Method of Moments for $\Lambda_b^0 \to pK^-\ell^+\ell^-$

Flavour at the Crossroads 26/04/2022

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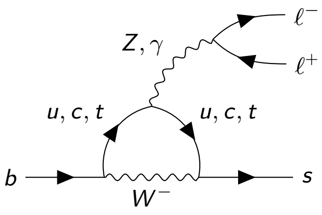
What are we interested in?

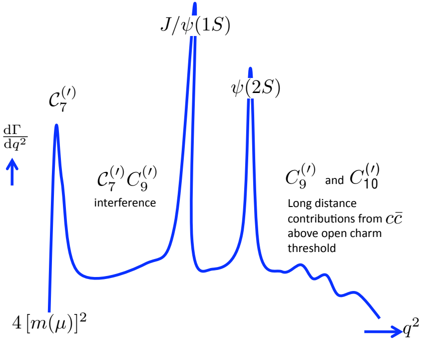
$\Lambda_b^0 \to \Lambda^* \ell^+ \ell^- \quad \leftrightarrow \quad b \to s \ell^+ \ell^-$ FCNC

In the SM: electroweak penguin decay



Angular measurement available for the transition via the weakly-decaying ground state:

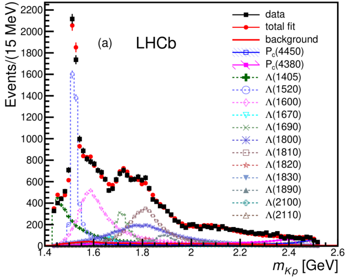
$$\Lambda_b^0 \to \Lambda^0 (\to p \pi^-) \mu^+ \mu^-$$ LHCb-PAPER-2015-009



Effective Hamiltonian described by the Wilson coefficients:
$\mathcal{C}_7$: how much tensor current (photon-like)?
$\mathcal{C}_9$: how much vector current $\bar{u}\gamma_\mu v$?
$\mathcal{C}_{10}$: how much axialvector current $\bar{u}\gamma_\mu\gamma_5 v$?

The Λ^* jungle

Strong Λ^* decay to pK^- is easiest to reconstruct
$\Rightarrow$ many overlapping states
Full fit to all states possible for $\Lambda_b^0 \to pK^- J/\psi$ and $\Lambda_b^0 \to pK^- \gamma$



$\Lambda_b^0 \to pK^- J/\psi\,(\to \mu^+\mu^-)$

LHCb-PAPER-2015-029

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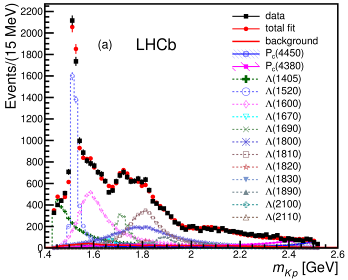
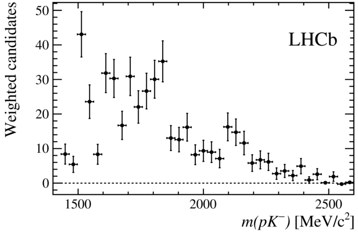


$\Lambda_b^0 \to pK^- J/\psi\,(\to \mu^+\mu^-)$

LHCb-PAPER-2015-029

Not yet enough data at non-resonant q^2 in $\Lambda_b^0 \to pK^-\ell^+\ell^-$

a) focus on one resonance: $\Lambda_b^0 \to \Lambda^*(1520)\mu^+\mu^-$ angular fit (see Felicia's talk)
b) avoid fitting: moment analysis in $\Lambda_b^0 \to pK^-\ell^+\ell^-$



$\Lambda_b^0 \to pK^-\mu^+\mu^-$

Suppl. material to LHCb-PAPER-2019-040

Method of moments + orthonormalization arXiv:1503.04100

1) Find angular basis $f_i(\Omega)$:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2\mathrm{d}m_{pK}\mathrm{d}\Omega} = \sum_i K_i(q^2, m_{pK}) f_i(\Omega)$$

$\rightarrow$ orthonormal bases have nicer properties than arbitrary ones

2) Find weighting functions $w_j(\Omega)$ orthonormal to the basis:

$$\int_\Omega f_i(\Omega) w_j(\Omega)\,\mathrm{d}\Omega = \delta_{ij}$$

3) Enjoy your fit-free measurement:

$$K_j(q^2, m_{pK}) = \int_\Omega \underbrace{\sum_i K_i(q^2, m_{pK}) f_i(\Omega)}_{\mathrm{d}\Gamma/\mathrm{d}q^2\mathrm{d}m_{pK}\mathrm{d}\Omega} w_j(\Omega)\,\mathrm{d}\Omega = \sum_{\substack{\text{data}\\ \text{points } n}} w_j(\Omega_n)$$

- 👍 model-independent observables
- 👍 robust independently of the size of the dataset
- 👍 well-defined statistical properties
- 👎 10-30% larger uncertainties compared to a (good) fit

Final aim of ongoing LHCb analysis: measure K_i in bins of m_{pK} and q^2

What does the decay rate look like?

Generic decay rate for 4-body $\Lambda_b^0 \to pK^-\ell^+\ell^-$ phase space

$$\mathrm{d}\Gamma = \frac{\overline{|\mathcal{M}|}^2}{m_{\Lambda_b^0}^2} \frac{1}{2^6(2\pi)^7} \frac{p_\Lambda^{\Lambda_b^0} p_p^{pK} p_\ell^{\ell\ell}}{\sqrt{q^2}} \underbrace{\mathrm{d}\cos\theta \mathrm{d}\cos\theta_p \mathrm{d}\phi_p \mathrm{d}\cos\theta_\ell \mathrm{d}\phi_\ell}_{\mathrm{d}\Omega} \mathrm{d}m_{pK} \mathrm{d}q^2$$

Matrix element

$$\overline{|\mathcal{M}|}^2 = \sum_\lambda P_\lambda \sum_{\lambda_{\bar{\ell}}, \lambda_\ell, \lambda_p} \left| \sum_{\Lambda, \lambda_\Lambda} \mathcal{M}^{\Lambda, \lambda_\Lambda}_{\lambda, \lambda_p, \lambda_{\bar{\ell}}, \lambda_\ell} \right|^2$$

Indices $=$ helicities

$P_+ + P_- = 1 \qquad P_+ - P_- = \; \Lambda_b^0$ polarization

Matrix element from helicity formalism

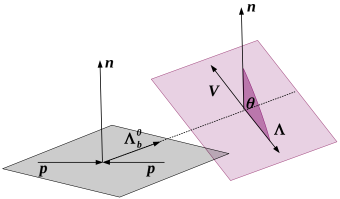
Helicity formalism

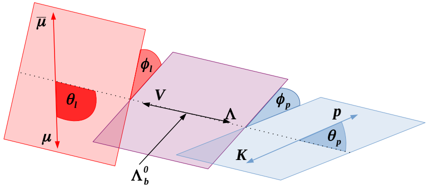
Calculate amplitude for $\Lambda_b^0 \to \Lambda^* V$, $\Lambda^* \to pK^-$, and $V \to \ell\ell$ in convenient systems
Connect systems via Wigner D-matrix elements (rotates spin bases)

Matrix element

$$\mathcal{M}^{\Lambda,\lambda_\Lambda}_{\lambda,\lambda_p,\lambda_{\bar{\ell}},\lambda_\ell}(q^2, m_{pK}, \Omega) = \underbrace{e^{i\alpha_\Lambda}}_{\text{global phase}} \times \underbrace{D^{J_{\Lambda_b^0}\,*}_{\lambda,\lambda_\Lambda-\lambda_{\ell\ell}}(0,\theta,0)}_{\text{polarization angle}} \times \underbrace{H^{m_{\ell\ell},J_{\ell\ell},J_\Lambda,P_\Lambda}_{\lambda_\Lambda,\lambda_{\ell\ell}}(q^2, m_{pK})}_{\Lambda_b^0\to\Lambda^* V\text{ amplitude}}$$

$$\times \underbrace{h^{m_{\ell\ell},J_{\ell\ell}}_{\lambda_{\bar{\ell}},\lambda_\ell}(q^2)}_{\text{leptonic amplitude}} \underbrace{D^{J_{\ell\ell}*}_{\lambda_{\ell\ell},\lambda_{\bar{\ell}}-\lambda_\ell}(\phi_\ell,\theta_\ell,-\phi_\ell)}_{\text{leptonic angles}} \times \underbrace{h^{\Lambda}_{\lambda_p,0}(m_{pK})}_{\text{hadronic amplitude}} \underbrace{D^{J_\Lambda *}_{\lambda_\Lambda,\lambda_p}(\phi_p,\theta_p,-\phi_p)}_{\text{hadronic angles}}$$





Helicity amplitudes

$$\mathcal{M}_\Lambda(q^2, m_{pK}, \Omega) = \overset{\text{global phase}}{e^{i\alpha_\Lambda}} \times \overset{\text{polarization angle}}{D^{J_{\Lambda_b^0}\,*}_{\lambda,\lambda_\Lambda-\lambda_{\ell\ell}}(0,\theta,0)} \times \overset{\Lambda_b^0\to\Lambda^* V\text{ amplitude}}{H^{m_{\ell\ell},J_{\ell\ell},J_\Lambda,P_\Lambda}_{\lambda_\Lambda,\lambda_{\ell\ell}}(q^2, m_{pK})}$$

$$\times \underbrace{h^{m_{\ell\ell},J_{\ell\ell}}_{\lambda_{\bar\ell},\lambda_\ell}(q^2)}_{\text{leptonic amplitude}} \; \underbrace{D^{J_{\ell\ell}\,*}_{\lambda_{\ell\ell},\lambda_{\bar\ell}-\lambda_\ell}(\phi_\ell,\theta_\ell,-\phi_\ell)}_{\text{leptonic angles}} \times \underbrace{h^\Lambda_{\lambda_p,0}(m_{pK})}_{\text{hadronic amplitude}} \; \underbrace{D^{J_\Lambda\,*}_{\lambda_\Lambda,\lambda_p}(\phi_p,\theta_p,-\phi_p)}_{\text{hadronic angles}}$$

Lepton side: $V \to \ell\ell$

$$h^V(q^2) \propto \bar u(\lambda_\ell)\gamma^\mu v(\lambda_{\bar\ell})$$

$$h^A(q^2) \propto \bar u(\lambda_\ell)\gamma^\mu\gamma_5 v(\lambda_{\bar\ell})$$

Hadron side: $\Lambda^* \to pK^-$

$$h^\Lambda_{\lambda_p}(m_{pK}) = \pm\frac{1}{m_\Lambda^2 - m_{pK}^2 - i m_\Lambda \Gamma(m_{pK})} \times g_{\text{QCD}}(m_{pK})(p_p^{pK})^\mu \; \bar u(\lambda_p)\gamma_5 u_\mu(\lambda_\Lambda)$$

Helicity amplitudes

$$\mathcal{M}_\Lambda(q^2, m_{pK}, \Omega) = \overset{\text{global phase}}{e^{i\alpha_\Lambda}} \times \overset{\text{polarization angle}}{D^{J_{\Lambda_b^0}\,*}_{\lambda,\lambda_\Lambda-\lambda_{\ell\ell}}(0,\theta,0)} \times \overset{\Lambda_b^0\to\Lambda^* V \text{ amplitude}}{H^{m_{\ell\ell},J_{\ell\ell},J_\Lambda,P_\Lambda}_{\lambda_\Lambda,\lambda_{\ell\ell}}(q^2, m_{pK})}$$

$$\times \underbrace{h^{m_{\ell\ell},J_{\ell\ell}}_{\lambda_{\bar\ell},\lambda_\ell}(q^2)}_{\text{leptonic amplitude}} \underbrace{D^{J_{\ell\ell}\,*}_{\lambda_{\ell\ell},\lambda_{\bar\ell}-\lambda_\ell}(\phi_\ell,\theta_\ell,-\phi_\ell)}_{\text{leptonic angles}} \times \underbrace{h^{\Lambda}_{\lambda_p,0}(m_{pK})}_{\text{hadronic amplitude}} \underbrace{D^{J_\Lambda\,*}_{\lambda_\Lambda,\lambda_p}(\phi_p,\theta_p,-\phi_p)}_{\text{hadronic angles}}$$

Λ_b^0 decay

$$H^V \propto -\frac{2m_b}{q^2}\Big(\mathcal{C}_7 \langle\Lambda^*|\,\bar s i\sigma_{\mu\nu}q^\nu P_R b\,|\Lambda_b^0\rangle + \mathcal{C}_{7'} \langle\Lambda^*|\,\bar s i\sigma_{\mu\nu}q^\nu P_L b\,|\Lambda_b^0\rangle \Big) \quad \text{tensor } (\to \text{vector})$$

$$+ \; \mathcal{C}_9 \langle\Lambda^*|\,\bar s\gamma_\mu P_L b\,|\Lambda_b^0\rangle + \mathcal{C}_{9'} \langle\Lambda^*|\,\bar s\gamma_\mu P_R b\,|\Lambda_b^0\rangle \quad \text{vector}$$

$$H^A \propto \mathcal{C}_{10} \langle\Lambda^*|\,\bar s\gamma_\mu P_L b\,|\Lambda_b^0\rangle + \mathcal{C}_{10'} \langle\Lambda^*|\,\bar s\gamma_\mu P_R b\,|\Lambda_b^0\rangle \quad \text{axialvector}$$

Quark model form factor predictions available for some Λ^*, given in the form: arxiv:1506.04106

$$\langle\Lambda^*|\,\bar s\Gamma_\mu b\,|\Lambda_b^0\rangle \propto \bar u(\Lambda)\left[F_1^\Gamma(q^2)\gamma_\mu + F_2^\Gamma(q^2)v_\mu + F_3^\Gamma(q^2)v'_\mu\right] u(\Lambda_b^0)$$

F_i^Γ: form factors, $v_\mu^{(\prime)}$: velocity of Λ_b^0 (Λ^*)

The unpredictable global phase

$$\mathcal{M}_\Lambda(q^2, m_{pK}, \Omega) = \underset{\text{global phase}}{e^{i\alpha_\Lambda}} \times \overset{\text{polarization angle}}{D^{J_{\Lambda_b^0}\,*}_{\lambda,\lambda_\Lambda-\lambda_{\ell\ell}}(0,\theta,0)} \times \overset{\Lambda_b^0\to\Lambda^* V \text{ amplitude}}{H^{m_{\ell\ell},J_{\ell\ell},J_\Lambda,P_\Lambda}_{\lambda_\Lambda,\lambda_{\ell\ell}}(q^2, m_{pK})}$$

$$\times \underbrace{h^{m_{\ell\ell},J_{\ell\ell}}_{\lambda_{\bar\ell},\lambda_\ell}(q^2)}_{\text{leptonic amplitude}} \underbrace{D^{J_{\ell\ell}\,*}_{\lambda_{\ell\ell},\lambda_{\bar\ell}-\lambda_\ell}(\phi_\ell,\theta_\ell,-\phi_\ell)}_{\text{leptonic angles}} \times \underbrace{h^\Lambda_{\lambda_p,0}(m_{pK})}_{\text{hadronic amplitude}} \underbrace{D^{J_\Lambda\,*}_{\lambda_\Lambda,\lambda_p}(\phi_p,\theta_p,-\phi_p)}_{\text{hadronic angles}}$$

Single states $\mathcal{M}_\Lambda = e^{i\alpha} H_\Lambda \Rightarrow \mathcal{M}_\Lambda \mathcal{M}_\Lambda^* = H_\Lambda H_\Lambda^*$

Several states $\mathcal{M}_{\Lambda_1}\mathcal{M}_{\Lambda_2}^* = H_{\Lambda_1} H_{\Lambda_2}^* \underbrace{e^{i(\alpha_1-\alpha_2)}}_{\text{generally } \neq 1}$

The unpredictable global phase

$$\mathcal{M}_\Lambda(q^2, m_{pK}, \Omega) = \underset{\text{global phase}}{e^{i\alpha_\Lambda}} \times \underset{\text{polarization angle}}{D^{J_{\Lambda_b^0}\,*}_{\lambda,\lambda_\Lambda-\lambda_{\ell\ell}}(0,\theta,0)} \times \underset{\Lambda_b^0\to\Lambda^* V \text{ amplitude}}{H^{m_{\ell\ell},J_{\ell\ell},J_\Lambda,P_\Lambda}_{\lambda_\Lambda,\lambda_{\ell\ell}}(q^2, m_{pK})}$$

$$\times \underset{\text{leptonic amplitude}}{h^{m_{\ell\ell},J_{\ell\ell}}_{\lambda_{\bar\ell},\lambda_\ell}(q^2)} \; \underset{\text{leptonic angles}}{D^{J_{\ell\ell}\,*}_{\lambda_{\ell\ell},\lambda_{\bar\ell}-\lambda_\ell}(\phi_\ell,\theta_\ell,-\phi_\ell)} \times \underset{\text{hadronic amplitude}}{h^\Lambda_{\lambda_p,0}(m_{pK})} \; \underset{\text{hadronic angles}}{D^{J_\Lambda\,*}_{\lambda_\Lambda,\lambda_p}(\phi_p,\theta_p,-\phi_p)}$$

Single states $\quad \mathcal{M}_\Lambda = e^{i\alpha} H_\Lambda \Rightarrow \mathcal{M}_\Lambda \mathcal{M}^*_\Lambda = H_\Lambda H^*_\Lambda$

Several states $\quad \mathcal{M}_{\Lambda_1}\mathcal{M}^*_{\Lambda_2} = H_{\Lambda_1} H^*_{\Lambda_2} \underbrace{e^{i(\alpha_1-\alpha_2)}}_{\text{generally } \neq 1}$

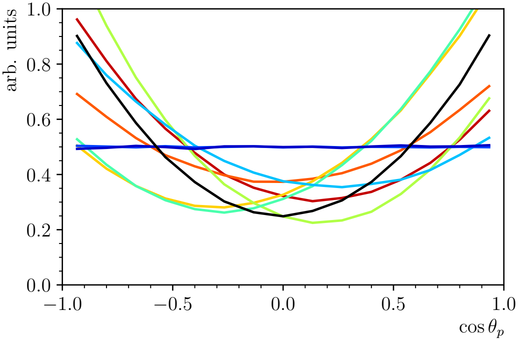
Example

Black: individual $\Lambda(1520)$
Dark blue: individual $\Lambda(1405)$, $\Lambda(1600)$

Other colours: $\underset{3/2^-}{\Lambda(1520)} + \underset{1/2^-}{\Lambda(1405)} + \underset{1/2^+}{\Lambda(1600)}$

0) no relative phases (all $\alpha_i = 0$)
1-5) randomly chosen phases

Different quantum numbers
$\Rightarrow$ interference effects visible in $\cos\theta_p$



Effect of the global phases on angular observables

Example: moments $K_{2,5,8}$, coming with $\cos\theta_\ell$

$w_2(\Omega) = \overbrace{\mathcal{P}_0(\cos\theta_p)}^{1} \cos\theta_\ell$

Independent of interferences and phases

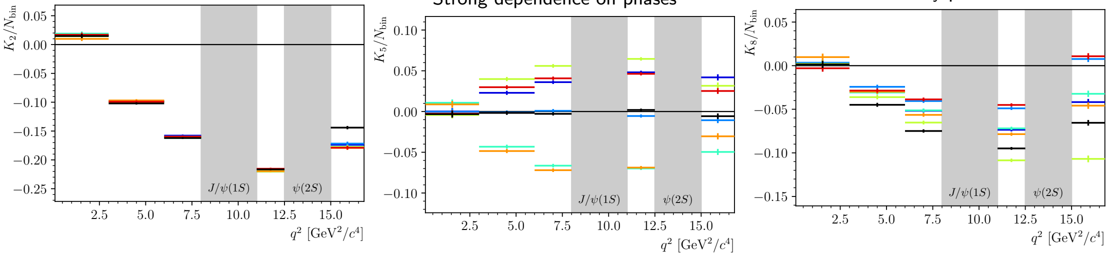
$w_5(\Omega) = \overbrace{\mathcal{P}_1(\cos\theta_p)}^{\cos\theta_p} \cos\theta_\ell$

Only arise with interferences

Strong dependence on phases

$w_8(\Omega) = \overbrace{\mathcal{P}_2(\cos\theta_p)}^{(3\cos^2\theta_p - 1)/2} \cos\theta_\ell$

Affected by phases



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Other colours: $\underset{3/2^-}{\Lambda(1520)} + \underset{1/2^-}{\Lambda(1405)} + \underset{1/2^+}{\Lambda(1600)}$

0) no relative phases (all $\alpha_i = 0$)

1-5) randomly chosen phases

New Physics in a realistic pK spectrum

Take all resonances from $\Lambda_b^0 \to pK^- J/\psi$ with FF predictions: $\frac{1}{2}^\pm, \frac{3}{2}^\pm, \frac{5}{2}^+$

Use PDG values for $\mathcal{B}(\Lambda^* \to pK^-)$

Generate samples with different Wilson coefficients and Λ_b^0 polarizations

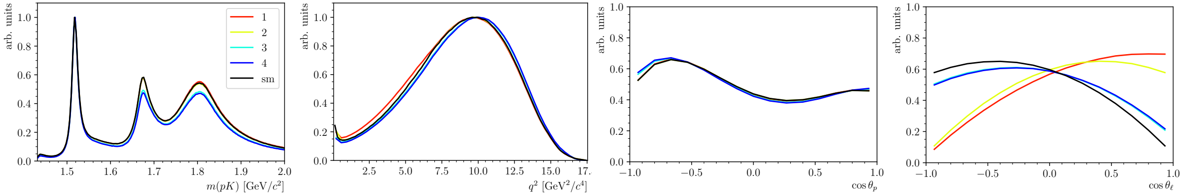
1) $\mathcal{C}_9 = -\mathcal{C}_9^{\text{SM}}$

2) $\mathcal{C}_{10} = -\mathcal{C}_{10}^{\text{SM}}$

3) $\mathcal{C}_{9'} = -\mathcal{C}_9^{\text{SM}}$

4) $\mathcal{C}_{10'} = -\mathcal{C}_{10}^{\text{SM}}$

Λ_b^0 polarization: 0 or 1 (plots show 0)

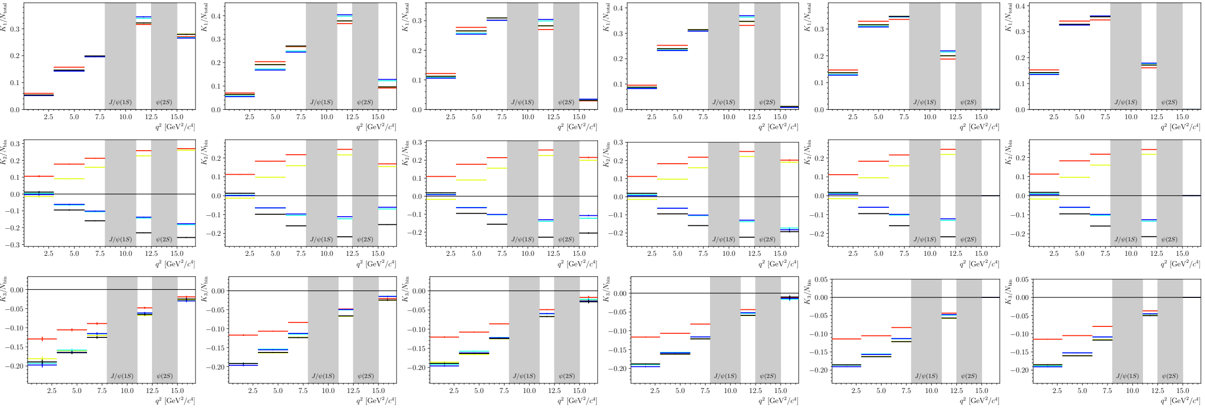


The distributions here are normalized and NOT to scale wrt. each other. The phases are all set to 0.

$\Rightarrow$ changes in the Wilson coefficients mainly affect $\cos\theta_\ell$

Three classic observables, $\frac{d\Gamma}{dq^2}$, A_{FB}^{ℓ}, F_i, in bins of m_{pK}

$m_{pK} \in [1.40, 1.50]$ $m_{pK} \in [1.50, 1.55]$ $m_{pK} \in [1.55, 1.65]$ $m_{pK} \in [1.65, 1.75]$ $m_{pK} \in [1.75, 1.85]$ $m_{pK} \in [1.85, 2.00]$



K_1 comes with angular function $w_1(\Omega) \propto 1 \Rightarrow d\Gamma/dq^2$
K_2 comes with angular function $w_2(\Omega) \propto \mathcal{P}_1(\cos\theta_\ell) \Rightarrow A_{FB}^{\ell}$
K_3 comes with angular function $w_3(\Omega) \propto \mathcal{P}_2(\cos\theta_\ell) \Rightarrow$ lepton-polarizations F_i

Which other moments are sensitive to NP?

It's complicated

50 moments $\times$ 6 m_{pK} bins $\times$ 5 q^2 bins $=$ 1500 potential observables only for unpolarized Λ_b^0!
(198 moments for polarized Λ_b^0, even more for $J_\Lambda > 5/2$)
Analytical expressions become huge and very difficult to penetrate.

Investigation strategy ideas are very welcome!

Trends we identified

- moments with $w(\Omega) \propto \cos\theta_\ell$ show sensitivity to $\mathcal{C}_9$ or $\mathcal{C}_{10}$ (think A_{FB}^{ℓ})
- moments with $w(\Omega) \propto \mathcal{P}_2(\cos\theta_\ell)$ show sensitivity to $\mathcal{C}_9$, especially at low q^2 $\rightarrow$ possibly $\mathcal{C}_9 - \mathcal{C}_7$ interference (think P_5')
- moments with $w(\Omega) \propto \cos\theta\mathcal{P}_2(\cos\theta_\ell)$ show sensitivity to $\mathcal{C}_{9'}$ and $\mathcal{C}_{10'}$
- moments with $w(\Omega) \propto \sin\theta_p \sin\theta_\ell$ may be particularly useful experimentally: NP models change magnitude AND shape across q^2 in all m_{pK} bins

A particularly nice set of moments

Moments 31, 32: change shape across q^2

Angular functions:

$$w_{31}(\Omega) \propto \cos\theta_\ell \sin\theta_\ell \cos\theta_p \sin\theta_p \sin(\varphi_\ell + \varphi_p)$$

$$w_{32}(\Omega) \propto \sin\theta_\ell \cos\theta_p \sin\theta_p \sin(\varphi_\ell + \varphi_p)$$

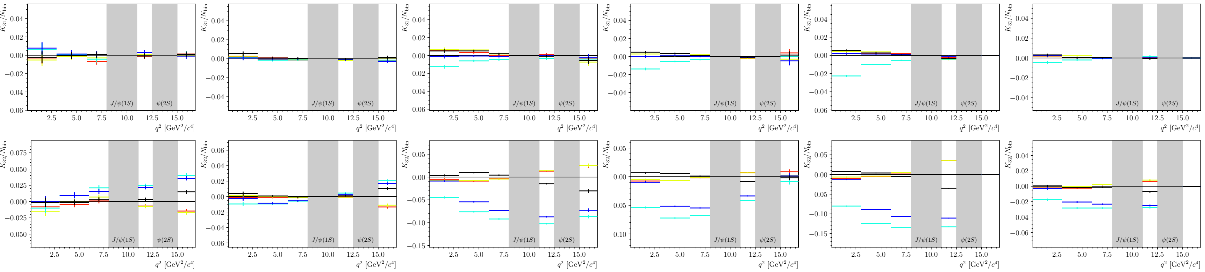
1) $\mathcal{C}_9 = -\mathcal{C}_9^{SM}$

2) $\mathcal{C}_{10} = -\mathcal{C}_{10}^{SM}$

3) $\mathcal{C}_{9'} = -\mathcal{C}_9^{SM}$

4) $\mathcal{C}_{10'} = -\mathcal{C}_{10}^{SM}$

$m_{pK} \in [1.40, 1.50]$ $m_{pK} \in [1.50, 1.55]$ $m_{pK} \in [1.55, 1.65]$ $m_{pK} \in [1.65, 1.75]$ $m_{pK} \in [1.75, 1.85]$ $m_{pK} \in [1.85, 2.00]$



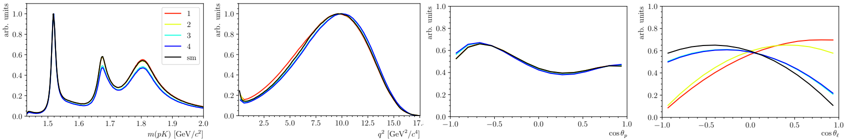
Wilson coefficients change the shape of the distributions along q^2.

What about the interferences?

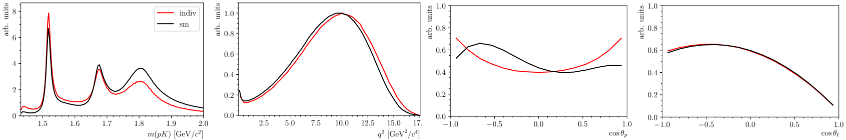
Global phases in the interferences affect the moments (see previous slides and Felicia's talk)

Could they also be useful New Physics observables?
$\Rightarrow$ check by generating individual resonances with the exact same settings and then mix them with correct ratios (ie. no interferences at all)

Different NP scenarios
(same plots as before)
$\Rightarrow$ large effect on $\cos\theta_\ell$



Mix with interferences
vs. sum of individual states
$\Rightarrow$ strong effect on $\cos\theta_p$

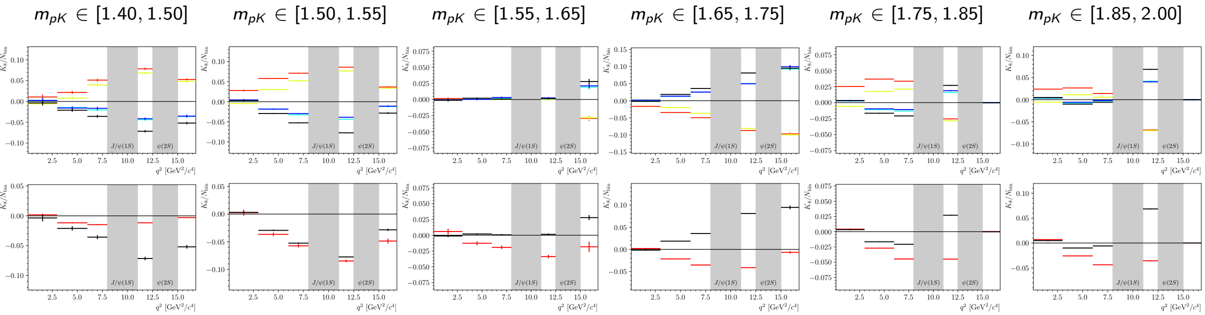


Interference or New Physics?

Example: K_8 comes with angular function $w_8(\Omega) \propto \mathcal{P}_2(\cos\theta_p)\cos\theta_\ell$

K_8 is non-zero in the SM for $J \geq \frac{3}{2}$ (black lines in plots)

Sensitive to NP due to $\cos\theta_\ell$ dependence (see top row plots: different colours = different NP scenarios)
Sensitive to the interference terms
(see bottom plots: mix with/without interferences, see prev. slides and Felicia's talk)



How can we distinguish boring phases from interesting New Physics?

Summary

Derived the decay rate for $\Lambda_b^0 \to pK^-\ell^+\ell^-$ for a realistic pK^- spectrum

Created a 7D $(m_{pK},\ q^2,\ \theta, \theta_p, \theta_\ell, \phi_p, \phi_\ell)$ MC generator for $\Lambda_b^0 \to pK^-\ell^+\ell^-$

Studied sensitivity to New Physics
- identified moments that are sensitive to different (combinations of) Wilson coefficients
- interference effects dilute NP sensitivity

Plans and WIP
- Create complete documentation (and make it public)
- Add option to include efficiencies in the generator
- Make generator publicly available
- Measure the moments in LHCb data

Input and ideas are very welcome!

Appendix: the orthonormal basis