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No-core shell model calculations of the nuclear structure corrections to super-allowed nuclear beta decays

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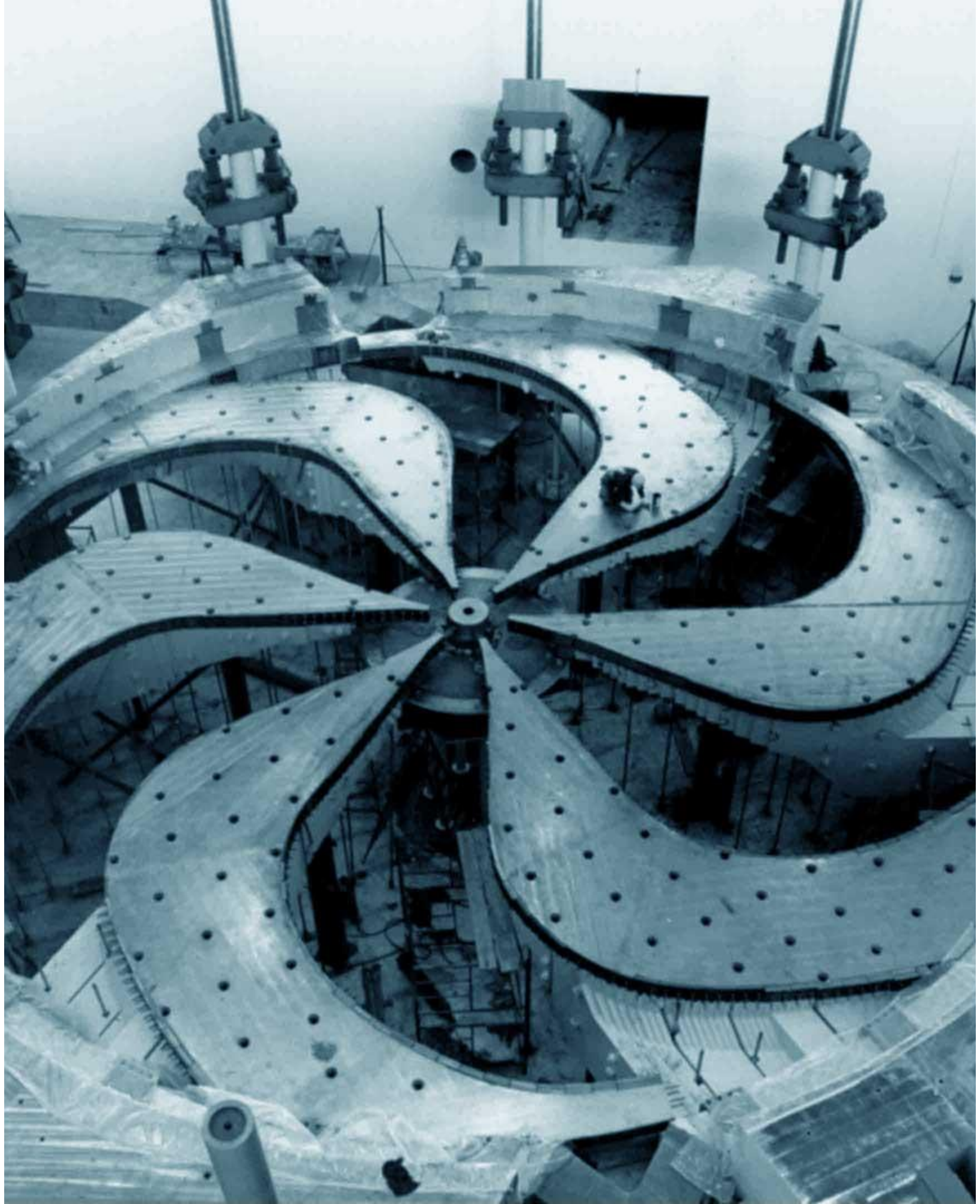


Discovery, accelerated

$\mathrm{V_{ud}}$ element of CKM matrix

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}(\bar{u}_L, \bar{c}_L, \bar{t}_L)\gamma^{\mu}W_{\mu}^{+}V_{CKM}\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$$

CKM matrix

- CKM unitarity sensitive probe of BSM physics
  - $\mathrm{V_{ud}}$ element from super-allowed Fermi transitions **[1,2]**
  - theoretical uncertainties dominant

$$|V_{ud}|^2 = \frac{\hbar^7}{G_F^2 m_e^5 c^4}\frac{\pi^3 \ln(2)}{\mathcal{F}t}$$

G_F ≡ Fermi coupling constant
determined from muon β decay

Fermi transitions required nuclear theory input

Shift in the unitarity landscape

- New dispersion integral approach indicates discrepancy **[3,4]**
- Disagreement is $(2-3)\sigma$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985\;(3)_{V_{ud}}(4)_{V_{us}}$$

[1] C. Y. Seng (2022)
[2] P.A. Zyla et al. (2020)
[3] C. Y. Seng et al. (2018)
[4] Gorchtein et al. (2019)

Nuclear Fermi transitions

- **CVC hypothesis:** Pure Fermi transitions give nucleus independent ft values

[4] $$\mathcal{F}t = \frac{K}{G_V^2 |M_{F0}|^2 (1 + \Delta_R^V)}$$

$G_V \equiv$ **vector coupling constant** for nuclear beta decay

$|M_{F0}|^2 \equiv \left|\langle \phi | T_\pm | \psi \rangle\right|^2$

Fermi transition corrections

$$\mathcal{F}t(1 + \Delta_R^V) = ft(1 + \delta'_R)\underline{(1 - \delta_C + \delta_{NS})}$$

- NS corrections
  - hadronic matrix elements modified by nuclear environment
  - renormalization of Fermi matrix element due to INC forces

[4] Gorchtein et al. (2019)

Nuclear Fermi transitions

$$\mathcal{F}t(1+\Delta_R^V) = ft(1+\delta'_R)\underline{(1-\delta_C+\delta_{NS})}$$

- NS corrections
  - hadronic matrix elements modified by nuclear environment
  - renormalization of Fermi matrix element due to INC forces

Historical treatment (Hardy and Towner) **[5]**

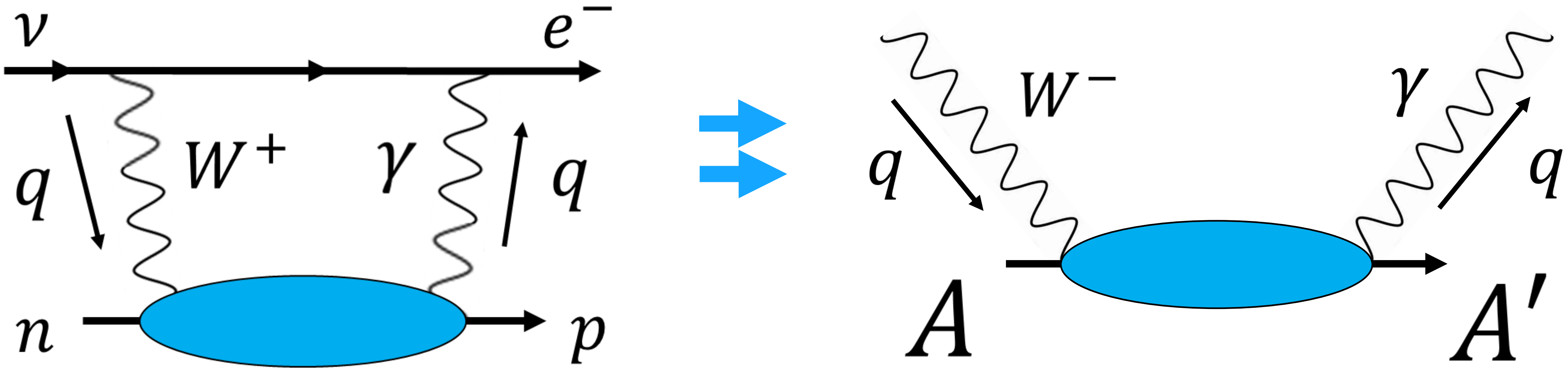
- δ_{NS} from shell model and approximate single-nucleon currents
- δ_C from shell model with Woods-Saxon potential
- Dominant approach for a decade!

[5] Hardy et al. (2020)

γW-box

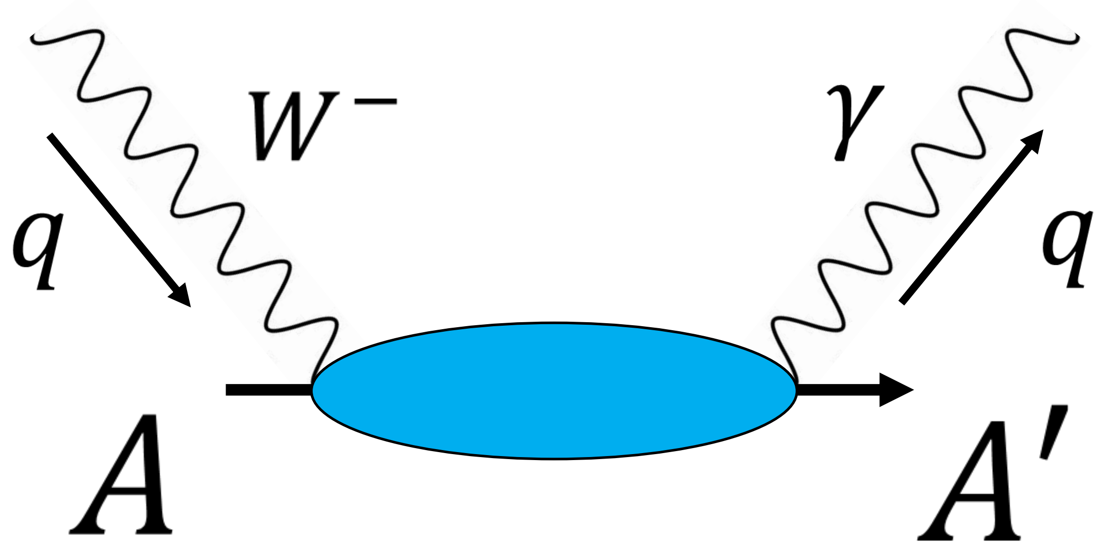
- Inner radiative correction governed by axial γW-box

$$\delta M = -i\sqrt{2}G_F e^2 L^\lambda \int \frac{d^4q}{(2\pi)^4} \frac{M_W^2}{M_W^2 - q^2} \frac{\epsilon^{\mu\nu\alpha\lambda}}{[(p_e - q)^2 - m_e^2]q^2} T_{\mu\nu}(p', p, q)$$



Δ_R^V to δ_{NS}

- Nuclear environment modifies hadronic matrix elements in Δ_R^V
- δ_{NS} parameterizes nuclear structure correction to γW-box



$$\delta_{NS} = 2[\Box_{\gamma W}^{VA,\mathrm{nuc.}} - \Box_{\gamma W}^{VA,\mathrm{free\ n}}]$$

$$T^{\mu\nu}_{\gamma W,\mathrm{nuc.}}(p,q) = \frac{1}{2}\int d^4x\; e^{iq\cdot x}\langle\phi_f(p)\big|T\big[J^{\mu}_{\mathrm{em}}(x)J^{\nu}_{W}(0)^{\dagger}\big]\big|\phi_i(p)\rangle$$

δ_{NS}

- Want to evaluate with NCSM eigenstates
- Express currents in momentum space **[6]**
  1) Fourier transform 3-currents
  2) Relate plane-wave states to QM states
  3) Multipole expansion of invariant amplitude T_3

$$J(\vec{q}) = \int d^3r \; e^{-i\vec{q}\cdot\vec{r}} J(0,\vec{r})$$

$$T_3(q_0, Q^2) = -4\pi i \frac{q_0}{q}\sqrt{M_i M_f}\sum_{J=1}^{\infty}(2J+1)$$
$$\times \langle A\lambda_f J_f M_f \Big| \Big[T_{J0}^{mag}(q)\, G(M_f + q_0 + i\epsilon)\, T_{J0}^{5,el}(q) + T_{J0}^{el}(q)\, G(M_f + q_0 + i\epsilon)\, T_{J0}^{5,mag}(q)$$
$$+ T_{J0}^{5,mag}(q)\, G(M_i - q_0 + i\epsilon)\, T_{J0}^{el}(q) + T_{J0}^{5,el}(q)\, G(M_i - q_0 + i\epsilon)\, T_{J0}^{mag}(q) \Big] \Big| A\lambda_i J_i M_i \rangle$$

[6] Haxton et al. (2007)

δ_{NS}

- Want to evaluate with NCSM eigenstates
- Express currents in momentum space **[6]**
  1) Fourier transform 3-currents
  2) Relate plane-wave states to QM states
  3) Multipole expansion of invariant amplitude T_3

$$J(\vec{q}) = \int d^3r \; e^{-i\vec{q}\cdot\vec{r}} J(0,\vec{r})$$

How do we efficiently compute nuclear Green's functions?

$$T_3(q_0, Q^2) = -4\pi i \frac{q_0}{q} \sqrt{M_i M_f} \sum_{J=1}^{\infty} (2J+1)$$
$$\times \langle A\lambda_f J_f M_f | \Big[T^{mag}_{J0}(q) \, G(M_f + q_0 + i\epsilon) \, T^{5,el}_{J0}(q) + T^{el}_{J0}(q) \, G(M_f + q_0 + i\epsilon) \, T^{5,mag}_{J0}(q)$$
$$+ T^{5,mag}_{J0}(q) \, G(M_i - q_0 + i\epsilon) \, T^{el}_{J0}(q) + T^{5,el}_{J0}(q) \, G(M_i - q_0 + i\epsilon) \, T^{mag}_{J0}(q) \Big] | A\lambda_i J_i M_i \rangle$$

[6] Haxton et al. (2007)

No-core shell model (NCSM)

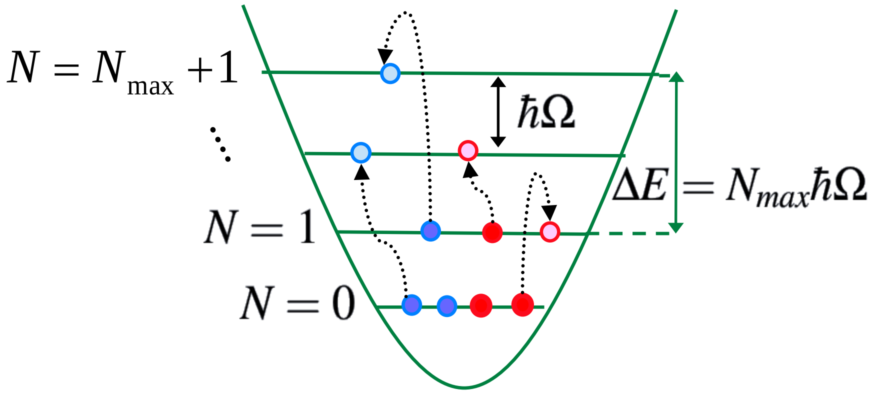
- *Ab initio* approach to many-body Schrödinger equation for bound states and narrow resonances **[7]**

$$H\left|\Psi_A^{J^\pi T}\right\rangle = E^{J^\pi T}\left|\Psi_A^{J^\pi T}\right\rangle$$

$$|\Psi_A^{J^\pi T}\rangle = \sum_{N=0}^{N_{max}} \sum_{\alpha} c_{N\alpha}^{J^\pi T} |\Phi_{N\alpha}^{J^\pi T}\rangle$$

Anti-symmetrized products of many-body HO states

  - NN+3N interactions are sole input
  - Two body: NN-N^4LO(500) **[8]**
  - Three body: $3N_{lnl}$ **[9]**



Accessible transitions

^{10}C → ^{10}B **^{14}O → ^{14}N**

[7] Barrett et al. (2013)
[8] Entem et al. (2017)
[9] Somà et al. (2020)

Lanczos continued fractions method

- Reformulate as inhomogeneous Schrödinger equation **[10]**

$$(H - E\mathbb{1})\left|\Phi_A^{J^\pi T}\right\rangle = \hat{O}\left|\Psi_A^{J^\pi T}\right\rangle$$

$$\begin{aligned}
H\mathbf{v}_1 &= \alpha_1\mathbf{v}_1 + \beta_1\mathbf{v}_2 \\
H\mathbf{v}_2 &= \beta_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \beta_2\mathbf{v}_3 \\
H\mathbf{v}_3 &= \beta_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 + \beta_3\mathbf{v}_4 \\
H\mathbf{v}_4 &= \beta_3\mathbf{v}_3 + \alpha_4\mathbf{v}_4 + \beta_4\mathbf{v}_5
\end{aligned}$$

$$\left|v_1\right\rangle = \frac{\hat{O}\left|\Psi_A^{J^\pi T}\right\rangle}{\left\langle\Psi_A^{J^\pi T}\middle|\hat{O}^\dagger\hat{O}\middle|\Psi_A^{J^\pi T}\right\rangle}$$

Choose specific starting vector

[10] Marchisio et al. (2003)
[11] Haydock (1974)

Lanczos continued fractions method

- Reformulate as inhomogeneous Schrödinger equation **[10]**

$$(H - E\mathbb{1})\left|\Phi_A^{J^\pi T}\right\rangle = \hat{O}\left|\Psi_A^{J^\pi T}\right\rangle$$

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H\mathbf{v}_1 &= \alpha_1\mathbf{v}_1 + \beta_1\mathbf{v}_2 \\
H\mathbf{v}_2 &= \beta_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \beta_2\mathbf{v}_3 \\
H\mathbf{v}_3 &= \beta_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 + \beta_3\mathbf{v}_4 \\
H\mathbf{v}_4 &= \beta_3\mathbf{v}_3 + \alpha_4\mathbf{v}_4 + \beta_4\mathbf{v}_5
\end{aligned}$$

- Resolvent cast in terms of Lanczos basis vectors with continued fraction coefficients **[11]**
- Avoids direct calculation of intermediate nuclear states

[10] Marchisio et al. (2003)
[11] Haydock (1974)

δ_{NS} in NCSM

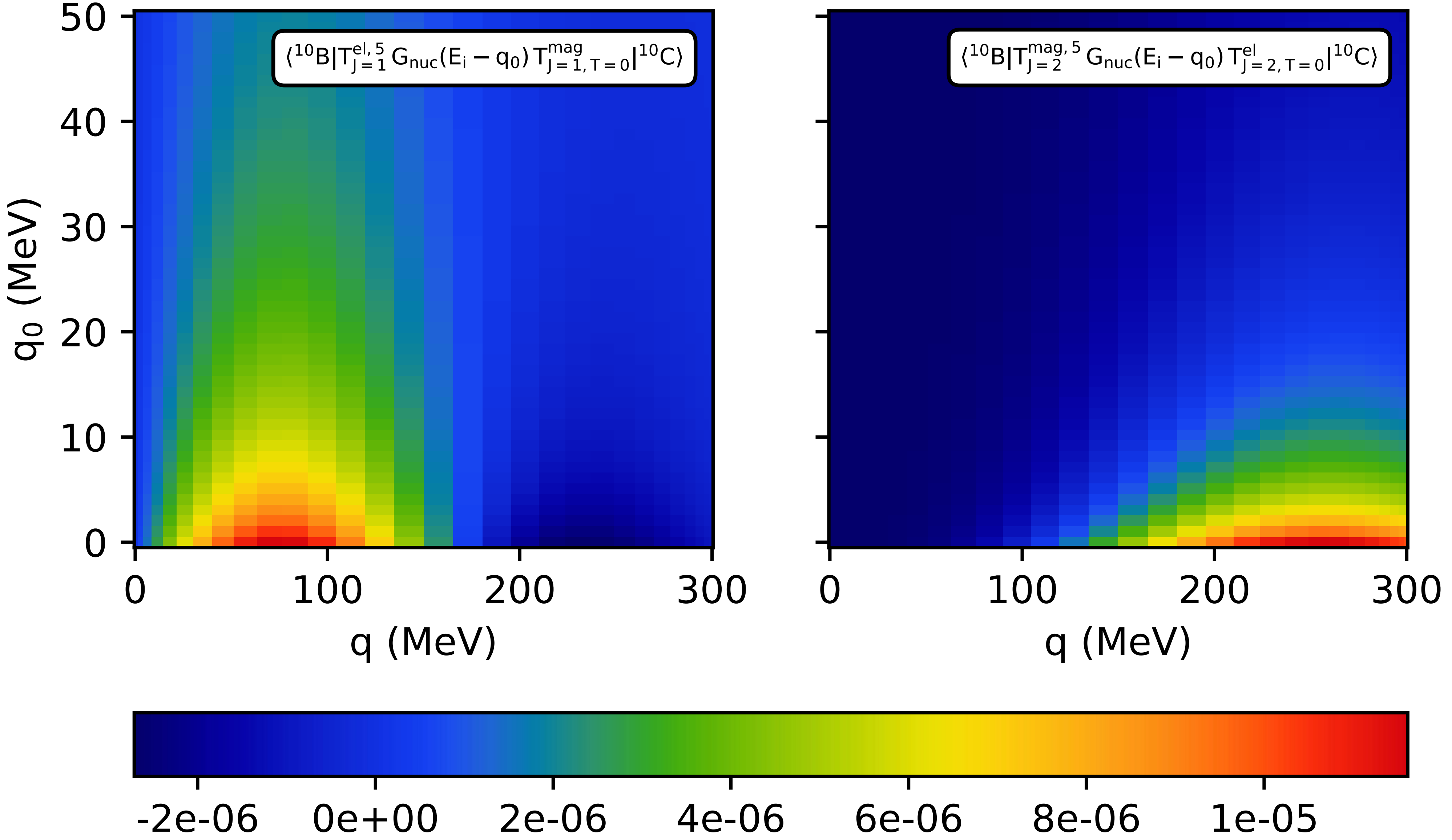
$$T_3(q_0, Q^2) = -4\pi i \frac{q_0}{q} \sqrt{M_i M_f} \sum_{J=1}^{\infty} (2J+1)$$

$$\times \langle A\lambda_f J_f M_f | \Big[T_{J0}^{mag}(q)\, G(M_f + q_0 + i\epsilon)\, T_{J0}^{5,el}(q) + T_{J0}^{el}(q)\, G(M_f + q_0 + i\epsilon)\, T_{J0}^{5,mag}(q)$$

$$+ T_{J0}^{5,mag}(q)\, G(M_i - q_0 + i\epsilon)\, T_{J0}^{el}(q) + T_{J0}^{5,el}(q)\, G(M_i - q_0 + i\epsilon)\, T_{J0}^{mag}(q) \Big] | A\lambda_i J_i M_i \rangle$$

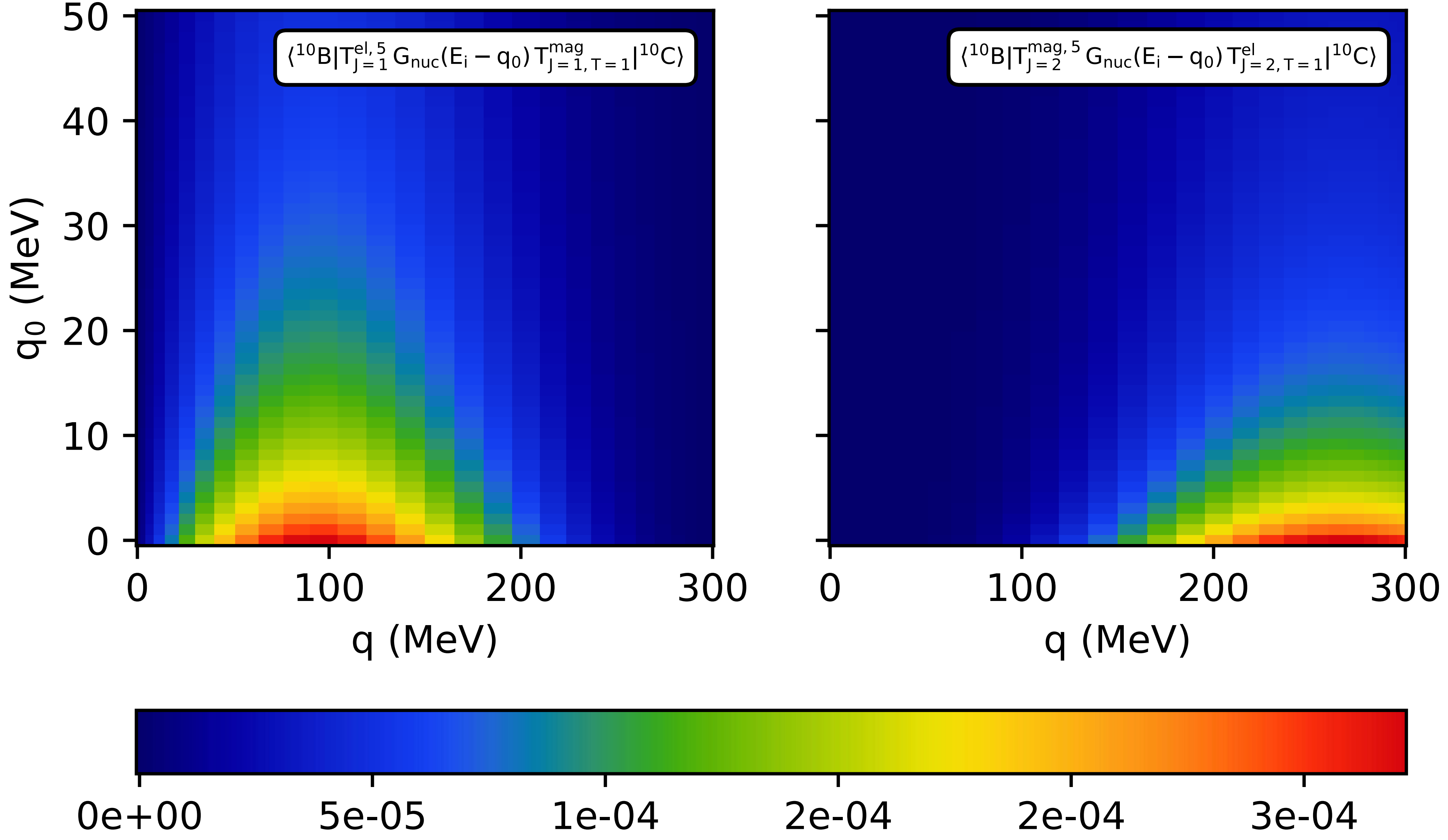
$G(M_i - q_0 + i\epsilon)$ terms: $T = 0$ EM current

Preliminary



$G(M_i - q_0 + i\epsilon)$ terms: $T = 1$ EM current

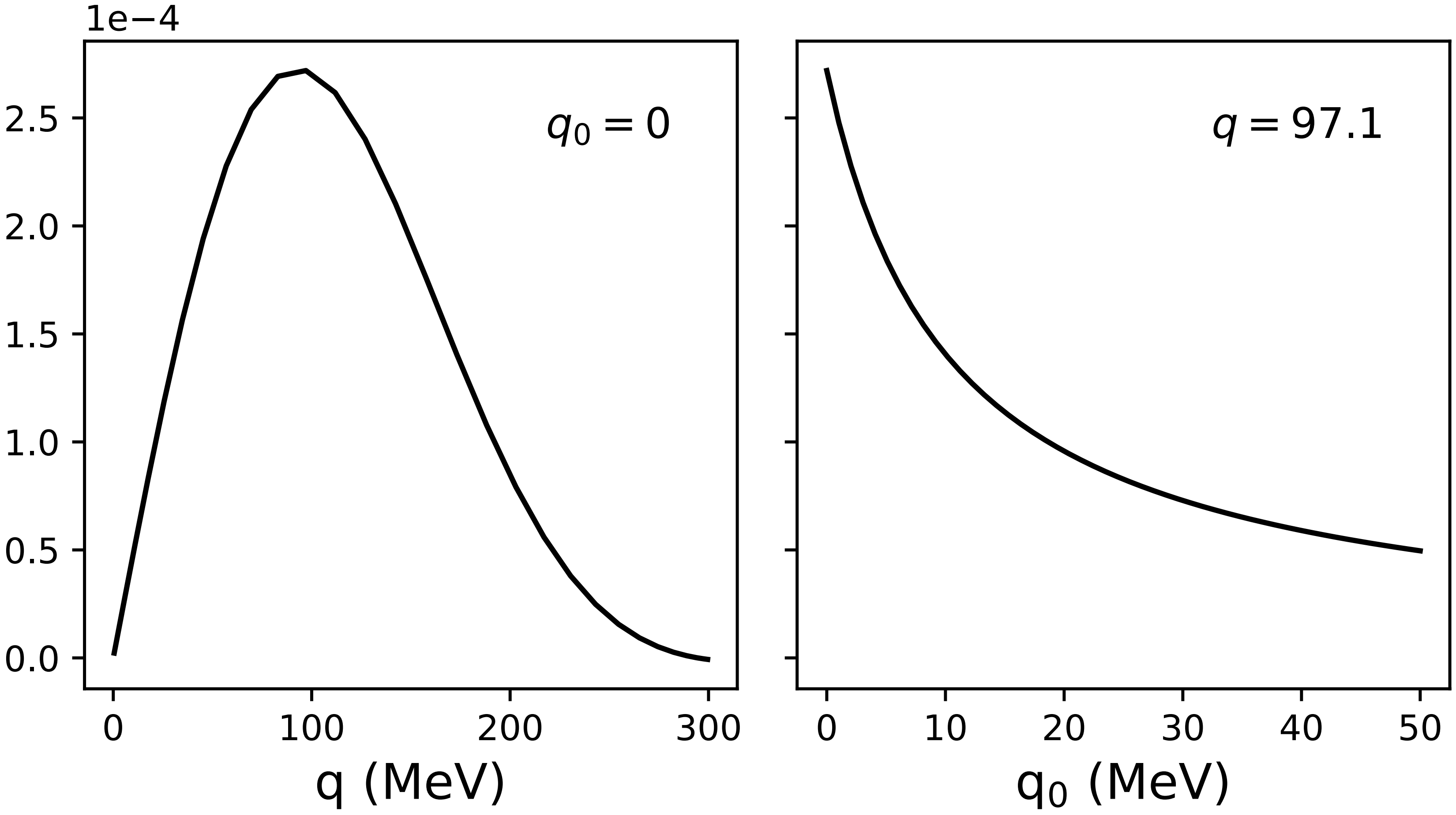
Preliminary



$G(M_i - q_0 + i\epsilon)$ terms: $T = 1$ EM current

Preliminary

$$\langle ^{10}\mathrm{B} | T^{el,5}_{J=1}\, G_{nuc}(E_i - q_0)\, T^{mag}_{J=1,T=1} | ^{10}\mathrm{C} \rangle$$



δ_{NS} in NCSM

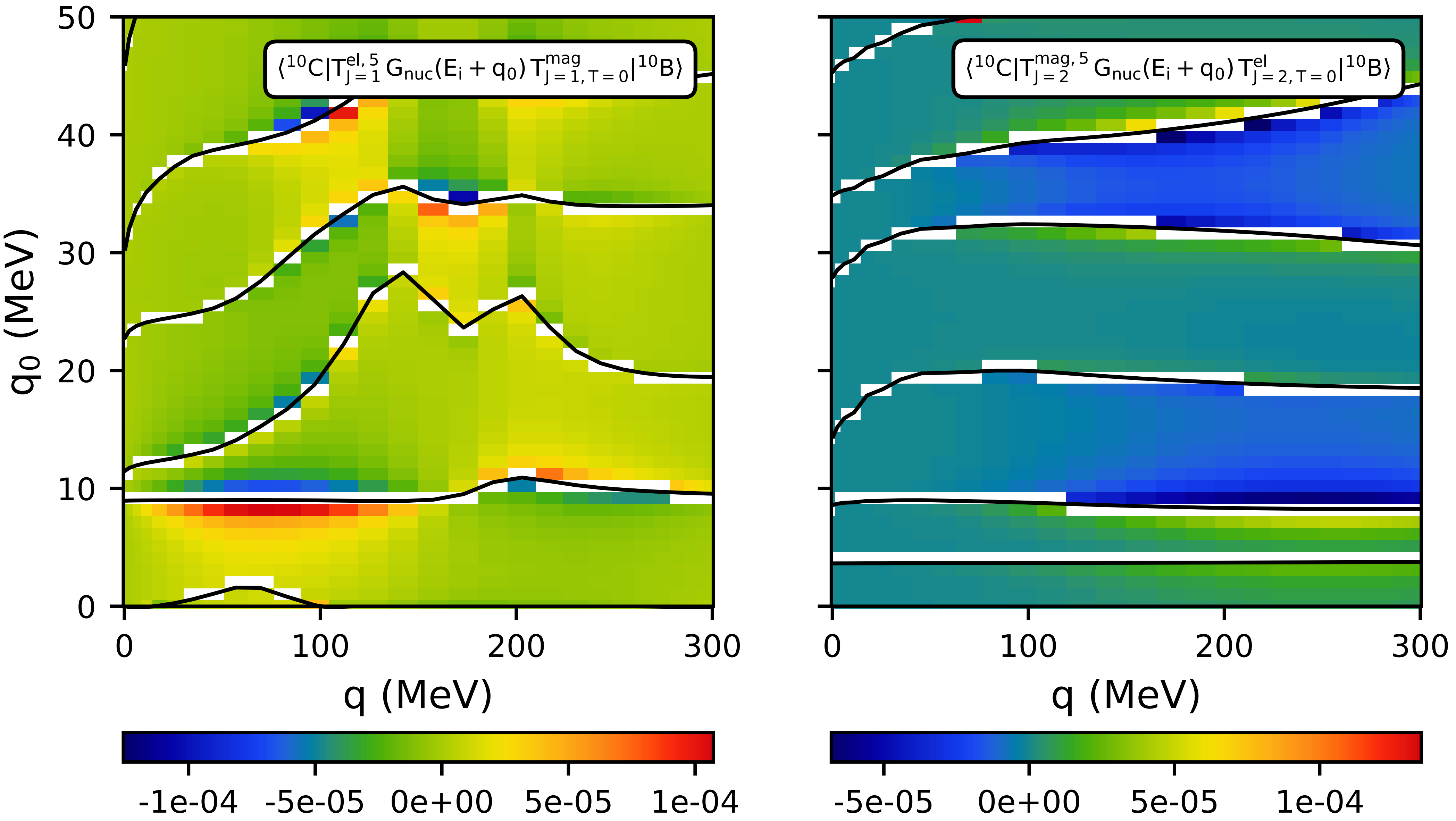
$$T_3(q_0, Q^2) = -4\pi i \frac{q_0}{q} \sqrt{M_i M_f} \sum_{J=1}^{\infty} (2J+1)$$

$$\times \langle A\lambda_f J_f M_f | \Big[T_{J0}^{mag}(q)\, G(M_f + q_0 + i\epsilon)\, T_{J0}^{5,el}(q) + T_{J0}^{el}(q)\, G(M_f + q_0 + i\epsilon)\, T_{J0}^{5,mag}(q)$$

$$+ T_{J0}^{5,mag}(q)\, G(M_i - q_0 + i\epsilon)\, T_{J0}^{el}(q) + T_{J0}^{5,el}(q)\, G(M_i - q_0 + i\epsilon)\, T_{J0}^{mag}(q) \Big] | A\lambda_i J_i M_i \rangle$$

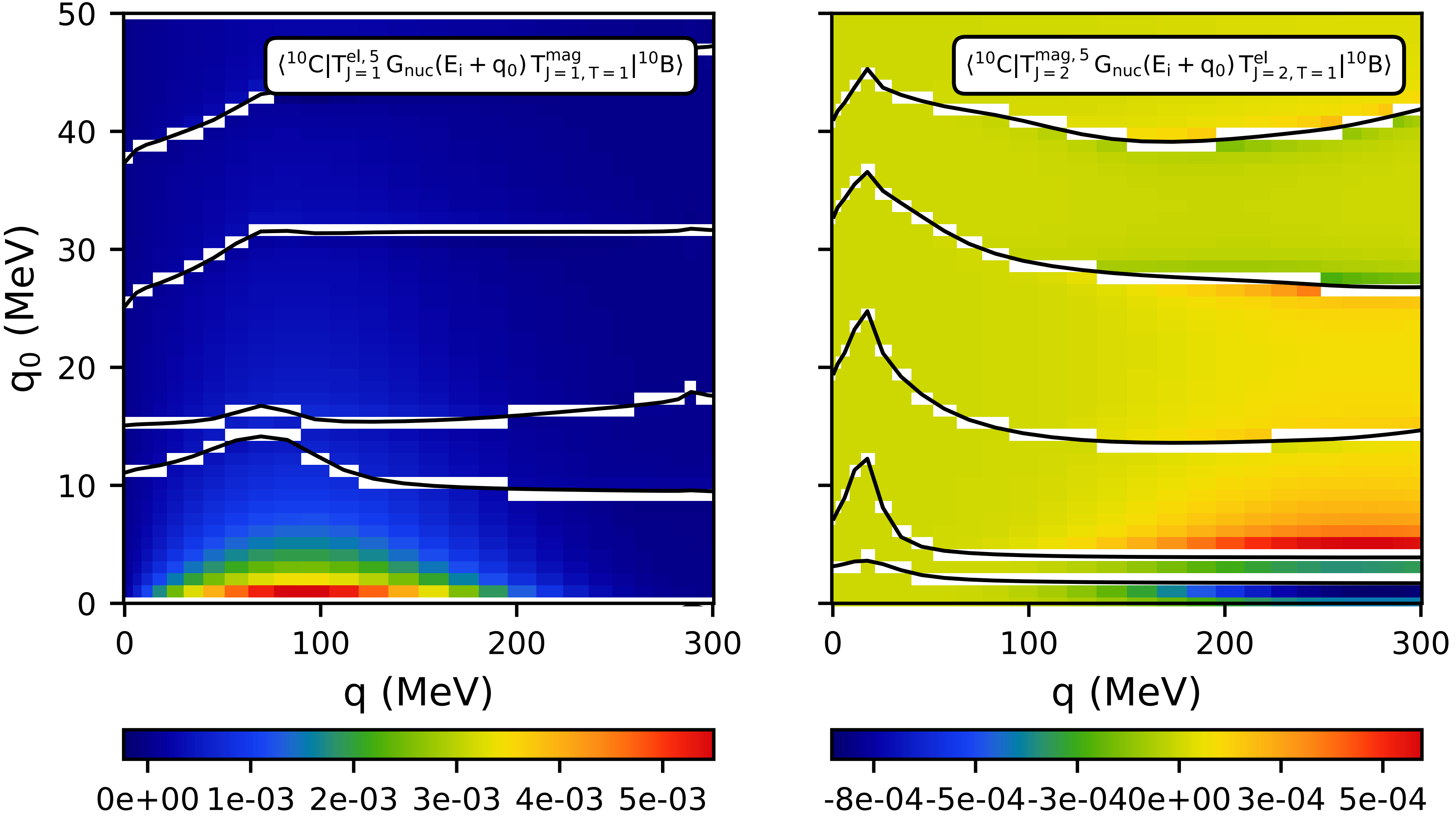
$G(M_i + q_0 + i\epsilon)$ terms: $T = 0$ EM current

Preliminary



$G(M_i + q_0 + i\epsilon)$ terms: $T = 1$ EM current

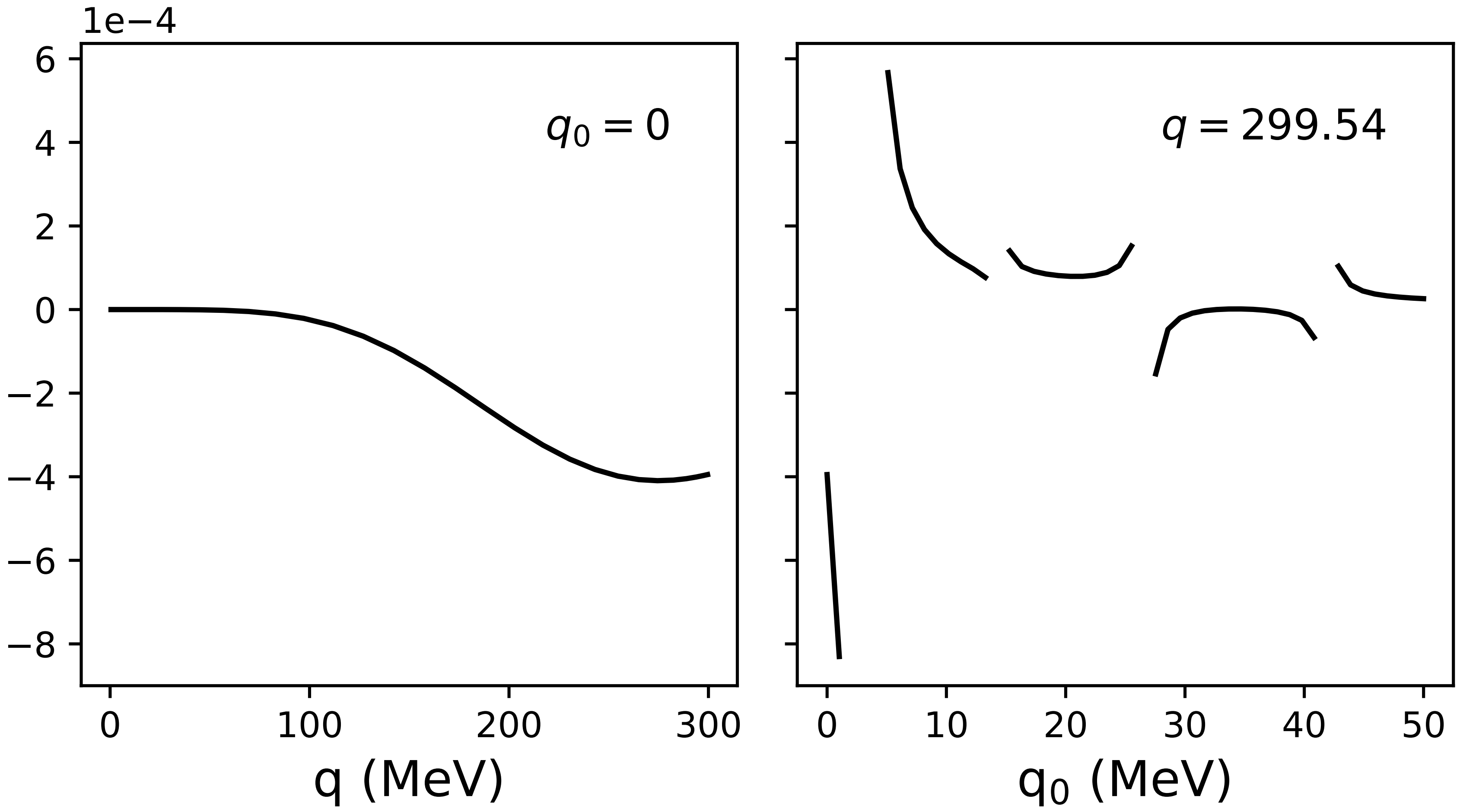
Preliminary



$G(M_i + q_0 + i\epsilon)$ terms: $T = 1$ EM current

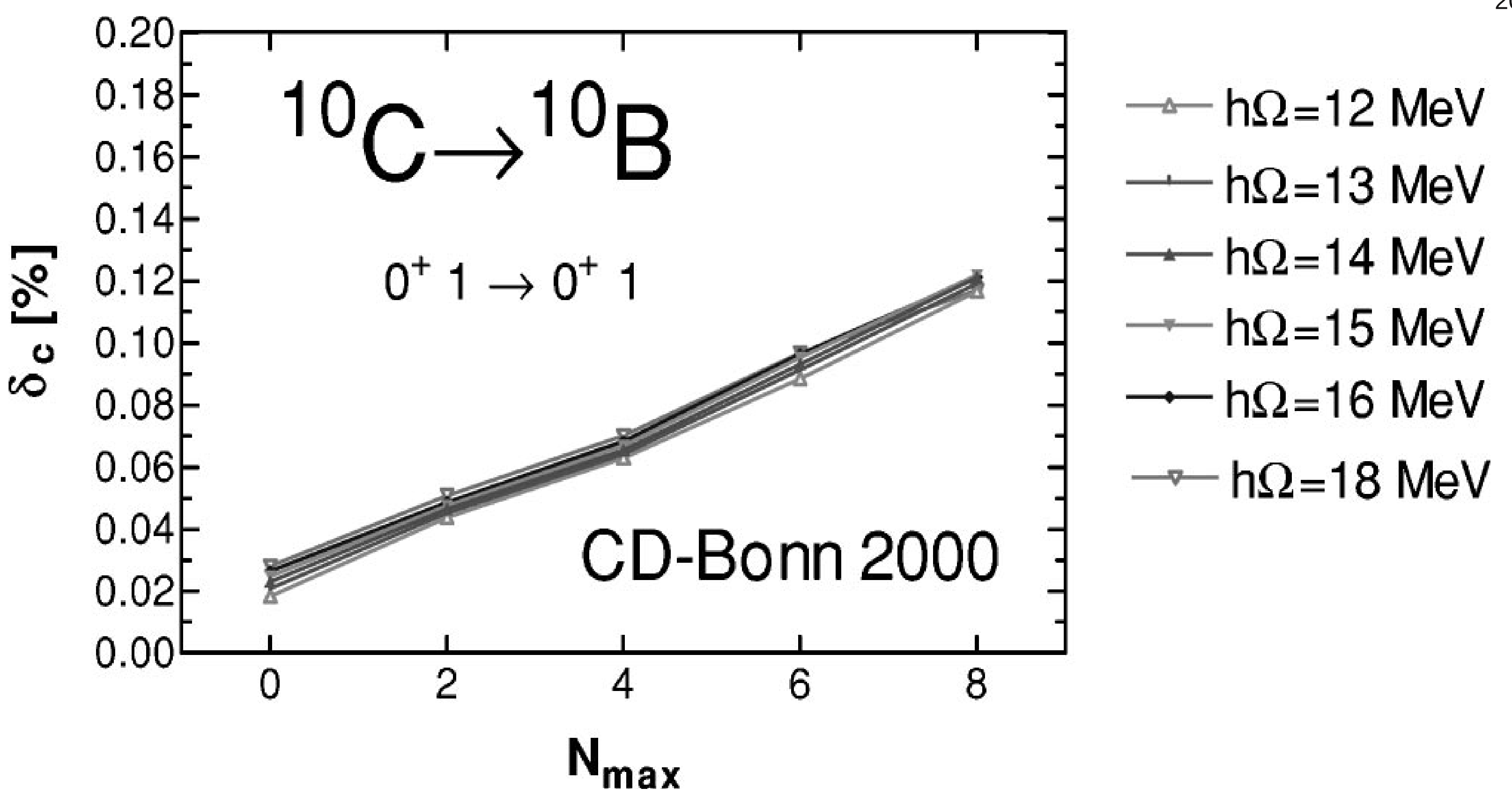
Preliminary

$$\langle {}^{10}\mathrm{C} | T^{\mathrm{mag},5}_{J=2}\, G_{\mathrm{nuc}}(E_i + q_0)\, T^{\mathrm{el}}_{J=2,T=1} | {}^{10}\mathrm{B} \rangle$$



δ_C in NCSM with continuum (NCSMC)

- N_{max} convergence for δ_C very poor
- Greater correlations in bound states required



PHYSICAL REVIEW C **66**, 024314 (2002)

***Ab initio* shell model for *A* = 10 nuclei**

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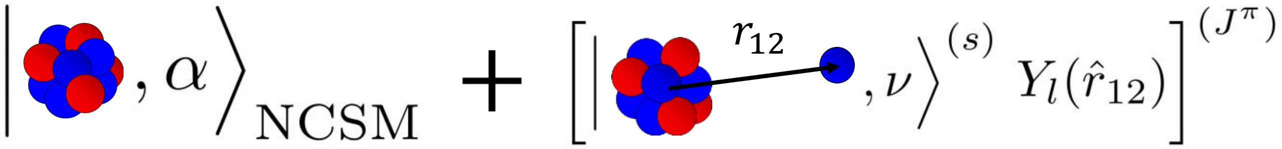
(Received 10 May 2002; published 13 August 2002)

δ_C in NCSM with continuum (NCSMC) **[12]**

$$|^{10}\mathrm{C}\rangle = \sum_{\alpha} c_{\alpha} \, |^{10}\mathrm{C}, \alpha\rangle_{\mathrm{NCSM}} + \sum_{\nu} \int dr \, \gamma_{\nu}(r) \mathcal{A}_{\nu} \, |^{9}\mathrm{B} + p, \nu\rangle$$



$$M_F = \left\langle \Psi^{J^{\pi} T_f M_{T_f}} \middle| T_+ \middle| \Psi^{J^{\pi} T_i M_{T_i}} \right\rangle \longrightarrow |M_F|^2 = |M_{F0}|^2 (1 - \delta_C)$$

Conclusions

- Both δ_{NS} and δ_{C} calculations underway in NCSM/NCSMC
- Goal is more consistent framework for nuclear structure corrections to Fermi transitions
- $^{14}\mathrm{O} \rightarrow {}^{14}\mathrm{N}$ transition

Outlook

- Further checks of NS calculation
- More serious uncertainty quantification

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δ_{NS} in NCSM

$$
\begin{aligned}
T_3(q_0, Q^2) = -4\pi i \frac{q_0}{q}\sqrt{M_i M_f}\sum_{J=1}^{\infty}(2J+1) \\
\times \langle A\lambda_f J_f M_f \big| \Big[T_{J0}^{mag}(q)\, G(M_f + q_0 + i\epsilon)\, T_{J0}^{5,el}(q) + T_{J0}^{el}(q)\, G(M_f + q_0 + i\epsilon)\, T_{J0}^{5,mag}(q) \\
+ T_{J0}^{5,mag}(q)\, G(M_i - q_0 + i\epsilon)\, T_{J0}^{el}(q) + T_{J0}^{5,el}(q)\, G(M_i - q_0 + i\epsilon)\, T_{J0}^{mag}(q) \Big] \big| A\lambda_i J_i M_i \rangle
\end{aligned}
$$

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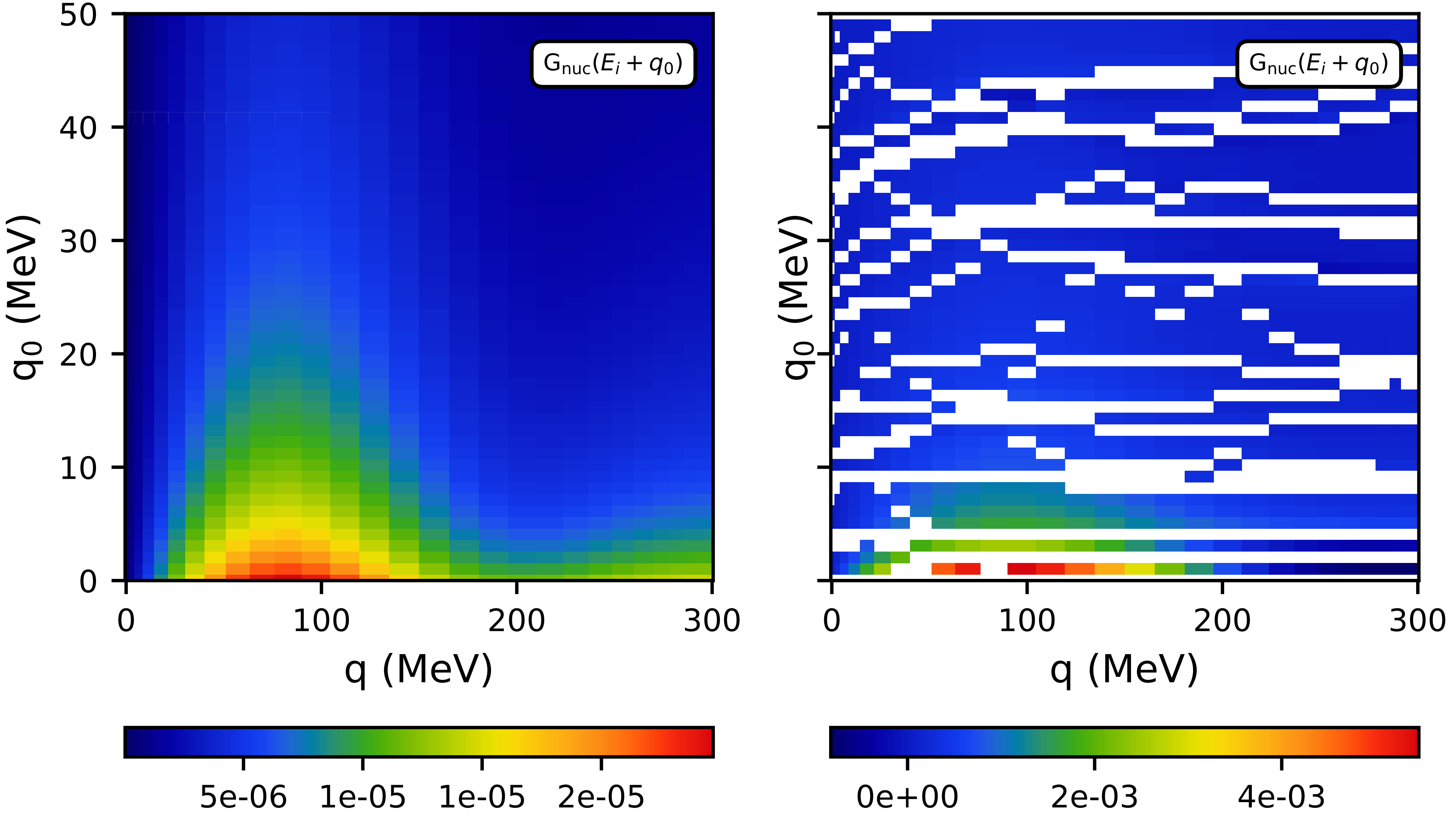
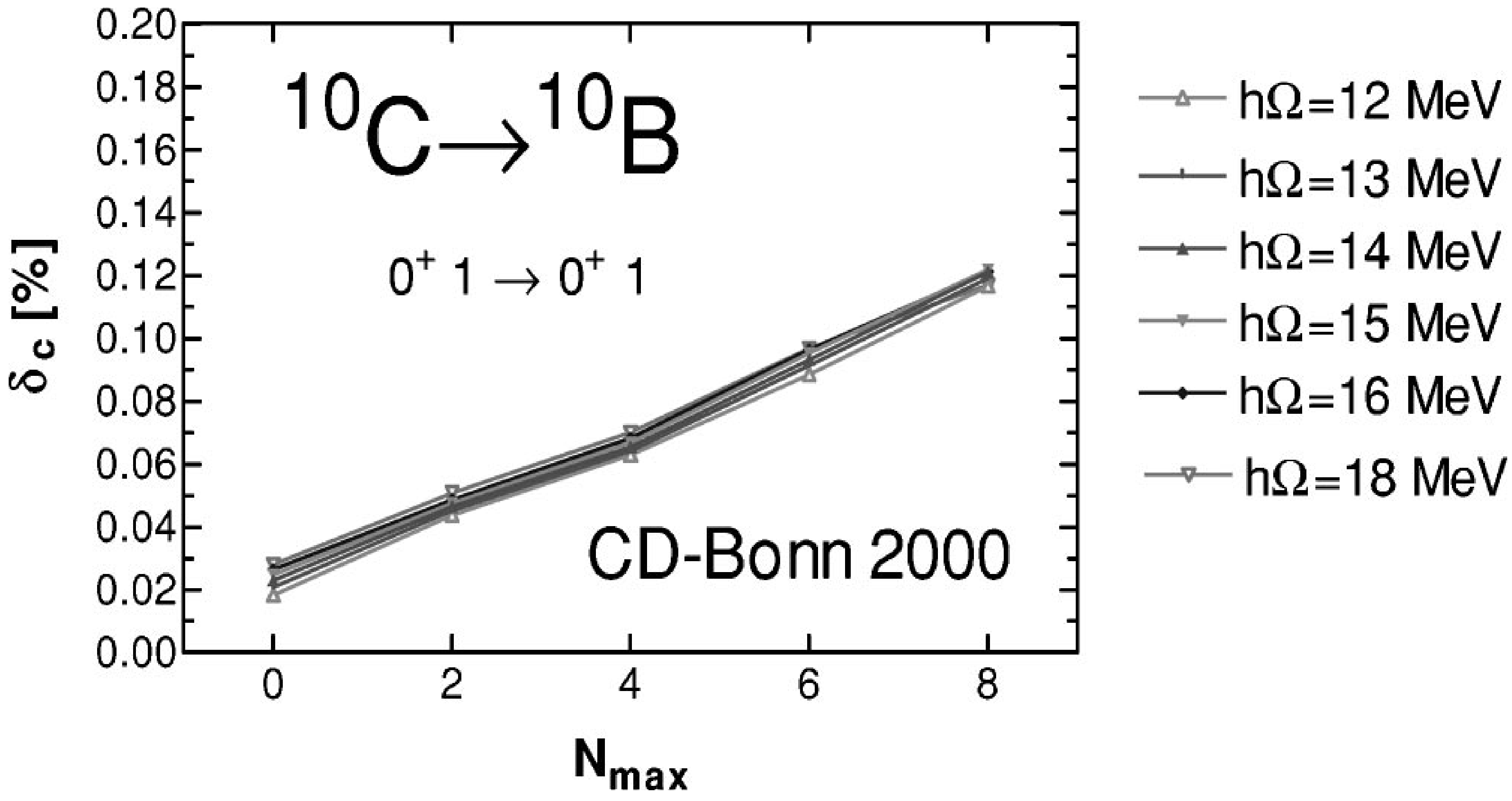
δ_{NS} in NCSM



δ_c in NCSM

- Successive N_{max} calculations show no convergence for δ_C
- Need greater correlations in bound states
- Require improved description of excited states



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Ab initio shell model for $A=10$ nuclei

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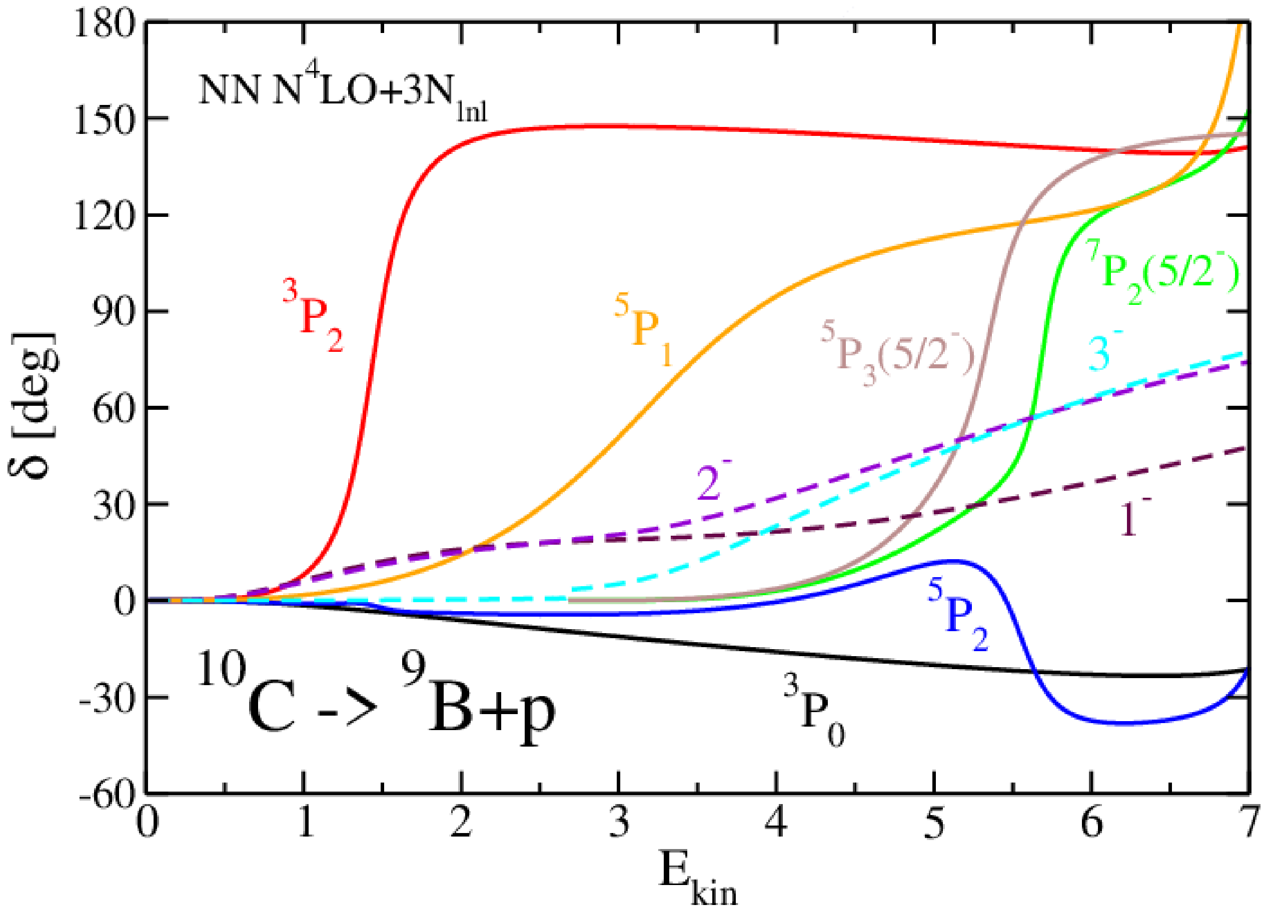
[3]*Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011*
(Received 10 May 2002; published 13 August 2002)

^{10}C structure result

$$|{}^{10}\mathrm{C}\rangle = \sum_\alpha c_\alpha \,|{}^{10}\mathrm{C},\alpha\rangle_{\mathrm{NCSM}} + \sum_\nu \int dr\,\gamma_\nu(r)\mathcal{A}_\nu\,|{}^{9}\mathrm{B}+p,\nu\rangle$$



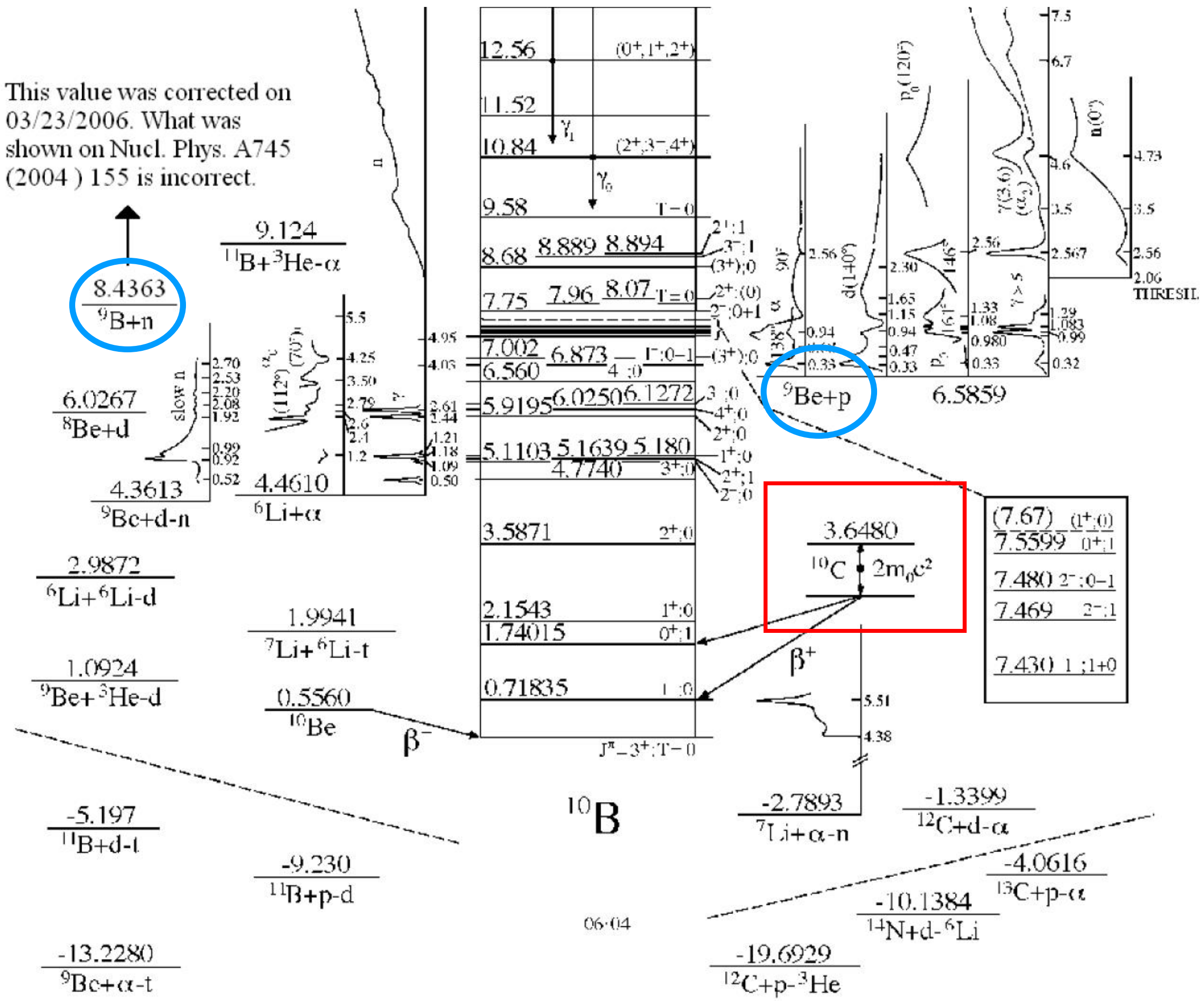
$\mathrm{NN}-\mathrm{N}^4\mathrm{LO}(500)+3\mathrm{N}_{\mathrm{lnl}}$ interaction with $\hbar\Omega = 18.0$ MeV and $\lambda_{SRG} = 1.8\ \mathrm{fm}^{-1}$

- Phase and eigenphase shifts for $\mathrm{p} + {}^9\mathrm{B}(3/2^-, 5/2^-, 1/2^-)$ scattering
- Two known experimental bound states 0^+ and 2^+ captured well
- NCSMC provides good description of 0^+ for calculation of δ_C

State	Energy (MeV)	Excitation energies
0^+	-3.62 (Exp. -4.006)	0.0
2^+	-0.11 (Exp. -0.652)	3.54 (Exp. 3.3536)

^{10}B structure result

$$|^{10}\mathrm{B}\rangle = \sum_\alpha c_\alpha \, |^{10}\mathrm{B}, \alpha\rangle_{\mathrm{NCSM}} + \sum_\nu \int dr \, \gamma_\nu(r) \mathcal{A}_\nu \, |^9\mathrm{Be} + p, \nu\rangle + \sum_\mu \int dr \, \gamma_\mu(r) \mathcal{A}_\mu \, |^9\mathrm{B} + n, \mu\rangle$$



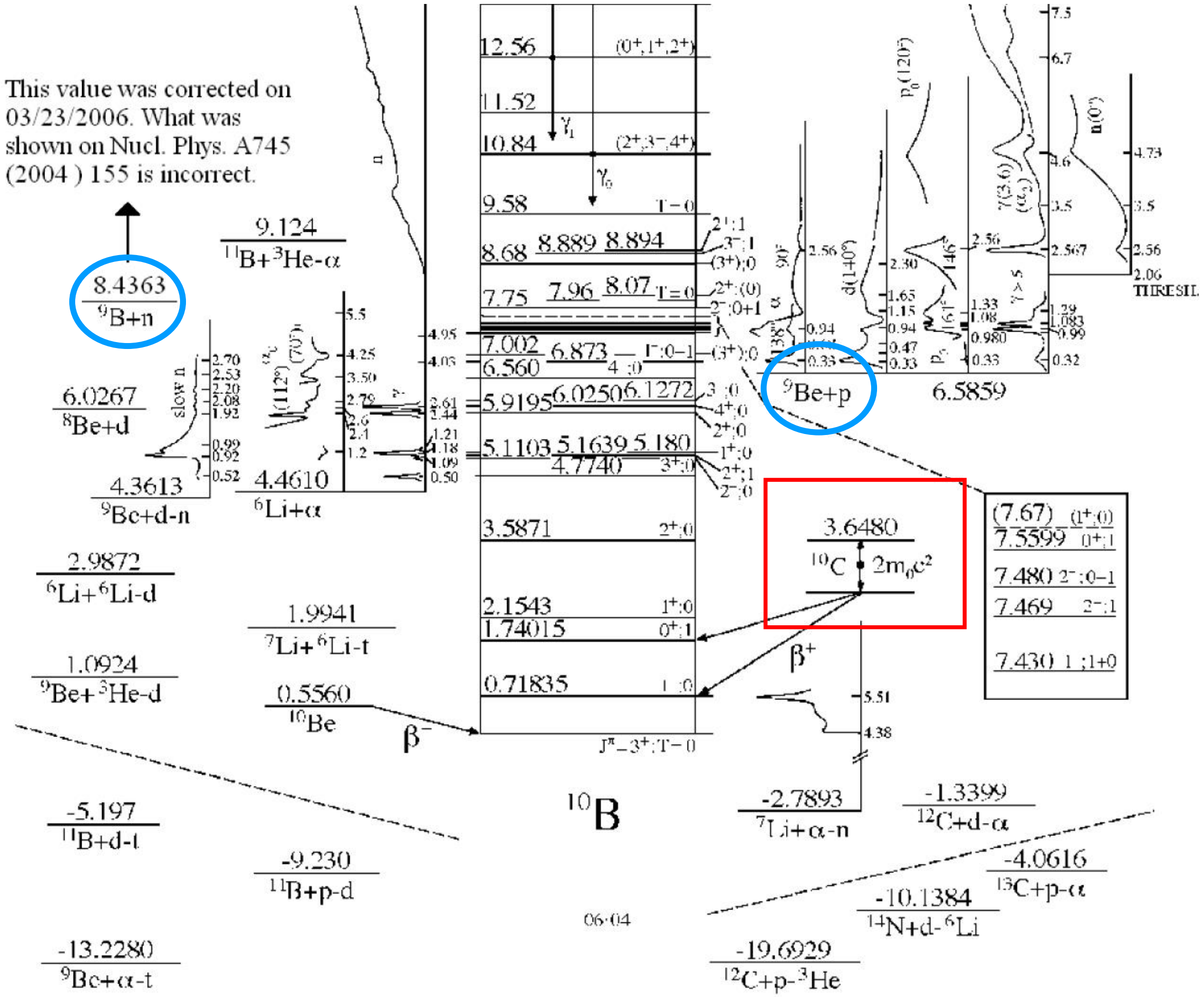
- Eight of twelve bound states predicted using $(3/2^-, 5/2^-)$ states of ^{9}B and ^{9}Be
- Novel charge exchange partitions give quality 0^+ structure for δ_C calculation

State	Energy (MeV)	Excitation energies
3^+	-5.75 (Exp. -6.5859)	0.0
1^+	-5.33 (Exp. -5.8676)	0.43 (Exp. 0.7184)
0^+	-4.30 (Exp. -4.8458)	1.45 (Exp. 1.7402)
1^+	-4.26 (Exp. -4.4316)	1.49 (Exp. 2.1543)
2^+	-2.69 (Exp. -2.9988)	3.06 (Exp. 3.5871)
2^+	-0.93 (Exp. -1.4220)	4.82 (Exp. 5.1639)
2^+	-0.70 (Exp. -0.6664)	5.05 (Exp. 5.9195)
4^+	-0.19 (Exp. -0.5609)	5.56 (Exp. 6.0250)

[13] Caurier et al. (2002)
[14] Navrátil et al. (2004)

^{10}B structure result

$$|{}^{10}\mathrm{B}\rangle = \sum_\alpha c_\alpha\,|{}^{10}\mathrm{B},\alpha\rangle_{\mathrm{NCSM}} + \sum_\nu \int dr\,\gamma_\nu(r)\mathcal{A}_\nu\,|{}^{9}\mathrm{Be}+p,\nu\rangle + \sum_\mu \int dr\,\gamma_\mu(r)\mathcal{A}_\mu\,|{}^{9}\mathrm{B}+n,\mu\rangle$$



- Correct ordering of 3^+ and excited 1^+
- Ordering sensitive to three-body part of nuclear Hamiltonian **[13,14]**

State	Energy (MeV)	Excitation energies
3^+	-5.75 (Exp. -6.5859)	0.0
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2^+	-0.93 (Exp. -1.4220)	4.82 (Exp. 5.1639)
2^+	-0.70 (Exp. -0.6664)	5.05 (Exp. 5.9195)
4^+	-0.19 (Exp. -0.5609)	5.56 (Exp. 6.0250)

Fermi matrix element in NCSMC

- Using NCSMC wavefunction compute Fermi matrix element M_F

$$M_F = \left\langle \Psi^{J^\pi T_f M_{T_f}} \middle| T_+ \middle| \Psi^{J^\pi T_i M_{T_i}} \right\rangle$$

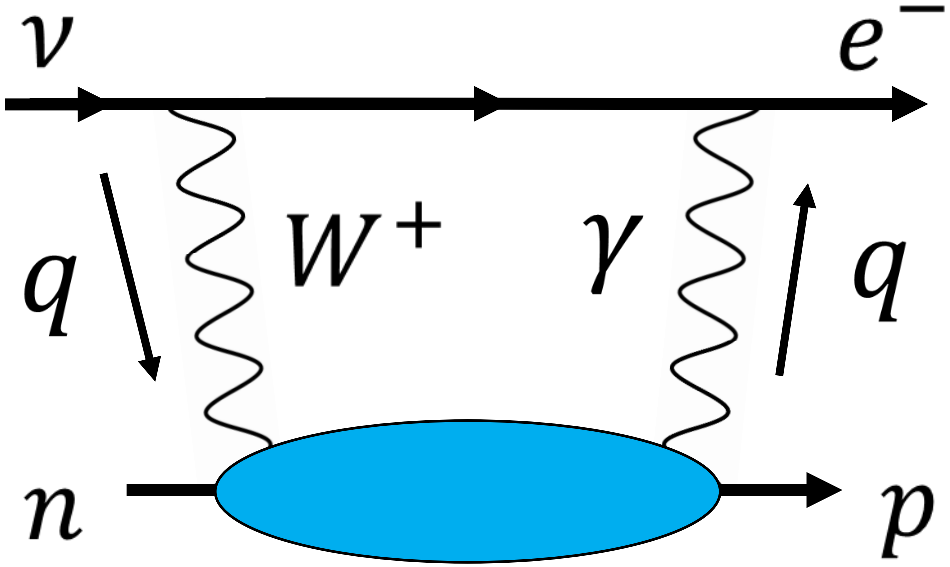
- Total isospin operator $T_+ = T_+^{(1)} + T_+^{(2)}$ for partitioned system
  - exact isospin symmetry gives $|M_F|^2 = 2$ for $T = 1$ systems
  - isospin is approximate symmetry → T_i and T_f approximate
- Expression derived by Dr. Mack Atkinson (TRIUMF)

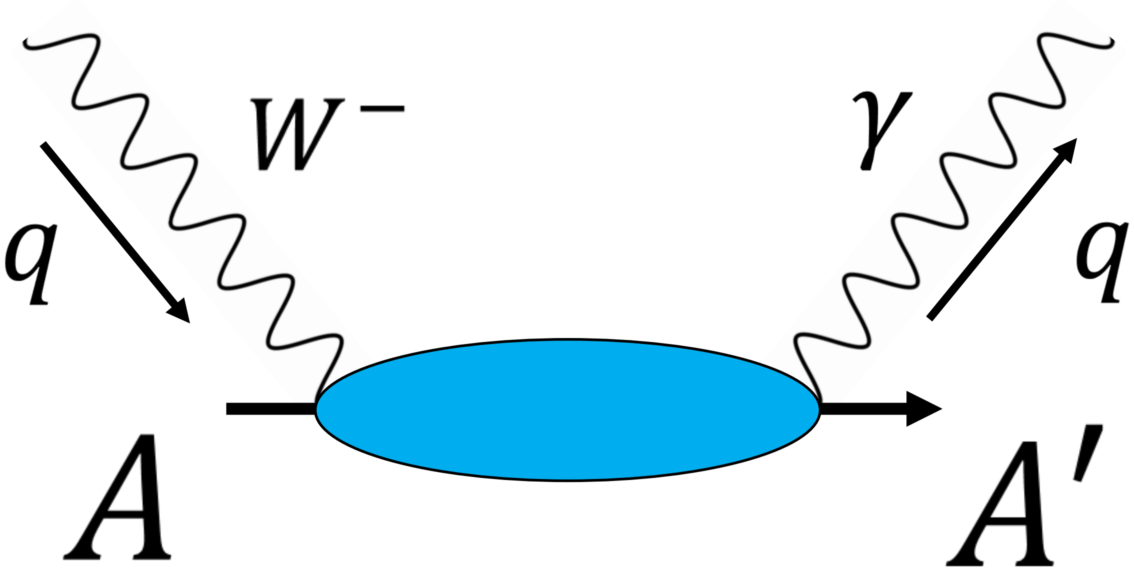
$$M_F \sim \langle A\lambda_f J_f T_f M_{T_f} | T_+ | A\lambda J_i T_i M_{T_i} \rangle + \langle A\lambda J_f T_f M_{T_f} | T_+ \mathcal{A}_{\nu i} | \Phi_{\nu r}^{J_i T_i M_{T_i}} \rangle$$
$$+ \langle \Phi_{\nu r}^{J_f T_f M_{T_f}} | \mathcal{A}_{\nu f} T_+ | A\lambda_i J_i T_i M_{T_i} \rangle + \langle \Phi_{\nu r}^{J_f T_f M_{T_f}} | \mathcal{A}_{\nu f} T_+ \mathcal{A}_{\nu i} | \Phi_{\nu r}^{J_i T_i M_{T_i}} \rangle$$

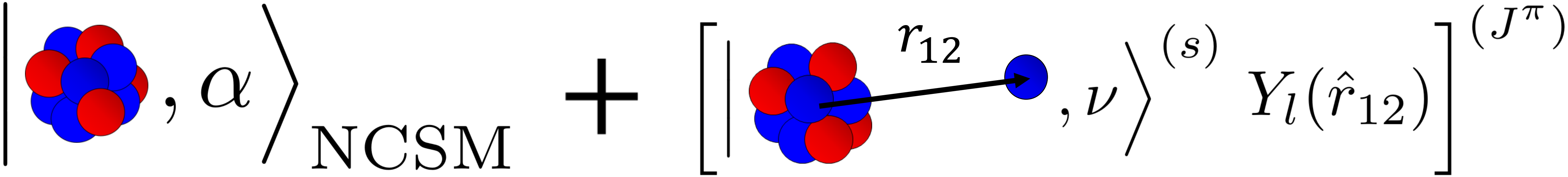
NCSM matrix element

NCSM-Cluster matrix elements

Continuum (cluster) matrix element


ν
e⁻
q
W⁺
γ
q
n
p


W⁻
γ
q
q
A
A′


, α
NCSM
+
r12
, ν
(s)
Yl(r̂12)
(Jπ)