Taming Hadronic and Nuclear Uncertainties in Vud

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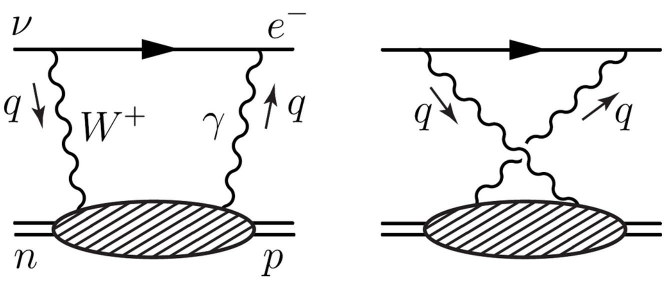
MITP workshop: "Precision Tests with Neutral-Current Coherent Interactions with Nuclei"
25 May, 2022

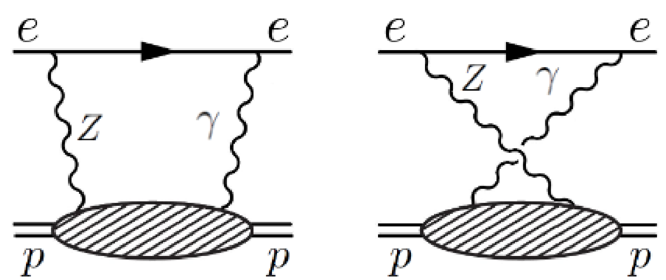
Vud measured from **beta decays** → **charged-current** process!

Anything to do with this workshop???

1)Similarities in higher-order SM corrections:



Beta decay: Vud



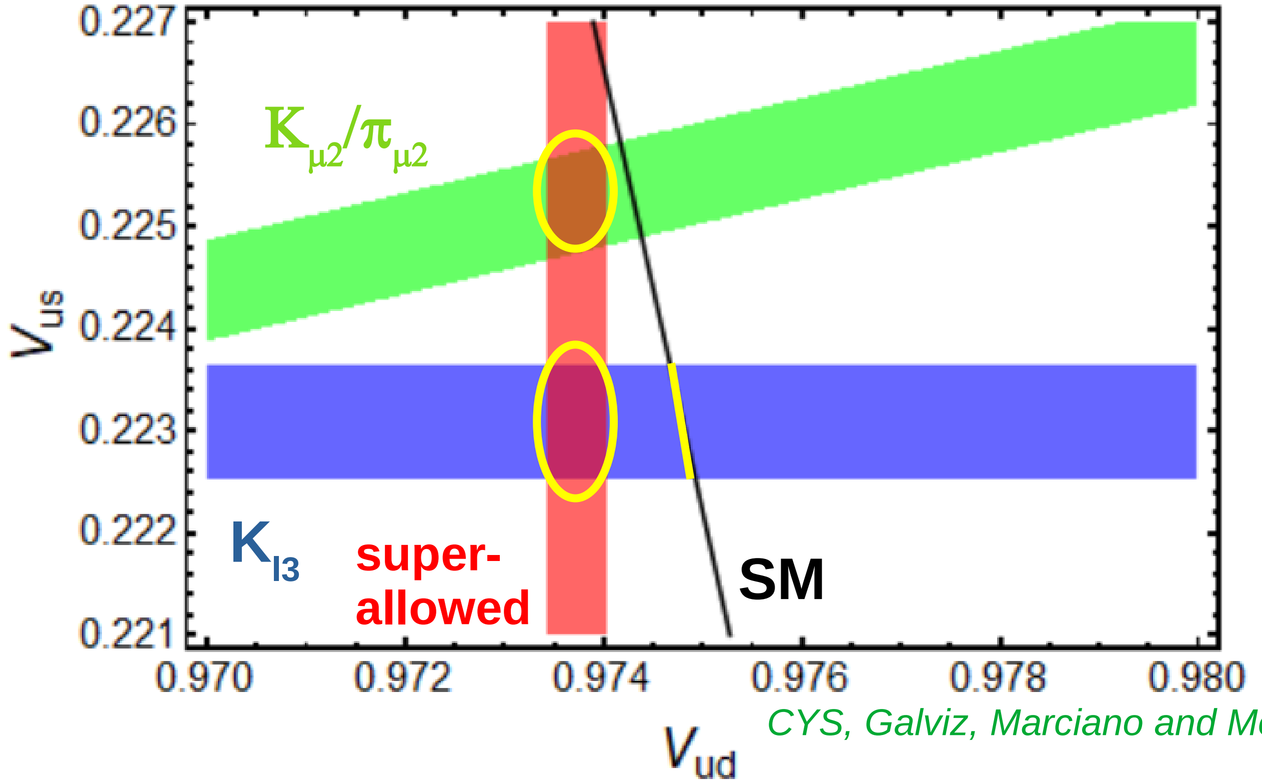
ep-scattering: weak mixing angle

Many techniques in common!

2)Observables in neutral-current processes may provide
useful inputs to charged-current processes, e.g. ISB effects

Anomalies in beta decays

Inconsistencies between different measurements of V_{ud} , V_{us} and SM predictions



CYS, Galviz, Marciano and Meißner, 2022 PRD

"Cabibbo Angle Anomaly (CAA)" ~3σ

Anomalies in beta decays

First-row CKM unitarity with $|V_{ud}|$ from superallowed ($0^+ \to 0^+$) beta decays and $|V_{us}|$ from semileptonic kaon decays (K_{l3})

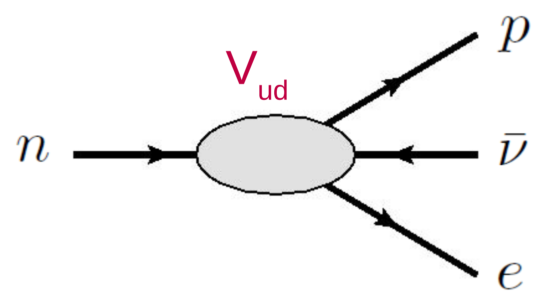
$$|V_{ud}|^2_{0^+} + |V_{us}|^2_{K_{\ell 3}} + \cancel{|V_{ub}|^2} - 1 = -0.0021(7)$$

$\lvert V_{ud}\rvert^2_{0^+} + \lvert V_{us}\rvert^2_{K_{\ell 3}} - 1$	-2.1×10^{-3}
$\delta\lvert V_{ud}\rvert^2_{0^+}$, **exp**	2.1×10^{-4}
$\delta\lvert V_{ud}\rvert^2_{0^+}$, **RC**	1.8×10^{-4}
$\delta\lvert V_{ud}\rvert^2_{0^+}$, **NS**	5.3×10^{-4}
$\delta\lvert V_{us}\rvert^2_{K_{\ell 3}}$, **exp+th**	1.8×10^{-4}
$\delta\lvert V_{us}\rvert^2_{K_{\ell 3}}$, **lat**	1.7×10^{-4}
Total uncertainty	6.5×10^{-4}
Significance level	3.2σ

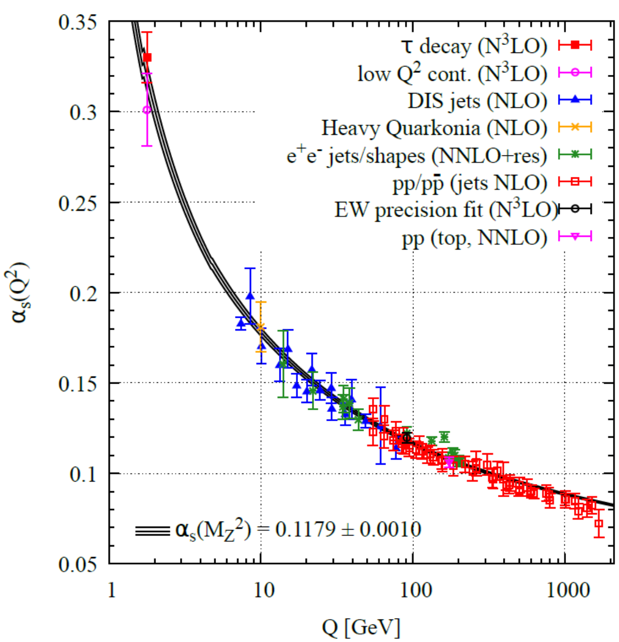
CYS, Galviz, Marciano and Meißner, 2022 PRD

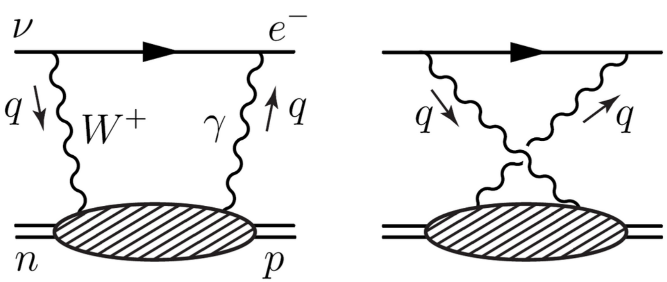
Single-nucleon radiative corrections: Box diagram

Avenues for **V_{ud}** extraction:
Free neutron and **nuclear beta decays**



A primary source of theory uncertainties in the **single-nucleon sector**:
the **"single-nucleon (antisymmetric) γW-box diagram"**





$Q^2 = -q^2$

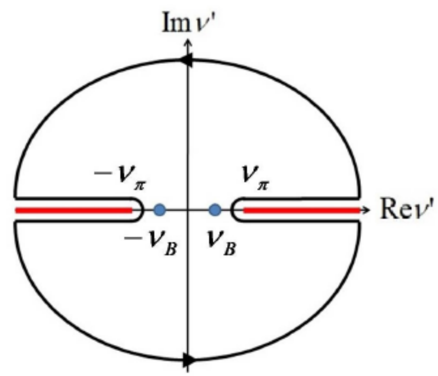
Main issue: Strong interactions governed by QCD become non-perturbative at $Q^2 \sim 1\ \text{GeV}^2$

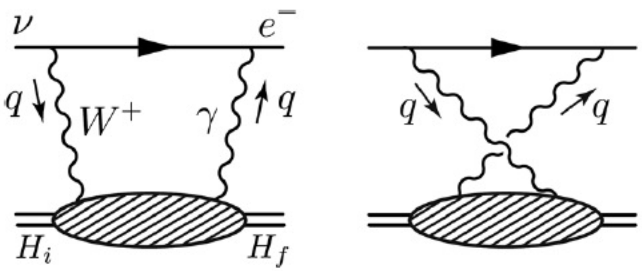
Major theory challenge in the single-nucleon sector for the past 4 decades

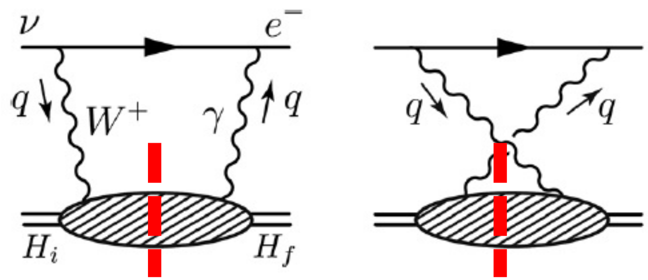
Sirlin, 1978 Rev.Mod.Phys

Single-nucleon radiative corrections: Box diagram

Year 2018: **Dispersion relation (DR)** treatment --- relate the loop integral to experimentally-measurable structure functions *CYS, Gorchtein, Patel and Ramsey-Musolf, 2018 PRL*







Generalized Compton tensor ⟷ **On-shell hadronic tensor**

$$\frac{1}{2}\int d^4x e^{iq\cdot x}\langle H_f(p)|T[J^\mu_{\rm em}(x)J^\nu_W(0)]|H_i(p)\rangle \longleftrightarrow \frac{1}{8\pi}\int d^4x e^{iq\cdot x}\langle H_f(p)|[J^\mu_{\rm em}(x),J^\nu_W(0)]|H_i(p)\rangle$$

Inspired by the **DR treatment of γZ-box**!

Gorchtein and Horowitz, 2009 PRL
Gorchtein, Horowitz and Ramsey-Musolf, 2011 PRC
…. and many more!

Single-nucleon radiative corrections: Box diagram

DR representation of the γW-box diagram correction to the **neutron g_V and g_A**

Fermi GT

$$\langle p | J_W^\mu | n \rangle = \bar{u}_p \gamma^\mu \left(g_V + g_A \gamma_5 \right) u_n$$

$$\Box_{\gamma W}^V = \frac{\alpha_{em}}{\pi \mathring{g}_V} \int_0^\infty \frac{dQ^2}{Q^2} \frac{M_W^2}{M_W^2 + Q^2} \int_0^1 dx \frac{1+2r}{(1+r)^2} F_3^{(0)}(x, Q^2)$$

$$\Box_{\gamma W}^A = -\frac{2\alpha_{em}}{\pi \mathring{g}_A} \int_0^\infty \frac{dQ^2}{Q^2} \frac{M_W^2}{M_W^2 + Q^2} \int_0^1 \frac{dx}{(1+r)^2} \left[\frac{5+4r}{3} g_1^{(0)}(x, Q^2) - \frac{4M^2 x^2}{Q^2} g_2^{(0)}(x, Q^2) \right]$$

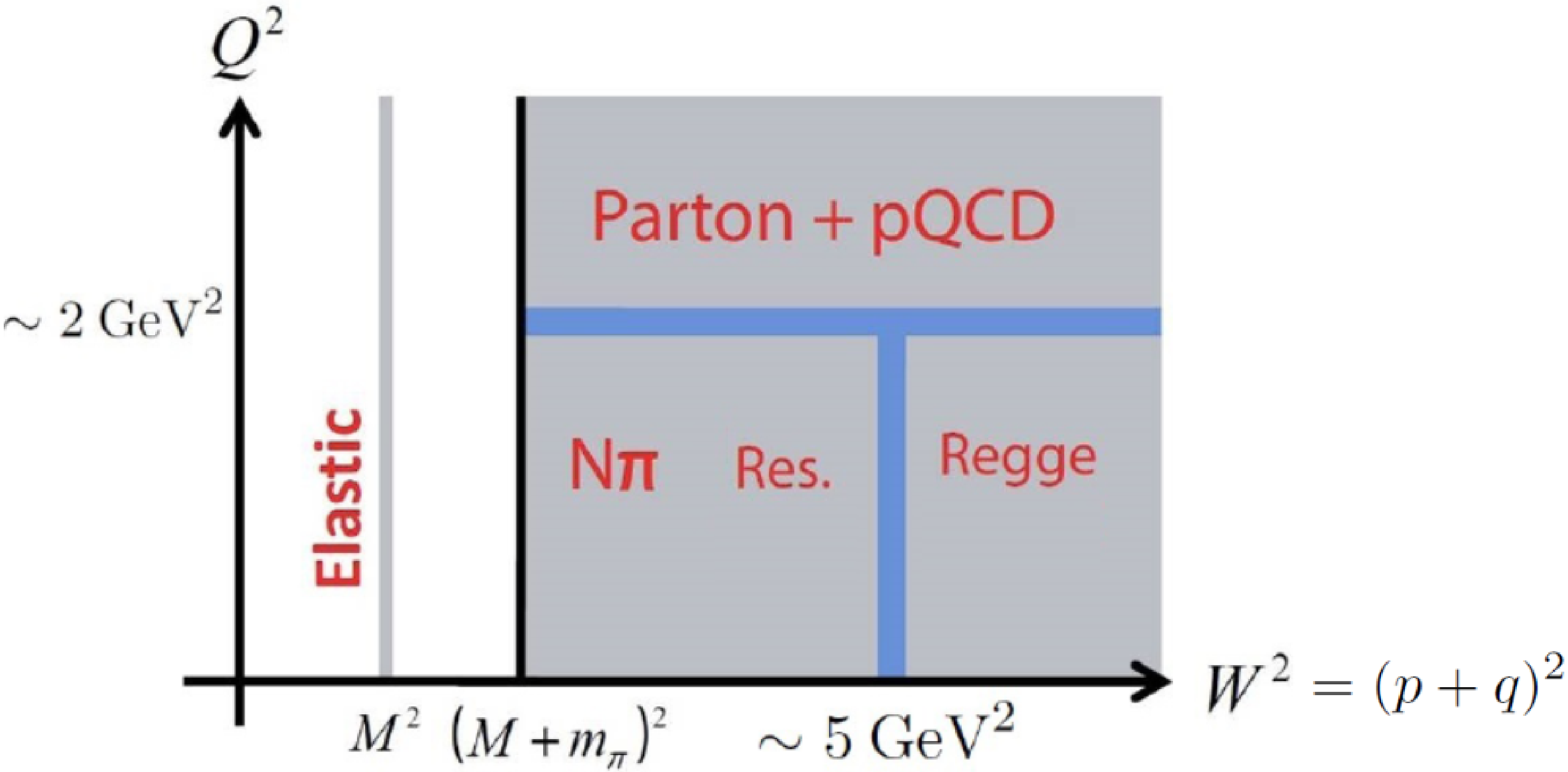
$$W_{\mu\nu}^{\gamma W} = \frac{1}{4\pi} \sum_X (2\pi)^4 \delta^{(4)}(p + q - p_X) \langle p | J_\mu^{em}(0) | X \rangle \langle X | J_\nu^W(0) | n \rangle$$

$$= -\frac{i\epsilon_{\mu\nu\alpha\beta} q^\alpha p^\beta}{2(p \cdot q)} F_3 + \frac{i\epsilon_{\mu\nu\alpha\beta} q^\alpha}{(p \cdot q)} \left[S^\beta g_1 + \left(S^\beta - \frac{(S \cdot q)}{p \cdot q} p^\beta \right) g_2 \right] + \dots$$

$$J_{\rm em}^\mu = J_{\rm em}^{(0)\mu} + J_{\rm em}^{(1)\mu}$$ (isoscalar + isovector)

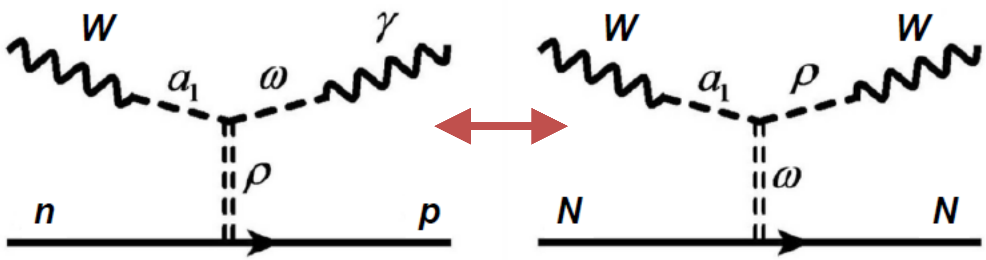
Single-nucleon radiative corrections: Box diagram

Dominant intermediate state contributions in different kinematic regions:

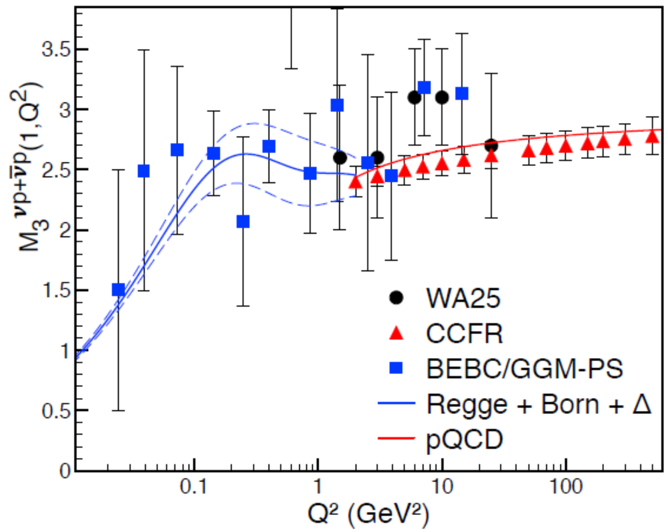


Single-nucleon radiative corrections: Box diagram

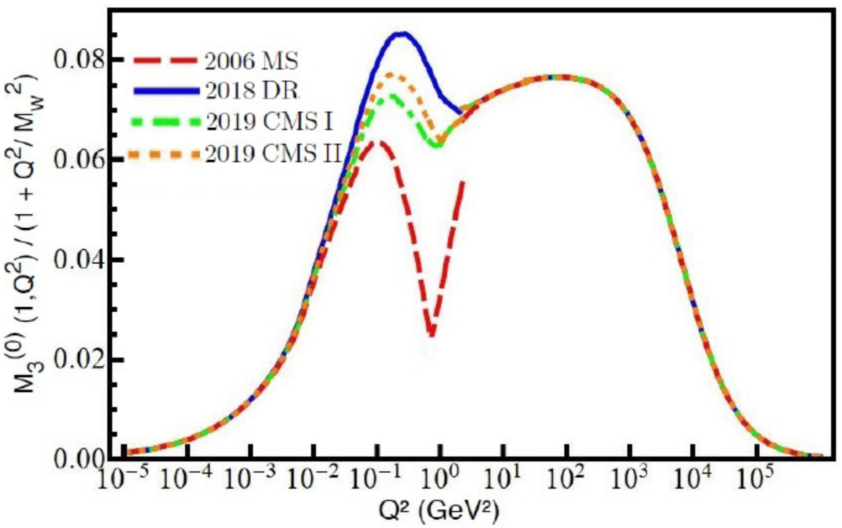
Inelastic contribution to g_V RC: data obtained from **neutrino-nucleus scattering**



Neutrino scattering data



Free neutron γW box



New treatment led to **reduced uncertainty** and **shifted central value:**

$$\Box^V_{\gamma W} = 3.26(19) \times 10^{-3} \rightarrow 3.79(10) \times 10^{-3}$$

Pre-2018 *2018*

Single-nucleon radiative corrections: Box diagram

Major limiting factor of the DR treatment: **low quality of the neutrino data** in the most interesting region: **$Q^2 \sim 1\text{GeV}^2$**

Next Step: Calculate the box diagram directly with **lattice QCD**

Year 2020: First realistic lattice QCD calculation of the simpler **pion** axial γW-box diagram, in collaboration with the **RBC-UKQCD** members

Feng, Gorchtein, Jin, Ma and CYS, 2020 PRL

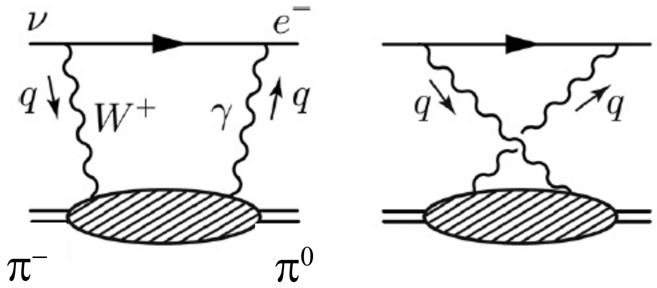
Pion semileptonic decay

$$\pi^+ \to \pi^0 + e^+ + \nu_e$$

$$\Gamma_{\pi\ell 3} = \frac{G_F^2 |V_{ud}|^2 m_\pi^5 |f_+^\pi(0)|^2}{64\pi^3} \overset{\text{R.C}}{(1+\delta)} I_\pi$$

Single-nucleon radiative corrections: Box diagram

Charged pion γW-box diagrams



$$\square^{VA}_{\gamma W}\Big|_{\pi} = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}M_\pi(Q^2)$$

Integral sensitive to **all values of Q^2**

LQCD not applicable at **large Q^2** (> 2 GeV^2) due to large lattice artifacts. But **perturbative QCD** works well:

$$M_\pi(Q^2) = \frac{1}{12}\left[1 - \tilde{C}_1\left(\frac{\alpha_S}{\pi}\right) - \tilde{C}_2\left(\frac{\alpha_S}{\pi}\right)^2 - \tilde{C}_3\left(\frac{\alpha_S}{\pi}\right)^3 - \tilde{C}_4\left(\frac{\alpha_S}{\pi}\right)^4 - \ldots\right]$$

$$\begin{aligned}\tilde{C}_1 &= 1\\ \tilde{C}_2 &= 4.583 - 0.333n_f\\ \tilde{C}_3 &= 41.44 - 7.607n_f + 0.177n_f^2\\ \tilde{C}_4 &= 479.4 - 123.4n_f + 7.697n_f^2 - 0.1037n_f^3\end{aligned}$$

Baikov, Chetyrkin and Kuhn, 2010 PRL

Single-nucleon radiative corrections: Box diagram

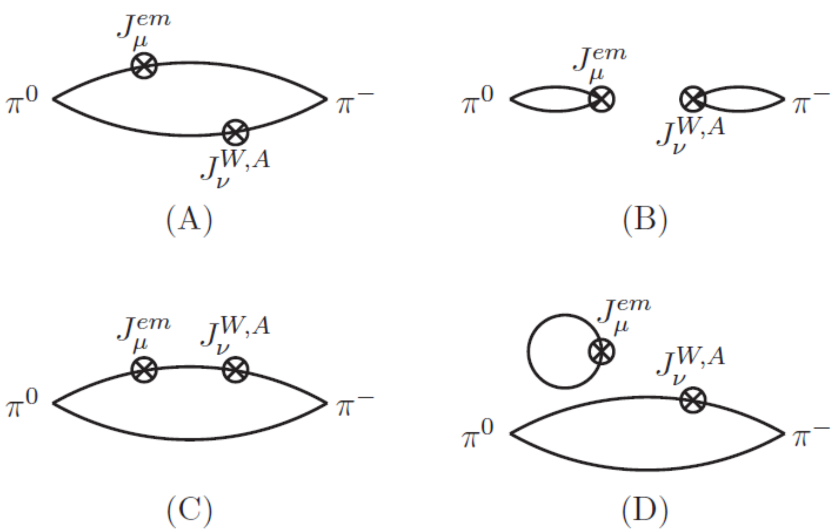
At **low Q^2** (< 2 GeV^2): **direct lattice computation** of the generalized Compton tensor

$$\mathcal{H}^{VA}_{\mu\nu}(x) = \langle \pi^0(p) \big| T[J^{\rm em}_\mu(x) J^{W,A}_\nu(0)] \big| \pi^-(p) \rangle$$

$$M_\pi(Q^2) = -\frac{1}{6\sqrt{2}} \frac{\sqrt{Q^2}}{m_\pi} \int d^4x \omega(Q,x) \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}^{VA}_{\mu\nu}(x)$$

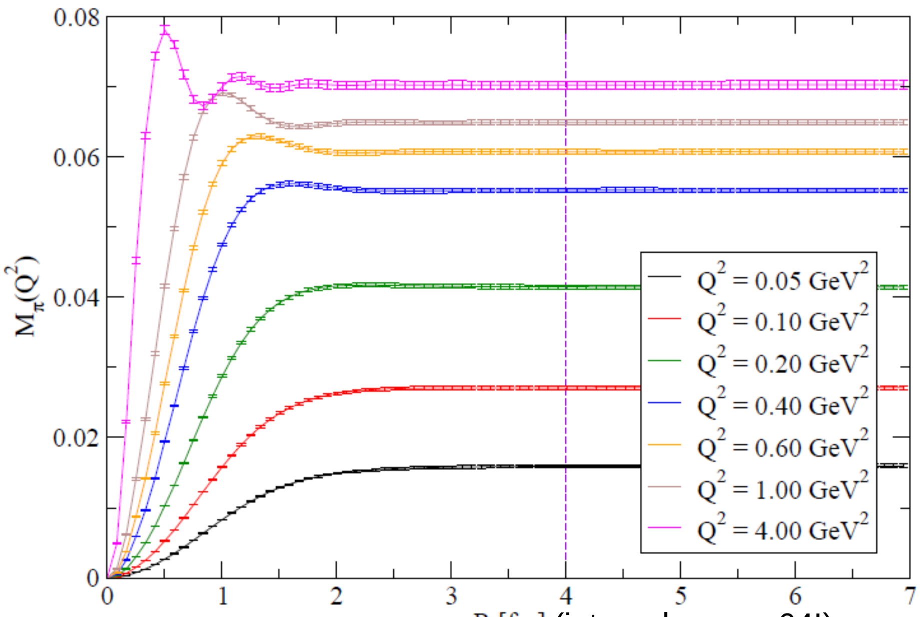
Lattice setup:
Five lattice QCD gauge ensembles at the **physical pion mass**, generated by **RBC** and **UKQCD** Collaborations using 2+1 flavor **domain wall fermion**.

Ensemble	m_π [MeV]	L	T	a^{-1} [GeV]	$N_{\rm conf}$	N_r	$\Delta t/a$
24D	141.2(4)	24	64	1.015	46	1024	8
32D	141.4(3)	32	64	1.015	32	2048	8
32D-fine	143.0(3)	32	64	1.378	71	1024	10
48I	135.5(4)	48	96	1.730	28	1024	12
64I	135.3(2)	64	128	2.359	62	1024	18

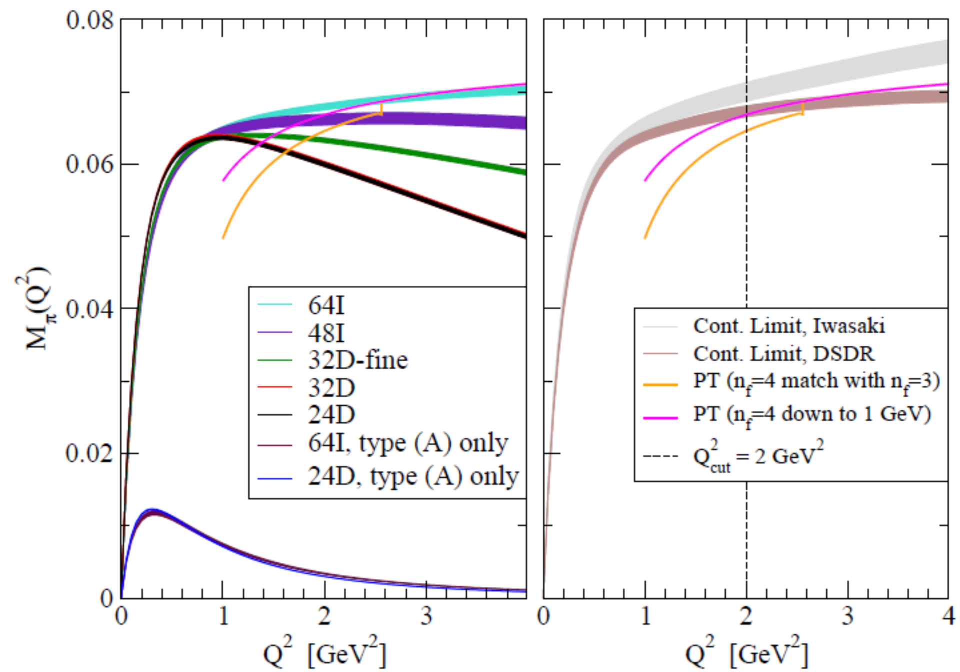


"4-point functions"

Single-nucleon radiative corrections: Box diagram



(integral range, 64I)



(before cont. extrapolation) (after cont. extrapolation)

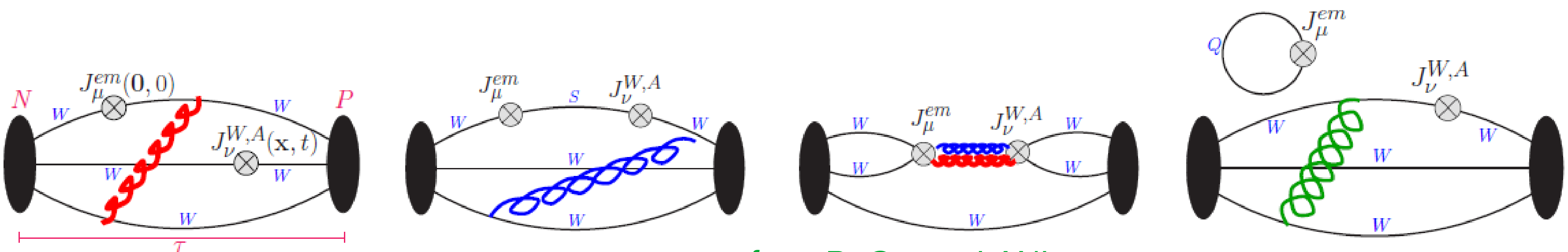
Final result: $\square^{VA}_{\gamma W}\Big|_\pi = 2.830(11)_{\rm stat}(26)_{\rm syst} \times 10^{-3}$ 1% precision!

Consequences:

- **3-fold reduction** of the theory uncertainty in **pion semileptonic decay (π_{e3})**
- Becomes a major theory motivation for the **next-generation rare-pion decay experiment (PIONEER)** *Hertzog, talk in TAU2021*
  Aguilar-Arevalo et al, SnowMass 2021 LoI

Single-nucleon radiative corrections: Box diagram

- **Direct lattice calculation of the neutron γW-box diagram**
  - Similar but more challenging than the pion box diagram



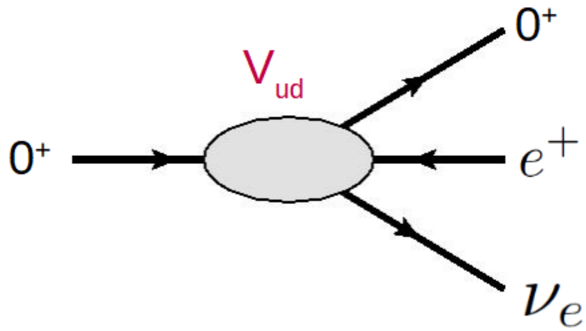
from R. Gupta, LANL

Extra challenges compared to the pion γW-box diagrams:

- The quark contraction becomes more complicated
- Much noisier data due to the exponentially-suppressed signal-to-noise ratio at large Euclidean time
- The full control of systematic effects (e.g. excited-state contamination) becomes more challenging

Nuclear structure effects in superallowed decays

Superallowed $0^+ \rightarrow 0^+$ nuclear beta decays provides the **best measurement of V_{ud}**



Master formula:

$$|V_{ud}|^2 = \frac{2984.43\,s}{\mathcal{F}t\,(1+\Delta_R^V)}.$$

Single-nucleon RC

Corrected ft (half-life*statistical function)-value:

$$\mathcal{F}t = ft\,(1+\delta'_{\mathrm{R}})\,(1+\delta_{\mathrm{NS}}-\delta_{\mathrm{C}})$$

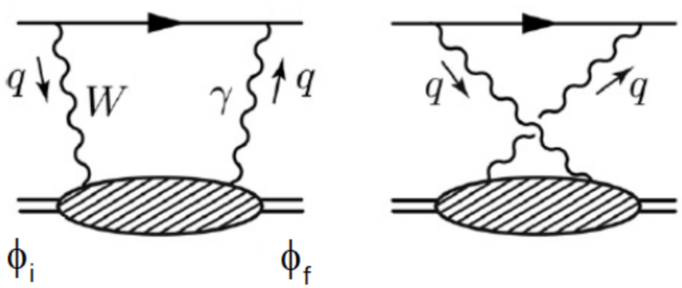
Nucleus-dependent "outer corrections" (under control)

Nuclear structure effects in inner RC

Isospin-breaking corrections

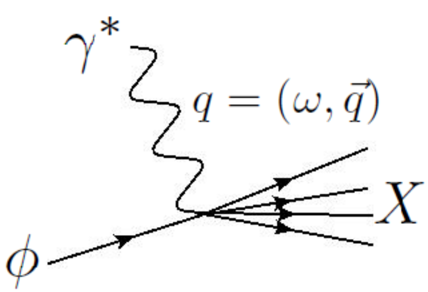
Nuclear structure effects in superallowed decays

δ_{NS}: nuclear modifications of the free-nucleon inner RC

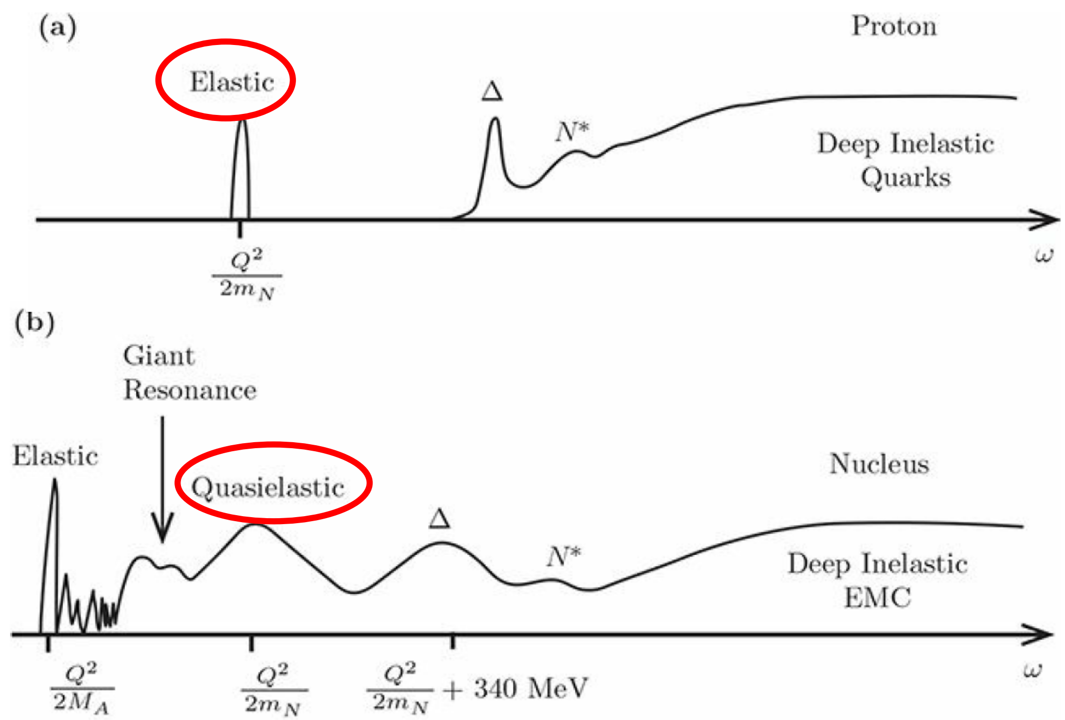


½ δ_{NS}

$$\Box^{\text{nucl.}}_{\gamma W} = \Box^{n}_{\gamma W} + \left[\Box^{\text{nucl.}}_{\gamma W} - \Box^{n}_{\gamma W}\right]$$



Nuclear modifications of absorption spectrum:



Donnelly, Formaggio, Holstein, Milner and Surrow, "Foundations of Nuclear and Particle Physics"

Nuclear structure effects in superallowed decays

Electron energy is also non-negligible due to small nuclear energy splittings!

Gorchtein, 2019 PRL

Energy-dependent nuclear axial γW-box:

$$\Box_{\gamma W}(E) = \frac{e^2}{2}\int \frac{d^4q}{(2\pi)^4}\frac{M_W^2}{M_W^2-q^2}\frac{1}{(p_e-q)^2q^2}\frac{Q^2+M\nu\frac{p_e\cdot q}{p\cdot p_e}}{M\nu}\frac{T_3(\nu,Q^2)}{f_+(0)}$$

Dispersive representation:

$$\Box^{\rm even}_{\gamma W}(E) = \frac{\alpha}{\pi}\frac{1}{Mf_+(0)}\int_0^\infty dQ^2\frac{M_W^2}{M_W^2+Q^2}\int_0^\infty\frac{d\nu}{\nu}F_3^{(0)}(\nu,Q^2)\times \frac{1}{4E}\left\{\ln\left|\frac{E+E_m}{E-E_m}\right|+\frac{\nu}{2E}\ln\left|1-\frac{E^2}{E_m^2}\right|\right\}$$

$$\Box^{\rm odd}_{\gamma W}(E) = -\frac{\alpha}{\pi}\frac{1}{Mf_+(0)}\int_0^\infty dQ^2\frac{M_W^2}{M_W^2+Q^2}\int_0^\infty\frac{d\nu}{\nu}F_3^{(1)}(\nu,Q^2)\times \frac{1}{4E}\left\{\ln\left|1-\frac{E^2}{E_m^2}\right|+\frac{\nu}{2E}\ln\left|\frac{E+E_m}{E-E_m}\right|-\frac{\nu}{E_m}\right\}$$

Nuclear structure function:

$$\frac{1}{4\pi}\sum_X(2\pi)^4\delta^{(4)}(p+q-p_X)\langle\phi_f(p)|J^{(I)\mu}_{\rm em}|X\rangle\langle X|J_A^\nu|\phi_i(p)\rangle = -\frac{i\epsilon^{\mu\nu\alpha\beta}q_\alpha p_\beta}{2p\cdot q}F^{(I)}_{3,\rm nucl}(\nu,Q^2)$$

Nuclear structure effects in superallowed decays

Ab-initio calculations of nuclear axial box diagram needed!

(1) Nuclear forces:

$$H = \sum_i T_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

AV18, IL7, CD-Bonn, ChEFT...

(2) Solve the many-body Schrödinger equation and compute matrix elements:

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

QMC, NCSM, NLEFT...

Calculations up to **medium-size nucleus (A≈40)** will cover **60%** of the superallowed transitions in the global average!

$T_z = -1$	
${}^{10}_{6}\text{C} \to {}^{10}_{5}\text{B}$	●
${}^{14}_{8}\text{O} \to {}^{14}_{7}\text{N}$	●
${}^{18}_{10}\text{Ne} \to {}^{18}_{9}\text{F}$	
${}^{22}_{12}\text{Mg} \to {}^{22}_{11}\text{Na}$	●
${}^{26}_{14}\text{Si} \to {}^{26}_{13}\text{Al}$	●
${}^{30}_{16}\text{S} \to {}^{30}_{15}\text{P}$	
${}^{34}_{18}\text{Ar} \to {}^{34}_{17}\text{Cl}$	●
${}^{38}_{20}\text{Ca} \to {}^{38}_{19}\text{K}$	●
${}^{42}_{22}\text{Ti} \to {}^{42}_{21}\text{Sc}$	
${}^{46}_{24}\text{Cr} \to {}^{46}_{23}\text{V}$	
${}^{50}_{26}\text{Fe} \to {}^{50}_{25}\text{Mn}$	
${}^{54}_{28}\text{Ni} \to {}^{54}_{27}\text{Co}$	

$T_z = 0$	
${}^{26m}_{13}\text{Al} \to {}^{26}_{12}\text{Mg}$	●
${}^{34}_{17}\text{Cl} \to {}^{34}_{16}\text{S}$	●
${}^{38m}_{19}\text{K} \to {}^{38}_{18}\text{Ar}$	●
${}^{42}_{21}\text{Sc} \to {}^{42}_{20}\text{Ca}$	●
${}^{46}_{23}\text{V} \to {}^{46}_{22}\text{Ti}$	●
${}^{50}_{25}\text{Mn} \to {}^{50}_{24}\text{Cr}$	●
${}^{54}_{27}\text{Co} \to {}^{54}_{26}\text{Fe}$	●
${}^{62}_{31}\text{Ga} \to {}^{62}_{30}\text{Zn}$	●
${}^{66}_{33}\text{As} \to {}^{66}_{32}\text{Ge}$	
${}^{70}_{35}\text{Br} \to {}^{70}_{34}\text{Se}$	
${}^{74}_{37}\text{Rb} \to {}^{74}_{36}\text{Kr}$	●

Hardy and Towner, 2020 PRC

● : Lifetime precision better than 0.23%

${}^{10}\text{C} \to {}^{10}\text{B}$ transition: the first, important prototype!

Nuclear structure effects in superallowed decays

Ongoing efforts to study $^{10}C \rightarrow {}^{10}B$

1. Quantum Monte Carlo (QMC)

Collaboration: Saori Pastore's group (WUSTL)

Minimize the expectation value of H using a trial wavefunction:

$$\frac{\langle \Psi_T | H | \Psi_T \rangle}{\langle \Psi_T | \Psi_T \rangle} = E_T \geq E_0 \qquad |\Psi_T\rangle = \left(1 - \sum_{i<j<k} F_{ijk}\right)\left(\mathcal{S}\prod_{i<j} F_{ij}\right)|\Phi_J\rangle$$

Multi-dimensional integration over particle positions done with **Monte Carlo techniques**

$$\langle \mathcal{O} \rangle = \frac{\int d\mathbf{R}\Psi_T^\dagger(\mathbf{R})\mathcal{O}\Psi_T(\mathbf{R})}{\int d\mathbf{R}\Psi_T^\dagger(\mathbf{R})\Psi_T(\mathbf{R})}$$

Carlson et al., 2015 RMP; Gandolfi, Lonardoni, Lovato and Piarulli, 2020 Front.Phys

Nuclear structure effects in superallowed decays

Strategy: To directly compute the **on-shell nuclear tensor**:

$$\frac{1}{4\pi}\sum_X (2\pi)^4\delta^{(4)}(p+q-p_X)\langle{}^{10}\mathrm{B}(p)|J_{\mathrm{em}}^{(I)\mu}|X\rangle\langle X|J_A^\nu|{}^{10}\mathrm{C}(p)\rangle$$
$$= \frac{i\epsilon^{\mu\nu\alpha\beta}p_\alpha q_\beta}{2p\cdot q}F_3^{(I)}$$

similar to their past work in the computation of **nuclear response functions** for inclusive neutrino-deuteron scattering:

$$R_{xy}(q,\omega)=\frac{2}{3}\sum_M\sum_f \delta(\omega+m_d-E_f)$$
$$\times \mathrm{Im}[\langle f|j^x(\mathbf{q},\omega)|d,M\rangle\langle f|j^y(\mathbf{q},\omega)|d,M\rangle^*].$$

Shen, Marcucci, Carlson, Gandolfi and Schiavilla, 2012 PRC

Nuclear structure effects in superallowed decays

Calculations more readily executable with **physical currents** and **diagonal external states**, which are obtained through **isospin rotation**

E-even term (isosinglet EM current):

Neutral current matrix elements!

$$\begin{aligned}\langle 1,0|J_{\mathrm{em}}^{(0)}(J_W^\dagger)_A|1,1\rangle &= \langle 1,-1|J_{\mathrm{em}}^{(0)}(J_W^\dagger)_A|1,0\rangle \\ &= \frac{1}{2\sqrt{2}}\{\langle 1,1|J_{\mathrm{em}}(J_Z)_A|1,1\rangle - \langle 1,-1|J_{\mathrm{em}}(J_Z)_A|1,-1\rangle\}\end{aligned}$$

E-odd term (isotriplet EM current):

$$\langle 1,0|J_{\mathrm{em}}^{(1)}(J_W^\dagger)_A|1,1\rangle = \frac{1}{\sqrt{2}}\left\{\langle 1,1|(J_W)_V(J_W^\dagger)_A|1,1\rangle - \langle 1,0|(J_W)_V(J_W^\dagger)_A|1,0\rangle\right\}$$

$$\langle 1,-1|J_{\mathrm{em}}^{(1)}(J_W^\dagger)_A|1,0\rangle = \frac{1}{\sqrt{2}}\left\{\langle 1,0|(J_W)_V(J_W^\dagger)_A|1,0\rangle - \langle 1,-1|(J_W)_V(J_W^\dagger)_A|1,-1\rangle\right\}$$

Example: the **A=10** isotriplet:

$$|1,1\rangle = |{}^{10}\mathrm{C}\rangle\ ,\ |1,0\rangle = |{}^{10}\mathrm{B}\rangle\ ,|1,-1\rangle = |{}^{10}\mathrm{Be}\rangle$$

Nuclear structure effects in superallowed decays

Ongoing efforts to study $^{10}C \rightarrow {}^{10}B$

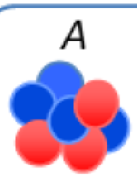
2. No-Core Shell Model (NCSM)

Collaboration: Petr Navratil's group (TRIUMF)

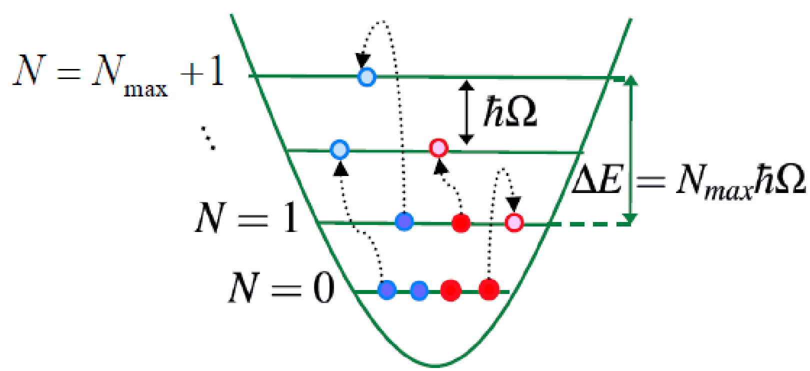
More in Michael Gennari's talk!

- All nucleons are active, no inert core like in standard shell model
- Expand in antisymmetrized products of **harmonic oscillator (HO)** states
- Expansion parameters $\mathbf{N_{max}}$ and $\mathbf{\hbar\Omega}$ (HO length)

Barrett, Navratil and Vary, 2013 Prog.Part.Nucl.Phys



$$\Psi^A = \sum_{N=0}^{Nmax} \sum_i c_{Ni} \Phi^A_{Ni}$$



Nuclear structure effects in superallowed decays

Strategy: To directly compute the **Generalized Compton tensor**:

$$T^{\mu\nu}(p,q) = -i\int d^3x e^{-i\vec{q}\cdot\vec{x}} \langle\phi_f(p)|\, J^\mu_{\rm em}(0,\vec{x}) G_{\rm nucl}(M+q_0+i\epsilon) J^{\dagger\nu}_W(0,\vec{0})\, |\phi_i(p)\rangle$$
$$-i\int d^3x e^{-i\vec{q}\cdot\vec{x}} \langle\phi_f(p)|\, J^{\dagger\nu}_W(0,\vec{0}) G_{\rm nucl}(M-q_0+i\epsilon) J^\mu_{\rm em}(0,\vec{x})\, |\phi_i(p)\rangle$$

at **zero and finite q**, with the nuclear Green's function:

$$G_{\rm nucl}(E) \equiv \sum_n \frac{|n\rangle\langle n|}{E_n - E}$$

treated using the "Lanczos continued fractions method".

Marchisio et al., 2003 Few-Body Systems;
Haydock, 1974 Journal of Physics A

q=0 points are related to moments of the structure function, e.g.:

$$\int_{\nu_0}^{\infty} d\nu \frac{F_3^{(0)}(\nu,0)}{\nu^2} = -\frac{1}{2}\int d^3x z \Big\{ \langle\phi_f(p)|J^x_{\rm em(0)}(\vec{x}) G_{\rm nucl}(M+i\epsilon)(J^{\dagger y}_W(0))_A|\phi_i(p)\rangle$$
$$+\langle\phi_f(p)|(J^{\dagger y}_{\rm W}(0))_A G_{\rm nucl}(M+i\epsilon)\ J^x_{\rm em(0)}(\vec{x})|\phi_i(p)\rangle \Big\}$$

Nuclear structure effects in superallowed decays

One-body current operators readily obtainable in **multipole expansion** (Coulomb, longitudinal, electric, magnetic):

$$
\begin{aligned}
M^{V}_{Jm_J;Im_I}(q,\vec{r}) &\equiv F_1^{(I)}(Q^2)\mathfrak{M}_J^{m_J}(q,\vec{r})\Gamma_{Im_I} \\
T^{V,\mathrm{el}}_{Jm_J;Im_I}(q,\vec{r}) &\equiv \frac{q}{m_N}\left\{F_1^{(I)}(Q^2)\Delta_J'^{m_J}(q,\vec{r}) + \frac{1}{2}G_M^{(I)}(Q^2)\Sigma_J^{m_J}(q,\vec{r})\right\}\Gamma_{Im_I} \\
T^{V,\mathrm{mag}}_{Jm_J;Im_I}(q,\vec{r}) &\equiv -i\frac{q}{m_N}\left\{F_1^{(I)}(Q^2)\Delta_J^{m_J}(q,\vec{r}) - \frac{1}{2}G_M^{(I)}(Q^2)\Sigma_J'^{m_J}(q,\vec{r})\right\}\Gamma_{Im_I} \ ,
\end{aligned}
$$

vector

$$
\begin{aligned}
M^{A}_{Jm_J;Im_I}(q,\vec{r}) &\equiv -i\frac{q}{m_N}\left\{G_A^{(I)}(Q^2)\Omega_J^{m_J}(q,\vec{r}) + \frac{1}{2}\left[G_A^{(I)}(Q^2) - \frac{q^0}{2m_N}G_P^{(I)}(Q^2)\right]\Sigma_J''^{m_J}(q,\vec{r})\right\}\Gamma_{Im_I} \\
L^{A}_{Jm_J;Im_I}(q,\vec{r}) &\equiv i\left\{G_A^{(I)}(Q^2) + \frac{q^2}{4m_N^2}G_P^{(I)}(Q^2)\right\}\Sigma_J''^{m_J}(q,\vec{r})\Gamma_{Im_I} \\
T^{A,\mathrm{el}}_{Jm_J;Im_I}(q,\vec{r}) &\equiv iG_A^{(I)}(Q^2)\Sigma_J'^{m_J}(q,\vec{r})\Gamma_{Im_I} \\
T^{A,\mathrm{mag}}_{Jm_J;Im_I}(q,\vec{r}) &\equiv G_A^{(I)}(Q^2)\Sigma_J^{m_J}(q,\vec{r})\Gamma_{Im_I} \ .
\end{aligned}
$$

axial

of which matrix elements with respect to HO states are **analytically known**

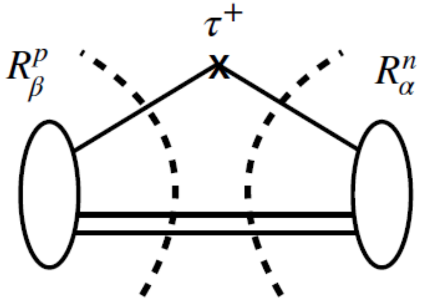
Donnelly and Haxton, 1979 Atom.Data Nucl.Data Tabl.

Isospin-breaking corrections in superallowed decays

δ_C: isospin-breaking (ISB) corrections to nuclear WFs

Essential to **align the Ft-values** of different superallowed transitions!

$$\mathcal{F}t = ft\,(1+\delta'_{\rm R})\,(1+\delta_{\rm NS}-\delta_{\rm C}) \qquad |V_{ud}|^2 = \frac{2984.43\ s}{\mathcal{F}t\,(1+\Delta_R^V)}.$$



If isospin symmetry is exact:

$$\int drr^2|R_\alpha^{i,p}(r)|^2 = \int drr^2|R_\alpha^{f,n}(r)|^2 = \int drr^2 R_\alpha^{i,p*}(r)R_\alpha^{f,n}(r) = 1$$

With isospin breaking effect:

$$\int drr^2 R_\alpha^{i,p*}(r)R_\alpha^{f,n}(r) = 1-\delta_C/2$$

Isospin-breaking corrections in superallowed decays

- Current input adopted in global analysis: Shell model + Woods-Saxon (WS) potential by Hardy and Towner (HT)
- Successful in aligning Ft value of different superallowed transitions

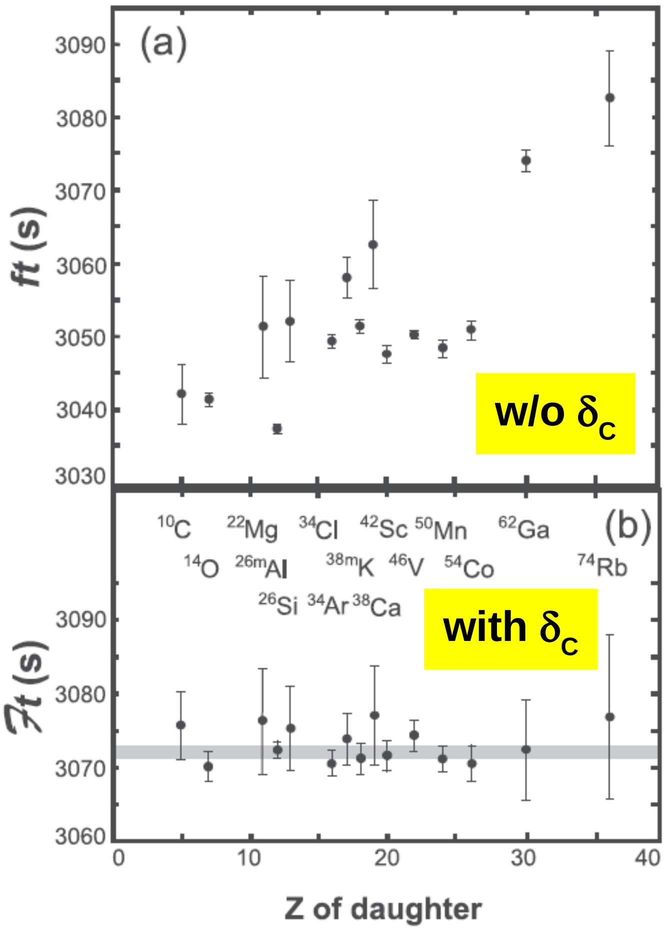
Caveats:

- Same degree of alignment failed to be reproduced by other nuclear approaches
- Theory inconsistencies, e.g. not using the correct isospin operator

Miller and Schwenk, 2008 PRC; 2009 PRC

- Recent study shows significant reduction of δ_C due to nucleon-nucleon short-range correlations

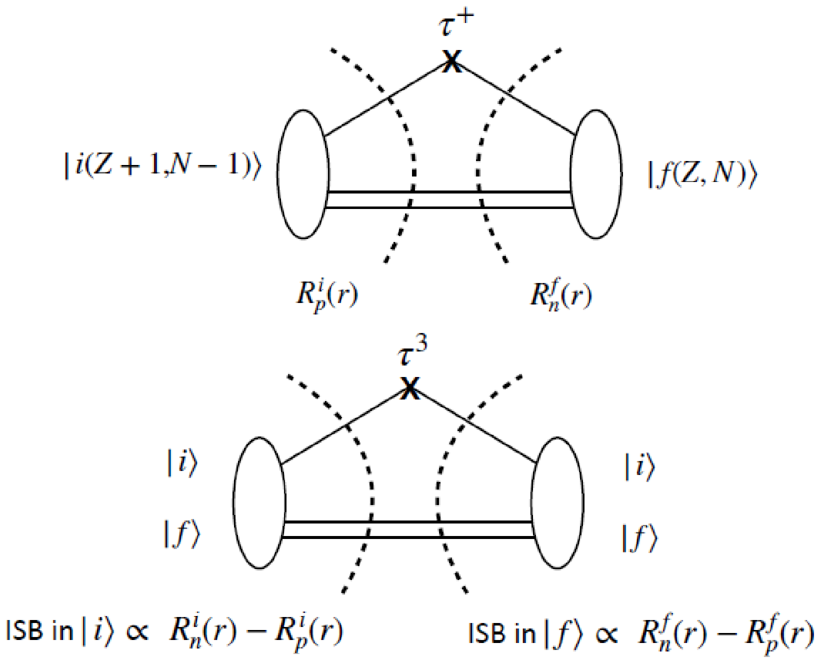
Condren and Miller, 2201.10651



Hardy and Towner, 2020 PRC

Isospin-breaking corrections in superallowed decays

Proposal: Expand the pool of observables related to δ_C

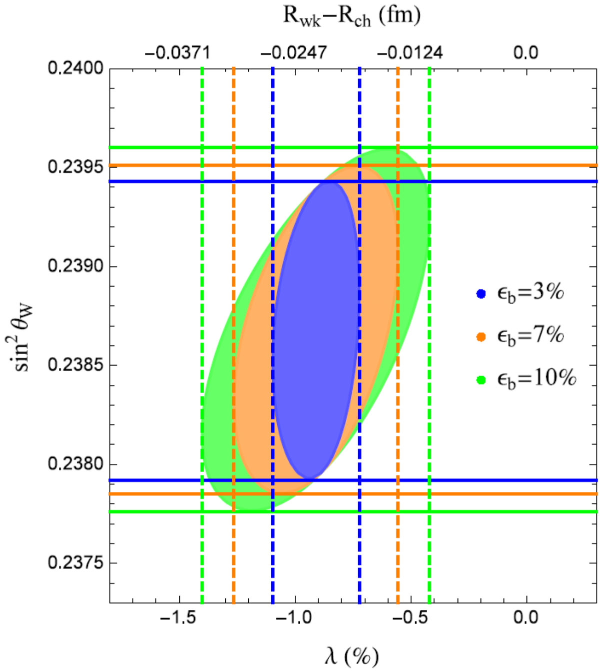


δ_C is not observable, but ISB effects in neutral currents are observable (neutron skin), and can be accessed with parity-violating elastic electron-nucleus scattering

$$\left(R_{n-p}^{i,f}\right)^2 \propto \int r^4 dr \left(|R_\alpha^{i,f\,n}(r)|^2 - |R_\alpha^{i,f\,p}(r)|^2\right)$$

Isospin-breaking corrections in superallowed decays

Exploratory study: **neutron skin in ^{12}C**



0.3% precise forward-angle asymmetry +
3% precise backward-angle asymmetry
in electron-^{12}C scattering

Determination of the ^{12}C **weak charge** and **weak radius** to 0.3% and 0.2%

Koshchii, Erler, Gorchtein, Horowitz, Piekarewicz, Roca-Maza, CYS and Spiesberger, 2020 PRC

Possibly strategies to proceed:

- Derive **model-independent sum rules** to relate δ_C and neutron skin, or
- Make use of **schematic models** (e.g. the Miller-Schwenk formalism) to study their relations
- In any case, **ab-initio calculation** is needed to translate the measured asymmetries into ISB parameters

Summary

- I describe several research topics in the field of beta decays that await precise hadron and nuclear physics calculations.
- New lattice QCD calculations are needed to pin down the single-nucleon box diagrams.
- Ab-initio calculations are needed to reduce the dominant nuclear uncertainties in V_{ud} extracted from superallowed beta decays.
- Parity-violating electron-nucleus scattering may provide a model-independent constraint on the isospin breaking corrections in nuclear beta decays.

Thanks for listening!