Gravitational wave background from non-Abelian reheating after axion-like inflation

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MITP Scientific Program on
Probing New Physics with Gravitational Waves

arXiv:2201.02317

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Gravitational waves constrain Axion-like inflation

$$\mathcal{L} \supset \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V_0(\varphi) - G_{\mu\nu}G^{\mu\nu} - \frac{\varphi}{f_a}\chi \qquad \chi = \frac{\alpha_s}{16\pi}\widetilde{G}_{\mu\nu}G^{\mu\nu}$$

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- Shift symmetry protects flat potential

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What about non-Abelian case?

Gauge self-interactions suppress GW production

1. non-Abelian gauge fields self-interact
   $\Rightarrow$ Fast equilibration (compared to Abeldian case)

We consider GW production rate in thermal equilibrium (after reheating)
$\Rightarrow$ reasonable lower bound

Other approaches are *e.g.* arXiv:1708.02944 / arXiv:1911.06827 / arXiv:2006.15122

Gauge self-interactions suppress GW production

1. non-Abelian gauge fields self-interact
   $\Rightarrow$ Fast equilibration (compared to Abeldian case)

2. Any viscous plasma generates GW but out-of-equilibrium production is much more efficient

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Other approaches are *e.g.* arXiv:1708.02944 / arXiv:1911.06827 / arXiv:2006.15122

GW production rate in thermal equilibrium

We compute new physics contributions to $G^{\mathcal{A}}$ at finite T:

$$\frac{\mathrm{d}\dot{e}_{\mathrm{gw}}}{\mathrm{d}^3k/(2\pi)^3} = \frac{4\pi}{m_{\mathrm{Pl}}^2} n_B G^{\mathcal{A}} \quad G^{\mathcal{A}} = \int \mathrm{d}^4x \; e^{\mathrm{i}(kt - \boldsymbol{kx})} K^{ij;kl} \frac{1}{2} \langle [T_{ij}(0), T_{kl}(x)] \rangle$$

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- After inflation $\Rightarrow$ Minkowksi space + Small fields ($V(\varphi) \to \frac{1}{2}m^2\varphi^2$)
- Lengthscale determines computational strategy:
  - Small distances $k \sim \pi T \Leftrightarrow$ Plasma is particle gas (Boltzmann domain)
  - Large distances $k \ll \alpha_s^2 T \Leftrightarrow$ Plasma is fluid (hydrodynamic domain)

Small distances $k \sim \pi T$: Particle picture

- Particle picture: Inflaton to matter coupling $\propto h_{ij} T^{ij}$

  $\Rightarrow$ Boltzmann equation gives production rate:

$$\frac{\mathrm{d}\dot{e}_{gw}}{\mathrm{d}^3 k/(2\pi)^3} = 2k\dot{f} \qquad \dot{f} = \Gamma\left(n_\mathrm{B} - f\right) \approx n_\mathrm{B}\Gamma \qquad n_\mathrm{B} = \frac{1}{e^{\beta k} - 1}$$

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- Optical theorem: Matrix element $\Leftrightarrow$ Spectral function $G^{\mathcal{A}}$

$$\Gamma = \frac{4\pi}{m_\mathrm{Pl}^2 k} G^{\mathcal{A}} \qquad G^{\mathcal{A}} = \int \mathrm{d}^4 x\; e^{\mathrm{i}(kt - \boldsymbol{k}\boldsymbol{x})} K^{ij;kl} \frac{1}{2} \langle [T_{ij}, T_{kl}] \rangle$$

Large distances $k \ll \alpha^2 T$: Fluid picture

- Viscous fluid in local thermal equilibrium:

$$T_{00} = e \qquad T_{0i} = (e+p)v_i$$

$$T_{ij} = \delta_{ij}\left(p - \boldsymbol{\xi}\nabla \boldsymbol{v}\right) - \boldsymbol{\eta}(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\nabla \boldsymbol{v})$$

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- Shear viscosity generates gravitational waves:

$$8\boldsymbol{\eta}T = \lim_{k\to 0} G^{<}$$

Connecting domains: Computing η from QFT

- Straightforward in principle:

$$8\eta T \overset{k \lll T}{=} n_B(k) G^{\mathcal{A}}$$

Problem: fixed order perturbation theory breaks down

Solution: Use resummed propagators

$$\frac{\mathrm{i}}{p^2 - m^2 - \mathrm{i}\,\epsilon} \to \frac{\mathrm{i}}{p^2 - M^2 - \mathrm{i}\,p_0 \Upsilon}$$

Long range $\Rightarrow$ on-shell contribution dominates:

$$M^2 = m^2 + \mathrm{Re}\,\Pi(m) \qquad p_0 \Upsilon = \mathrm{Im}\,\Pi(m)$$

$$\Rightarrow \quad \eta \overset{k \lll \Upsilon}{\approx} T^4/\Upsilon$$

Weak interactions $\Leftrightarrow$ large shear viscosities

Summarizing the Computational Approaches

1. Hydrodynamic domain ($k \ll \alpha^2 T$):

2. Boltzmann domain ($k \sim \pi T$):

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   - No interactions, but finite width propagators:

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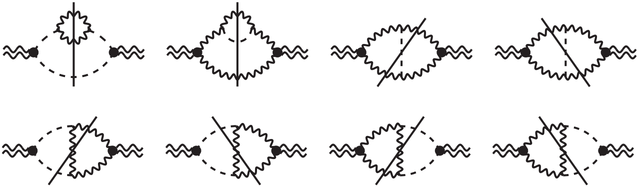
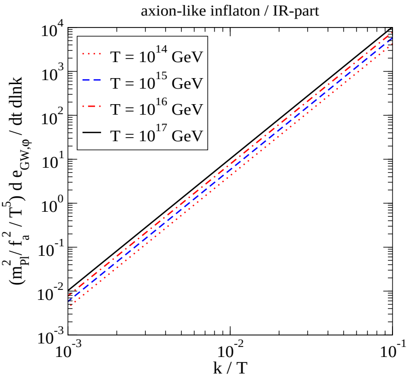
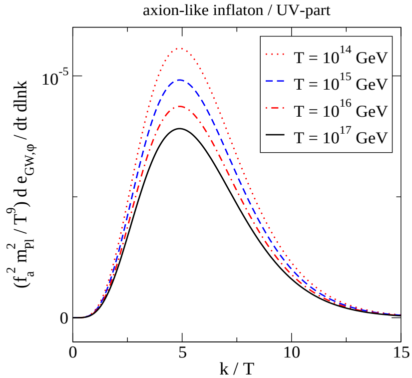
2. Boltzmann domain ($k \sim \pi T$):
   - No resummation
   - Full stress-energy tensor $T_{\mu\nu} = \partial_\mu \varphi \partial_\nu \varphi - G_{\mu\lambda} G_\nu{}^\lambda$

$G^{\mathcal{A}} =$



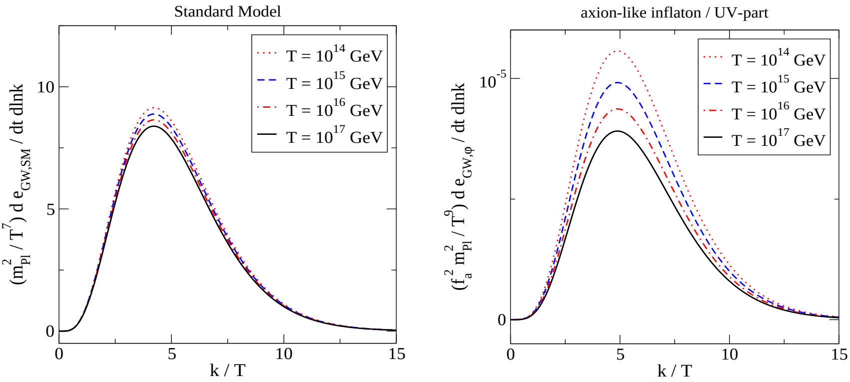
Takeaway 1: Boltzmann domain most important





- $k \ll \Upsilon \Rightarrow \text{GW} \propto k^3/\Upsilon$ and $\Upsilon \ll k \ll \alpha^2 T \Rightarrow \text{GW} \propto k\Upsilon$
- Small signal for $k \ll \alpha^2 T$, peak at $k \sim \pi T$

Takeaway 2: SM dominates over NP contribution



- NP relevant for $T_{\max} \gtrsim 10^3 f_a$ but simulations give $T_{\max} \lesssim 200 f_a$

  $\Rightarrow$ No overproduction issues as in Abelian case
  (*cf.* arXiv:1909.12842 arXiv:1909.12842)

Summary and Outlook

- Computed GW background from reheating after axion-like inflation
- Main assumptions: Thermal equilibrium + Minkowski space
  $\Rightarrow$ Reasonable lower bound

Main results

1. Bolzmann domain most important
2. SM contribution dominates over NP contribution
   $\Rightarrow$ Model avoids overproduction issue from Abelian case

Future prospects: Include Hubble rate H, full potential $V(\varphi)$,
Out-of-equilibrium contributions, etc.

Thank you for your attention!