# Q-Monopole-Ball:
## A Topological and Nontopological Soliton

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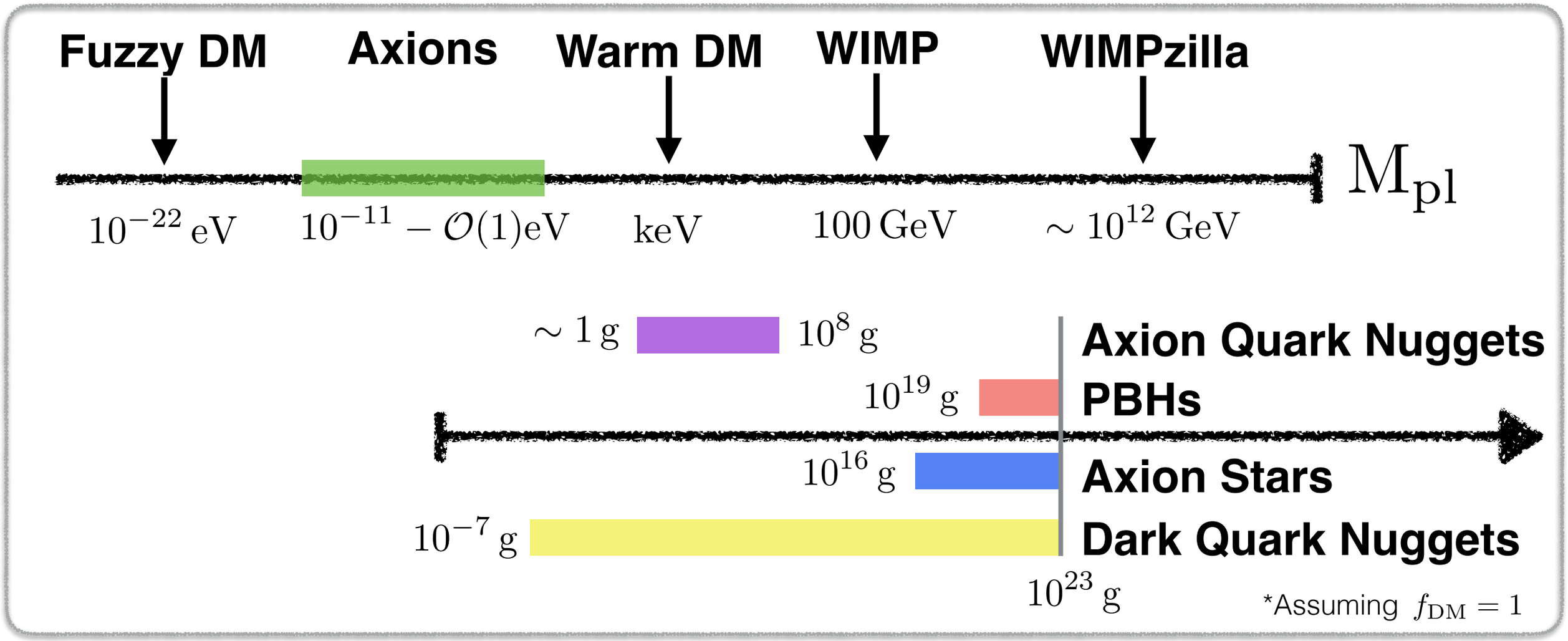
@MITP Mega Dark Matter workshop, 18.05.2022

2111.10360 + on going work, w/ Yang Bai and Nicholas Orlofsky

# DM Zoo




- The formation of MEGA DM sometimes need other ingredients, but not necessary
- Field theory itself provides interesting candidates: <u>the solitons</u>

# Topological meets Non-Topological

**Monopoles**

- Dirac quantization
  $q_{\rm mag} = 2\pi q/e$
- charged topologically
  $\pi_2(SU(2)/U(1)) = \mathbb{Z}$

**Q-balls**

- charged non-topologically
- protected by charge conservation

# Topological meets Non-Topological

**Monopoles** **Q-balls**

- ✦ Dirac quanti non-topologically
  $q_{\mathrm{mag}} = 2$
- ✦ charged top d by charge
  $\pi_2(SU(2)/U($ ation

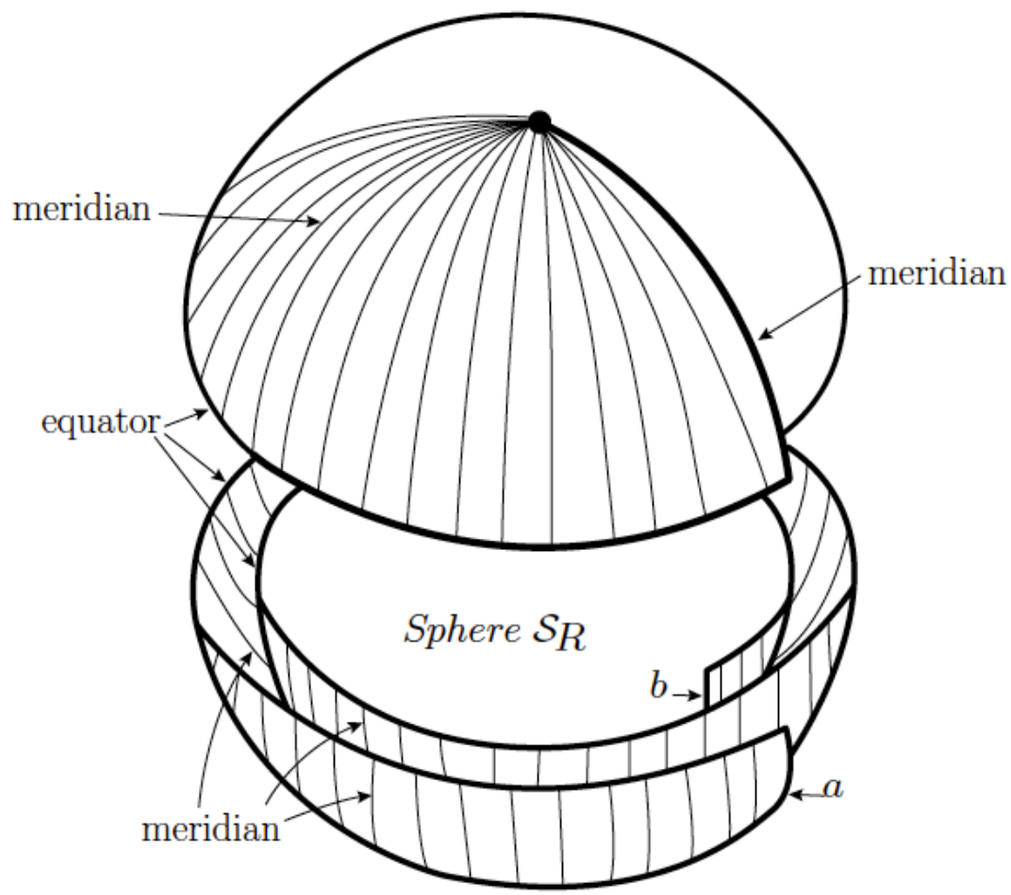
**Q-monopole-balls**

# Outline

- **Brief intro to monopoles and non-topological solitons**
- **Q-monopole-balls of unity magnetic charge (q=2)**
  - small-$Q$ regime
  - large-$Q$ regime
  - stability
- **Q-monopole-balls of multiple magnetic charges**
- **Phenomenology**
  - possible constraints
  - formation & relic abundance
- **Concluding Marks**

# Magnetic Monopoles

- **Magnetic charge is topological charge**
  - ➡ Consider a spontaneously broken gauge symmetry
  - ➡ 't Hooft-Polyakov: $\pi_2(SU(2)/U(1)) = \mathbb{Z}$ $\qquad q_{\rm mag} = 2\pi q/e$




[M. Shifman, *Advanced Topics in Quantum Field Theory*]

# Magnetic Monopoles

- **Magnetic charge is topological charge**
  - ➡ Consider a spontaneously broken gauge symmetry
  - ➡ 't Hooft-Polyakov: $\pi_2(SU(2)/U(1)) = \mathbb{Z}$ $\qquad q_{\rm mag} = 2\pi q/e$
- **EOM for q=2**
  - ➡ Hedgehog gauge:

$$\phi^a = \hat{r}^a\, v\, f(r)\,,\ A_0 = 0\,,\quad A_i^a = \epsilon^{aij}\,\frac{\hat{r}^j}{e\,r}\,a(r)$$

  - ➡ Let $\rho = e\,v\,r$

$$a'' - \frac{1}{\rho^2}\,a(1-a)(2-a) + (1-a)\,f^2 = 0\,,$$

$$f'' + \frac{2}{\rho}\,f' - \frac{2}{\rho^2}\,(1-a)^2\,f - \frac{1}{2}\,\lambda_\phi\,f\,(f^2-1) = 0\,,$$

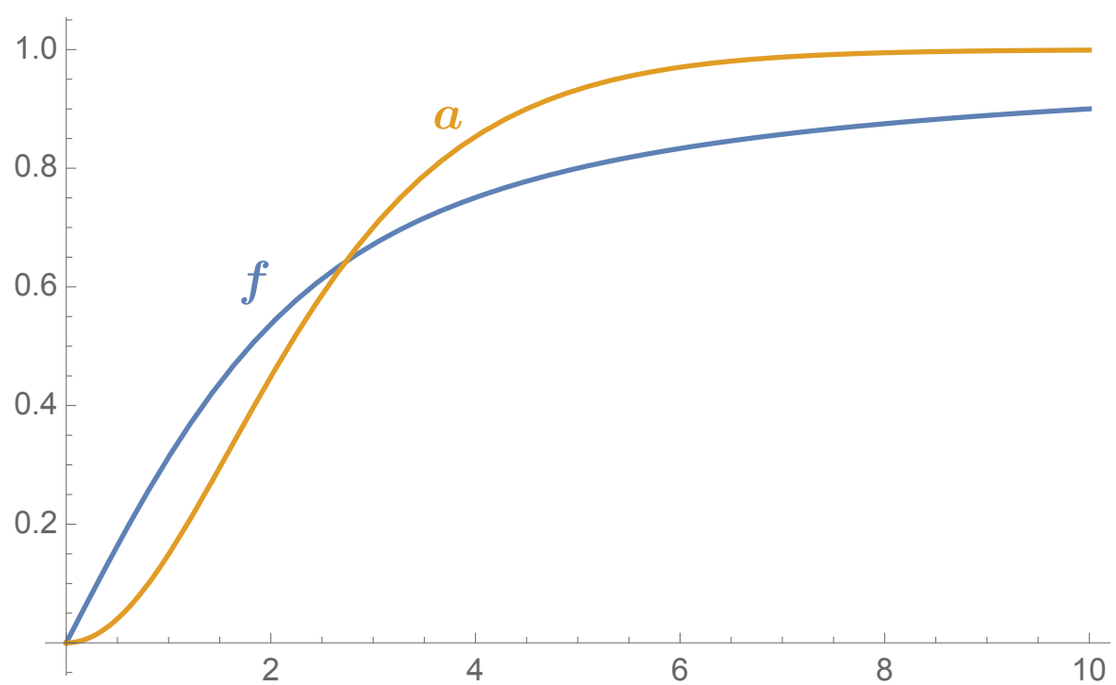
**Higgs quartic coupling**

# Magnetic Monopoles

- **BPS limit**
  - ➡ The EOM has analytic solutions when $\lambda_\phi = 0$

$$a = 1 - \frac{\rho}{\sinh\rho}\,, \quad f = \frac{\cosh\rho}{\sinh\rho} - \frac{1}{\rho}$$

$$M_{\rm mono} = \frac{4\pi v}{e}$$



# Magnetic Monopoles

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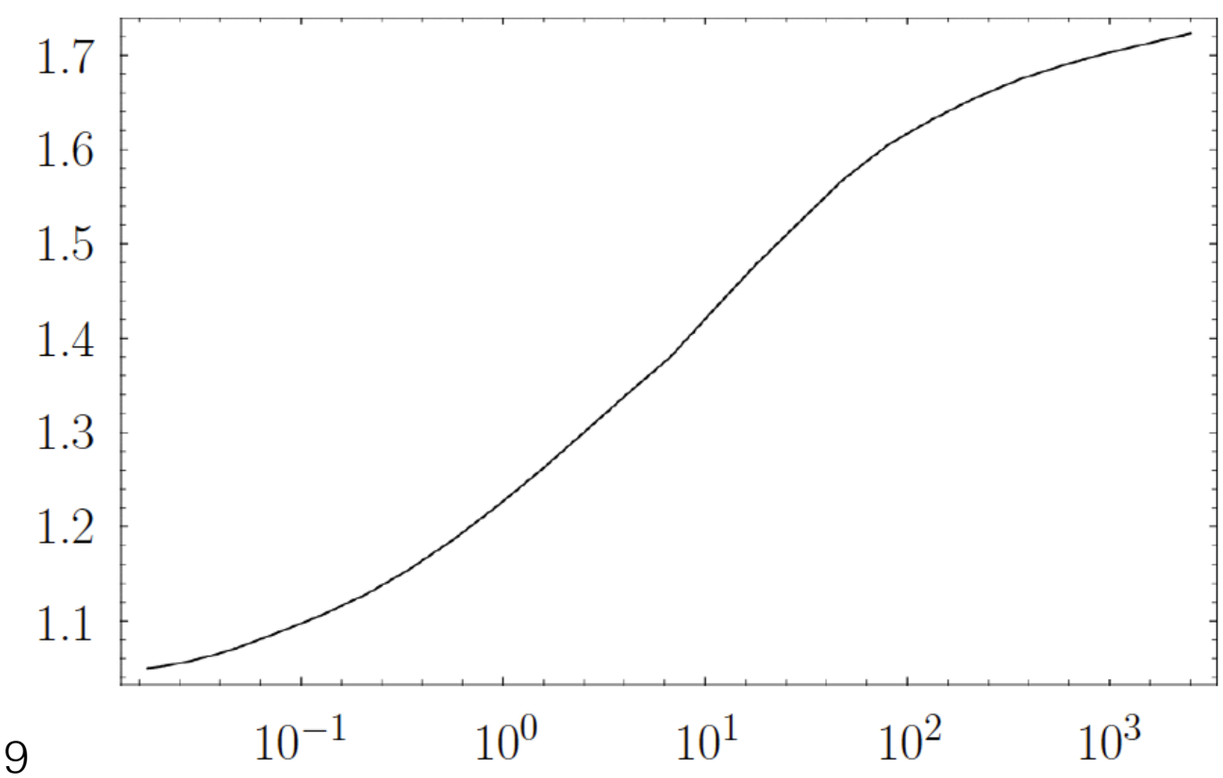
- **Beyond the BPS limit**

$$M_{\rm mono} = \frac{4\pi v}{e}\, Y\left(\frac{\lambda_\phi}{e^2}\right)$$

$$Y(0) = 1,\ Y(\infty) \approx 1.787$$

[T. Kirkman, C. Zachos,
Phys. Rev. D 24, 999 (1981)]



# Non-topological Solitons

- **Stable macroscopic bound states may exist in a theory with reasonable amount of nonlinearity**
  - ➡ A (global) symmetry to protect the stability
  - ➡ A scalar potential providing an attractive force

[See T. D. Lee and Y. Pang, Phys. Rept. 221 (1992) 251-350 for a review]

- **Examples**
  - ➡ Coleman's Q-ball [S. Coleman, Nucl. Phys. B 262 (1985) 2 263 ]
  - ➡ Baryon-ball/lepton-ball in MSSM [A. Kusenko, Phys. Lett. B 405 (1997) 108 ]
  - ➡ Electroweak symmetric DM ball [E. Ponton, Y. Bai, B. Jain, JHEP 09 (2019) 011 ]

# Non-topological Solitons

- **Let's use Q-ball with a *global* U(1) as an example**

[See e.g. 2103.06905 for examples of gauged Q-balls]

  - Let $\Phi = \phi(r)\, e^{-i\omega t}/\sqrt{2}$
  - From the Lagrangian we immediately have

$$E = \int d^3r \left[\frac{1}{2}\omega^2\phi^2 + \frac{1}{2}(\nabla\phi)^2 + U(\phi)\right],\ Q = \omega \int d^3r\, \phi^2$$

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + \underbrace{\omega^2\phi - \frac{dU'}{d\phi}}_{\text{effective potential}} = 0$$

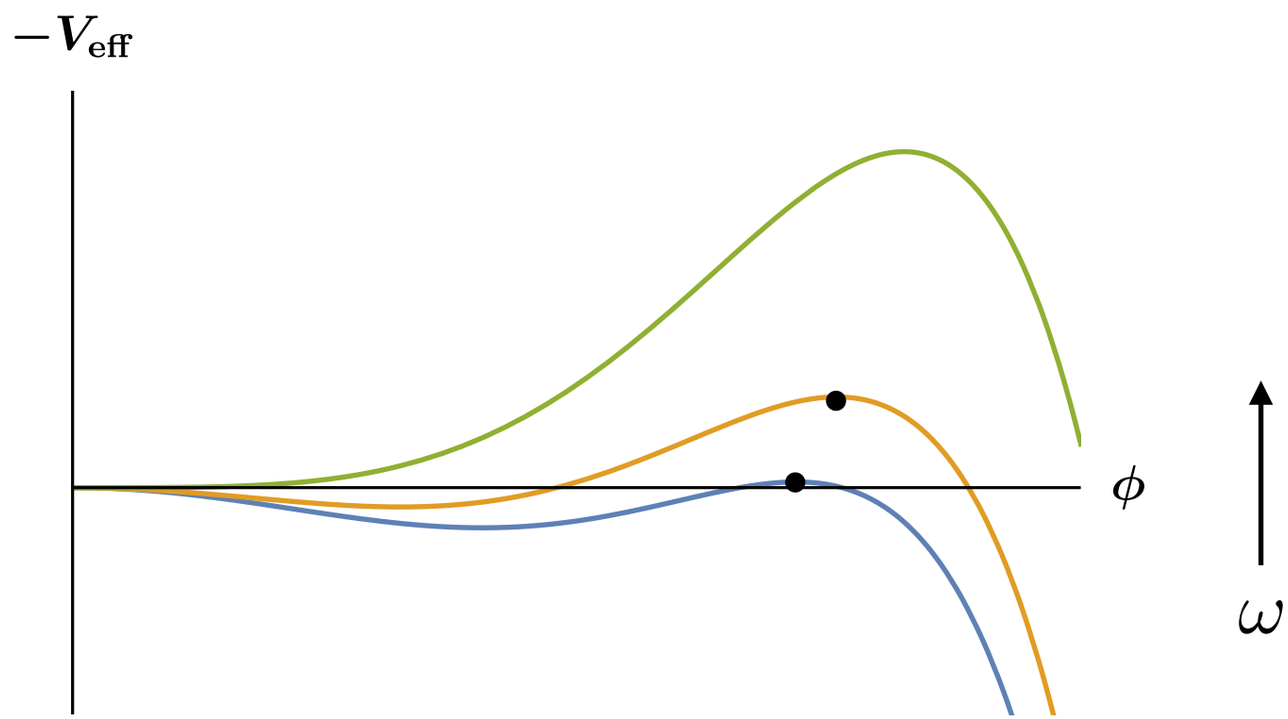
$$V_{\text{eff}} = U - \frac{1}{2}\omega^2\phi^2$$

  - By defining an effective potential, the EOM has a Newtonian interpretation if we take $r \to t,\ \phi \to x$

# Non-topological Solitons

- **A particle moving along -V**

$$\phi(r=0)=\phi_0,\ \phi(r=\infty)=0 \rightarrow x(t=0)=x_0,\ x(t=\infty)=0$$




➡ There must be a local minimum and a local maxima

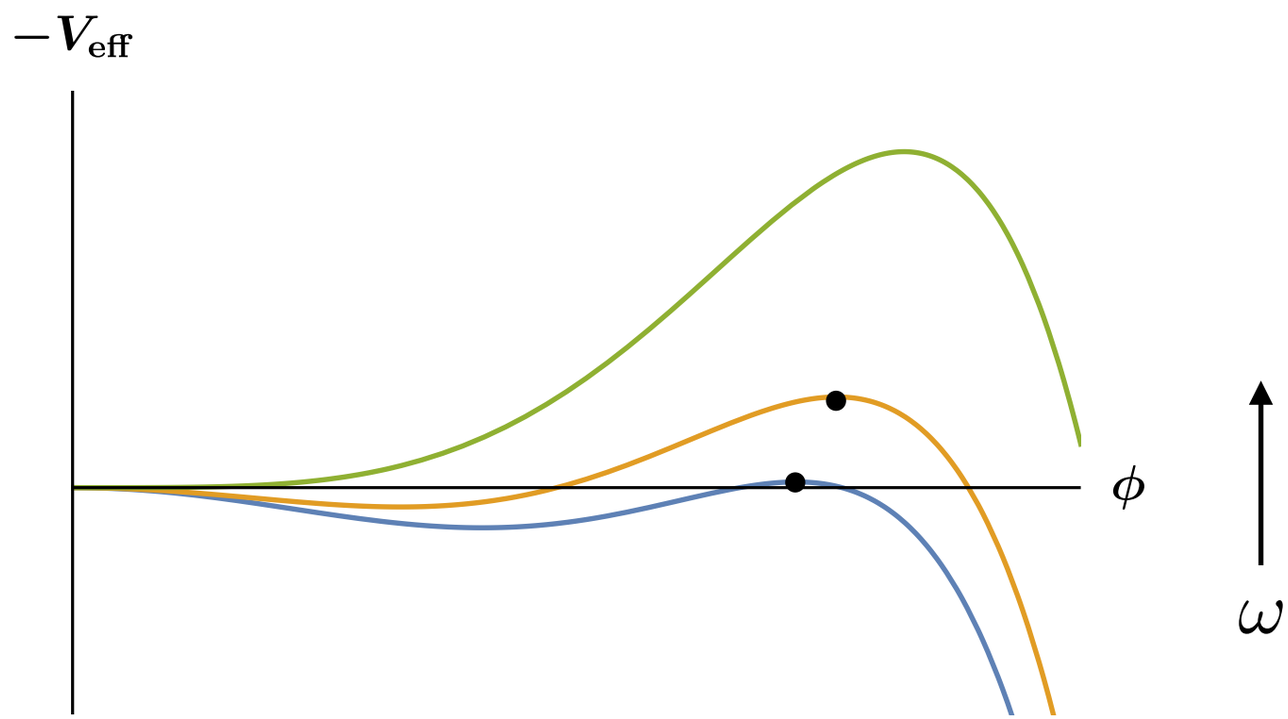
➡ The local maxima must be greater than zero

so cannot be achieved with a single field at quartic order

# Non-topological Solitons

- **A particle moving along -V**

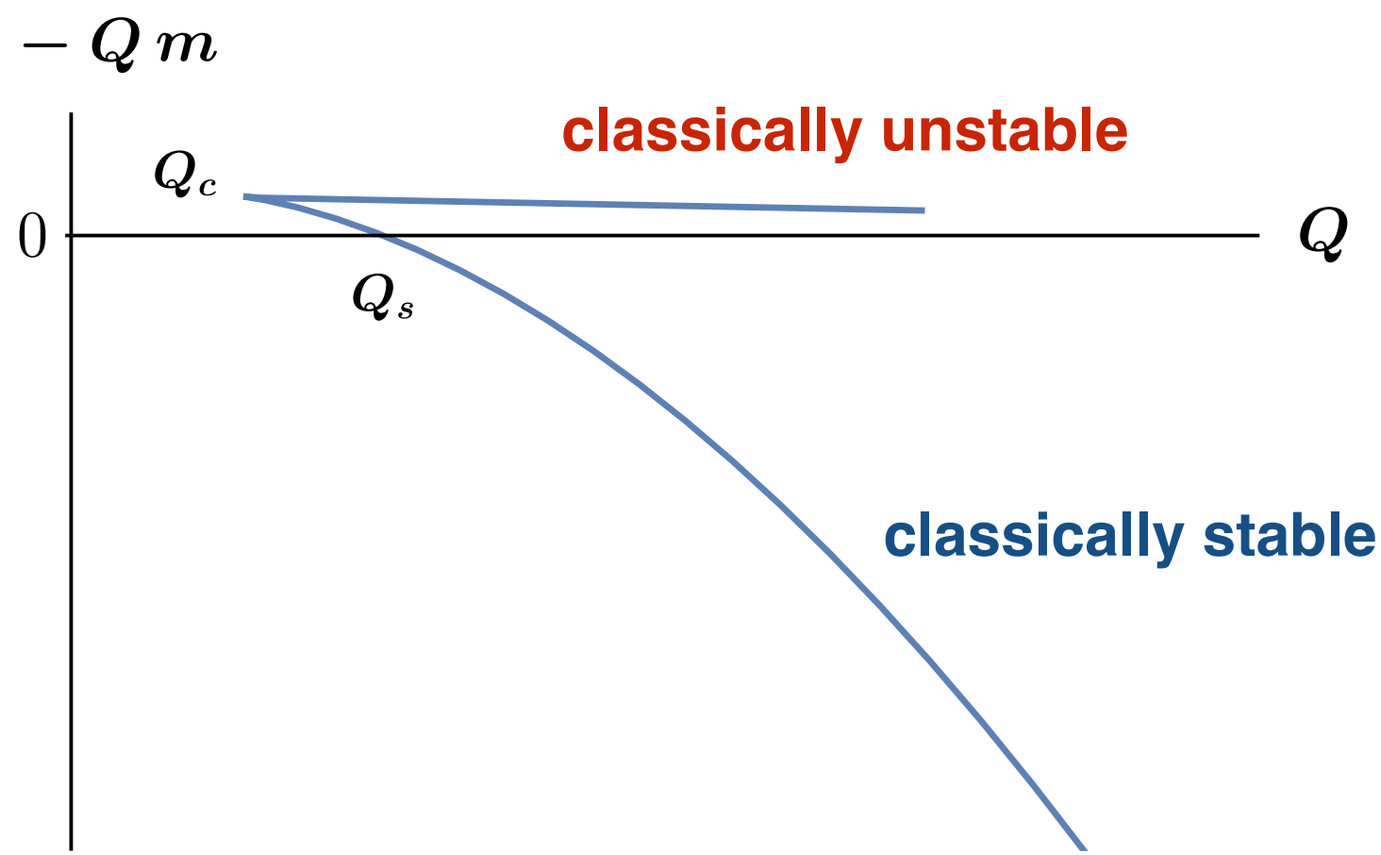
$$\phi(r=0)=\phi_0,\ \phi(r=\infty)=0 \rightarrow x(t=0)=x_0,\ x(t=\infty)=0$$




➡ The larger $\omega$, the smaller charge for the Q-ball (and *vice versa*)

# Non-topological Solitons

- **Two branches of solutions**
  - ➡ Can a Q-ball with charge *Q* and mass *M* may decay into *Q* free U(1) quanta (with mass *m*)?



# Outline

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- **Q-monopole-balls of unity magnetic charge (q=2)**
  - small-$Q$ regime
  - large-$Q$ regime
  - stability
- **Q-monopole-balls of multiple magnetic charges**
- **Phenomenology**
  - possible constraints
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- **Concluding Marks**

# Basics of QMBs

- **A Q-monopole-ball (QMB) is charged both topologically and non-topologically**
  - ➡ Consider gauged SU(2) × global U(1). Introduce a quartic coupling between the scalar fields

$$\mathcal{L} = |\partial_\mu S|^2 + \frac{1}{2}(D_\mu \phi^a)^2 - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - V(S,\phi)\,,$$

$$V(S,\phi) = \frac{1}{8}\lambda_\phi(\phi^a\phi^a - v^2)^2 + \frac{1}{2}\lambda_{\phi S}|S|^2(\phi^a\phi^a) + \lambda_s|S|^4 + \underline{m^2_{s,0}|S|^2}$$

**taken to be zero**

$$m_S = v\sqrt{\lambda_{\phi S}/2}$$

  - ➡ Again, rewrite everything dimensionlessly

$$\phi^a = \hat{r}^a\, v\, f(r)\,, \quad S = e^{-i\omega t}\frac{v}{\sqrt{2}}\, s(r)\,, \quad A_0 = 0\,, \quad A^a_i = \epsilon^{aij}\frac{\hat{r}^j}{e\, r}\, a(r)$$

$$\Omega = \omega/v,\ r = \overline{r}/v$$

# Basics of QMBs

- **A Q-monopole-ball (QMB) is charged both topologically and non-topologically**
  - ➡ A slightly more complicated EOM

$$a'' - \frac{1}{\bar{r}^2}\, a(1-a)(2-a) + e^2\,(1-a)\, f^2 = 0\,,$$
$$f'' + \frac{2}{\bar{r}}\, f' - \frac{2}{\bar{r}^2}\,(1-a)^2\, f - \frac{1}{2}\,\lambda_\phi\, f\,(f^2-1) - \frac{1}{2}\,\lambda_{\phi S}\, s^2 f = 0\,,$$
$$s'' + \frac{2}{\bar{r}}\, s' + \Omega^2\, s - \frac{1}{2}\,\lambda_{\phi S}\, f^2\, s - \lambda_S\, s^3 - \mu_0^2\, s = 0\,,$$

  - ➡ BCs remain the same as those of monopole/Q-ball

$$f(0) = 0\,,\ f(\infty) = 1\,,\ s'(0) = 0\,,\ s(\infty) = 0\,,\ a(0) = 0\,,\ a(\infty) = 1$$

# Basics of QMBs

- **A Q-monopole-ball (QMB) is charged both topologically and non-topologically**
  - ➡ The range of $\Omega$:

$$\Omega_c^2 \equiv \sqrt{\frac{1}{2}\lambda_s\lambda_\phi} < \Omega^2 < \frac{\lambda_{\phi S}}{2}$$

**-V_eff maxima > 0** **-V_eff has a minima**

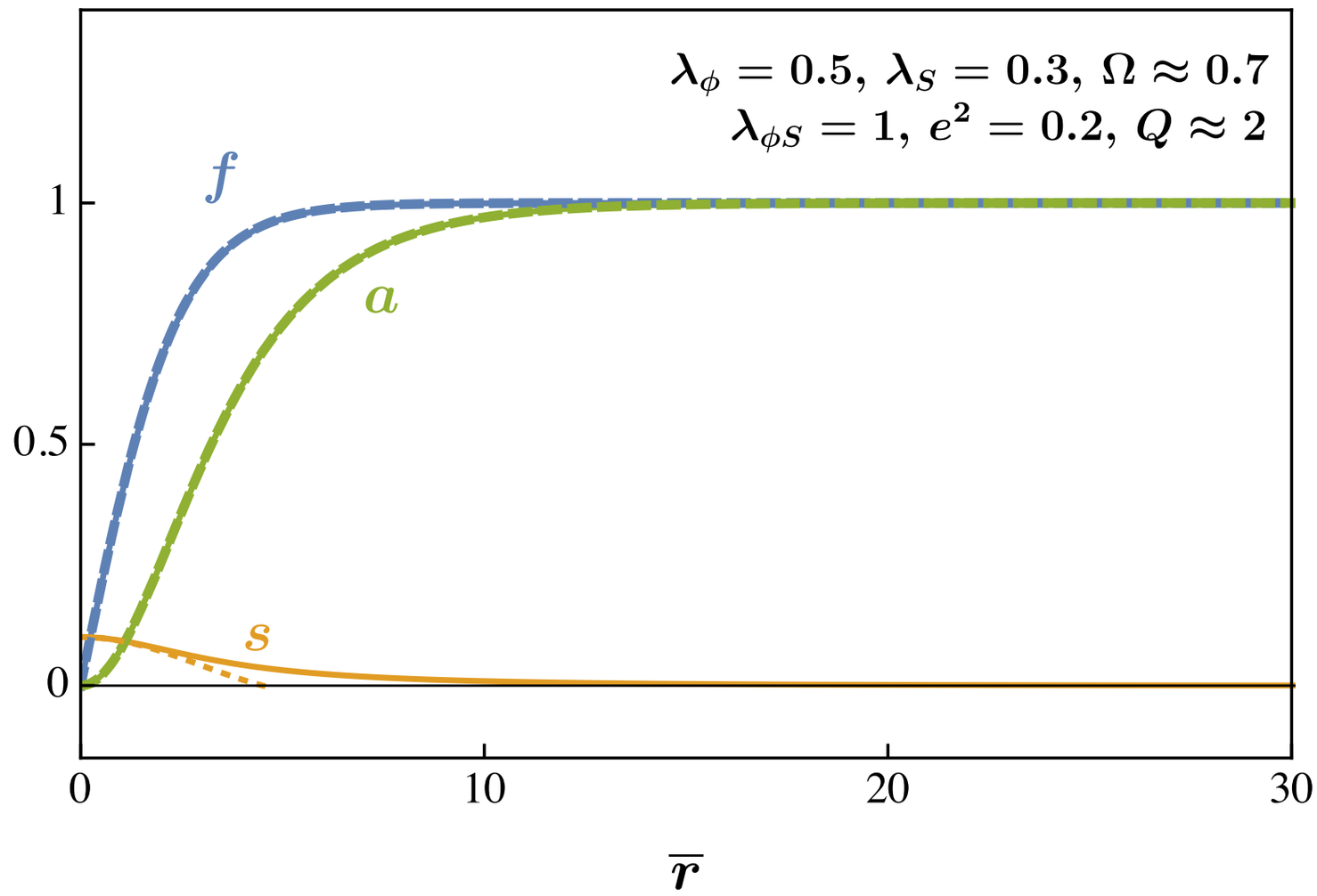
  - ➡ Charge and mass

$$Q = 4\pi\,\Omega \int_0^\infty d\bar{r}\,\bar{r}^2 s^2\,,$$

$$M = 4\pi v \int_0^\infty d\bar{r} \left\{ \frac{1}{2e^2}\left[2a'^2 + \frac{1}{\bar{r}^2}\left(2a - a^2\right)^2\right] + (1-a)^2 f^2 + \bar{r}^2\left[\frac{1}{2}f'^2 + \frac{1}{8}\lambda_\phi(f^2-1)^2\right] \right.$$
$$\left. + \bar{r}^2\left[\frac{1}{2}s'^2 + \frac{1}{2}\Omega^2 s^2 + \frac{1}{4}\lambda_{\phi S} f^2 s^2 + \frac{1}{4}\lambda_s s^4 + \frac{1}{2}\mu_0^2 s^2\right] \right\}$$

# Small-*Q* Regime

- **Small-*Q* is obtained at large $\Omega$**
  - ➡ The monopole profiles remain (almost) untouched

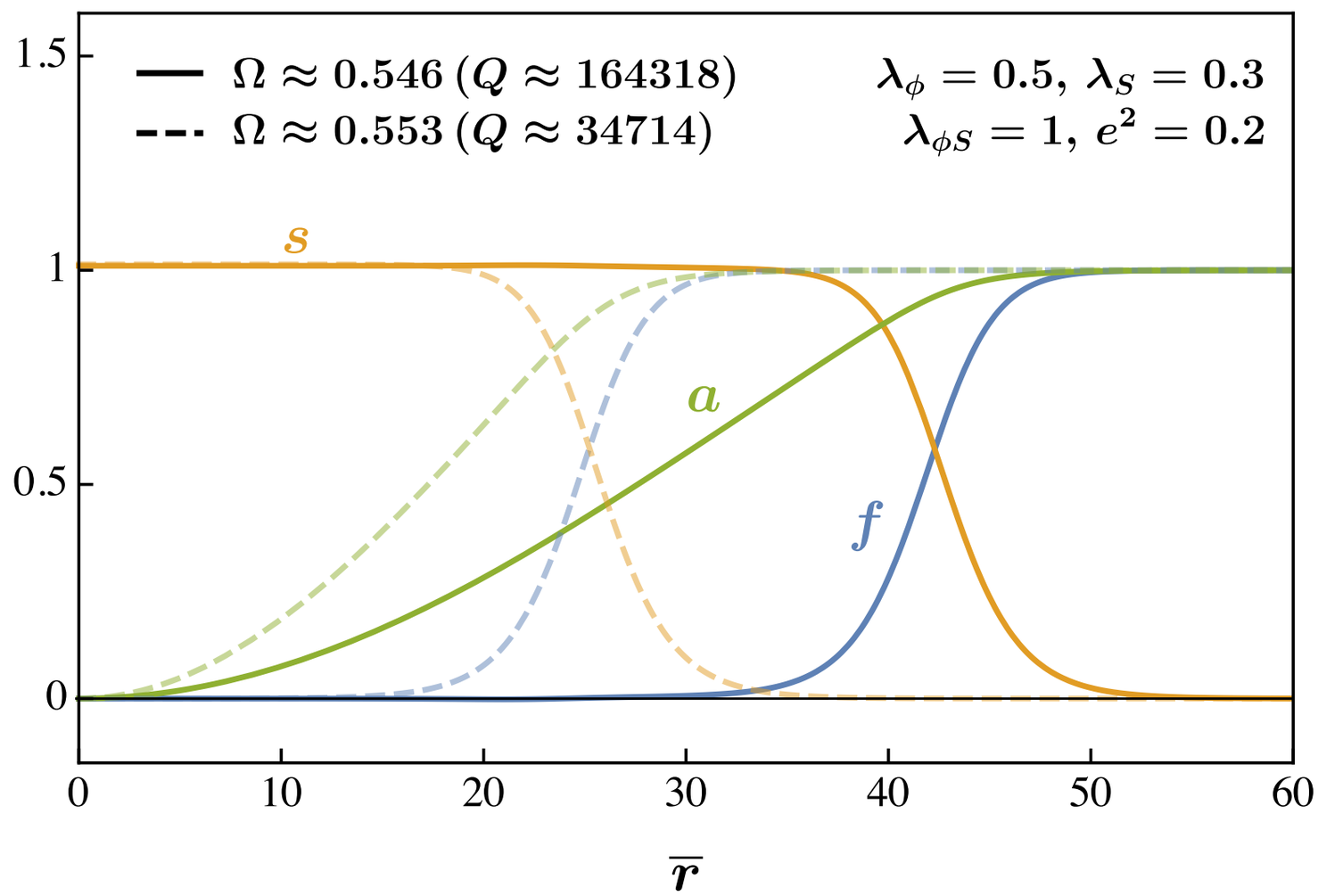


# Small-*Q* Regime

- **Some qualitative understandings**
  - ➡ Adding global charges to the monopole from *Q*=0 to form a QMB could save the vacuum energy of the Q-ball (Q-ball living in a monopole)
  - ➡ It should stop when the monopole solution starts to be significantly modified (which involves a competition between $e$ and $\lambda_{\phi S}^{1/2}$ )
  - ➡ Increasing $\lambda_\phi$ shrinks the region where the monopole field is small, and hence makes it harder to add *Q* to the system
- **Analytic expressions are difficult to obtain, though**

# Large-*Q* Regime

- ***Q* grows as we decrease $\Omega$**
  - ➡ Scalar field profiles are like step-functions (thin-wall)



# Large-*Q* Regime

- ***Q* grows as we decrease $\Omega$**
  - ➡ Scalar field profile is largely like step function (thin-wall)

$$f(\overline{r}) \approx \begin{cases} 0\,, & \overline{r} < \overline{r}_b \\ 1\,, & \overline{r} > \overline{r}_b \end{cases}\,, \quad s(\overline{r}) \approx \begin{cases} s_0\,, & \overline{r} < \overline{r}_b \\ 0\,, & \overline{r} > \overline{r}_b \end{cases}\,, \quad a(\overline{r}) \approx \begin{cases} \overline{r}^2/\overline{r}_b^2\,, & \overline{r} < \overline{r}_b \\ 1\,, & \overline{r} > \overline{r}_b \end{cases}$$

$$Q \approx \frac{4\pi}{3}\,\overline{r}_b^3\,\Omega\,s_0^2\,, \quad M_{(2,Q)} \approx \underbrace{\frac{304\,\pi\,v}{35\,e^2\,\overline{r}_b}}_{\text{magnetic energy}} + \frac{4\pi}{3}\overline{r}_b^3 v\left(\frac{1}{4}\lambda_S s_0^4 + \frac{1}{8}\lambda_\phi + \frac{1}{2}\Omega^2 s_0^2\right)$$

  - ➡ Minimize M with respect to $s_0$ and $\overline{r}_b$ , then to the leading order and in the limit of $\lambda_S^{1/3} e^2 Q^{4/3} \gg 60$

$$\overline{r}_b \approx \frac{(3/\pi)^{1/3}}{2^{5/12}}\,\frac{\lambda_S^{1/12}}{\lambda_\phi^{1/4}} Q^{1/3}\,, \quad s_0 \approx \left(\frac{\lambda_\phi}{2\,\lambda_S}\right)^{1/4}\,, \quad \Omega \approx \Omega_c\,,$$

$$M_{(2,Q)} \approx M_{(0,Q)} \approx Q\,v\,\Omega \approx Q\,v\,\left(\Omega_c + c_2\,Q^{-1/3}\right)$$

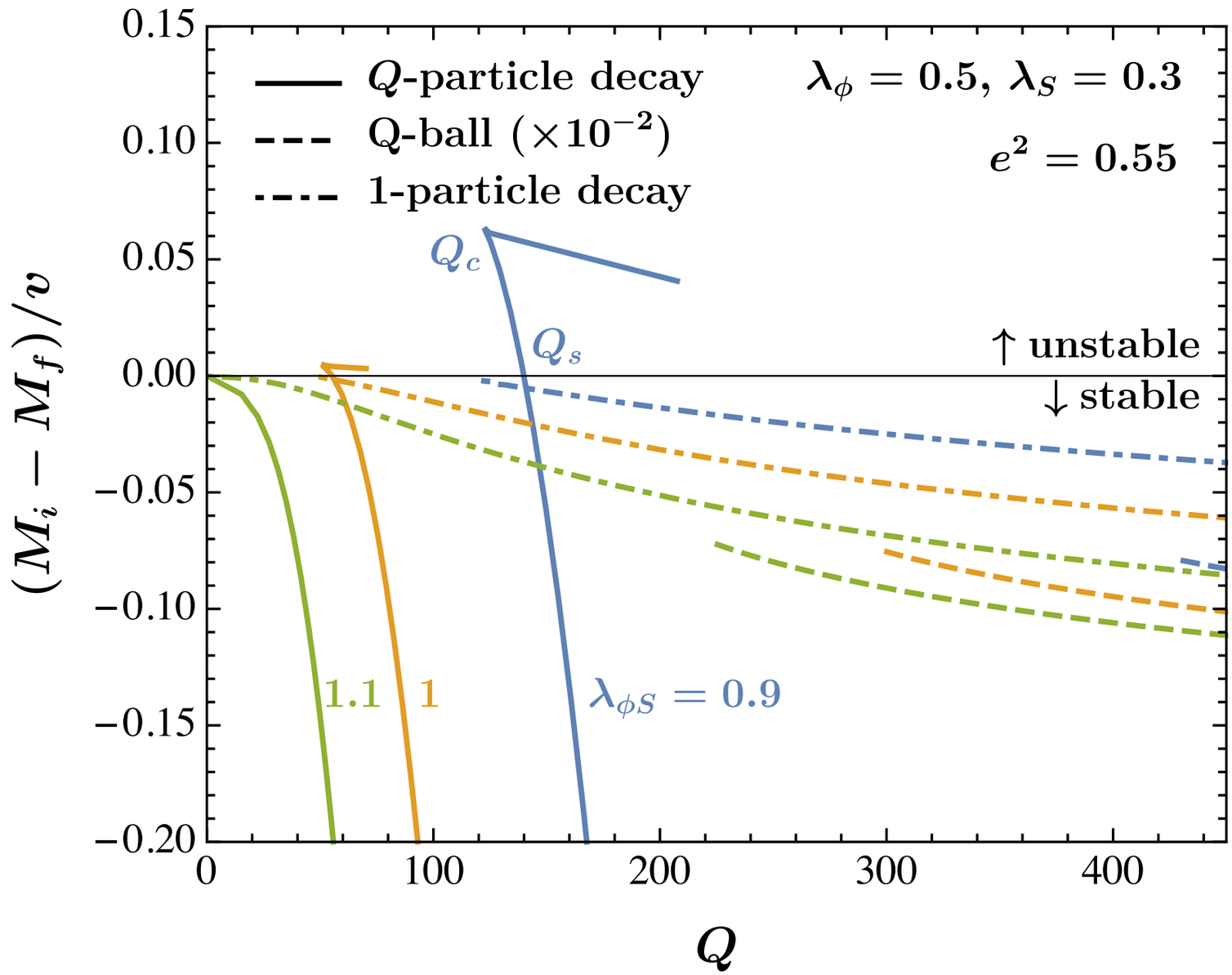
# Stability

- **Three possible decay channels for q=2**

$$
\begin{aligned}
Q\text{-particle decay}: &\quad (2,Q) \to Q \times (0,1) + (2,0)\,, \\
\text{Q-ball decay}: &\quad (2,Q) \to (0,Q) + (2,0)\,, \\
\text{1-particle decay}: &\quad (2,Q) \to (2,Q-1) + (0,1)\,.
\end{aligned}
$$

- **At q>2 we would also worry about emitting a monopole**

# Numeric Results



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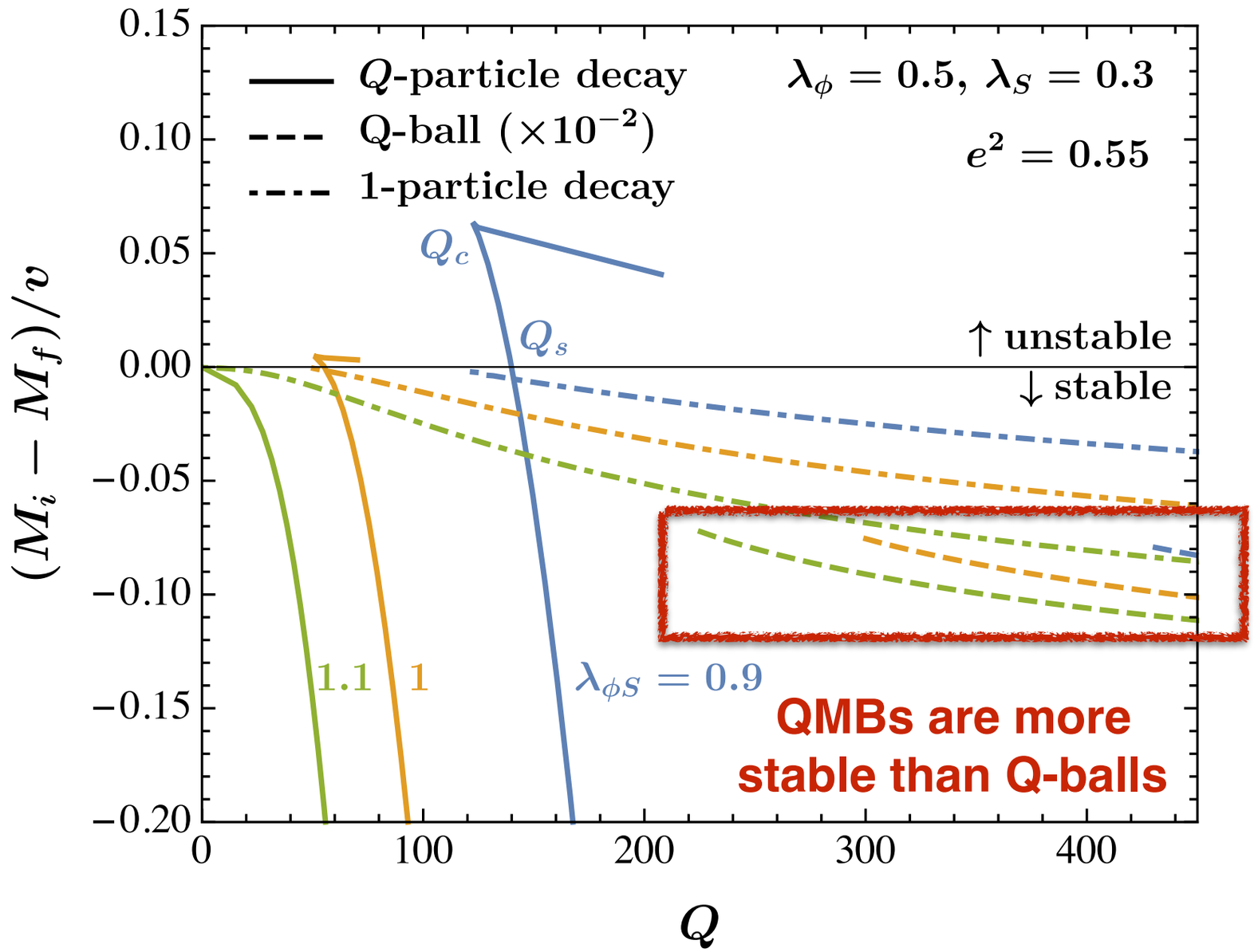

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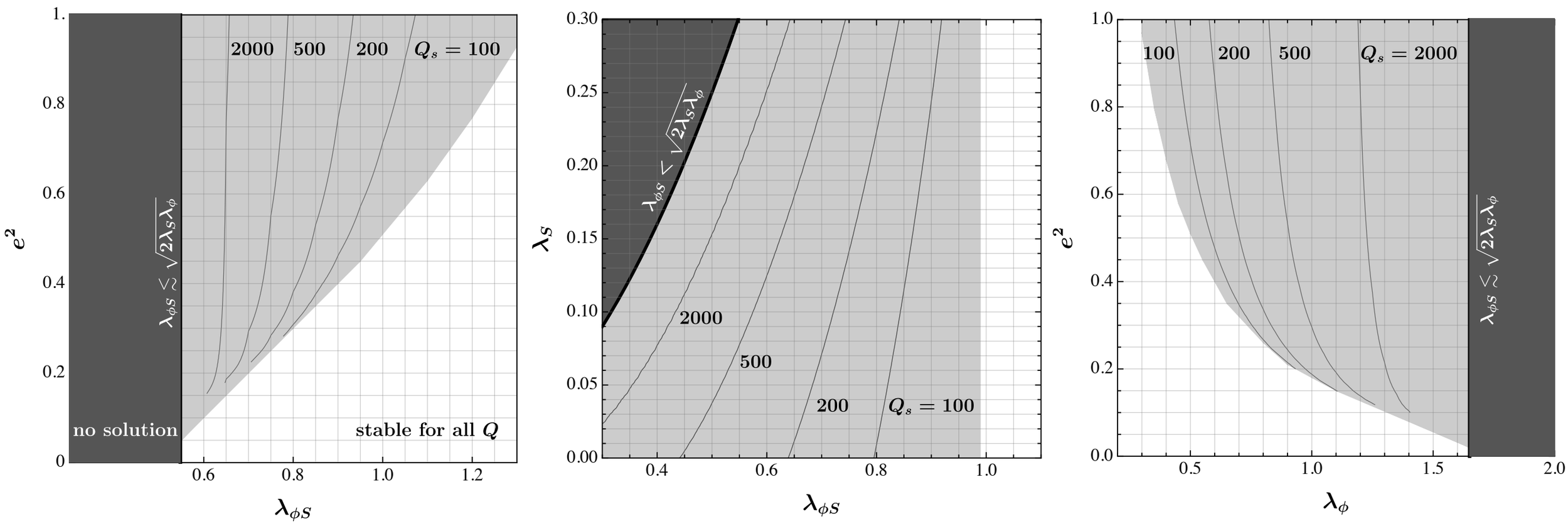


$\lambda_\phi = 0.5\,,\quad \lambda_{\phi S} = 1\,,\quad \lambda_S = 0.3\,,\quad e^2 = 0.5$ if not a scanning parameter

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# More Magnetic Charges?

- **Multiple magnetic charges**
  - ➡ Spherical solution doesn't exist for q>2

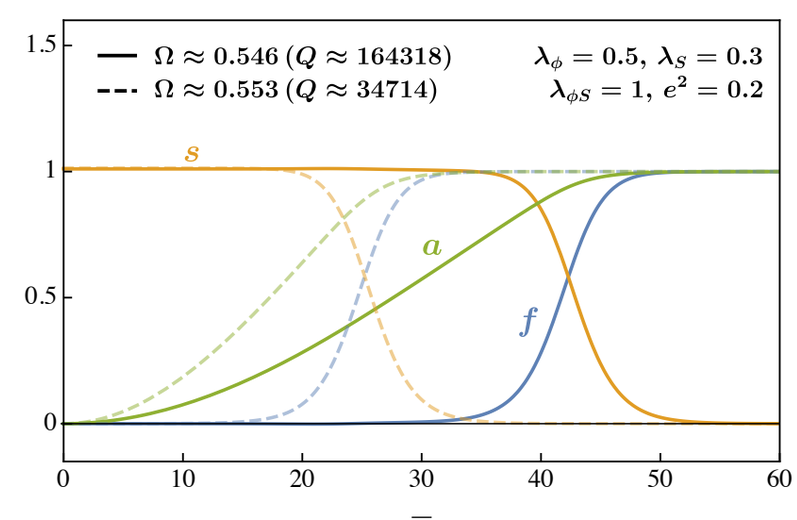
[E. Weinberg, A. Guth, Phys. Rev. D 14 1660 (1976)]

  - ➡ In the BPS limit, there exists arguments claiming that at large q the configuration should be quasi-spherical, which is sometimes called "magnetic bag"

[S. Bolognesi, Nucl. Phys. B 752 (2006) 93–123; K. Lee, E. Weinberg, Phys. Rev. D 79 (2009) 025013 ]

# The Existence

- **It's hard (if not unlikely) to obtain a numeric solution for q>2. But there are qualitative arguments why they should exist**
  - ➡ Consider a large Q-ball. Adding in a monopole will hardly influence the Q-ball profiles, so having a non-zero magnetic charge reduces the vacuum energy of the monopoles
  - ➡ Meanwhile, the magnetic energy should also decrease due to the symmetry restoration inside the QMB



# An Educated Guess

- **A back-of-envelope estimation on the mass**
  - ➡ magnetic energy scale as $(q/2)^2$ for a $(q, Q)$
  - ➡ simply update the previous large-$Q$ expressions

$$M_{(q,Q)} \sim \frac{304\pi v (q/2)^2}{35 e^2 \overline{r}_b} + \frac{4\pi}{3}\overline{r}_b^3 v \left(\frac{1}{4}\lambda_S s_0^4 + \frac{1}{8}\lambda_\phi + \frac{1}{2}\Omega^2 s_0^2\right)$$

  – check the stability of emitting one monopole

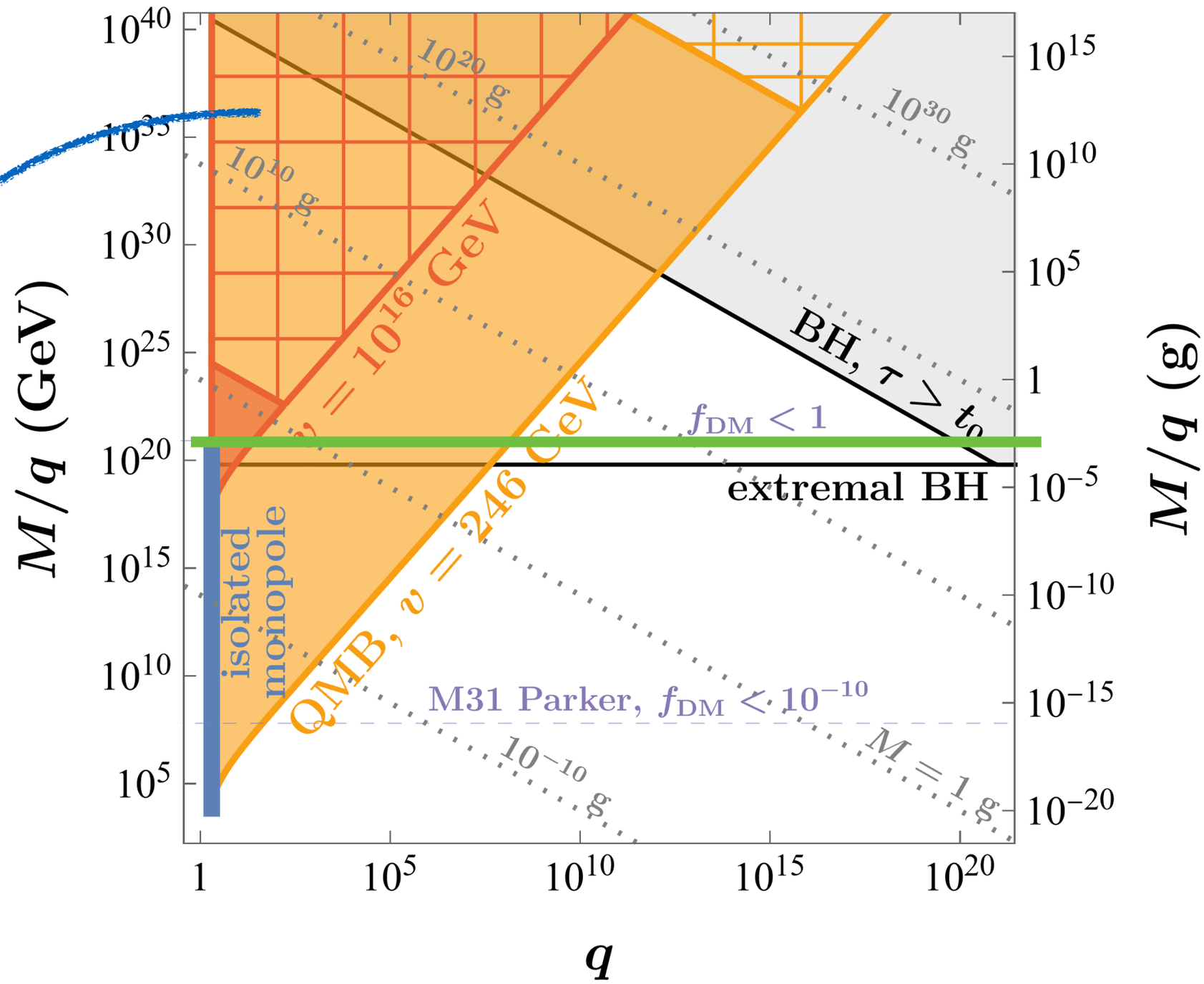
$$q \lesssim \frac{35(3/\pi)^{1/3}}{76 \times 2^{5/12}} e\, Y \frac{\lambda_S^{1/12}}{\lambda_\phi^{1/4}} Q^{1/3}$$

  – mass at the boundary of stability

$$M_{(q,Q)} \gtrsim 21 \frac{\lambda_\phi}{e^3 Y^3} q^3 v$$

# Parameter Space

Schwarzschild radius larger than QMB radius



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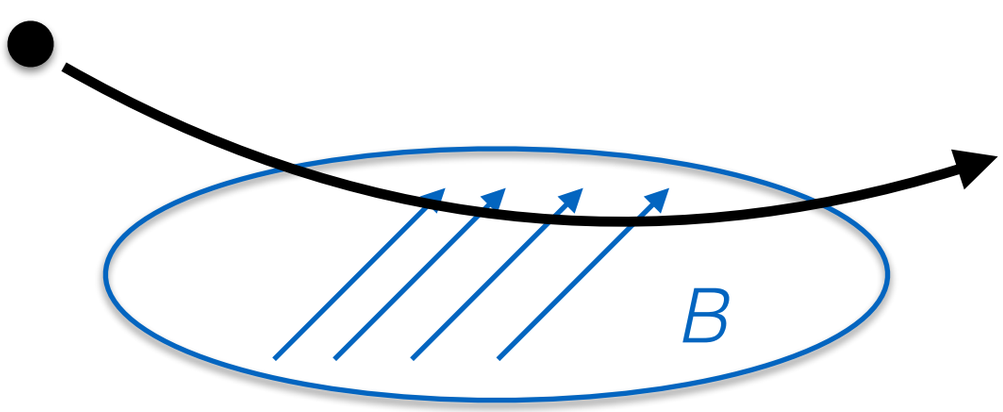
# Constraints from Astrophysics

- **The Parker bound**
  - ➡ galactic magnetic accelerate magnetic objects, which extract magnetic energy from the galaxy
  - ➡ energy drained through this process should not deplete galactic magnetic field within a regeneration time

$$\Delta E \times F \times (\pi\, \ell_c^2) \times (4\pi\, \mathrm{sr}) \times t_{\mathrm{reg}} \lesssim \frac{B^2}{2} \frac{4\pi\, \ell_c^2}{3}, \quad \Delta E \sim M\, \Delta v^2/2$$




[E. Parker, Astrophys. J. 160 (1970) 383;
M. Turner, E. Parker, T. Bogdan, Phys. Rev. D 26 (1982) 1296 ]

# Constraints from Astrophysics

- **The Parker bound**
  - coherent galactic magnetic field accelerates the magnetic object to a speed of

$$v_{\rm mag} \simeq \min\left[1, \sqrt{\frac{2B(2\pi q/e)\ell_c}{M}}\right] \simeq \min\left[1, 4\times 10^{-5}\sqrt{\frac{q}{M}5.2 M_{\rm pl}\ell_{21}B_3}\right]$$

  - The bound depends on whether the magnetic object is virialized

$$f_{\rm DM} \lesssim \begin{cases} 6\times 10^{-3}\left(\frac{M}{5.2\,q\,M_{\rm pl}}\right)^2 \frac{v_{-3}}{\rho_{0.4}(\ell_{21}/30)(t_{15}/300)}\,, & v_{\rm mag} \lesssim 10^{-3} \\ 10^2\left(\frac{M}{5.2\,q\,M_{\rm pl}}\right)\frac{B_3/(5/3)}{v_{-3}(\rho_{0.4}/10^{-6})(t_{15}/300)}\,, & v_{\rm mag} \gtrsim 10^{-3} \end{cases}$$

# Direct Detection?

- **Seems hopeless…**
  - ➡ Sensitivity of the magnetic anomaly detectors (MADs)

$$S_{\rm MAD} \sim 1 \text{ to } 10^3 \ \mathrm{fT}/\sqrt{\mathrm{Hz}}$$

  - ➡ To be above the threshold

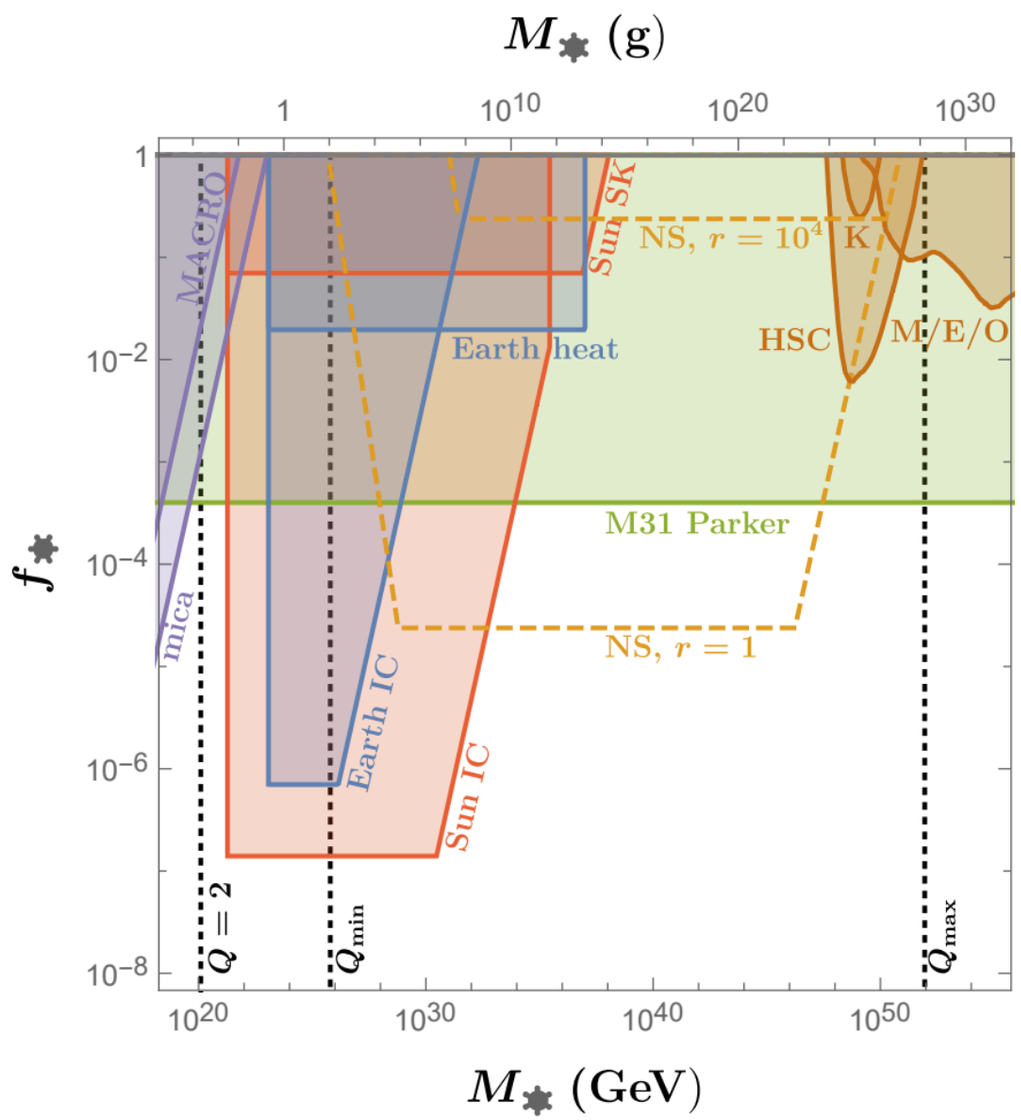
$$B_m \sqrt{t_{\rm pass}} > S_{\rm MAD}$$

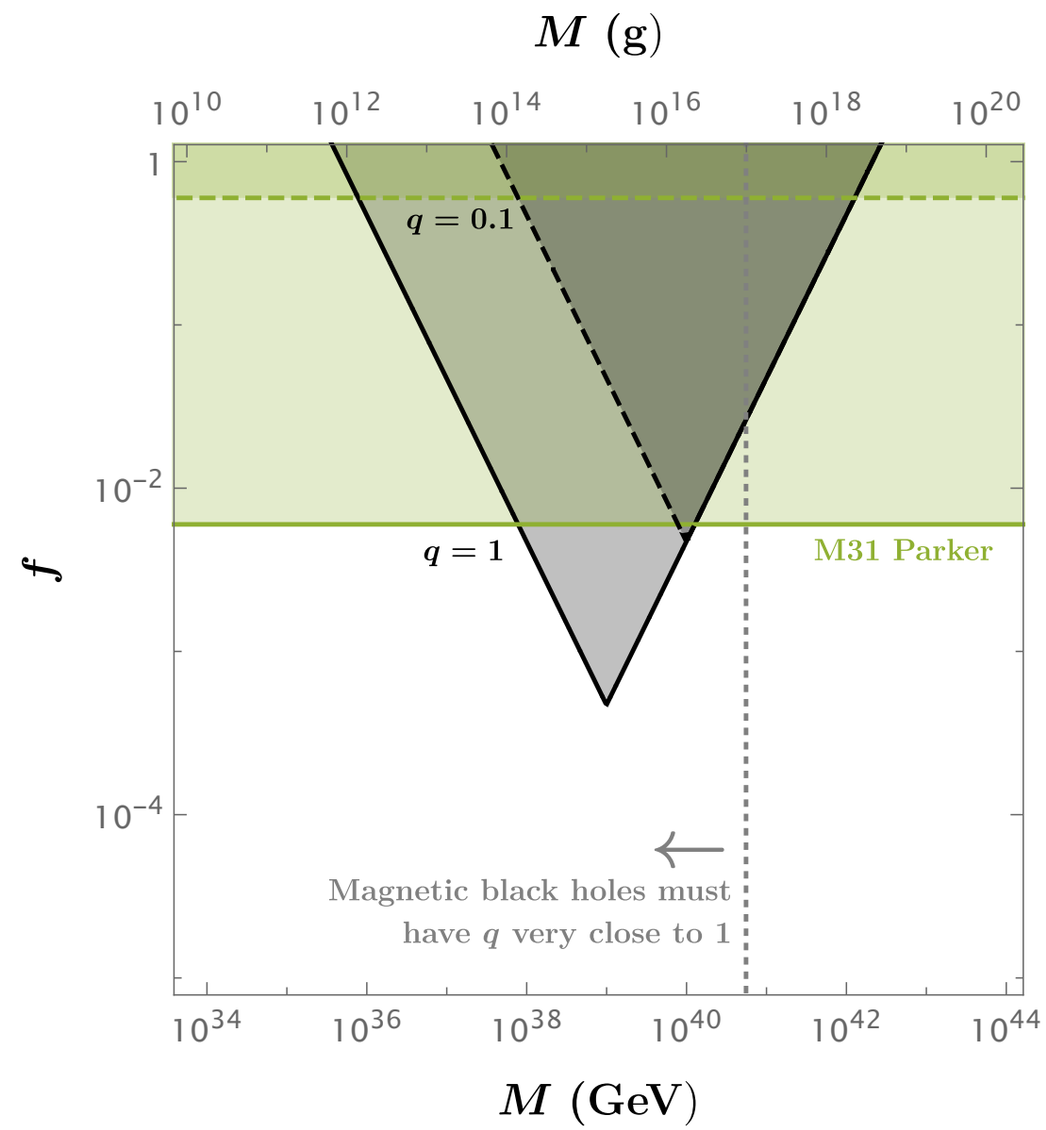
  - ➡ With $N_{\rm MAD}$ detectors

$$N_{\rm det} \sim 7 \times 10^{-7} \, N_{\rm MAD} \, f_{\rm DM} \, q^{4/3} \left( \frac{10^{17}\,\mathrm{GeV}}{M} \right) \left( \frac{\mathrm{fT}/\sqrt{\mathrm{Hz}}}{S_{\rm MAD}} \right)^{4/3} \left( \frac{T}{\mathrm{yr}} \right) (v_{-3})^{1/3}$$

# Other Possible Constraints?

- See Nicholas' talk yesterday

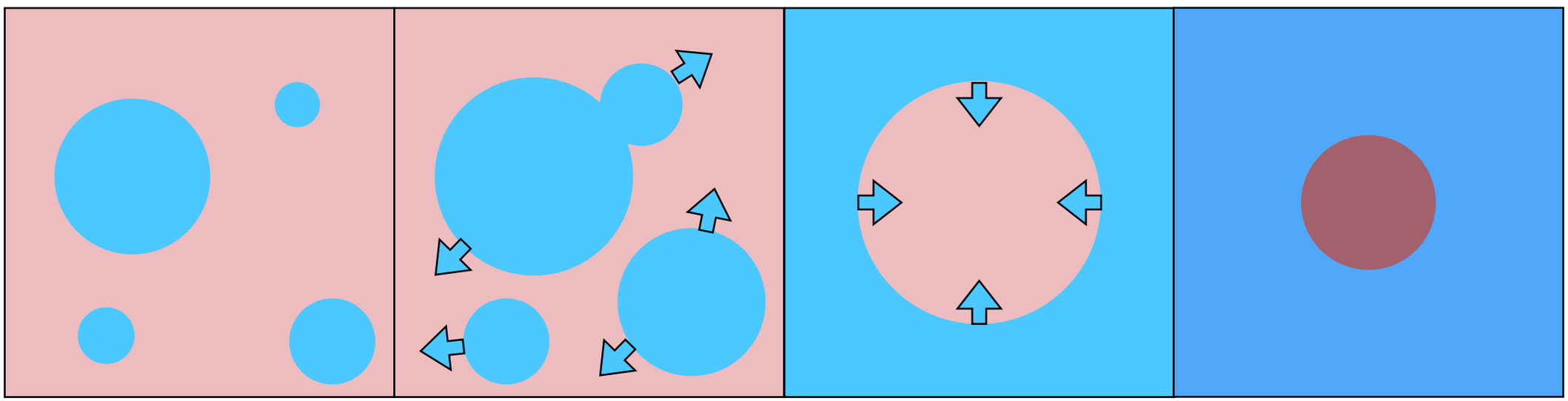



[Y. Bai, J. Berger, M. Korwar, N. Orlofsky, *JHEP* 10 (2020) 210
Y. Bai, *SL*, N. Orlofsky, Phys. Rev. Lett. 127 (2021) 10 101801]

# Formation of QMB

- **QMB can be formed through cosmic phase transitions**
  - ➡ The picture is clear for first-order phase transitions



  - ➡ More tricky for second-order
    - monopole possibly formed where multiple bubble walls collider
    - Q-ball arises from local thermal fluctuation

[T. Kibble, J. Phys. A 9, 1387 (1976);
W. Zurek, Nature 317 (1985) 505–508 ]

# Relic Abundance from PT

- **QMB can be formed through cosmic phase transitions**
  - ➡ The average charge in a QMB can be estimated from the number density of global charge and nucleation sites. It also depends on the asymmetry of Q

$$\langle Q_{\rm QMB}\rangle \sim \max\left[(3\times 10^{16})\, v_3^{-3/2}\left(\frac{\lambda_{\phi S}}{3}\right)^{7}, \eta_Q\,(1\times 10^{33})\, v_3^{-3}\left(\frac{\lambda_{\phi S}}{3}\right)^{14}\right]$$

  - ➡ In the large-*Q* limit we then know the mass, and hence the relic abundance

$$\begin{cases} \text{asymmetric dominates:} \quad \eta_Q \sim (6\times 10^{-11} v_3^{-1})\Omega_c^{-1} \quad \text{and} \quad v \lesssim (3\times 10^5\ \text{GeV})\left(\frac{\lambda_{\phi S}}{3}\right)^{14/5}\Omega_c^{-2/5} \\ \text{symmetric dominates:} \quad v \sim (3\times 10^5\ \text{GeV})\left(\frac{\lambda_{\phi S}}{3}\right)^{14/5}\Omega_c^{-2/5} \quad \text{and} \quad \eta_Q \lesssim (2\times 10^{-13})\Omega_c^{-1}\left(\frac{3}{\lambda_{\phi S}}\right)^{14/5} \end{cases}$$

# Solitosynthesis

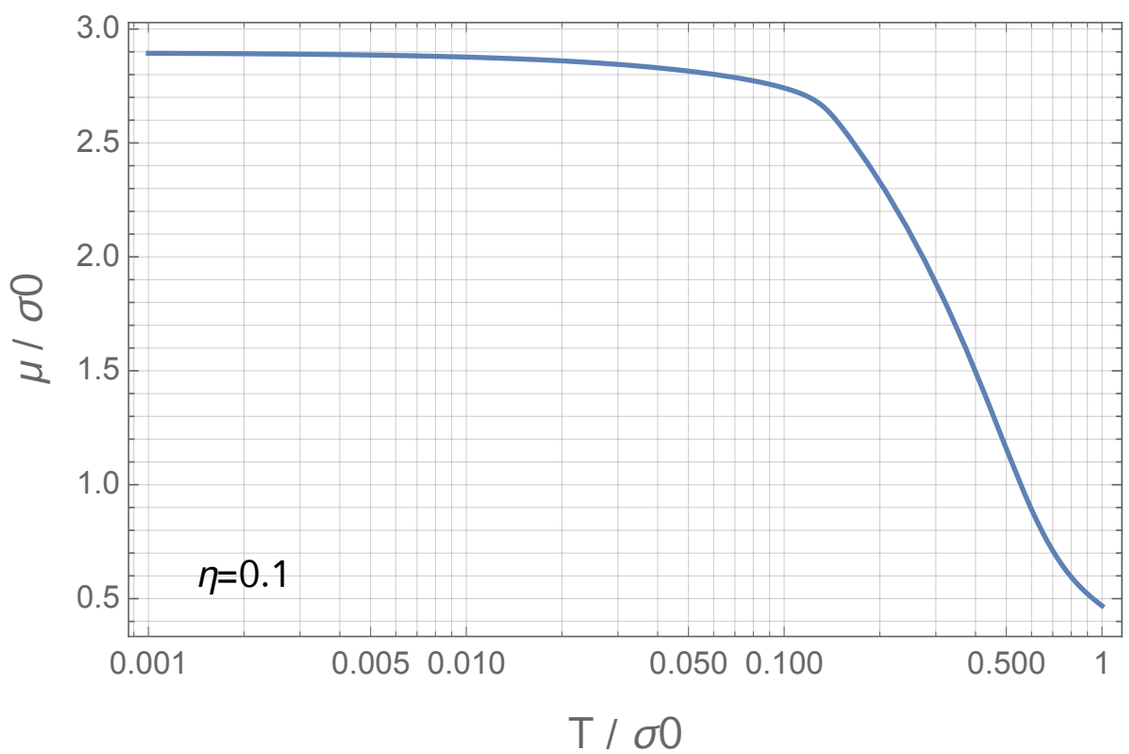
- **Late universe evolution could change the story**
  - ➡ The soliton may absorb or release free particle/anti-particles and change their charges and sizes
  - ➡ Known as “solitosynthesis”

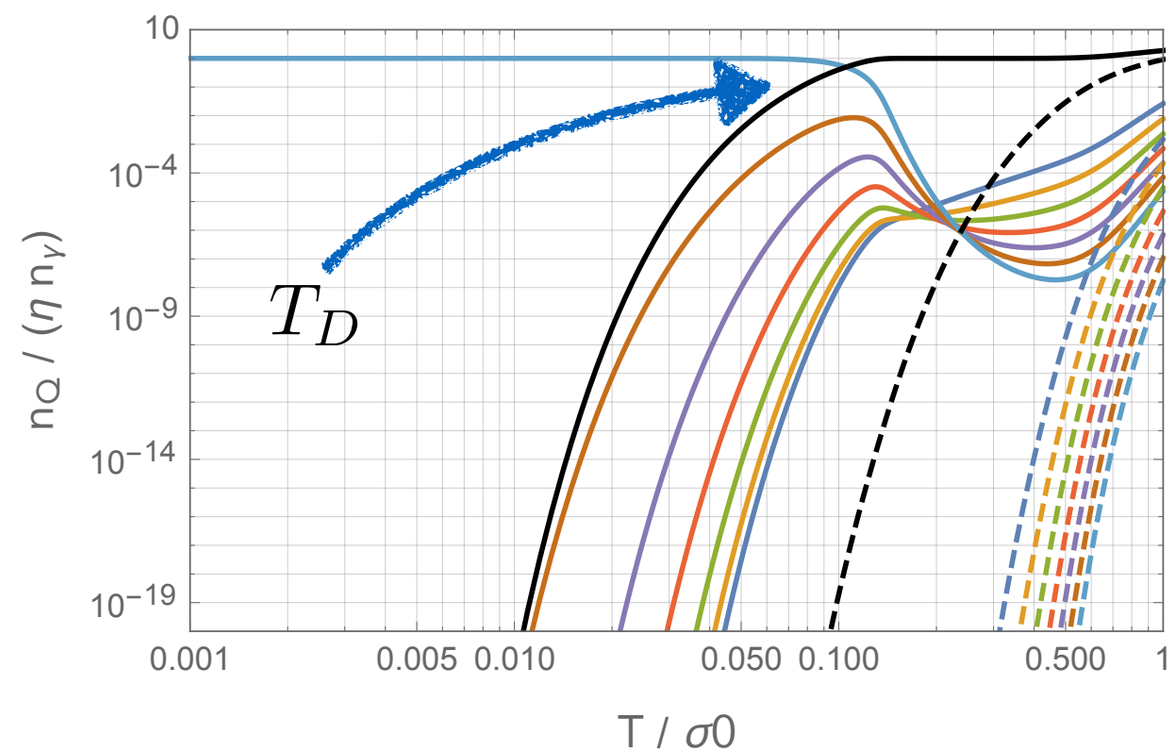
$$s + \bar{s} \to X + X$$
$$\mathcal{Q}_{\rm min} + \bar{s} \to (Q_{\rm min} - 1)s + X$$
$$\mathcal{Q} + s \to (\mathcal{Q} + 1) + X$$
$$\mathcal{Q} + \bar{s} \to (\mathcal{Q} - 1) + X$$

[K. Griest, E. Kolb, *Phys.Rev.D* 40 (1989) 3231]

# Solitosynthesis

- **Assuming certain amount of asymmetry within the dark sector**
  - ➡ In equilibrium and with a reasonable* M(Q) vs. Q, the binding energy will push the Q charges into larger Q-balls




$$\eta = [n_s - \bar{n}_s + \sum_Q Q(n_Q - \bar{n}_Q)]/n_\gamma$$

*Say $M \propto Q^p$, “reasonable” means p<1

# Solitosynthesis

- **The evolution should finally freeze-out**
  - ➡ Solve a set of Boltzmann equations for each component and determine the freeze-out temperature $T_F$
  - ➡ For solitons to be the main DM components, we should at least expect $T_D > T_F$

**what Griest & Kolb assumed to determine the freeze-out**

$$\boxed{s + \bar{s} \to X + X}$$

$$\mathcal{Q}_{\min} + \bar{s} \to (Q_{\min} - 1)s + X$$

$$\mathcal{Q} + s \to (\mathcal{Q} + 1) + X$$

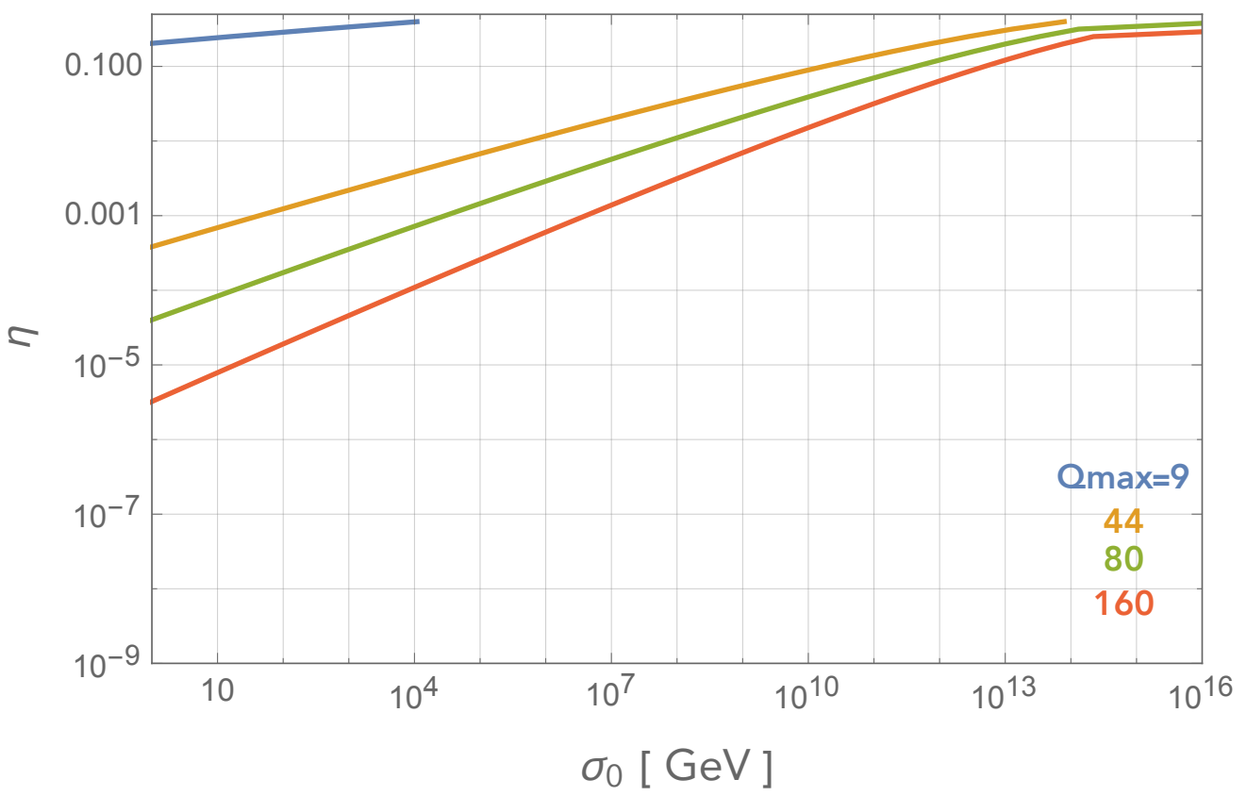
$$\mathcal{Q} + \bar{s} \to (\mathcal{Q} - 1) + X$$

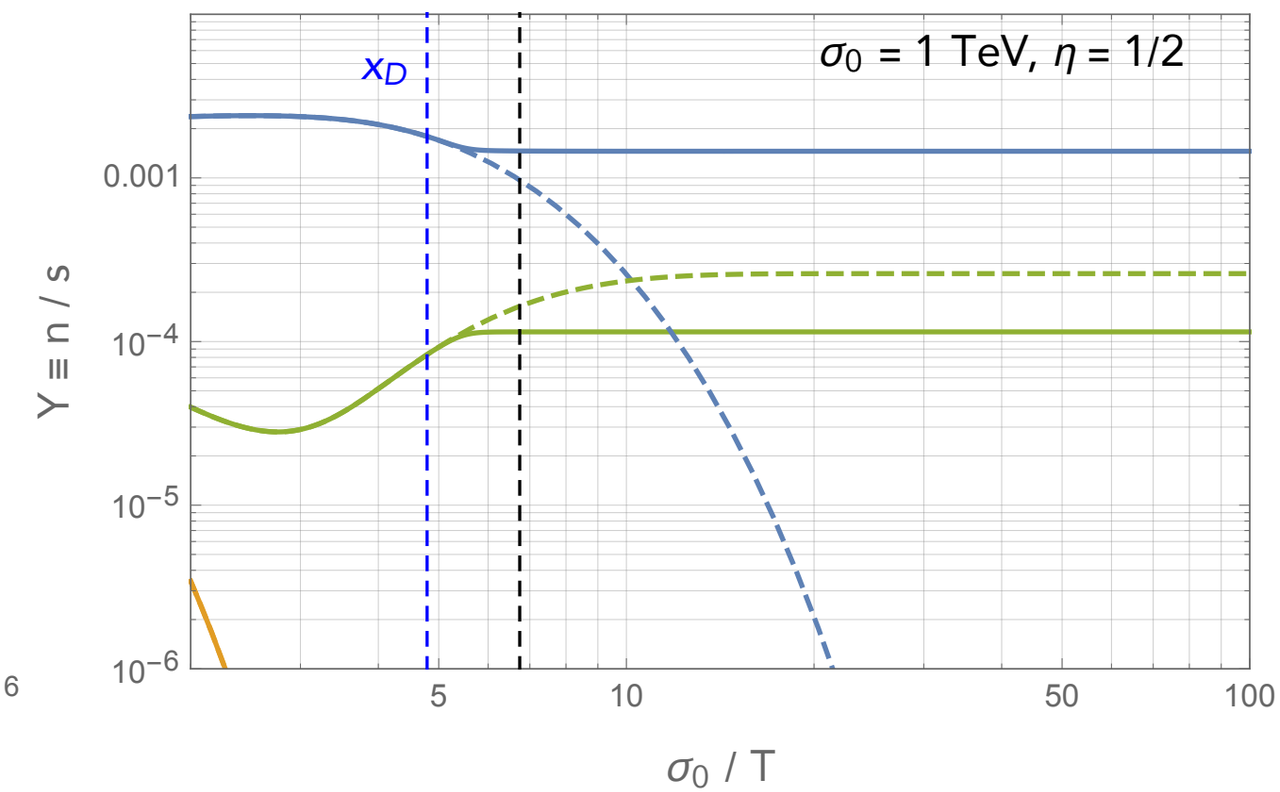
  - ➡ Also they assemble solitons from free particles

[K. Griest, E. Kolb, *Phys.Rev.D* 40 (1989) 3231]

# Solitosynthesis

- **The evolution should finally freeze-out**
  - ➡ Solve a set of Boltzmann equations for each component and determine the freeze-out temperature $T_F$




[K. Griest, E. Kolb, *Phys.Rev.D* 40 (1989) 3231]

numerically we can at least deal with Qmax~50, thanks to modern computers

# Solitosynthesis

- **But when writing down all the Boltzmann equations**

$$\dot{n}_Q + 3Hn_Q = -\delta_{Q,Q_{\rm min}}\sigma v(Q_{\rm min})\left(n_{Q_{\rm min}}n_{\bar{\phi}} - n^{\rm eq}_{Q_{\rm min}}n^{\rm eq}_{\bar{\phi}}\left(\frac{n_\phi}{n^{\rm eq}_\phi}\right)^{Q_{\rm min}-1}\right)$$
$$-(1-\delta_{Q,Q_{\rm max}})\sigma v(Q)\left(n_Q n_\phi - n^{\rm eq}_Q n^{\rm eq}_\phi\left(\frac{n_{Q+1}}{n^{\rm eq}_{Q+1}}\right)\right)$$
$$+(1-\delta_{Q,Q_{\rm min}})\sigma v(Q)\left(n_{Q-1}n_\phi - n^{\rm eq}_{Q-1}n^{\rm eq}_\phi\left(\frac{n_Q}{n^{\rm eq}_Q}\right)\right)$$

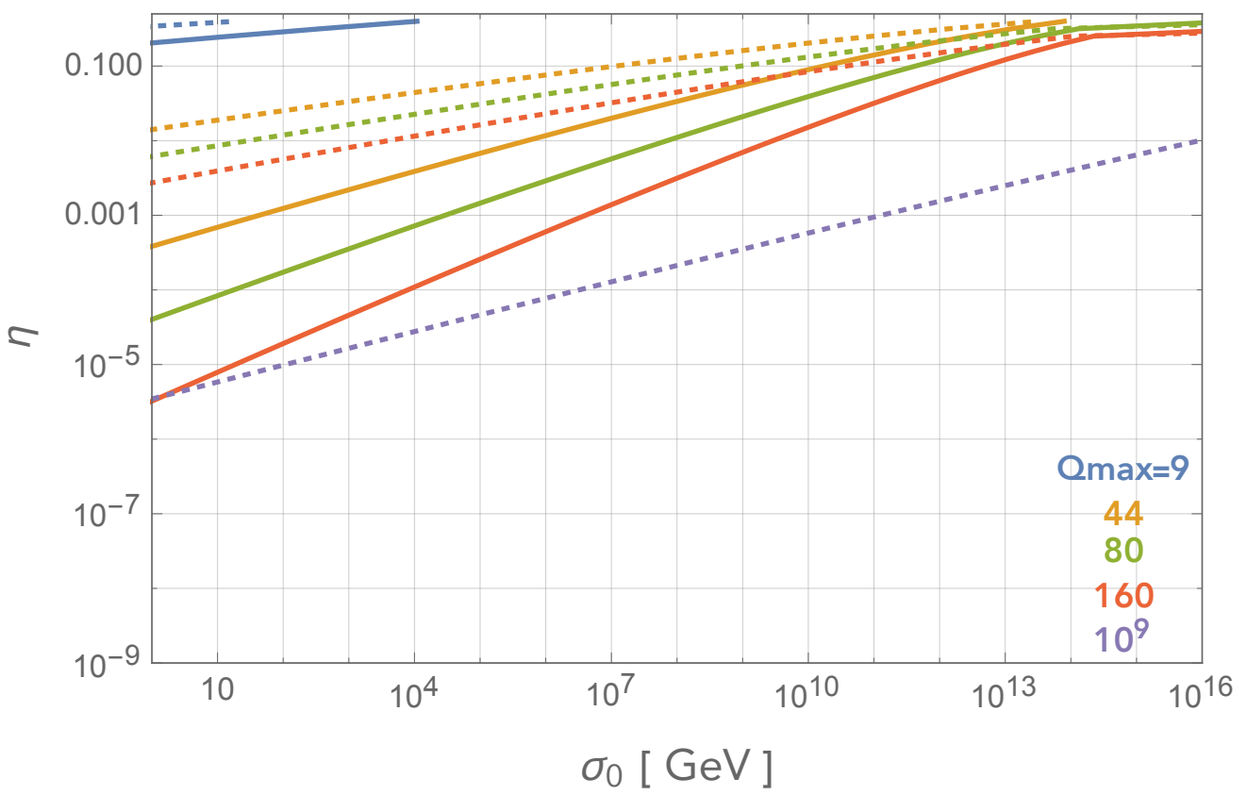
➡ Summing the expressions of all possible Q

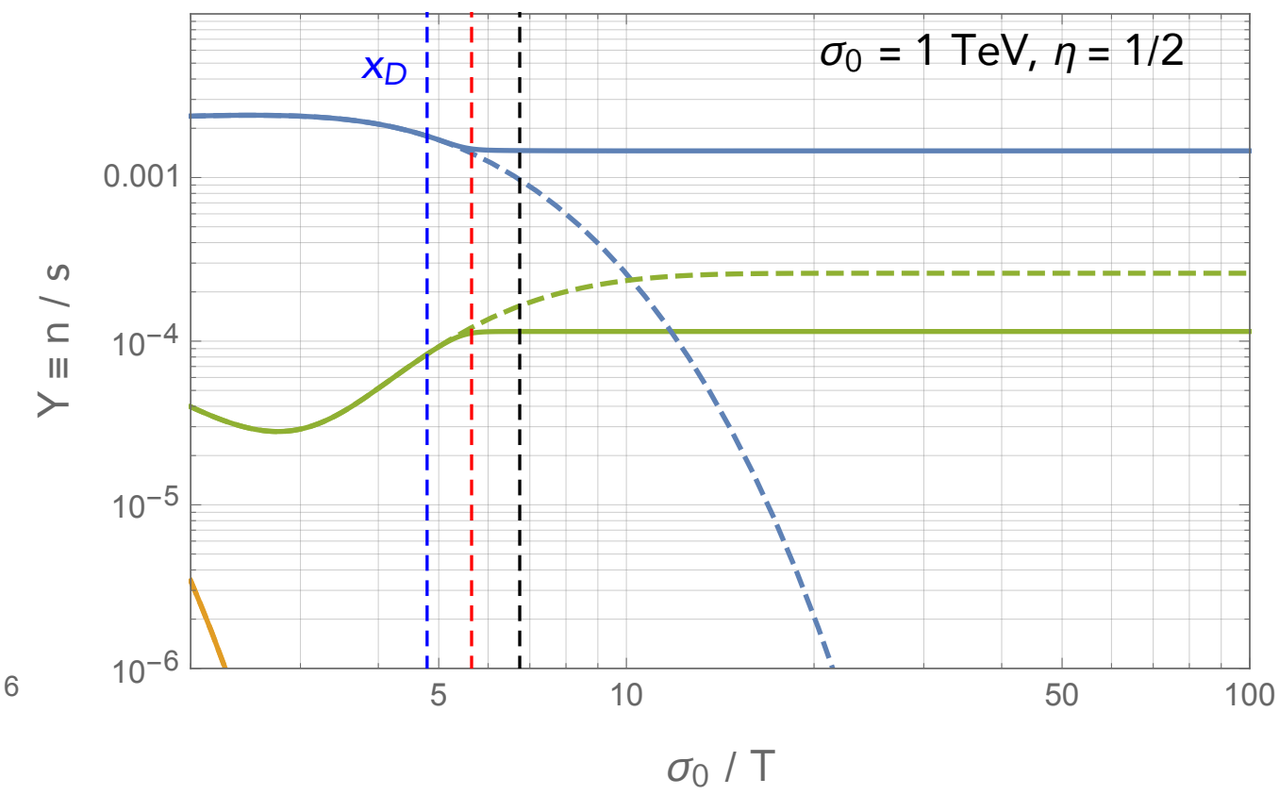
$$\dot{n}_{\rm NTS} + 3Hn_{\rm NTS} = -\sigma v(Q_{\rm min})\left(n_{Q_{\rm min}}n_{\bar{\phi}} - n^{\rm eq}_{Q_{\rm min}}n^{\rm eq}_{\bar{\phi}}\left(\frac{n_\phi}{n^{\rm eq}_\phi}\right)^{Q_{\rm min}-1}\right)$$

$$\Rightarrow Hn_{\rm NTS} \sim \sigma v(Q_{\rm min})n_{Q_{\rm min}}n_{\bar{\phi}}\,|_{T=T_F}$$

# Solitosynthesis

- **Writing down all the Boltzmann equations**
  - ➡ Also from phase transition, we can expect larger Qmax



# Conclusion

- We proposed a macroscopic state, the Q-monopole-ball (QMB), which is charged both topologically and non-topologically

- It may be energetically preferred for QMBs to possess a large number of magnetic charge, which is therefore a new non-gravitational approach to form highly magnetic objects beyond the BPS-limit

- Q-monopole-balls can be formed through a cosmic phase transition and account for all the dark matter, but this may depend on the evolution after the phase transition

Thank you!

# Backup

# Small-*Q* Regime

- **Competition between $e$ and $\lambda_{\phi S}^{1/2}$**
  - ➡ To start with, we consider the situation where a Q-ball "lives in" a monopole, i.e. $e \lesssim \lambda_{\phi S}^{1/2}$
  - ➡ Monopole fields should be small near the origin

$$s(\overline{r}) \approx \begin{cases} s_0 \dfrac{\sin(\Omega\,\overline{r})}{\Omega\,\overline{r}}\,, & \overline{r} < \overline{r}_b = \dfrac{\pi}{\Omega} \\ 0\,, & \overline{r} > \overline{r}_b \end{cases}$$

recall $\Omega_c^2 \equiv \sqrt{\frac{1}{2}\lambda_s\lambda_\phi} < \Omega^2 < \frac{\lambda_{\phi S}}{2}$

and $m_S = v\sqrt{\lambda_{\phi S}/2}$

  - ➡ To do some solid calculation, we use

$$f = f_{\rm BPS} = \coth(e\overline{r}) - 1/(e\overline{r}) < e\overline{r}/3 \;\Rightarrow M_S \lesssim \frac{2(4\pi^2-6)^{1/4}}{3\sqrt{3}} Q\, e^{1/2}\, \lambda_{\phi S}^{1/4}\, v$$

# Non-zero Bare Mass

- **A non-zero $\mu_0$ corresponds to a rescale of $\lambda_{\phi S}$**
  - ➡ $\mu_0$ immediately shifts $\Omega$ but not the field profiles

$$a'' - \frac{1}{\bar{r}^2}\, a(1-a)(2-a) + e^2\,(1-a)\, f^2 = 0\,,$$

$$f'' + \frac{2}{\bar{r}}\, f' - \frac{2}{\bar{r}^2}\,(1-a)^2\, f - \frac{1}{2}\,\lambda_\phi\, f\,(f^2-1) - \frac{1}{2}\,\lambda_{\phi S}\, s^2 f = 0\,,$$

$$s'' + \frac{2}{\bar{r}}\, s' + \underline{\Omega^2\, s} - \frac{1}{2}\,\lambda_{\phi S}\, f^2\, s - \lambda_s\, s^3 - \underline{\mu_0^2\, s} = 0$$

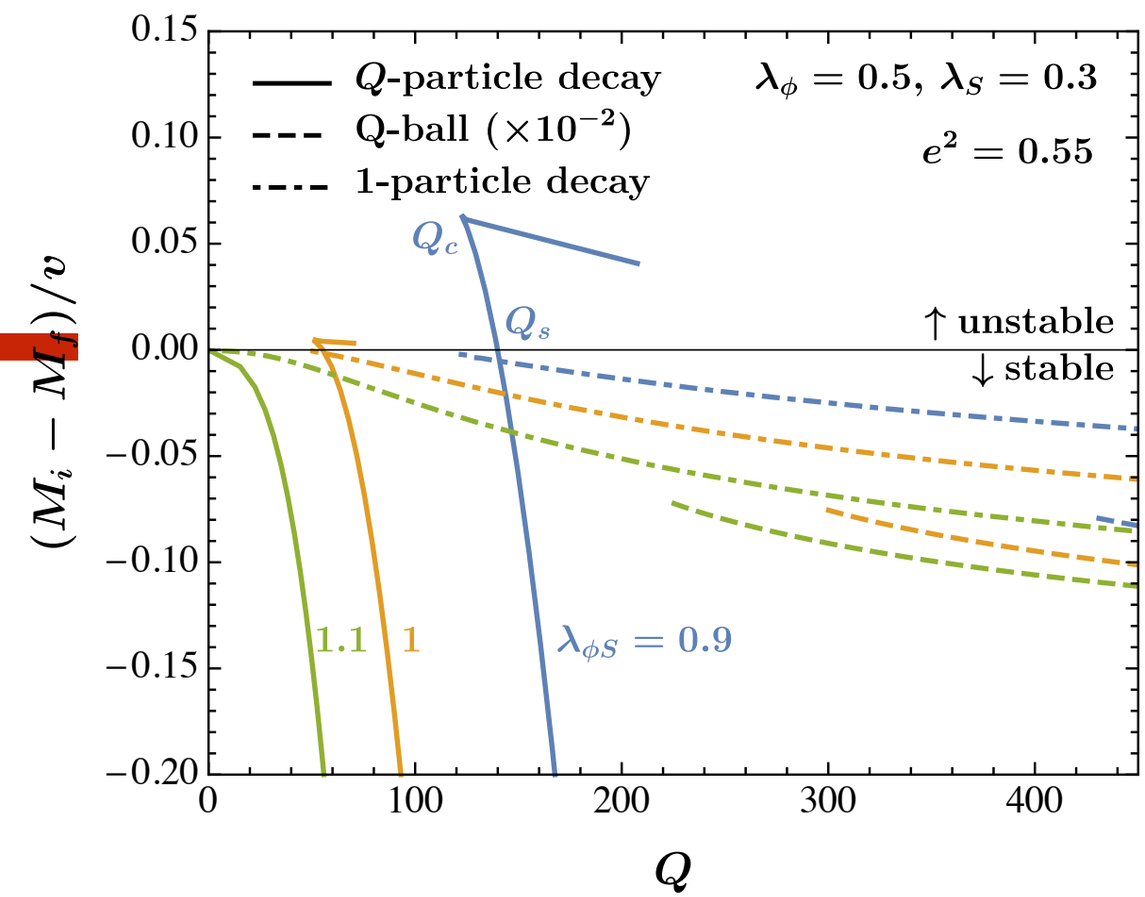
  - ➡ For a set of solution, denoting the $\Omega$ achieving it at $\mu_0 = 0$ as $\overline{\Omega}$ , then the additional mass is proportional to *Q*

$$\Delta M = \begin{cases} M_0(\overline{\Omega}_Q) - \sqrt{\frac{\lambda_{\phi S}}{2}}\, Q\, v - M_{(2,0)}\,, & \mu_0 = 0 \\ M_0(\overline{\Omega}_Q) - \left(\sqrt{\frac{\lambda_{\phi S}}{2} + \mu_0^2} - \frac{\mu_0^2}{\sqrt{\overline{\Omega}_Q^2 + \mu_0^2}}\right) Q\, v - M_{(2,0)}\,, & \mu_0 > 0 \end{cases}$$

# Non-zero Bare Mass

- **A non-zero $\mu_0$ corresponds to a rescale of $\lambda_{\phi S}$**
  - ➡ on the other hand, having a $\mu_0$ doesn't influence the boundary of stability

**being stable means the mass curve goes smoothly to *Q*=0, while on the other hand the additional mass is proportional to *Q***



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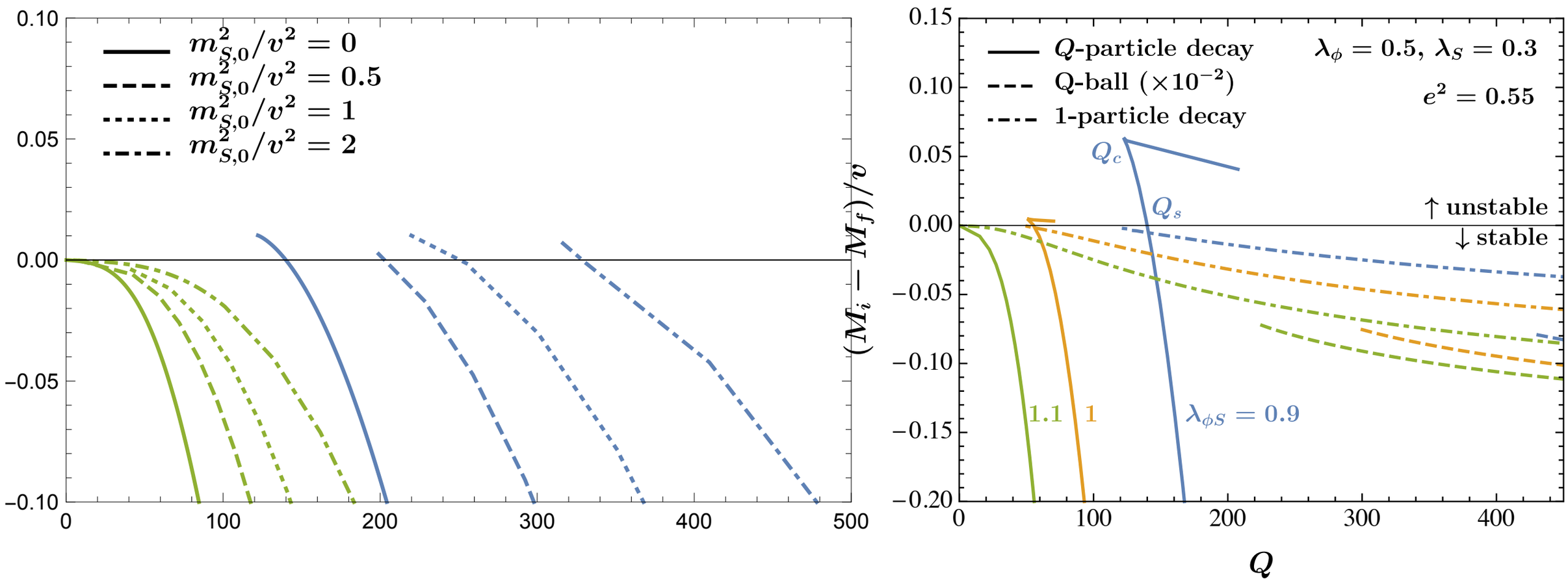

# QMB Number Density

- **Bubble nucleation rate**

$$\gamma \approx \zeta T^4 \left(\frac{S_3}{2\pi T}\right)^{3/2} e^{-S_3/T}$$

- **Fraction of space in the unbroken phase**

$$f_{\rm unbroken}(t) = \exp\left[-\frac{4\pi}{3}\int_{t_c}^{t} dt'\, \beta_{\rm w}^3\, (t-t')^3\, \gamma(t')\right]$$

- **Number density of nucleation sites**

$$n_{\rm nucleartion} = \int_{t_c}^{\infty} dt' \gamma(t') f_{\rm unbroken}(t')$$

[E. Ponton, Y. Bai, B. Jain, JHEP 09 (2019) 011 ]