Neutral-Current *B* anomalies

Status and New Developments

Javier Virto

Universitat de Barcelona

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Rare *B* Decays:

- FCNCs (leptonic, rare semileptonic, rare hadronic)
- Lepton-Universality-Violating observables
- Lepton-Flavor-Violating modes
- Lepton-Number-Violating modes

Strong suppression of these decays in the SM $\Rightarrow$ Smoking guns of NP

BUT: FCNCs are no longer "rare" at the LHC!

e.g. $N^{LHCb}_{events}(Run\,1) = 2398$; $N^{LHCb}_{events}(2016) = 2187$ for $B \to K^* \mu\mu$

And will become "common" decays at **Belle-2** and **LHCb Upgrade II**.

$\to$ Opportunity to study **BOTH** *New Physics* and *QCD* in rare decays.

B mesons mix and decay due to $\mathcal{L}_{Weak} + \mathcal{L}_{BSM?}$

For $m_B \ll M_W, M_{BSM}$ we use an EFT : $\mathcal{L}_{EFT} = \mathcal{L}_{QCD+QED} + \sum_i C_i \, \mathcal{O}_i$

Class	Flavour structure	Number of Ops.	Other flavours	ADM	Example process
Class I	$\bar{s}b\,\bar{s}b$	5+3	$\bar{d}b\,\bar{d}b$	$\hat{\gamma}_{\rm I}$	$B_q - \overline{B}_q$ mixing
Class II	$\bar{u}b\,\bar{\ell}\nu_{\ell'}$	$(2+3)\times 9$	$\bar{c}b\,\bar{\ell}\nu_{\ell'}$	$\hat{\gamma}_{\rm II}$	$\overline{B}_d \to \pi^+\mu^-\bar{\nu}$
Class III	$\bar{s}b\,\bar{u}c$	10+10	$\bar{s}b\,\bar{c}u$ $\bar{d}b\,\bar{u}c$ $\bar{d}b\,\bar{c}u$	$\hat{\gamma}_{\rm III}$	$B^- \to \overline{D}^0 K^-$
Class IV	$\bar{s}b\,\bar{s}d$	5+5	$\bar{d}b\,\bar{d}s$ $\bar{b}s\,\bar{b}d$	$\hat{\gamma}_{\rm IV}$	$B^- \to \overline{K}^0 K^-$
Class V	$\bar{s}b\,\bar{q}q$ $\bar{s}b\,F$, $\bar{s}b\,G$ $\bar{s}b\,\bar{\ell}\ell$	57+57	$\bar{d}b\,\bar{q}q$ $\bar{d}b\,F$, $\bar{d}b\,G$ $\bar{d}b\,\bar{\ell}\ell$	$\hat{\gamma}_{\rm V}$	$\overline{B}_d \to D^+ D_s^-$ $\overline{B}_d \to X_s\gamma$ $B^- \to K^-\mu^+\mu^-$
Class Vb	$\bar{s}b\,\bar{\ell}\ell'$, $\ell \neq \ell'$	$(5+5)\times 6$	$\bar{d}b\,\bar{\ell}\ell'$	$\hat{\gamma}_{\rm Vb}$	$\overline{B}_s \to \mu^-\tau^+$
Class Vν	$\bar{s}b\,\bar{\nu}_\ell\nu_{\ell'}$	$(1+1)\times 9$	$\bar{d}b\,\bar{\nu}_\ell\nu_{\ell'}$	zero	$B^- \to K^-\bar{\nu}\nu$

Aebischer, Fael, Greub, Virto 2017

Relevant part of the W$_{\text{eak}}$E$_{\text{ffective}}$T$_{\text{theory}}$ for $b \to s\ell\ell$ transitions:

$$\mathcal{L}_W = \mathcal{L}_{\rm QCD} + \mathcal{L}_{\rm QED} + \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^\star \sum_i \mathcal{C}_i(\mu)\, \mathcal{O}_i(\mu)$$

$$\mathcal{O}_1 = (\bar{c}\gamma_\mu P_L b)(\bar{s}\gamma^\mu P_L c) \qquad \mathcal{O}_2 = (\bar{c}\gamma_\mu P_L T^a b)(\bar{s}\gamma^\mu P_L T^a c)$$

$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_R b) F^{\mu\nu} \qquad \mathcal{O}_{7'} = \frac{e}{16\pi^2} m_b (\bar{s}\sigma_{\mu\nu} P_L b) F^{\mu\nu}$$

$$\mathcal{O}_{9\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell) \qquad \mathcal{O}_{9'\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell)$$

$$\mathcal{O}_{10\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell) \qquad \mathcal{O}_{10'\ell} = \frac{\alpha}{4\pi} (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell),$$

Currently, global determinations of $\mathcal{C}_9$ (and -maybe- $\mathcal{C}_{10}$) seem discrepant with SM predictions, with an important statistical significance.

Anomalies in $b \to s\ell^+\ell^-$

- **Anomalies in $b \to s\mu^+\mu^-$**: (LHCb, Belle, ATLAS, CMS)

$$P_5'(B \to K^*\mu^+\mu^-) \sim 3\sigma\ ; \quad \mathcal{B}(B_s \to \phi\mu^+\mu^-) \sim 2\sigma$$

$$\mathcal{B}(B \to K^{(*)}\mu^+\mu^-) \sim 1\sigma\ ; \quad \text{many others} \sim 0\sigma$$

Combined (180 Observables) $\sim 5\sigma$

Significantly alleviated if — Descotes, Matias, Virto 2013

$$\boxed{\mathcal{L}_{NP} \simeq (35\,\mathrm{TeV})^{-2}\ [\bar{s}\gamma_\nu P_L b][\bar{\mu}\gamma^\nu \mu]}$$

- **LFNU**: (LHCb, Belle)

$$R_K,\, R_{K^*} \gtrsim 2\sigma\ ; \quad Q_5 \equiv P'_{5\mu} - P'_{5e} \gtrsim 1\sigma$$

Combined $\sim 4\sigma$

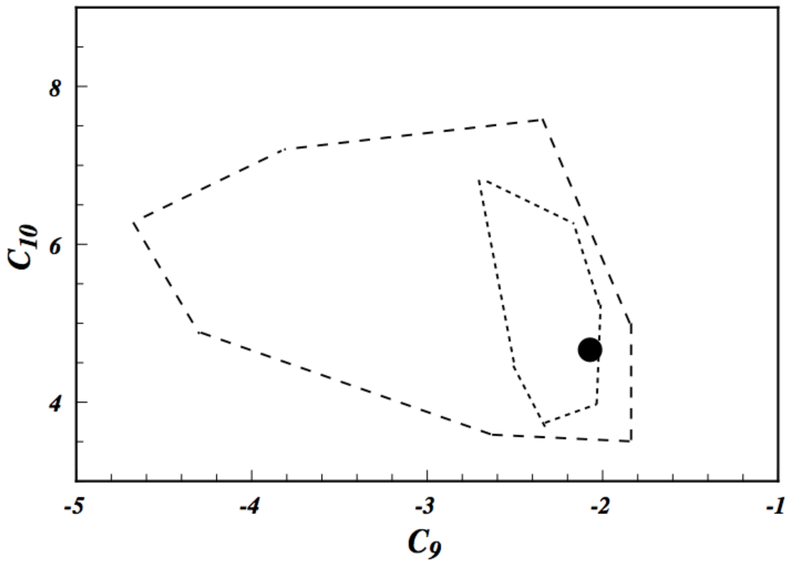
Consistent$^{(*)}$ with $b \to s\mu^+\mu^-$ — Alonso, Camalich, Grinstein 2014

NP interpretation requires accurate TH predictions of $B \to M\ell^+\ell^-$ obs

Historical prelude

Fits to $b \to s$ transitions is not a new business

"Towards a Model-Independent Analysis of Rare B Decays", Ali, Giudice, Mannel, 1994

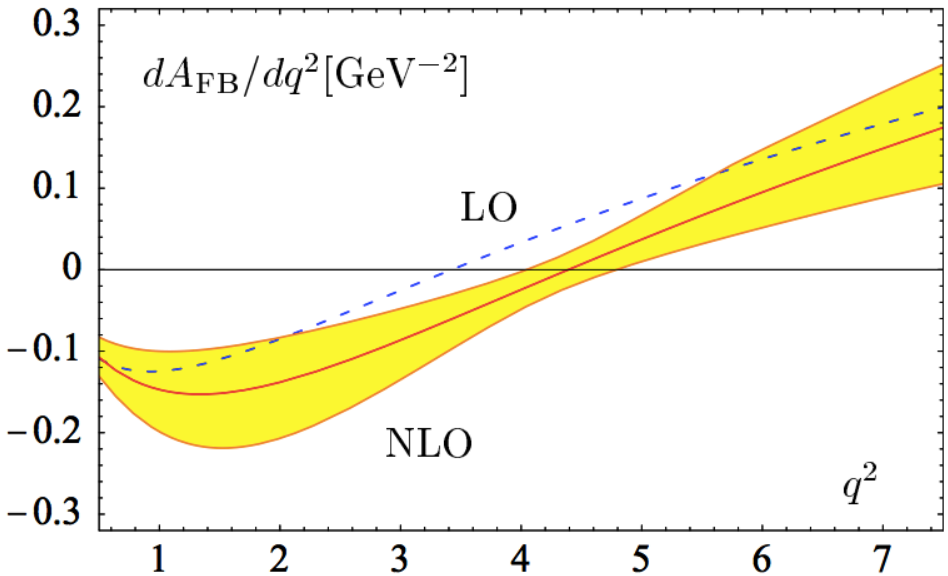


But measurements of key modes ($B_s \to \mu\mu$, $B \to K^{(*)}\ell\ell$) awaited LHC(b)

These measurements were anticipated by theorists.

Clean observables: $A_{\rm FB}$ zero

Beneke, Feldmann, Seidel 2001



Cancellation of hadronic uncertainties in the zero-crossing

At LO: $\quad C_9 + \mathrm{Re}(Y(q_0^2)) = -\dfrac{2M_B m_b}{q_0^2} C_7^{\rm eff}$

Hadronic Form Factors at Large Recoil

Charles et al 1998; Beneke, Feldmann 2000

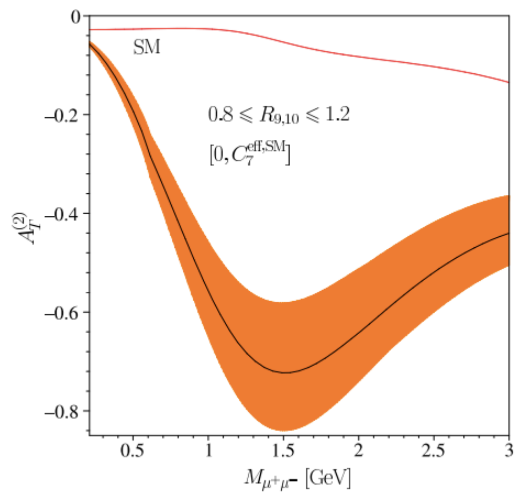
$$
\begin{aligned}
V(q^2) &= \frac{m_B + m_{K*}}{m_B}\, \xi_\perp(q^2) + \Delta V^{\alpha_s}(q^2) + \Delta V^{\Lambda}(q^2)\,, \\
A_1(q^2) &= \frac{2E}{m_B + m_{K*}}\, \xi_\perp(q^2) + \Delta A_1^{\alpha_s}(q^2) + \Delta A_1^{\Lambda}(q^2)\,, \\
A_2(q^2) &= \frac{m_B}{m_B - m_{K*}} \left[\xi_\perp(q^2) - \xi_\|(q^2)\right] + \Delta A_2^{\alpha_s}(q^2) + \Delta A_2^{\Lambda}(q^2)\,, \\
A_0(q^2) &= \frac{E}{m_{K*}}\, \xi_\|(q^2) + \Delta A_0^{\alpha_s}(q^2) + \Delta A_0^{\Lambda}(q^2)\,, \\
T_1(q^2) &= \xi_\perp(q^2) + \Delta T_1^{\alpha_s}(q^2) + \Delta T_1^{\Lambda}(q^2)\,, \\
T_2(q^2) &= \frac{2E}{m_B}\, \xi_\perp(q^2) + \Delta T_2^{\alpha_s}(q^2) + \Delta T_2^{\Lambda}(q^2)\,, \\
T_3(q^2) &= \left[\xi_\perp(q^2) - \xi_\|(q^2)\right] + \Delta T_3^{\alpha_s}(q^2) + \Delta T_3^{\Lambda}(q^2)\,,
\end{aligned}
$$

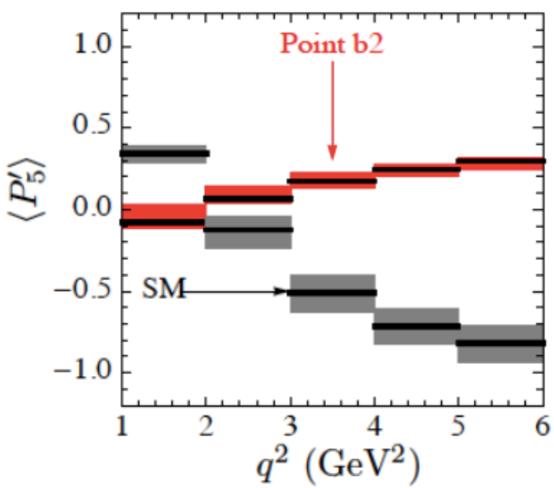
Only two independent structures at leading order + power

Clean observables for all q^2 (Cancellation as functions of q^2)

Kruger,Matias 2002

Descotes-Genon, Matias, Ramon, Virto 2012





Also: clean observables at large q^2

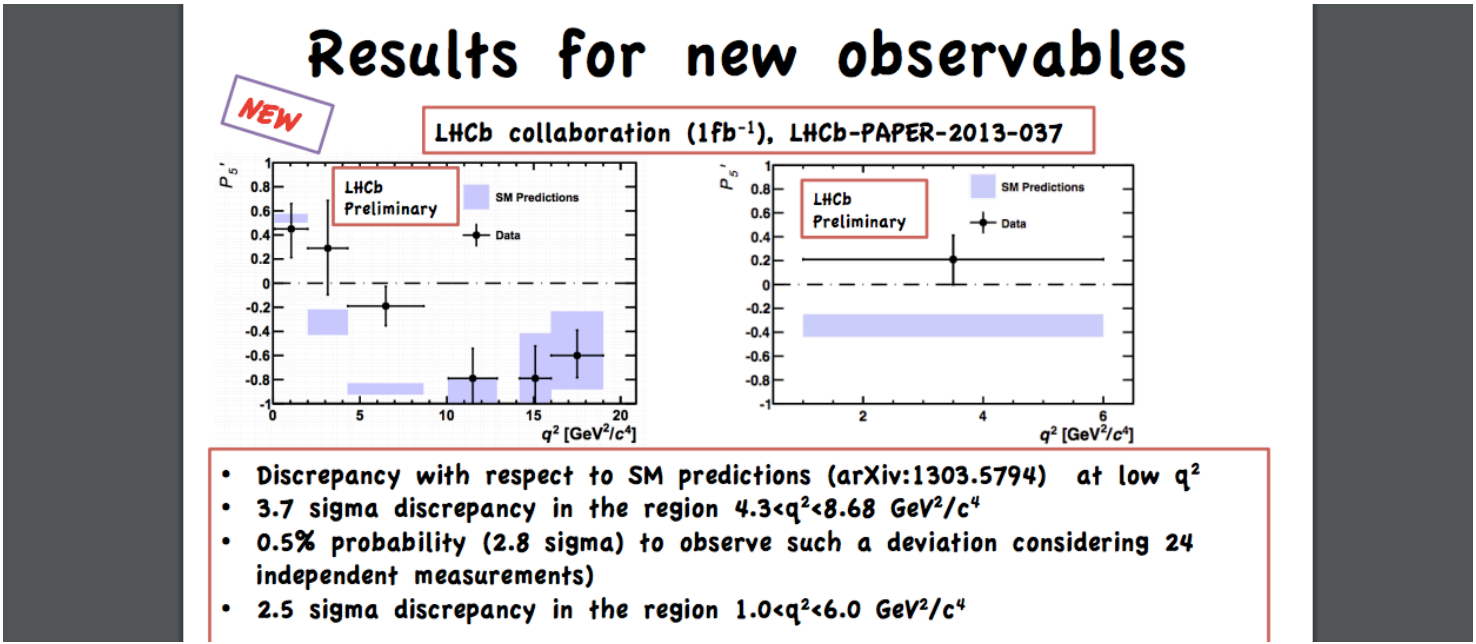
Bobeth, Hiller, van Dyk

Full basis of "all-q^2-clean" observables

Descotes-Genon, Hurth, Matias, Virto 2013

2013 LHCb measurements

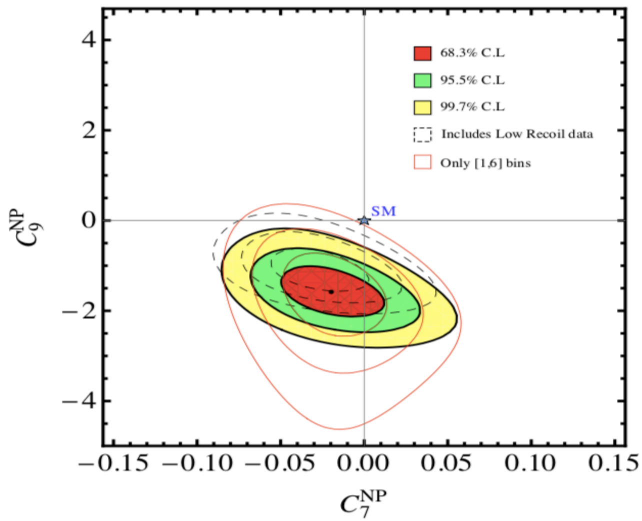
LHCb 2013 (2 papers)



N. Serra, talk at EPS-HEP 2013

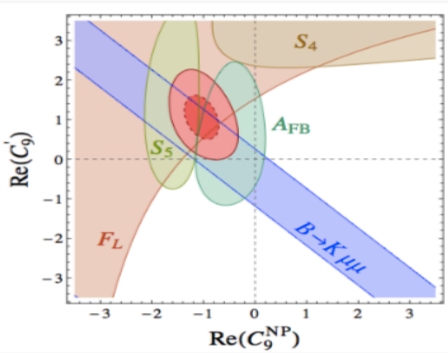
The $B \to K^* \mu\mu$ Anomaly

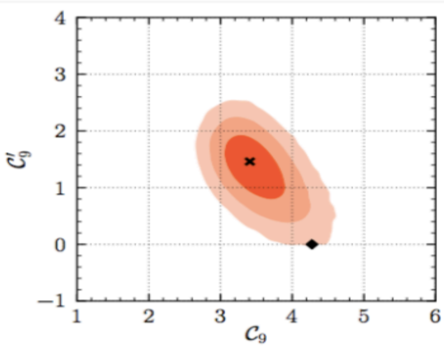
Descotes-Genon, Matias, Virto 2013

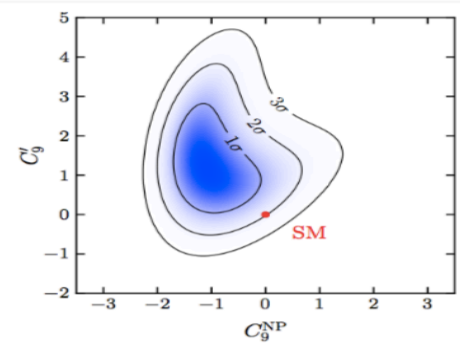


Indication for $\mathcal{C}_9^{\rm NP} \sim -1$

We have combined the recent LHCb measurements of $B \to K^* \mu^+ \mu^-$ observables [19, 20] with other radiative modes in a fit to Wilson coefficients, using the framework of our previous works [15, 21]. We have found a strong indication for a negative NP contribution to the coefficient $\mathcal{C}_9$, at 4.5 σ using large-recoil data (3.9 σ using both large- and low-recoil data). Our results correspond to $\mathcal{C}_9$ inside a 68 % C.L. range $2.2 \le \mathcal{C}_9 \le 2.8$ to be compared with $\mathcal{C}_9^{\rm SM} = 4.07$ at the scale $\mu_b = 4.8$ GeV. This is the main conclusion of our analysis of LHCb $B \to K^* \mu^+ \mu^-$ measurements.







Altmannshofer, Straub 1308.1501, Beaujean, Bobeth, van Dyk 1310.2478, Horgan et al. 1310.3887

A new “twist”: Lepton Flavor Non-Universality

$$R_K \equiv \frac{\mathcal{B}(B^+ \to K^+ \mu\mu)_{[1,6]\text{GeV}^2}}{\mathcal{B}(B^+ \to K^+ ee)_{[1,6]\text{GeV}^2}}; \quad R_K^{\text{SM}} = 1; \quad R_K^{\text{LHCb 2014}} \simeq 0.75 \pm 0.1$$

Hiller,Kruger 2004; Bobeth, Hiller, Piranishvili 2007; Bordone, Isidori, Pattori 2016; LHCb 2014

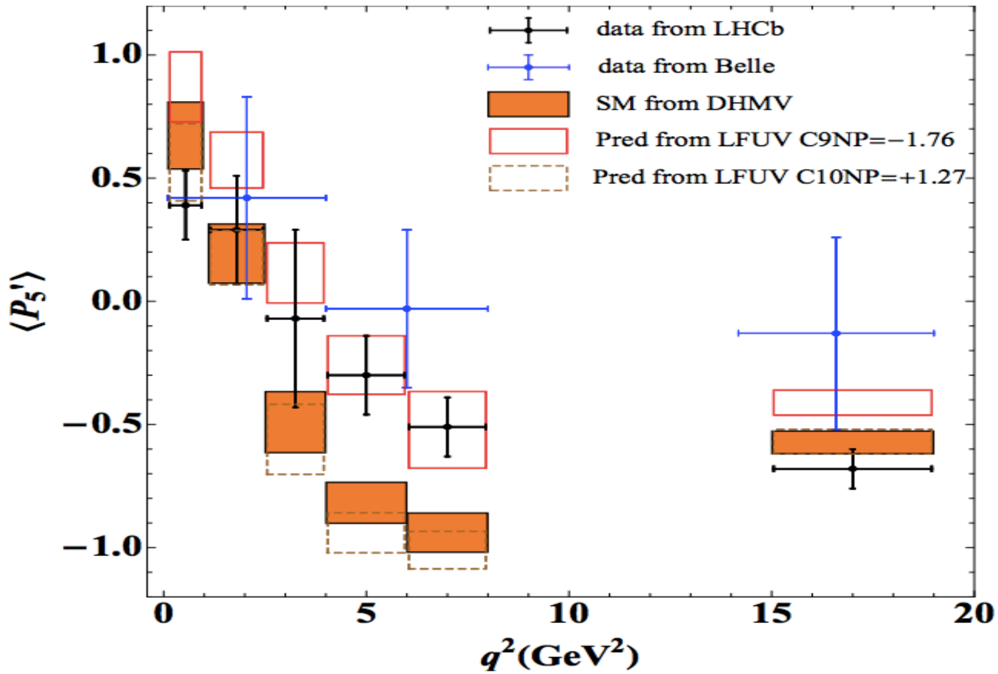
not very well bound, especially for the electronic case, so different scenarios of NP could currently explain (15). For example one could entertain the possibility of a sizable and negative effect in C_9 affecting only the muonic mode, $\delta C_9^{\mu} = -1$. In this scenario one obtains $R_K \simeq 0.79$. As a side remark, it is worth emphasizing that such a negative NP contribution to $\mathcal{O}_9^{(\prime)}$ has been argued to be necessary to understand the current $b \to s\mu\mu$ data set [27–30].

From Alonso, Grinstein, Camalich 2014

See also Hiller, Schmaltz 2014; Gosh, Nardecchia, Renner 2014

A new "twist": Lepton Flavor Non-Universality

Consistency of P_5' and LFNU in 2017 Capdevila, Crivellin, Descotes-Genon, Matias, Virto 2017



Fit to LFNU observables only (2017) predicted correct LHCb P_5' measurements

“Anomalies” (as of 2020)

Observable	Experiment	SM prediction	pull
$R_K^{[1.1,6]}$	0.85 ± 0.06	1.00 ± 0.01	$+2.5\sigma$
$R_{K^*}^{[0.045,1.1]}$	$0.66^{+0.11}_{-0.07}$	0.92 ± 0.02	$+2.3\sigma$
$R_{K^*}^{[1.1,6]}$	$0.69^{+0.12}_{-0.08}$	1.00 ± 0.01	$+2.6\sigma$
$\langle P_5' \rangle_{[4,6]}$	-0.44 ± 0.12	-0.82 ± 0.08	-2.7σ
$\langle P_5' \rangle_{[6,8]}$	-0.58 ± 0.09	-0.94 ± 0.08	-2.9σ
$\mathcal{B}_{\phi\mu\mu}^{[2,5]}$	0.77 ± 0.14	1.55 ± 0.33	$+2.2\sigma$
$\mathcal{B}_{\phi\mu\mu}^{[5.8]}$	0.96 ± 0.15	1.88 ± 0.89	$+2.2\sigma$

Global fit should accommodate these deviations within all other measurements

Current Status

New measurements 2021

LFUV

$$R_K^{\text{LHCb 2020}} = 0.846^{+0.060+0.016}_{-0.054-0.014} \qquad R_{K_S}^{\text{LHCb 2021}} = 0.66^{+0.20+0.02}_{-0.14-0.04}$$

$$R_K^{\text{LHCb 2021}} = 0.846^{+0.042+0.013}_{-0.039-0.012} \qquad R_{K^*}^{\text{LHCb 2021}} = 0.70^{+0.18+0.13}_{-0.03-0.04}$$

$B_s \to \mu\mu$

$$10^9 \mathcal{B}(B_s \to \mu\mu)_{\text{WA 2020}} = 2.69^{+0.37}_{-0.35}$$ (> 2σ below SM)

$$10^9 \mathcal{B}(B_s \to \mu\mu)_{\text{LHCb 2017}} = 3.0 \pm 0.6^{+0.3}_{-0.2}$$

$$10^9 \mathcal{B}(B_s \to \mu\mu)_{\text{LHCb 2021}} = 3.09^{+0.46+0.15}_{-0.43-0.11}$$

$$10^9 \mathcal{B}(B_s \to \mu\mu)_{\text{AS 2021}} = 2.93 \pm 0.35$$ (2.3σ below SM)

$$10^9 \mathcal{B}(B_s \to \mu\mu)_{\text{GGJLCS 2021}} = 2.84 \pm 0.33$$

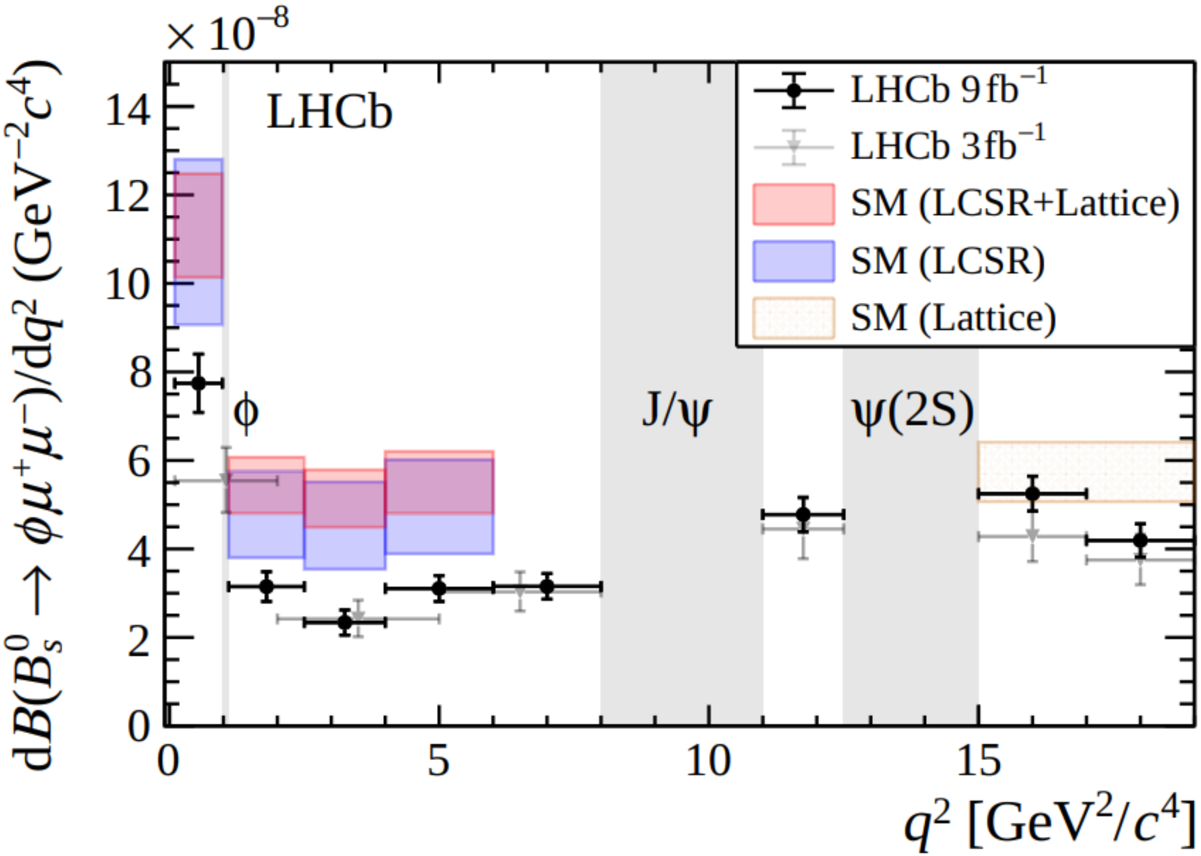
$$10^9 \mathcal{B}(B_s \to \mu\mu)_{\text{SM}} = 3.63 \pm 0.13$$ (Beneke, Bobeth, Szafron 2019)

AO's

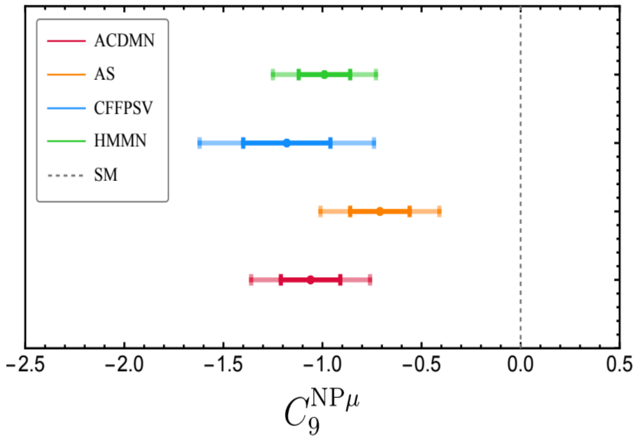
$B^+ \to K^{*+}\mu^+\mu^-$ (AOs) $\qquad \mathcal{B}(B^s \to \phi\mu^+\mu^-)$

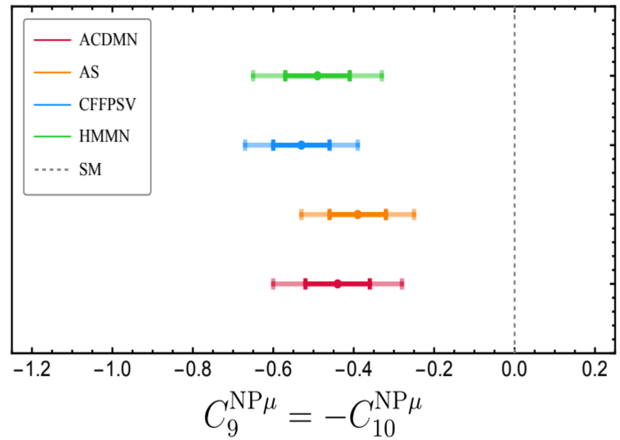
New measurements 2021

2105.14007



1-dimensional global fits



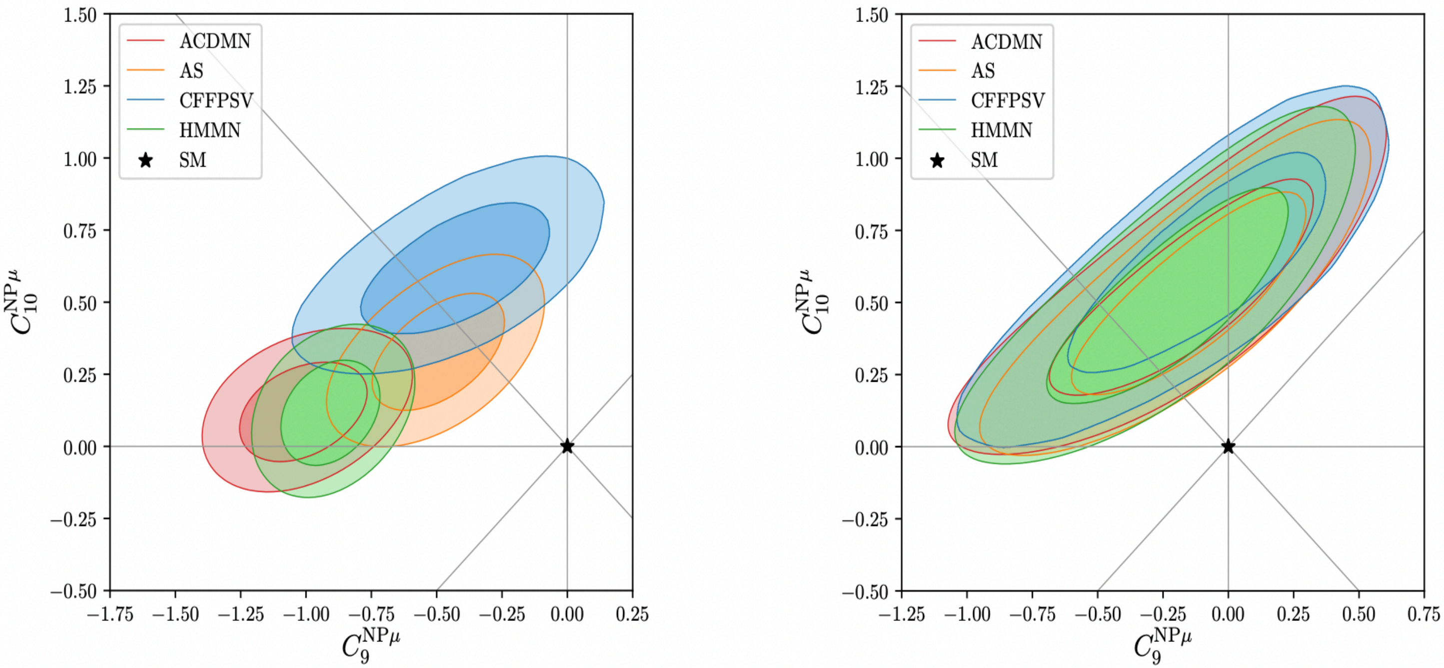


- NP scenarios preferred over SM with $\mathrm{Pull}_{\mathrm{SM}}{}^{*} > 5\sigma$
- Different results due to different assumptions about non-local matrix elements, different choices of form factors and observables, etc.
- Remarkable agreement between fits of different groups despite different approaches
  $\Rightarrow$ ***b*** $\rightarrow$ ***s$\ell\ell$* global analyses are robust**

*: $\mathrm{Pull}_{\mathrm{SM}} \neq$ global significance; conservative global significance $\simeq 4.3\sigma$ determined in Isidori, Lancierini, Owen, Serra, arXiv:2104.05631

B. Capdevila, M. Fedele, S. Neshatpour, P. Stangl   Flavor at the Crossroads, 19-29 Apr. 2022

2-dimensional fits



global fit

fit to LFU observables + $B_s \to \mu\mu$

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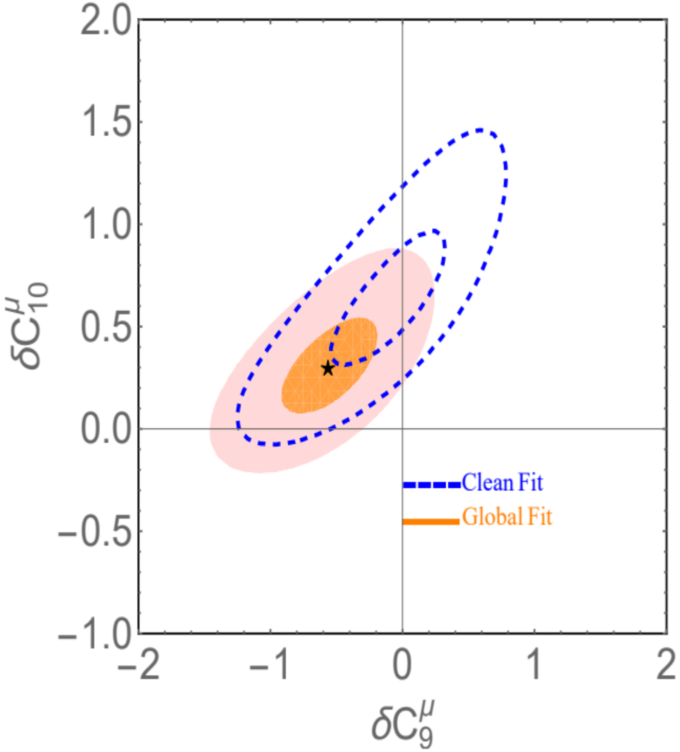
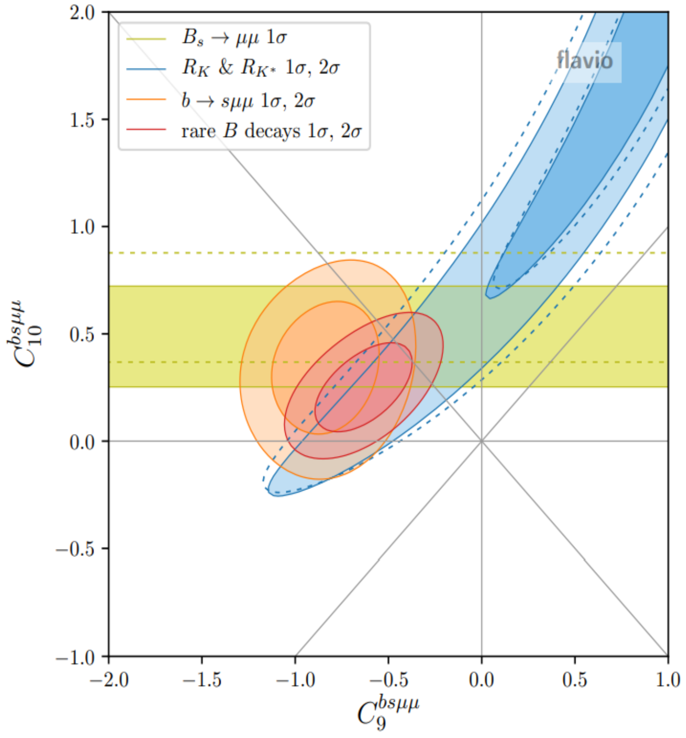
2021 Fits

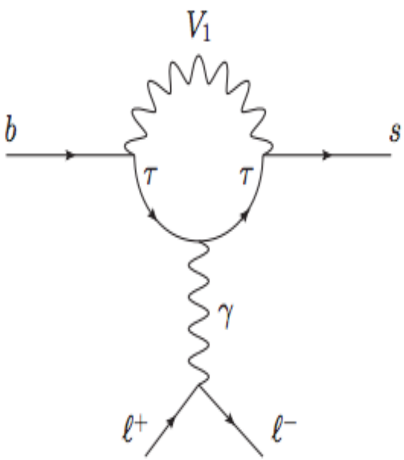
Geng et al 2021; Altmannshofer, Stangl 2021

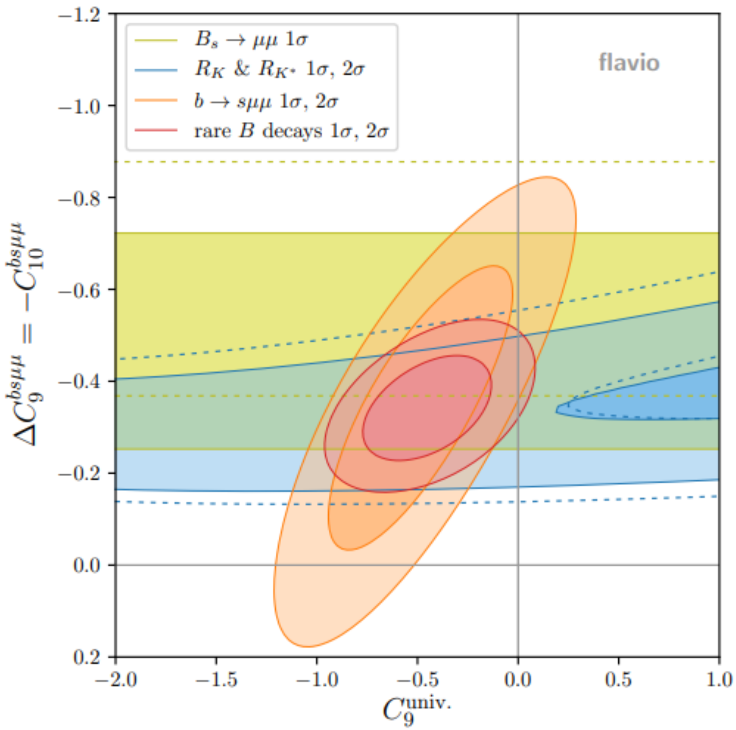




2021 Fits

Altmannshofer, Stangl 2021





Theory

Theory predictions

- TH predictions require very involved and nontrivial **perturbative** and **non-perturbative** calculations.

- The fact that $\Lambda_{QCD} \ll m_b \ll \Lambda_{EW}, \Lambda_{BSM}$ helps a lot

- Weak Effective Theory:

$$\mathcal{L}_{WET} = \mathcal{L}_{QCD+QED} + \sum_i C_i \, \mathcal{O}_i$$

Short distance C_i known in SM to NNLL — Bobeth, Misiak, Urban, Gorbahn, Haisch,...

BSM part of C_i is the target

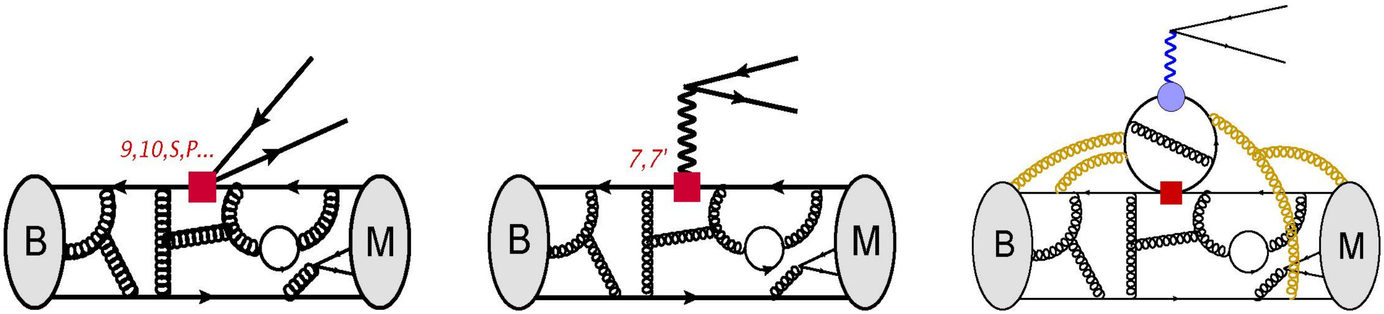
- This leaves non-perturbative MEs as the **leading challenge**

Theory predictions

What we need depends on **mode** and **observable**. E.g.:

- $B_q \to \ell^+\ell^-$ needs f_{B_q} (up to QED...)
- $B \to D^{(*)}\ell\nu$ needs **local** form factors (3 for D, 7 for D^*)
- $\mathcal{R}_{D^{(*)}}$ needs local form factor ratios
- $b \to s\ell\ell$ needs both **local** and **non-local** form factors
  - $\mathcal{B}$ needs all (3 for K, 7 for K^*, ϕ)
  - Ratios such as P_5' need only **ratios** (SCET & HQET relations)
  - $\mathcal{R}_{K,K^*,\ldots}$ (LFNU) almost exact cancellation of FFs (both local and non-local) within SM, but **NOT** in LFNU BSM in general
  - QED effects relevant in LFNU ratios and angular distributions

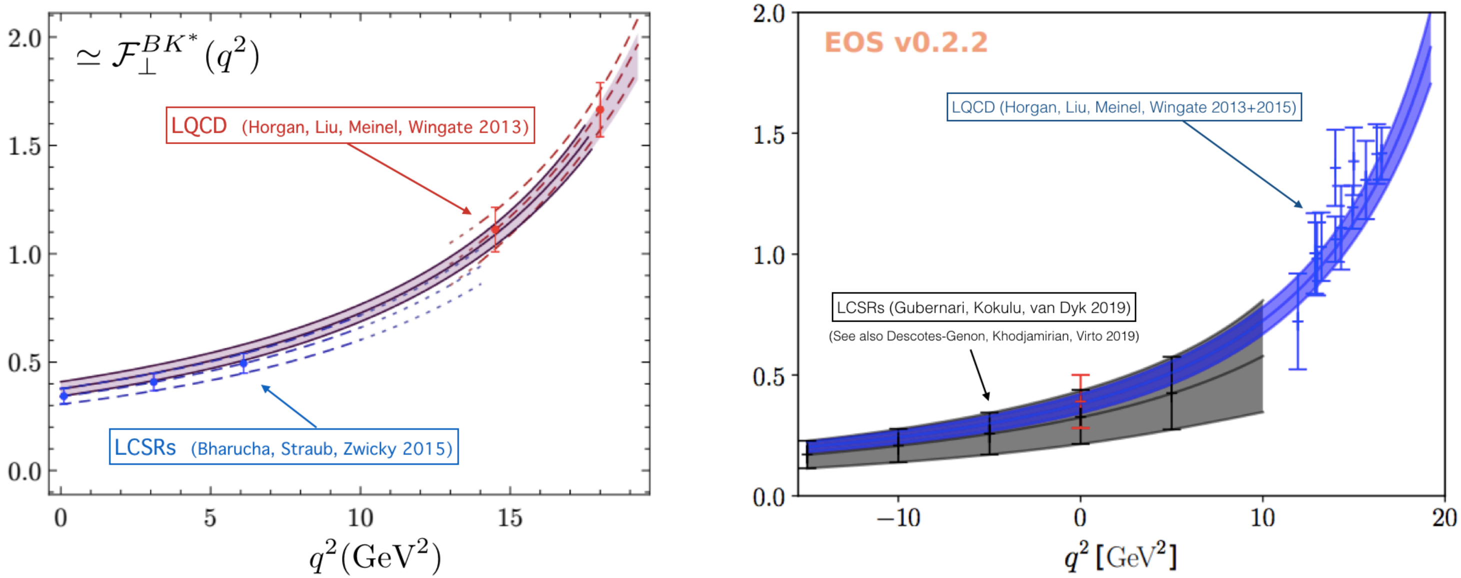
Anatomy of $B \to M_\lambda \ell^+ \ell^-$ EFT Amplitudes



$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- Local (Form Factors) : $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) | \, \bar{s} \, \Gamma_\lambda^{(T)} \, b \, | \bar{B}(k+q) \rangle$
- Non-Local : $\mathcal{H}_\lambda(q^2) = i \, \mathcal{P}_\mu^\lambda \int d^4x \, e^{iq\cdot x} \, \langle \bar{M}_\lambda(k) | T\{ \mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i \, \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$

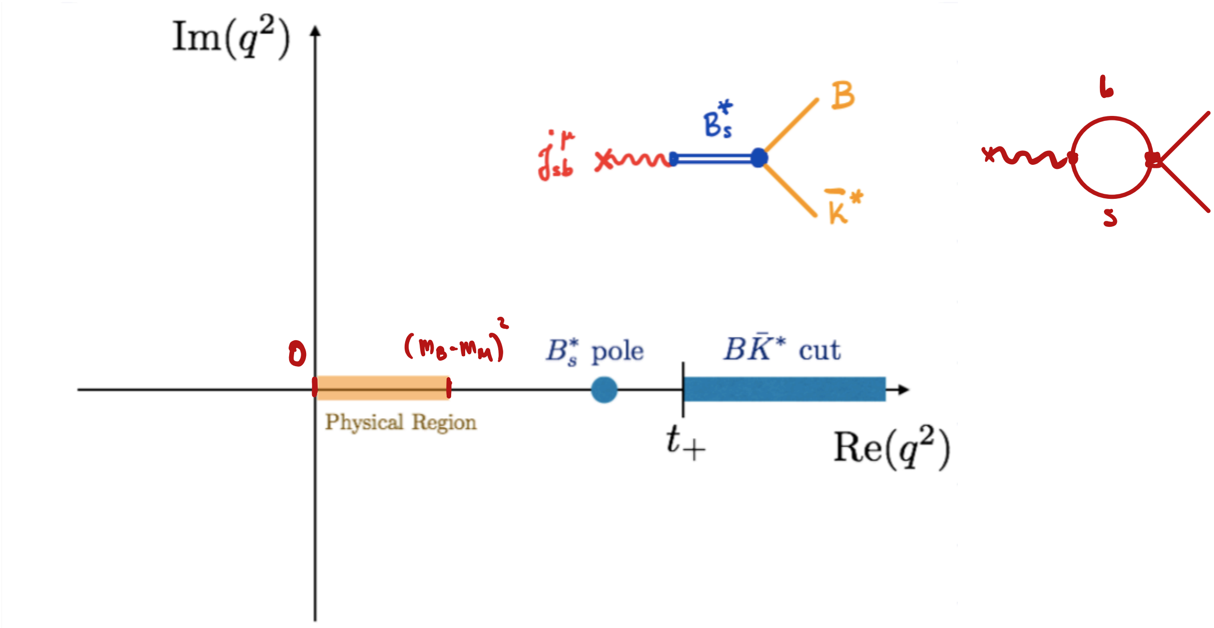
Local Form Factors



- Two main approaches: (1) Lattice QCD (large q^2) (2) LCSRs (low q^2)
- Two approaches to LCSRs, in terms of (1) K^* LCDAs (2) B LCDAs
- q^2 dependence can be parametrized model-independently

Local Form Factors : q^2-dependence from analyticity

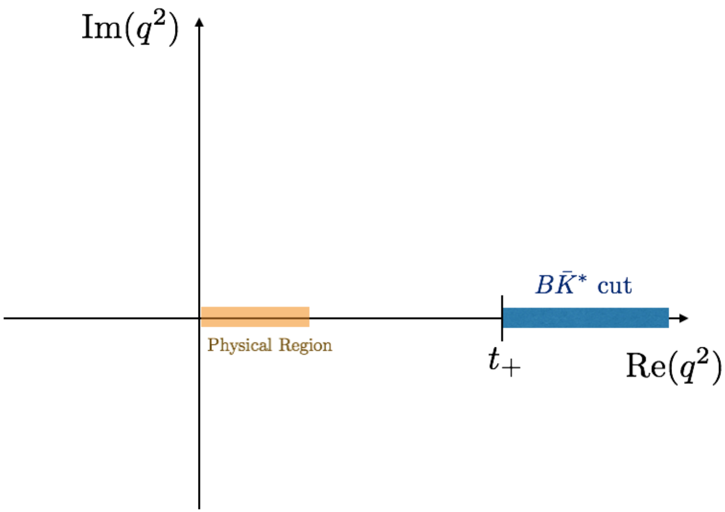
$\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k)|\,\bar{s}\,\Gamma_\lambda^{(T)}\,b\,|\bar{B}(k+q)\rangle$: Analytic structure in q^2 :

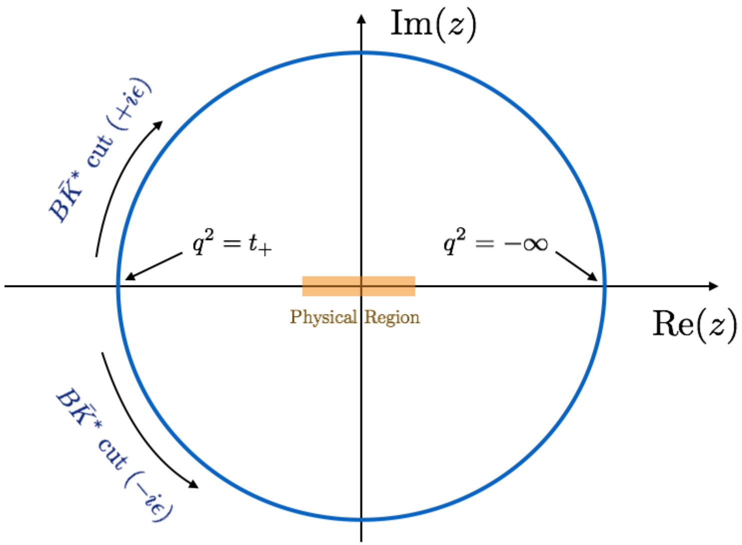


$\hat{\mathcal{F}}_\lambda^{(T)}(q^2) \equiv (q^2 - m_{B_s^*}^2)\,\mathcal{F}_\lambda^{(T)}(q^2)$ has no pole, only cut.

Local Form Factors : q^2-dependence from analyticity

- Conformal mapping : $z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$





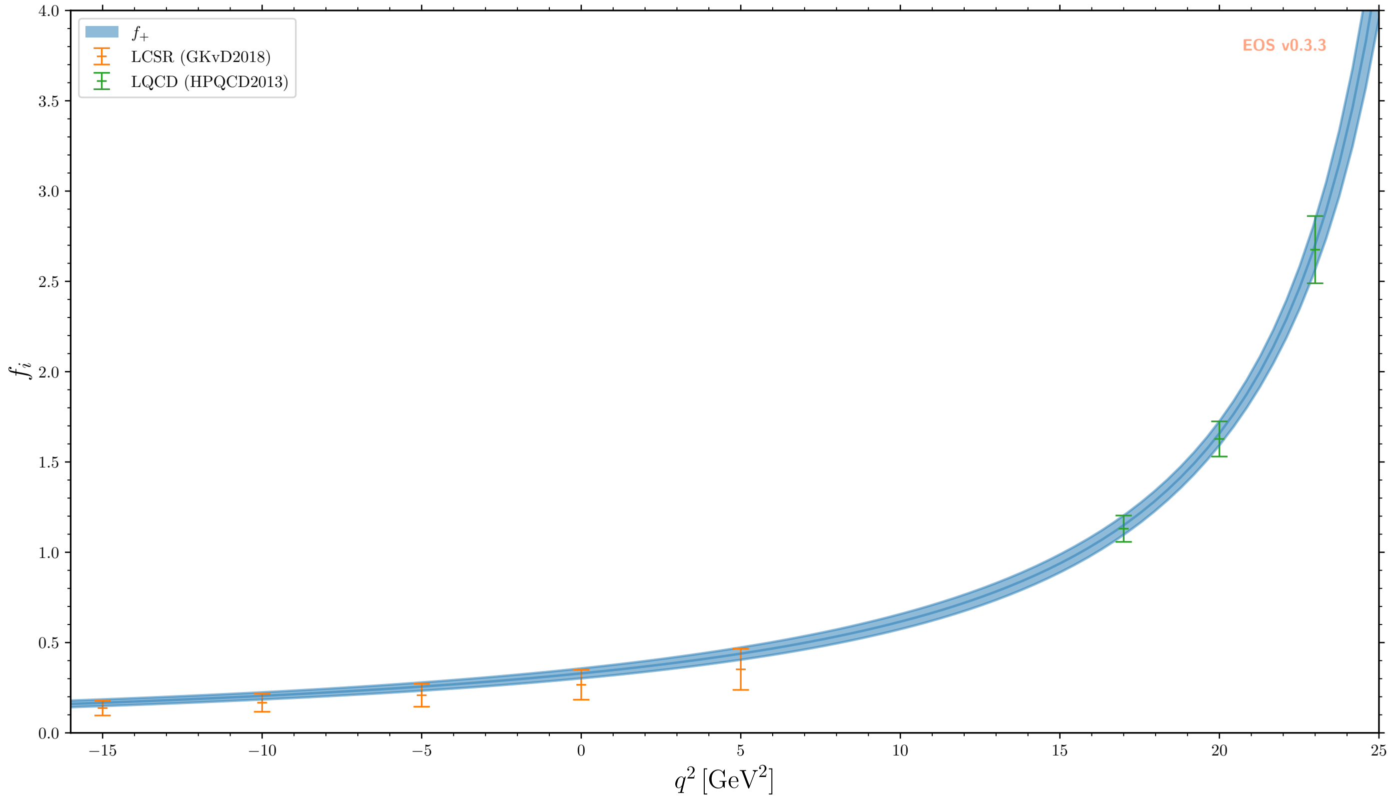
- "z-parametrization" : $\widehat{\mathcal{F}}_\lambda^{(T)}(q^2(z))$ is **analytic in** $|z| < 1$ $\quad(|z_{\rm phys}| < 0.15)$

$$\mathcal{F}_\lambda^{(T)}(q^2) = \frac{1}{(q^2 - m_{B_s^*}^2)} \sum_k \alpha_k \, z(q^2)^k$$

Bourrely, Caprini, Lellouch; Boyd, Grinstein, Lebed; Caprini, Lellouch, Neubert; ...

Local Form Factors : New Fits to (B-DAs) LCSRs + LQCD

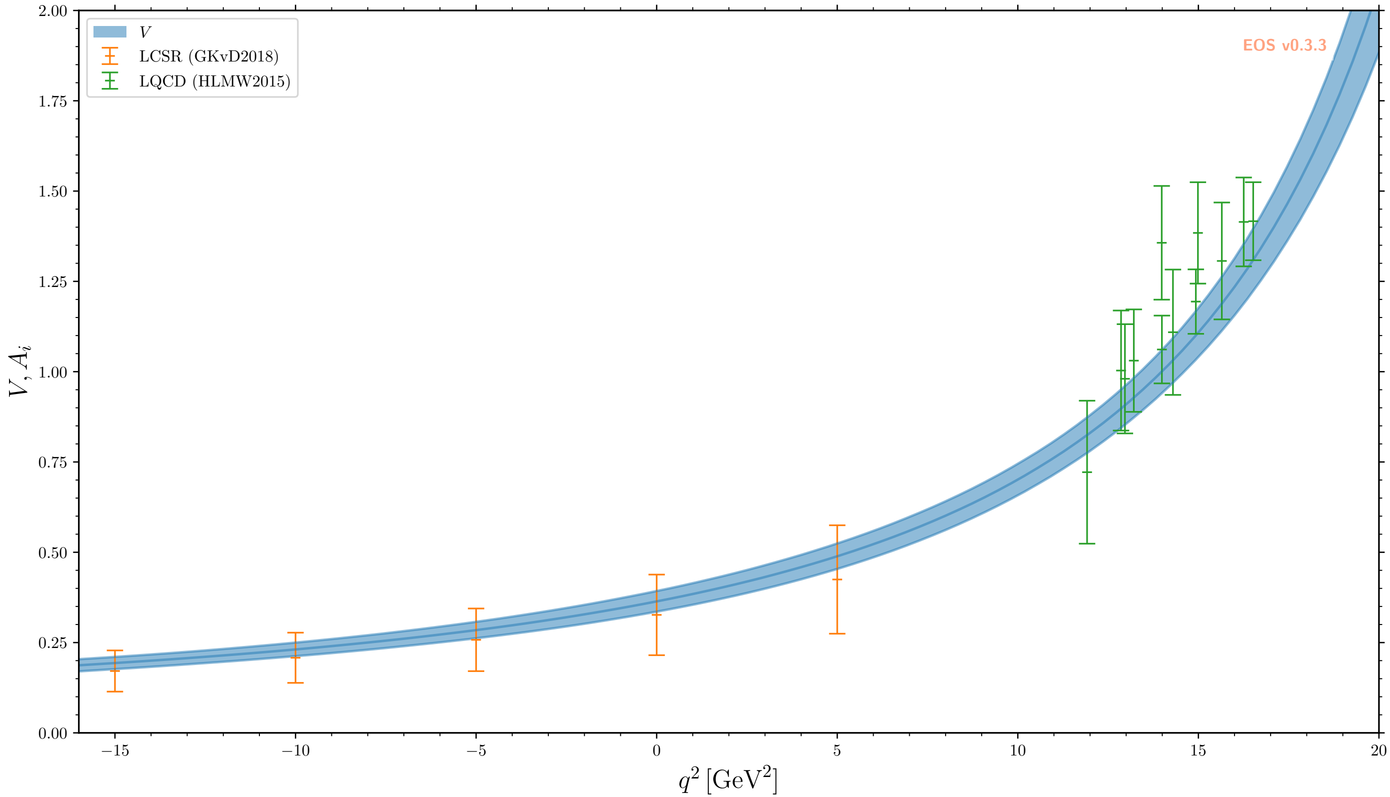
- $B \to K\ell\ell$



Gubernari, Reboud, van Dyk, Virto, w.i.p.

Local Form Factors : New Fits to (B-DAs) LCSRs + LQCD

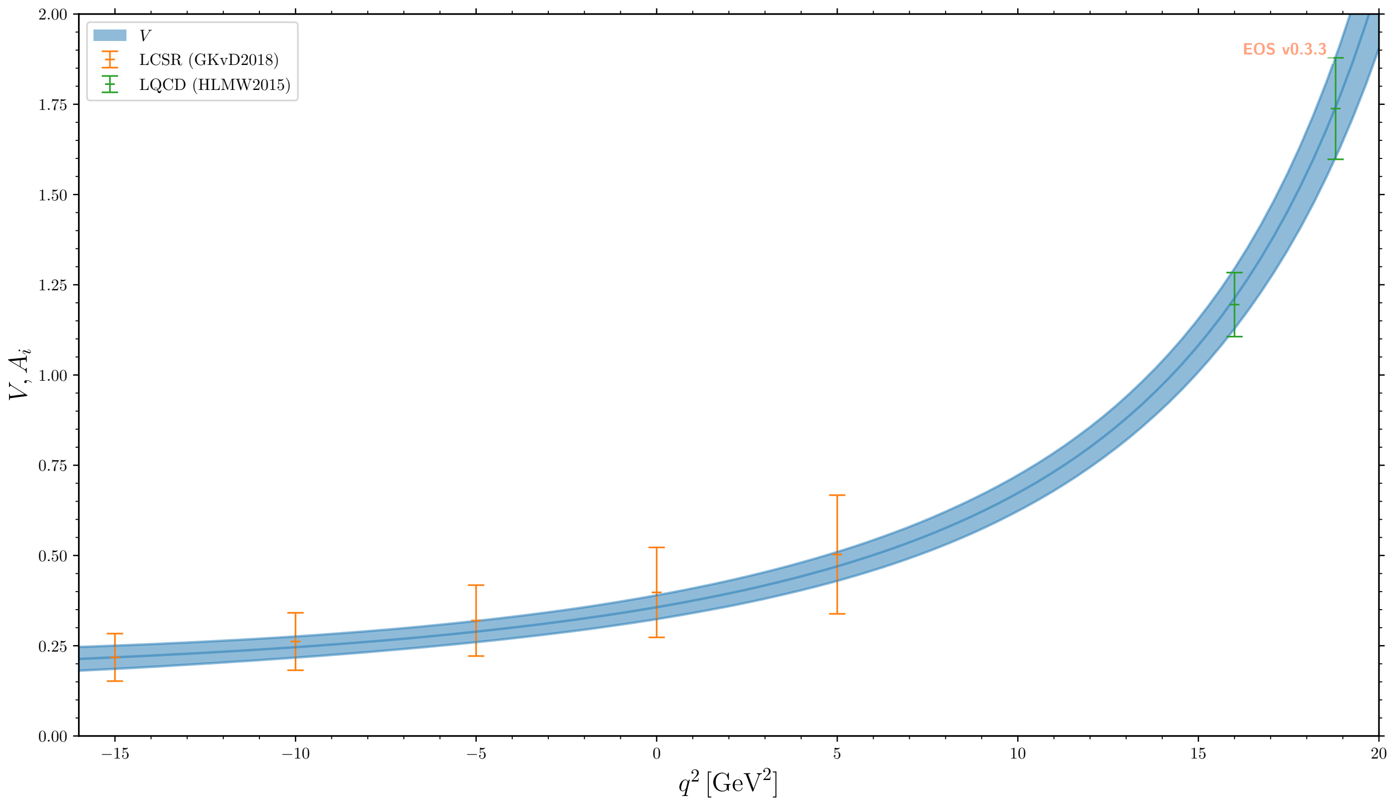
- $B \to K^* \ell\ell$



Gubernari, Reboud, van Dyk, Virto, w.i.p.

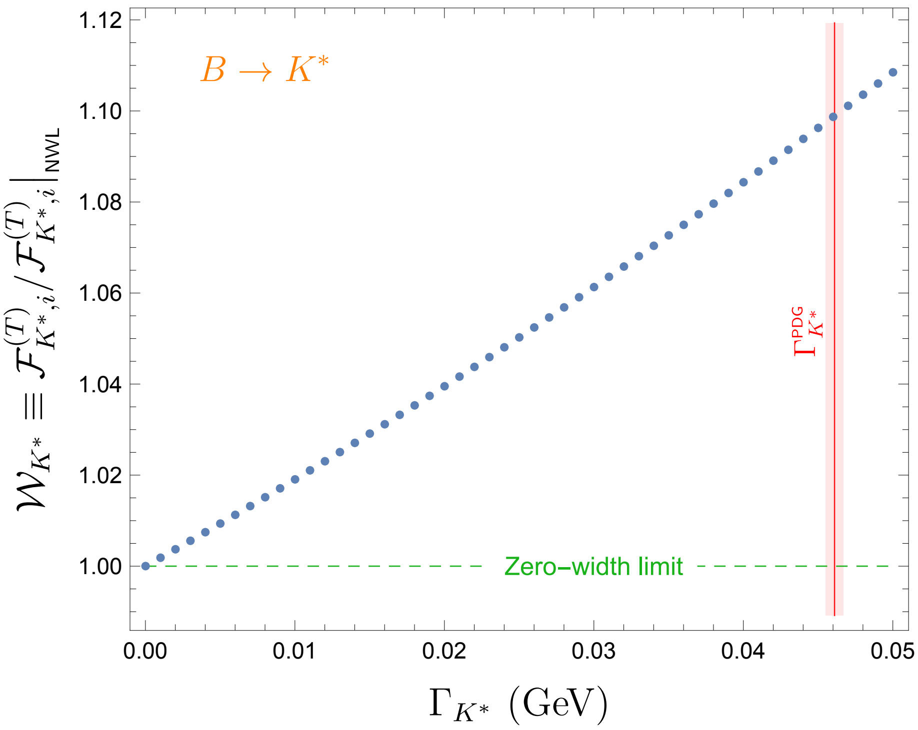
Local Form Factors : New Fits to (B-DAs) LCSRs + LQCD

- $B_s \to \phi\ell\ell$



Gubernari, Reboud, van Dyk, Virto, w.i.p.

Descotes-Genon, Khodjamirian, Virto, 1908.02267



(Crucial input: $\tau \to K\pi\nu$)

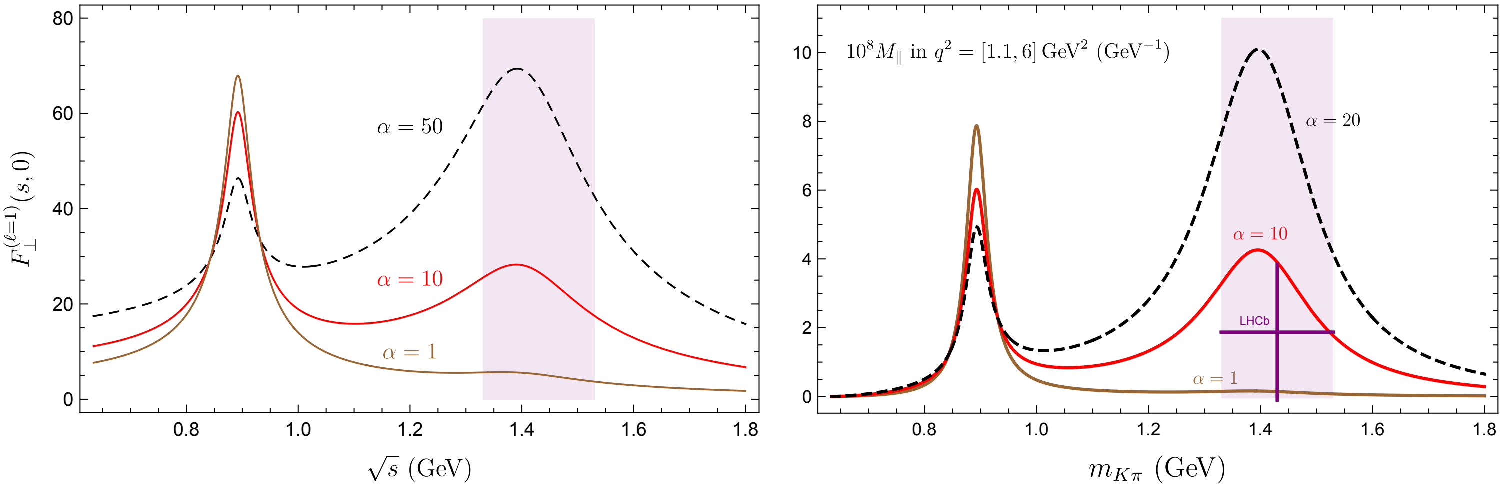
$$\mathcal{W}_{K*} \simeq 1 + 1.9\,\frac{\Gamma_{K*}}{m_{K*}}$$

$$\mathcal{W}_{K*} = 1.09 \pm 0.01$$

- $\mathcal{W}_{K*}$ is independent of the form factor type
- $\mathcal{W}_{K*}$ is indep. of q^2

$\Rightarrow$ BRs are corrected by a factor $|\mathcal{W}_{K*}|^2 \simeq 1.2$. Ratios unaffected.

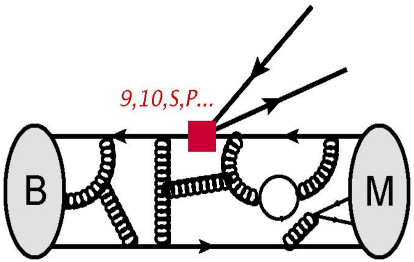
Set $\mathcal{F}_{K^*(1410)} = \alpha\, \mathcal{F}_{K^*(892)}$ with α a floating parameter

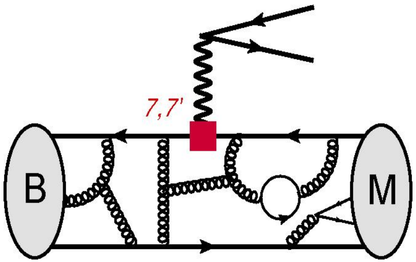


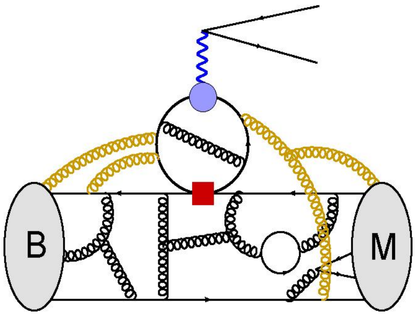
$\alpha = 1 : \mathcal{F}_{K^*,\perp}(0) = 0.28$; $\alpha = 10 : \mathcal{F}_{K^*,\perp}(0) = 0.22$; $\alpha = 50 : \mathcal{F}_{K^*,\perp}(0) = 0.11$.

Constrained by angular measurements on 1430 region (LHCb)

Non-Local Form Factors







$$\mathcal{A}_\lambda^{L,R} = \mathcal{N}_\lambda \left\{ (C_9 \mp C_{10}) \mathcal{F}_\lambda(q^2) + \frac{2 m_b M_B}{q^2} \left[C_7 \mathcal{F}_\lambda^T(q^2) - 16\pi^2 \frac{M_B}{m_b} \mathcal{H}_\lambda(q^2) \right] \right\}$$

- Local (Form Factors) : $\mathcal{F}_\lambda^{(T)}(q^2) = \langle \bar{M}_\lambda(k) |\, \bar{s}\, \Gamma_\lambda^{(T)}\, b \,| \bar{B}(k+q) \rangle$
- Non-Local : $\mathcal{H}_\lambda(q^2) = i\,\mathcal{P}_\mu^\lambda \int d^4x\, e^{iq\cdot x} \langle \bar{M}_\lambda(k) | \mathcal{T}\{ \mathcal{J}_{\rm em}^\mu(x), \mathcal{C}_i\, \mathcal{O}_i(0) \} | \bar{B}(q+k) \rangle$

Non-local form factors: Operator Product Expansion

$$\mathcal{H}^\mu(q,k) = i\int d^4x\, e^{iq\cdot x}\,\langle \bar{M}_\lambda(k)|\mathcal{T}\{\mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i\,\mathcal{O}_i(0)\}|\bar{B}(q+k)\rangle$$

- Large-q^2: Dominated by $x \sim 0$ (short-distance dominance - OPE)

  Grinstein, Pirjol; Beylich, Buchalla, Feldmann

- Low-q^2: Dominated by $x^2 \sim 0$ (light-cone dominance - LCOPE)

  Khodjamirian, Mannel, Pivovarov, Wang

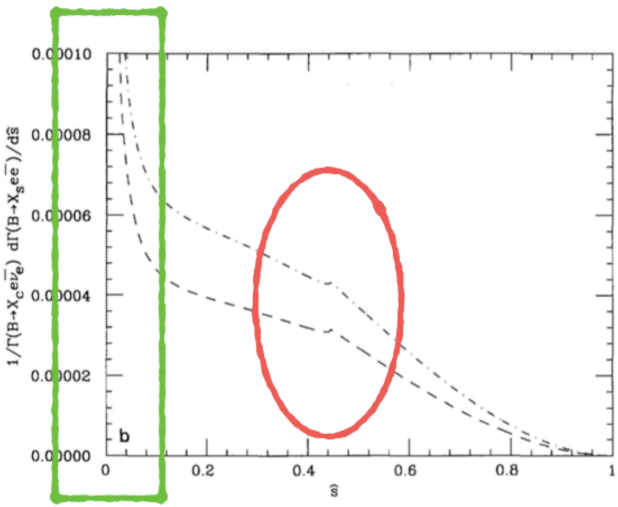
+ Must analytically-continue from OPE region to physical region

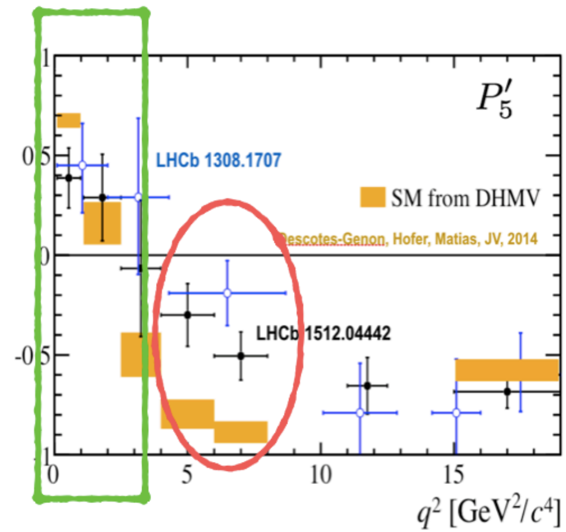
Non-local form factors: Importance of on-shell cuts

- QCD Factorization Beneke, Feldmann, Seidel 2001

$$\mathcal{H}_\lambda(q^2) \sim \Delta C_9^\lambda(q^2)\,\mathcal{F}_\lambda(q^2) + \frac{1}{q^2}\Delta C_7^\lambda(q^2)\,\mathcal{F}_\lambda^T(q^2) + HSS + \mathcal{O}(\Lambda/m_B, \Lambda/E)$$

- It is assumed that the charm loop is dominated by short distances



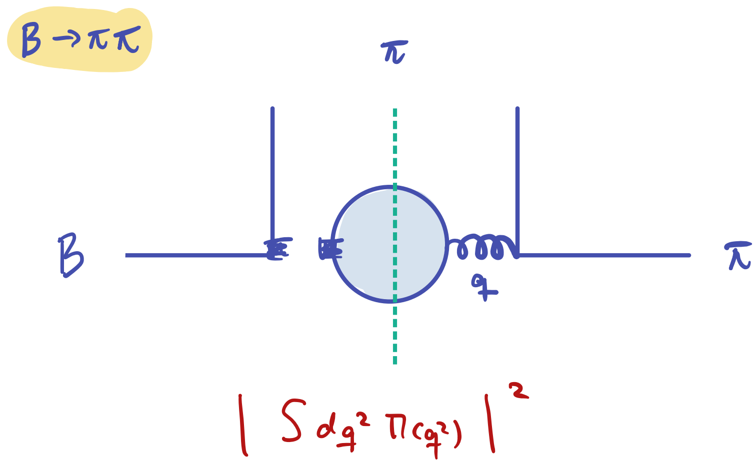


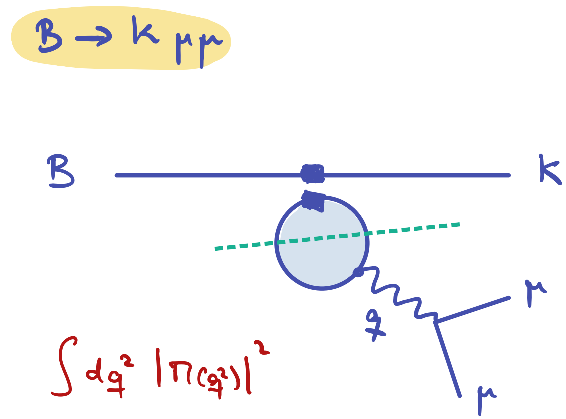
- Kink at $q^2 = 4m_c^2$ symptom of breaking of perturbativity

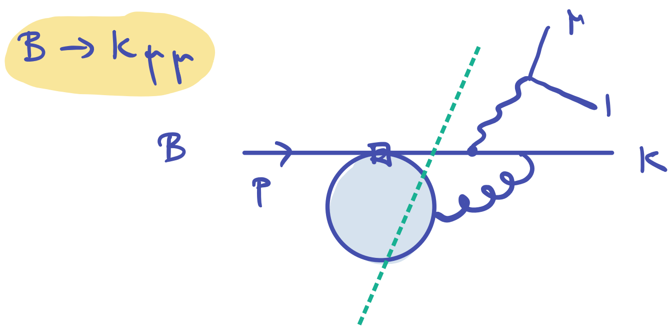
What is QCD Factorization doing for us?

Beneke, Feldmann, Seidel 2001

Beneke, Buchalla, Neubert, Sachrajda 2009







← This imaginary part is like the one in B→ππ.

(See also: Beylich, Buchalla, Feldmann.)

Approaches to the problem

- Use QCDF directly on the physical timelike q^2 region. Discussions whether to use the $[6,8]\text{GeV}^2$ bin or not.

- Use QCDF at very low q^2 (or better, $q^2 < 0$!), and extrapolate (or interpolate!) somehow to higher q^2.

- Do not use any theory and use data to fit to some given q^2 shape.

We write

$$\mathcal{H}^\mu(q,k) = \langle \bar{M}_\lambda(k)|\mathcal{K}^\mu(q)|\bar{B}(q+k)\rangle$$

With the operator $\mathcal{K}^\mu(q)$ given by

$$\mathcal{K}^\mu(q) = i\int d^4x\, e^{iq\cdot x}\, \mathcal{T}\{\mathcal{J}^\mu_{\rm em}(x), \mathcal{C}_i\, \mathcal{O}_i(0)\}$$

It turns out that: **Leading-power OPE = Leading-power LCOPE**

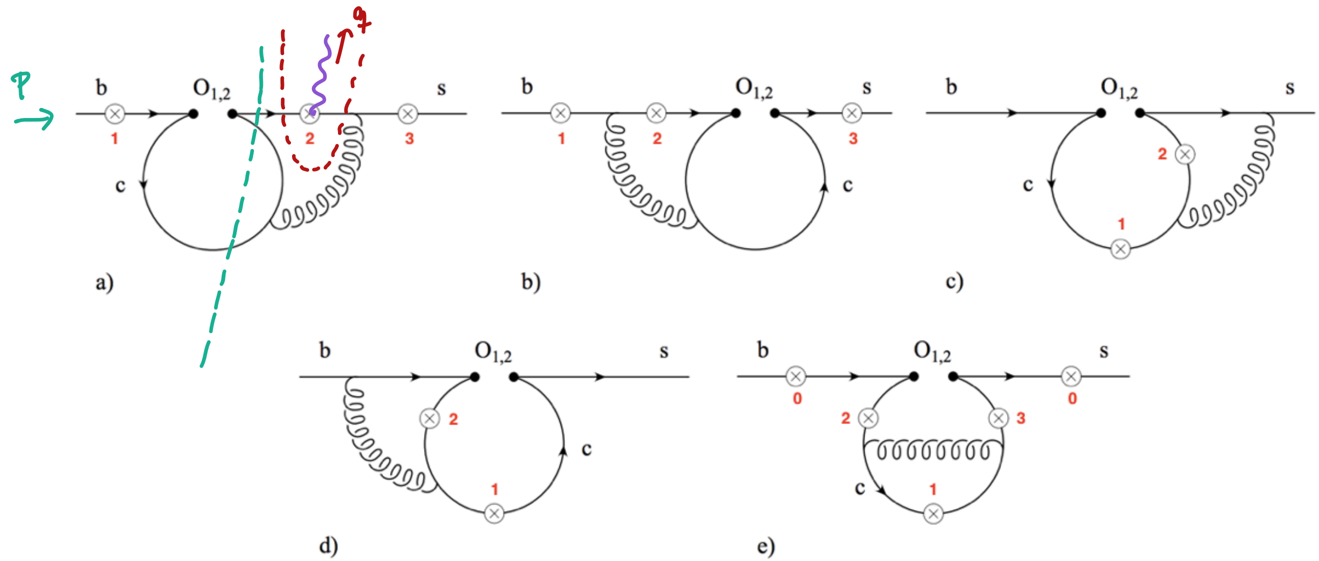
$$\mathcal{K}^\mu_{\rm OPE}(q) = \Delta C_9(q^2)\big(q^\mu q^\nu - q^2 g^{\mu\nu}\big)\, \bar{s}\gamma_\nu P_L b + \Delta C_7(q^2)\, 2im_b\, \bar{s}\sigma^{\mu\nu} q_\nu P_R b + \cdots$$

With this we have:

$$\mathcal{H}^\mu_{\rm OPE}(q,k) = \Delta C_9(q^2)\big(q^\mu q^\nu - q^2 g^{\mu\nu}\big)\mathcal{F}_\nu + 2im_b\, \Delta C_7(q^2)\mathcal{F}^{T\mu} + \cdots$$

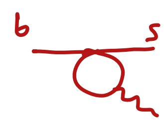
Asatrian, Greub, Virto 2019

Objective: Fully analytical calculation in two variables: q^2 and m_c.



$\Rightarrow$ Gives $\Delta C_9(q^2)$ and $\Delta C_7(q^2)$ at $\mathcal{O}(\alpha_s^2)$.

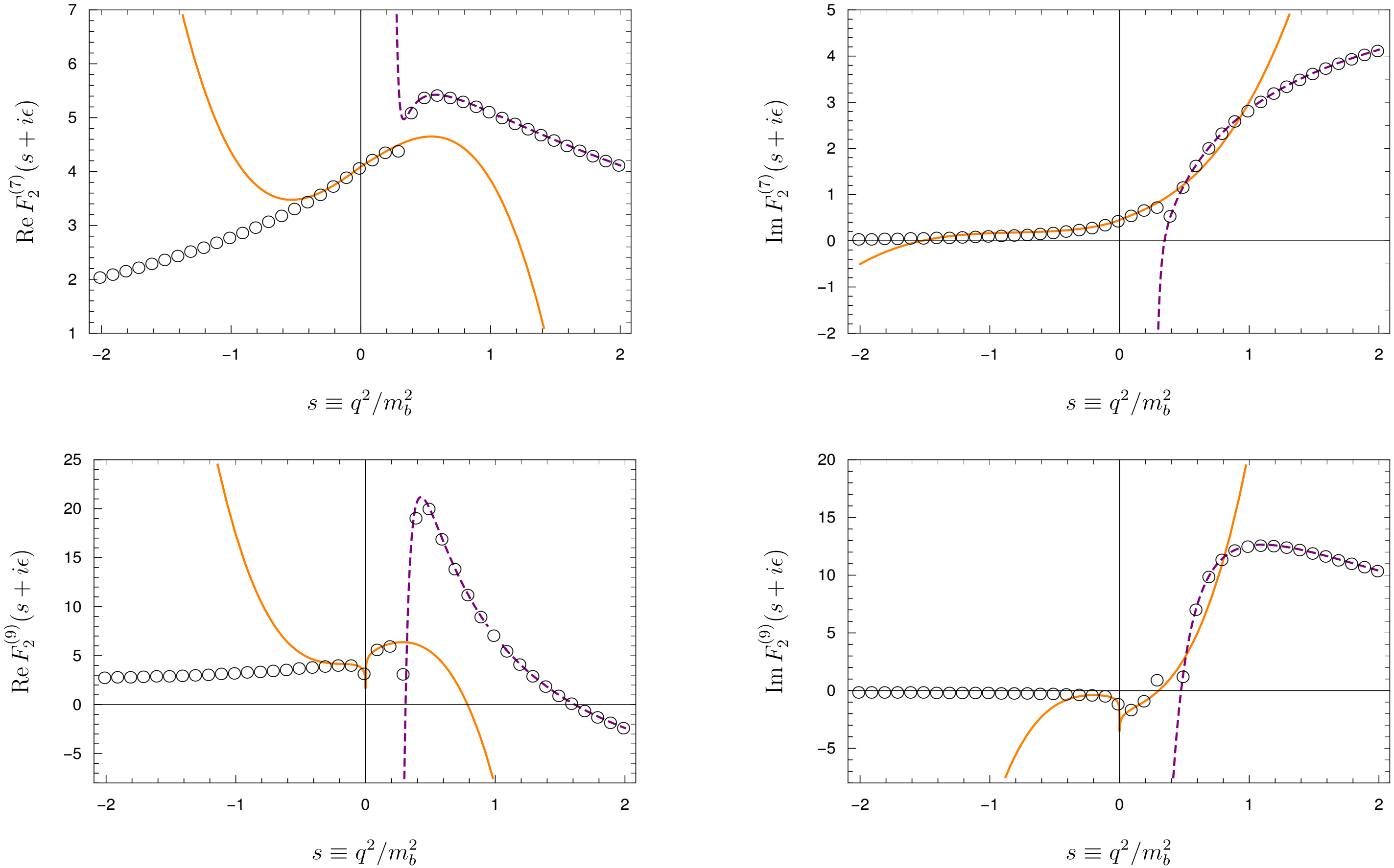
$\Delta C_7 = \mathcal{O}(\alpha_s)$ and $\Delta C_9^{LO} =$



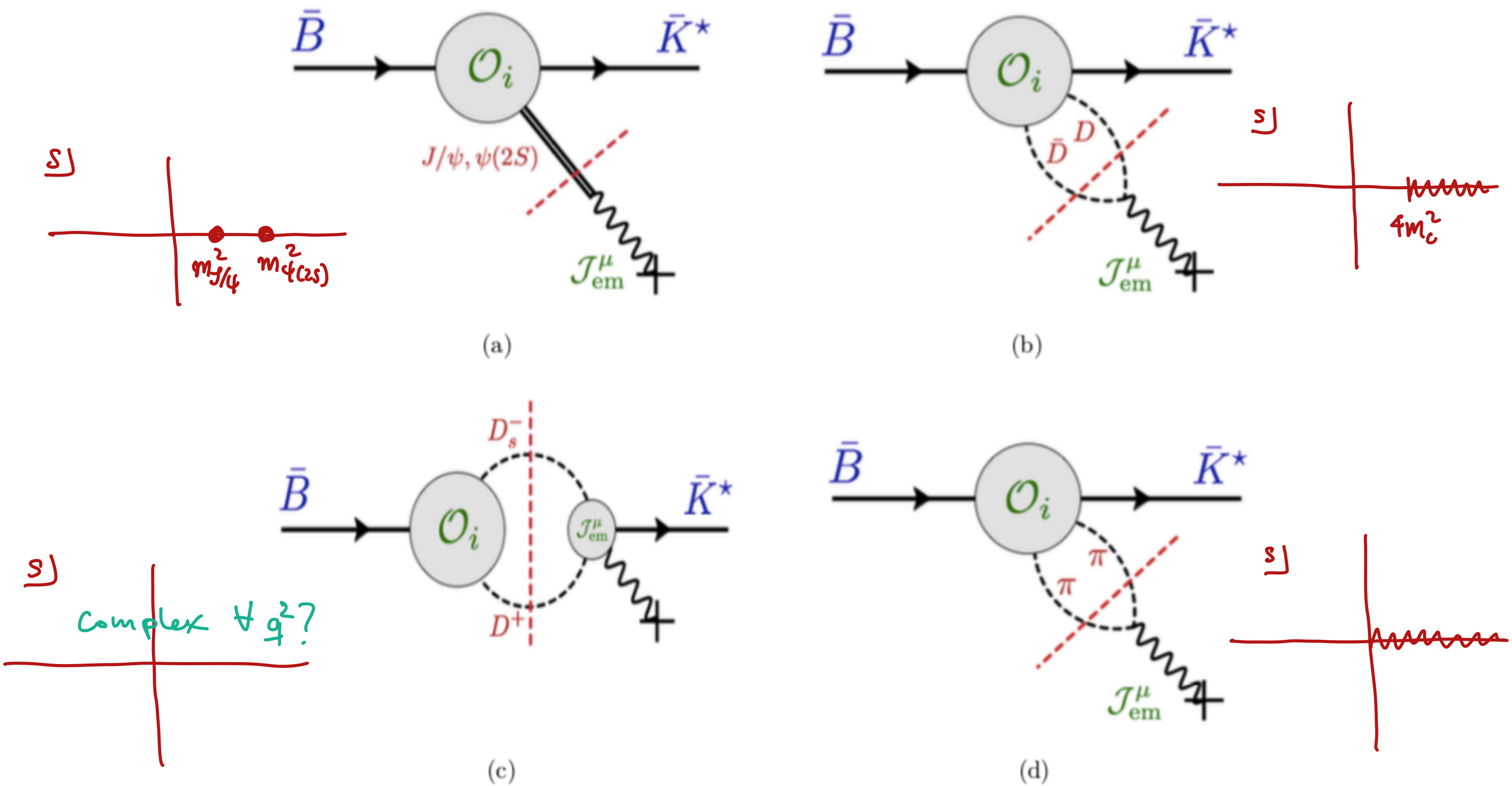
$\sim \log(4m_c^2 - q^2)$

Asatrian, Greub, Virto 2019

Results: Comparison to previous calculations: (Asatrian et al)

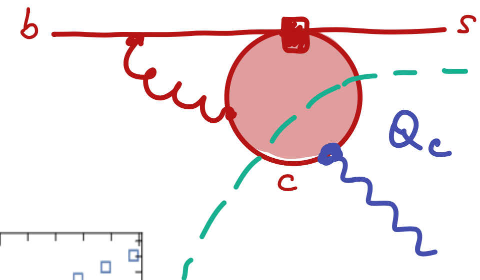


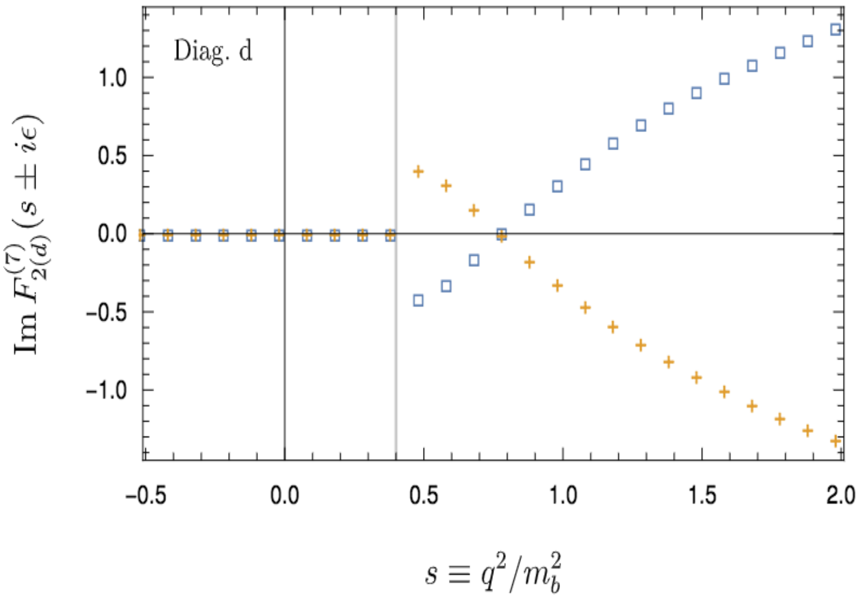
Checking analytic structure of $\mathcal{H}(q^2)$



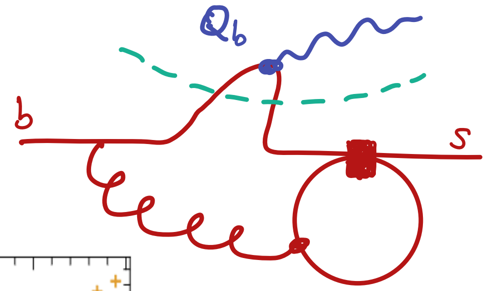
Asatrian, Greub, Virto 2019

Checking analytic structure of $\mathcal{H}(q^2)$

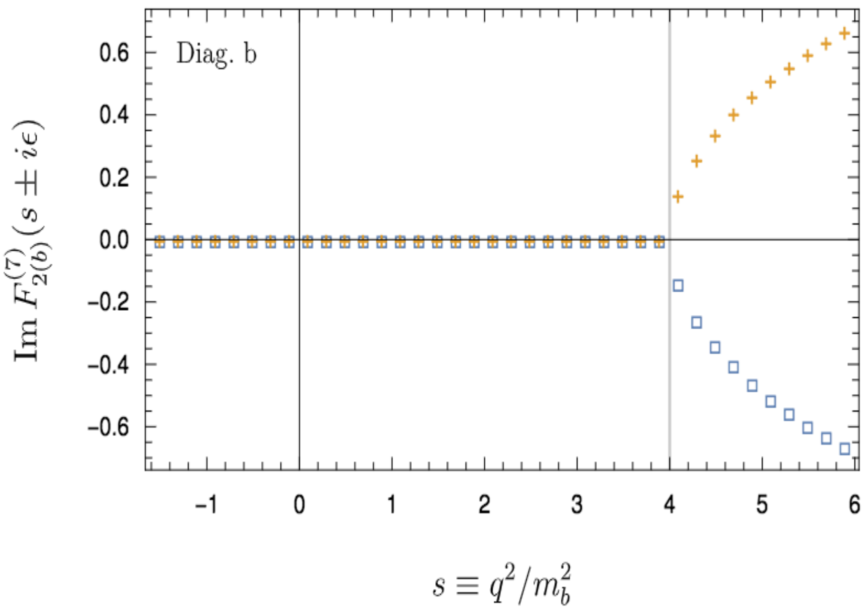




Asatrian, Greub, Virto 2019

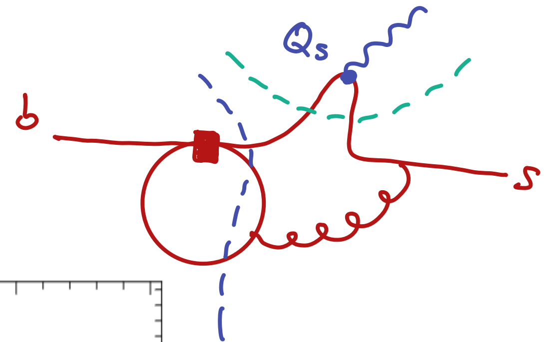


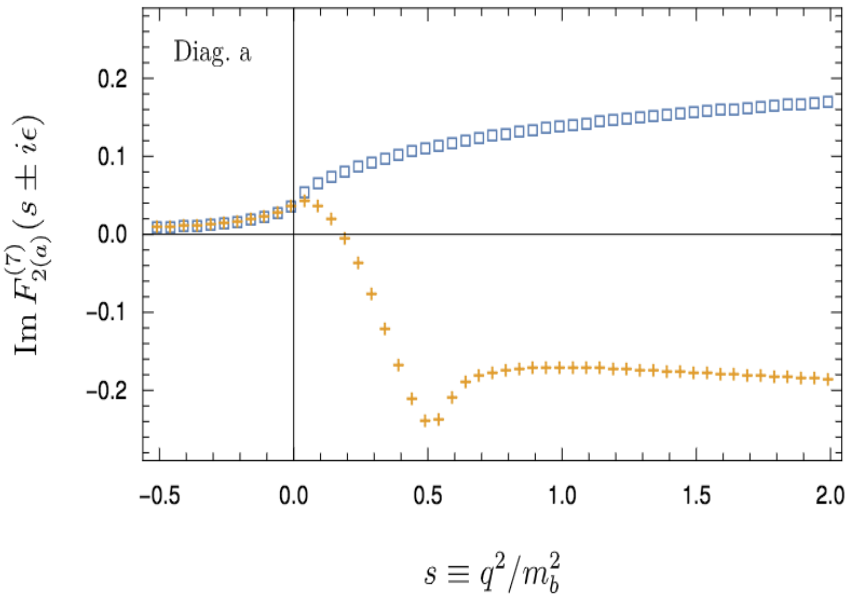
Checking analytic structure of $\mathcal{H}(q^2)$



Asatrian, Greub, Virto 2019

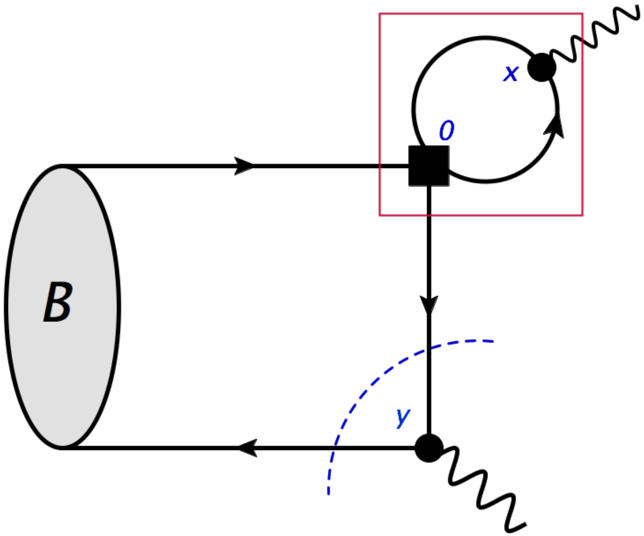
Checking analytic structure of $\mathcal{H}(q^2)$





Subleading LCOPE contributions

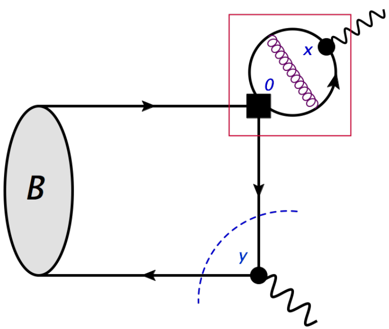
- LCSRs with B-meson DAs — Khodjamirian, Mannel, Pivovarov, Wang

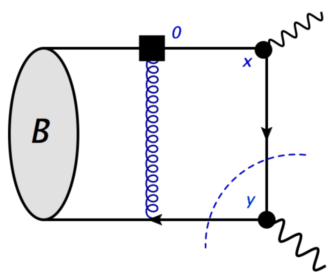


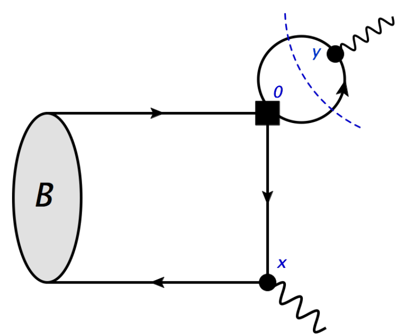
LC exp. of charm prop. Balitsky, Braun 1989

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{matching coeff}} [\bar{s}\,\Gamma\, b] + \cdots$$

$$\Rightarrow \; \mathcal{H}_\lambda = (\text{matching coeff}) \times \mathcal{F}_\lambda^{LCSR}$$







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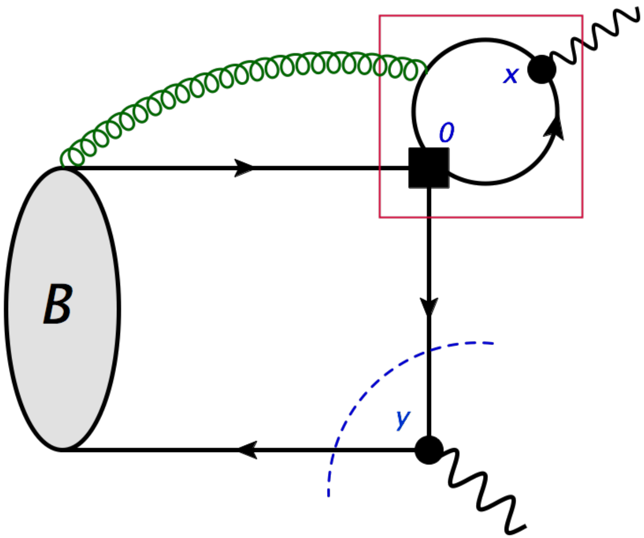
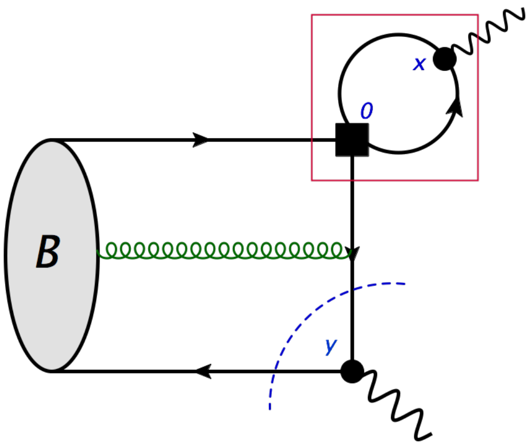


LC exp. of charm prop. Balitsky, Braun 1989

$$\xrightarrow{q^2 \ll 4m_c^2} \underbrace{\left(\frac{C_1}{3} + C_2\right) g(m_c^2, q^2)}_{\text{matching coeff}} [\bar{s}\,\Gamma\, b] +$$

$$+\ (\text{coeff}) \times [\,\bar{s}_L \gamma^\alpha (in_+ \cdot \mathcal{D})^n \tilde{G}_{\beta\gamma} b_L\,]\ + \cdots$$

3-particle correction to $\mathcal{F}_\lambda \longrightarrow$



Subleading LCOPE contributions

Recalculation of charm-loop effect Gubernari, van Dyk, Virto, 2011.09813

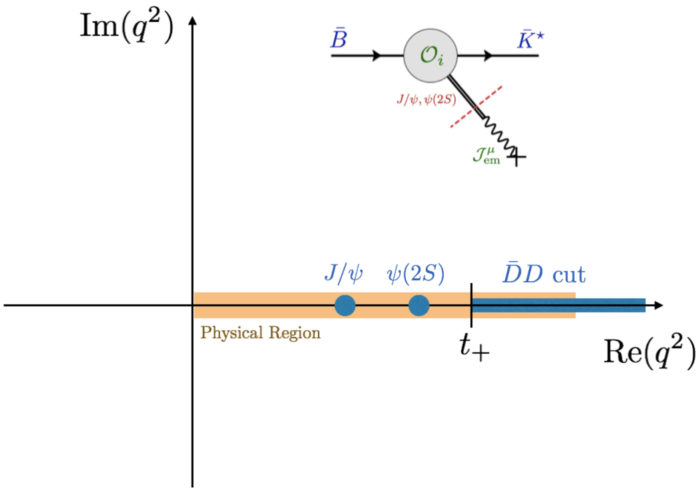
Transition	$\tilde{\mathcal{V}}(q^2 = 1\,\mathrm{GeV}^2)$	This work	Ref. [11]
$B \to K$	$\tilde{\mathcal{A}}$	$(+4.9 \pm 2.8) \cdot 10^{-7}$	$(-1.3^{+1.0}_{-0.7}) \cdot 10^{-4}$
$B \to K^*$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 3.6) \cdot 10^{-7}\,\mathrm{GeV}$	$(-1.5^{+1.5}_{-2.5}) \cdot 10^{-4}\,\mathrm{GeV}$
	$\tilde{\mathcal{V}}_2$	$(+3.3 \pm 2.0) \cdot 10^{-7}\,\mathrm{GeV}$	$(+7.3^{+14}_{-7.9}) \cdot 10^{-5}\,\mathrm{GeV}$
	$\tilde{\mathcal{V}}_3$	$(+1.1 \pm 1.0) \cdot 10^{-6}\,\mathrm{GeV}$	$(+2.4^{+5.6}_{-2.7}) \cdot 10^{-4}\,\mathrm{GeV}$
$B_s \to \phi$	$\tilde{\mathcal{V}}_1$	$(-4.4 \pm 5.6) \cdot 10^{-7}\,\mathrm{GeV}$	—
	$\tilde{\mathcal{V}}_2$	$(+4.3 \pm 3.1) \cdot 10^{-7}\,\mathrm{GeV}$	—
	$\tilde{\mathcal{V}}_3$	$(+1.7 \pm 2.0) \cdot 10^{-6}\,\mathrm{GeV}$	—

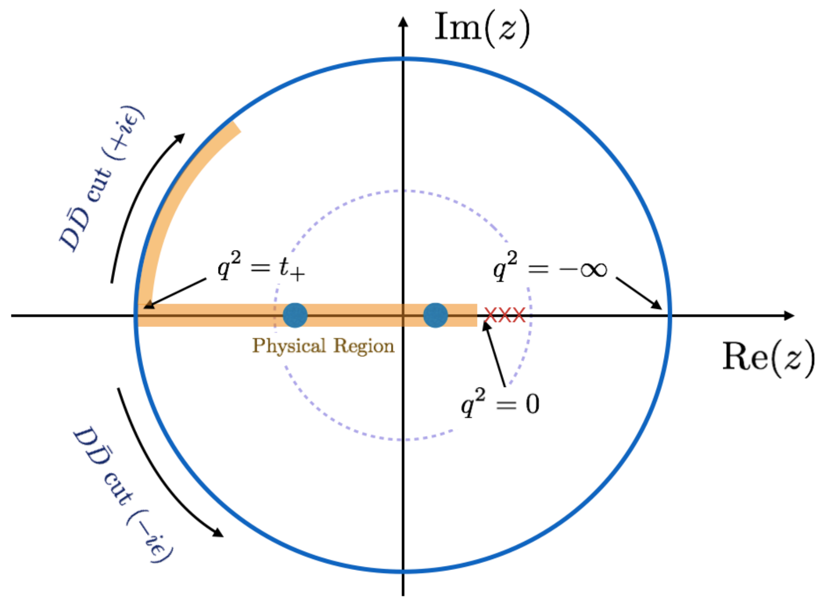
- We reproduce the result of KMPW'2010
- We incude complete set of 3-particle LCDAs Braun,Li,Manashov 2017
- Cancellations + Parametric lead to a reduction of the effect of **two orders of magnitude**
- Local matrix elements $\lambda_{E,H}$ crucial in this cancellation. Revisit (see e.g. Rahimi, Wald 2020)

Analytic continuation to physical q^2

Bobeth, Chrzaszcz, van Dyk, Virto 2017

z-parametrisation for $\mathcal{H}_\lambda(q^2)$





- $\hat{\mathcal{H}}_\lambda(q^2(z)) = (q^2 - M^2_{J/\psi})(q^2 - M^2_{\psi(2S)})\,\mathcal{H}_\lambda(q^2)$ is **analytic in** $|z| < 1$
- Taylor expand $\hat{\mathcal{H}}_\lambda(z)$ around $z = 0$: $\hat{\mathcal{H}}_\lambda(z) = \left[\sum_{k=0}^{K} \alpha_k^{(\lambda)} z^k\right] \mathcal{F}_\lambda(z)$ (*)
- Expansion needed for $|z| < 0.52$ ($-7\,\text{GeV}^2 \leq q^2 \leq 14\text{GeV}^2$) (*****)

Experimental constraints on z parametrisation

Bobeth, Chrzaszcz, van Dyk, Virto 2017

Experimental constraints :

- The residues of the poles are given by $B \to K^* \psi_n$:

$$\mathcal{H}_\lambda(q^2 \to M_{\psi_n}^2) \sim \frac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2 (q^2 - M_{\psi_n}^2)} + \cdots$$

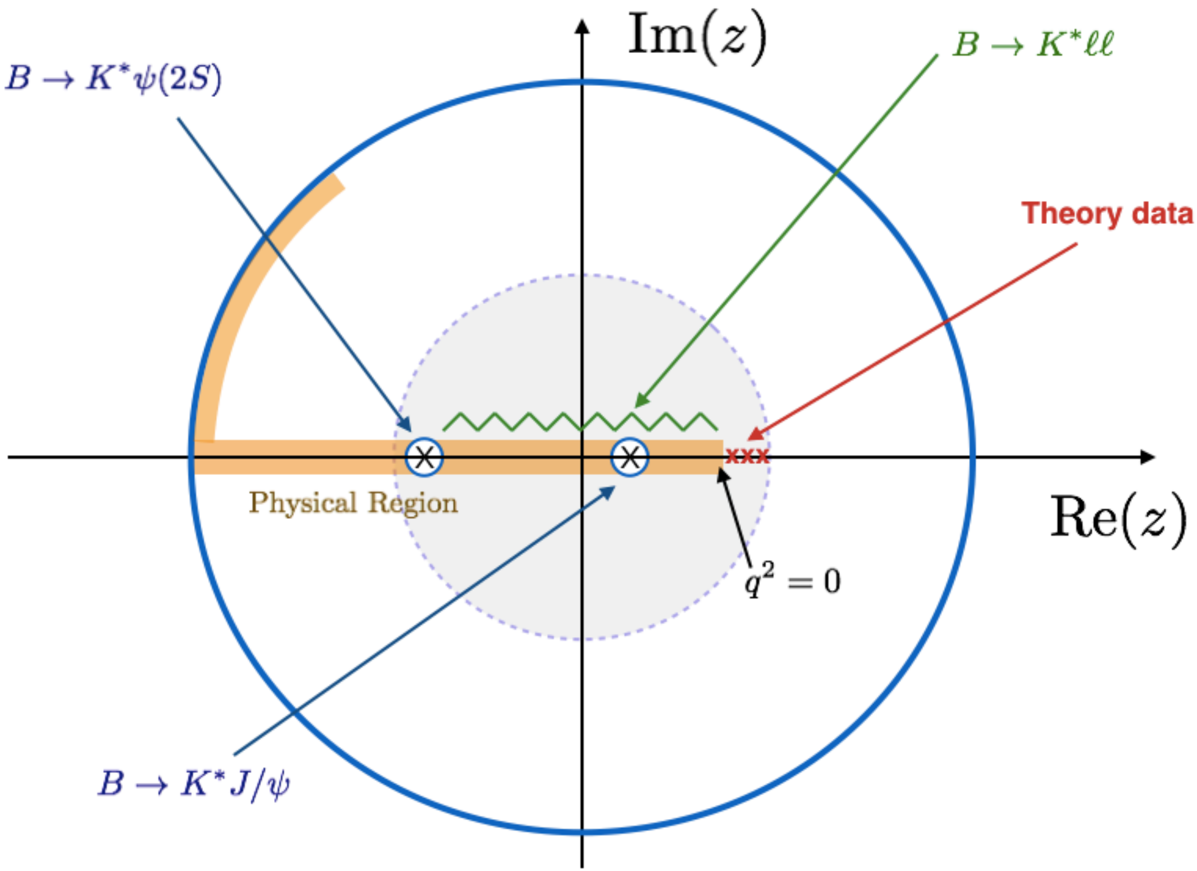
- Angular analyses Belle, Babar, LHCb determine :

$$|r_\perp^{\psi_n}|,\ |r_\parallel^{\psi_n}|,\ |r_0^{\psi_n}|,\ \arg\{r_\perp^{\psi_n} r_0^{\psi_n *}\},\ \arg\{r_\parallel^{\psi_n} r_0^{\psi_n *}\},$$

where $$r_\lambda^{\psi_n} \equiv \underset{q^2 \to M_{\psi_n}^2}{\text{Res}} \frac{\mathcal{H}_\lambda(q^2)}{\mathcal{F}_\lambda(q^2)} \sim \frac{M_{\psi_n} f_{\psi_n}^* \mathcal{A}_\lambda^{\psi_n}}{M_B^2\, \mathcal{F}_\lambda(M_{\psi_n}^2)}$$

Fit to z-parametrisation

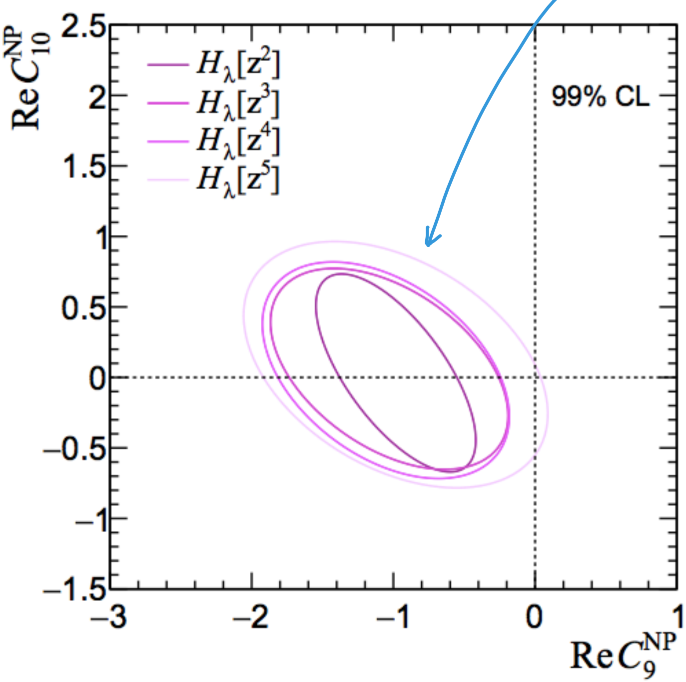
Bobeth, Chrzaszcz, van Dyk, Virto 2017



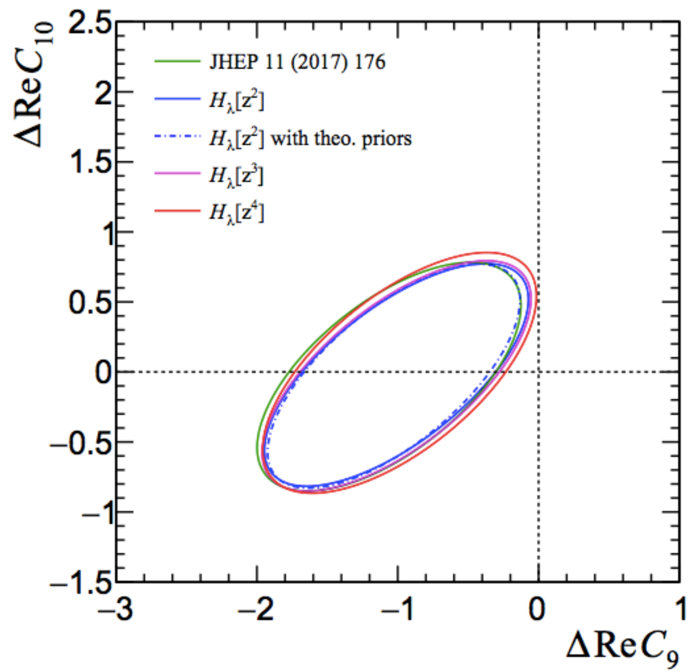
Prospects: LHC Run-2 unbinned fits to z-parametrization

Would be interesting to have a bound.

Chrzaszcz, Mauri, Serra, Coutinho, van Dyk 1805.06378



Mauri, Serra, Coutinho 1805.06401



Unbinned fits to $B \to K^* \mu\mu$ (Left) and $B \to K^* \ell\ell$ (Right)

Dispersive bound

$$\Pi^{\mu\nu}(q) \equiv i\int d^4x\, e^{iq\cdot x}\langle 0|T\left\{O^\mu(q;x), O^{\nu,\dagger}(q;0)\right\}|0\rangle = \left(\frac{q^\mu q^\nu}{q^2} - g^{\mu\nu}\right)\Pi(q^2)$$

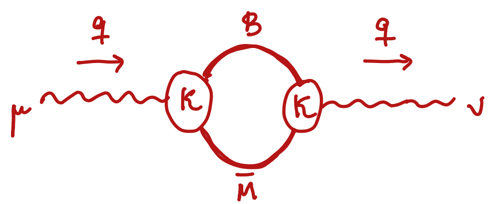
Here, the operators $O^\mu(q;x)$ and $O^{\dagger,\nu}(q;0)$ are defined as

$$O^\mu(q;x) = \left(\frac{-16\pi^2 i}{q^2}\right)\int d^4y\, e^{+iq\cdot y}\, T\{j^\mu_{\text{em}}(x+y), (C_1\mathcal{O}_1 + C_2\mathcal{O}_2)(x)\}\,,$$

$$O^{\nu,\dagger}(q;0) = \left(\frac{+16\pi^2 i}{q^2}\right)\int d^4z\, e^{-iq\cdot z}\, T\{j^\nu_{\text{em}}(z), (C_1\mathcal{O}_1 + C_2\mathcal{O}_2)^\dagger(0)\}\,. = \frac{K^\nu(q)}{q^2}$$

Idea:

$$\text{Im}\left[\quad\right]$$



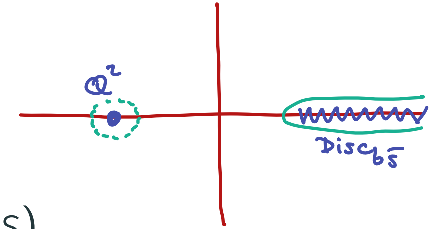
$$\sim \left|\mathcal{H}^{B\to M}(q^2)\right|^2 + \underbrace{\dots}_{\text{positive}}$$

or : $|\mathcal{H}^{B\to M}|$ constrained by $e^+e^- \to b\bar{s}$ inclusive rate.

Dispersive bound

Gubernari, van Dyk, Virto 2020

Twice-subtracted dispersion relation:



$$\chi^{\rm OPE}(Q^2) \equiv \frac{1}{2i\pi}\int_0^\infty ds\, \frac{{\rm Disc}_{b\bar{s}}\Pi^{\rm had}(s)}{(s-Q^2)^3}$$

$$\frac{3}{32i\pi^3}\,{\rm Disc}_{b\bar{s}}\Pi^{\rm had}(s) = \frac{2M_B^4\,\lambda^{3/2}(M_B^2,M_K^2,s)}{s^4}\left|\mathcal{H}_0^{B\to K}(s)\right|^2\theta(s-s_{BK})$$

$$+\frac{2M_B^6\sqrt{\lambda(M_B^2,M_{K*}^2,s)}}{s^3}\left(\left|\mathcal{H}_\perp^{B\to K^*}(s)\right|^2+\left|\mathcal{H}_\parallel^{B\to K^*}(s)\right|^2+\frac{M_B^2}{s}\left|\mathcal{H}_0^{B\to K^*}(s)\right|^2\right)\theta(s-s_{BK^*})$$

$$+\frac{M_B^6\sqrt{\lambda(M_{B_s}^2,M_\phi^2,s)}}{s^3}\left(\left|\mathcal{H}_\perp^{B_s\to\phi}(s)\right|^2+\left|\mathcal{H}_\parallel^{B_s\to\phi}(s)\right|^2+\frac{M_{B_s}^2}{s}\left|\mathcal{H}_0^{B_s\to\phi}(s)\right|^2\right)\theta(s-s_{B_s\phi})$$

$+$ further positive terms (e.g. $\Lambda_b\to\Lambda$, $B\to K\pi\pi\pi\pi\pi\pi$, ...)

Redefine $\mathcal{H}_i$ as before:

"Blaschke factor" $\sim \prod_{J/\psi,\,\psi(2S)} (z - z_\psi)$

$$\hat{\mathcal{H}}_0^{B\to P}(z) \equiv \phi_0^{B\to P}(z)\,\mathcal{P}(z)\,\mathcal{H}_0^{B\to P}(z)\,,$$

$$\hat{\mathcal{H}}_\lambda^{B\to V}(z) \equiv \phi_\lambda^{B\to P}(z)\,\mathcal{P}(z)\,\mathcal{H}_\lambda^{B\to P}(z)\,,$$

"outer functions": Among other things contain $\chi(Q^2)$.

Expand in ortogonal polynomials in unit circle:

$$\hat{\mathcal{H}}_\lambda^{B\to M}(z) = \sum_{n=0}^{\infty} a_{\lambda,n}^{B\to M}\, p_n^{B\to M}(z)$$

Better for dispersive bound than z-Taylor expansion.

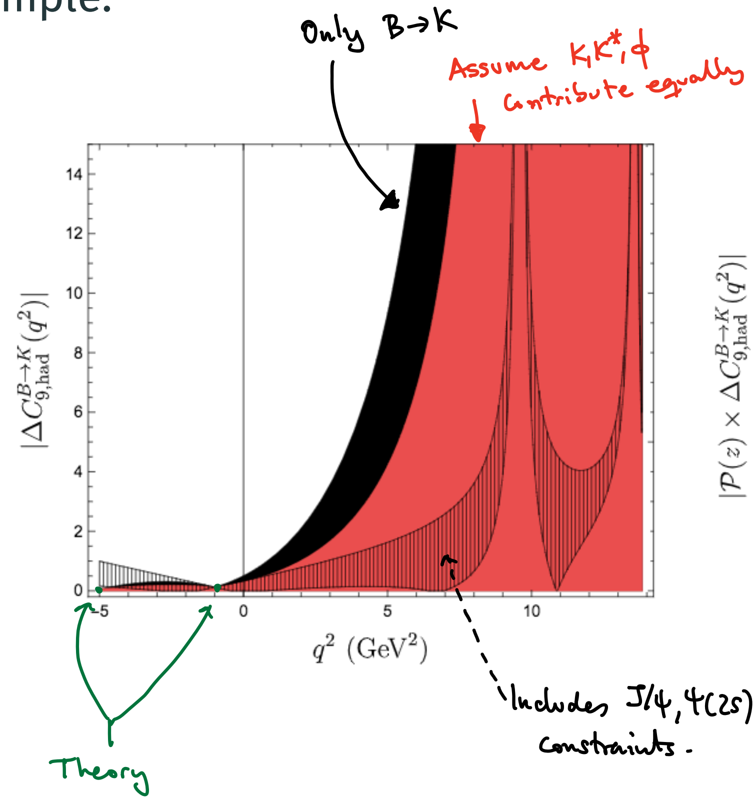
The dispersive bound then takes the simple form

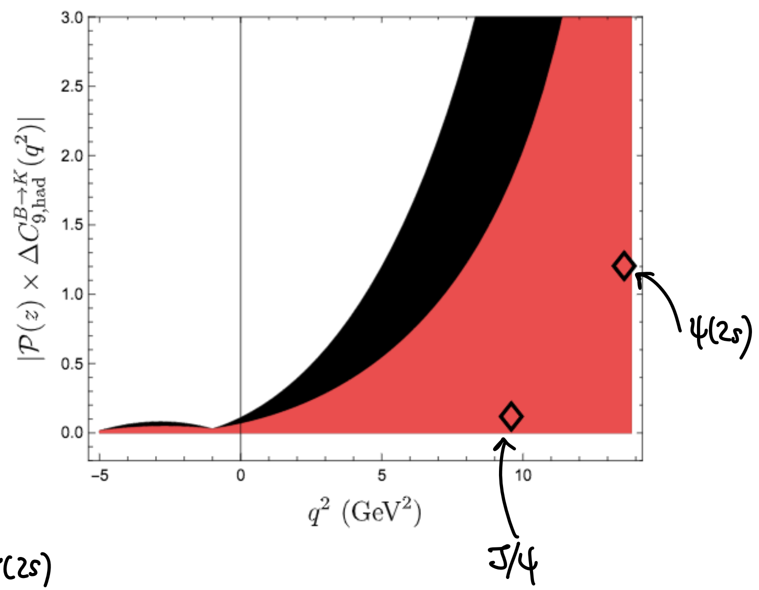
$$\boxed{\sum_{n=0}^{\infty} \left\{ 2\left|a_{0,n}^{B\to K}\right|^2 + \sum_{\lambda=\perp,\parallel,0} \left[2\left|a_{\lambda,n}^{B\to K^*}\right|^2 + \left|a_{\lambda,n}^{B_s\to\phi}\right|^2 \right] \right\} < 1\,.}$$

Further positive terms can be included

Example:

$$\Delta C_{9,\text{had}}^{B\to K}(q^2) = \frac{32\pi^2 M_B^2}{q^2}\,\frac{\mathcal{H}_0^{B\to K}(q^2)}{\mathcal{F}_0^{B\to K}(q^2)}$$





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OBJECTIVE:

Perform a global analysis of $B_{u,d} \to K^{(*)}\mu^+\mu^-$ and $B_s \to \phi\mu^+\mu^-$ including:

- Updated unified fits to LQCD and LCSRs with B-meson LCDAs for all Local Form Factors.
- Finite-width effect for local $B_{u,d} \to K^*$ form factors.
- Global determination of z-expansion parameters for all Non-Local Form factors, including:
  - Updated calculations of theory points at $q^2 < 0$.
  - (Up-to-date) Experimental data on $B \to M J/\psi$ to fix $\mathcal{H}_{c,\lambda}^{B\to M}(M_{J/\psi}^2)$.
  - Unitarity bound for z-expansion coefficients
- Global NP fit to $C_{9,10}$ using the Improved SM predictions from above.

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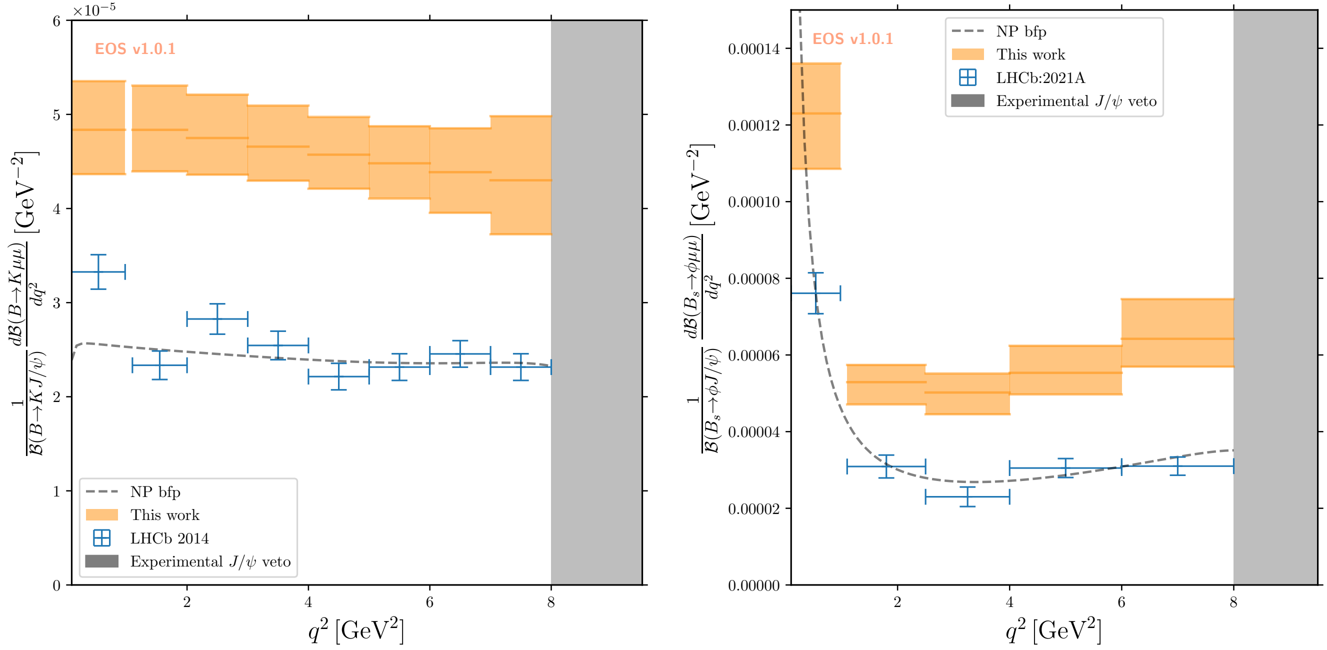
PRELIMINARY



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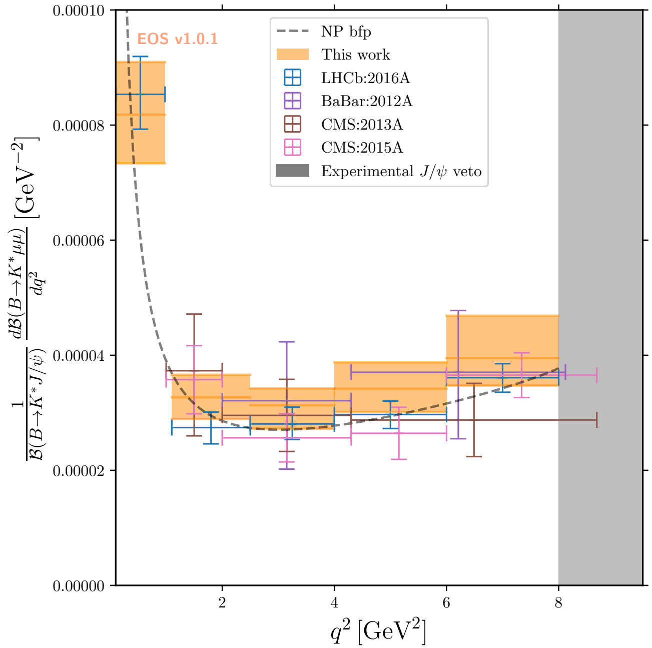

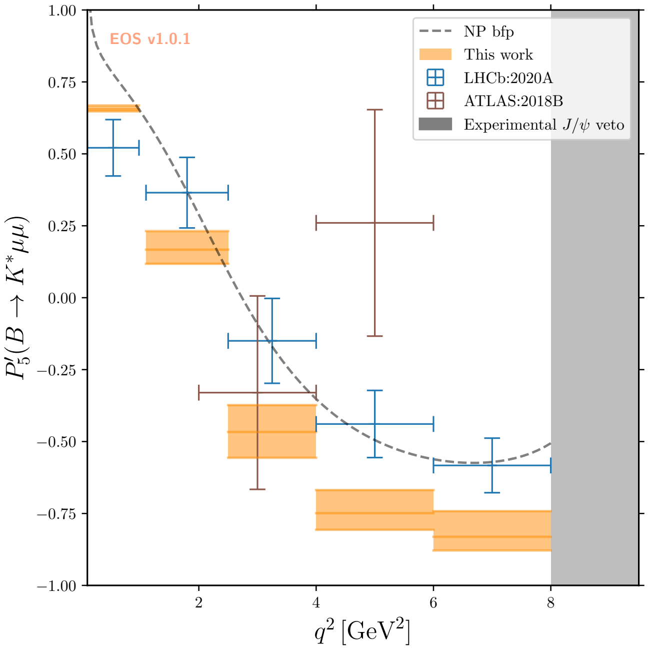

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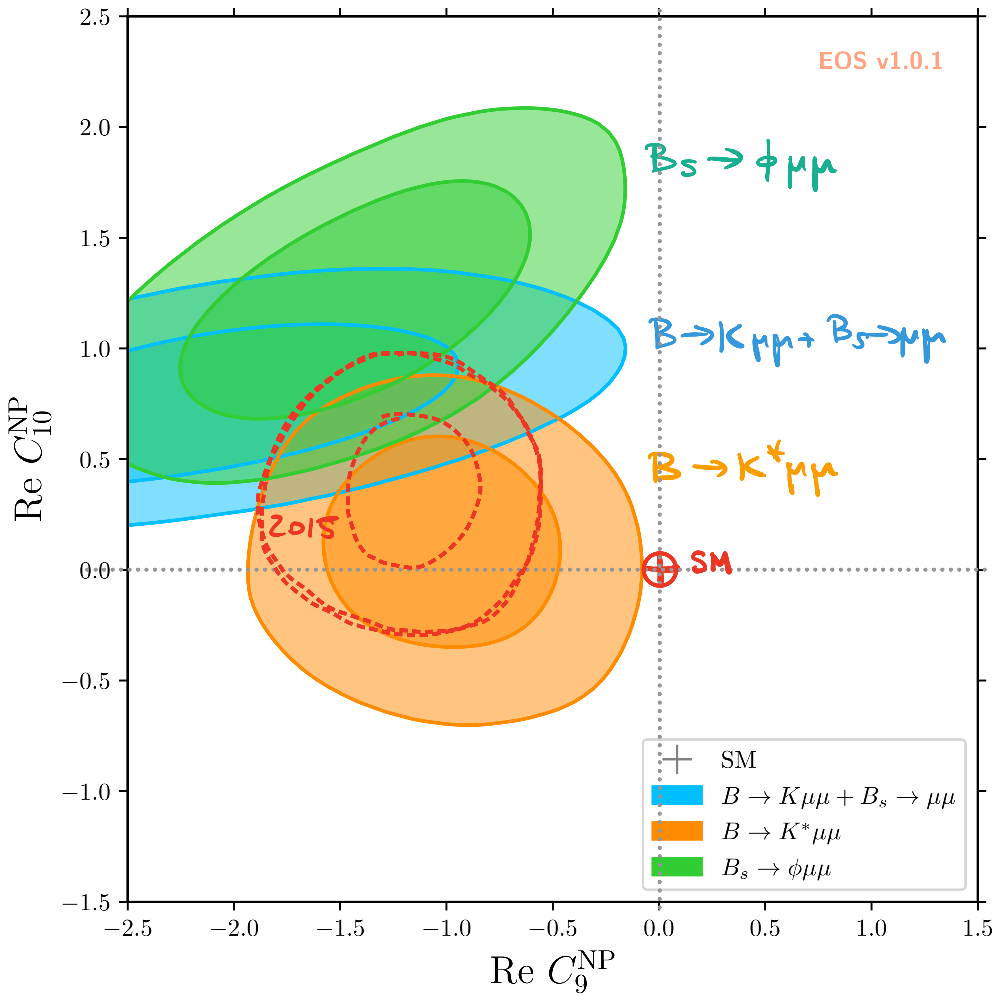
PRELIMINARY



Summary

- NP PATTERNS BEHIND $b \to s\mu\mu$ ANOMALIES $\simeq$ 2014.
- UNDERSTANDING NP in $b \to s\ell^+\ell^-$ REQUIRES A GOOD CONTROL OF NON-LOCAL FORM FACTORS.

  — ALSO FOR LFUV OBSERVABLES —

- NEED: – THEORY! (IN THE CORRECT PLACE).

  – A RIGOROUS (NOT AD-HOC) PARAM. OF q^2 SHAPE.

  – INTERPOLATION → NOT EXTRAPOLATION

  ⇒ DATA ON J/ψ, $\psi(2S)$.