, \mu_B, \mu_Q, \{\mu_\alpha\})} = l_\alpha, \quad \alpha \in e,\, \mu,\, \tau$$

$$\frac{n_B(T, \mu_B, \mu_Q)}{s(T, \mu_B, \mu_Q, \{\mu_l\})} = b$$

Constraints from cosmology:

$$b = \frac{n_B}{s} \approx 8 \times 10^{-11}$$

$$l = \frac{n_L}{s} \lesssim 0.012$$

from CMB and number of effective neutrinos

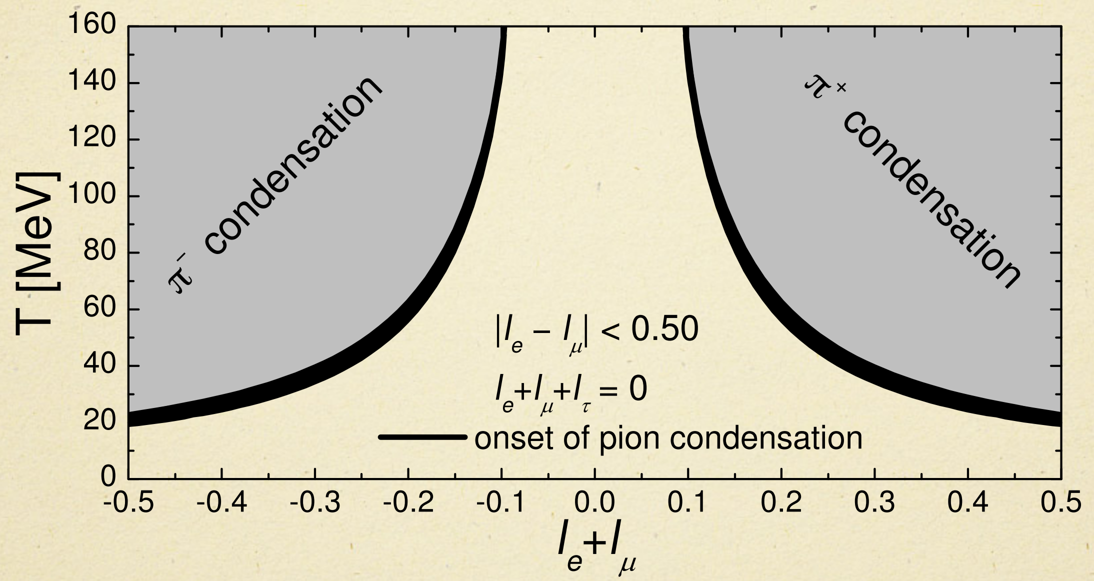
Pressure (and equation of state) in the thermal bath of the early Universe:

$$p = p_{\mathrm{QCD}}(T, \mu_B, \mu_Q) + p_L(T, \mu_Q, \{\mu_l\}) + p_\gamma(T)$$

M. Wygas et al., 1807.10815 Planck collab., 1502.01589 Oldengott, Schwarz, 1706.01705

# Pion Condensation

Using Thermal-FIST package for hadron resonance gas model the onset of pion condensation is computed:




It is done by scanning over $|l_e - l_\mu|$ and $|l_e + l_\mu|$ .

Pion-pion interactions implemented into the HRG model to account for the pion condensed phase (using effective mass model).

Pion condensation (Bose-Einstein condensation of pions) can happen if

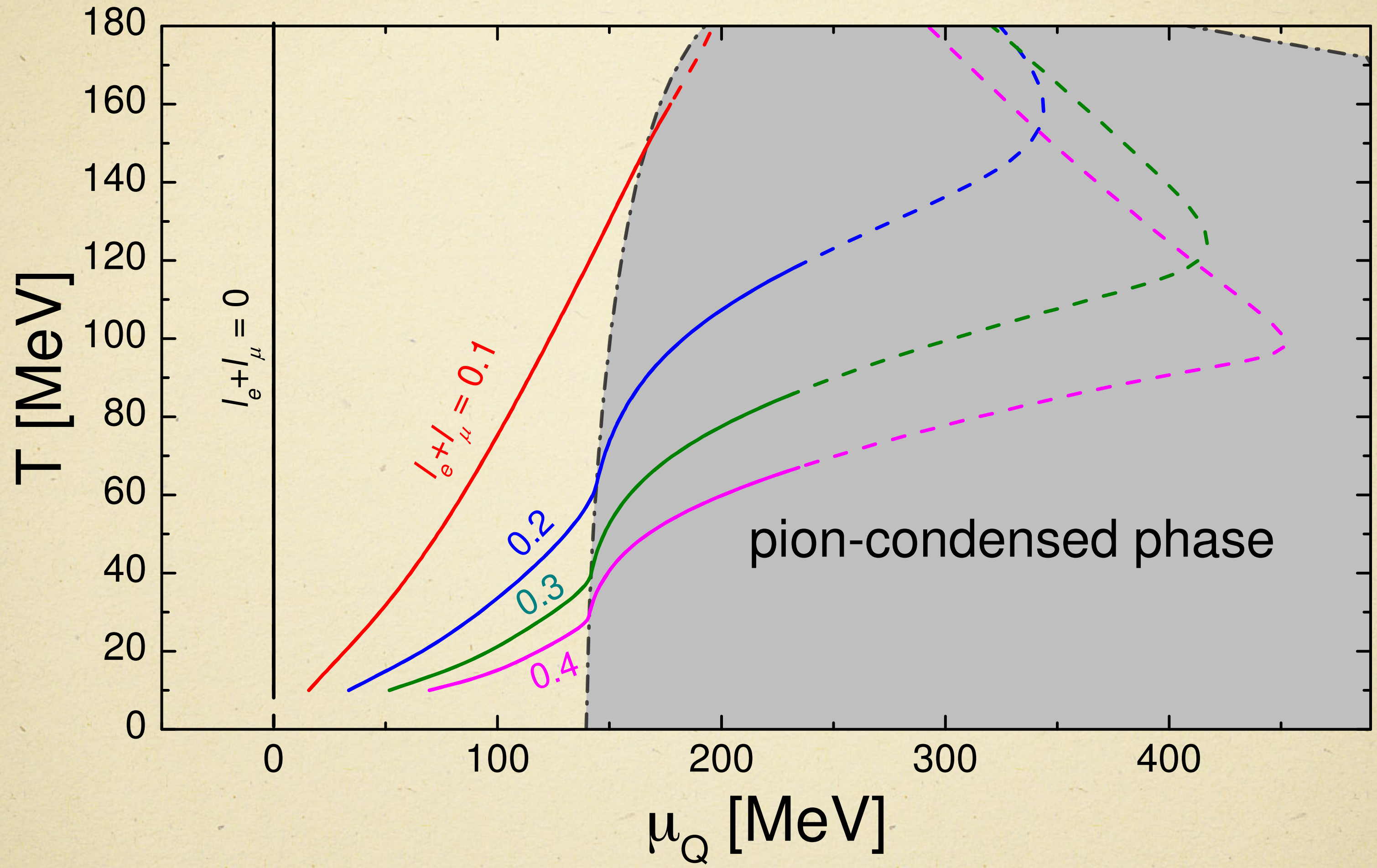
$T < 160\ MeV$ at $|\mu_Q| > m_\pi$.

V.Vovchenko, H. Stocker 1901.05249



V.Vovchenko et al. 2009.02309

Cosmic trajectories for different lepton flavour asymmetries:




The lower bound on the lepton flavour asymmetry that can lead to the pion condensation formation can be found:

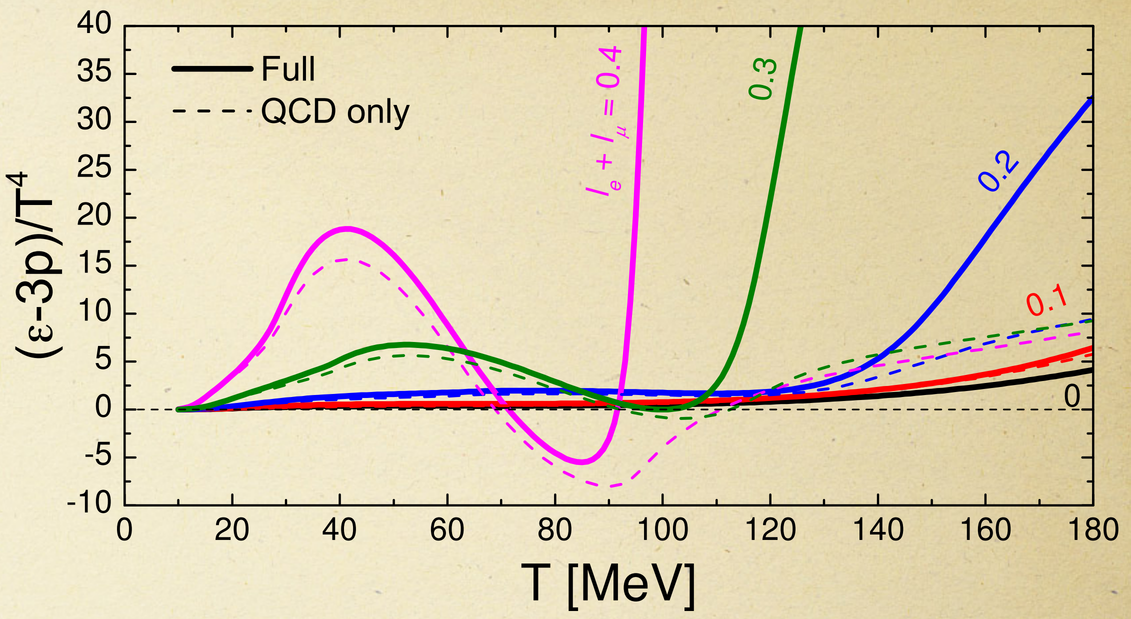
$$|l_e + l_\mu| \gtrsim 0.1$$

Larger lepton asymmetry blocks the fermi levels for leptonic decay products of charged pions and boost the pion condensed phase formation!

V.Vovchenko et al. 2009.02309 Middeldorf-Wygas et al. 2009.00036

Trace anomaly versus temperature:

The trace anomaly can be negative inside the pion condensation regime!

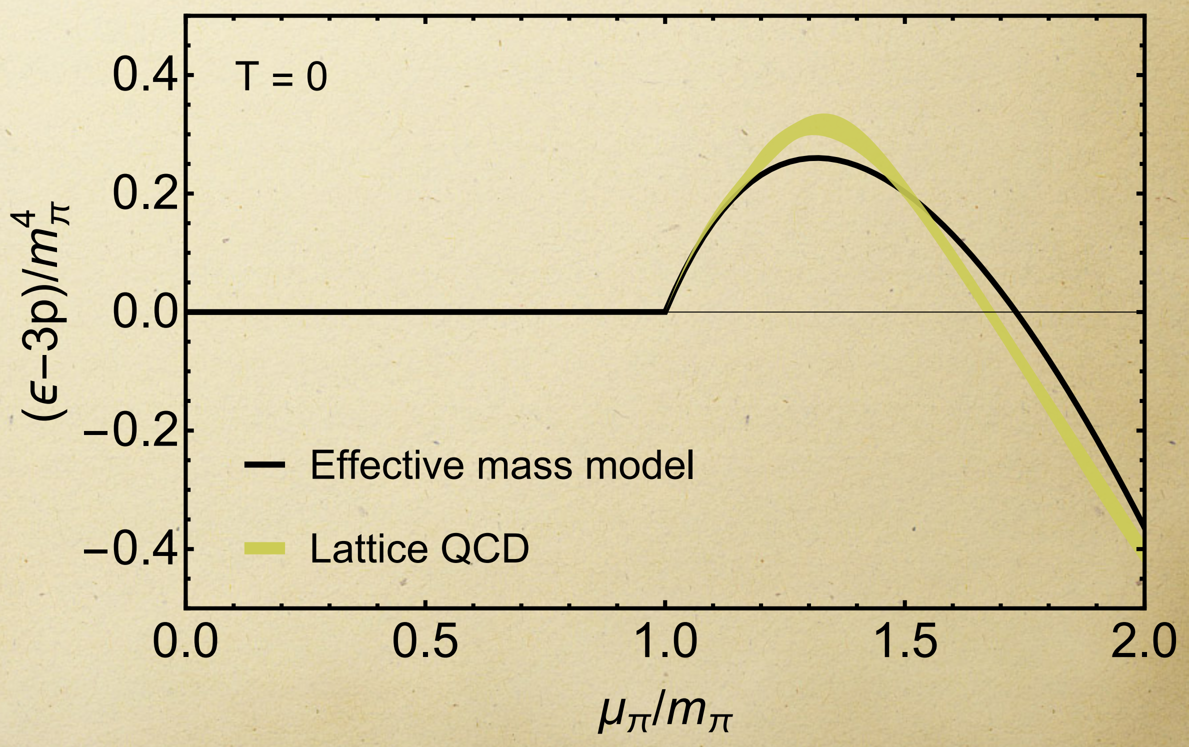


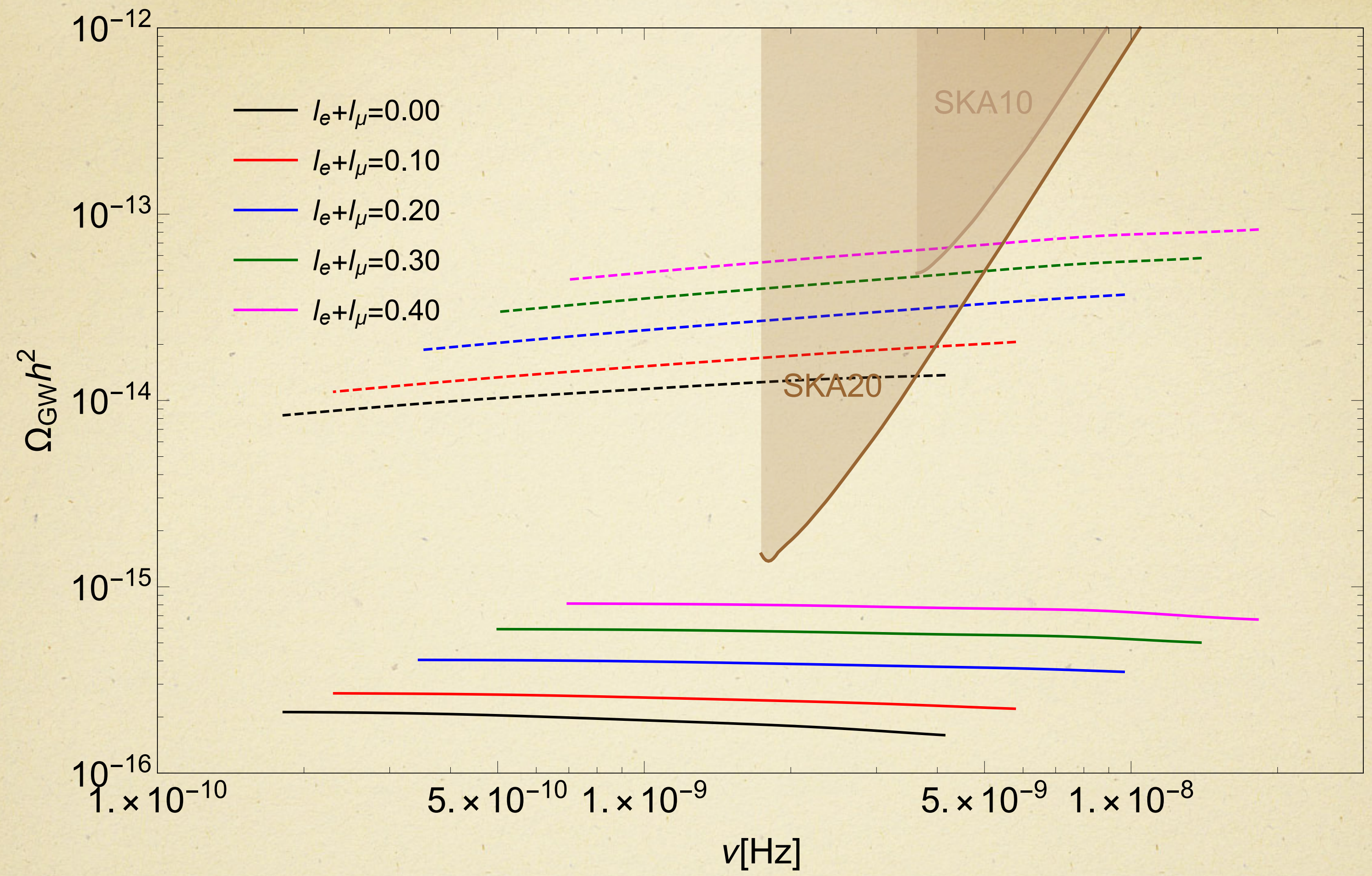

We can not have first order QCD phase transition even based on reasonably large values of non vanishing lepton flavour asymmetry! So only the modification of Hubble factor due to the change of equation of state around QCD transition can affect cosmology.

Trace anomaly versus pion chemical potential at zero temperature:

The pion chemical potential becomes equal to pion effective mass at the start of pion condensation.

At T=0 the effective mass model is matched with $\chi PT$.






The effect of non vanishing lepton asymmetry on the PGW comes from the increasing of value of entropy density which changes the scale factor and temperature relation very much and also from the change of the equation of state parameter that modifies the Hubble rate.

For frequencies around nanohertz the PGW spectrum can be measured by Pulsar Timing Array experiments (SKA, EPTA, NanoGRAV).

Recently the NanoGRAV experiment reported a signal of gravitational wave with cosmic origin however probably it is from some astrophysical background.

N. Bernal, F. Hajkarim 1905.10410



V.Vovchenko et al. 2009.02309

# Primordial Black Holes

High density regions in the early universe can collapse to form PBH.

Density perturbation:

$$\delta = \left(\frac{M_{\rm BH}}{K M_h}\right)^{\frac{1}{\gamma}} + \delta_c$$

PBH mass

Threshold for primordial black hole formation

Parameter from PBH formation simulation

Horizon mass

Fraction of PBH w.r.t. total dark matter abundance:

$$f_{\rm PBH}(M_{\rm BH}) = \frac{1}{\Omega_{\rm CDM}} \int_0^{\infty} \frac{2 dM_h}{\sqrt{2\pi\sigma(M_h)^2}} \frac{M_{\rm BH}}{\gamma M_h} \exp\left[-\frac{\delta^2(M_h)}{2\sigma^2(M_h)}\right] \left(\frac{M_{\rm BH}}{K M_h}\right)^{\frac{1}{\gamma}} \sqrt{\frac{M_{eq}}{M_h}}$$

Parameter from PBH formation simulation

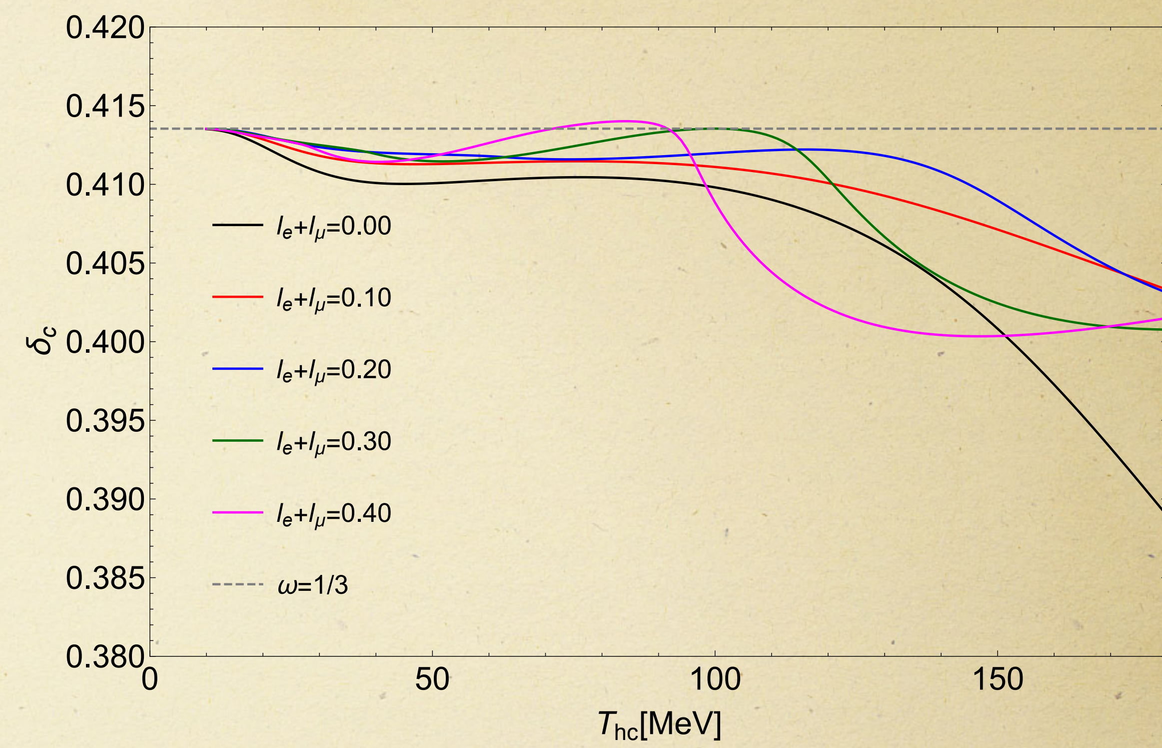
Horizon mass at matter radiation equality

$$M_h = \frac{4\pi}{3} H^{-3} \varepsilon$$

Energy density of the universe

V.Vovchenko et al. 2009.02309

Variation of the threshold of PBH formation versus horizon crossing temperature:

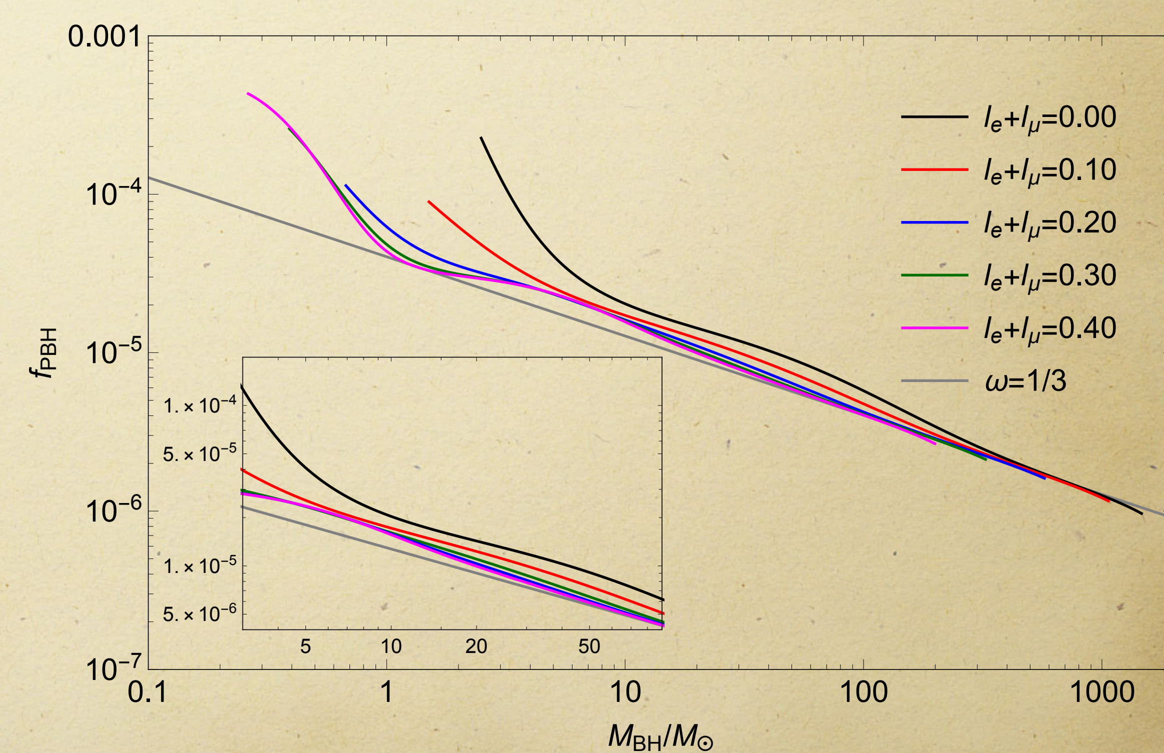



Threshold for PBH formation for a radiation like fluid from simulation and analytic estimation: $\delta_c \simeq 0.41$

Analytical formula for PBH threshold:

$$\delta_c = \frac{3(1+\omega)}{5+3\omega} \sin^2 \left[ \frac{\pi\sqrt{\omega}}{1+3\omega} \right]$$

Different experiments can put upper bounds on the PBH parameter space. Fraction of PBH versus BH mass:




Recent LIGO observation can be from the merger of PBHs formed in the pion condensed phase! => LIGO GW190521

T. Herada, C. Yoo, K. Kohri 1309.4201



V.Vovchenko et al. 2009.02309

# Nonstandard cosmologies and the PGW

- The history of universe before big bang nucleosynthesis is unknown.
- UV completion theories predict nonstandard cosmologies (by new scalar fields) beyond the standard radiation dominated era before BBN and after inflationary epoch.
- The production mechanism of dark matter in the early universe is unknown. No hints for DM produced from the standard radiation dominated scenario!



Laine & Meyer, 2015 Borsanyi et al. 2016 N. Bernal, F. H., arXiv:1905.10410

Friedmann equations in nonstandard cosmology:

Equation of state parameter for the dominant component

$$\frac{d\rho_\phi}{dt} + 3(1+\omega_\phi)H\rho_\phi = -\Gamma_\phi \rho_\phi$$

$$\frac{ds_R}{dt} + 3\,H\,s_R = +\frac{\Gamma_\phi\,\rho_\phi}{T}$$

$$H^2 = \frac{\rho_\phi + \rho_R + \rho_m + \rho_\Lambda}{3\,M_{Pl}^2}$$

Temperature at which $\phi$ decays:

$$T_{\rm dec}^4 = \frac{90}{\pi^2\,g_\star(T_{\rm dec})}\,M_{Pl}^2\,\Gamma_\phi^2$$

SM degrees of freedom

Decay width

Ratio of initial densities:

$$\xi \equiv \left.\frac{\rho_\phi}{\rho_R}\right|_{T=T_{\rm max}}$$

Initial value for temperature in the numerical calculation (not physical only to cover all possible cases that cross the experimental constraints):

$$T_{\rm max} = 10^{14}\ GeV$$



Tashiro & Shiba & Sasaki 2003 Nakayama & Takahashi 2010 Durrer & Hessenkamp 2011 Ramberg & Visinelli 2019 D'Eramo & Schmitz 2019

Hubble rate during radiation domination:

$$H_R \propto a^{-2}$$

Hubble rate in $\phi$ domination:

$$H_\phi \propto a^{-\frac{3}{2}(1+\omega_\phi)}$$

The PGW relic for modes come inside the horizon during $\phi$ domination: $\Omega_{GW} \propto k^{-2\frac{1-3\omega_\phi}{1+3\omega_\phi}}$

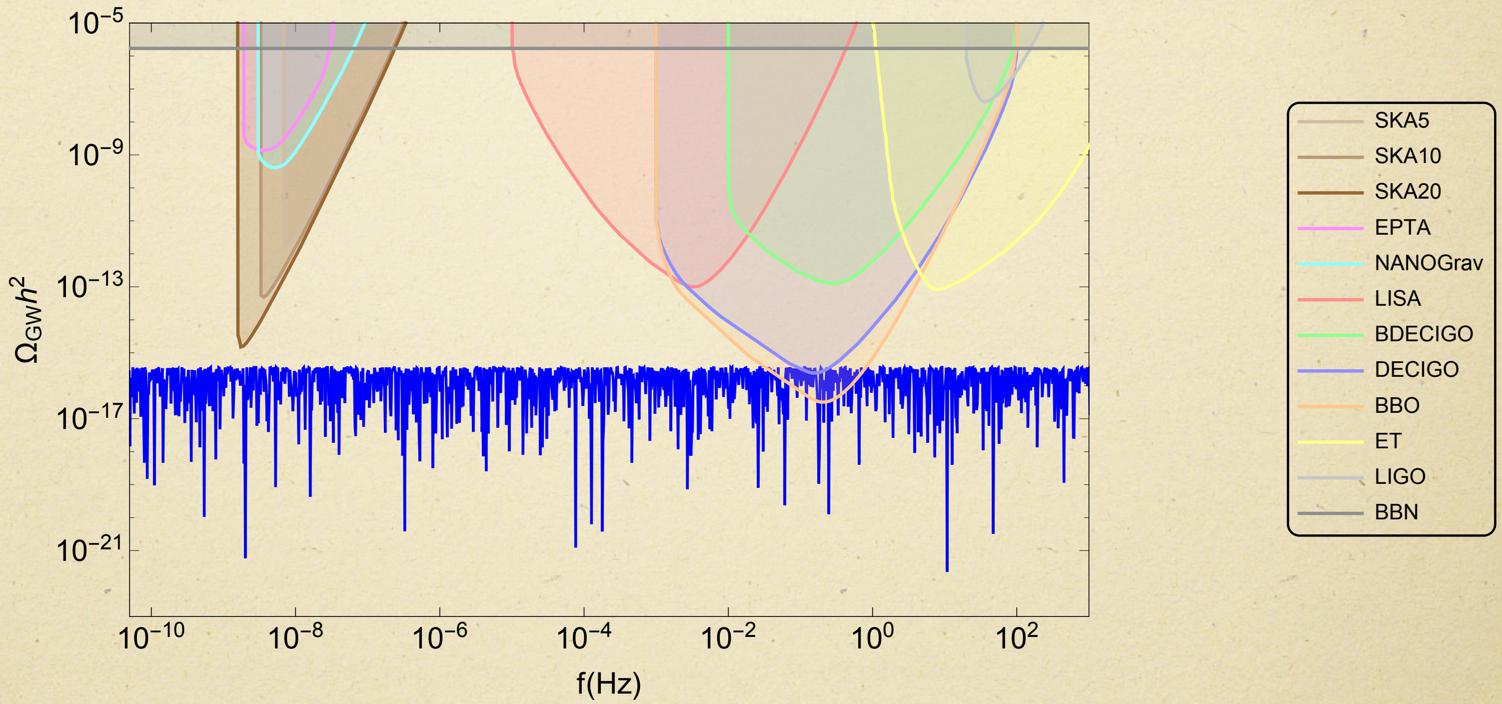
The minimum value of ratios to have a modified radiation density due to nonstandard cosmological scenario ( $\rho_R \propto a^{-4} \to a^{-\frac{3}{2}(1+\omega_\phi)}$):

$$\xi_{\min} \approx \left[ \left( \frac{g_\star(T_{\max})}{g_\star(T_{\text{dec}})} \right)^{\frac{1}{4}} \frac{T_{\max}}{T_{\text{dec}}} \right]^{3\omega_\phi - 1}$$

$$\omega_\phi = 1/3, \xi = 10^{25}$$

$$T_{\rm dec} = 10 \; MeV$$

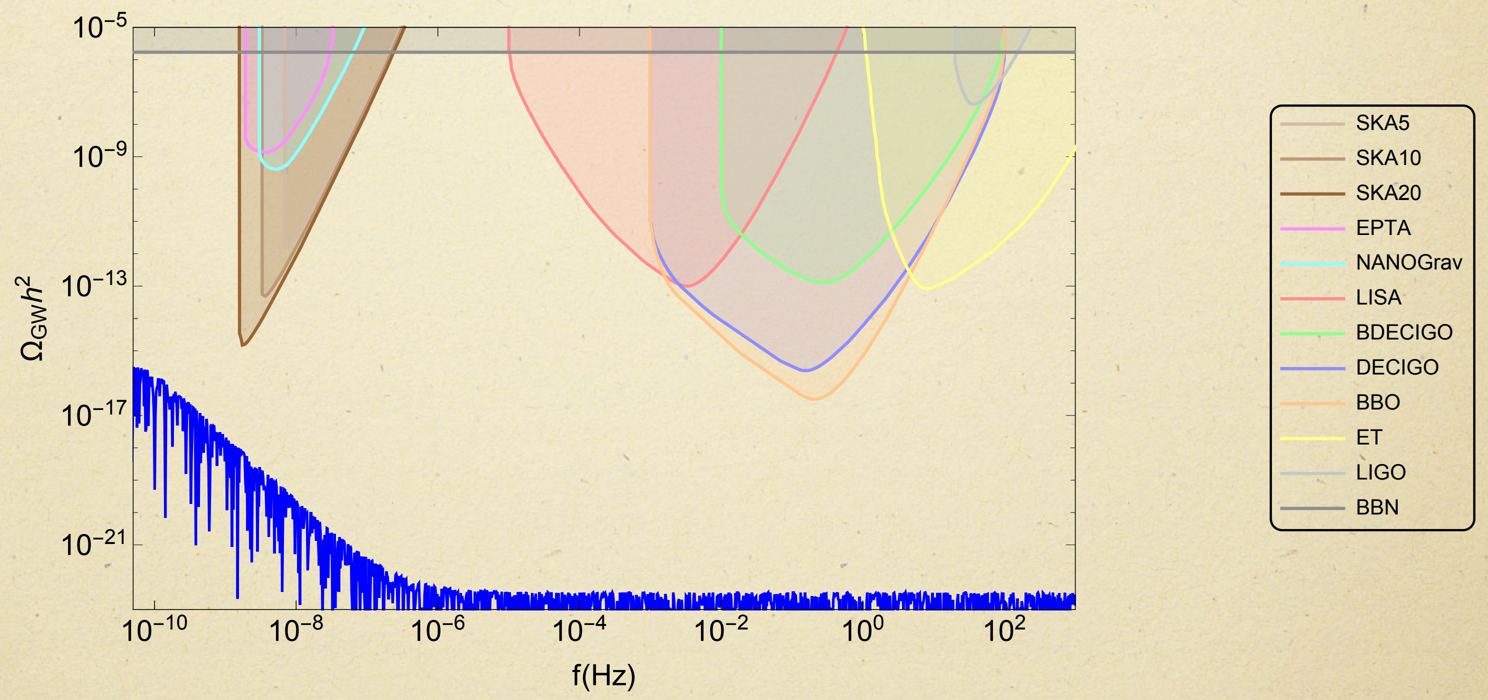
Radiation domination like scenario



$$\omega_\phi = 0, \xi = 10^{-11}$$

$$T_{\rm dec} = 10\ MeV$$

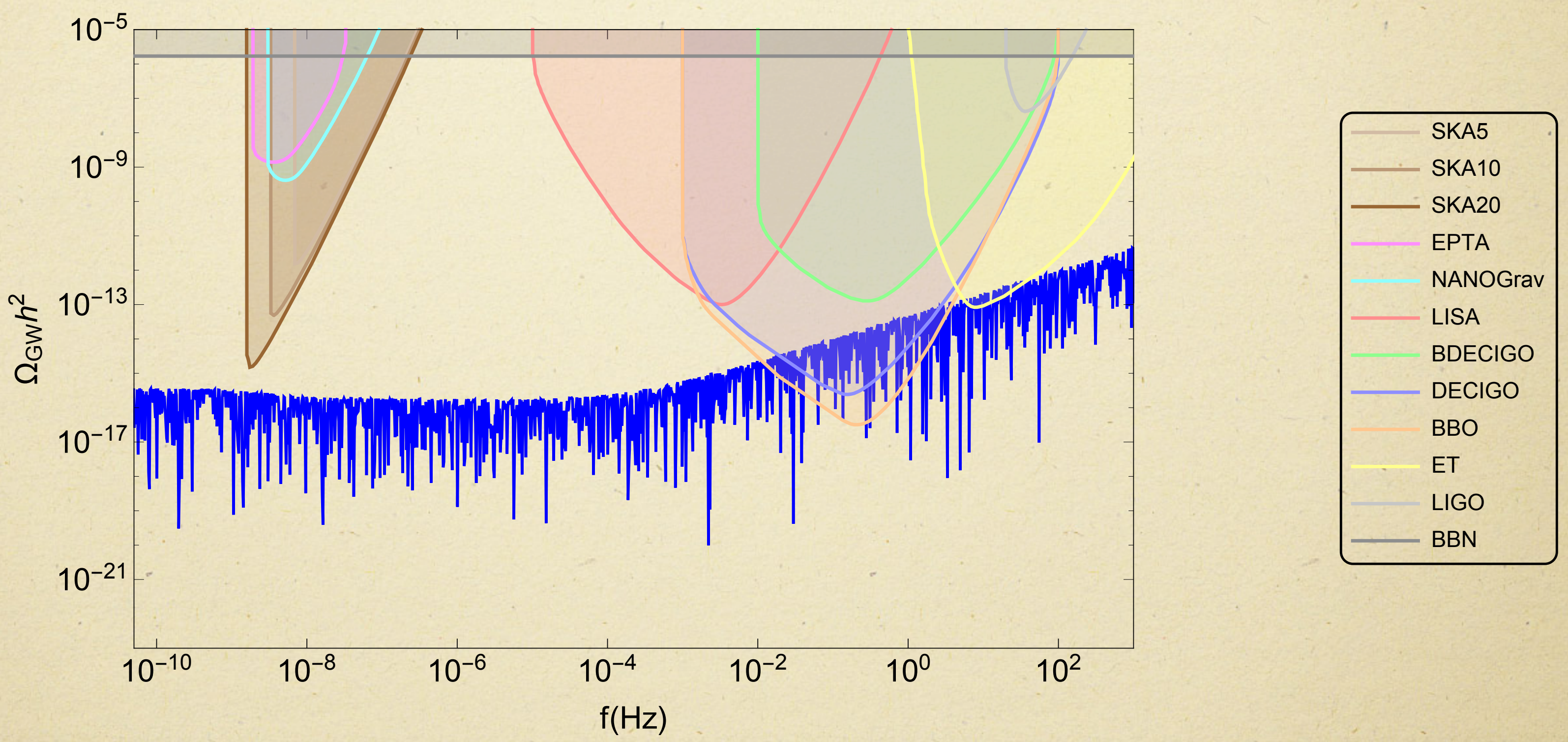
Modulus or matter domination like scenario



$$\omega_\phi = 2/3, \xi = 10^{10}$$

$$T_{\rm dec} = 10\ MeV$$

Kination domination like scenario



# Primordial Gravitational Waves in Modified Cosmologies

Modified gravity theories can modify the Hubble in the pre BBN epoch. Then they can affect the PGW spectrum.

Scalar-Tensor theory of gravity:

Jordan frame:

$$S_{\rm ST} = \frac{1}{16\pi\, G_*} \int d^4x \sqrt{-g}\, [F(\phi)\, R(g) - Z(\phi)\, g^{\mu\nu}\, \partial_\mu \phi\, \partial_\nu \phi - 2V(\phi)]$$

Conformal factor

$$g_{\mu\nu} = A_C^2(\phi_*)\, g_{*\mu\nu} \qquad A_C(\phi_*) = e^{\frac{1}{2}\beta\, \phi_*^2}$$

Einstein frame:

$$S_{\rm ST} = \frac{1}{16\pi G_*} \int d^4x_* \sqrt{-g_*}\, [R_*(g_*) - 2g_*^{\mu\nu} \partial_\mu \phi_* \partial_\nu \phi_* - 4V_*(\phi_*)]$$



N. Bernal, F. H., A. Ghoshal, G. Lambiase arXiv:2008.04959

## Hubble rate:

$$H = \frac{A_C(\phi_*)}{A_C(\phi_{*0})} \frac{1+\alpha(\phi_*)\frac{d\phi_*}{dN}}{\sqrt{1+\alpha^2(\phi_{*0})}\sqrt{1-\frac{1}{3}\left(\frac{d\phi_*}{dN}\right)^2}} H_{\rm GR}$$

## Scalar field equation of motion:

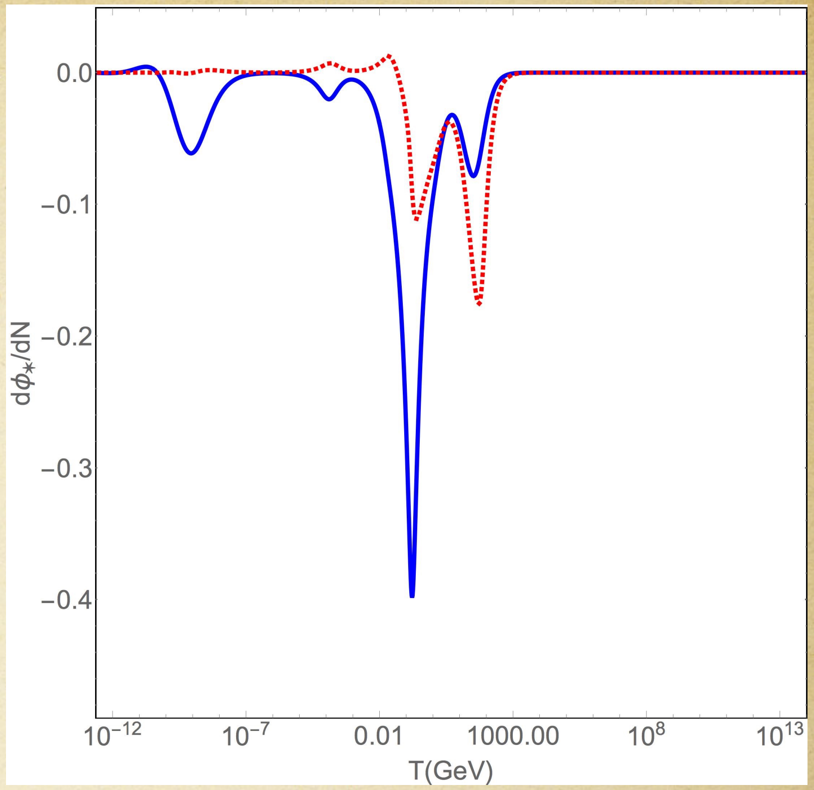
$$\frac{2}{3-\left(\frac{d\phi_*}{dN}\right)^2}\frac{d^2\phi_*}{dN^2} + [1-\omega]\frac{d\phi_*}{dN} + \alpha(\phi_*)\,[1-3\omega] = 0$$

Equation of state parameter where thermal history plays role!

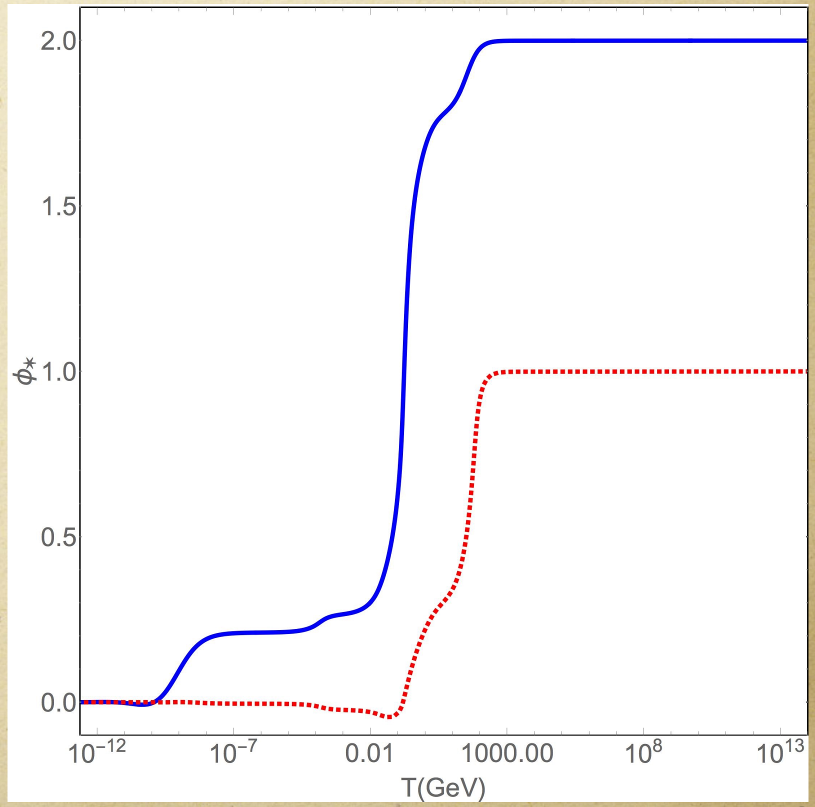
$$G = G_*\,A_C^2(\phi_{*0})\,\left[1+\alpha^2(\phi_{*0})\right]$$

Gravitational constant

$$\alpha(\phi_*) \equiv \frac{d\ln A_C(\phi_*)}{d\phi_*}$$

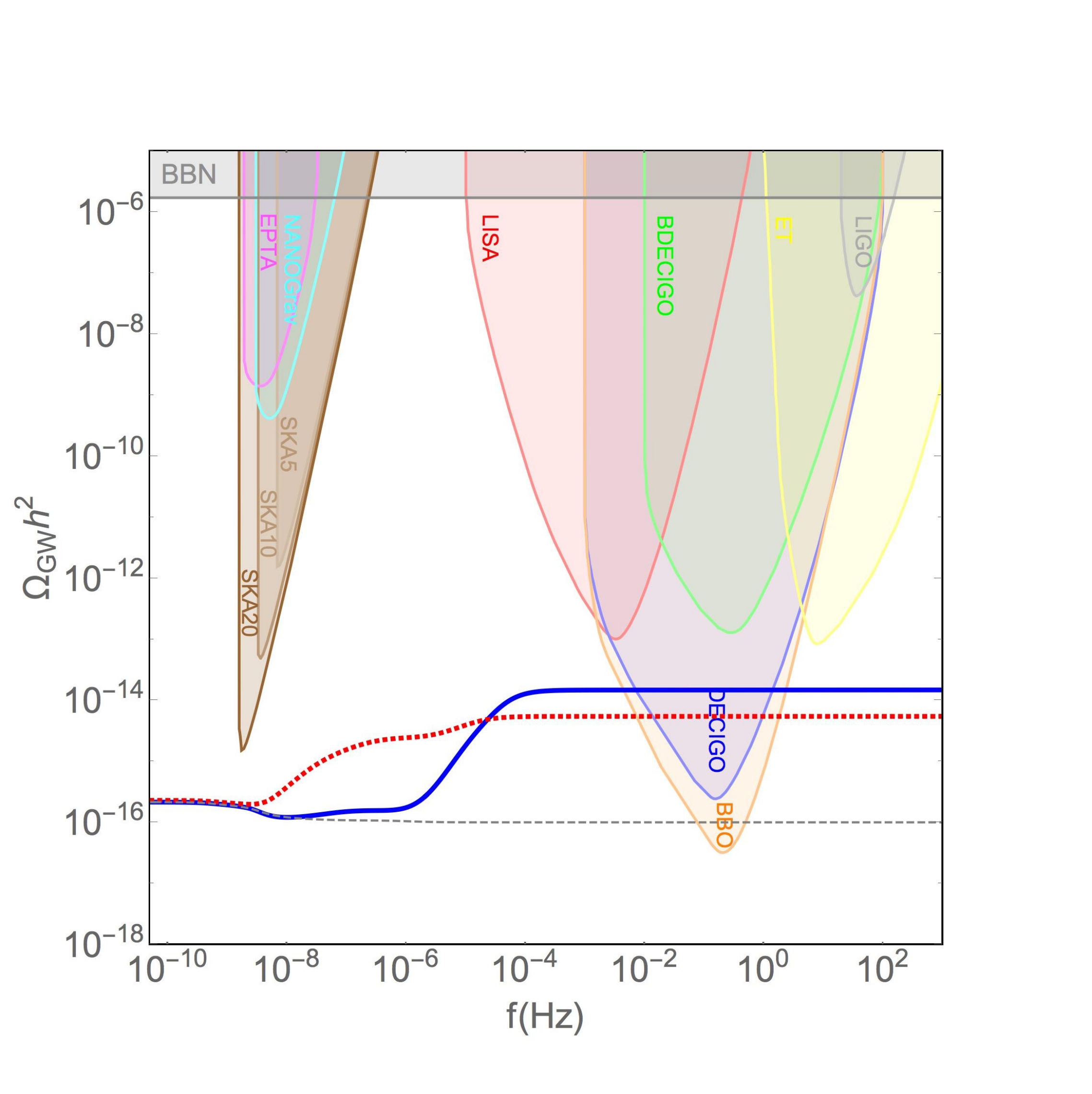



$$[\beta, \phi_{*\rm in}] = [1,2]\&[5,1], \quad (d\phi/dN)_{\rm in} = 0$$






N. Bernal, F. H., A. Ghoshal, G. Lambiase arXiv:2008.04959




PGW spectrum in specific scalar tensor scenarios

$$[\beta, \phi_{*\text{in}}] = [1,2]\&[5,1], \quad (d\phi/dN)_{\text{in}} = 0$$



N. Bernal, F. H., A. Ghoshal, G. Lambiase arXiv:2008.04959

# Brane world cosmology:

Hubble rate:

Modified term from extra dimension

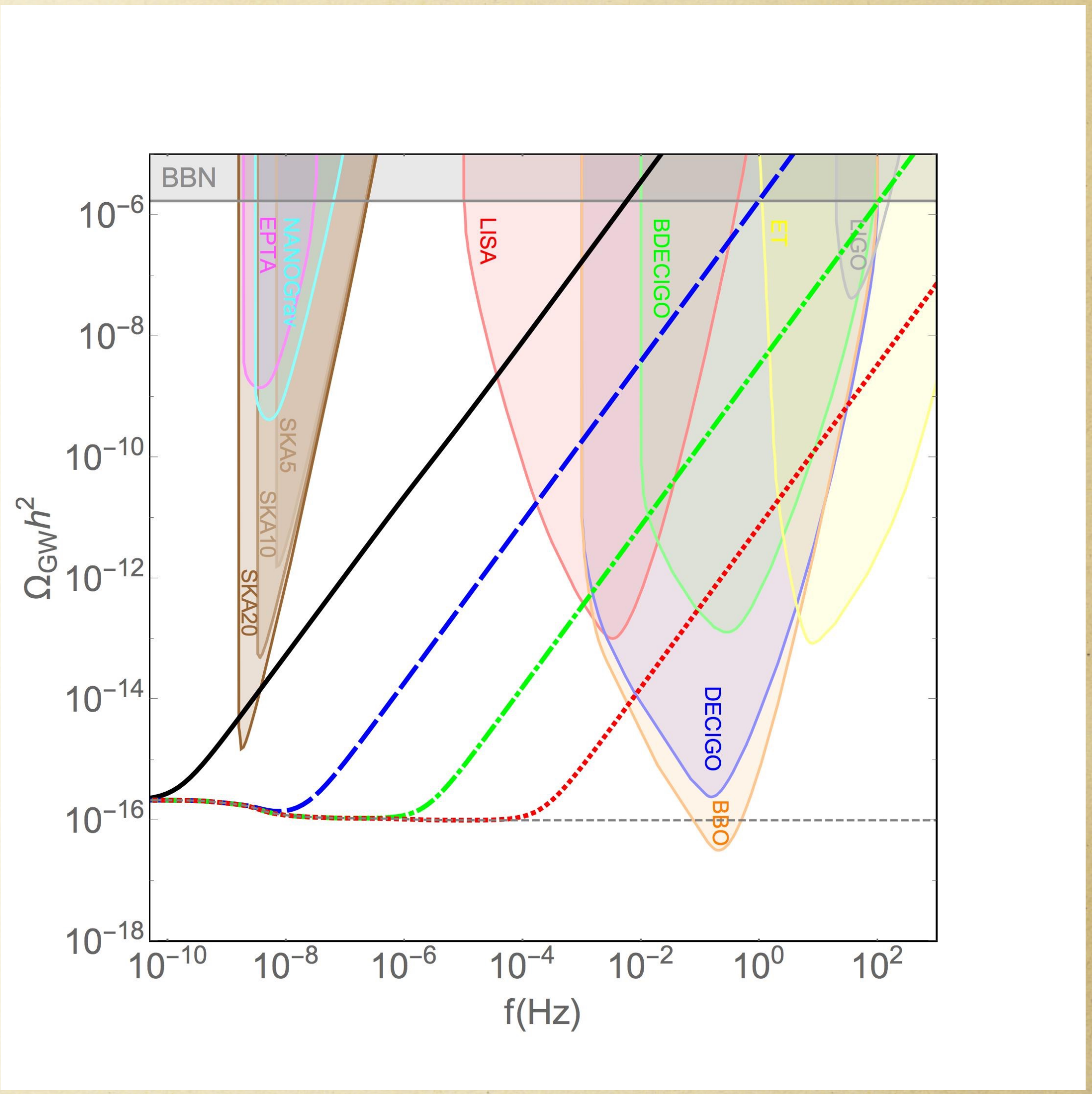
$$H^2 = \frac{8\pi G}{3}\rho\left(1+\frac{\rho}{\sigma}\right)$$

$$\sigma \equiv 96\pi\, G\, M_5^6$$

Brane tension

Five dimensional Planck's mass

$$\sigma = \frac{\pi^2}{30}\, g(T_\sigma)\, T_\sigma^4$$




$T_\sigma = 10\ \text{MeV}, 1\ \text{GeV}, 100\ \text{GeV}, 10\ \text{TeV}$

PGW relic density for brane world scenario dominated regime before BBN:

$$\Omega_{\rm GW}(\tau_0,\, k) = \frac{P_T(k)}{24\, a_0^4\, H_0^2}\left[H(a_\sigma)\, k^2\, a_\sigma^4\right]^{\frac{2}{3}} \propto \mathcal{P}_T(k)\, k^{\frac{4}{3}}$$



N. Bernal, F. H., A. Ghoshal, G. Lambiase arXiv:2008.04959

# Induced (2nd order) PGW from Scalar Perturbations

- In case the tensor-to-scalar ratio is very small then the first order PGW will be negligible and might not be in the range accessible by future experiments.
- There is not any lower limit on "r" at first order.
- The second order tensor perturbations induced from scalar perturbation can be the lower limit on the GW background.

$$h''_{\mathbf{k}}(\eta) + 2\mathcal{H}h'_{\mathbf{k}}(\eta) + k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$$



F. H., J. Schaffner-Bielich, arXiv:1910.12357

# 2nd Order Tensor Perturbation Sourced by Scalar Perturbation

Tensor perturbation equation at second order:

$$h''_{\mathbf{k}}(\eta) + 2\mathcal{H}h'_{\mathbf{k}}(\eta) + k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$$

Source from second order scalar perturbation ($\omega = \frac{p}{\rho}$):

$$S_{\mathbf{k}} = \int \frac{\mathrm{d}^3 q}{(2\pi)^{3/2}} e_{ij}(\mathbf{k}) q_i q_j \left( 2\Phi_{\mathbf{q}}\Phi_{\mathbf{k}-\mathbf{q}} + \frac{4}{3(1+w)} \left(\mathcal{H}^{-1}\Phi'_{\mathbf{q}} + \Phi_{\mathbf{q}}\right) \left(\mathcal{H}^{-1}\Phi'_{\mathbf{k}-\mathbf{q}} + \Phi_{\mathbf{k}-\mathbf{q}}\right) \right)$$

Second order tensor perturbation in the comoving frame:

$$a(\eta) h_{\mathbf{k}}(\eta) = 4 \int^{\eta} \mathrm{d}\bar{\eta}\, G_{\mathbf{k}}(\eta, \bar{\eta}) a(\bar{\eta}) S_{\mathbf{k}}(\bar{\eta})$$

↓ Green's function: solution of source free tensor perturbation equation



Kohri, Terada, 2018 Baumann, Steinhardt, Takahashi, Ichiki 2007 Ananda, Clarkson, Wands 2006

# 2nd Order Tensor Perturbation Sourced by Scalar Perturbation

Evolution of scalar perturbation ($c_s^2 = \frac{dp}{d\rho}$):

Speed of sound

$$\Phi_{\mathbf{k}}'' + 3\mathcal{H}(1 + c_{\rm s}^2)\Phi_{\mathbf{k}}' + (2\mathcal{H}' + (1 + 3c_{\rm s}^2)\mathcal{H}^2 + c_{\rm s}^2 k^2)\Phi_{\mathbf{k}} = 0$$

Second order tensor power spectrum:

$$\mathcal{P}_T(\eta, k) = 4\int_0^\infty \int_{|1-v|}^{1+v} {\rm d}v\ {\rm d}u \left[\frac{4v^2 - (1 + v^2 - u^2)^2}{4vu}\right]^2 I^2(v, u, x)\mathcal{P}_{\mathcal{R}}(kv)\mathcal{P}_{\mathcal{R}}(ku)$$

$$I(v, u, x) = \int_0^x {\rm d}\bar{x} \frac{a(\bar{\eta})}{a(\eta)} k G_k(\eta, \bar{\eta}) f(v, u, \bar{x})$$

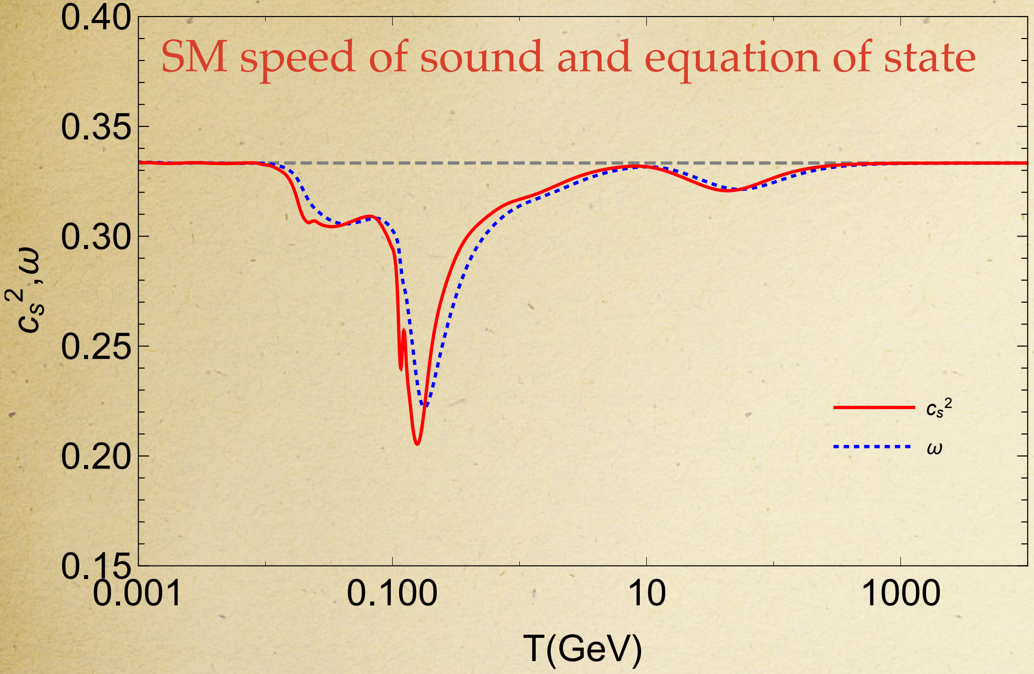
$$f(v, u, \bar{x}) = \frac{6(w+1)}{3w+5}\Phi(v\bar{x})\Phi(u\bar{x}) + \frac{12(w+1)}{(3w+5)^2\mathcal{H}}\left(\partial_{\bar{\eta}}\Phi(v\bar{x})\Phi(u\bar{x}) + \partial_{\bar{\eta}}\Phi(u\bar{x})\Phi(v\bar{x})\right) + \frac{12(1+w)}{(3w+5)^2\mathcal{H}^2}\partial_{\bar{\eta}}\Phi(v\bar{x})\partial_{\bar{\eta}}\Phi$$

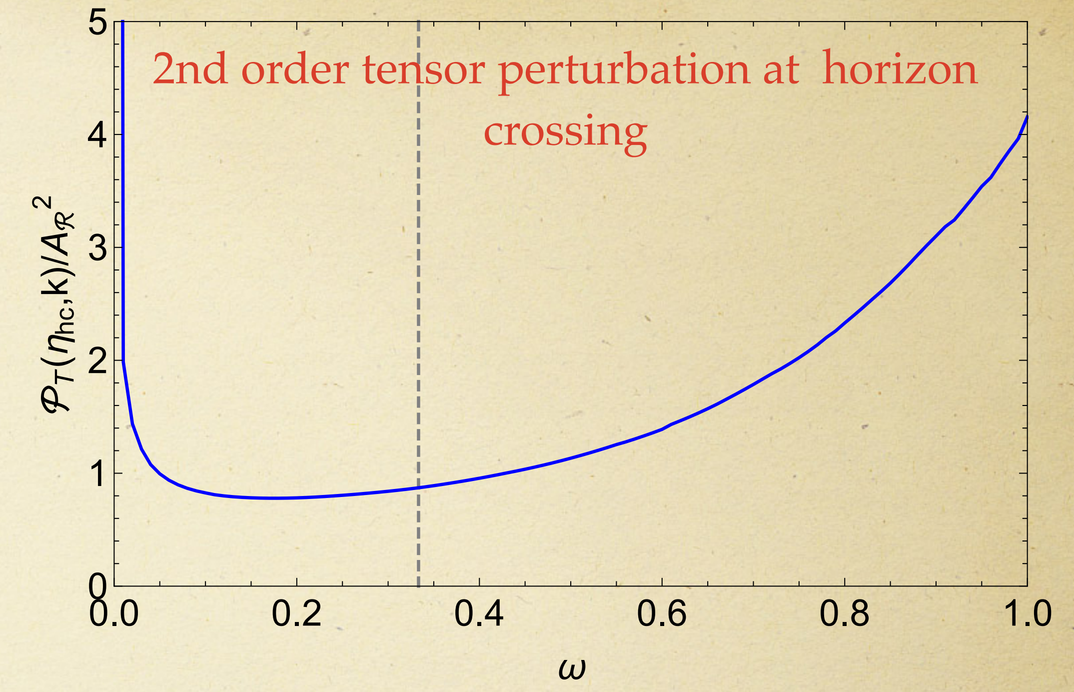
Assuming scale invariant curvature power spectrum:

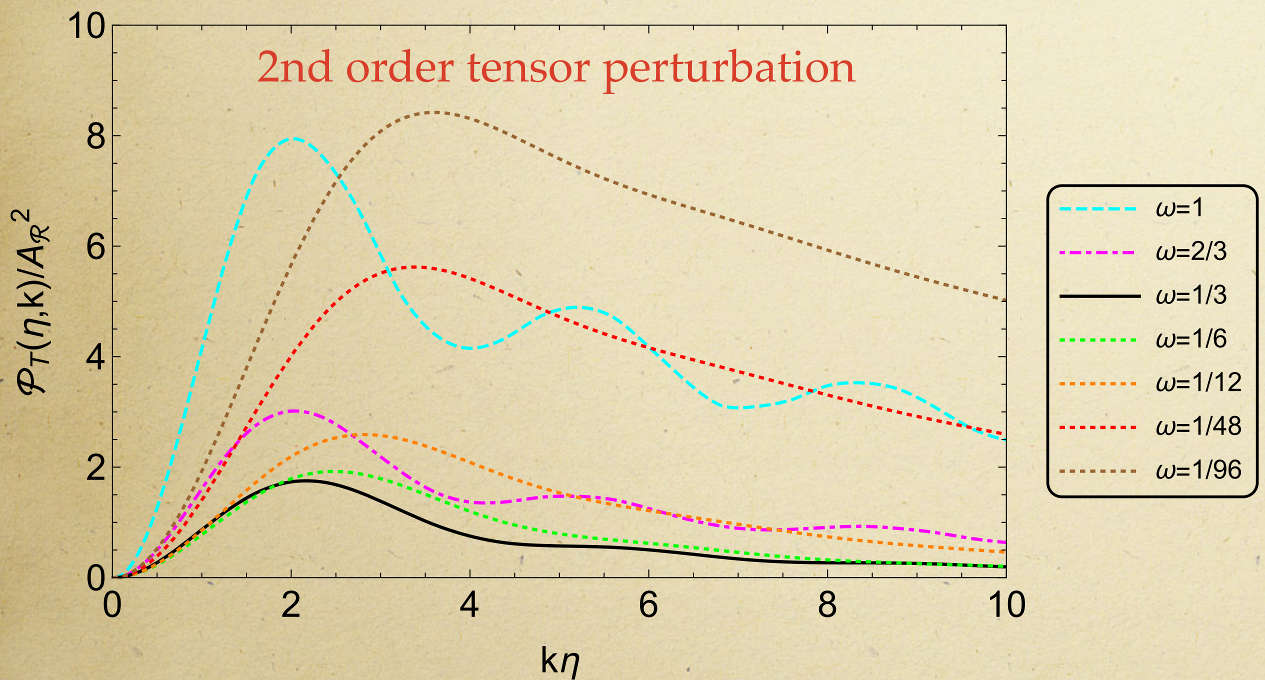
$$\mathcal{P}_{\mathcal{R}} = A_{\mathcal{R}}$$

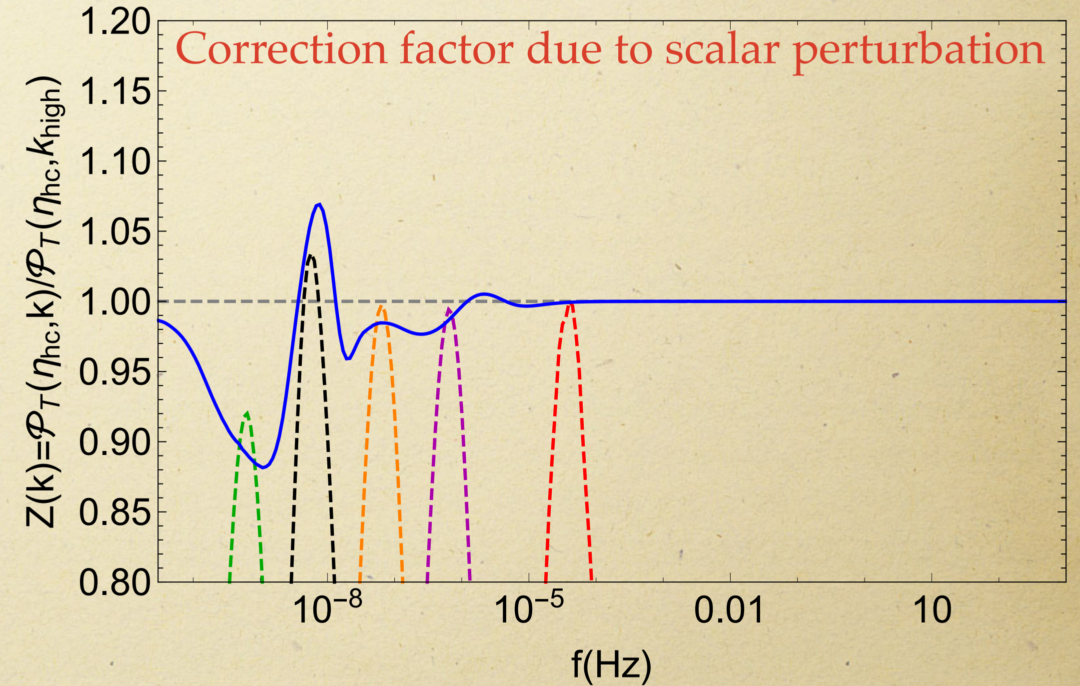


Kohri, Terada, 2018 Baumann, Steinhardt, Takahashi, Ichiki 2007 Ananda, Clarkson, Wands 2006



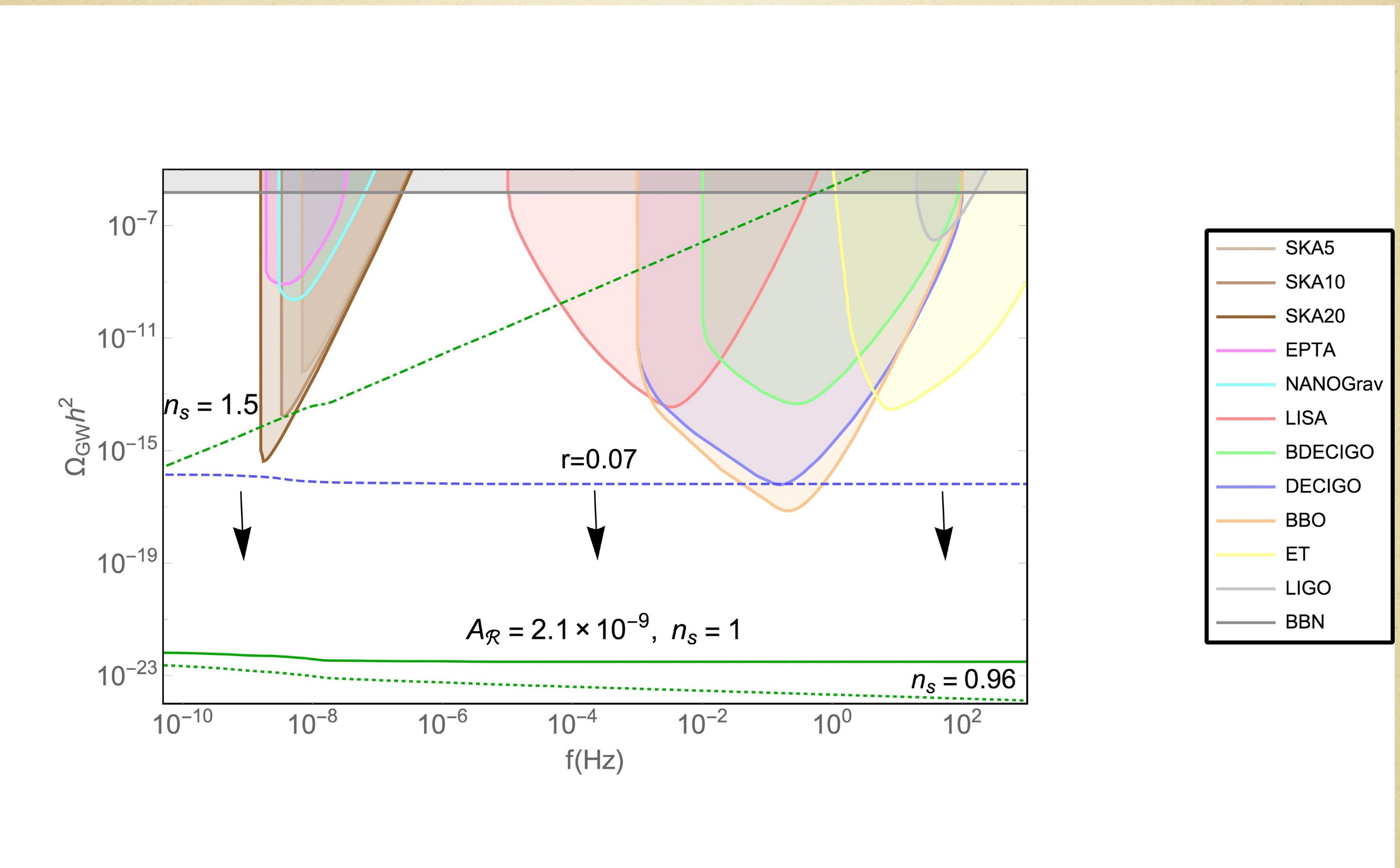




$$\Omega_{GW}(k,\eta_0) \propto Z(T_{hc})\rho_{tot}(T_{hc})s_{tot}(T_{hc})^{-\frac{4}{3}}$$

a correction factor from scalar perturbation and SM DoF



Kohri, Terada, 2018 Baumann, Steinhardt, Takahashi, Ichiki 2007 Ananda, Clarkson, Wands 2006 M. Drees, F. H., E. R. Schmitz arXiv:1503.03513

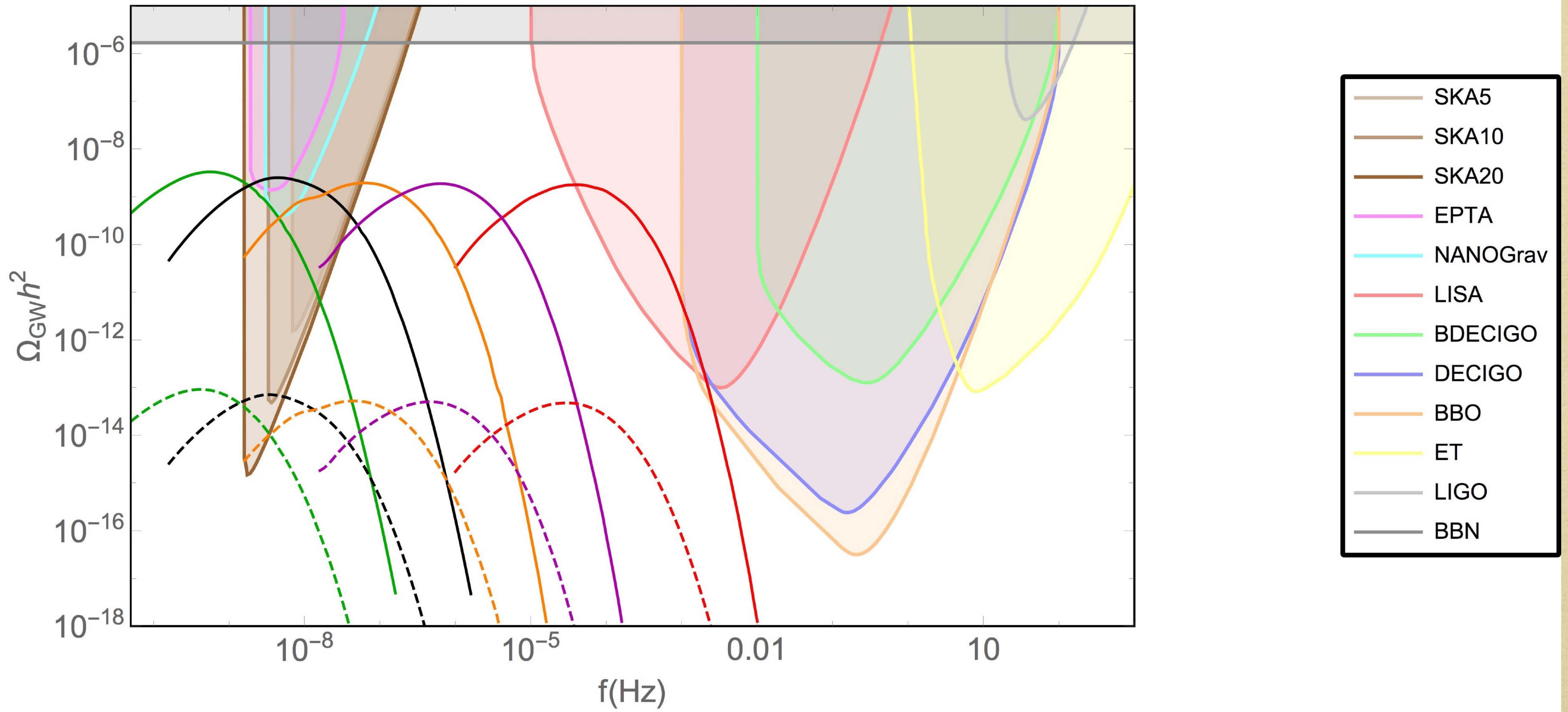



Scale dependent curvature power spectrum:

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\mathcal{R}} \left(\frac{k}{\tilde{k}}\right)^{n_s - 1}$$



Kohri, Terada, 2018 Baumann, Steinhardt, Takahashi, Ichiki 2007 Ananda, Clarkson, Wands 2006 F. H., J. Schaffner-Bielich, arXiv:1910.12357




Non-gaussianity effect can enhance the spectrum as some peaks with large relic density accessible by planned experiments.



Kohri, Terada, 2018 Baumann, Steinhardt, Takahashi, Ichiki 2007 Ananda, Clarkson, Wands 2006

# Primordial Black Holes

- Density fluctuations in the early universe can collapse and form a PBH.

- By emitting all particles via Hawking evaporation → PBH will lose mass and have a black body spectrum.

- They should radiate all SM particles + DM particles!

- In case PBHs dominate the energy density of the universe they should completely evaporate before Big Bang Nucleosynthesis!

- Constraint on PBH domination from BBN: $T_{ev} \gtrsim 4$ MeV, $M_{\rm in} \lesssim 2 \times 10^8$ g



B.Carr, S.Hawking 1974 B.Carr 1975 S.Hawking 1975 arXiv:2006.02838 arXiv:1908.10189

# Formation and Evaporation of PBH

Initial mass of PBH at the time of formation:

$$M_{\rm in} \equiv M_{bh}(T_{\rm in}) = \frac{4\pi}{3}\gamma \frac{\rho_R(T_{\rm in})}{H_R^3(T_{\rm in})}, \qquad H_R^2 = \rho_R/3M_P^2$$

Reduced Planck's mass

Hawking temperature of primordial black hole:

$$T_{\rm BH} = \frac{M_P^2}{M_{\rm BH}} \simeq 10^{13}\,{\rm GeV}\left(\frac{1\,{\rm g}}{M_{\rm BH}}\right)$$

Ratio of PBH w.r.t. radiation energy density at formation time: $\beta \equiv \dfrac{\rho_{\rm bh}(T_{\rm in})}{\rho_R(T_{\rm in})}$

Temperature of the universe at equal densities $\rho_{\rm R}(T_{\rm eq}) = \rho_{\rm bh}(T_{\rm eq})$: $T_{\rm eq} = \beta\, T_{\rm in}\left(\dfrac{g_{\star s}(T_{\rm in})}{g_{\star s}(T_{\rm eq})}\right)^{1/3}$

Start of radiation domination due to PBH evaporation: $T_{\rm c} \simeq \left[\dfrac{g_\star(T_{\rm in})\,\pi}{5760}\,\dfrac{M_P^{10}\,T_{\rm eq}}{M_{\rm in}^6}\right]^{1/5}$

Matter Domination $\longrightarrow$ $T \lesssim T_{\rm eq}$



arXiv:2006.02838 arXiv:0912.5297 arXiv:2011.12306

From non-vanishing isocurvature perturbation:

$$\Phi'' + 3\mathscr{H}(1+c_s^2)\Phi' + (\mathscr{H}^2(1+3c_s^2) + 2\mathscr{H}')\Phi - c_s^2\Delta\Phi = \frac{a^2\rho_{bh}}{2M_{\rm pl}^2}c_s^2 S$$

$$c_s^2 \equiv \frac{4}{9}\frac{\rho_r}{\rho_{bh} + \frac{4}{3}\rho_r}$$

$$S = \frac{\delta\rho_{bh}}{\rho_{bh}} - \frac{3}{4}\frac{\delta\rho_r}{\rho_r}$$

From BBN constraint the induced gravity waves from PBH domination has an upper bound:

$$\beta \lesssim 3.3 \times 10^{-8}\left(\frac{\gamma}{0.2}\right)^{-\frac{1}{2}}\left(\frac{g_\star(T_{bh})}{108}\right)^{\frac{7}{16}}\left(\frac{g_\star(T_{ev})}{106.75}\right)^{\frac{1}{16}}\left(\frac{M_{in}}{10^4\ \mathrm{g}}\right)^{-\frac{7}{8}}$$

Similar constraint from future GW experiment can be found!

arXiv:2012.08151

# PBH Domination Scenario

Friedmann equations assuming early PBH dominated era:

Hubble rate ⟶ $H^2 = (\rho_R + \rho_{bh})/3M_P^2 \rightarrow$

$$\frac{d\rho_{bh}}{dt} + 3\,H\,\rho_{bh} = +\frac{\rho_{bh}}{M_{bh}}\frac{dM_{bh}}{dt}$$ ⟶ It is similar to a scalar domination epoch with time dependent decay rate!

$$\frac{d\rho_R}{dt} + 4\,H\,\rho_R = -\frac{\rho_{bh}}{M_{bh}}\frac{dM_{bh}}{dt}$$ ⟶ Entropy production from evaporation of PBH dilutes the axion relic density

Time evolution of PBH mass due to evaporation:

$$\frac{dM_{\rm bh}}{dt} = -\frac{\pi g_\star(T_{\rm bh})}{480}\frac{M_P^4}{M_{\rm bh}^2}$$

Temperature of the universe at the time of evaporation completion:

$$T_{\rm ev} \simeq \left(\frac{g_\star(T_{\rm in})}{640}\right)^{\frac{1}{4}}\left(\frac{M_P^5}{M_{\rm in}^3}\right)^{\frac{1}{2}}$$

Condition for PBH domination epoch: $\beta > \beta_c\,, \qquad \beta_c \equiv \frac{T_{\rm ev}}{T_{\rm in}}$



arXiv:2003.01100 arXiv:1801.06090 arXiv:2107.00013 arXiv:1511.00672 arXiv:1905.01301 arXiv:2012.08151

# QCD Axion and Strong CP problem

- QCD axion is a solution for the strong CP problem.

- Connection to the UV completion physics through Peccei-Quinn symmetry restoration/breaking at high scales →giving mass to axions as pseudo Nambu-Goldstone bosons.

- Possible contribution to dark matter or dark radiation depending on their mass, production process and pre-BBN thermal history.

- There are some constraints from laboratory, astrophysical and cosmological experiments. The supernova constraint put a lower bound on axion decay constant $f_a$. The upper bound on that comes from the condition on the overclosure of the universe. The valid range of $f_a$ in standard case: $10^8$ GeV $\lesssim f_a \lesssim 10^{12}$ GeV.



arXiv:2003.01100 R. Peccei, H. Quinn, 1977

A. R. Zhitnitsky, 1980 M. Dine, W. Fischler, and M. Srednicki, 1981 J. E. Kim, 1979 M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, 1980

# Axion DM in Standard Cosmology

Axion mass in terms of axion decay constant at zero temperature:

$$m_a \simeq 5.7 \times 10^{-6} \left( \frac{10^{12}\ \text{GeV}}{f_a} \right) \text{eV}$$

Thermal evolution of axion effective mass above and below $T_{QCD} \simeq 150$ MeV:

$$\tilde{m}_a(T) \simeq m_a \times \begin{cases} (T_{QCD}/T)^4 & \text{for } T \geq T_{QCD} \\ 1 & \text{for } T \leq T_{QCD} \end{cases}$$

Axion oscillation starts at:

$$3\,H(T_{osc}) \equiv \tilde{m}_a(T_{osc}) \qquad \rho_a(T_{\text{osc}}) \simeq \frac{1}{2} \tilde{m}_a^2(T_{\text{osc}}) f_a^2\, \theta_i^2$$

→ Initial misalignment angle

Energy and entropy density of thermal bath:

Degrees of freedom of SM

$$\rho_R = \frac{\pi^2}{30} g_\star T^4 \qquad s_R = \frac{2\pi^2}{45} g_{\star s} T^3$$

$$\Omega_a h^2 \equiv \frac{\rho_a(T_0)}{\rho_c/h^2} \simeq 0.12 \text{ from PLANCK observation}$$

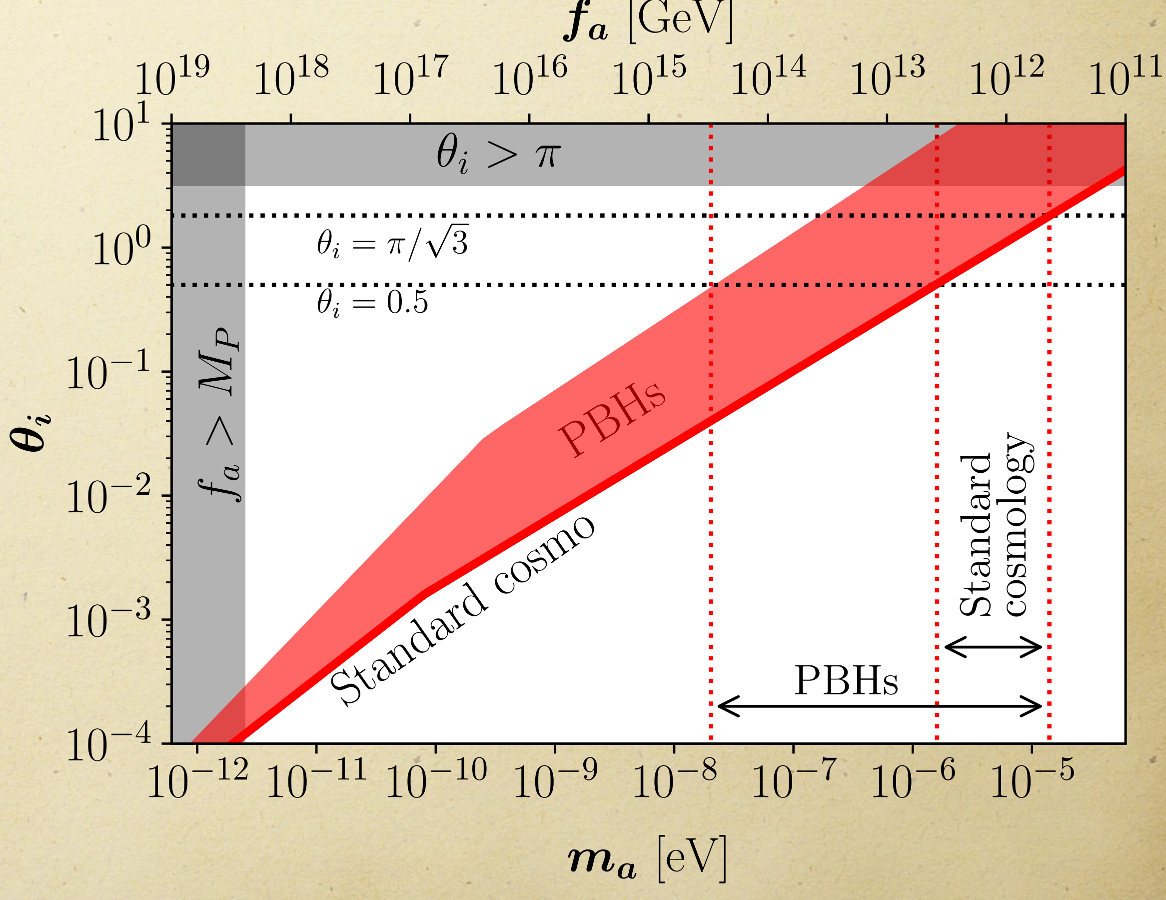


arXiv:2107.13575 arXiv:1503.03513

# Axion Relic Density

Assuming the misalignment mechanism only, with the initial angle in the range $\theta_i \in [0.5, \sqrt{\pi}]$, in order to include both, the pre- and post-inflationary scenarios. The allowed axion mass for the correct relic density in standard case and PBH domination cases: $1.6 \times 10^{-6}$ eV $\lesssim m_a \lesssim 1.4 \times 10^{-5}$ eV and $2 \times 10^{-8}$ eV $\lesssim m_a \lesssim 1.4 \times 10^{-5}$ eV

1) The oscillation temperature of axions reduces if there is a PBH dominated era.
2) PBH evaporation injects entropy to the standard model, diluting the axion relic abundance originally produced.



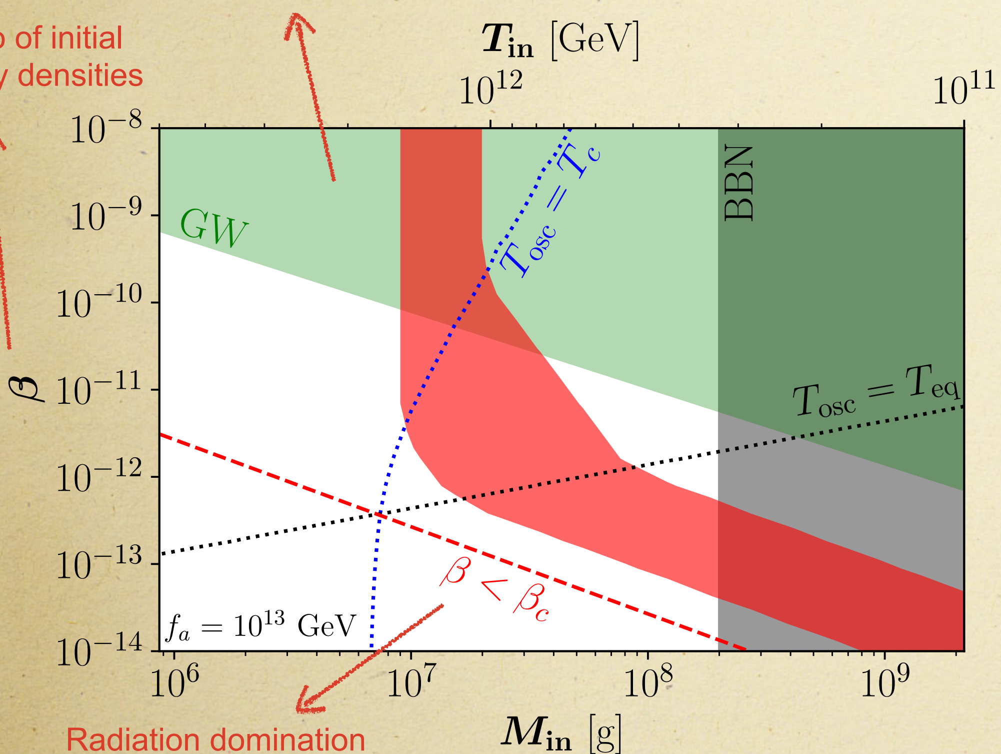



arXiv:2107.13575

# Constraints and Allowed Parameter Space

Constraint from BBN on the upper limit of induced Gravitational Waves by enhanced scalar power spectrum that leads to the formation of PBH

Ratio of initial energy densities

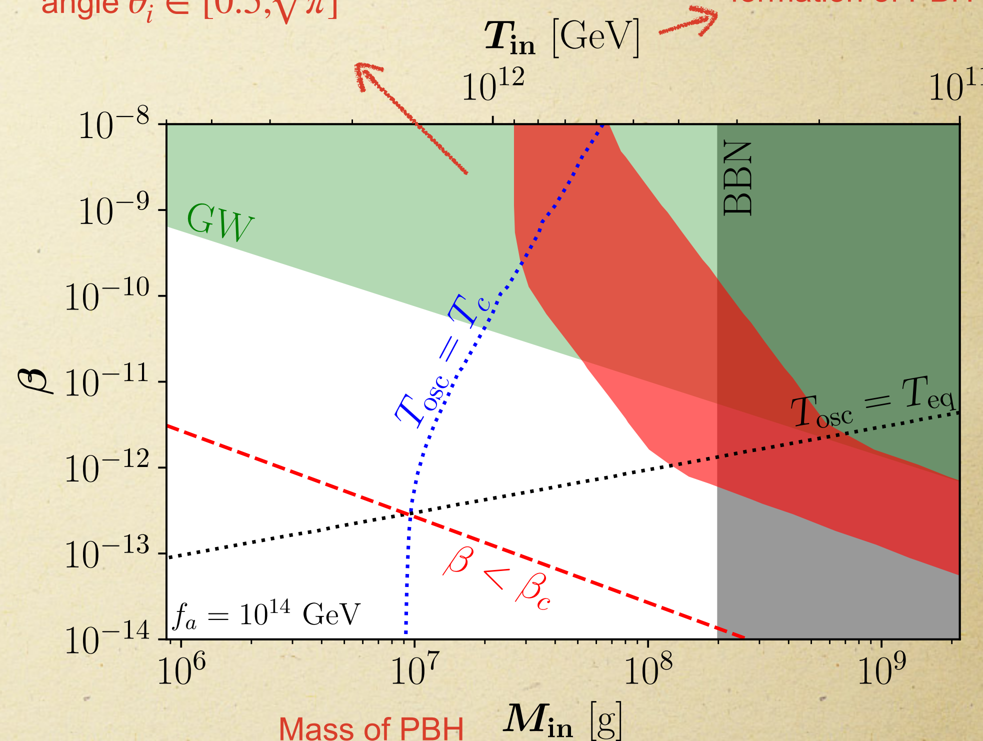



Radiation domination condition

$f_a = 10^{13}$ GeV, $m_a \simeq 6 \times 10^{-7}$ eV

Band for initial misalignment angle $\theta_i \in [0.5, \sqrt{\pi}]$

Temperature at formation of PBH




Mass of PBH

$f_a = 10^{14}$ GeV, $m_a \simeq 6 \times 10^{-8}$ eV



arXiv:2012.08151 arXiv:2107.13575

# Summary

- Cosmology prior to the BBN can be affected by the thermodynamic and type of the background fluid that have imprints on the GW from the early universe.

- GW observatories can indirectly put constraints on baryon-lepton asymmetry, dark matter models, non-standard and modified cosmologies, etc.

Thank you for your attention!

# Scale Dependent Power Spectrum and Constraints

Tensor power spectrum and its scale dependence:

$$\mathcal{P}_T(k) = A_T \left(\frac{k}{\tilde{k}}\right)^{n_T}$$

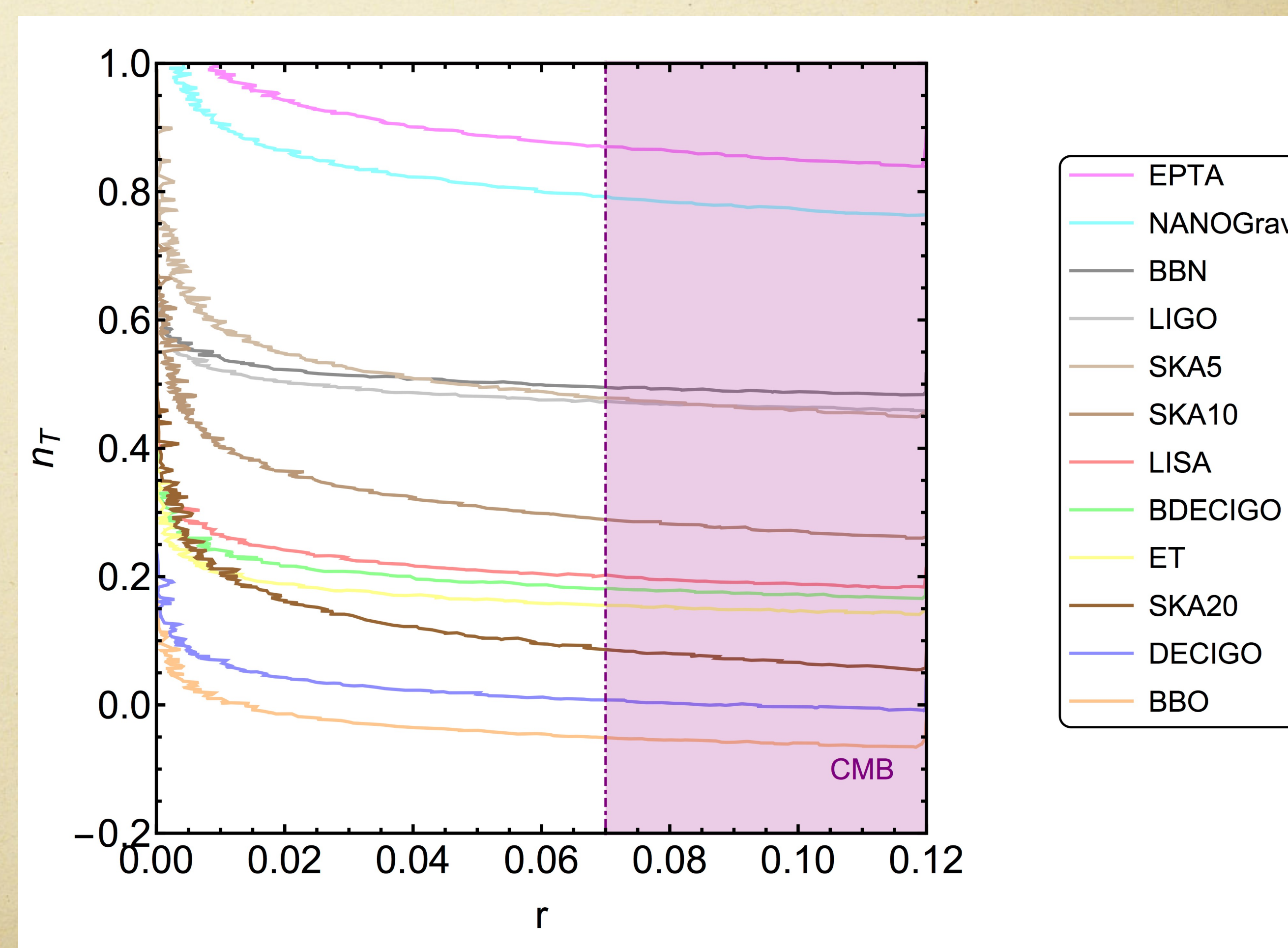
Tensor tilt

Pivot scale

Tensor to scalar perturbation ratio:

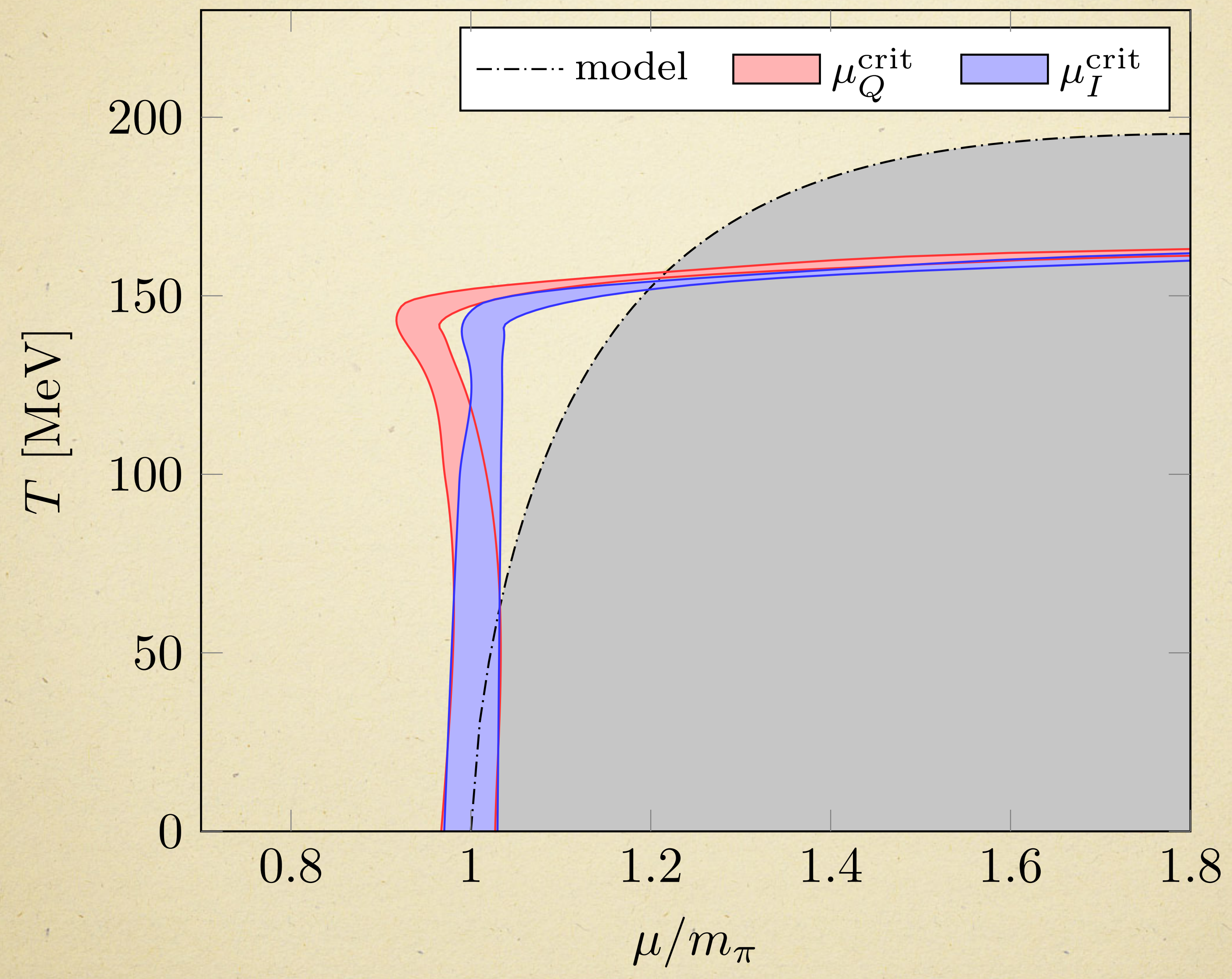
$$r \equiv \frac{A_T}{A_S}$$

By fixing the scalar perturbation amplitude from Planck data:




N. Bernal, F. H., arXiv:1905.10410

Critical lines (boundaries) for effective mass model, charge and isospin chemical potentials (from lattice):



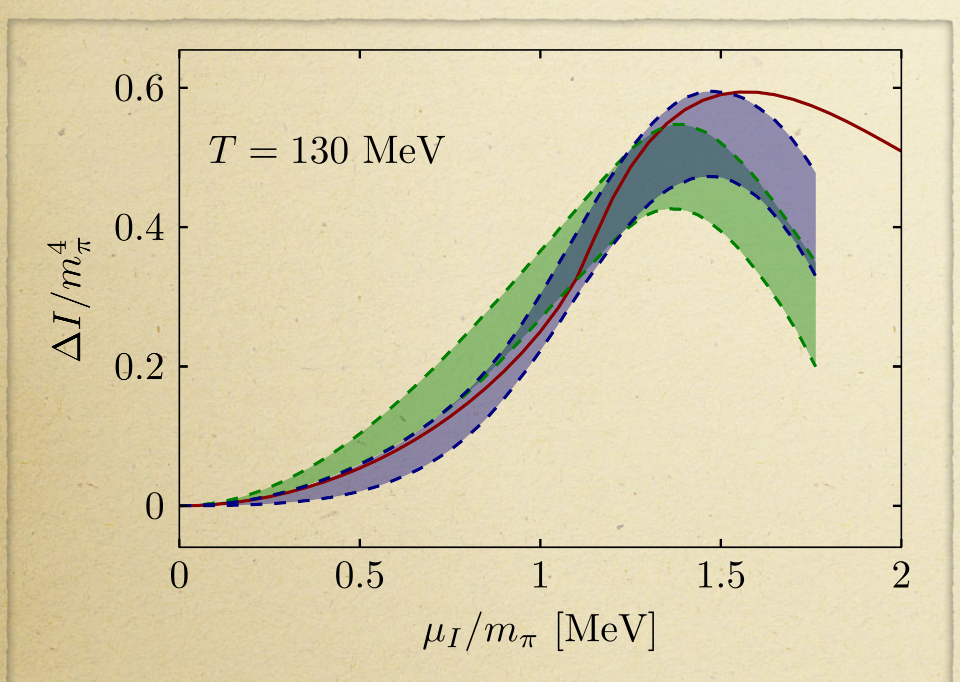

We can not have first order QCD phase transition even based on reasonably large values of non vanishing lepton flavour asymmetry! So only the modification of Hubble factor due to the change of equation of state around QCD transition can affect cosmology.

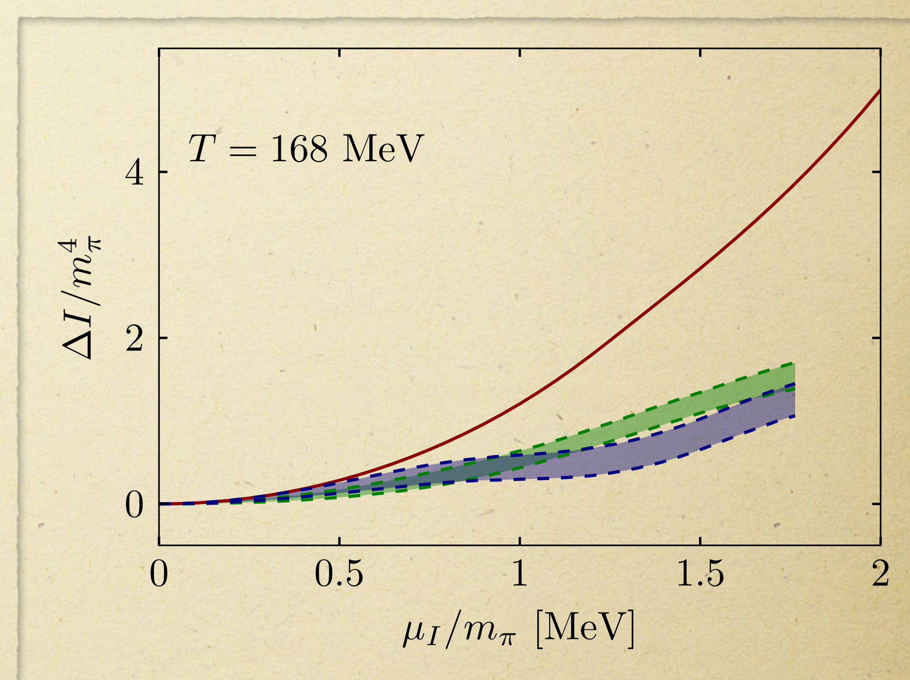
V.Vovchenko et al. 2009.02309

green blue

Trace anomaly difference considering two lattice ensembles for $N_t = 10$ and $N_t = 12$:

At higher temperatures there is a deviation between two models!




V.Vovchenko et al. 2009.02309