Universität Zürich[UZH]

Renormalization Group Evolved 4321 and intriguing behaviors in the UV

MITP Scientific Program, Flavor at the Crossroads, 27 April 2022

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Based on

R. Houtz, J.Pagès and *S. Trifinopoulos*, arXiv:2204.06440

1 Laudau poles
2 Radiative Electroweak Symmetry Breaking
3 UV Unification
Renormalization Group Evolution of the 4321 Scalar sector

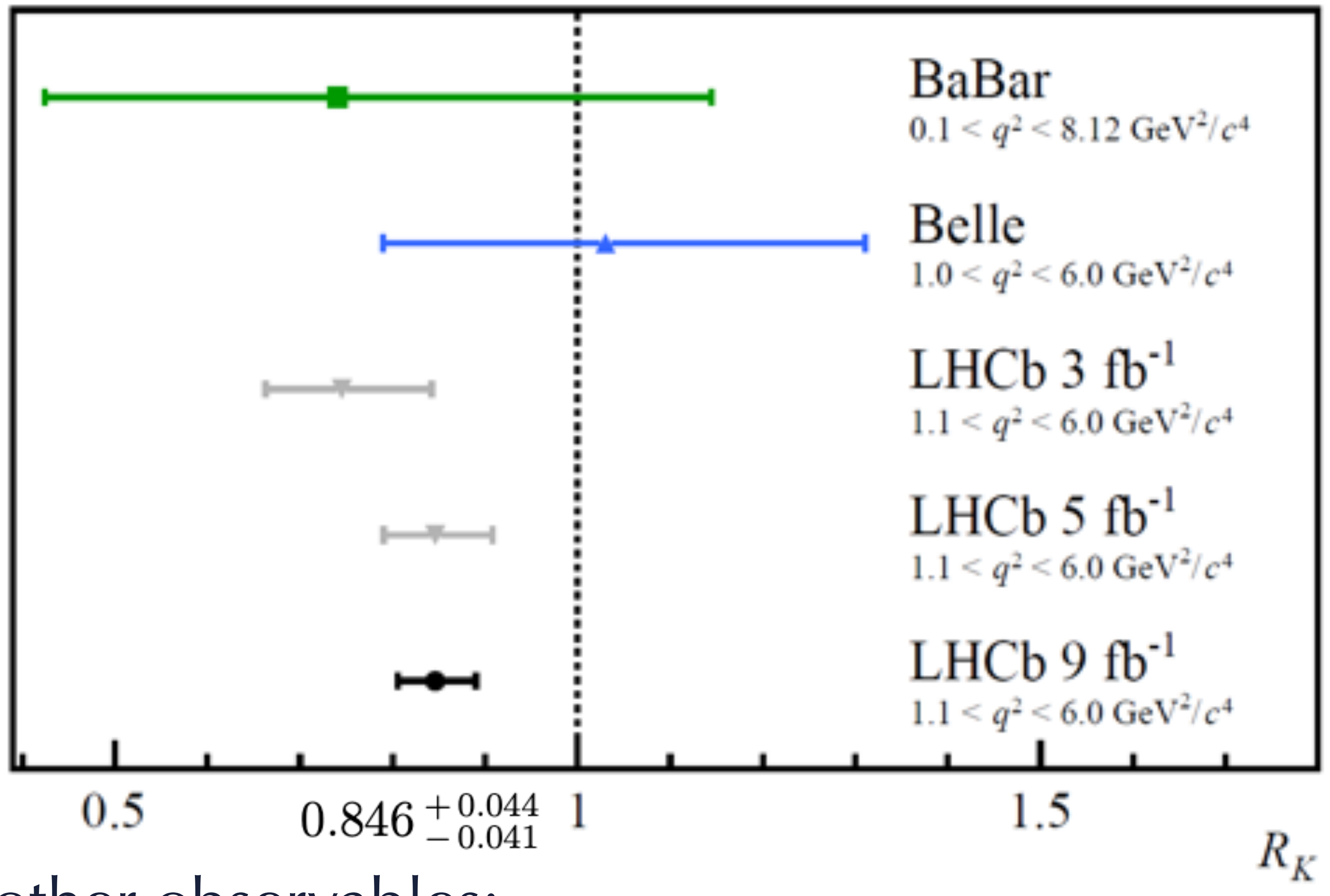
B anomalies

Neutral currents

$b \to s\ell^+\ell^-$: ~~universality~~ in μ vs. e

$$R_K = \frac{\Gamma(B \to K\mu^+\mu^-)}{\Gamma(B \to K e^+e^-)}$$

3.1 σ



+ other observables:

$R_{K^{(*)}}$, P_5', $B \to K^{(*)}\mu^+\mu^-$,
$B_s \to \mu^+\mu^-$, $B_s \to \phi\mu^+\mu^-$, ...

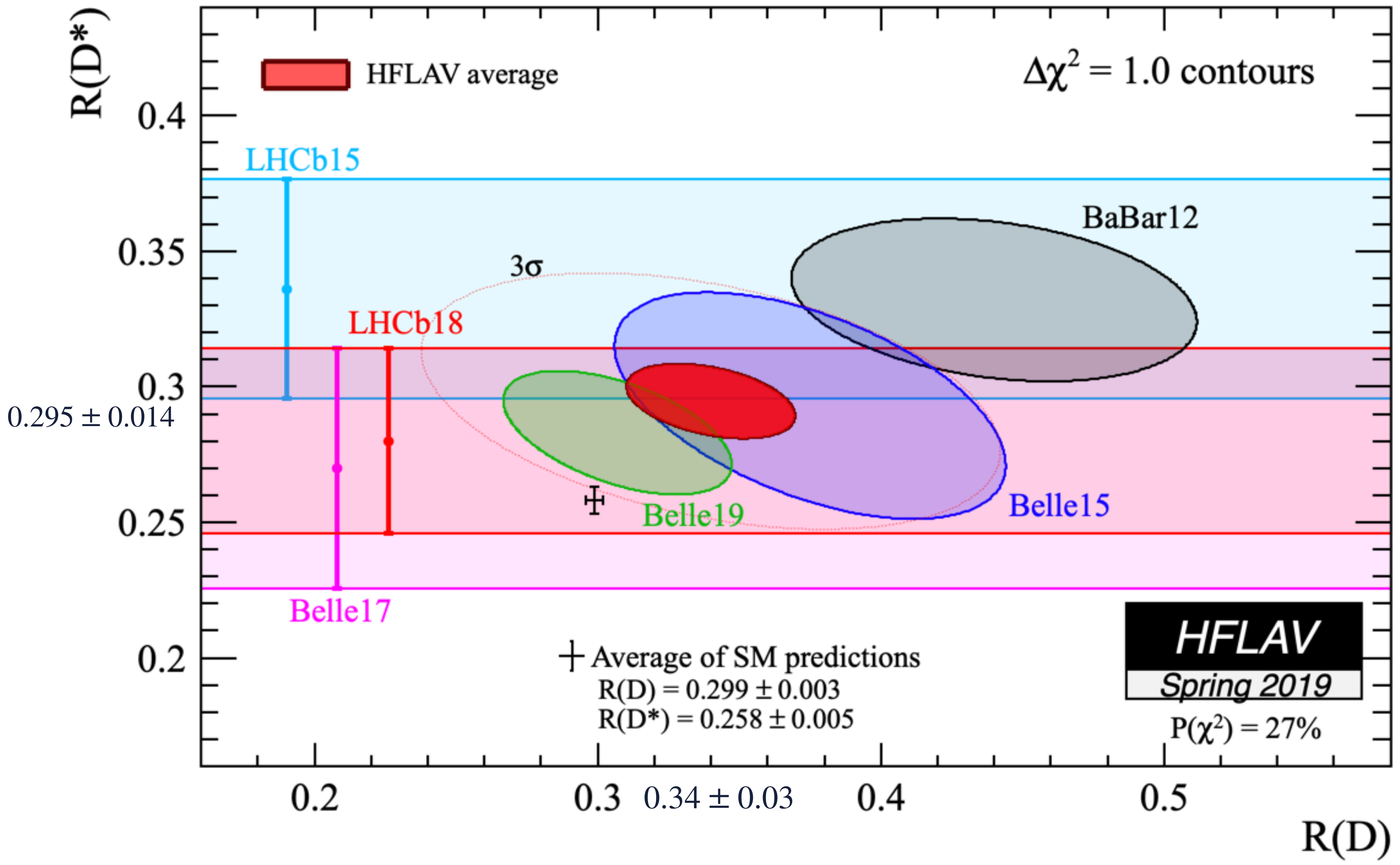
$> 4\ \sigma$

Charged currents

$b \to c\ell\nu$: ~~universality~~ in τ vs. μ,e

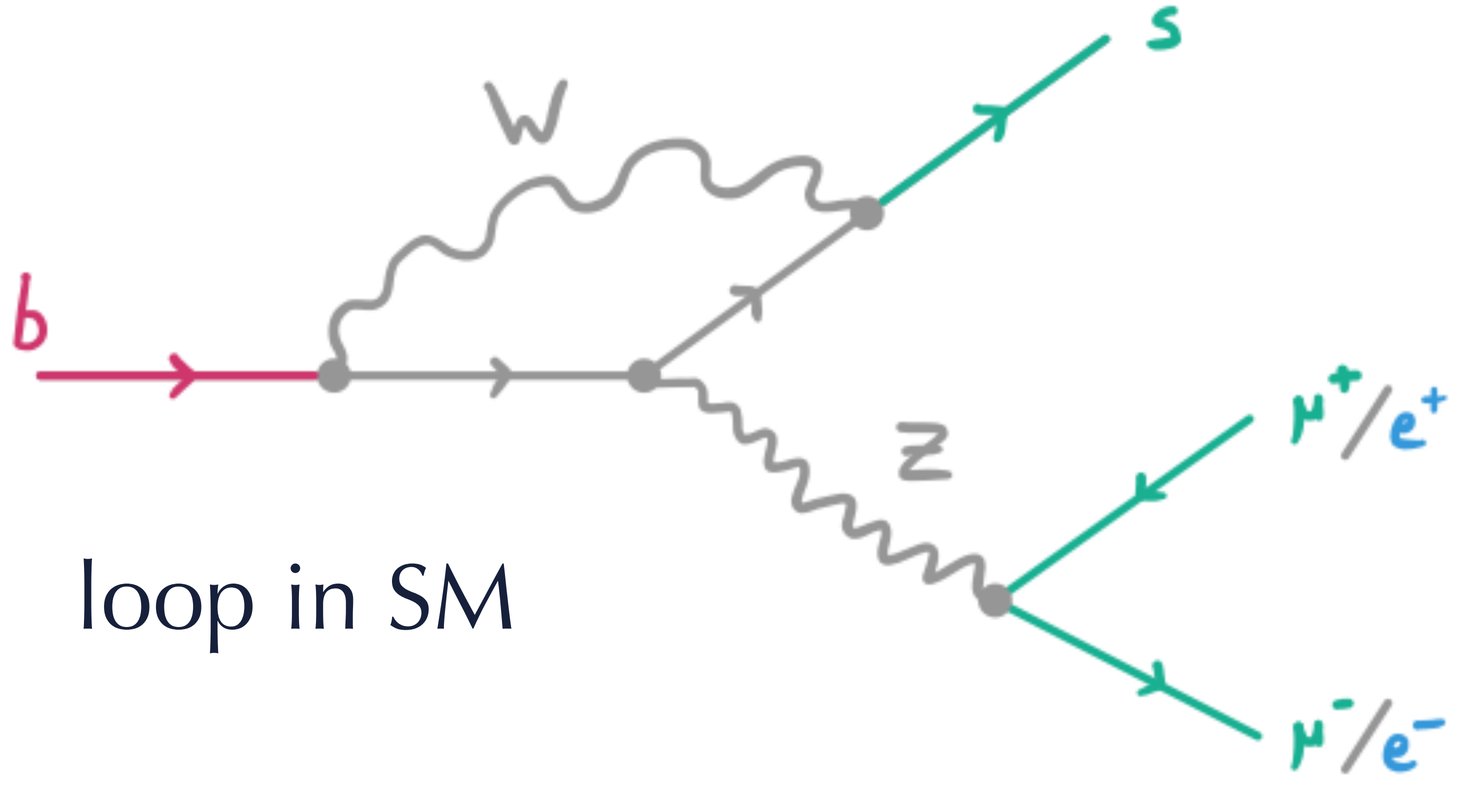
$$R_{D^{(*)}} = \frac{\Gamma(B \to D^{(*)}\tau\nu)}{\Gamma(B \to D^{(*)}\ell\nu)}$$

3.4 σ



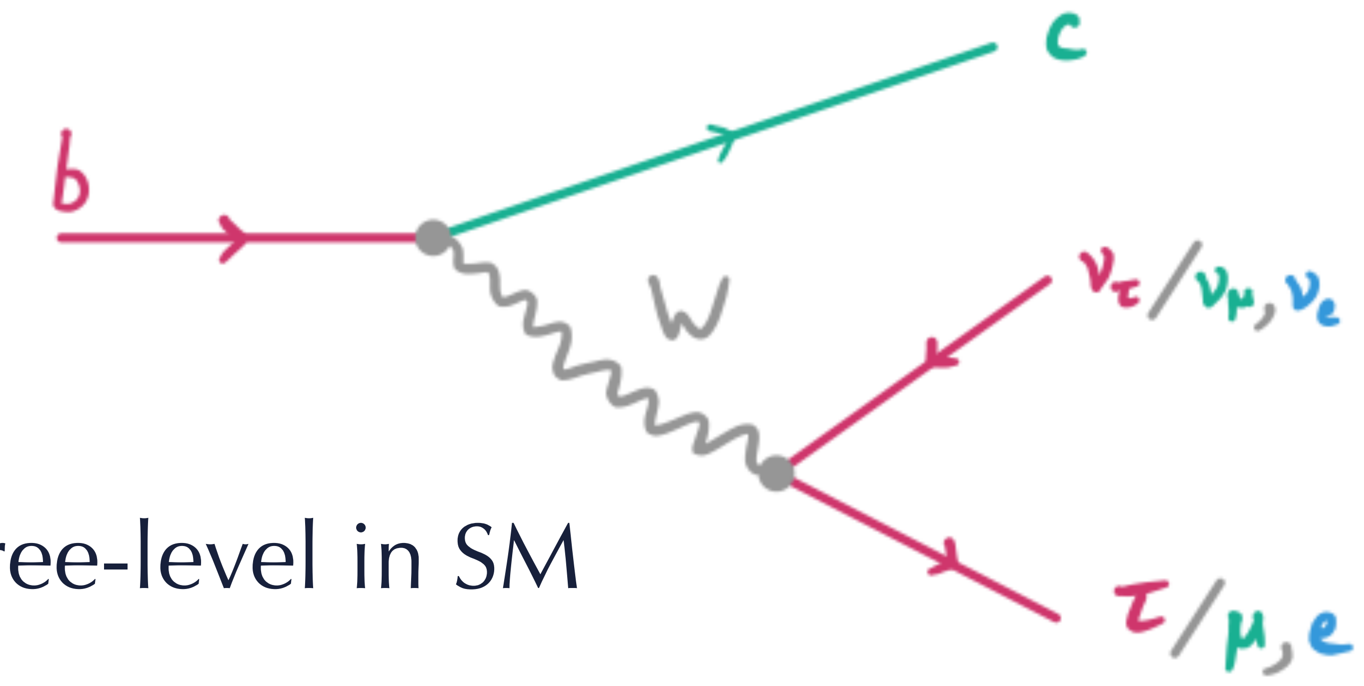
Vector LQ mediator for the B anomalies

Neutral currents: $b \to s\ell^+\ell^-$



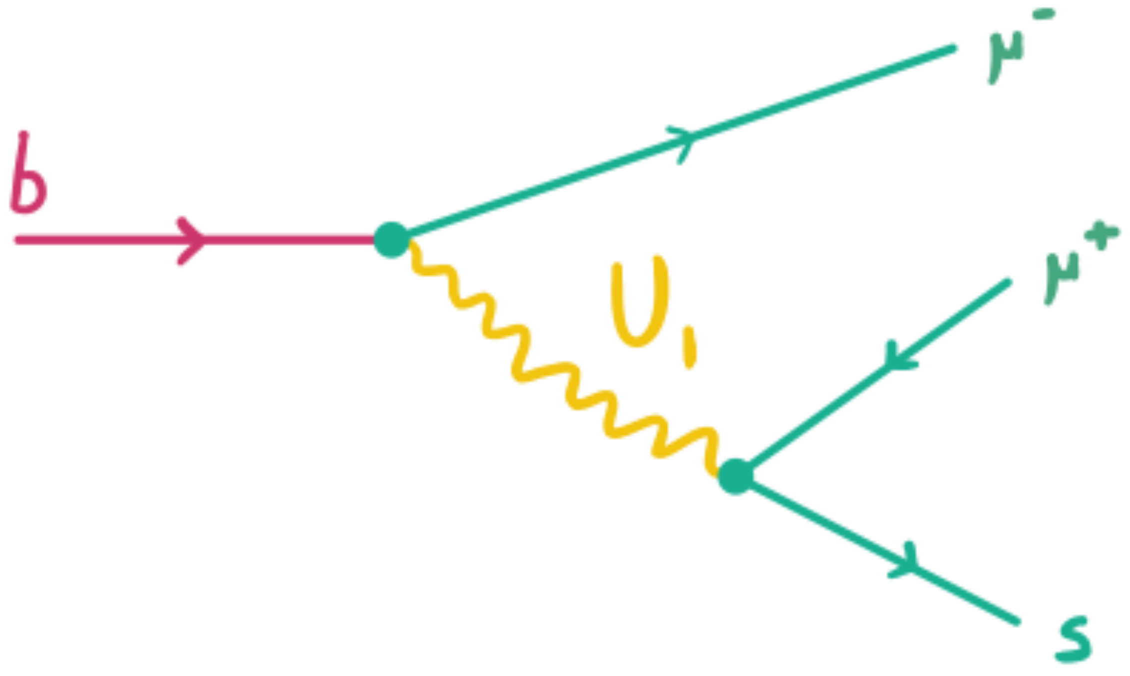
loop in SM

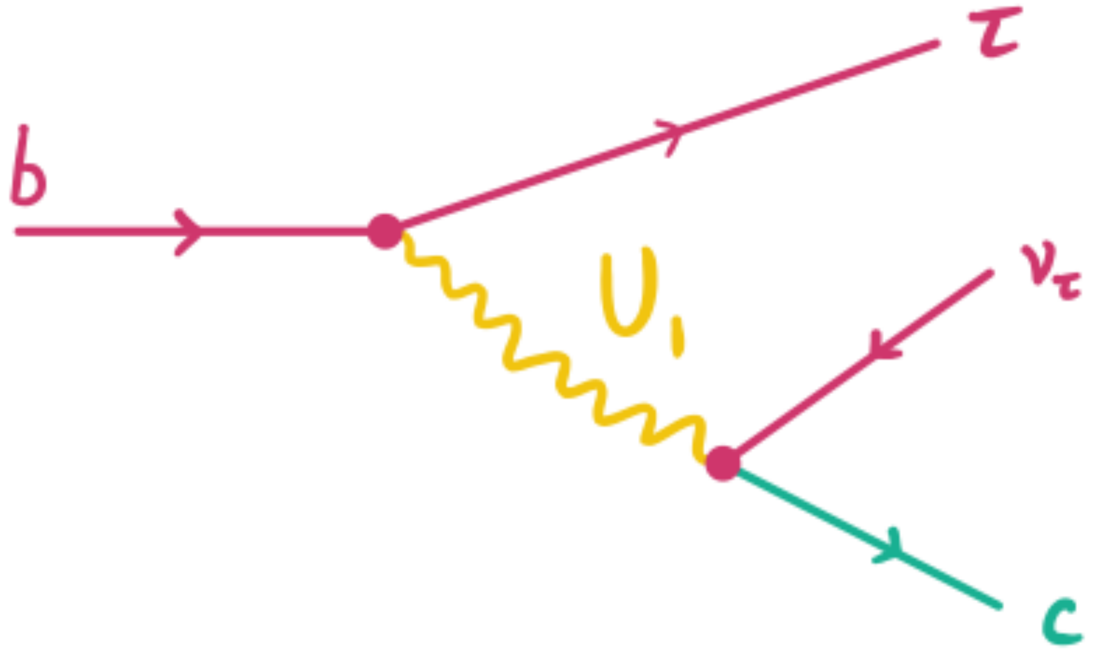
Charged currents: $b \to c\ell\nu$



tree-level in SM

Combined single mediator solution: the $U_1 \sim (3,1)_{2/3}$ vector leptoquark





UV origin of the U_1 leptoquark

$U_1 \sim (3,1)_{2/3}$ vector leptoquark

$SU(4)$

gauged → fundamental LQ

global → composite LQ

$$A^a_\mu \sim \begin{pmatrix} G' & U_1 \\ U_1^\dagger & Z' \end{pmatrix}$$

fundamental LQ

symmetry breaking sector

→ fundamental

[Diaz, Schmaltz, Zhong, 1706.05033]
[Di Luzio, Greljo, Nardecchia, 1708.08450]

→ composite

[Fuentes-Martín, Stangl, 2004.11376]
[Fuentes-Martin, Isidori, Lizana, Selimovic, Stefanek, 2203.01952]

composite LQ

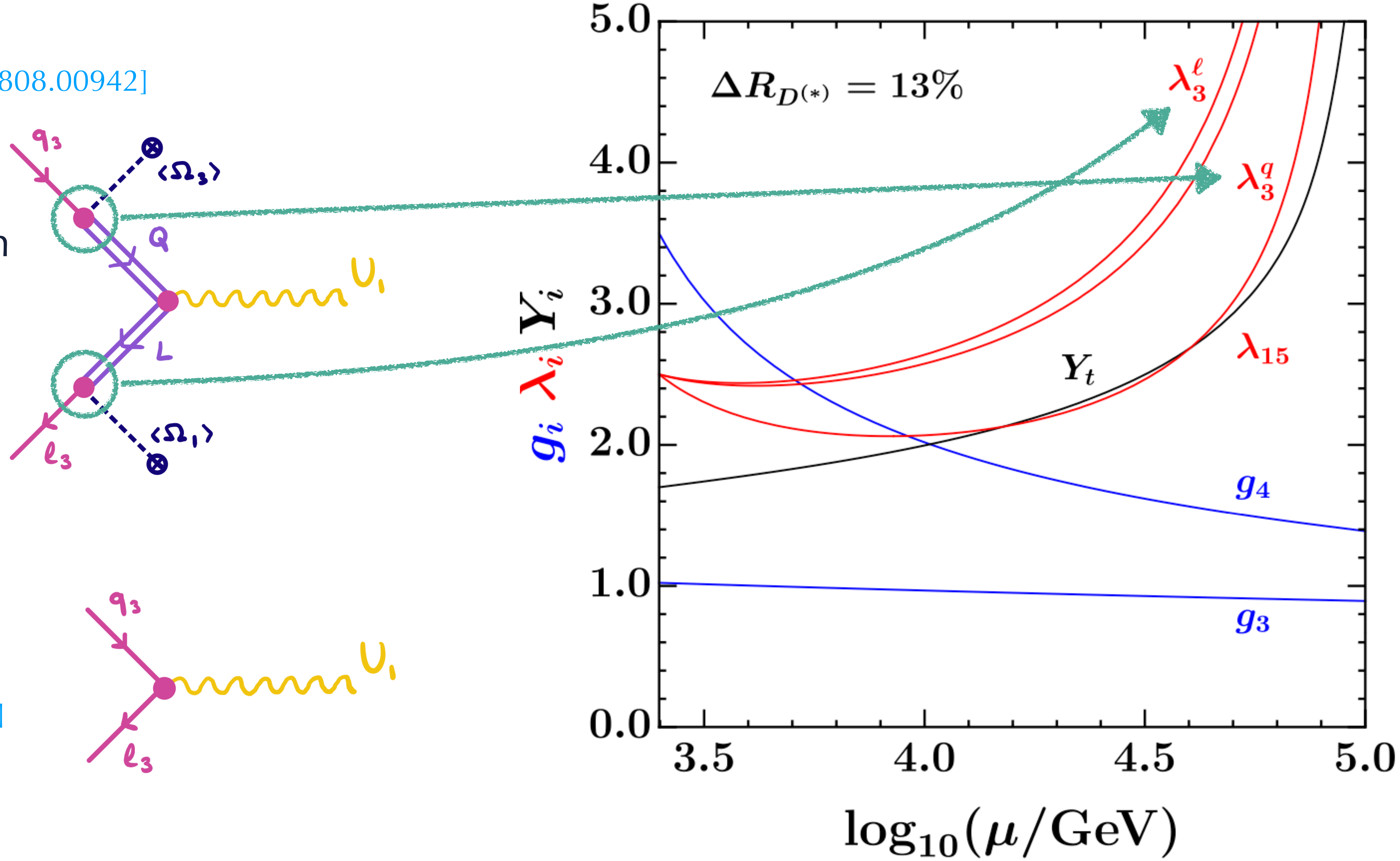
[Barbieri, Murphy, Senia, 1611.04930]
similarly for scalar LQs [Marzocca, 1803.10972]

4321 model

- Flavor universal

[Di Luzio, Greljo, Nardecchia, 1708.08450]
[Di Luzio, Fuentes-Martín, Greljo, Nardecchia, Renner, 1808.00942]

➡ large mixing between 3rd generation SM fields and vector-like partners
↪ Landau poles $\lesssim$ 100 TeV



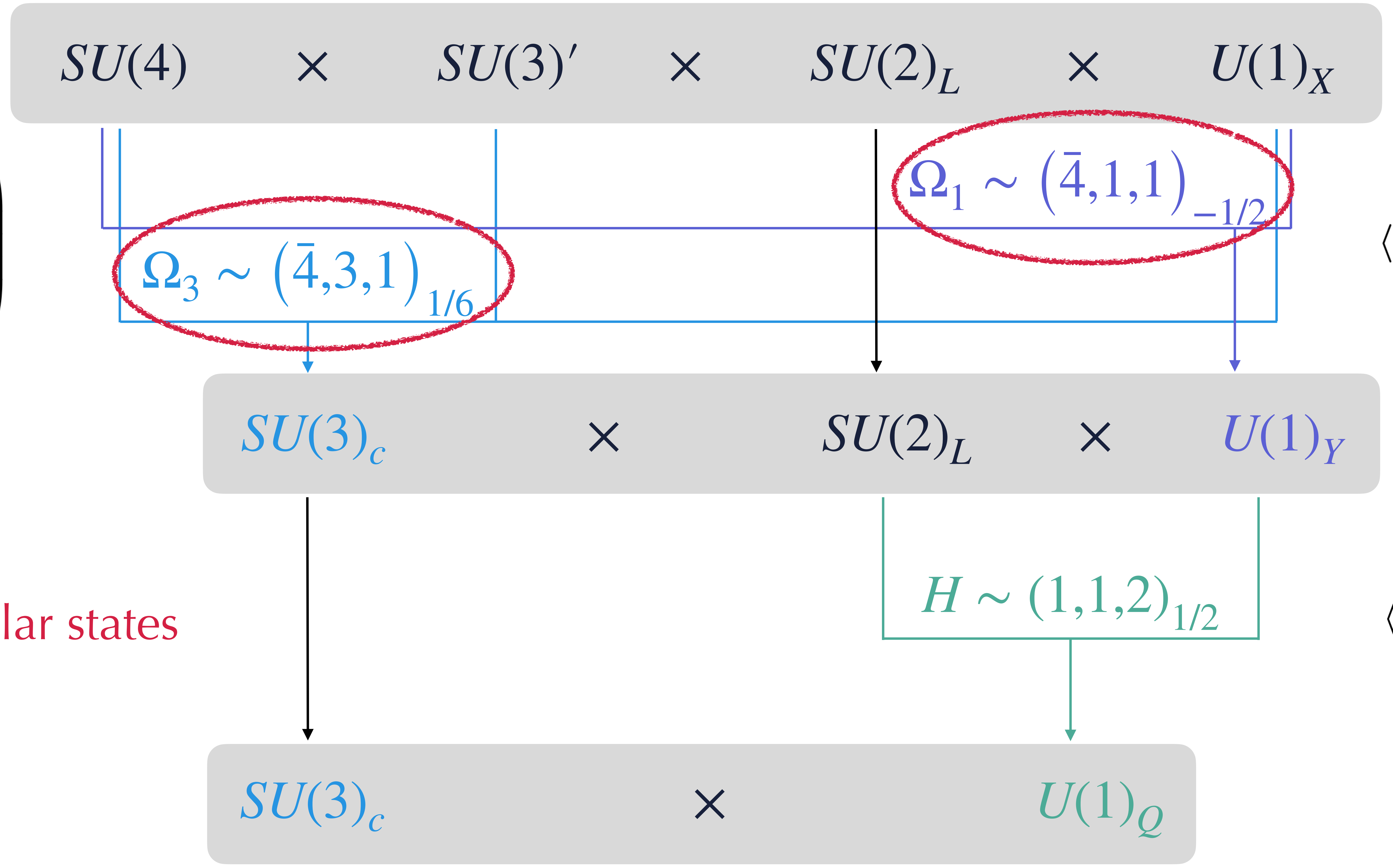
- Flavor non-universal

[Greljo, Stefanek, 1802.04274]
[Bordone, Cornella, Fuentes-Martín, Isidori, 1805.09328]
[Cornella, Fuentes-Martin, Isidori, 1903.11517]

see Claudia's talk

[Di Luzio, Fuentes-Martín, Greljo, Nardecchia, Renner, 1808.00942]

Spontaneous symmetry breaking of the 4321 model



$$\langle \Omega_3 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} \omega_3 & 0 & 0 \\ 0 & \omega_3 & 0 \\ 0 & 0 & \omega_3 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\langle \Omega_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ 0 \\ \omega_1 \end{pmatrix}$$

TeV-scale new scalar states

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Scalar potential in the 4321 model

Omega potential

with quartics ρ_i

$$V_\Omega = m^2_{\Omega_3}\mathrm{Tr}[\Omega_3^\dagger\Omega_3] + m^2_{\Omega_1}\Omega_1^\dagger\Omega_1 + \frac{\rho_1}{2}(\Omega_1^\dagger\Omega_1)^2 + \frac{\rho_2}{2}\mathrm{Tr}[\Omega_3^\dagger\Omega_3]^2 + \frac{\rho_2'}{2}\mathrm{Tr}[\Omega_3^\dagger\Omega_3\Omega_3^\dagger\Omega_3]$$
$$+ \rho_3\mathrm{Tr}[\Omega_3^\dagger\Omega_3]\Omega_1^\dagger\Omega_1 + \rho_4\,\Omega_1^\dagger\Omega_3\Omega_3^\dagger\Omega_1 + \frac{\rho_5}{3!}\left(\epsilon_{\alpha\beta\gamma\delta}\epsilon^{abc}(\Omega_3)^\alpha_a(\Omega_3)^\beta_b(\Omega_3)^\gamma_c(\Omega_1)^\delta + \mathrm{h.c.}\right)$$

Higgs potential

with quartic λ

$$V_H = \mu_H^2 H^\dagger H + \frac{\lambda}{2}(H^\dagger H)^2$$

$H - \Omega_{1,3}$ mixing

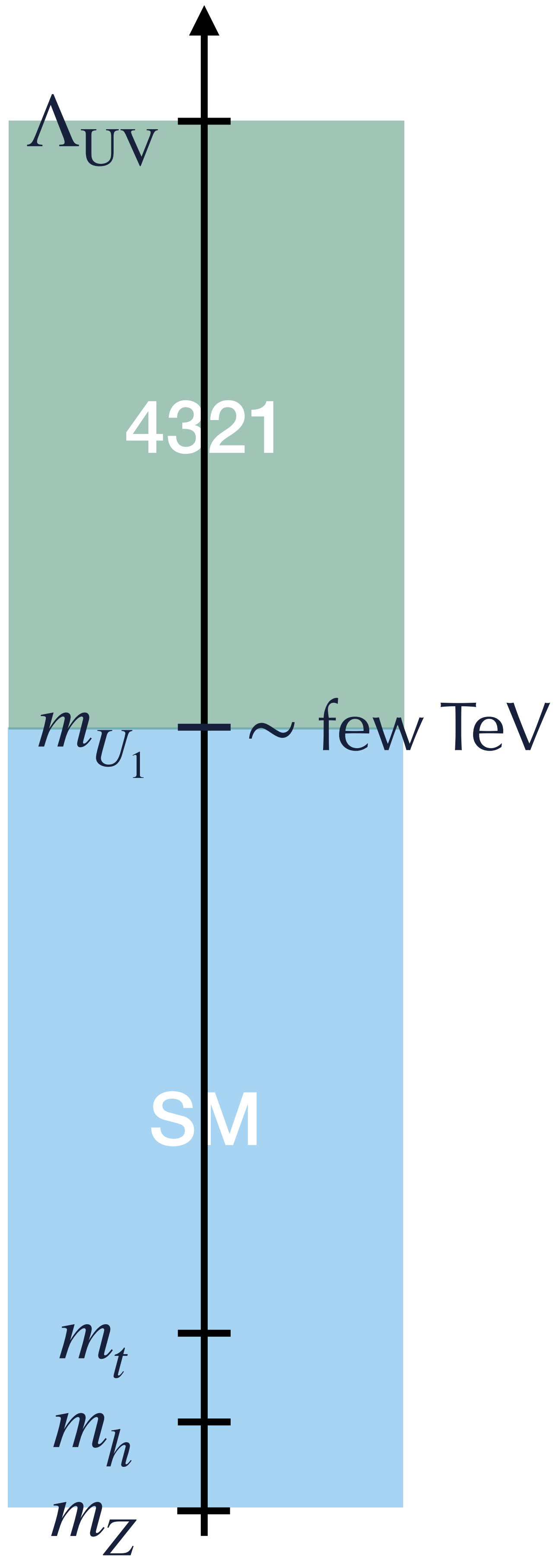
with quartics η_i

$$V_{\Omega H} = \eta_1\, H^\dagger H\, \Omega_1^\dagger\Omega_1 + \eta_3\, H^\dagger H\, \mathrm{Tr}[\Omega_3^\dagger\Omega_3]$$

Renormalization Group Evolution of the 4321

$$V = V_\Omega + V_H + V_{\Omega H}$$

$\{\rho_i\}, m_{\Omega_{1,3}}$ λ, μ_H $\{\eta_i\}$



RGE study:

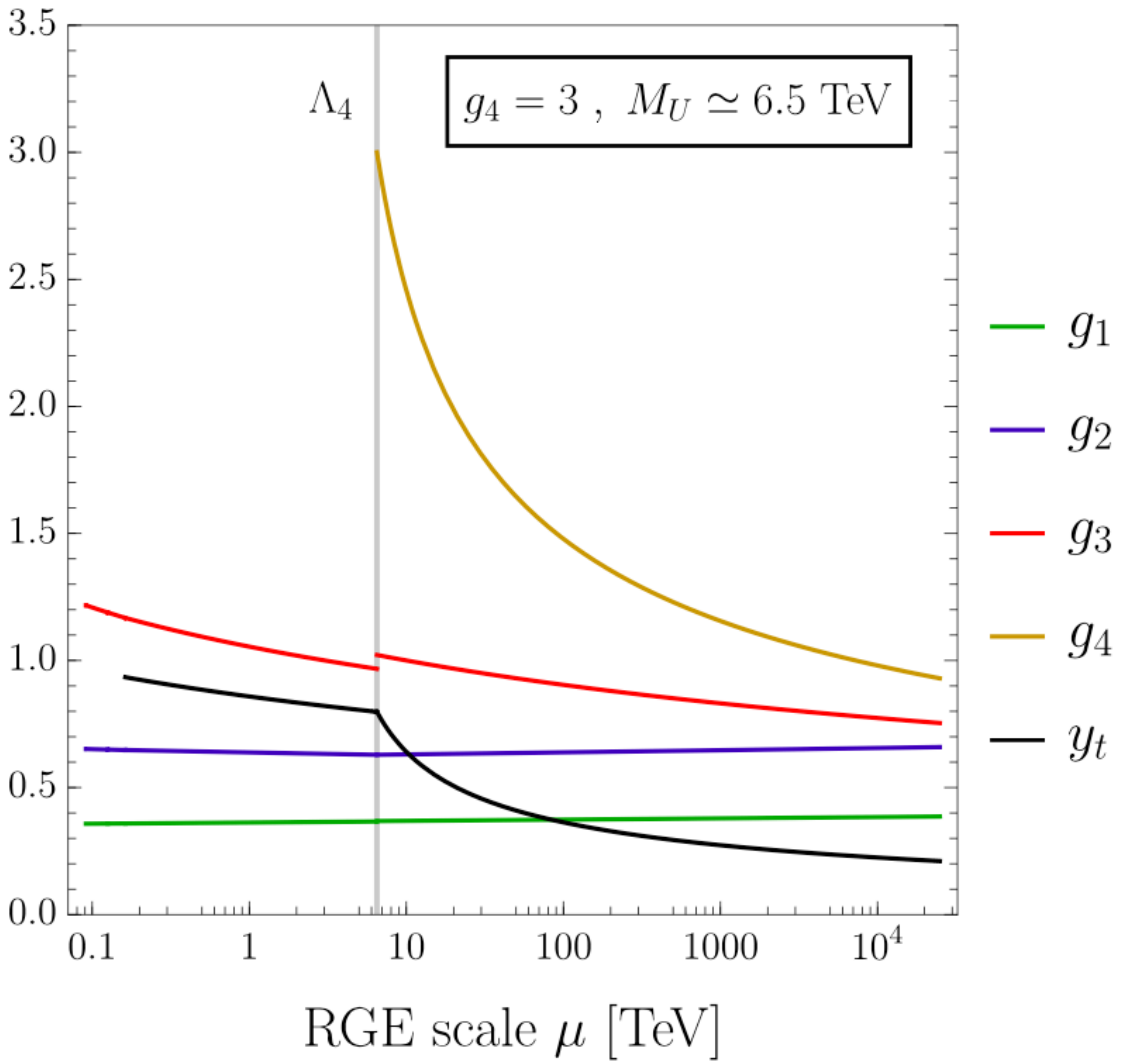
1) Fix boundary conditions from phenomenology constraints
↪ since under-constrained, pick a benchmark satisfying the constraints

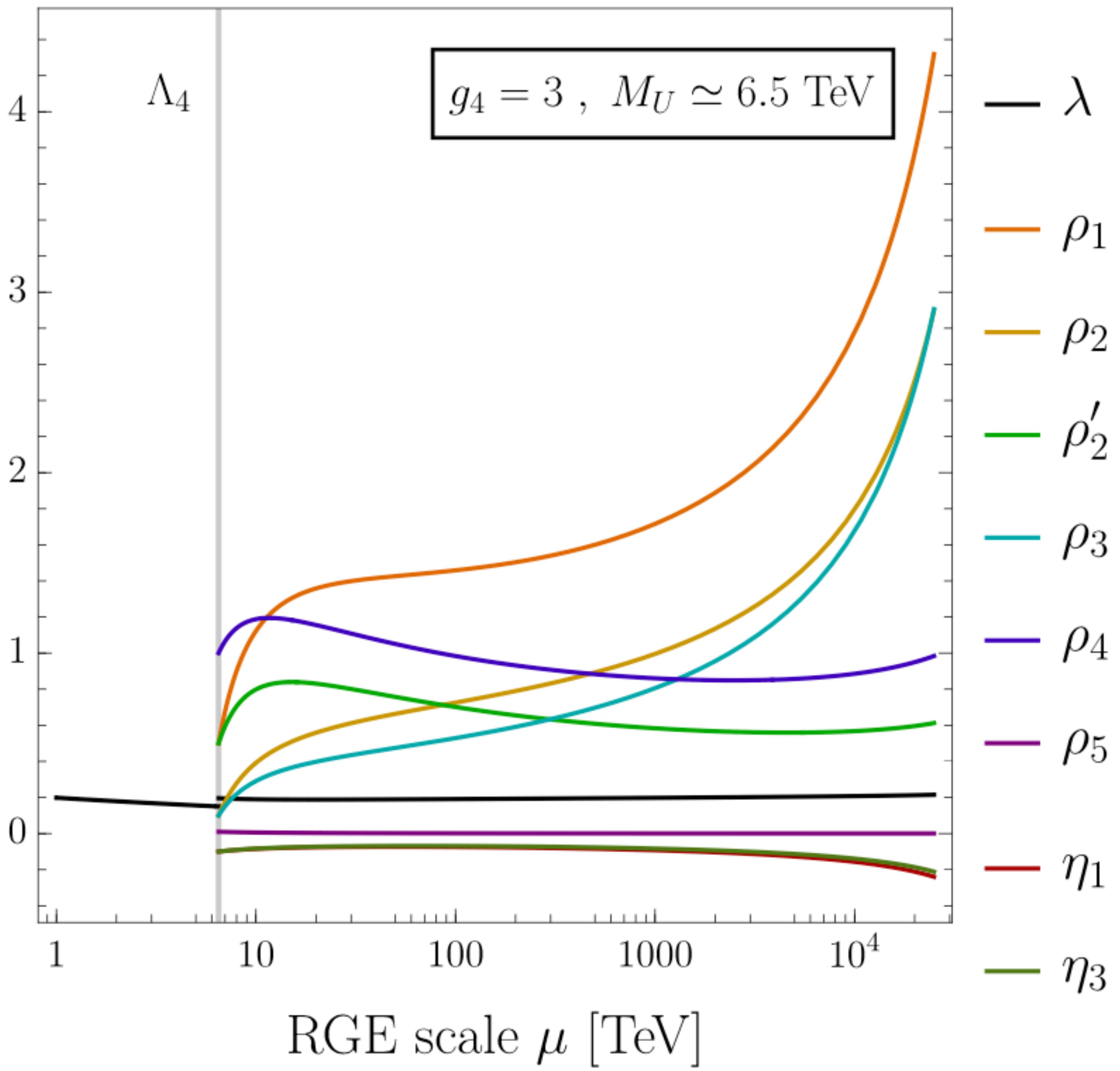
2) Derive all β-functions for each theory/energy range at one-loop

crosschecked/computed with RGBeta [Thomsen, 2101.08265]

3) Run, Match (at tree-level), Run, etc…

1. Landau poles





"Asymptotic freedom" in gauge couplings but Landau poles develop in the Ω potential quartics

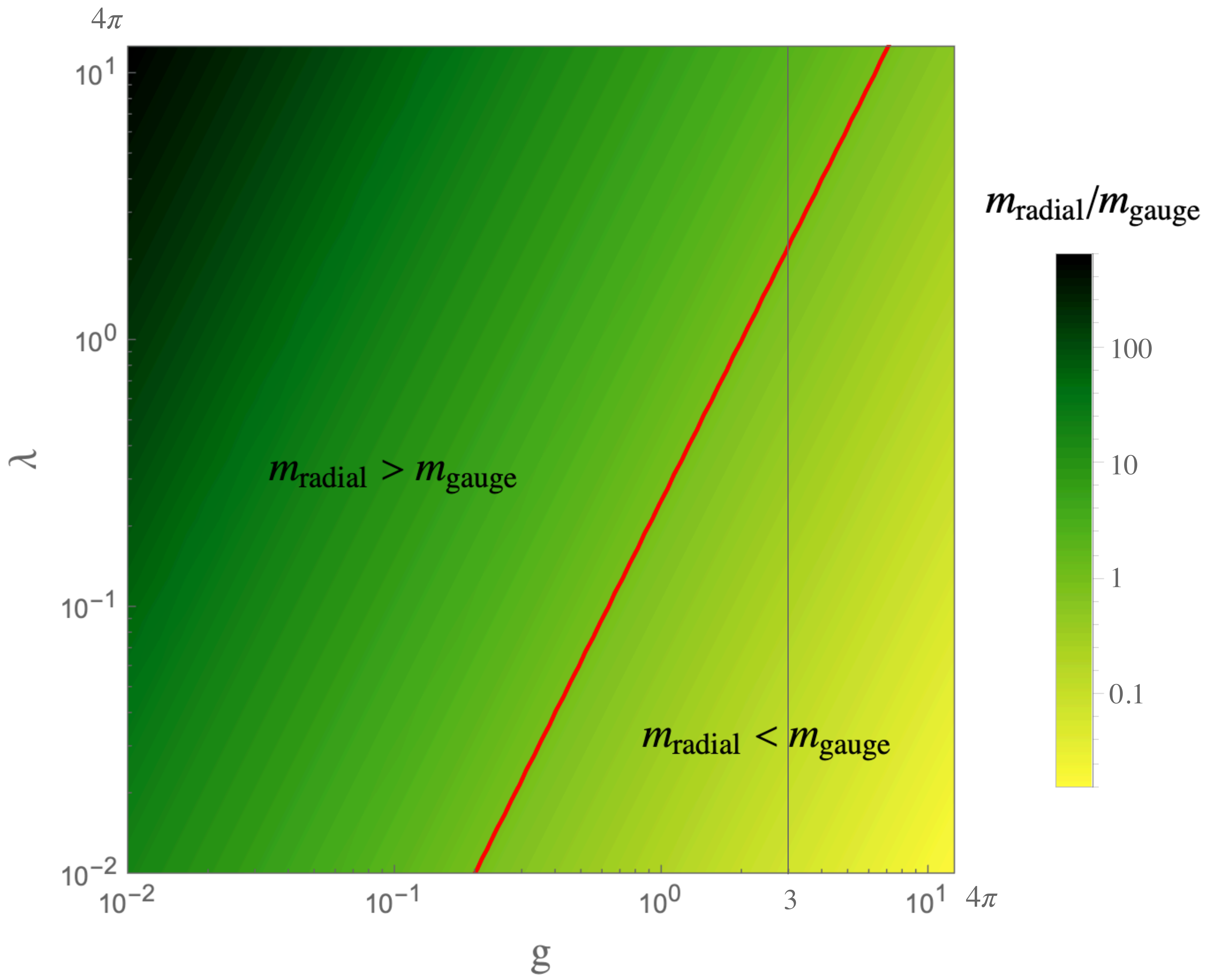
Masses of the radials

Decoupling of radial modes
is not easily realised with $g_4 = 3$

$$m_{\text{radial}} = \sqrt{\lambda}\, v \qquad m_{\text{gauge}} = \frac{g\, v}{2}$$

$$\frac{m_{\text{radial}}}{m_{\text{gauge}}} = 2\frac{\sqrt{\lambda}}{g} \lesssim 2.4 \left(\frac{3}{g}\right)$$

Decoupling limit corresponds to $\lambda \gg 2$



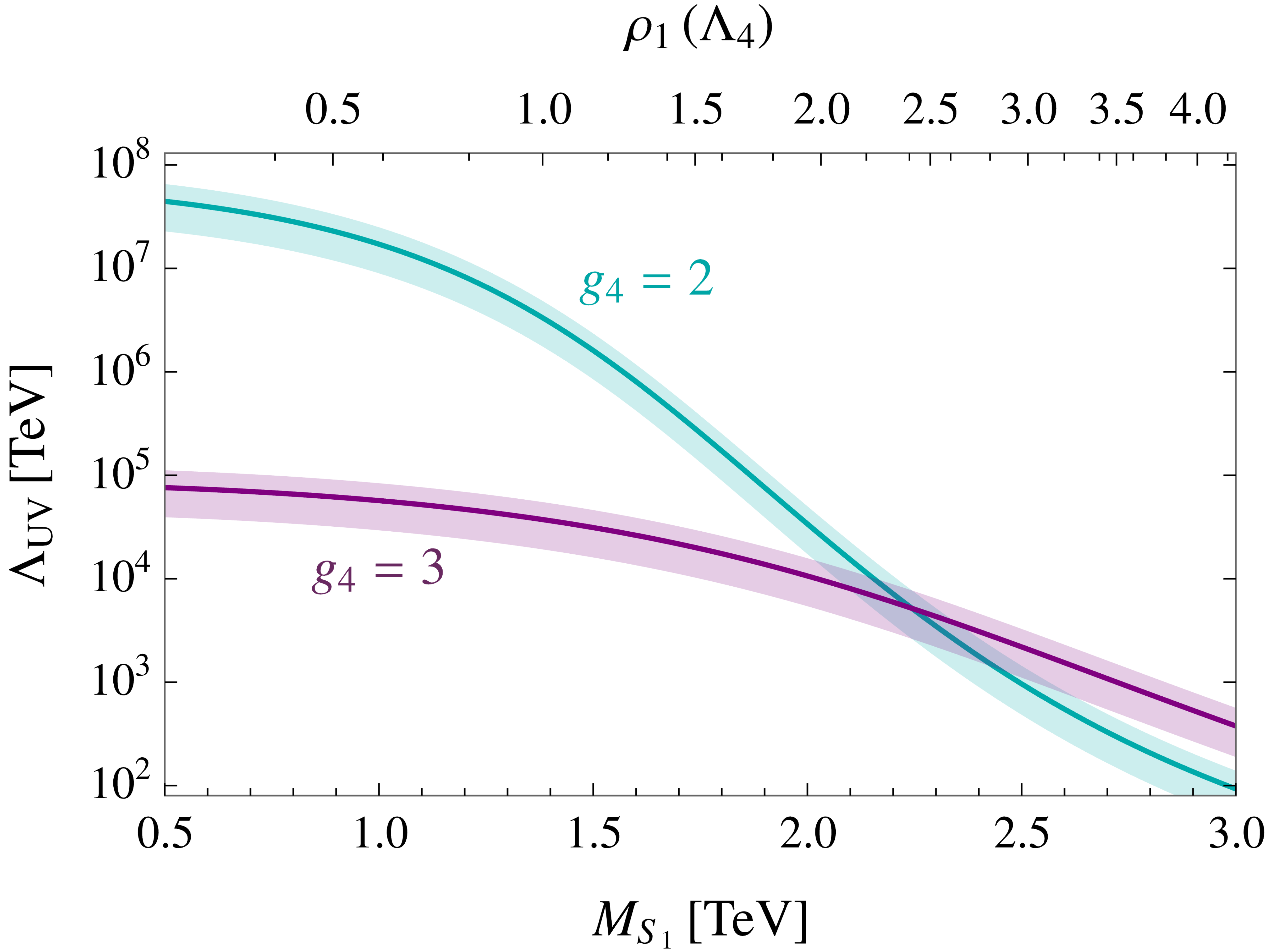
Landau poles and masses of the radials

Running of the quartic ρ_1

$$\beta_{\rho_1} = \left(16\rho_1 - 3g_1^2 - \frac{45}{2}g_4^2\right)\rho_1 + 24\rho_3^2 + 12\rho_3\rho_4 + 6\rho_4^2 + 4\eta_1^2 + \frac{3}{4}g_1^4 + \frac{9}{4}g_1^2 g_4^2 + \frac{99}{16}g_4^4$$

Large quartic couplings
=
heavier radials
=
lower cut-off scale

Cut-off scale where $\rho_1(\Lambda_{UV}) \geq 4\pi$

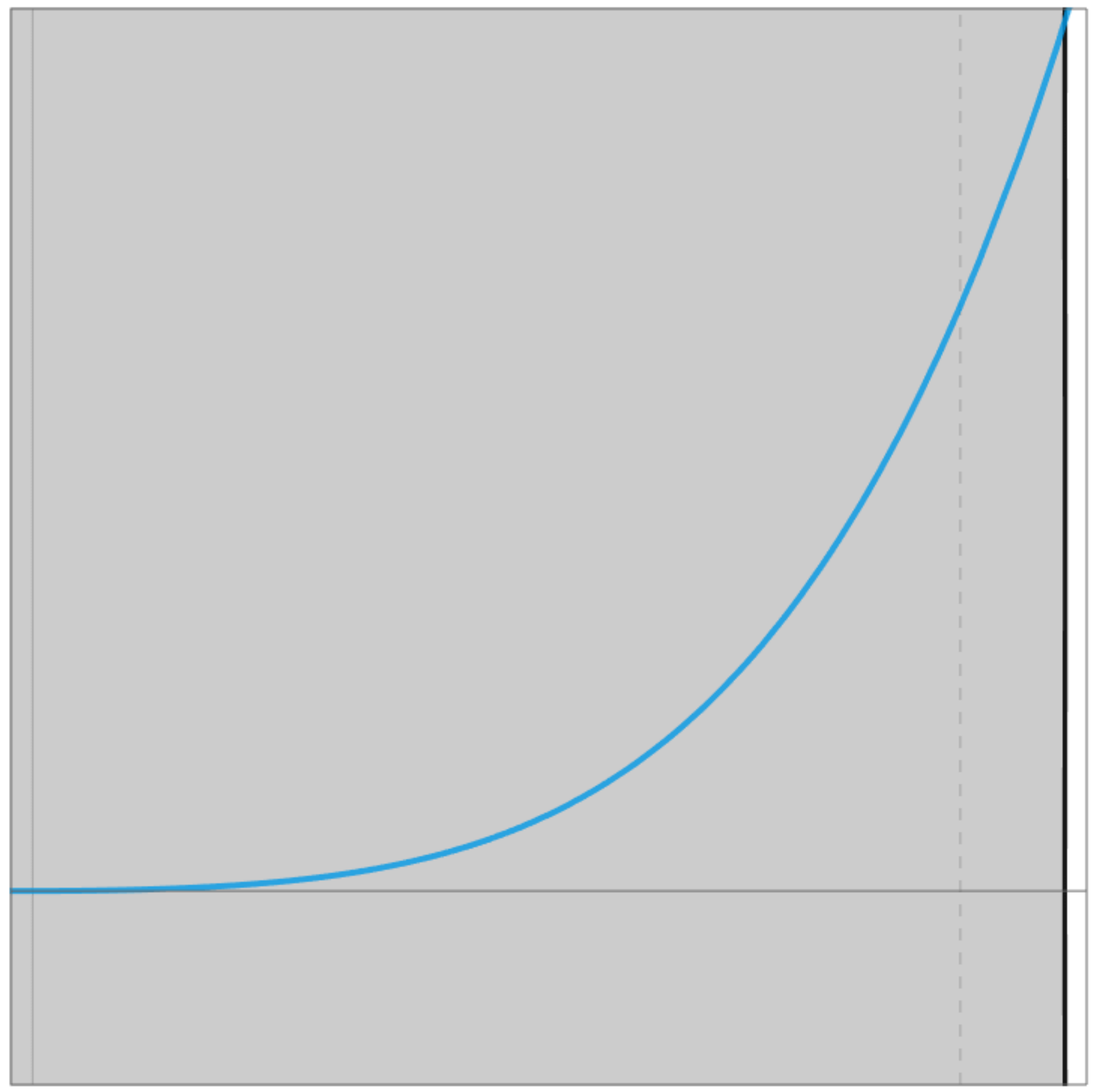


2. Radiative Electroweak symmetry breaking

Radiative electroweak symmetry breaking:

Electroweak symmetry is conserved at the classical level,
but loop corrections to the mass parameter of the Higgs boson trigger its spontaneous breaking.

$\Rightarrow$ A positive Higgs mass parameter at high field value can turn negative at lower scale via the renormalization group flow.



Examples: [Babu, Gogoladze, Khan, 1512.05185]

- Standard Model $\beta_H^{\rm SM} \propto m_H^2$ ✗
- Type-I seesaw model $\beta_H^{\rm SS1} = \beta_H^{\rm SM} - 4|y_\nu|^2|m_R|^2$ ✗
- Scalar singlet dark matter model $\beta_H^{\rm sDM} = \beta_H^{\rm SM} + \lambda_3 m_s^2$ ✓

Necessary ingredients:

new states
positive contribution
$\Rightarrow$ TeV-scale new scalars

Radiative Electroweak Symmetry Breaking

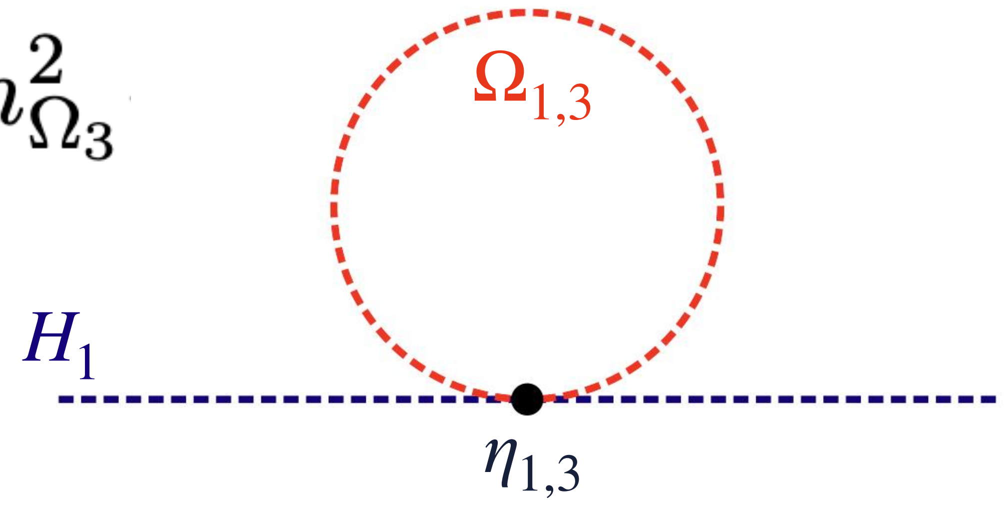
All ingredients are here!

Field	$SU(4)$	$SU(3)'$	$SU(2)_L$	$U(1)_X$
Ω_1	$\bar{4}$	1	1	$-1/2$
Ω_3	$\bar{4}$	3	1	$1/6$

TeV-scale new scalar states

$$\beta_{\mu_H^2} = \left(6\lambda + 8y_t^2 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2\right)\mu_H^2$$

$$+\, 8\eta_1 m_{\Omega_1}^2 + 24\eta_3 m_{\Omega_3}^2$$



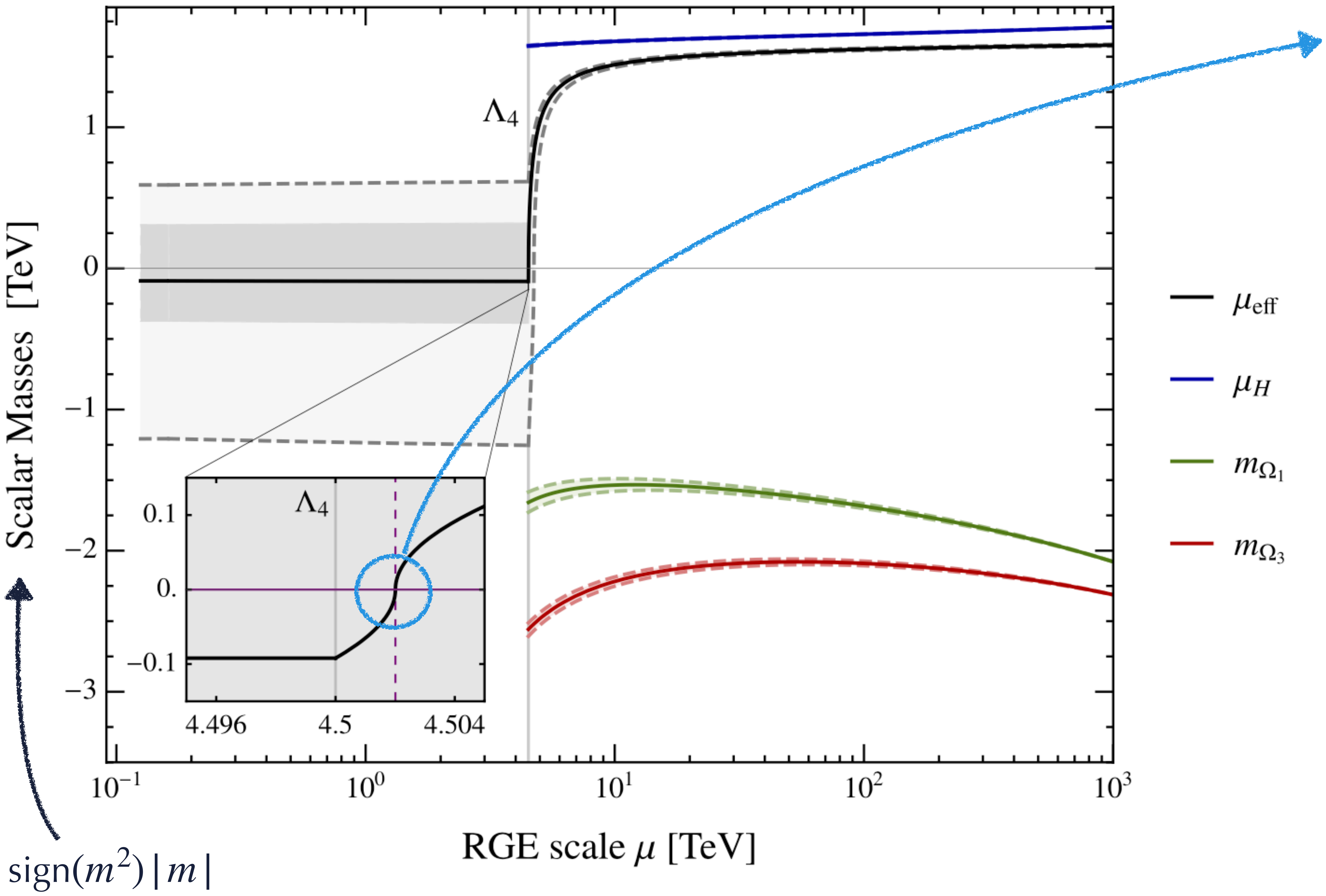
positive contribution for $\eta_{1,3} < 0$

Diagonalization of the Hessian (equivalent to integrating out $\Omega_{1,3}$) gives the effective SM Higgs mass:

$$\mu_{\text{eff}}^2 = -\frac{\lambda_{\text{eff}}}{2}v^2 = \mu_H^2 - \frac{\eta_1 D_1 - 3\eta_3\rho_3}{D_{12}} m_{\Omega_1}^2 - 3\frac{\eta_3\rho_1 - \eta_1\rho_3}{D_{12}} m_{\Omega_3}^2$$

$$D_1 = 3\rho_2 + \rho_2' > 0$$
$$D_{12} = \rho_1 D_1 - 3\rho_3^2 > 0$$
$$D_{123} = \lambda D_{12} - (3\eta_3^2\rho_1 - 6\eta_1\eta_3\rho_3 + \eta_1^2 D_1) > 0$$

Radiative Electroweak Symmetry Breaking



Radiative EWSB realised when μ_{eff} flips sign

Checked:

✓ perturbativity

✓ bounded from below conditions

Some observations:

✓ undeniably an RG effect

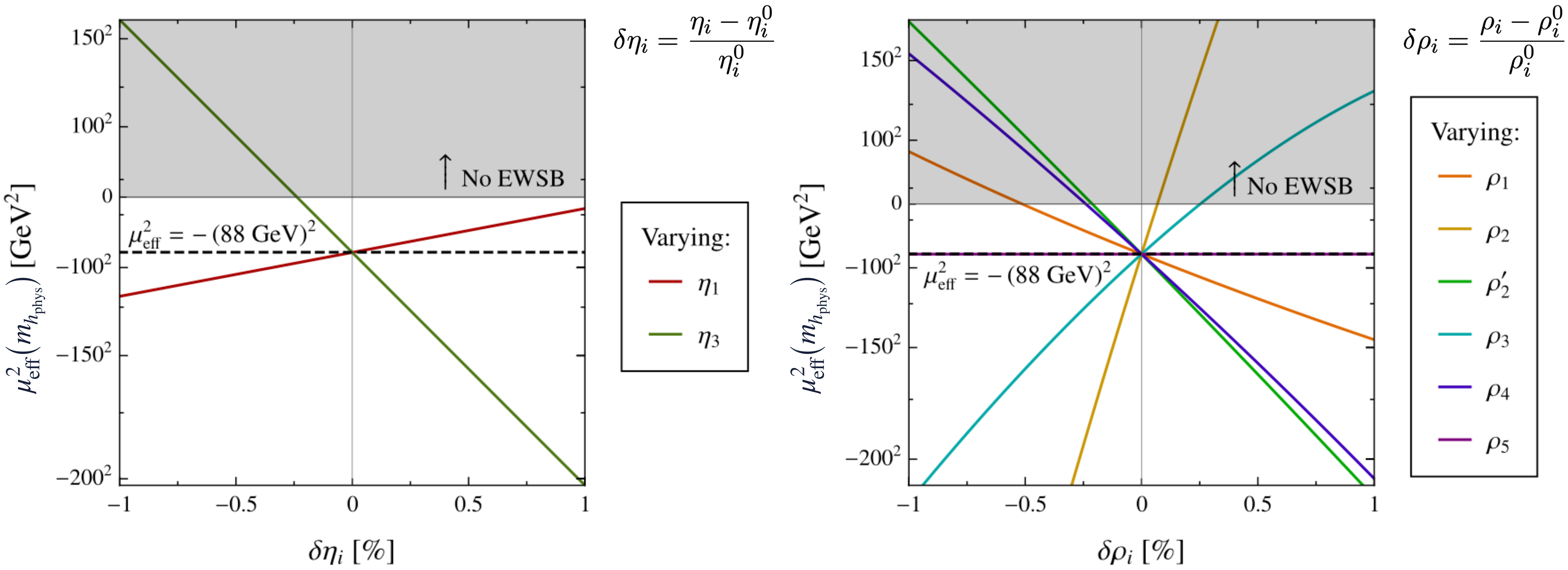
↪ without running, positive eigenvalues of the hessian stay positive.

⊖ unavoidable fine-tuning

↪ strong acceleration near the 4321 breaking scale

Fine-tuning

Quantifying the fine-tuning and identifying sensitivity of $m_{h_{\text{phys}}}$ to variation of the quartic by $\pm 1\,\%$

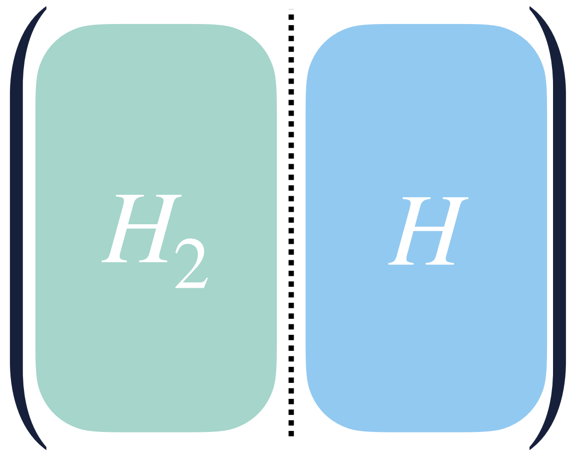


3. Unification in the UV

In some UV completion of 4321 see Ben's talk

[Fuentes-Martín, Isidori, Lizana, Selimovic, Stefanek, 2203.01952]
[Bordone, Cornella, Fuentes-Martín, Isidori, 1712.01368]

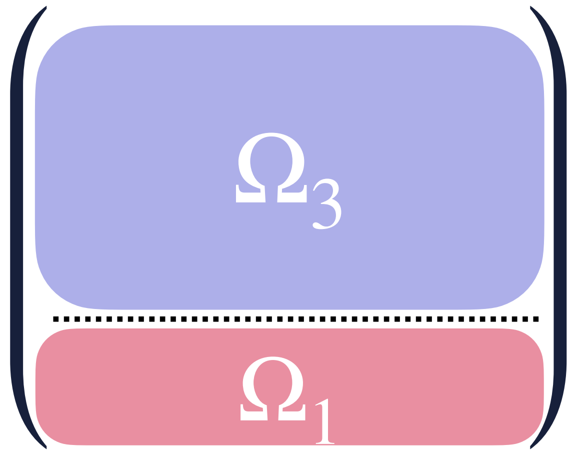
- the doublet Higgs comes from a bi-doublet field under $SU(2)_L$ and $SU(2)_R$ $\Phi \sim$



  leading to a two-Higgs-doublet-model after 4321 breaking scale with relations:

$$\mu_H = \mu_2 \qquad \text{and} \qquad \lambda = \lambda_2 = \lambda_3$$

- the two $\Omega_{1,3}$ come from the same "bi-quadruplet" under $SU(4)_{\text{light}}$ and $SU(4)_3$ $\Omega \sim$



  leading to the relations after SSB:

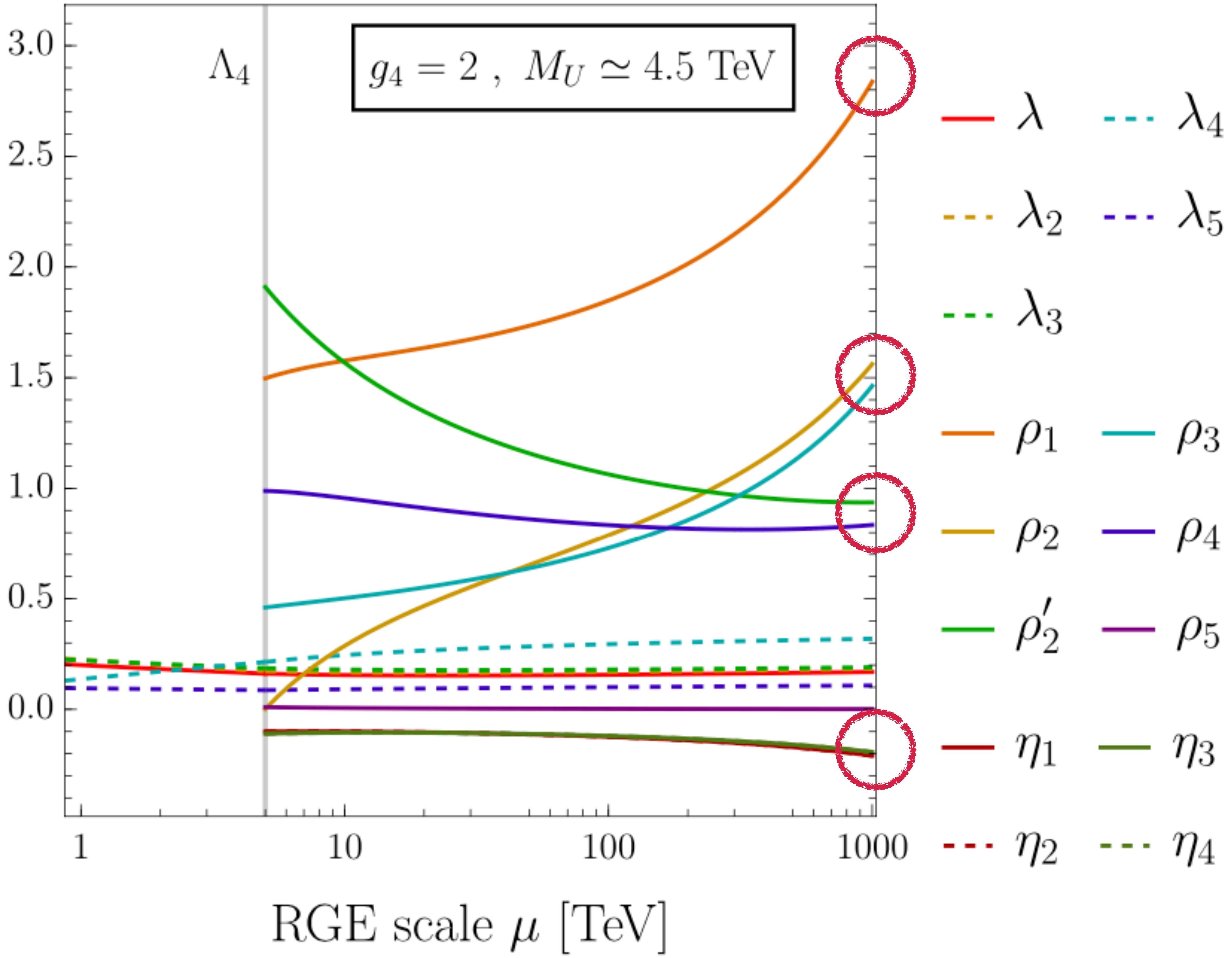
$$m_{\Omega_1} = m_{\Omega_3}, \qquad \rho_1 = \rho_2 + \rho_2' , \qquad \rho_2 = \rho_3 \qquad \text{and} \qquad \rho_2' = \rho_4$$

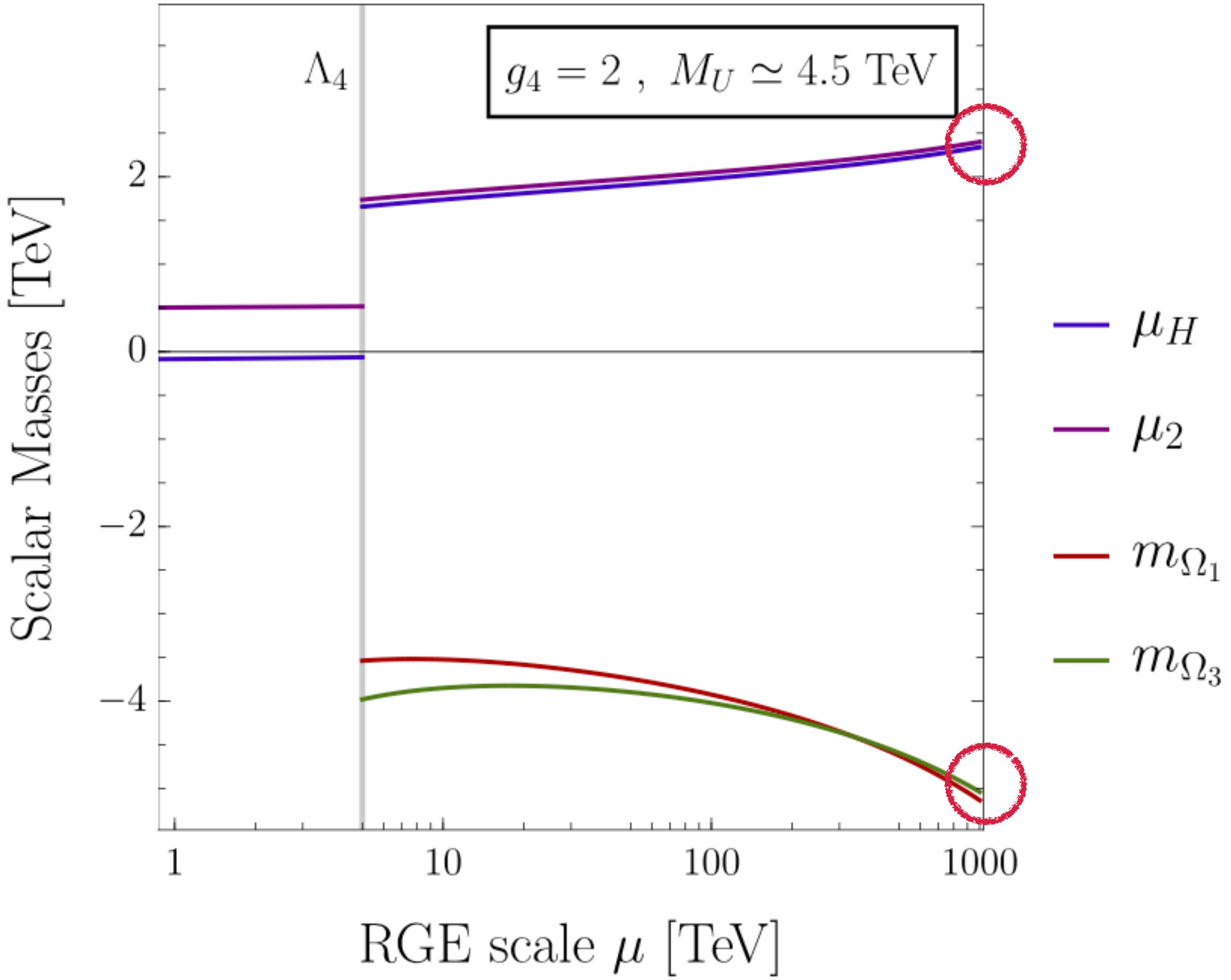
$$\eta_1 = \eta_2 = \eta_3 = \eta_4$$

UV unification

Unification in the UV can be realised allowing for 10 % threshold corrections at ~ 1000 TeV

$$\lambda = \lambda_2 = \lambda_3 \qquad \rho_1 = \rho_2 + \rho_2' \qquad \mu_H = \mu_2$$
$$\eta_1 = \eta_2 = \eta_3 = \eta_4 \qquad \rho_2 = \rho_3 \qquad m_{\Omega_1} = m_{\Omega_3}$$
$$\rho_2' = \rho_4$$





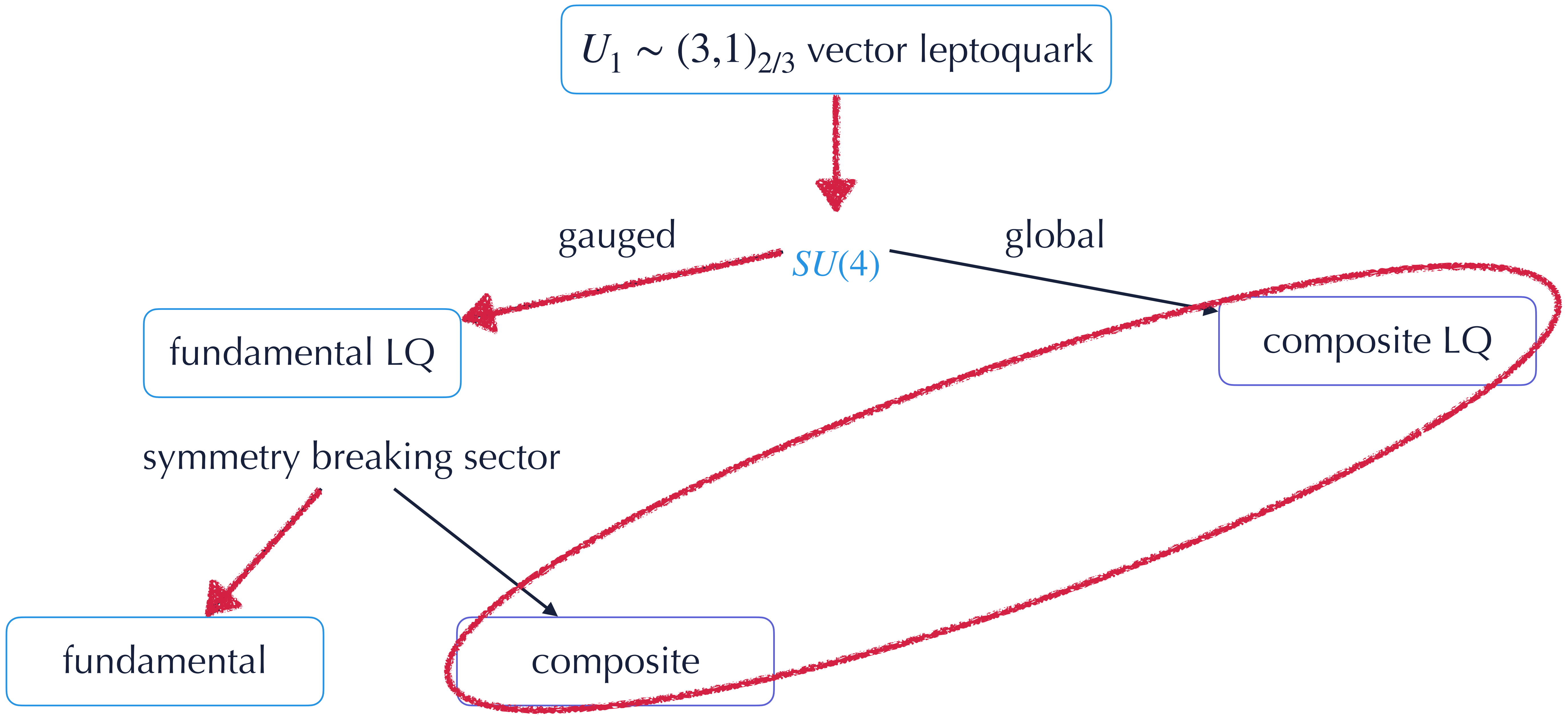
UV unification scale

Number of benchmark points satisfying the UV conditions per energy scale
↪ unification scale between $10^2 - 10^4$ TeV

Conclusion

The RGE of the 4321 scalar sector revealed *intriguing* features:

- Laudau poles appear as early as 100 TeV for heavy scalar radial modes
- Radiative EWSB can happen but fine-tuning seems ineluctable
- Couplings can unify in the UV between $10^2 - 10^4$ TeV



Thank you for your attention!

Back-up

U_1, G' and Z' masses and couplings

Gauge bosons masses

$$M_U = \frac{g_4}{2}\sqrt{\omega_1^2 + \omega_3^2}$$

$$M_{G'} = \sqrt{\frac{g_4^2 + g_3^2}{2}}\,\omega_3$$

$$M_{Z'} = \frac{1}{2\sqrt{6}}\sqrt{\left(3g_4^2 + 2g_1^2\right)\left(3\omega_1^2 + \omega_3^2\right)}$$

Gauge bosons couplings

$$g_U = g_4$$

$$g_Y = \frac{g_1 g_4}{\sqrt{g_4^2 + \frac{2}{3}g_1^2}}$$

$$g_{G'} = \sqrt{g_4^2 - g_s^2}$$

$$g_s = \frac{g_4 g_3}{\sqrt{g_4^2 + g_3^2}}$$

$$g_{Z'} = \frac{1}{2\sqrt{6}}\sqrt{g_4^2 - \frac{2}{3}g_Y^2}$$

Radial modes spectrum

$\Omega_{1,3}$ radials

$$\Omega_1^\dagger = \begin{pmatrix} \tilde{T}_1 \\ \frac{\omega_1}{\sqrt{2}} + \tilde{S}_1^* \end{pmatrix} \qquad \Omega_3^\dagger = \begin{pmatrix} \left(\frac{\omega_3}{\sqrt{2}} + \frac{\tilde{S}_3^*}{\sqrt{3}}\right)\mathbb{1}_3 + \tilde{O}_3^{a*}\, t^a \\ \tilde{T}_3^\dagger \end{pmatrix}$$

- $M^2_{O_R} = \omega_3(\rho_2'\omega_3 - \rho_5\omega_1)$ with $\rho_2'\omega_3 > \rho_5\omega_1$
- $M^2_{T_R} = \frac{1}{2}\left(\rho_4 - \rho_5\frac{\omega_3}{\omega_1}\right)(\omega_1^2 + \omega_3^2)$ with $\rho_4\omega_1 > \rho_5\omega_3$
- $M^2_{S_0} = \frac{\rho_5}{2}\frac{\omega_3}{\omega_1}(3\omega_1^2 + \omega_3^2)$ with $\rho_5 > 0$
- $M^2_{S_1} = \frac{1}{2}\left(\rho_1\omega_1^2 + (3\rho_2 + \rho_2')\omega_3^2 + \frac{\rho_5}{2}\frac{\omega_3}{\omega_1}(\omega_1^2 - \omega_3^2)\right) - \frac{u}{2\omega_1}$
- $M^2_{S_2} = \frac{1}{2}\left(\rho_1\omega_1^2 + (3\rho_2 + \rho_2')\omega_3^2 + \frac{\rho_5}{2}\frac{\omega_3}{\omega_1}(\omega_1^2 - \omega_3^2)\right) + \frac{u}{2\omega_1}$

$$u^2 = \left[\rho_1\omega_1^3 + (3\rho_2 + \rho_2')\omega_1\omega_3^2 + \frac{\rho_5}{2}\omega_3(\omega_1^2 - \omega_3^2)\right]^2 - 4\omega_1\omega_3\left[\rho_1\omega_1^3\left(\frac{\rho_5}{2}\omega_1 + (3\rho_2 + \rho_2')\omega_3\right)\right.$$
$$\left. - 3\omega_3\left(\rho_3^2\omega_1^3 + \rho_3\rho_5\omega_3\omega_1^2 + \frac{\rho_5^2}{3}\omega_3^2\omega_1 + \frac{\rho_5}{6}(3\rho_2 + \rho_2')\omega_3^3\right)\right].$$

H, H_2 radials

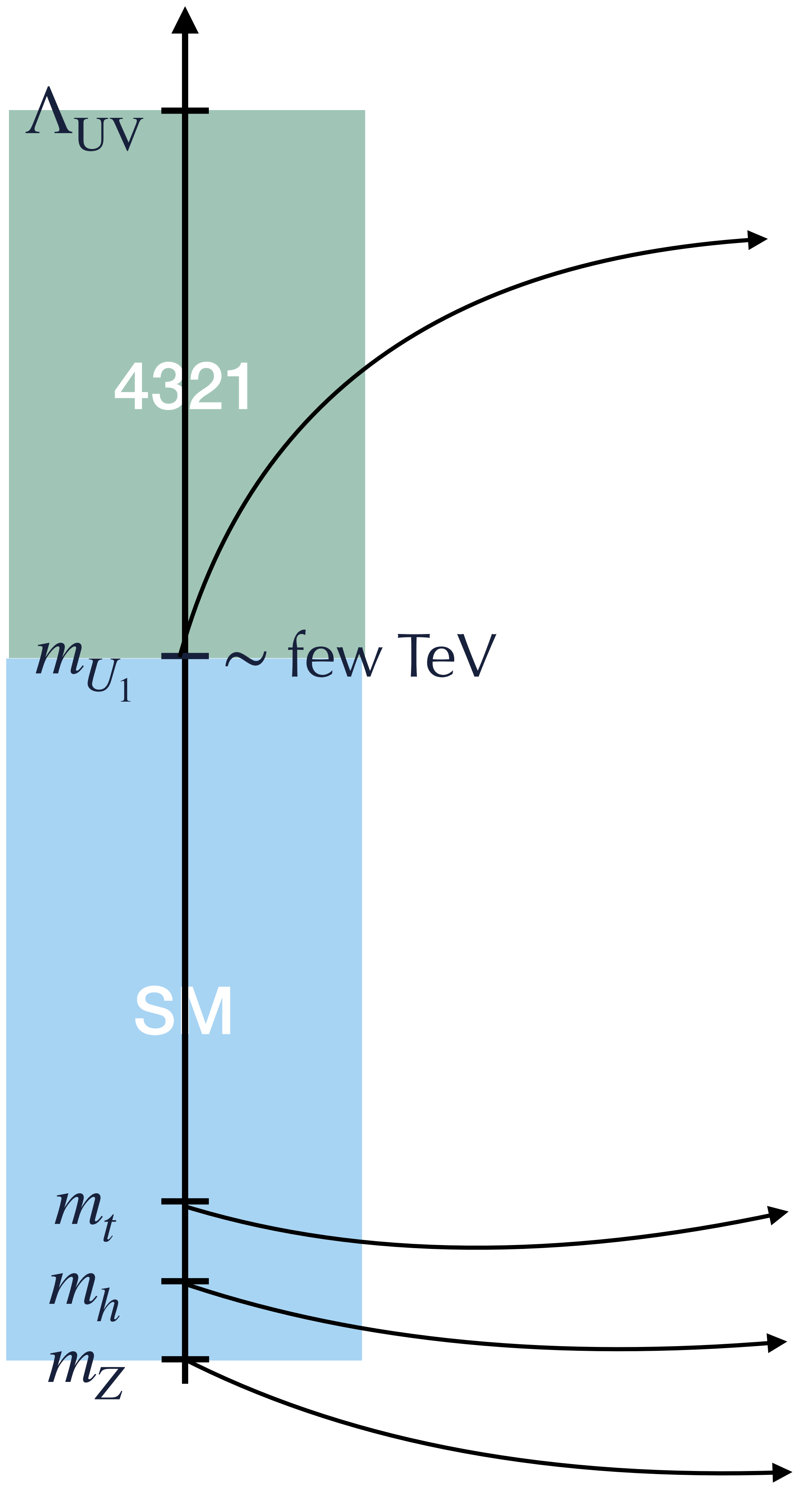
$$H = \begin{pmatrix} \eta_W^+ \\ \frac{v + h}{\sqrt{2}} + i\,\eta_Z \end{pmatrix} \qquad H_2 = \begin{pmatrix} h^+ \\ \frac{1}{\sqrt{2}}(h_R + i\,h_I) \end{pmatrix}$$

$$m_h^2 = \lambda v^2\ ,$$
$$m_\pm^2 = \mu_2^2 + \frac{\lambda_3}{2}v^2\ ,$$
$$m_R^2 = \mu_2^2 + \frac{\lambda_3 + \lambda_4 + \lambda_5}{2}v^2$$
$$m_I^2 = \mu_2^2 + \frac{\lambda_3 + \lambda_4 - \lambda_5}{2}v^2$$

$$V_H \supset \mu_2^2 H_2^\dagger H_2 + \frac{\lambda_2}{2}(H_2^\dagger H_2)^2 + \lambda_3(H^\dagger H)(H_2^\dagger H_2)$$
$$+ \lambda_4(H^\dagger H_2)(H_2^\dagger H) + \frac{\lambda_5}{2}\left((H^\dagger H_2)^2 + \text{h.c}\right)$$

$$V_{\Omega H} \supset \eta_2\, H_2^\dagger H_2\, \Omega_1^\dagger\Omega_1 + \eta_4\, H_2^\dagger H_2\, \mathrm{Tr}[\Omega_3^\dagger\Omega_3]$$

Boundary conditions



- $g_U \approx (1.1 \pm 0.2) \times \left(\frac{M_U}{2\ \text{TeV}}\right) \Leftrightarrow \sqrt{\omega_1^2 + \omega_3^2} \in [3, 4.5]$ TeV for the B anomalies
- $\{\rho_i\}$and $\{\eta_i\}$ from $M_{G'} \gtrsim 4$ TeV and $M_U, M_{Z'} \gtrsim 3$ TeV (high-p_T bounds)

  $M_{O_R}, M_{T_R} \gtrsim 2$ TeV (QCD collider bounds)

  $M_{S_{0,1,2}} \gtrsim 500$ GeV (Higgs collider bounds)
- $\omega_3 > \omega_1$ for mesons mixing
- m_{Ω_3} and m_{Ω_1} from vacuum equation for Ω_3, Ω_1

α_t

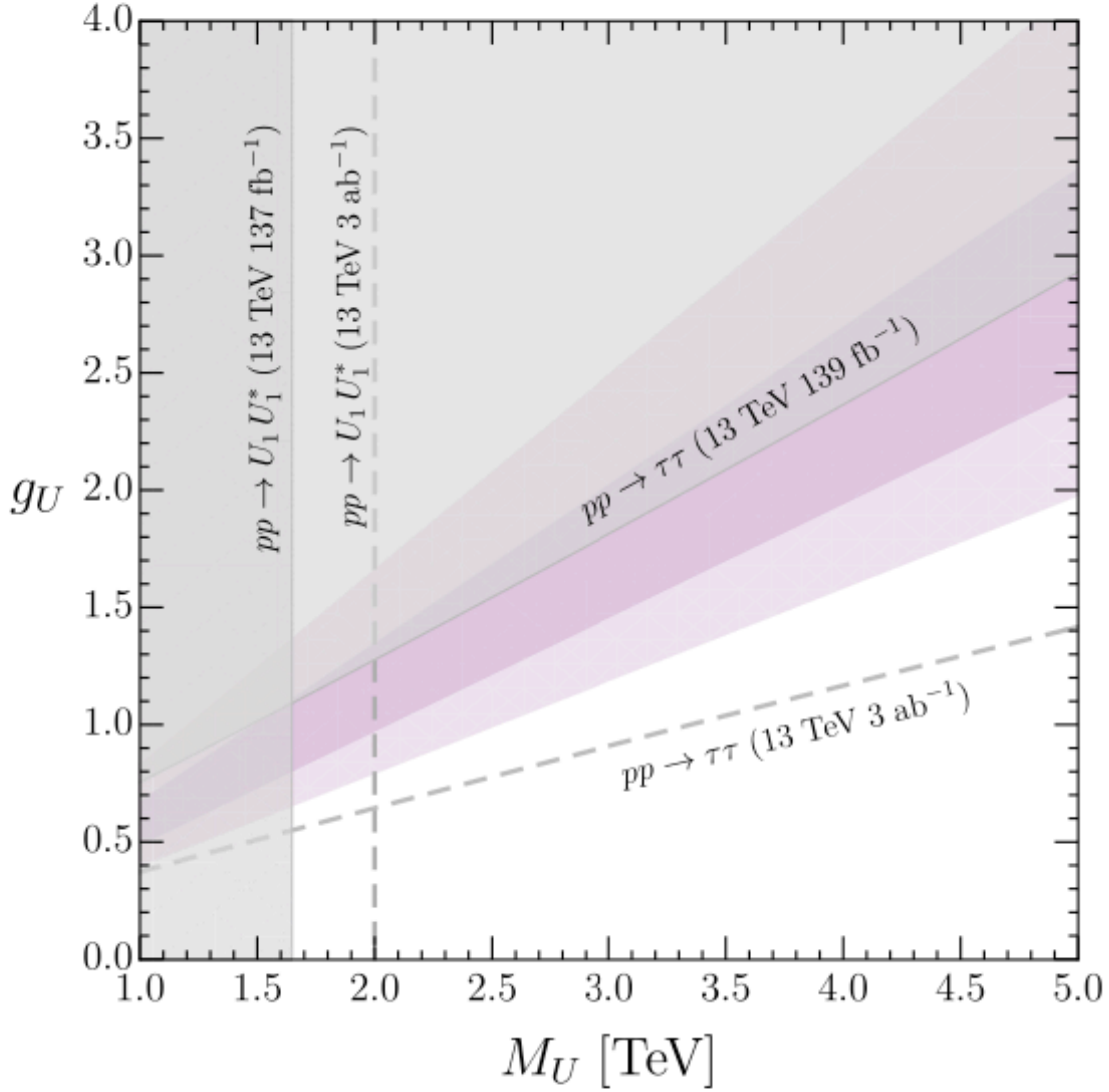
λ and μ_H from vacuum equation for H and physical Higgs mass

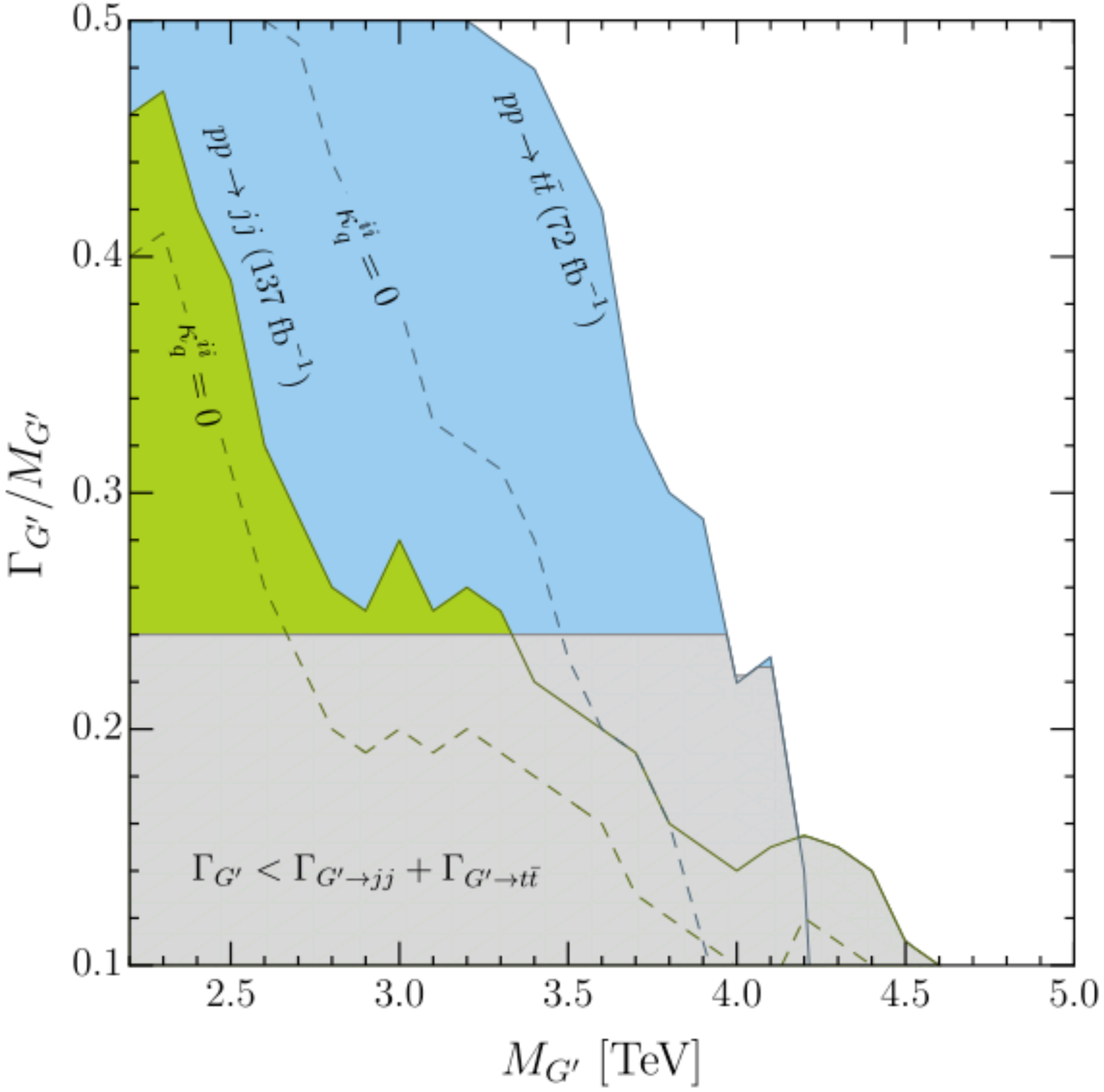
$\alpha_Y, \alpha_2, \alpha_s$

+ Bounded from below conditions

U_1, G' searches

[Cornella, Faroughy, Fuentes-Martín, Isidori, Neubert, 2103.16558]





Benchmark points

Ω VEVs	values	scalar couplings	values
ω_1	1.5 TeV	λ	0.2
ω_3	4 TeV	ρ_1	0.5
mass parameters		ρ_2	0.1
μ_H^2	$(1.6\text{ TeV})^2$	ρ_2'	0.5
$m_{\Omega_1}^2$	$-(1.8\text{ TeV})^2$	ρ_3	0.1
$m_{\Omega_3}^2$	$-(2.5\text{ TeV})^2$	ρ_4	1
gauge coupling		ρ_5	0.01
g_4	3	η_1	-0.1
		η_3	-0.1

Beta-functions - Gauge and Higgs potential

Gauge couplings:

$$\mu\frac{d\alpha_i}{d\mu} \equiv \frac{1}{16\pi^2}\beta_{\alpha_i} = -\frac{\alpha_i^2}{2\pi}b_i$$

$$U(1)': b_1 = -\frac{115}{18}\,, \qquad SU(2)_L : b_2 = \frac{19}{6} - \frac{8}{3}n_{\rm VL}$$

$$SU(3)_{1+2}: b_3 = \frac{23}{3}\,, \qquad SU(4)_3 : b_4 = \frac{38}{3} - \frac{4}{3}n_{\rm VL}$$

Top yukawa:

$$\beta_{y_t} = \left(\frac{11}{2}y_t^2 - \frac{9}{4}g_2^2 - \frac{45}{4}g_4^2 - \frac{3}{4}g_1^2\right)y_t$$

Higgs masses:

$$\beta_{\mu_H^2} = \left(6\lambda + 8y_t^2 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2\right)\mu_H^2 + 8\eta_1 m_{\Omega_1}^2 + 24\eta_3 m_{\Omega_3}^2 + (4\lambda_3 + 2\lambda_4)\mu_2^2$$

$$\beta_{\mu_2^2} = \left(6\lambda_2 + 8y_t^2 - \frac{3}{2}g_1^2 - \frac{9}{2}g_2^2\right)\mu_2^2 + (4\lambda_3 + 2\lambda_4)\mu_H^2 + 8\eta_2 m_{\Omega_1}^2 + 24\eta_4 m_{\Omega_3}^2$$

Higgs quartics:

$$\beta_\lambda = \left(12\lambda + 16y_t^2 - 3g_1^2 - 9g_2^2\right)\lambda + 8\eta_1^2 + 24\eta_3^2 - 16y_t^4 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2)$$
$$+ \frac{3}{4}g_1^4 + \frac{3}{2}g_1^2g_2^2 + \frac{9}{4}g_2^4$$

$$\beta_{\lambda_2} = \left(12\lambda_2 + 16y_t^2 - 3g_1^2 - 9g_2^2\right)\lambda_2 + 8\eta_2^2 + 24\eta_4^2 - 16y_t^4 + 4\lambda_3^2 + 4\lambda_3\lambda_4 + 2\lambda_4^2 + 2\lambda_5^2$$
$$+ \frac{3}{4}g_1^4 + \frac{3}{2}g_1^2g_2^2 + \frac{9}{4}g_2^4$$

$$\beta_{\lambda_3} = \left(4\lambda_3 + 6\lambda + 6\lambda_2 - 3g_1^2 - 9g_2^2 + 16y_t^2\right)\lambda_3 + 2\lambda_4(\lambda + \lambda_2) + 2\lambda_4^2 + 2\lambda_5^2 - 16y_t^4$$
$$+ 8\eta_1\eta_2 + 24\eta_3\eta_4 + \frac{3}{4}g_1^4 - \frac{3}{2}g_1^2g_2^2 + \frac{9}{4}g_2^4$$

$$\beta_{\lambda_4} = \left(4\lambda_4 + 2\lambda + 2\lambda_2 + 8\lambda_3 - 3g_1^2 - 9g_2^2 + 16y_t^2\right)\lambda_4 + 8\lambda_5^2 + 3g_1^2g_2^2 + 16y_t^4$$

$$\beta_{\lambda_5} = \left(2\lambda + 2\lambda_2 + 8\lambda_3 + 12\lambda_4 + 16y_t^2 - 9g_2^2 - 3g_1^2\right)\lambda_5$$

Beta-functions - Ω potential

Ω quartics:

$$\beta_{\rho_1} = \left(16\rho_1 - 3g_1^2 - \frac{45}{2}g_4^2\right)\rho_1 + 24\rho_3^2 + 12\rho_3\rho_4 + 6\rho_4^2 + 4\eta_1^2 + 4\eta_2^2 + \frac{3}{4}g_1^4 + \frac{9}{4}g_1^2 g_4^2 + \frac{99}{16}g_4^4$$

$$\beta_{\rho_2} = \left(32\rho_2 + 28\rho_2' - \frac{1}{3}g_1^2 - 16g_3^2 - \frac{45}{2}g_4^2\right)\rho_2 + 6{\rho_2'}^2 + 8\rho_3^2 + 4\rho_3\rho_4 + 4\rho_5^2 + 4\eta_3^2 + 4\eta_4^2 + \frac{1}{108}g_1^4 + \frac{11}{6}g_3^4 + \frac{27}{16}g_4^4 - \frac{1}{9}g_1^2 g_3^2 - \frac{1}{12}g_1^2 g_4^4 + \frac{13}{2}g_3^2 g_4^2$$

$$\beta_{\rho_2'} = \left(14\rho_2' + 12\rho_2 - \frac{1}{3}g_1^2 - 16g_3^2 - \frac{45}{2}g_4^2\right)\rho_2' + 2\rho_4^2 - 4\rho_5^2 + \frac{5}{2}g_3^4 + \frac{9}{2}g_4^4 + \frac{1}{3}g_1^2 g_3^2 + \frac{1}{3}g_1^2 g_4^2 - \frac{7}{2}g_3^2 g_4^2$$

$$\beta_{\rho_3} = \left(4\rho_3 + 10\rho_1 + 26\rho_2 + 14\rho_2' - \frac{5}{3}g_1^2 - 8g_3^2 - \frac{45}{2}g_4^2\right)\rho_3 + 2\rho_4^2 + 4\rho_5^2 + 2(\rho_1 + 3\rho_2 + \rho_2')\rho_4 + 4\eta_1\eta_3 + 4\eta_2\eta_4 + \frac{1}{12}g_1^4 + \frac{1}{4}g_1^2 g_4^2 + \frac{27}{16}g_4^4$$

$$\beta_{\rho_4} = \left(8\rho_4 + 2\rho_1 + 2\rho_2 + 6\rho_2' + 8\rho_3 - \frac{5}{3}g_1^2 - 8g_3^2 - \frac{45}{2}g_4^2\right)\rho_4 - 4\rho_5^2 - g_1^2 g_4^2 + \frac{9}{2}g_4^4$$

$$\beta_{\rho_5} = \left[6(\rho_2 - \rho_2' + \rho_3 - \rho_4) - g_1^2 - 12g_3^2 - \frac{45}{2}g_4^2\right]\rho_5$$

Ω masses:

$$\beta_{m^2_{\Omega_1}} = \left(10\rho_1 - \frac{3}{2}g_1^2 - \frac{45}{4}g_4^2\right)m^2_{\Omega_1} + (24\rho_3 + 6\rho_4)m^2_{\Omega_3} + 4\eta_1\mu_H^2 + 4\eta_2\mu_2^2$$

$$\beta_{m^2_{\Omega_3}} = \left(26\rho_2 + 14\rho_2' - \frac{1}{6}g_1^2 - 8g_3^2 - \frac{45}{4}g_4^2\right)m^2_{\Omega_3} + 4\eta_3\mu_H^2 + (8\rho_3 + 2\rho_4)m^2_{\Omega_1} + 4\eta_4\mu_2^2$$

Mixing quartics:

$$\beta_{\eta_1} = \left(4\eta_1 + 6\lambda + 10\rho_1 - 3g_1^2 - \frac{9}{2}g_2^2 - \frac{45}{4}g_4^2 + 8y_t^2\right)\eta_1 + 4\lambda_3\eta_2 + 2\lambda_4\eta_2 + 24\rho_3\eta_3 + 6\rho_4\eta_3 + \frac{3}{4}g_1^4$$

$$\beta_{\eta_2} = \left(4\eta_2 + 6\lambda_2 + 10\rho_1 - 3g_1^2 - \frac{9}{2}g_2^2 - \frac{45}{4}g_4^2 + 8y_t^2\right)\eta_2 + 4\lambda_3\eta_1 + 2\lambda_4\eta_1 + 24\rho_3\eta_4 + 6\rho_4\eta_4 + \frac{3}{4}g_1^4$$

$$\beta_{\eta_3} = \left(6\lambda + 26\rho_2 + 14\rho_2' - \frac{5}{3}g_1^2 - \frac{9}{2}g_2^2 - 8g_3^2 - \frac{45}{4}g_4^2 + 8y_t^2\right)\eta_3 + (4\lambda_3 + 2\lambda_4)\eta_4 + (8\rho_3 + 2\rho_4)\eta_1 + 4\eta_3^2 + \frac{1}{12}g_1^4$$

$$\beta_{\eta_4} = 6\lambda_2\eta_4 + (4\lambda_3 + 2\lambda_4)\eta_3 + 2(13\rho_2 + 7\rho_2')\eta_4 + (8\rho_3 + 2\rho_4)\eta_2 + 4\eta_4^2 - \left(\frac{5}{3}g_1^2 + \frac{9}{2}g_2^2 + 8g_3^2 + \frac{45}{4}g_4^2\right)\eta_4 + \frac{1}{12}g_1^4 + 8\eta_4 Y_{12}$$

Bounded from below conditions

V_Ω conditions

$$\rho_1 > 0 \ ,$$
$$\rho_2 + \rho_2' > 0 \ , \quad 3\rho_2 + \rho_2' > 0 \ ,$$
$$\rho_3 > -\sqrt{\rho_1(\rho_2 + \rho_2')} \ , \quad \rho_3 + \rho_4 > -\sqrt{\rho_1(\rho_2 + \rho_2')}$$
$$|\rho_5| < \frac{1}{4}(\rho_1 + 3(3\rho_2 + \rho_2') + 6\rho_3) \ ,$$

$V_{\Omega H}$ conditions

$$\eta_1 > -\sqrt{\lambda \rho_1} \ ,$$
$$\eta_3 > -\min\left[\sqrt{\lambda(\rho_2 + \rho_2')}, \sqrt{\lambda(3\rho_2 + \rho_2')}\right]$$
$$\eta_2 > -\sqrt{\lambda_2 \rho_1}$$
$$\eta_4 > -\min\left[\sqrt{\lambda_2(\rho_2 + \rho_2')}, \sqrt{\lambda_2(3\rho_2 + \rho_2')}\right]$$

$V_{2\mathrm{HDM}}$ conditions

$$\lambda > 0 \ , \qquad \lambda_2 > 0 \ , \qquad \lambda_3 > -\sqrt{\lambda\,\lambda_2} \ , \qquad \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda\,\lambda_2}$$

Effective quantities for EWSB

Potential minimization

$$\left.\frac{\partial V(s_1, s_3, h)}{\partial h_i}\right|_{\langle h\rangle, \langle s_1\rangle, \langle s_3\rangle} = 0 \quad \text{for } h_i = h, s_1, s_3$$

$$[M^2_{s_3,s_1,h}]_{ij} = \frac{\partial^2 V}{\partial h_i \partial h_j} \text{ with } h_i \in (s_3, s_1, h)$$

positive-definite:

$$M^2_{s_3,s_1,h} = \begin{pmatrix} 2(3\rho_2 + \rho_2')\omega_3^2 & 2\sqrt{3}\rho_3\omega_1\omega_3 & \sqrt{6}\eta_3 v\omega_3 \,, \\ 2\sqrt{3}\rho_3\omega_1\omega_3 & 2\rho_1\omega_1^2 & \sqrt{2}\eta_1 v\omega_1 \,, \\ \sqrt{6}\eta_3 v\omega_3 & \sqrt{2}\eta_1 v\omega_1 & \lambda v^2 \end{pmatrix}$$

$$D_1 = 3\rho_2 + \rho_2' > 0$$
$$D_{12} = \rho_1 D_1 - 3\rho_3^2 > 0$$
$$D_{123} = \lambda D_{12} - (3\eta_3^2\rho_1 - 6\eta_1\eta_3\rho_3 + \eta_1^2 D_1) > 0$$

Effective mass and quartic coupling:

$$\mu^2_{\text{eff}} = -\frac{\lambda_{\text{eff}}}{2}v^2 = \mu_H^2 - \frac{\eta_1 D_1 - 3\eta_3\rho_3}{D_{12}} m^2_{\Omega_1} - 3\frac{\eta_3\rho_1 - \eta_1\rho_3}{D_{12}} m^2_{\Omega_3}$$

$$m^2_{h_{\text{phys}}} = \lambda_{\text{eff}}\, v^2 = \frac{D_{123}}{D_{12}} v^2 + O\left(\frac{v^2}{\omega^2_{1,3}}\right)$$