Neutron Star Mergers Chirp About Vacuum Energy

[arXiv:1802.04813 [astro-ph.HE]]

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Probing New Physics with Gravitational Waves

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The Cosmological Constant Problem

If the accelerated expansion is explained by a cosmological constant,

$$\Lambda \sim (10^{-3}\,\mathsf{eV})^4 \ll \mathsf{TeV}^4, M_{\mathsf{Pl}}^4.$$

Lots of questions:

- Is it vacuum energy of the underlying QFT?
- Why so small? Why not zero?
- Is it always small? Is there an adjustment mechanism?

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The Evolution of Vacuum Energy

If the CC results from microphysics, we expect it to jump at every phase transition:

$$\Delta\Lambda \sim T_{\text{crit}}^4.$$

This means:

- In the past Λ was much larger than now
- It was always a subdominant contribution to the energy density before the Λ-dominated epoch
- Various jumps: for sure QCD, EW. Maybe SUSY? GUT?

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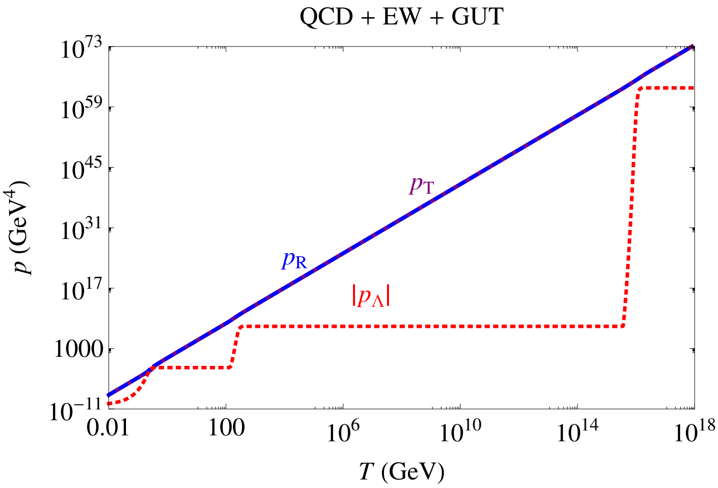
A Simple Sketch



B. Bellazzini, C. Csáki, J. Hubisz, J. Serra, J. Terning, "Cosmological and Astrophysical Probes of Vacuum Energy", *JHEP* **06**, 104 (2016) [`arXiv:1502.04702 [astro-ph.CO]`].

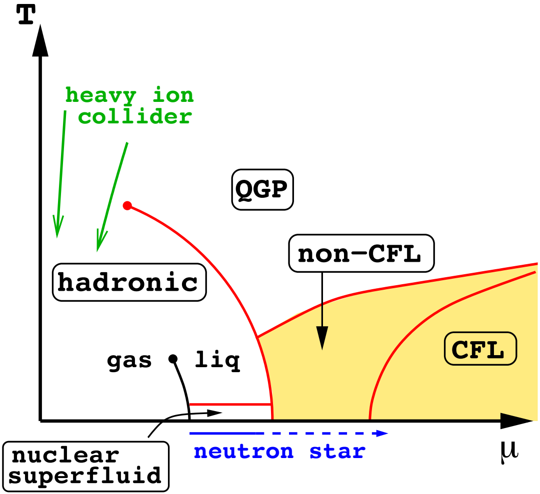
Testing the VE Picture

How to test phases of the SM different from the usual one?

NEUTRON STARS

- In the core there might be an unconventional QCD phase at low temperature T and large chemical potential μ
- The VE $\sim \Lambda^4_{\text{QCD}}$ is an $\mathcal{O}(1)$ fraction of the total energy
- Jump in VE vs adjustment mechanism

QCD Phase Diagram



- Sign problem in lattice simulations
- Unknown behavior in the intermediate regime
- Evidence supporting an exotic phase

M. G. Alford, A. Schmitt, K. Rajagopal, T. Schäfer, "Color Superconductivity in Dense Quark Matter", *Rev. Mod. Phys.* **80**, 1455 (2008) [arXiv:0709.4635 [hep-ph]].

Outline

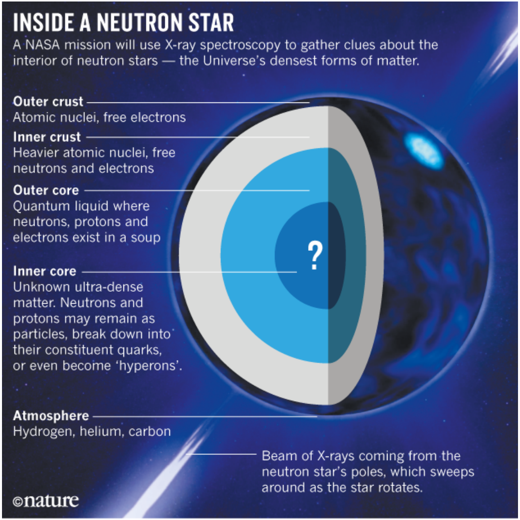
1. **Modeling High Density QCD**

2. **Modeling Neutron Stars**

3. **Results and Fits**

Outline

Dissecting Neutron Stars



- Many layers, complex structure
- Low density physics is well understood
- Mysterious inner core

E. Gibney, "Neutron Stars Set to Open Their Heavy Hearts", *Nature* **546**, 18 (2017).

Equation of State

The internal structure of neutron stars is very complicated:

- Hard to obtain the EoS from first principles, i.e. QCD
- Piecewise polytropic parametrization with 7 layers
- Relation between pressure and mass density
- Critical pressures and densities, p_i and ρ_i

For the outer 6 layers,

$$p = K_i \rho^{\gamma_i}, \qquad p_{i-1} \leq p \leq p_i.$$

The energy density enters the Einstein equations and can be calculated from the first law of thermodynamics:

$$\epsilon = (1 + a_i)\rho + \frac{K_i}{\gamma_i - 1}\rho^{\gamma_i}, \qquad \rho_{i-1} \leq \rho \leq \rho_i.$$

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Continuity on the Outer Layers

Lots of parameters, but many are fixed by continuity of pressure and energy density:

$$K_i = K_{i-1}\rho_{i-1}^{\gamma_{i-1}-\gamma_i}, \qquad a_i = \frac{\epsilon(\rho_{i-1})}{\rho_{i-1}} - 1 - \frac{K_i}{\gamma_i - 1}\rho_{i-1}^{\gamma_i - 1}.$$

Moreover:

- Additional conditions fix $p_0 = 0$ and $a_1 = 0$
- The only independent parameters are K_1, p_i, γ_i

Effects of Vacuum Energy in the Core

Let's assume that the core is in a different phase of QCD.

By definition we introduce a vacuum energy contribution as

$$p = K_7 \rho^{\gamma_7} - \Lambda,$$
$$\epsilon = (1 + a_7)\rho + \frac{K_7}{\gamma_7 - 1}\rho^{\gamma_7} + \Lambda.$$

Notice that:

- Λ defined by extrapolating to zero density
- First order phase transition: mass and energy density have to jump from ρ_- to ρ_+ and from ϵ_- to ϵ_+
- The pressure is still continuous: $p_6 = K_6 \rho_-^{\gamma_6} = K_7 \rho_+^{\gamma_7} - \Lambda$

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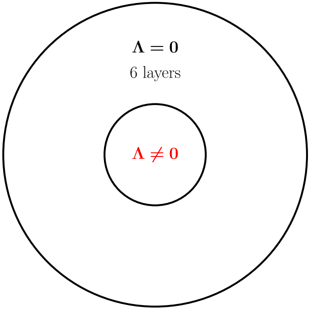
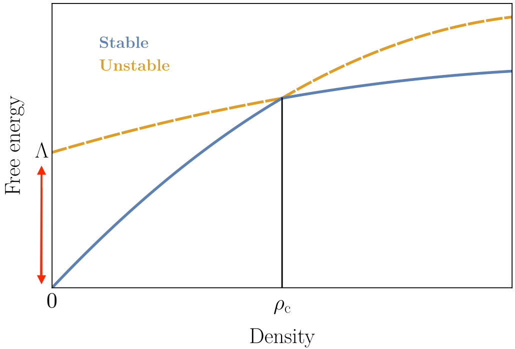
Modeling the Jump





- We parametrize the phase transition as $\epsilon_+ - \epsilon_- = \alpha|\Lambda|$
- We take γ_7, α and Λ as inputs
- For all our simulations we pick $\alpha = 3$

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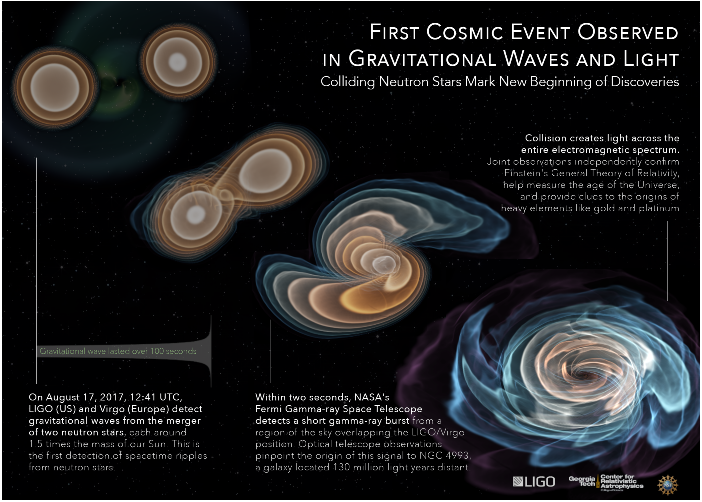
GW170817

Inspiral $\longrightarrow$ Merger $\longrightarrow$ Ringdown



Spherically Symmetric Solution

With a spherically symmetric metric ansatz, the Einstein equations become the TOV equations:

$$
\begin{aligned}
m'(r) &= 4\pi r^2 \epsilon(r), \\
p'(r) &= -\frac{p(r)+\epsilon(r)}{r(r-2Gm(r))} G\Big[m(r) + 4\pi r^3 p(r)\Big], \\
\nu'(r) &= -\frac{2p'(r)}{p(r)+\epsilon(r)}.
\end{aligned}
$$

These provide the unperturbed solutions for the stars.

Stability Conditions

We need to satisfy the following conditions:

- Causality: the speed of sound $v_{\mathsf{s}} = \sqrt{\mathrm{d}p/\mathrm{d}\epsilon}$ must be less than 1
- Both pressure and partial pressure must be positive: $-p_6 < \Lambda$
- The neutron star solution must be stable: $\partial M/\partial p_{\mathsf{center}} \geq 0$

In some cases, the last requirement is violated for $p > p_6$ but satisfied again for even larger pressures.

Different Models

Three parametrizations:

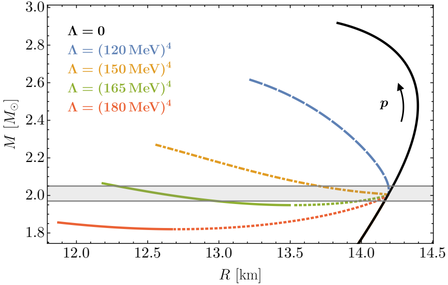
- The Hebeler et al. stiff equation of state[1]
- The SLy equation of state
- The AP4 equation of state

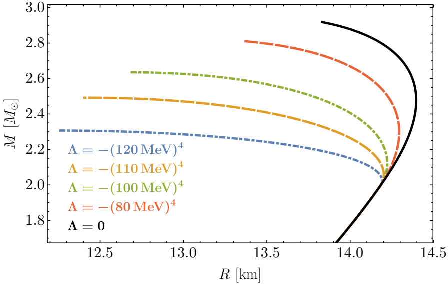
Will only show Hebeler et al. since SLy and AP4:

- Benchmark models for LIGO/Virgo
- Smaller masses, smaller deviations
- Qualitatively similar

[1] K. Hebeler, J. M. Lattimer, C. J. Pethick, A. Schwenk, "Equation of State and Neutron Star Properties Constrained by Nuclear Physics and Observation", *Astrophys. J.* **773**, 11 (2013) [`arXiv:1303.4662 [astro-ph.SR]`].

$M(R)$ Curves: Hebeler et al.





- Varying only the central pressure
- For a high enough pressure the core is in the exotic phase
- To date, the maximal NS mass is $(2.01 \pm 0.04) M_\odot$
- For some positive Λ we obtain disconnected branches characteristic of phase transitions

Tidal Deformability

The presence of the second neutron star acts as an external perturbation:

- During the inspiral we can use a multipole expansion
- The source is spherically symmetric
- The binary is axially symmetric

In the local rest frame and using Cartesian coordinates:

$$\frac{1+g_{tt}}{2} \approx \frac{GM}{r} + \frac{3GQ_{ij}}{2r^5}x^i x^j - \frac{1}{2}\mathcal{E}_{ij}x^i x^j \dots$$

$\mathcal{E}_{ij}$ is the external tidal gravitational field, Q_{ij} is the induced quadrupole moment.

The Love Number

To linear order in the response,

$$Q_{ij} = -\lambda \mathcal{E}_{ij}.$$

We define a dimensionless quantity, the $l = 2$ tidal Love number:

$$k_2 \equiv \frac{3}{2} \frac{G\lambda}{R^5}.$$

In particular, this quantity:

- Describes how the star deforms due to a perturbation
- Is determined by the internal structure, i.e. by the EoS
- Is one of the main physical observables

Determining the Tidal Deformability

We consider perturbation theory around the spherically symmetric solution:

$$h_{\alpha\beta} = \mathrm{diag}\Big(e^{\nu(r)} H(r), e^{\mu(r)} H(r), r^2 K(r), r^2 \sin^2\theta K(r)\Big) Y_2^0(\theta,\phi).$$

The Einstein equations eventually reduce to a second order differential equation:

$$H'' = 2He^{\mu}\left\{-2\pi G\left[5\epsilon + 9p + \frac{d\epsilon}{dp}(\epsilon + p)\right] + \frac{3}{r^2} + 2G^2 e^{\mu}\left(\frac{m(r)}{r^2} + 4\pi r p\right)^2\right\}$$
$$+ \frac{2}{r} H' e^{\mu}\left\{-1 + \frac{Gm(r)}{r} + 2\pi G r^2(\epsilon - p)\right\}.$$

Solving the Differential Equation

The final result for the tidal Love number is

$$
\begin{aligned}
k_2 = \frac{8C^5}{5}&(1-2C)^2[2+2C(y-1)-y] \\
&\times \left\{2C[6-3y+3C(5y-8)] + 4C^3[13-11y+C(3y-2)+2C^2(1+y)]\right. \\
&\left. + 3(1-2C)^2[2-y+2C(y-1)]\log(1-2C)\right\}^{-1} \equiv F(C,y),
\end{aligned}
$$

where:

- $C \equiv GM/R$ is the compactness parameter
- $y \equiv RH'(R)/H(R)$ is obtained from the solution for H

Another combination is also common in the literature:

$$
\bar{\lambda} \equiv \frac{2k_2}{3C^5} = \frac{\lambda}{G^4 M^5}.
$$

Outline

LIGO/Virgo Observables

The waveform of the gravitational wave emitted in a neutron star merger depends on various expansion parameters. The chirp mass is

$$\mathcal{M} = \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}}.$$

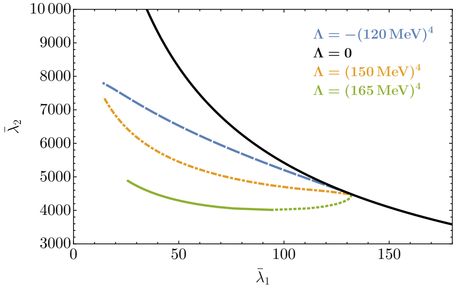
The combined dimensionless tidal deformability is

$$\tilde{\Lambda} = \frac{16}{13} \frac{(M_1 + 12 M_2) M_1^4 \bar{\lambda}_1 + (M_2 + 12 M_1) M_2^4 \bar{\lambda}_2}{(M_1 + M_2)^5}.$$

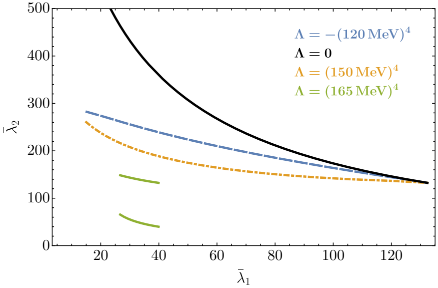
For GW170817:

- $\mathcal{M} = 1.188^{+0.004}_{-0.002} M_\odot$
- $\tilde{\Lambda} \leq 800$ (low spin) or $\tilde{\Lambda} \leq 700$ (high spin)

Tidal Deformabilities: Hebeler et al.



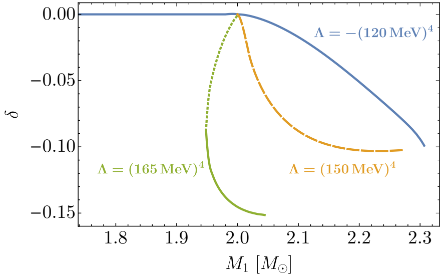
(a) $\mathcal{M} = 2^{-1/5} 1.4 M_\odot \approx 1.22 M_\odot$



(b) $\mathcal{M} = 2^{-1/5} 2.0 M_\odot \approx 1.74 M_\odot$

- We fix the chirp mass and vary the individual masses
- Again big deviations and disconnected branches
- For larger chirp masses both stars can contain the exotic phase

Relative Shift: Hebeler et al.



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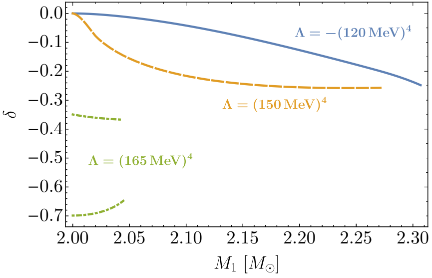

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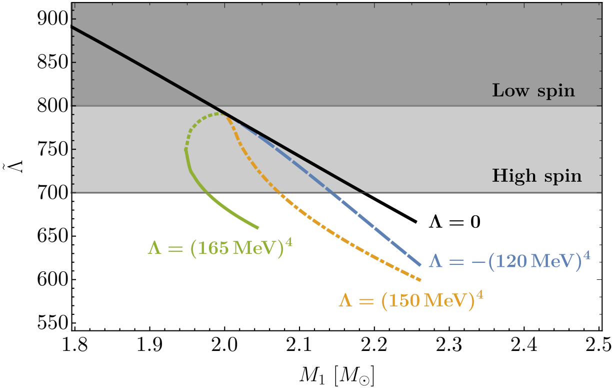
- We define the relative deviation $\delta \equiv \tilde{\Lambda}/\tilde{\Lambda}_0 - 1$
- We compare events with the same individual masses, so the effect is entirely due to the change in tidal deformabilities
- Up to $\mathcal{O}(1)$ deviations for larger chirp masses

Money Plot



- Hebeler et al. parametrization with the chirp mass of GW170817
- VE can significantly alter the allowed mass range
- It should be taken into account when comparing EoSs

What Next?

Many promising directions:

- Neutron star–black hole mergers: novel features like tidal disruption and cutoff frequency
- Rotating neutron stars: maybe interesting effects if the angular speed is significant
- Degeneracy of parameters: progress is possible if theoretical calculation of the EoS improve
- Increase in the number of events with the upgrades of LIGO/Virgo

Conclusions

- Vacuum energy is an important part of our standard picture of cosmology and particle physics, yet it is not very well understood
- It can contribute to the equation of state of neutron stars if the core contains a new phase of QCD at large densities
- This significantly affects the mass versus radius curves and LIGO/Virgo observables such as tidal deformabilities
- As the sensitivities of the experiments evolve and more events are observed, neutron star mergers can provide a new test of the gravitational properties of vacuum energy

Thank you!