On-shell methods for non-protected operators in N=4 SYM

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Based on 1410.8485 with Christoph Sieg, Matthias Wilhelm and Gang Yang and 1504.06323 with Florian Loebbert, Christoph Sieg, Matthias Wilhelm and Gang Yang

Outline

- Form factor review.
- Konishi form factors at 1 and 2 loops.
- Regularization for Konishi
- Cross section in N=4 SYM: two-point correlation function
- SU(2) sector dilatation operator and remainder function at 2 loops.
- Conclusion & Outlook

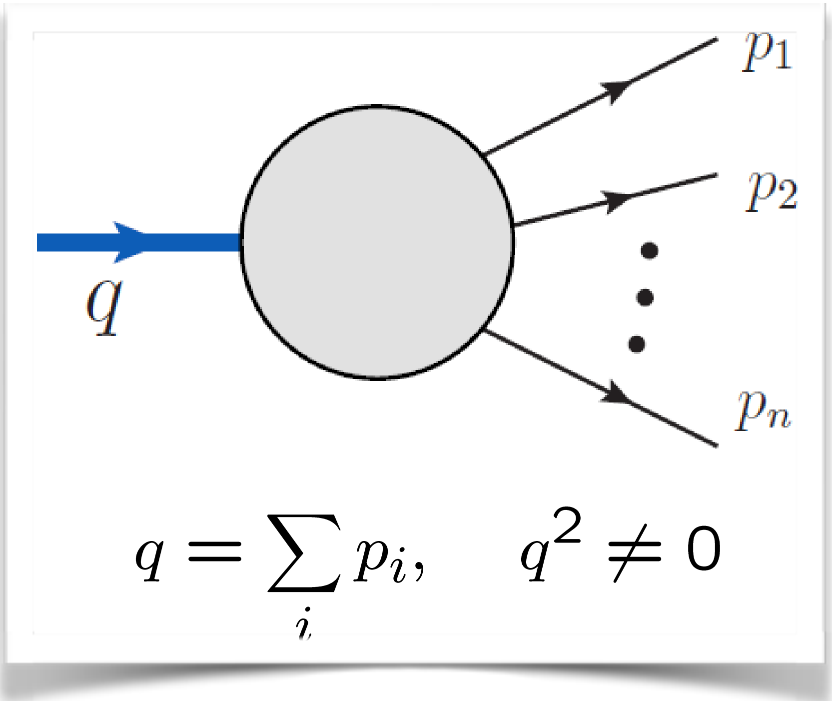
Form Factors

Review

- Hybrid of on-shell states and off-shell gauge invariant operators. Partial "off-shell".

$\langle 0|p_1 p_2 \cdots p_n\rangle$

scattering amplitude



$\langle 0\,|\mathcal{O}(x_1)\mathcal{O}(x_2)...\mathcal{O}(x_n)|\,0\rangle$

correlation function

$$\mathcal{F}_n = \int d^D x\, e^{iq\cdot x}\langle 1\cdots n|\mathcal{O}(x)|0\rangle = \delta^4(q - \sum_i p_i)\langle 1\cdots n|\mathcal{O}(0)|0\rangle$$

Form factor

Sudakov form factor gives the IR divergence in gauge theories

[Mueller, Collins, Sen]

This was used to write the IR behavior of the planar amplitudes in N=4 SYM

[Bern, Dixon, Smirnov]

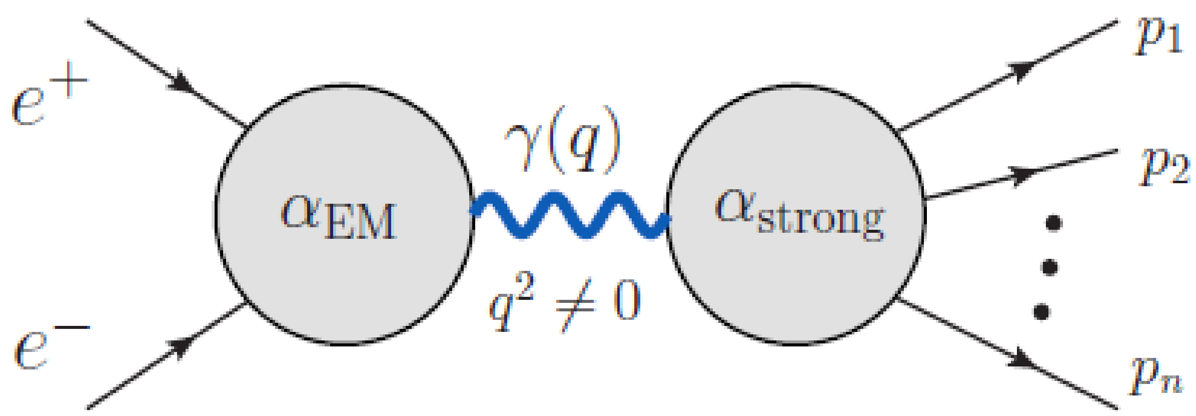
$$\mathcal{M}_n = \prod_{i=1}^{n} \left[\mathcal{M}^{[gg\to 1]} \left(\frac{s_{i,i+1}}{\mu^2}, \alpha_s, \epsilon \right) \right]^{1/2} \times h_n \left(k_i, \mu, \alpha_s, \epsilon \right)$$

$$\Downarrow$$

$$\exp\left[-\frac{1}{4} \sum_{l=1}^{\infty} a^l \left(\frac{\mu^2}{-Q^2} \right)^{l\epsilon} \left(\frac{\hat{\gamma}_K^{(l)}}{(l\epsilon)^2} + \frac{2\hat{\mathcal{G}}_0^{(l)}}{l\epsilon} \right) \right]$$

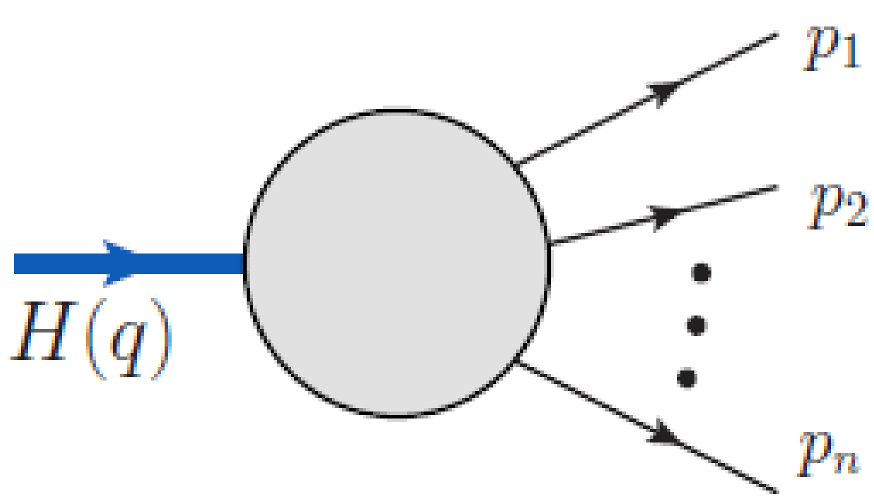
- Today we will focus on form factors in N=4 SYM

Examples of occurrence of form factor



electron-positron decay to create hadronic state :

Form factor of hadronic EM current



Higgs+multigluon amplitude in QCD:

Form factor of $Tr(F^2)$

SUSY relates to form factor of half-BPS operator in N=4 SYM

$$\mathcal{L}_{\text{eff.int.}} = -\frac{\lambda}{4} H \ \text{tr}\,(F_{\mu\nu}F^{\mu\nu})$$

[MHV formula by Dixon, Glover and Khoze]

Brandhuber, Gurdogan, Mopney, Travaglini, Yang

Form Factors in N=4 SYM

- Studied for almost 30 years now . Studied by Van Neerven (1985)
- Recent perturbative studies shows similarities to scattering amplitudes

[Brandhuber, Spence, Travaglini,,Yang]

non-color ordered form factors

color ordered form factors

$$\hat{\mathcal{F}}_{\mathcal{O},X} = \sum_{\ell=0}^{\infty} g^{2\ell} \hat{\mathcal{F}}^{(\ell)}_{\mathcal{O},X} \, .$$

$$\hat{\mathcal{F}}^{(\ell)}_{\mathcal{O},n}(\{a_i, p_i, \eta_i\}) = g_{\mathrm{YM}}^{n-2} \sum_{\sigma \in S_n/Z_n} \mathrm{tr}(\mathrm{T}^{a_{\sigma(1)}} \cdots \mathrm{T}^{a_{\sigma(n)}}) \mathcal{F}^{(\ell)}_{\mathcal{O},n}(\{p_i, \eta_i\})$$

Scattering amplitudes are color-ordered but operator is color singlet : can be inserted at any position

[Van Neerven, Travaglini, Brandhuber, Spence, Yang, Gurdogan, Mooney, Wen, Penante, Bork, Kazakov, Vartanov, Zhiboedov, Alday, Maldacena, Gao, Gehrmann, Henn, Huber, Korchemsky, Sokatchev, Belitsky, Hohenegger, Roiban, Engelund, Boels, Tarasov, Kniehl, Moch, Naculich, Young…]

Mueller, Collins, Sen, Korchemsky, Radyushkin, Magnea, Sterman, Tejeda-Yeomans

Nair's on-shell superspace in D=4

$$\Phi(p,\eta) = g_+(p) + \eta^A \psi_A(p) + \frac{\eta^A\eta^B}{2!}\phi_{AB}(p) + \frac{\varepsilon_{ABCD}\eta^A\eta^B\eta^C}{3!}\tilde{\psi}^D(p) + \eta^1\eta^2\eta^3\eta^4 g_-(p)$$

Packages all the fields of the theory into one super field. Get components by expanding in Grassmann parameters. SUSY packages operators in multiplets. We focus on half-BPS operator of stress-tensor supermultiplet.

BPS in SU(4)

$$\mathcal{O}_{\rm BPS} = {\rm tr}(\phi_{AB}\phi_{AB})$$

BPS in SO(6)

$$\mathcal{O}_{\rm BPS} = {\rm tr}(\phi_{(I}\phi_{J)})$$

MHV form factor for half-BPS operator

$$F_n^{\rm MHV}(1^+,..,i^-,..,j^-,..,n^+;{\rm tr}(\phi^2)) = \delta^4(\sum_{i=1}^n p_i - q)\frac{\langle ij\rangle^2}{\langle 12\rangle\cdots\langle n1\rangle}$$

$$q = \sum_i p_i, \quad p_i^2 = 0, \quad q^2 \neq 0$$

[Brandhuber, Spence, Travaglini, Yang]

Similarity of amplitude and form factor @ tree level

Parke-Taylor formula for MHV amplitude

$$A_n^{\rm MHV}(1^+,..,i^-,..,j^-,..,n^+) = \delta^4(\sum_{i=1}^n p_i)\frac{\langle ij\rangle^4}{\langle 12\rangle\cdots\langle n1\rangle}$$

$$0 = \sum_i p_i, \quad p_i^2 = 0$$

[Parke, Taylor]

Minimal form factor (n=2): just the delta function

Next Step: Form factors of non-protected operators

- Amplitudes and BPS Form factors are UV finite
- Konishi has UV divergence
- We will see new QCD-like features such as rational terms and spurious poles.

Konishi: Form factors and anomalous dimension from unitarity

Konishi in SO(6)

Konishi in SU(4)

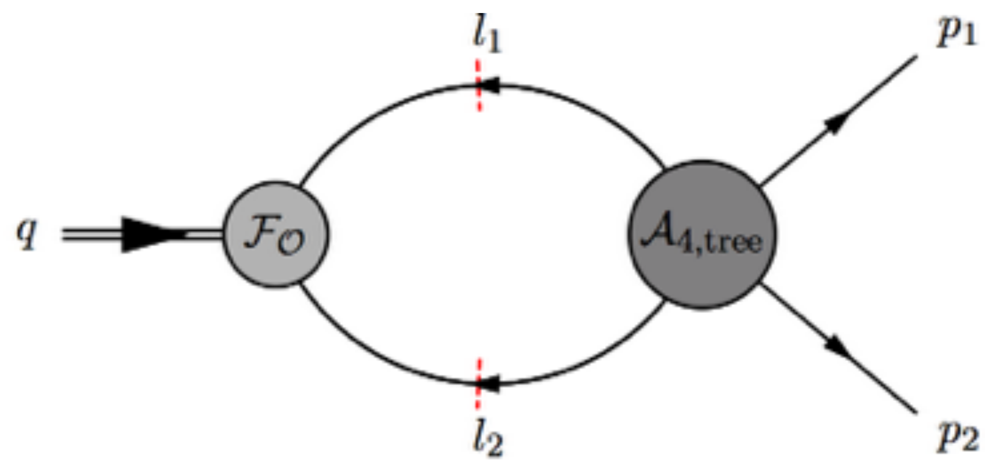
$$\mathcal{K} = \sum_{I,J} \delta^{IJ}\,\mathrm{tr}(\phi_I \phi_J) \quad \longleftrightarrow \quad \mathcal{K}_6 = \frac{1}{8}\varepsilon^{ABCD}\,\mathrm{tr}(\phi_{AB}\phi_{CD})$$

The difference in SO(6) and the SU(4) representation will be significant later

Form factor using unitarity cuts

One-loop two point Konishi

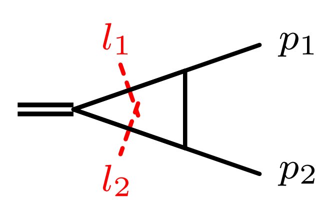
[Bern, Dixon, Dunbar, Kosower]



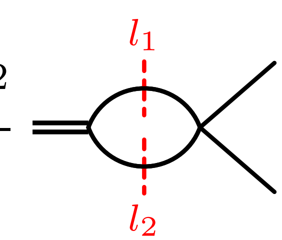
integrate over all particle species

$$F^{(1)}_{\mathcal{K}_6}(1_{\phi_{12}}, 2_{\phi_{34}})\Big|_{q^2\text{-cut}} = \int d\text{LIPS}(l_1, l_2)\, d^4\eta_{l_1} d^4\eta_{l_2} \mathcal{F}^{(0)}_{\mathcal{K}_6,2}(-l_1, -l_2) \times \mathcal{A}^{(0)}_4(p_1, p_2, l_2, l_1)\Big|_{(\eta_1^1\eta_1^2)(\eta_2^3\eta_2^4)}$$

$$= F^{(0)}_{\mathcal{K}_6}(1_{\phi_{12}}, 2_{\phi_{34}})\, i \left(-s_{12} \right.$$

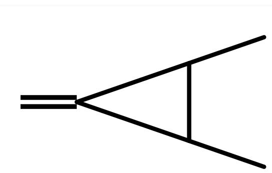


$$\left. + \, 6 \frac{(l_1 + p_2)^2}{s_{12}} \right.$$

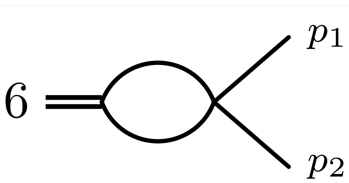


$$\left. \right)$$

$$\frac{F^{(1)}_{\mathcal{K}_6}(1_{\phi_{12}}, 2_{\phi_{34}})}{F^{(0)}_{\mathcal{K}_6}(1_{\phi_{12}}, 2_{\phi_{34}})} = f^{(1)}_{\mathcal{K}_6}(1_{\phi_{12}}, 2_{\phi_{34}}) = -2s_{12}$$



$$-\,6$$



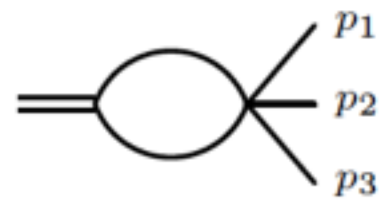
$f^{(1)}_{\text{BPS},2}$ (under the first term)

IR

UV

one-loop and two-loop Konishi form factor

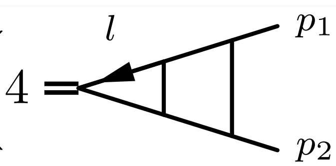
$$f^{(1)}_{\mathcal{K},(\phi,\phi,g)} = -\,3\left[1 + \frac{s_{13}^2 + s_{23}^2}{(s_{13}+s_{23})^2}\right]$$



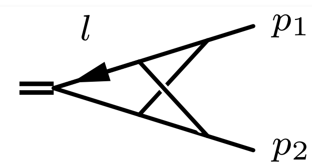
$$-\ \frac{6 s_{13} s_{23}}{(s_{13}+s_{23})^2}$$

$$+\ \frac{12 s_{13} s_{23}}{s_{12}(s_{13}+s_{23})} I_3^D[\ell_\epsilon^2] + f^{(1)}_{\mathrm{BPS},3}\ ,$$

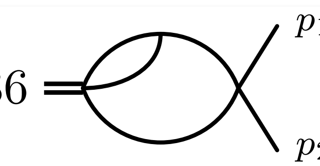
$$f^{(2)}_{\mathcal{K}_6}(1_{\phi_{12}}, 2_{\phi_{34}}) = -6(l+p_1)^2(l+p_2)^2 \Big(4$$



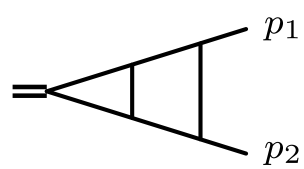
$$+$$

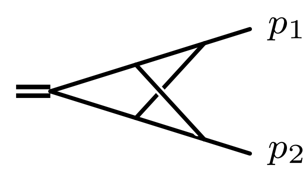


$$\Big) + 36$$



$$+\ \underbrace{s_{12}^2 \Big(4 \quad [\text{diagram}] \quad + \quad [\text{diagram}] \Big)}_{f^{(2)}_{\mathrm{BPS},2}}$$





- Form factors of different components are different
- Bubble integrals contain the UV divergences due to Konishi

More interesting features of one-loop three point Konishi

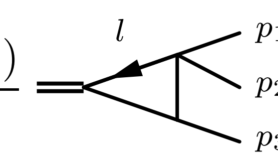
$$f^{(1)}_{\mathcal{K},(\phi,\phi,g)} = -3\left[1+\frac{s_{13}^2+s_{23}^2}{(s_{13}+s_{23})^2}\right]$$ 

$$-\frac{6s_{13}s_{23}}{(s_{13}+s_{23})^2}$$ 

$$+\frac{12s_{13}s_{23}}{s_{12}(s_{13}+s_{23})} I_3^D[\ell_\epsilon^2] + f^{(1)}_{\text{BPS},3}\,,$$

Rational terms!

$$I_3^D[\ell_\epsilon^2] = \tfrac{1}{2} + \mathcal{O}(\epsilon) \qquad \eta_{\mu\nu} l^\mu l^\nu = l^2_{(4)} = l^2 + l^2_\epsilon$$

- Rational terms from 4-D unitarity cut and PV reduction $\frac{3s_{1l}s_{2l}(s_{13}+s_{23})}{s_{12}^2}$ 

- Integral coefficients have unphysical poles which cancel

$$\frac{1}{s_{13}+s_{23}} = \frac{1}{s_{123}-s_{12}}$$

$$-6\frac{s_{13}s_{23}}{(s_{13}+s_{23})^2}\log\left(\frac{s_{123}}{s_{12}}\right) + 6\frac{s_{13}s_{23}}{s_{12}(s_{13}+s_{23})}$$ but it is regular for $s_{13}+s_{23}\to 0$

Features found in one-loop QCD amplitudes

Unitarity and Regularization for Konishi

Form factors computed using on-shell external fields in D=4.

Integrals are regulated by continuing to D = 4- 2e

$\mathcal{N} = 1 \quad \text{SYM in} \quad D = 10$

Dimensional Reduction *[Siegel]*

$\mathcal{N} = 4 \quad \text{SYM in} \quad D = 4 - 2\epsilon$

Preserves SUSY !

$A_M \quad M = 1, \ldots 10$

$A_\mu, \quad \mu = 1, ..., D = 4 - 2\epsilon$

$\phi_I, \quad I = 1, ..., N_\phi = 6 + 2\epsilon$

$$D + N_\phi = 10$$

Four-dimensional Helicity(FDH) *[Bern, Kosower]*

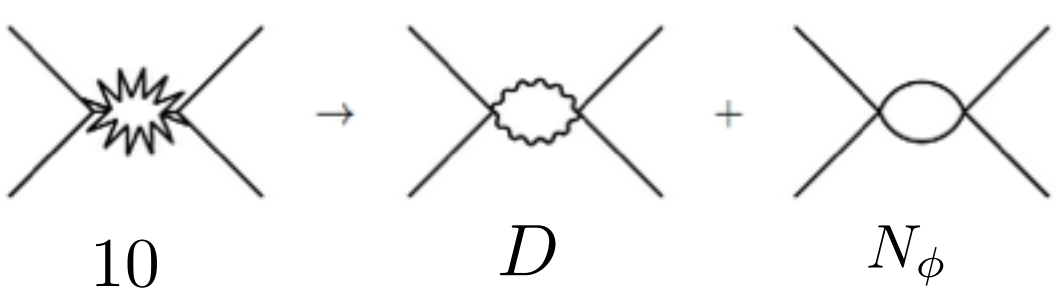
Used in scattering amplitudes and BPS form factors

Also preserves SUSY !

$A_\mu, \quad \mu = 1, ..., D = 4$

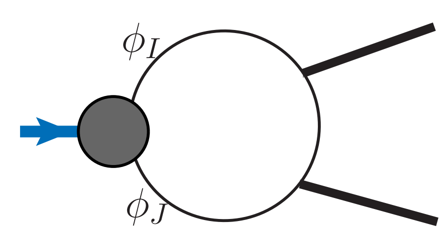
$\phi_I, \quad I = 1, ..., N_\phi = 6$

Internally Closed Index Loop



Operator not included in index loop

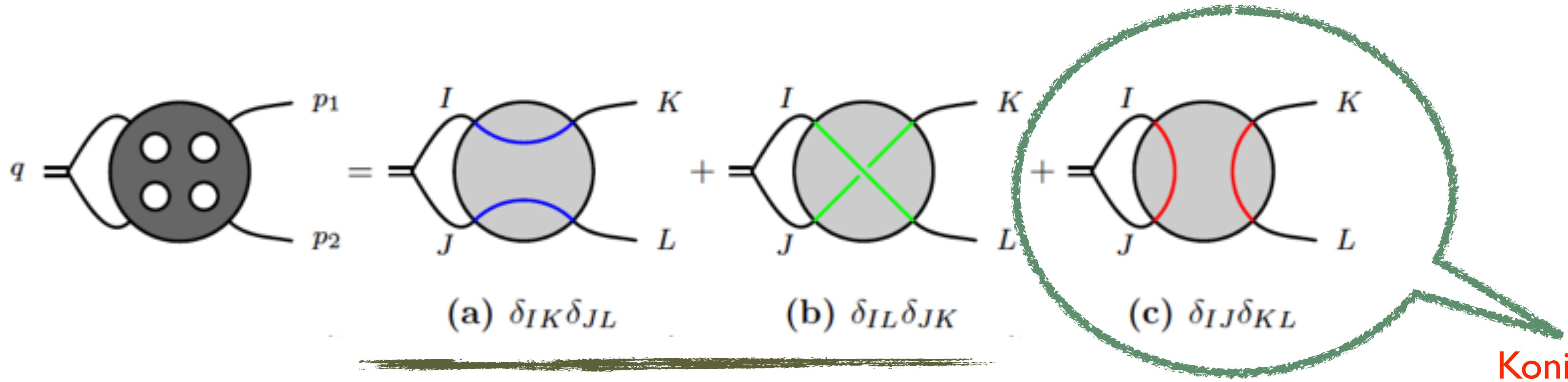
Externally Closed Index Loop



$$N_\phi = \delta_I^I = 6 + 2\epsilon$$

for Konishi in SO(6)

General 2 point multi-loop diagram with contraction of the flavor index



BPS contribution is universal - can be subtracted

This holds for our 3-point form factors and generally for n-points and l-loops

Modification to unitarity

$$f^{(\ell)}_{\mathcal{K}_6,n} = f^{(\ell)}_{\mathrm{BPS},n} + \tilde{f}^{(\ell)}_{\mathcal{K}_6,n} \quad \to \quad f^{(\ell)}_{\mathrm{BPS},n} + \frac{6+2\epsilon}{6}\tilde{f}^{(\ell)}_{\mathcal{K}_6,n} = f^{(\ell)}_{\mathcal{K},n}$$

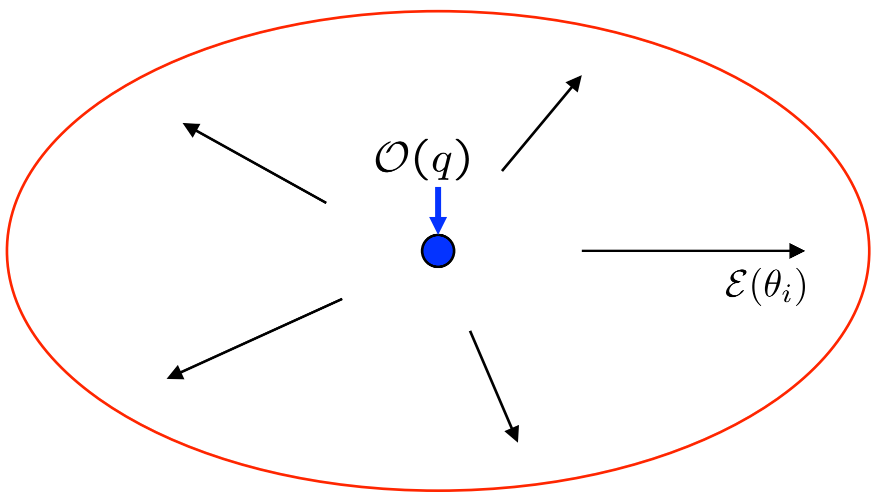
We can safely use unitarity with D=4 on shell fields.

Example: The one loop 3 point form factor only receives a rational term as a correction .

Cross-section in N=4 SYM

Can we define IR safe quantities in a CFT ?

- Scattering Amplitudes and Form factors are IR divergent
- We want to define IR safe "Cross-section" like quantity



$$\sigma_w(q) = \sum_X (2\pi)^4 \delta^{(4)}(q - k_X) w(X) |\langle X|O(0)|0\rangle|^2$$

Energy-Energy Correlation function

[Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov]

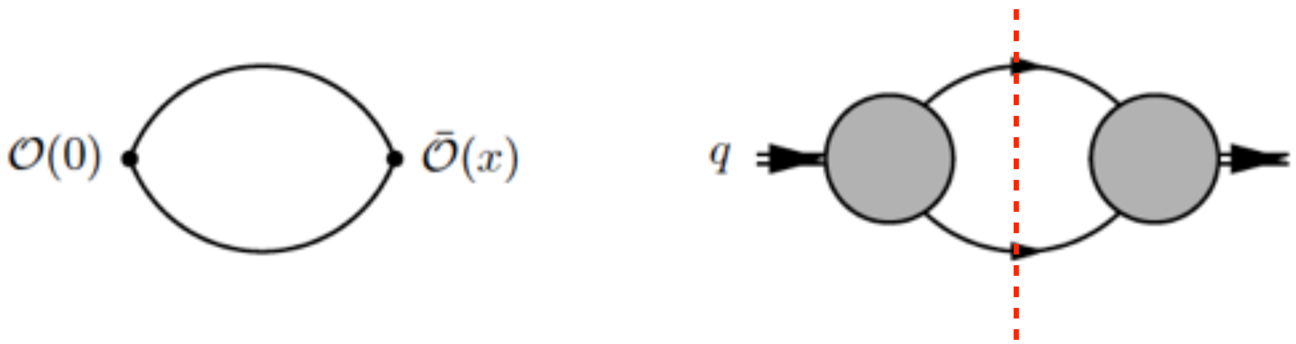
We study inclusive decay rate

$$\sigma(q) = \sum_X \delta^D(q - p_X) |\langle X|\mathcal{O}(0)|0\rangle|^2$$

$$w(X) = 1$$

Imaginary part of two-point function by optical theorem

$$\sigma_{\mathcal{O}_{\mathrm{R}}} = \Gamma_{\mathcal{O}_{\mathrm{R}}\to X} = \mathrm{Im}[2i\,\tilde{G}_{2,\mathrm{R}}(q^2)] = \sum \delta^D(q-p_X)\,|\langle X|\mathcal{O}_{\mathrm{R}}(0)|0\rangle|^2$$



How to read off anomalous dimension from cross-section

$$\frac{\sigma_R}{\sigma^{(0)}} \propto \left(\frac{q^2}{4\mu^2}\right)^{\gamma_{\mathcal{O}}}$$

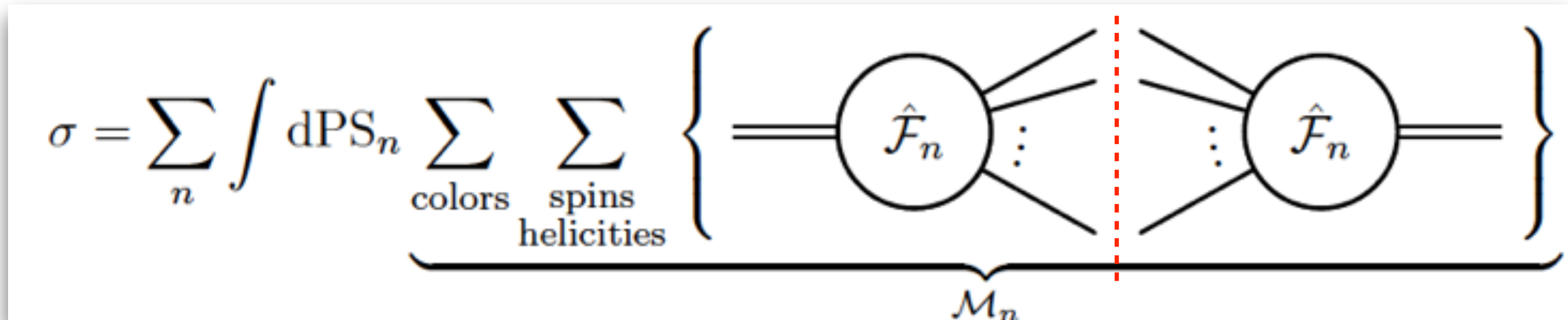
$$\log\frac{\sigma_{\mathrm{R}}}{\sigma^{(0)}} = \gamma(g)\,\log\frac{q^2}{\mu^2} + \mathrm{constants} + \mathcal{O}(\epsilon)$$

Renormalization constant related to UV divergence
gives the anomalous dimensions of operators

$$\mathcal{O}_{\mathrm{R}} = \mathcal{Z}_{\mathcal{O}}\mathcal{O}_{\mathrm{B}} = \left[1 + g^2\mathcal{Z}^{(1)} + g^4\mathcal{Z}^{(2)} + \mathcal{O}(g^6)\right]\mathcal{O}_{\mathrm{B}}$$

$$\mathcal{Z}_{\mathcal{O}} = \exp\left(\sum_{\ell=1}^{\infty}\frac{g^{2\ell}}{2\ell\epsilon}\gamma_{\mathcal{O}}^{(\ell)}\right) = 1 + g^2\frac{\gamma_{\mathcal{O}}^{(1)}}{2\epsilon} + g^4\left(\frac{(\gamma_{\mathcal{O}}^{(1)})^2}{8\epsilon^2} + \frac{\gamma_{\mathcal{O}}^{(2)}}{4\epsilon}\right) + \mathcal{O}(g^6)$$

Strategy for computing cross-section



- Compute Form factors
- Squared matrix element
- Integration over the Phase space of particles across cut

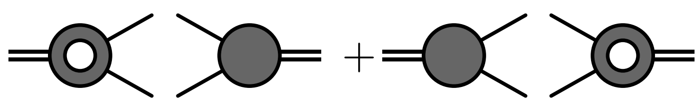
Phase space

$$\mathrm{dPS}_n = \prod_{\ell=1}^{n} \frac{\mathrm{d}^D p_\ell}{(2\pi)^D} 2\pi\delta_+(p_\ell^2) \cdot (2\pi)^D \delta^D\Big(q - \sum_{\ell=1}^{n} p_\ell\Big)$$

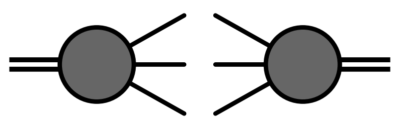
$$\delta_+(p^2) = \delta(p^2)\theta(p_0)$$

Cross-section for Konishi

IR safety by real and virtual cancelation

$\int dPS_2$ 

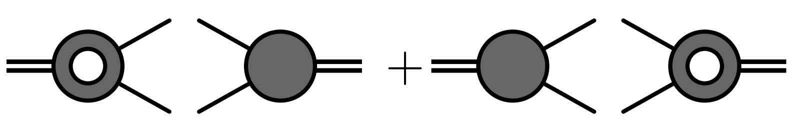
$$\sigma^{(1)}_{\rm BPS,2} = \int dPS_2\, \mathcal{M}^{(1)}_{\rm BPS,2} = \sigma^{(0)}_{\rm BPS} \left(\frac{\mu^2}{q^2}\right)^{\epsilon} \left(-\frac{4}{\epsilon^2} + \frac{7\pi^2}{3}\right) + \mathcal{O}(\epsilon)$$

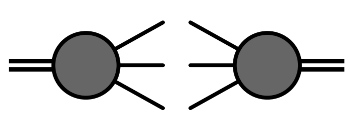
$\int dPS_3$ 

$$\sigma^{(1)}_{\rm BPS,3} = \int dPS_3\, \mathcal{M}^{(0)}_{\rm BPS,3} = \sigma^{(0)}_{\rm BPS} \left(\frac{\mu^2}{q^2}\right)^{\epsilon} \left(\frac{4}{\epsilon^2} - \frac{7\pi^2}{3}\right) + \mathcal{O}(\epsilon)$$

$$\sigma_{\rm BPS} = \sigma^{(0)}_{\rm BPS} + \mathcal{O}(\epsilon)$$

BPS is tree exact

$\int dPS_2$ 

$\int dPS_3$ 

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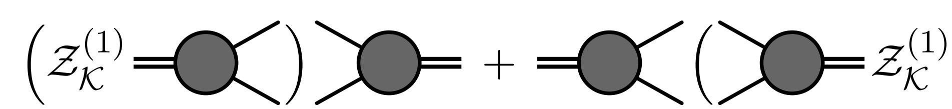
$\int dPS_2$ 

Renormalization for canceling UV divergence

$$\mathcal{Z}^{(1)}_{\mathcal{K}} = \frac{6}{\epsilon} \longrightarrow \gamma^{(1)}_{\mathcal{K}} = 12$$

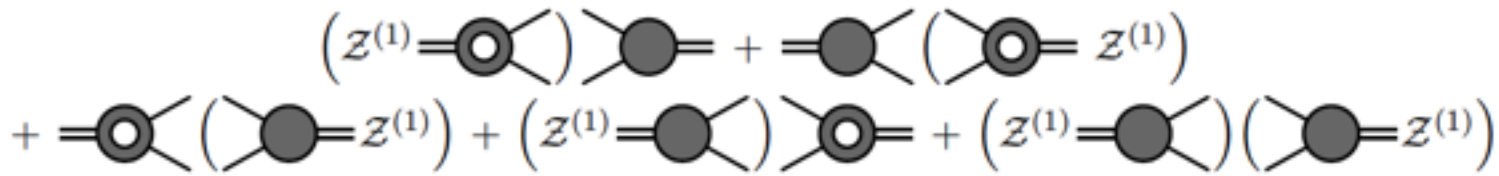
$$\sigma^{(1)}_{\mathcal{K},\rm R} = \sigma^{(1)}_{\mathcal{K}} + \sigma^{(1)}_{\mathcal{Z}^{(1)}\mathcal{K}} = \sigma^{(0)}_{\mathcal{K}} \left(12 \log \frac{q^2}{\mu^2} - 28\right) + \mathcal{O}(\epsilon)$$

Konishi anomalous dimension

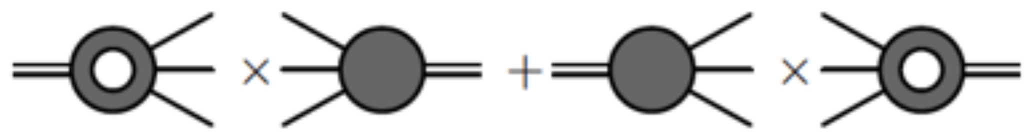
(a)

$\sigma^{(2)}_{\mathcal{K},2}$

Two-particle channel



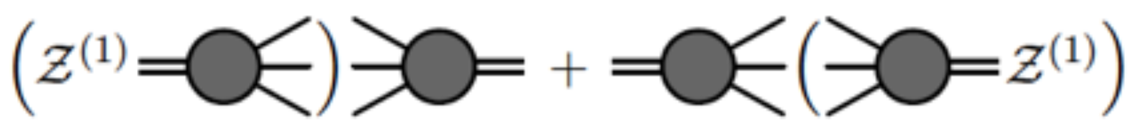
$\sigma^{(2)}_{\mathcal{Z}^{(1)}\mathcal{K},2}$



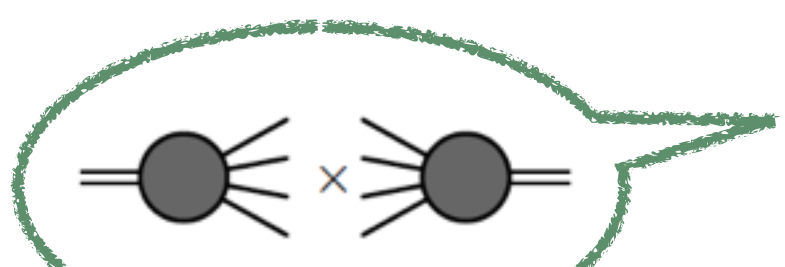
(b)

$\sigma^{(2)}_{\mathcal{K},\mathrm{3pt}}$

Three-particle channel



$\sigma^{(2)}_{\mathcal{Z}^{(1)}\mathcal{K},\mathrm{3pt}}$



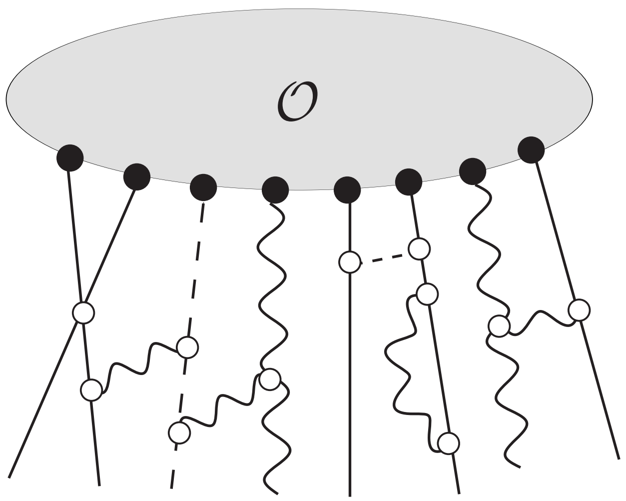
(c)

Included in the BPS

$$\log\left(\frac{\sigma_{\mathcal{K},\mathrm{R}}}{\sigma^{(0)}_{\mathcal{K}}}\right) = g^2\left(12\,\log\left(\frac{q^2}{\mu^2}\right) - 28\right) + g^4\left(-48\,\log\left(\frac{q^2}{\mu^2}\right) + 116 + 72\pi^2 + 336\zeta_3\right) + \mathcal{O}(g^6,\epsilon)$$

$\gamma^{(1)} = 12\,, \qquad \gamma^{(2)} = -48$

Unitarity, dilation operator and remainder function



Integrability picture and its connection to minimal form factors for general operators in N=4 SYM were given recently.
And the full one-loop dilation operators were (re)derived using unitarity cuts of form factors. *[Wilhelm]*

$$\delta\mathfrak{D} = \frac{\mathrm{d}}{\mathrm{d}\mu}\log\mathcal{Z} = 2\varepsilon g^2 \frac{\partial}{\partial g^2}\log\mathcal{Z} = \sum_{\ell=1}^{\infty} g^{2\ell}\mathfrak{D}^{(\ell)}$$

Form factors and Unitarity provide new promising tools to compute higher-loop dilation operators (for which the Konishi form factor provides a two-loop example), remainder functions as well as new insight to understand integrability in N=4 SYM.

[Koster, Mitev, Staudacher][Brandhuber, Penante, Travaglini, Young] [Zwiebel]

SU(2) unitarity and Leading transcendentality

Length of the operator is L

$$\{X=\phi_{14}, Y=\phi_{24}\} \qquad \mathcal{O}=\mathrm{tr}(XXYX\cdots)$$

n=L is for Minimal form factor

Operators eigenstate under renormalization eg. BPS or Konishi primary yields ratio of loop to tree. Otherwise, promote equation to operator.

$$\mathcal{F}_{\mathcal{O}}=(1+g^2\mathcal{I}^{(1)}+g^4\mathcal{I}^{(2)}+\ldots)\mathcal{F}^{(0)}_{\mathcal{O}}$$

l-loop interactions maximally involve (l+1) neighboring fields.

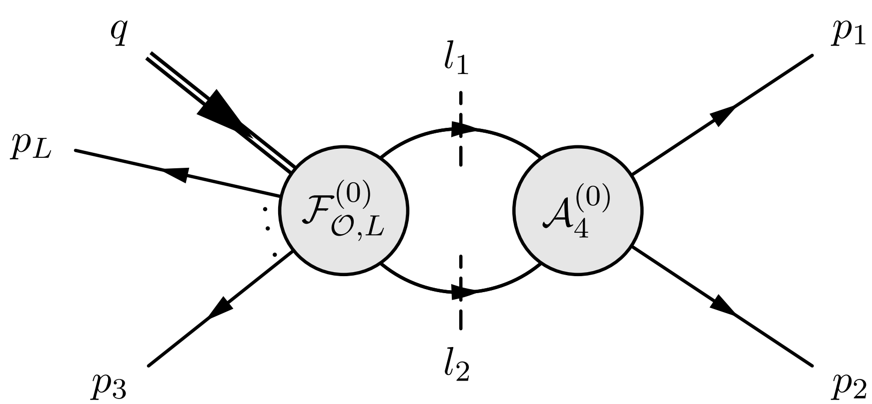
@ 1-loop

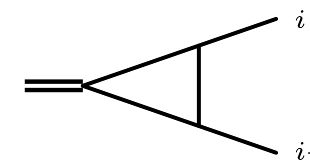
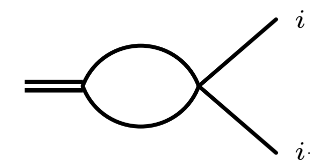
$$\mathcal{I}^{(1)}=\sum_{i=1}^{L} I^{(1)}_{i\,i+1}$$

$$\{XX\to XX, XY\to XY, XY\to YX\}$$

$$I^{(1)}_{i\,i+1}=\sum_{A,B,C,D=1}^{2}(I^{(1)}_i)^{Z_C Z_D}_{Z_A Z_B}\eta_i^C\frac{\partial}{\partial\eta_i^A}\eta_{i+1}^D\frac{\partial}{\partial\eta_{i+1}^B}$$

$$\{Z_A Z_B\to Z_C Z_D\}$$



$(I_i^{(1)})$	$\begin{smallmatrix} X X \\ X X \end{smallmatrix}$	$\begin{smallmatrix} X Y \\ X Y \end{smallmatrix}$	$\begin{smallmatrix} Y X \\ X Y \end{smallmatrix}$
 $s_{i\,i+1}$	-1	-1	0
	0	-1	+1

Renormalization and anomalous dimension

$$\mathcal{F}^{(0)}_{\mathcal{Z}\mathcal{O}}(1,\ldots,L;q) = \mathcal{Z}\mathcal{F}^{(0)}_{\mathcal{O}}(1,\ldots,L;q)$$

$$\underline{\mathcal{I}}^{(1)} = \mathcal{I}^{(1)} + \mathcal{Z}^{(1)}$$

$$\mathcal{Z}_{i\,i+1} = \frac{1}{\varepsilon}(\mathbb{1} - \mathbb{P})_{i\,i+1}$$

UV divergence needs renormalization. acts as operator . At 1-loop we get UV renormalized interaction canceling UV divergence of bubble integrals. We also rewrite dilatation operator as an operator acting on tree level form factors.

$$(\mathfrak{D}_i^{(1)})^{XX}_{XX} = 0\,, \qquad (\mathfrak{D}_i^{(1)})^{XY}_{XY} = 2\,, \qquad (\mathfrak{D}_i^{(1)})^{YX}_{XY} = -2$$

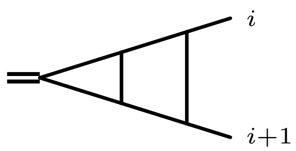
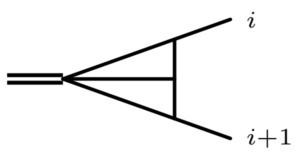
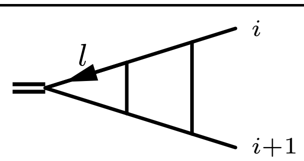
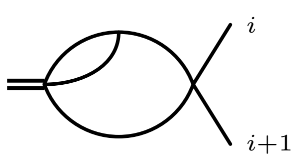
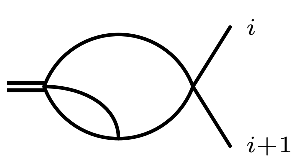
$$\mathfrak{D}^{(1)}_{i\,i+1} = 2(\mathbb{1} - \mathbb{P})_{i\,i+1}$$

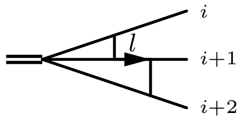
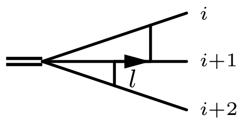
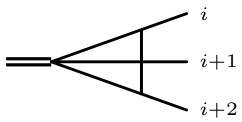
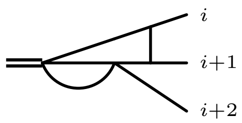
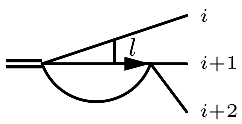
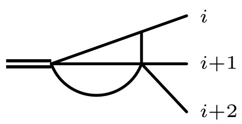
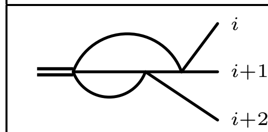
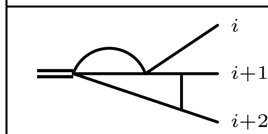
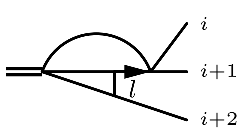
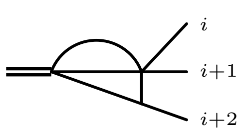
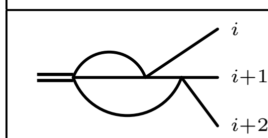
@ two loops - UV and IR entangled

The possible length of interactions are 2 or 3. Also 2 disconnected one loop interactions.

$$\mathcal{I}^{(2)} = \sum_{i=1}^{L} \left(I^{(2)}_{i\,i+1\,i+2} + I^{(2)}_{i\,i+1} + \frac{1}{2} \sum_{j=i+2}^{L+i-2} I^{(1)}_{i\,i+1} I^{(1)}_{j\,j+1} \right)$$

We need double and triple cuts to compute the loop corrections.

$(I_i^{(2)})$		$\begin{matrix}XX\\XX\end{matrix}$	$\begin{matrix}XY\\XY\end{matrix}$	$\begin{matrix}YX\\XY\end{matrix}$
	$s^2_{i\,i+1}$	+1	+1	0
	$s_{i\,i+1}$	+1	+1	0
	$s_{i\,i+1}s_{i\,l}$	0	+1	-1
		0	+1	-1
		0	+1	-1

$(I_i^{(2)})$		$\begin{matrix}XXX\\XXX\end{matrix}$	$\begin{matrix}XXY\\XXY\end{matrix}$	$\begin{matrix}XYX\\XYX\end{matrix}$	$\begin{matrix}XYX\\XXY\end{matrix}$	$\begin{matrix}XXY\\XYX\end{matrix}$	$\begin{matrix}YXX\\XXY\end{matrix}$
	$s_{i\,l}s_{i+1\,i+2}$	+1	+1	+1	0	0	0
	$s_{i\,i+1}s_{i+2\,l}$	+1	+1	+1	0	0	0
	$s_{i\,i+1\,i+2}$	-1	-1	-1	0	0	0
	$s_{i\,i+1}$	0	+1	+1	-1	-1	0
	$s_{i\,l}$	0	+1	+1	-1	-1	0
		0	-1	-1	+1	+1	0
		0	0	+1	-1	0	+1
	$s_{i+1\,i+2}$	0	0	+1	0	0	0
	$s_{i+2\,l}$	0	0	+1	0	0	0
		0	0	-1	0	0	0
		0	0	+1	0	-1	0

Master Integrals are known

IR divergences have well defined universal structure[BDS]. Subtract by BDS ansatz. We are only left with UV divergence which are renormalized.

$$\underline{\mathcal{I}}^{(2)} = \mathcal{I}^{(2)} + \mathcal{I}^{(1)}\mathcal{Z}^{(1)} + \mathcal{Z}^{(2)}$$

Like in 1-loop we can write the operator form of the renormalization operator. This leads to the two-loop dilatation operator in SU(2) sector.

$$\mathfrak{D}^{(2)}_{i\,i+1\,i+2} = -2\big(\mathbb{P}_{i\,i+1}\mathbb{P}_{i+1\,i+2} + \mathbb{P}_{i+1\,i+2}\mathbb{P}_{i\,i+1} - 3\mathbb{P}_{i\,i+1} - 3\mathbb{P}_{i+1\,i+2} + 4\big)$$

$$(\mathfrak{D}^{(2)}_i)^{XXX}_{XXX} = 0\,, \qquad (\mathfrak{D}^{(2)}_i)^{XYX}_{XYX} = -8\,, \qquad (\mathfrak{D}^{(2)}_i)^{XXY}_{XXY} = -2$$

$$(\mathfrak{D}^{(2)}_i)^{XXY}_{XYX} = 4\,, \qquad (\mathfrak{D}^{(2)}_i)^{XYX}_{XXY} = 4\,, \qquad (\mathfrak{D}^{(2)}_i)^{YXX}_{XXY} = -2$$

We can compute other interesting quantities !

Transcendentality of Remainder Function

Finite remainder function is obtained from the BDS ansatz of renormalized form factors. It was also computed earlier for the BPS form factor at 2-loops.

$$\mathcal{R}^{(2)} = \underline{\mathcal{I}}^{(2)}(\varepsilon) - \frac{1}{2}\left(\underline{\mathcal{I}}^{(1)}(\varepsilon)\right)^2 - f^{(2)}(\varepsilon)\underline{\mathcal{I}}^{(1)}(2\varepsilon) + \mathcal{O}(\varepsilon)$$

$$f^{(2)}(\varepsilon) = -2\zeta_2 - 2\zeta_3\varepsilon - 2\zeta_4\varepsilon^2$$

$$\mathcal{R}^{(2)} = \sum_{i=1}^{L} R^{(2)}_{i\,i+1\,i+2}$$

At 2-loops interaction length is 3, remainder function is also a density acting simultaneously on 3 neighboring points.

Remainder is a function of:

$$u_i = \frac{s_{i\,i+1}}{s_{i\,i+1\,i+2}}\,, \quad v_i = \frac{s_{i+1\,i+2}}{s_{i\,i+1\,i+2}}\,, \quad w_i = \frac{s_{i+2\,i}}{s_{i\,i+1\,i+2}}$$

$$s_{i\,i+1\,i+2} = s_{i\,i+1} + s_{i+1\,i+2} + s_{i+2\,i}$$

$$u_i + v_i + w_i = 1$$

For every triplet of points these are like cross-ratio's in amplitudes.

For scattering amplitudes and BPS form factors remainder function was of uniform transcendentally of degree (2l).

$$(R_i^{(2)})^{YXX}_{XXY} + (R_i^{(2)})^{XYX}_{XXY} + (R_i^{(2)})^{XXY}_{XXY} = (R_i^{(2)})^{XXX}_{XXX}$$
$$(R_i^{(2)})^{XYX}_{XYX} + (R_i^{(2)})^{YXX}_{XYX} + (R_i^{(2)})^{XXY}_{XYX} = (R_i^{(2)})^{XXX}_{XXX}$$
$$(R_i^{(2)})^{XYX}_{XXY} + (R_i^{(2)})^{YXX}_{XXY} = (R_i^{(2)})^{XXY}_{XYX} + (R_i^{(2)})^{XXY}_{YXX}$$

Relations exist between different component remainders as a consequence of SU(2) symmetry. Only 3 independent ones.

$$(R_i^{(2)})^{XXX}_{XXX},\ (R_i^{(2)})^{XYX}_{XXY} \text{ and } (R_i^{(2)})^{YXX}_{XXY}.$$

$$(R_i^{(2)})^{XXX}_{XXX} = (R_i^{(2)})^{XXX}_{XXX}\Big|_4 \quad \begin{aligned} &-\mathrm{Li}_4(1-u_i) - \mathrm{Li}_4(u_i) + \mathrm{Li}_4\left(\frac{u_i-1}{u_i}\right) - \log\left(\frac{1-u_i}{w_i}\right)\left[\mathrm{Li}_3\left(\frac{u_i-1}{u_i}\right) - \mathrm{Li}_3\left(1-u_i\right)\right] \\ &-\log\left(u_i\right)\left[\mathrm{Li}_3\left(\frac{v_i}{1-u_i}\right) + \mathrm{Li}_3\left(-\frac{w_i}{v_i}\right) + \mathrm{Li}_3\left(\frac{v_i-1}{v_i}\right) - \frac{1}{3}\log^3\left(v_i\right) - \frac{1}{3}\log^3\left(1-u_i\right)\right] \\ &-\mathrm{Li}_2\left(\frac{u_i-1}{u_i}\right)\mathrm{Li}_2\left(\frac{v_i}{1-u_i}\right) + \mathrm{Li}_2\left(u_i\right)\left[\log\left(\frac{1-u_i}{w_i}\right)\log\left(v_i\right) + \frac{1}{2}\log^2\left(\frac{1-u_i}{w_i}\right)\right] \\ &+\frac{1}{24}\log^4\left(u_i\right) - \frac{1}{8}\log^2\left(u_i\right)\log^2\left(v_i\right) - \frac{1}{2}\log^2\left(1-u_i\right)\log\left(u_i\right)\log\left(\frac{w_i}{v_i}\right) \\ &-\frac{1}{2}\log\left(1-u_i\right)\log^2\left(u_i\right)\log\left(v_i\right) - \frac{1}{6}\log^3\left(u_i\right)\log\left(w_i\right) \\ &-\zeta_2\left[\log\left(u_i\right)\log\left(\frac{1-v_i}{v_i}\right) + \frac{1}{2}\log^2\left(\frac{1-u_i}{w_i}\right) - \frac{1}{2}\log^2\left(u_i\right)\right] \quad +\zeta_3\log(u_i) + \frac{\zeta_4}{2} \\ &+ G\left(\{1-u_i, 1-u_i, 1, 0\}, v_i\right) \end{aligned}$$

Leading transcendental degree is 3.

$$(R_i^{(2)})^{XYX}_{XXY} = (R_i^{(2)})^{XYX}_{XXY}\Big|_3 + \mathrm{Li}_2\left(1-u_i\right) + \mathrm{Li}_2\left(1-v_i\right) + \log\left(u_i\right)\log\left(v_i\right) - \frac{1}{2}\log\left(-s_{i+1\,i+2}\right)\log\left(\frac{u_i}{v_i}\right) + 2\log\left(-s_{i\,i+1}\right) + \frac{\pi^2}{3} - 7$$

$$\begin{aligned}(R_i^{(2)})^{XYX}_{XXY}\Big|_3 = &\Bigg[\mathrm{Li}_3\left(-\frac{u_i}{w_i}\right) - \log\left(u_i\right)\mathrm{Li}_2\left(\frac{v_i}{1-u_i}\right) + \frac{1}{2}\log\left(1-u_i\right)\log\left(u_i\right)\log\left(\frac{w_i^2}{1-u_i}\right)\\ &- \frac{1}{2}\mathrm{Li}_3\left(-\frac{u_i v_i}{w_i}\right) - \frac{1}{2}\log\left(u_i\right)\log\left(v_i\right)\log\left(w_i\right) - \frac{1}{12}\log^3\left(w_i\right) + \left(u_i \leftrightarrow v_i\right)\Bigg]\\ &-\mathrm{Li}_3\left(1-v_i\right) + \mathrm{Li}_3\left(u_i\right) - \frac{1}{2}\log^2\left(v_i\right)\log\left(\frac{1-v_i}{u_i}\right) + \frac{1}{6}\pi^2\log\left(\frac{v_i}{w_i}\right) - \frac{1}{6}\pi^2\log\left(-s_{i\,i+1\,i+2}\right)\end{aligned}$$

Leading transcendental degree is 2

$$(R_i^{(2)})^{YXX}_{XXY} = \frac{1}{2}\log\left(-s_{i+1\,i+2}\right)\log\left(\frac{u_i}{v_i}\right) - \mathrm{Li}_2\left(1-u_i\right) - \log\left(u_i\right)\log\left(v_i\right) + \frac{1}{2}\log^2\left(v_i\right) + \log\left(-s_{i+1\,i+2}\right) - 2\log\left(-s_{i\,i+1}\right) + \frac{7}{2}$$

Rational terms of the remainder function are related to the dilatation operator !

$$\mathfrak{D}^{(2)}_{i\,i+1\,i+2} = -\frac{4}{7}\,R^{(2)}_{i\,i+1\,i+2}\Big|_0 .$$

For non-protected operator we have mixed transcendentality for minimal form factors.

Only 1 transcendental degree 4 function. It is same as in BPS case, would be true for any operator which is an identity in flavor space.

Leading degree of transcendentality t=4-s, is related to the shuffling number s of the remainder function density.

Only one function of transcendental degree 3 and two of degree 2 and less.

Conjecture: Maximal transcendental part of all two loop minimal form factors has same degree 4 part as for the BPS one.

Soft and collinear limit the remainder function of minimal form factors are non-vanishing as in BPS case.

Conclusions & Outlook

- Study of form factor of non-protected operator.
- New interesting features for Konishi- UV divergence, rational terms.
- Modification of Unitarity prescription in D=4 for non-protected operators
- Study Cross-section like quantity for CFT— IR safe.
- Anomalous Dimension @ 2 loops for Konishi.
- SU(2) operator 2 loop dilatation operator.
- SU(2) remainder function does not have uniform transcendentality.

- Three point functions from Unitarity — CFT data ?
- Cross -section for energy- energy Correlation function?
- Higher-loop Cross-section for Konishi?
- Dilatation operator for other operators at 2 loops?
- Other theories-ABJM?
- Bootstrap remainder function for all transcendentality and higher points?