Electroweak Nuclear Responses with Controlled Theory Uncertainty

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In collaboration with
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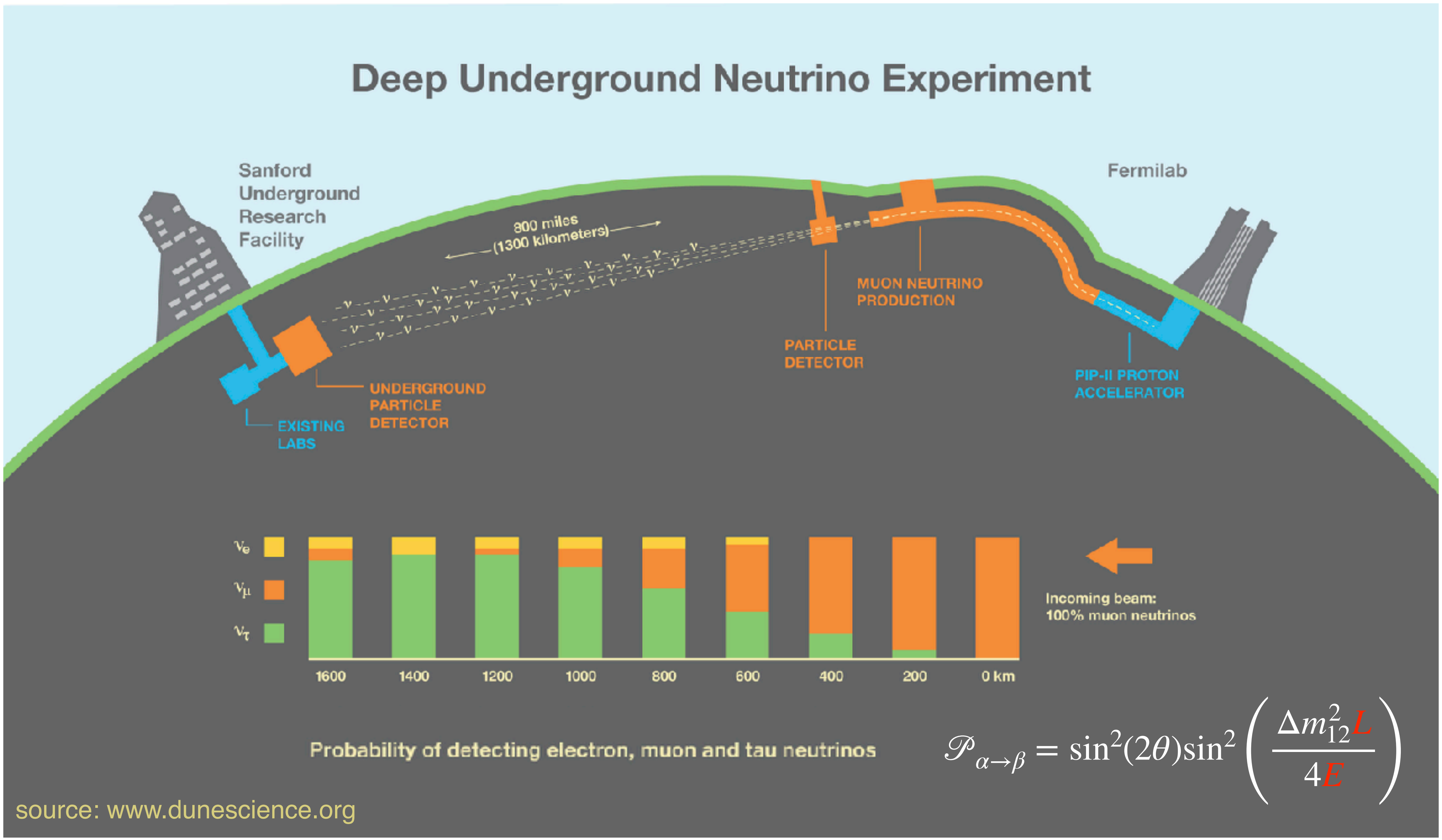
Bijaya Acharya
Gaute Hagen

Precision Tests with Neutral-Current Coherent Interactions with Nuclei, 25/05/2022

Cluster of Excellence PRISMA+ Precision Physics, Fundamental Interactions and Structure of Matter
Johannes Gutenberg Universität Mainz
JG|U
Alexander von Humboldt Stiftung/Foundation

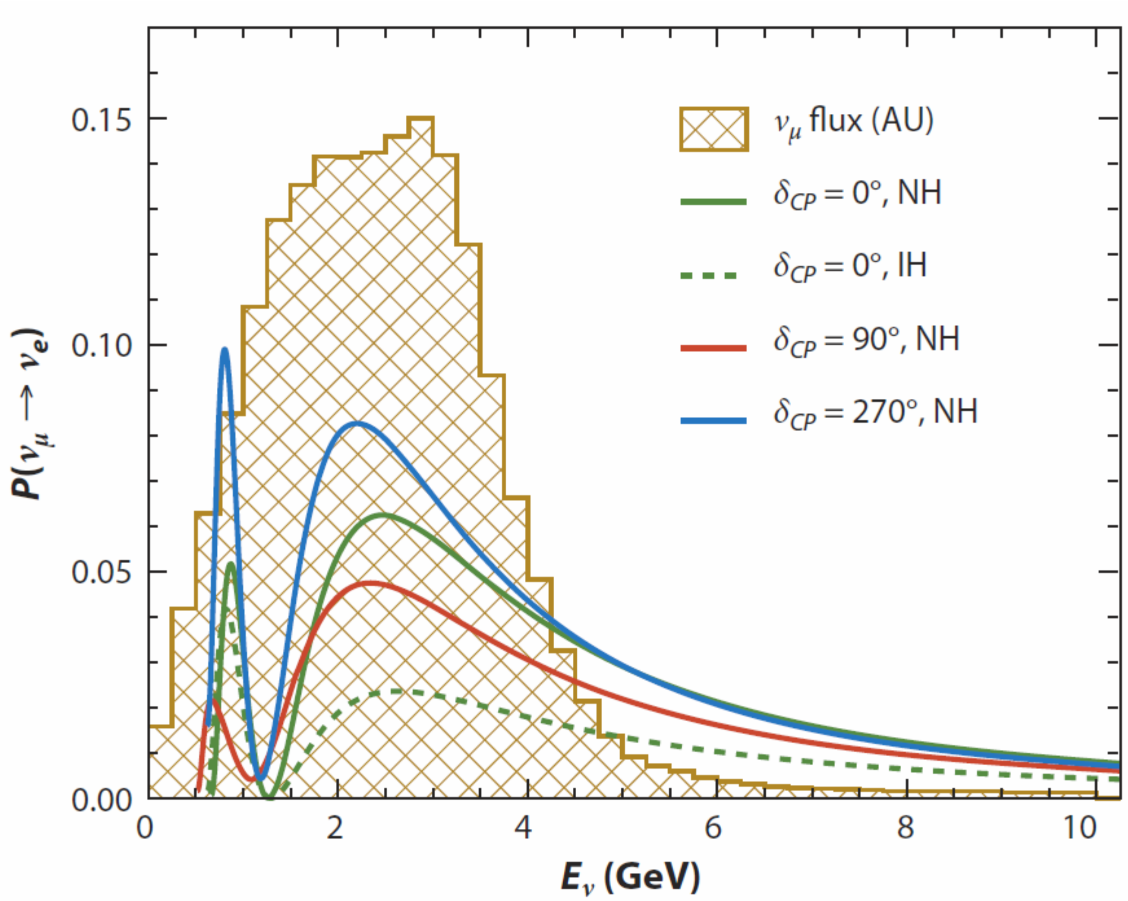
Neutrino oscillations



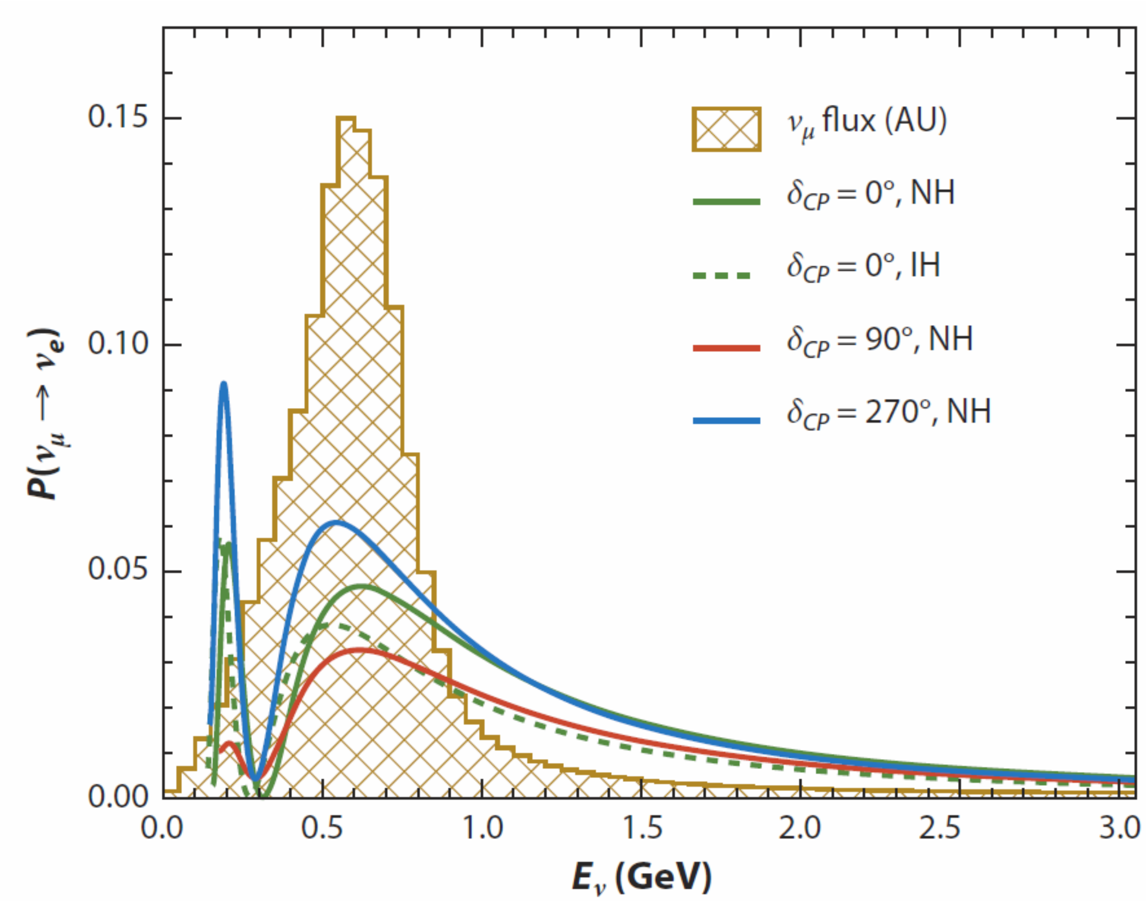
$$\mathscr{P}_{\alpha\to\beta} = \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2_{12} L}{4E}\right)$$

Aims & challenges

DUNE



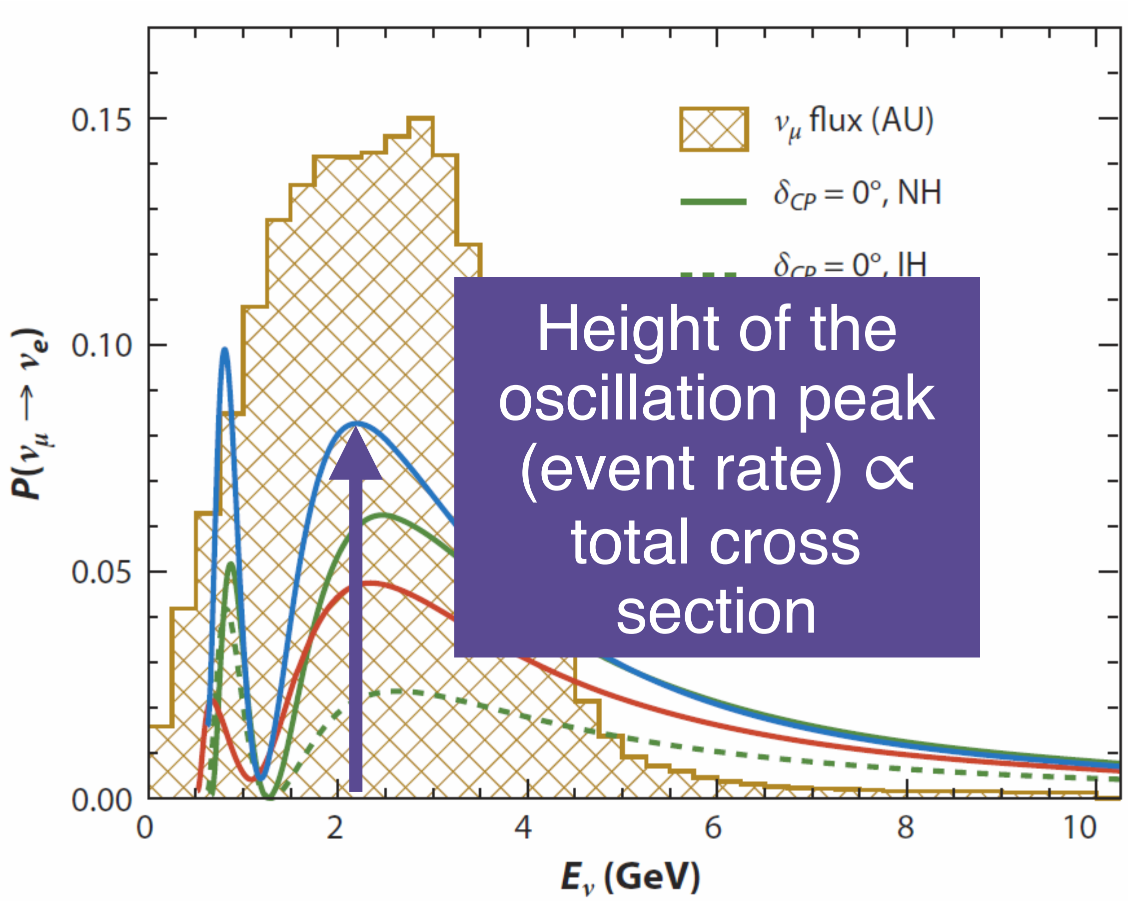
T2HK



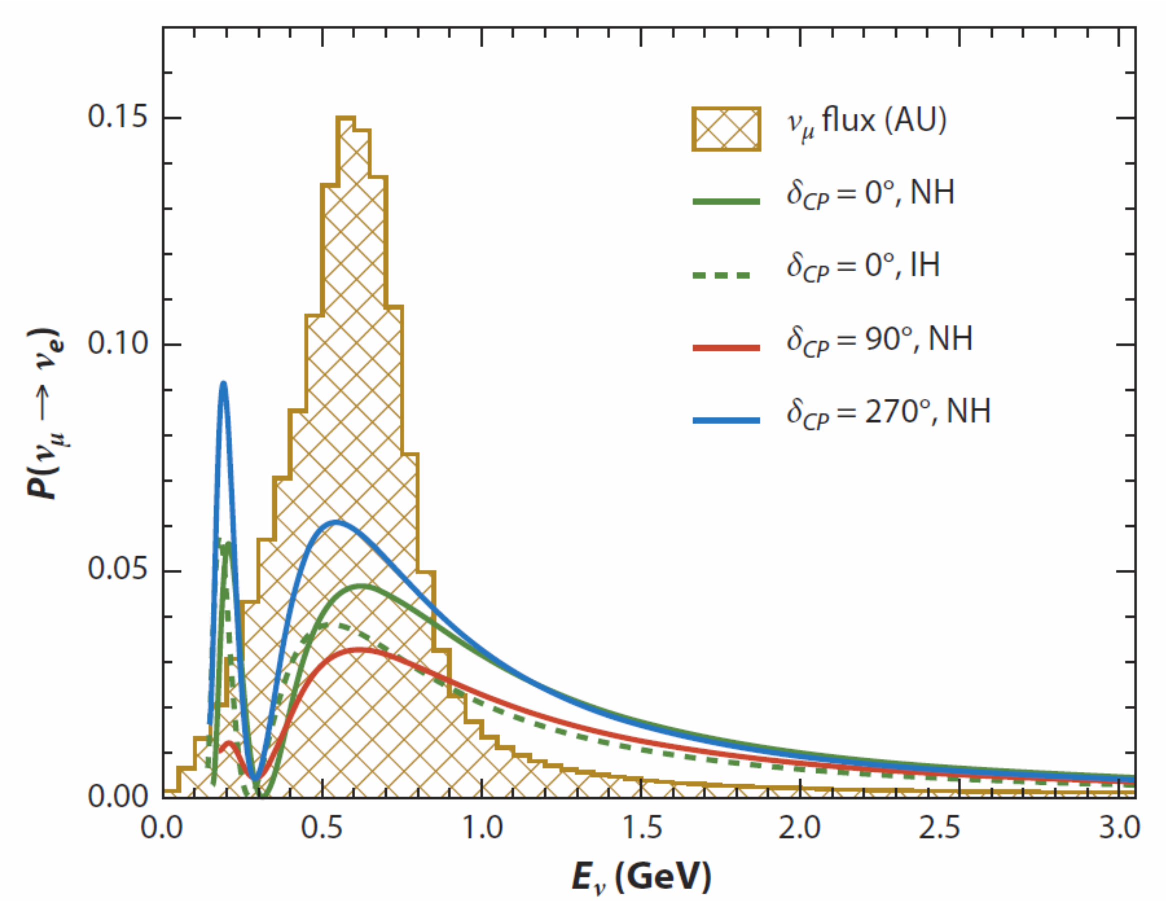
From: Diwan et al, Ann. Rev.Nucl. Part. Sci 66 (2016)

Aims & challenges

DUNE



T2HK



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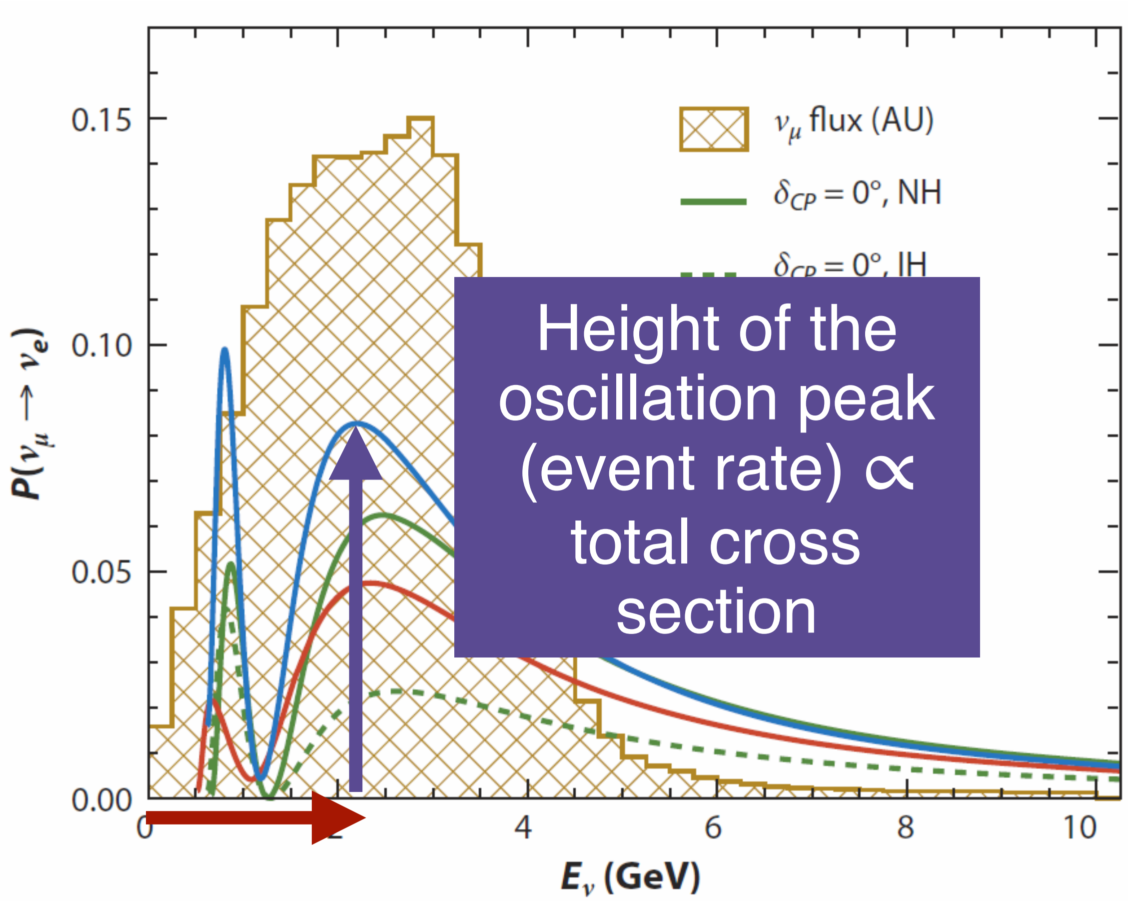

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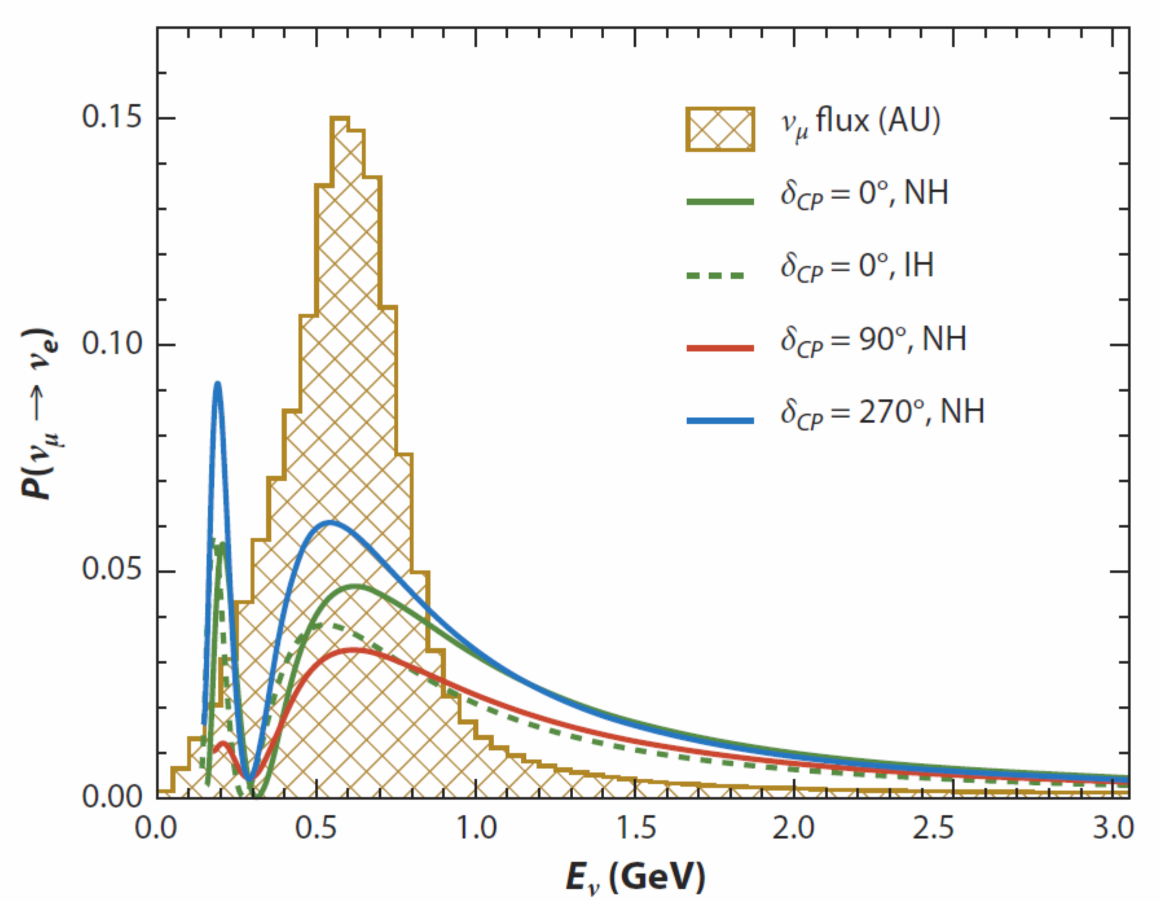
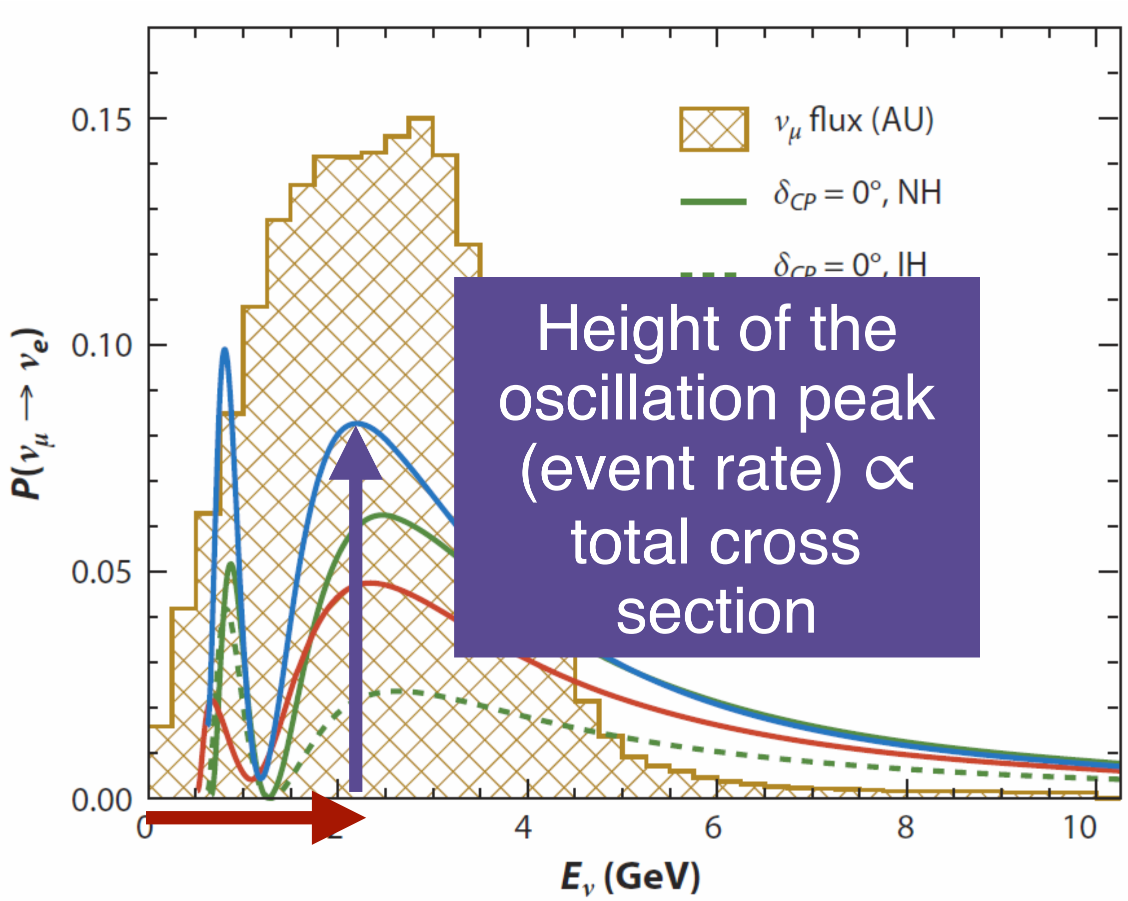


Position of the oscillation peak depends on energy reconstruction

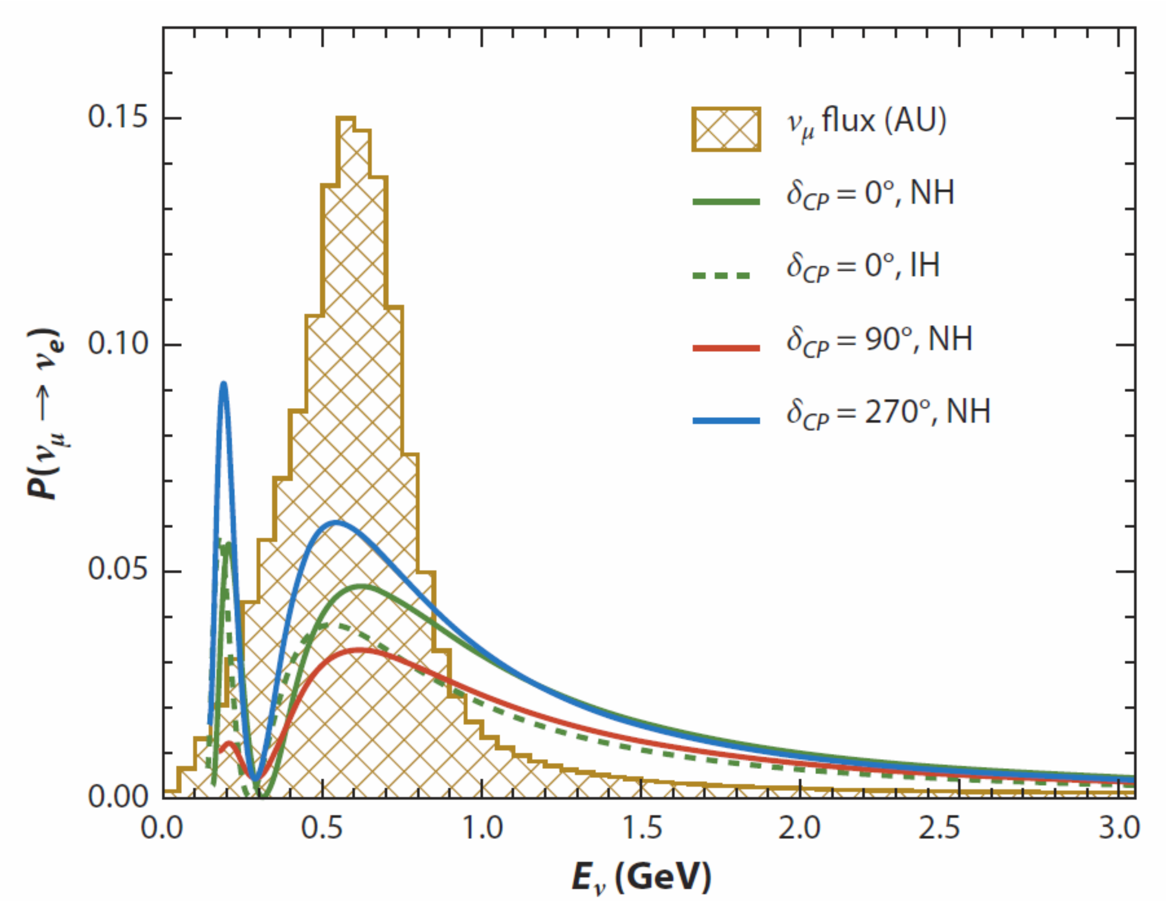
DUNE aims at uncertainties < 1% meaning O(25 MeV) precision of energy reconstruction

Aims & challenges

DUNE



T2HK



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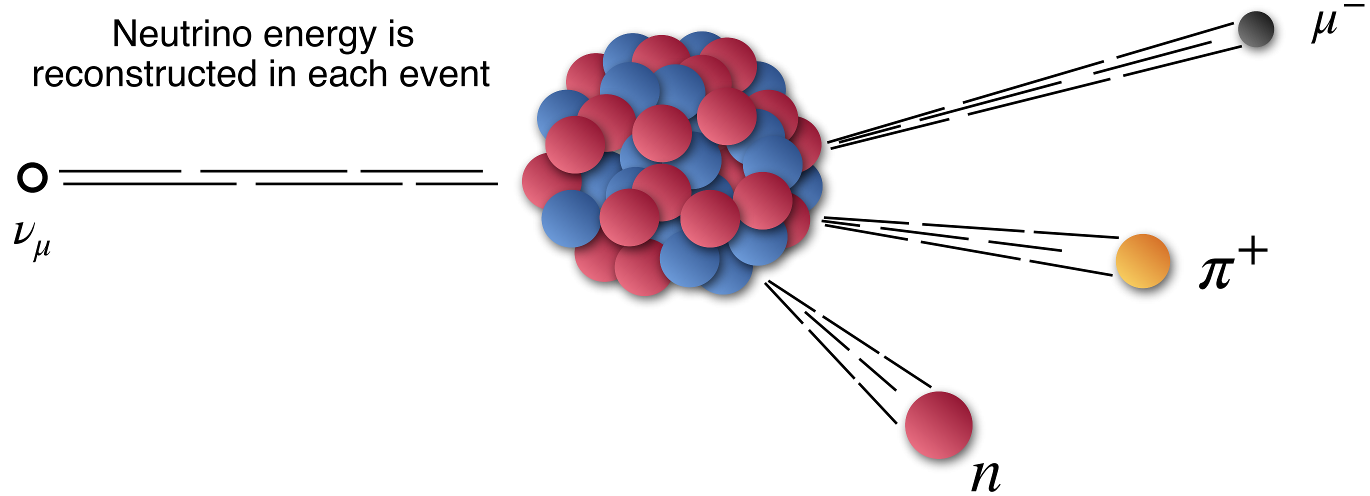
Position of the oscillation peak depends on energy reconstruction

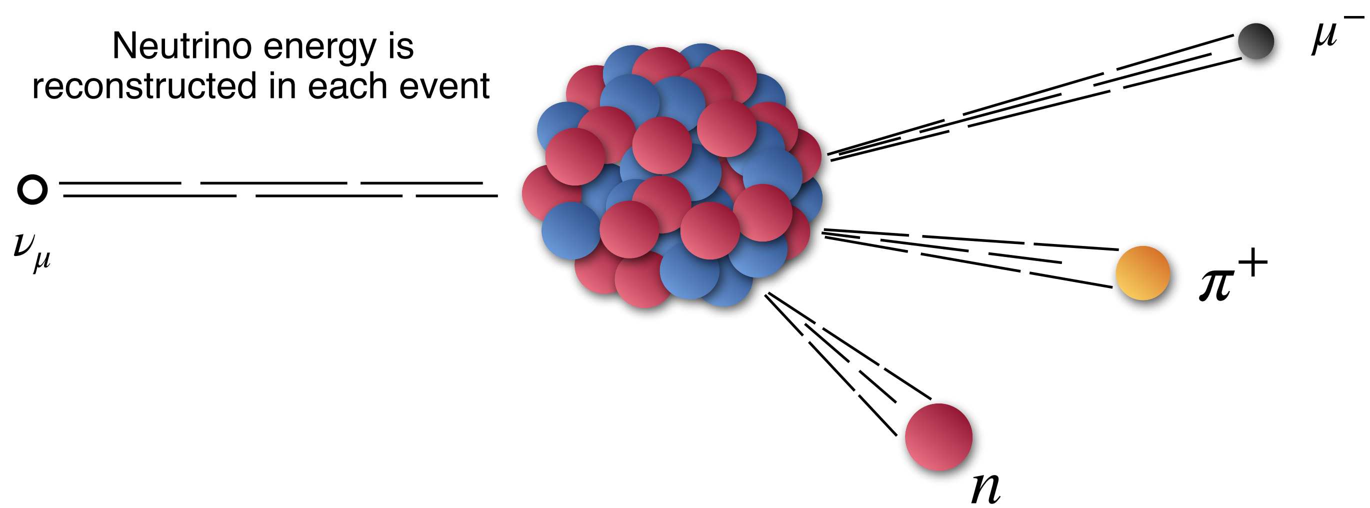
DUNE aims at uncertainties < 1% meaning O(25 MeV) precision of energy reconstruction

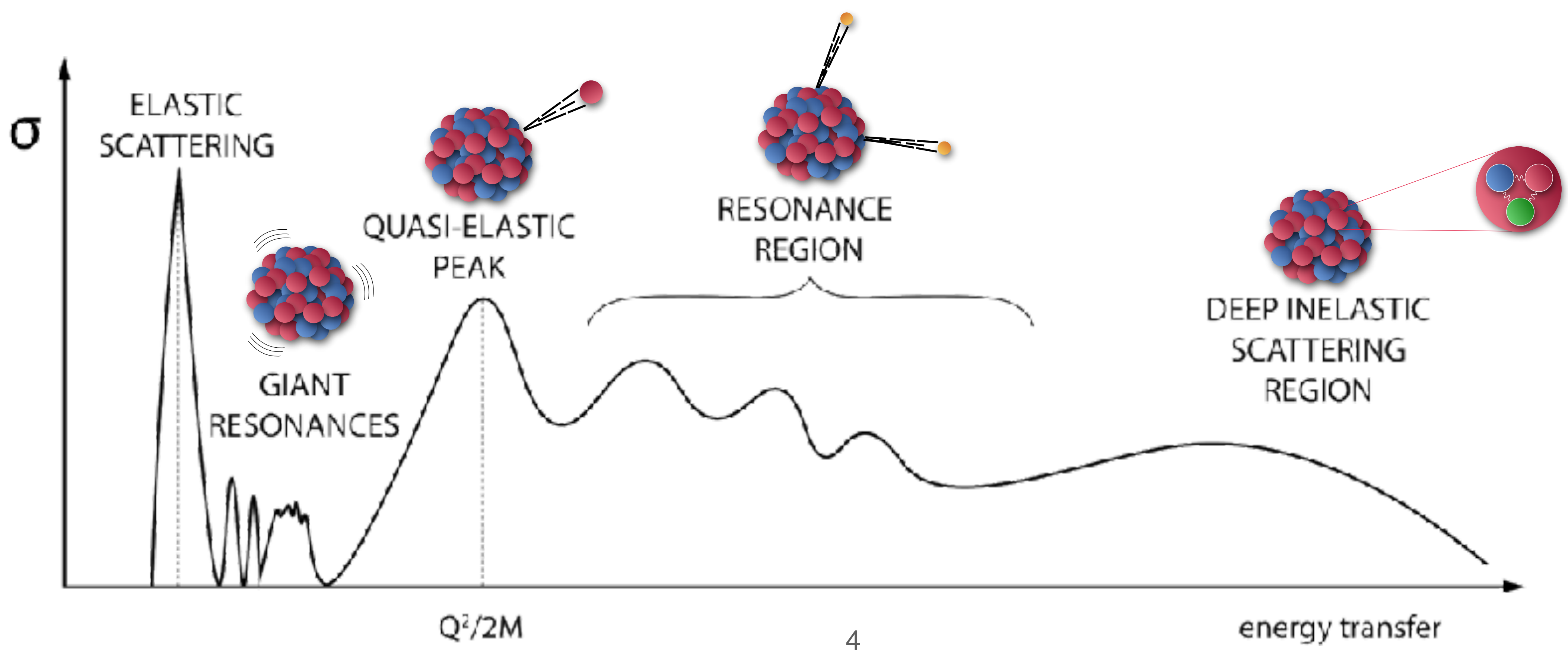
Systematic errors should be small since statistics will be high.

Motivation

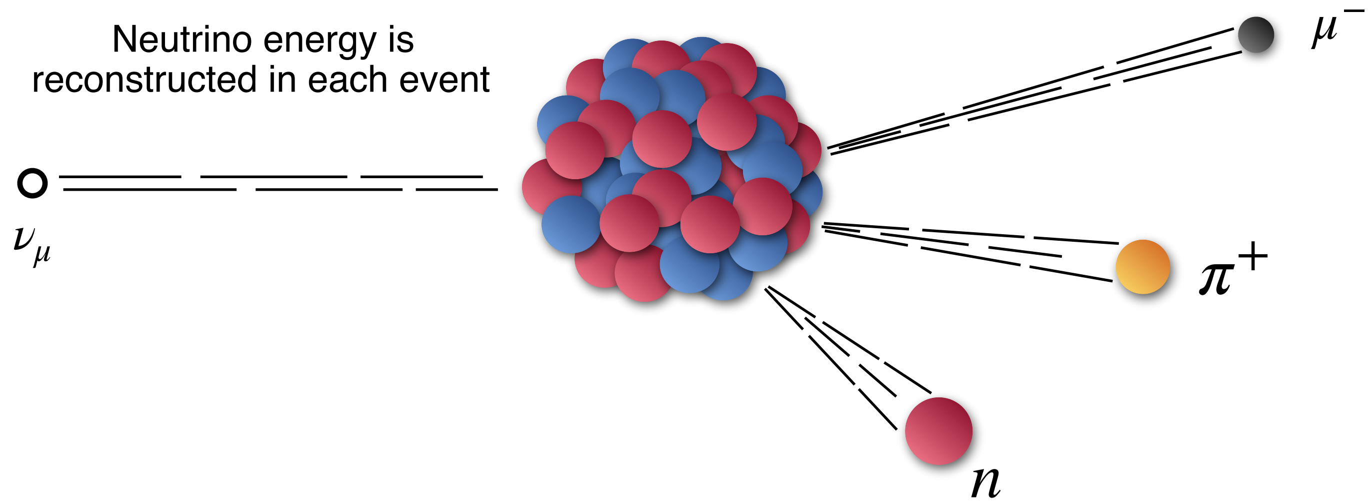


Motivation


Neutrino energy is
reconstructed in each event
ν_μ
μ^-
π^+
n


σ
ELASTIC
SCATTERING
GIANT
RESONANCES
QUASI-ELASTIC
PEAK
RESONANCE
REGION
DEEP INELASTIC
SCATTERING
REGION
Q²/2M
energy transfer

Motivation



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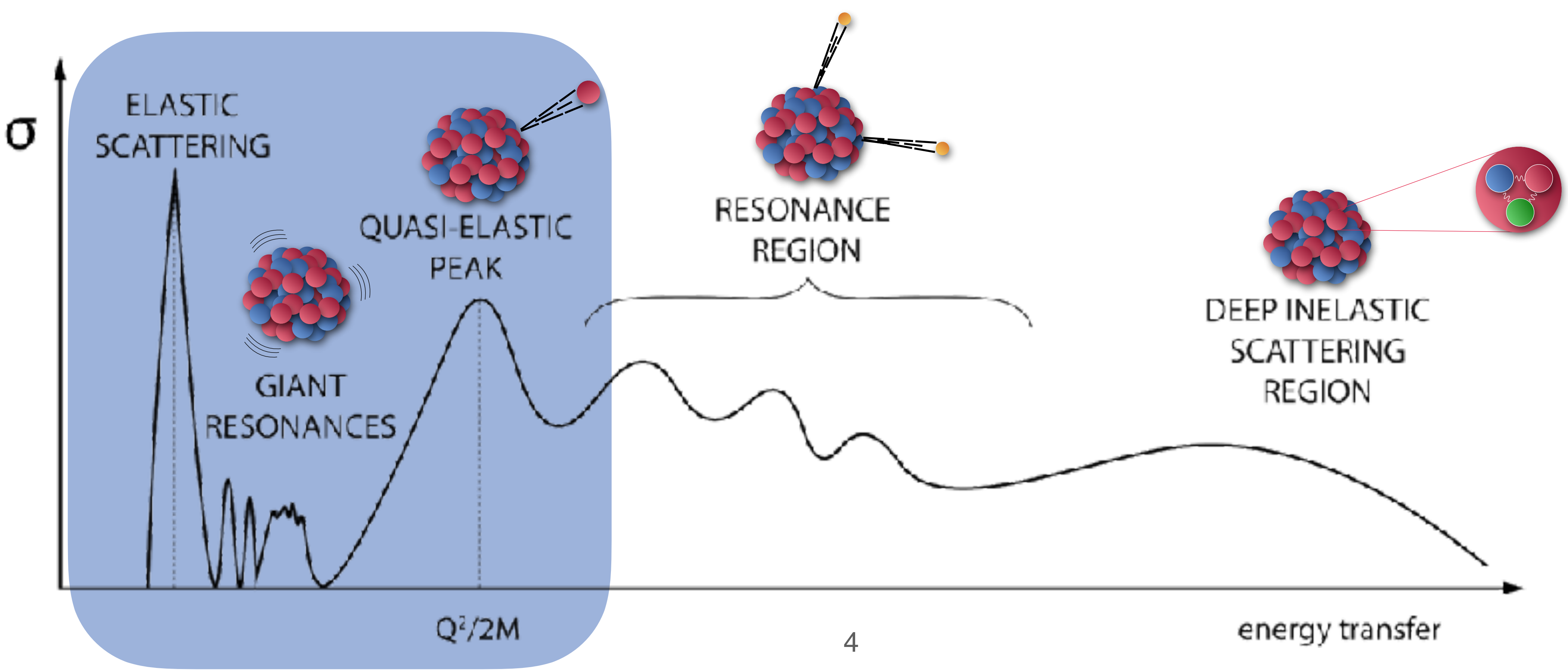

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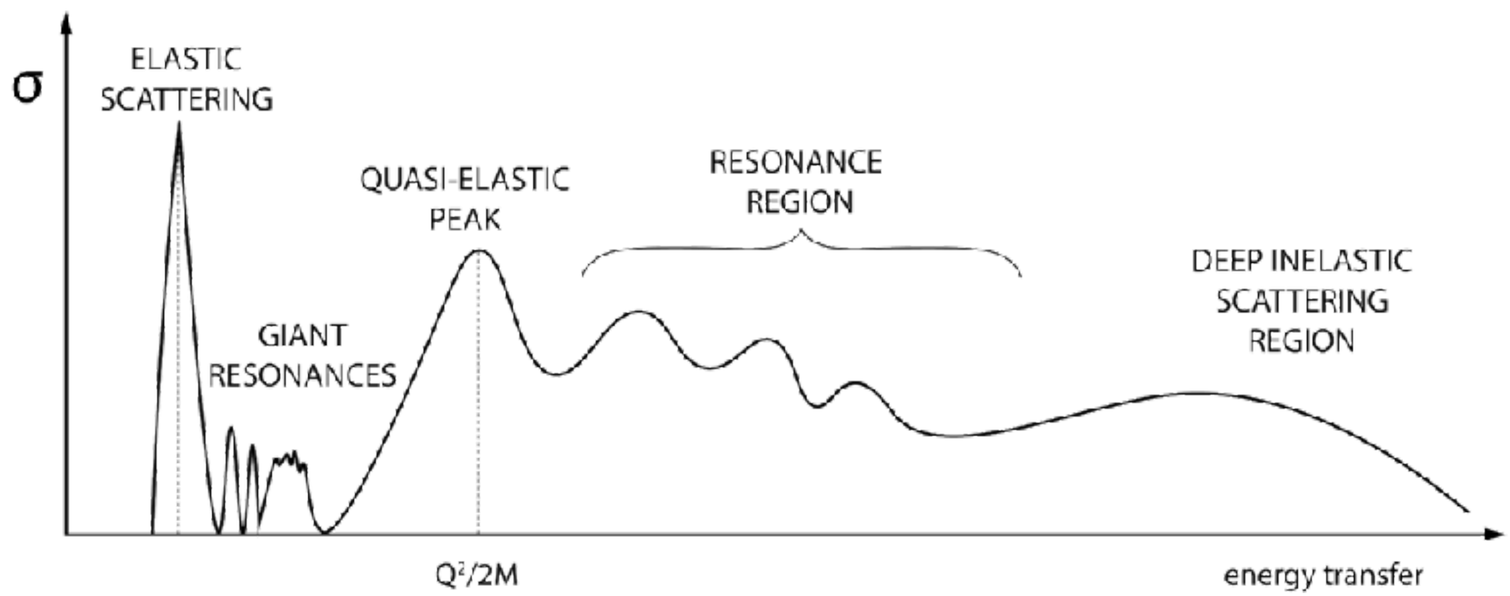
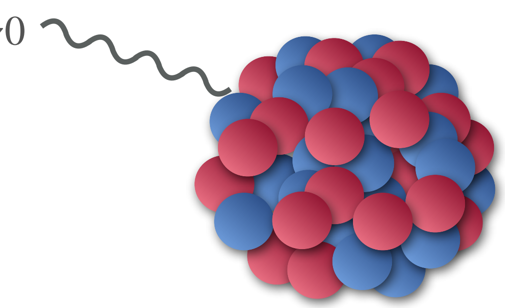
Nuclear response



$$J_\mu = (\rho, \vec{j}) \,|\, \Psi \rangle$$

$\gamma, W^\pm, Z^0$



$$\sigma \propto L^{\mu\nu} R_{\mu\nu}$$

lepton tensor — nuclear responses

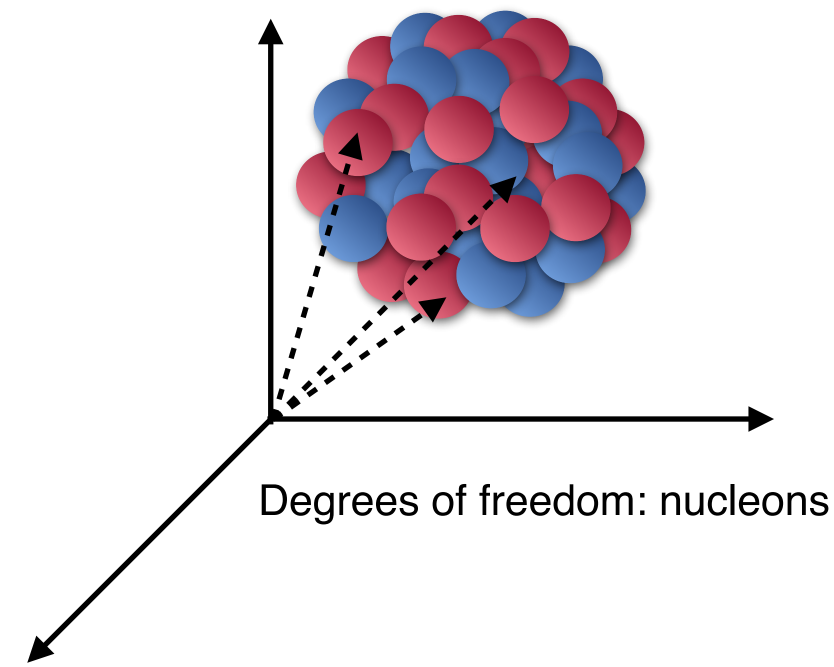
$$R_{\mu\nu}(\omega, q) = \sum_f \langle \Psi \,|\, J_\mu^\dagger(q) \,|\, \Psi_f \rangle \langle \Psi_f \,|\, J_\nu(q) \,|\, \Psi \rangle \delta(E_0 + \omega - E_f)$$

Ab initio nuclear theory for neutrinos

Nuclear chiral Hamiltonian

$$\mathscr{H}\,|\Psi\rangle = E\,|\Psi\rangle$$

➡ order of expansion
➡ low energy constants fit to data



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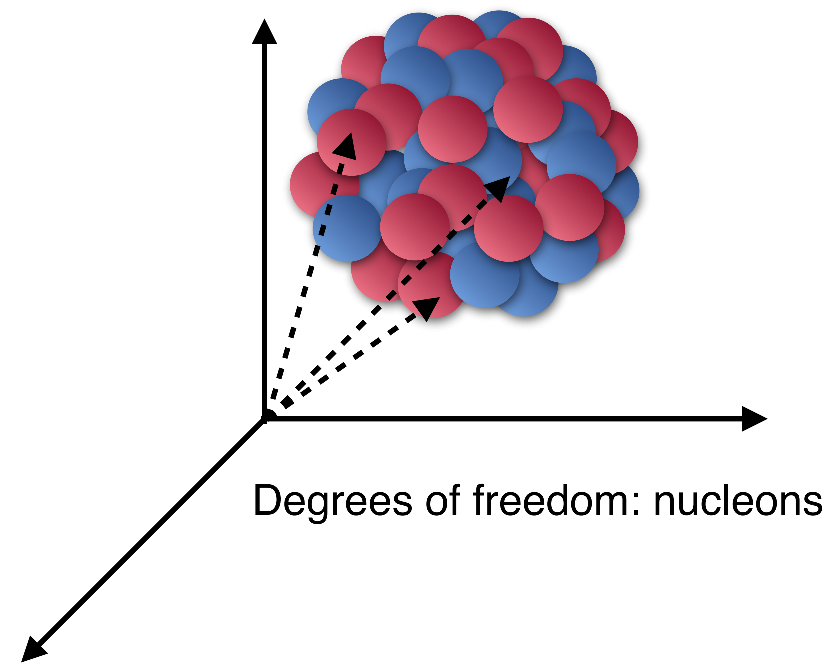

Electroweak currents

$$J^\mu = (\rho, \vec{j})$$

➡ order of expansion
➡ 2-body currents important

Ab initio nuclear theory for neutrinos

Nuclear chiral Hamiltonian

$$\mathscr{H}\,|\,\Psi\rangle = E\,|\,\Psi\rangle$$

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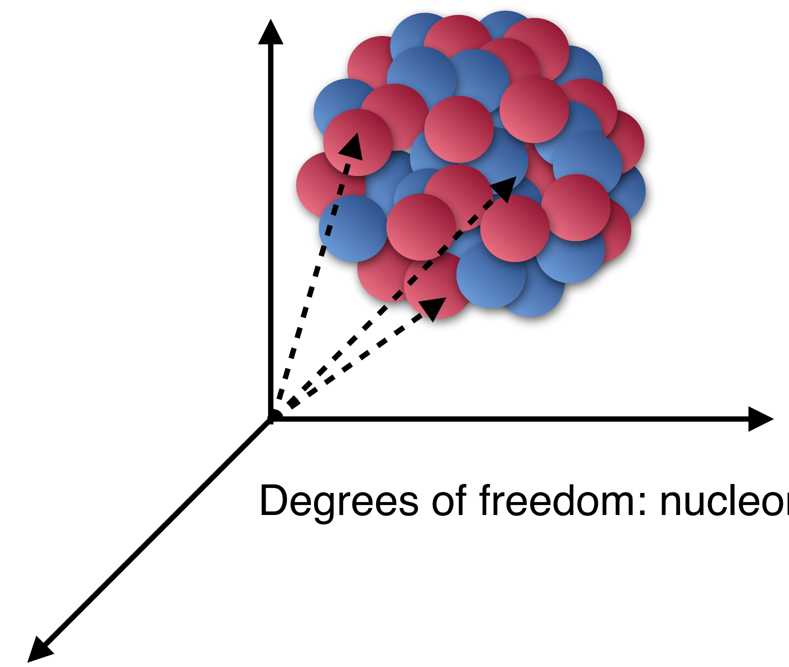


Electroweak currents

$$J^{\mu} = (\rho, \vec{j})$$

- ➡ order of expansion
- ➡ 2-body currents important

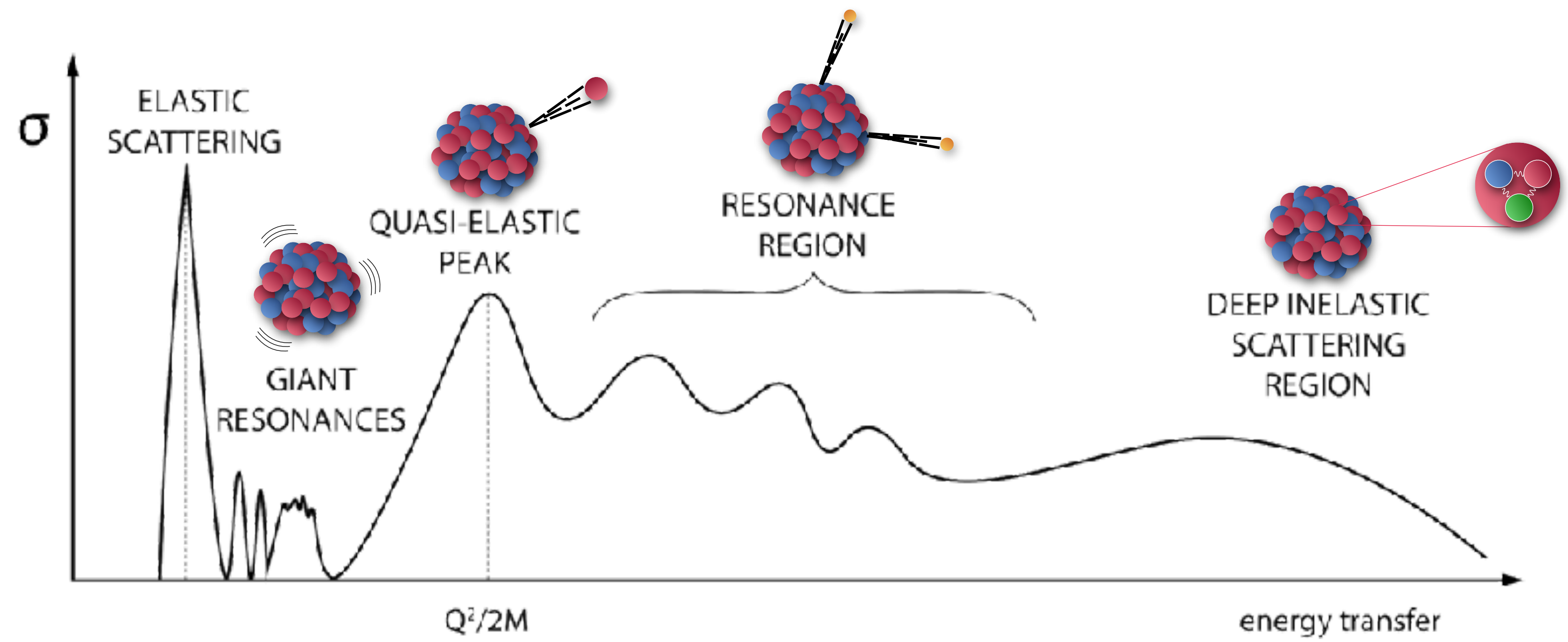
Coupled cluster method

$$\mathscr{A} = \langle \Psi_m \,|\, J_\mu \,|\, \Psi_n \rangle$$

- ➡ truncation in correlations
- ➡ model space dependence

Quasielastic response

- Momentum transfer ~hundreds MeV
- Upper limit for ab initio methods
- Important mechanism for T2HK, DUNE
- Role of final state interactions
- Role of 1-body and 2-body currents

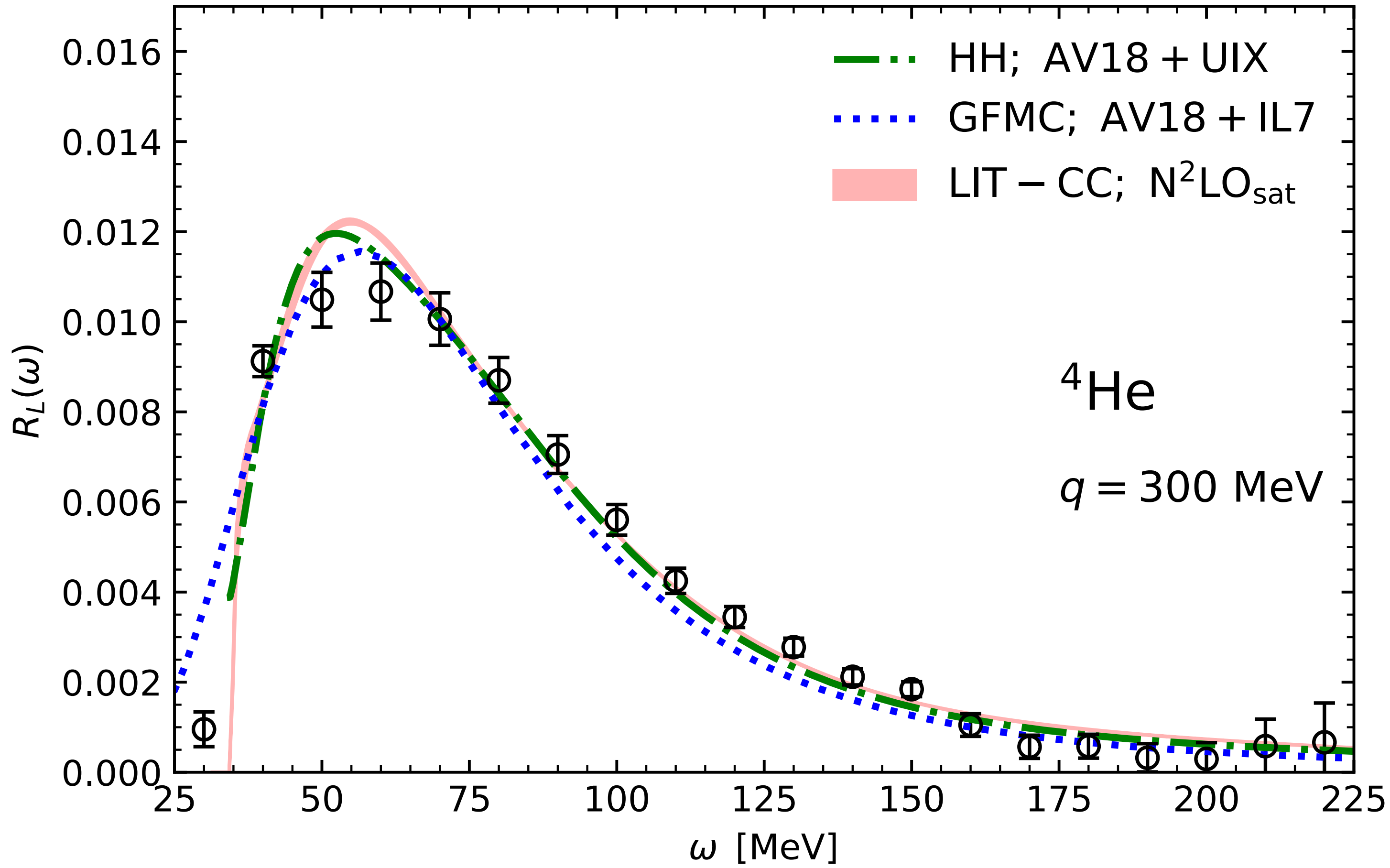


First step: analyse the longitudinal response

$$\left.\frac{d\sigma}{d\omega dq}\right|_e = \sigma_M\left(v_L R_L + v_T R_T\right)$$

charge operator $\hat{\rho}(q) = \sum_{j=1}^{Z} e^{iqz'_j}$

Longitudinal response



JES, B. Acharya, S. Bacca, G. Hagen; PRL 127 (2021) 7, 072501

Uncertainty band: inversion procedure

$$R_{\mu\nu}(\omega, q) = \sum\!\!\!\!\!\!\int_f \langle \Psi | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f)$$

Lorentz Integral Transform

$$R_{\mu\nu}(\omega, q) = \sum\!\!\!\!\!\!\int_f \langle \Psi | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f)$$

continuum spectrum

Integral transform

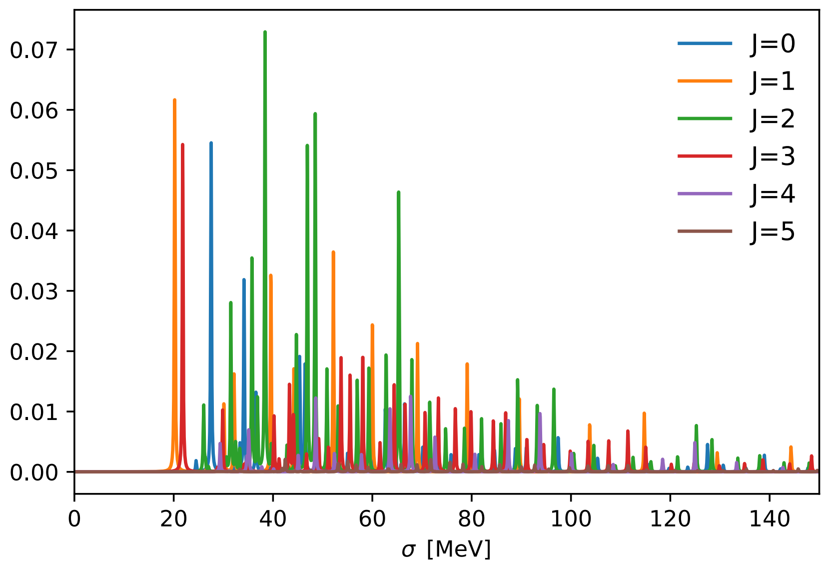
$$S_{\mu\nu}(\sigma, q) = \int d\omega K(\omega, \sigma) R_{\mu\nu}(\omega, q) = \langle \Psi | J_\mu^\dagger \, K(\mathscr{H} - E_0, \sigma) \, J_\nu | \Psi \rangle$$

Lorentzian kernel:

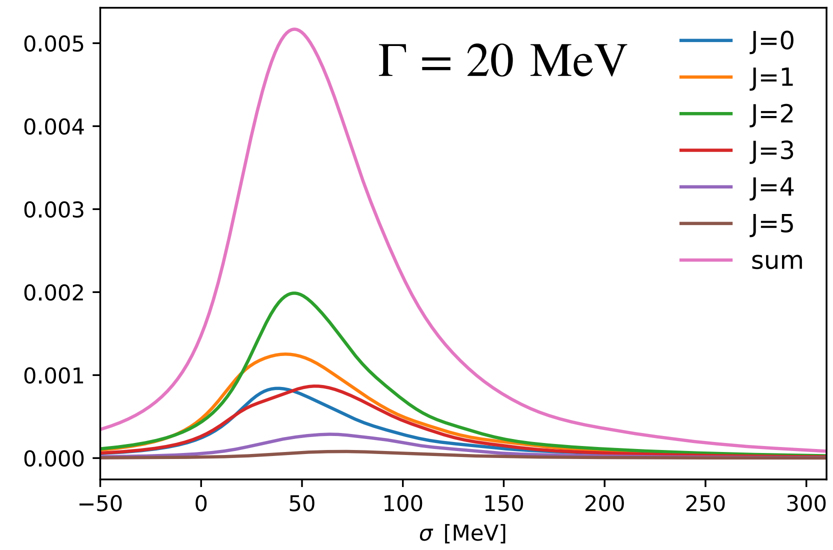
$$K_\Gamma(\omega, \sigma) = \frac{1}{\pi} \frac{\Gamma}{\Gamma^2 + (\omega - \sigma)^2}$$

$S_{\mu\nu}$ has to be inverted to get access to $R_{\mu\nu}$

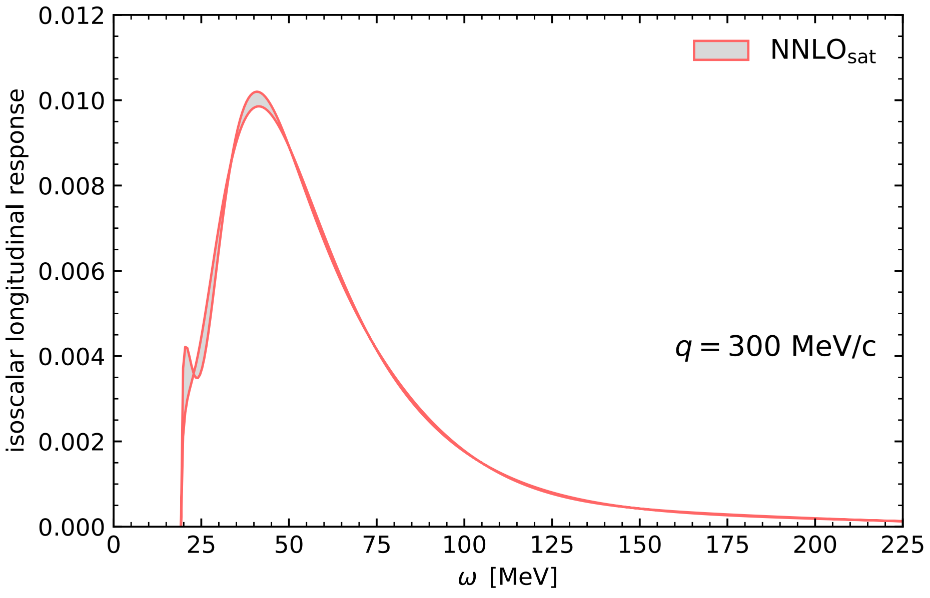
Lorentz Integral Transform



Integral transform

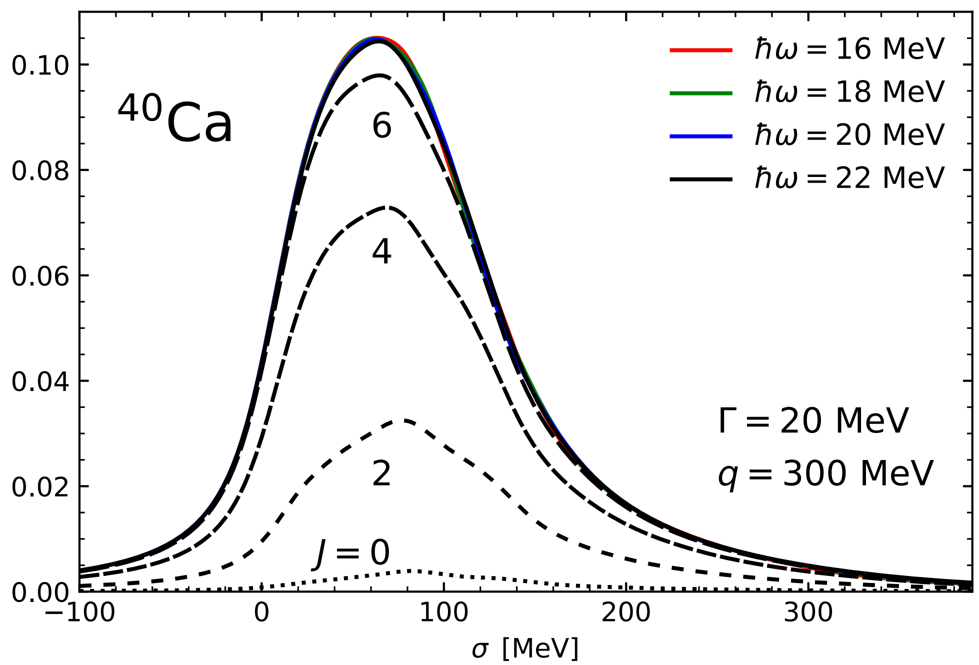


Inversion

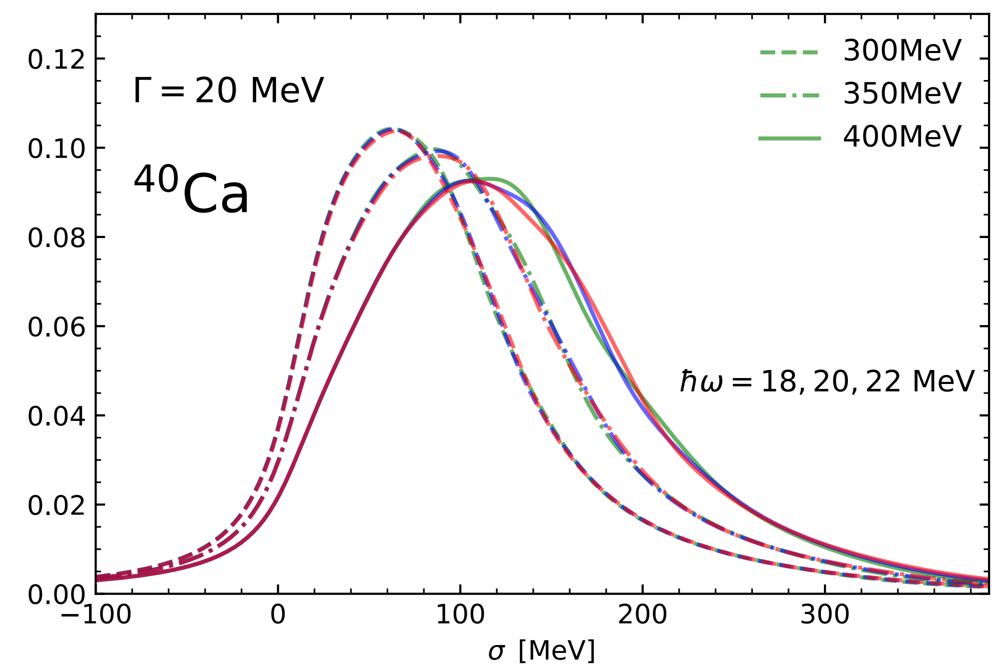


Longitudinal isoscalar response on ^{4}He at q=300 MeV

Longitudinal response ^{40}Ca

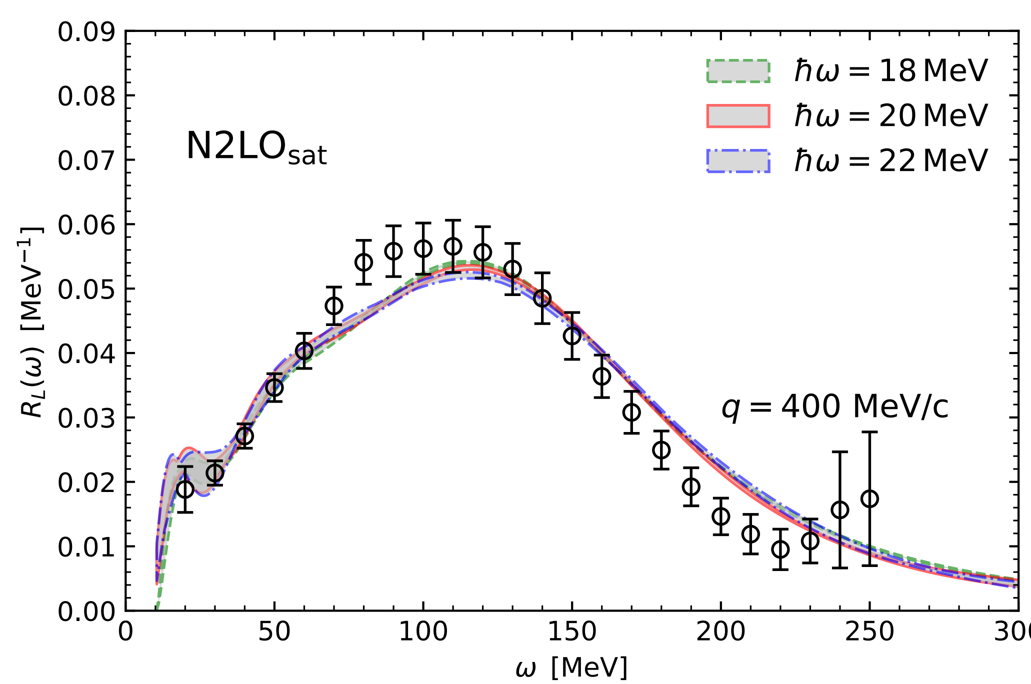


Sum over multipoles

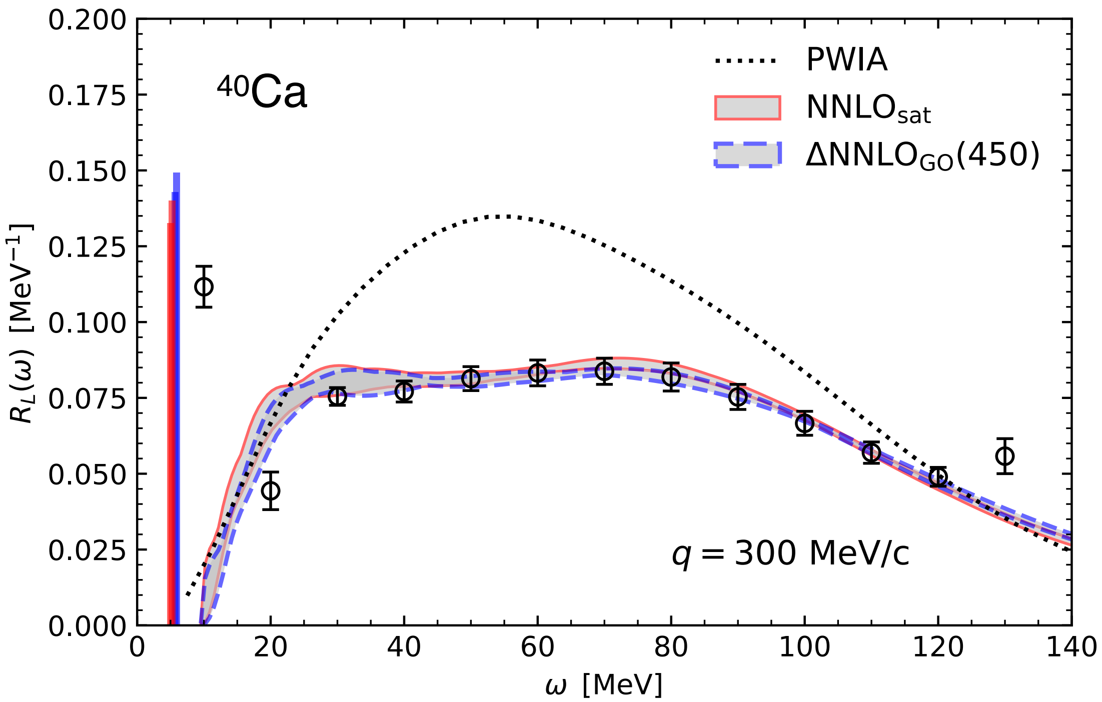


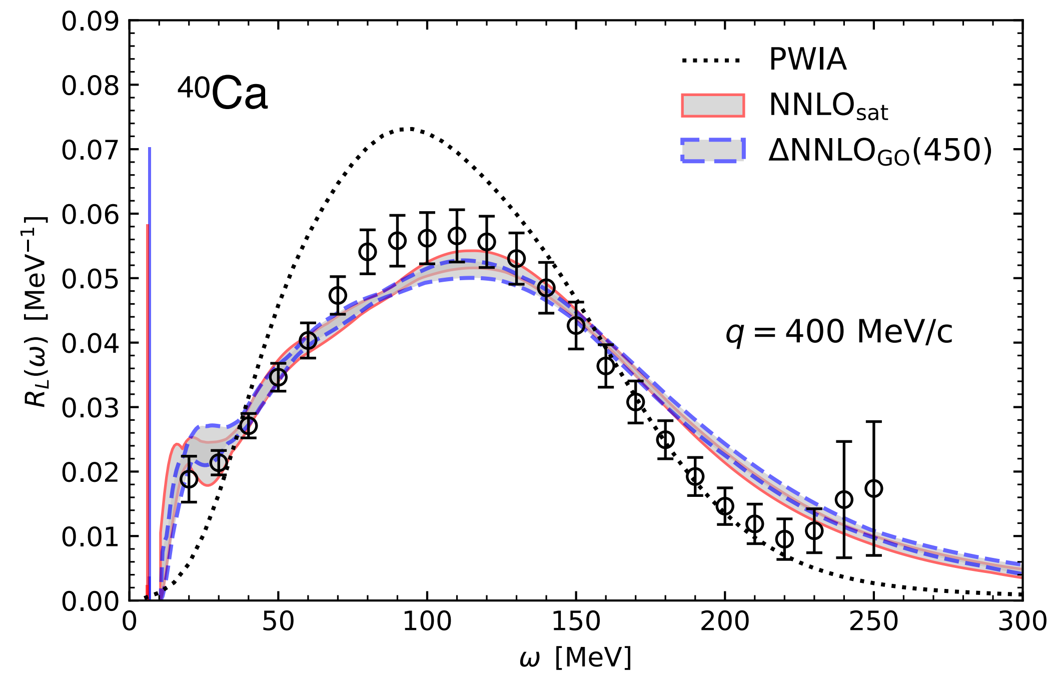
Underlying oscillator frequency

Inversion



Longitudinal response ^{40}Ca



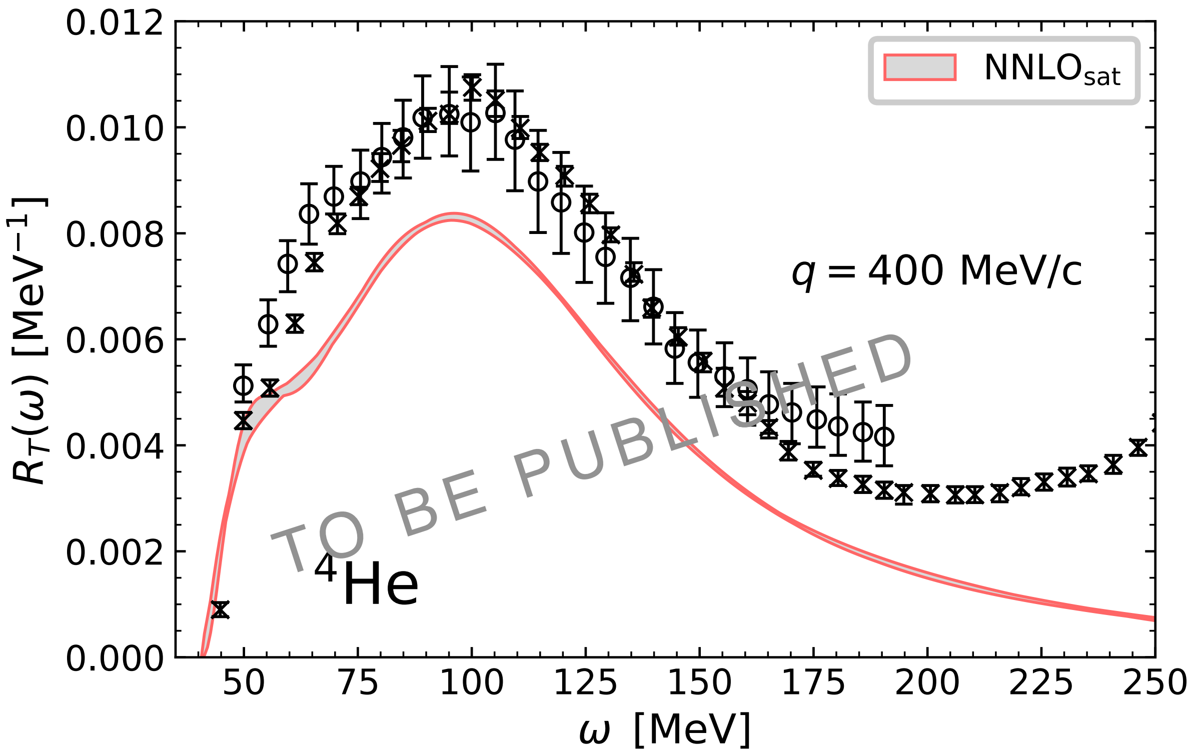


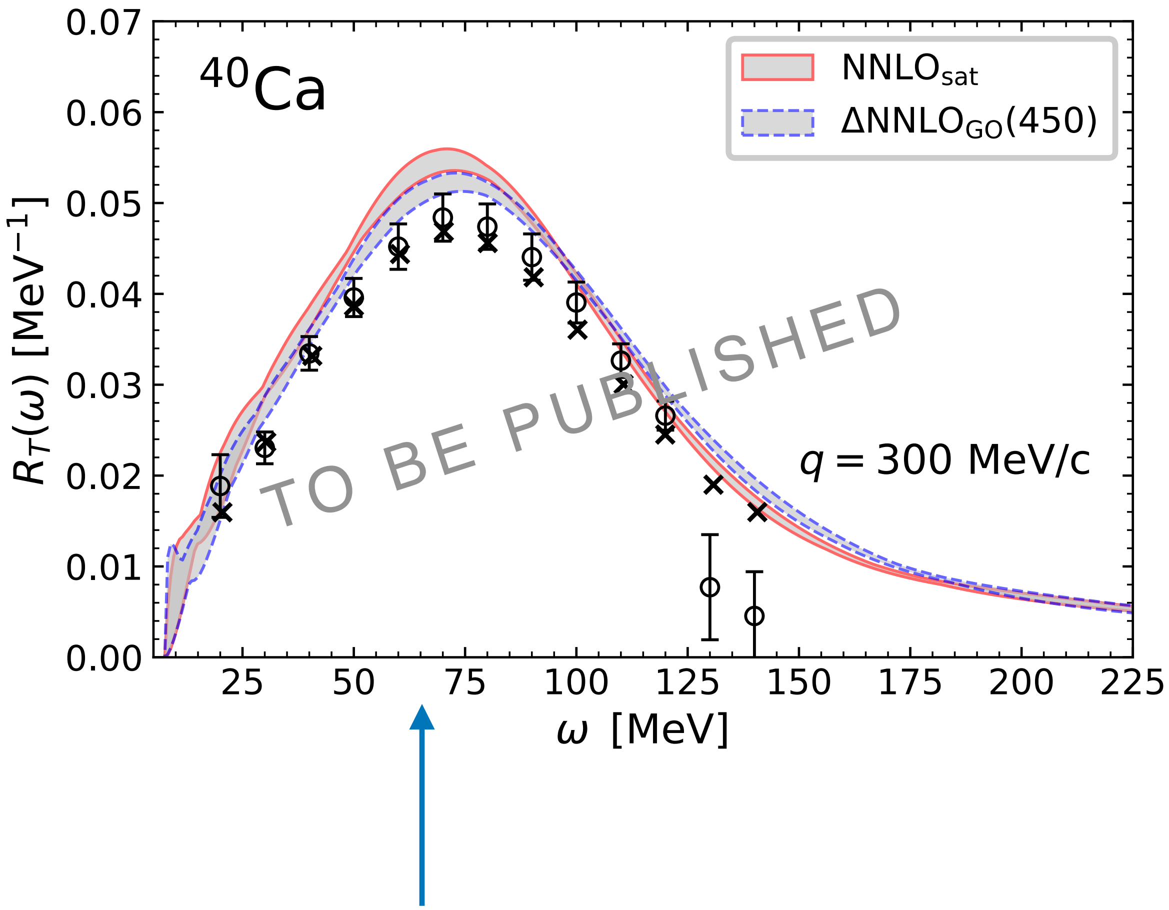
JES, B. Acharya, S. Bacca, G. Hagen; PRL 127 (2021) 7, 072501

- ✓ CC singles & doubles
- ✓ varying underlying harmonic oscillator frequency
- ✓ two different chiral Hamiltonians
- ✓ inversion procedure

First ab-initio results for many-body system of 40 nucleons

Transverse response





- ➡ This allows to predict electron-nucleus cross-section
- ➡ Currently only 1-body current

2-body currents important for 4He
→ more correlations needed?
→ 2-b currents strength depends on nucleus?

ChEK method

Chebyshev Expansion of integral Kernel

$$R_{\mu\nu}(\omega, q) = \sum\!\!\!\!\!\!\int_f \langle \Psi | J_\mu^\dagger | \Psi_f \rangle \langle \Psi_f | J_\nu | \Psi \rangle \delta(E_0 + \omega - E_f)$$

integral transform

$$S_{\mu\nu}(\sigma, q) = \int d\omega K(\omega, \sigma) R_{\mu\nu}(\omega, q) = \langle \Psi | J_\mu^\dagger \, K(\mathscr{H} - E_0, \sigma) \, J_\nu | \Psi \rangle$$

Gaussian kernel:

$$K_\Lambda(\omega, \sigma) = \frac{1}{\sqrt{2\pi}\Lambda} \exp\left(-\frac{(\omega - \sigma)^2}{2\Lambda^2} \right)$$

expansion in Chebyshev polynomials

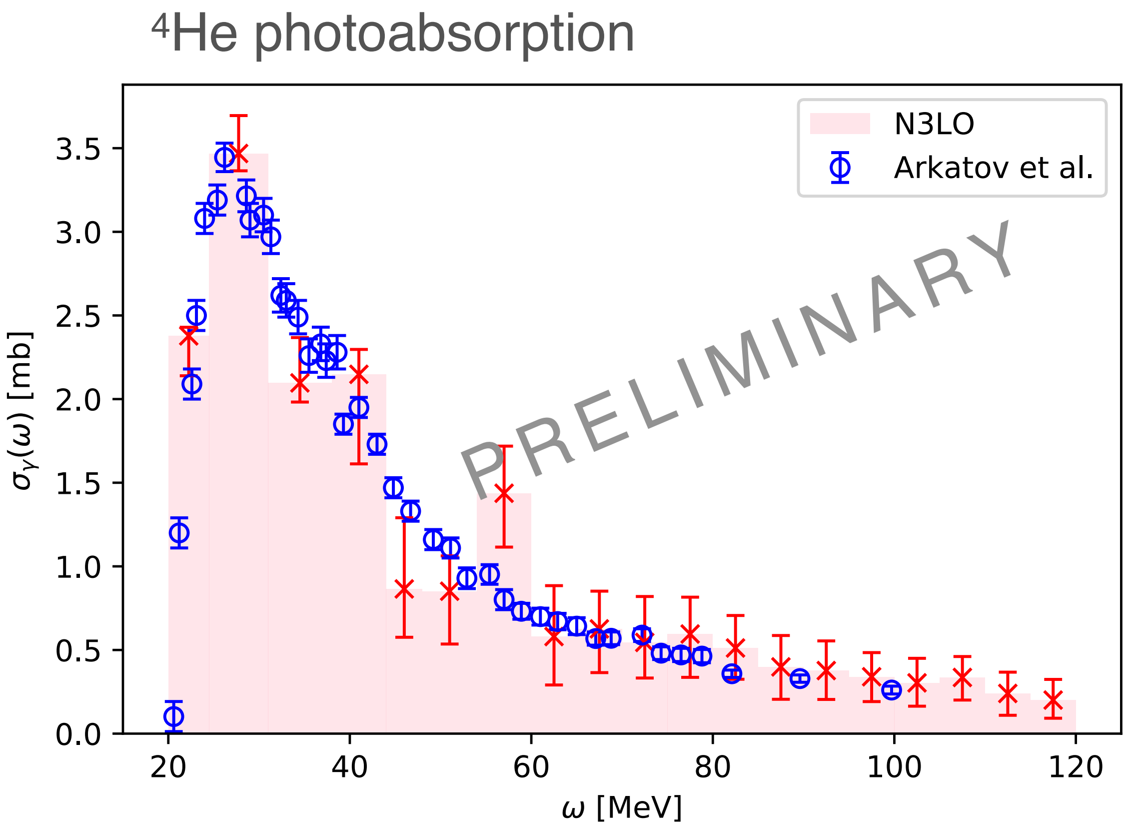
$$K(\mathscr{H}, \sigma) = \sum_k c_k(\sigma) T_k(\mathscr{H})$$

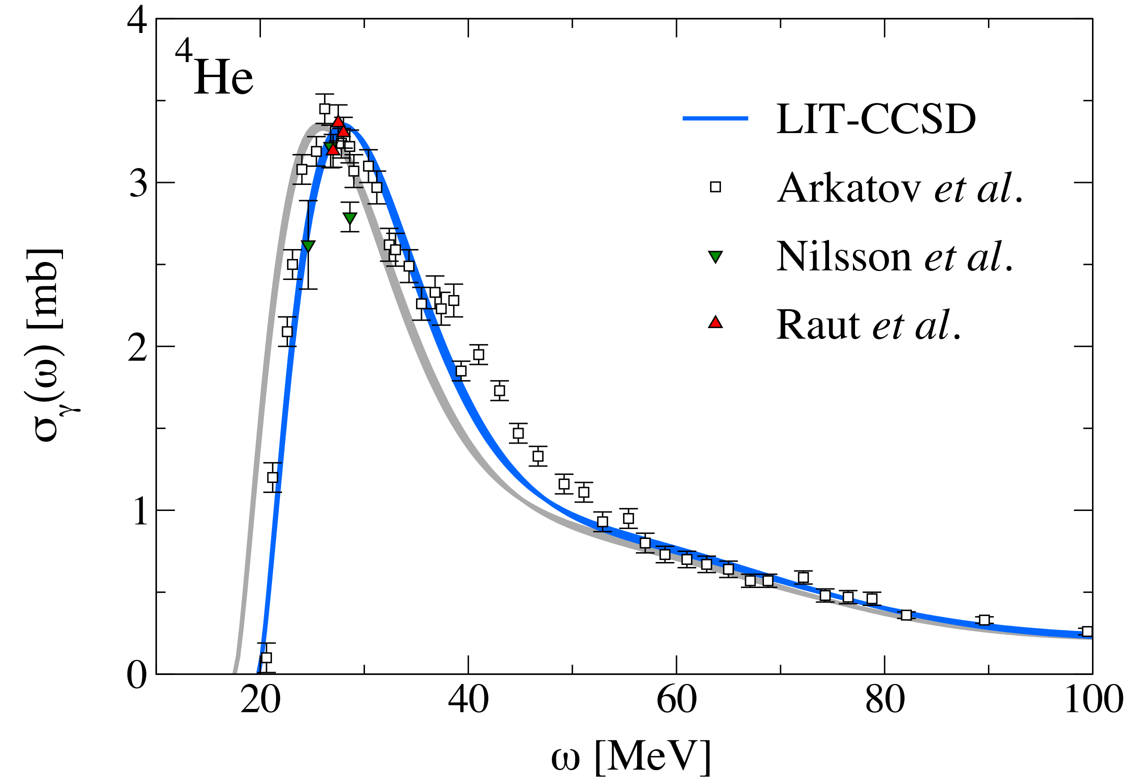
Response reconstruction as histogram

A. Roggero Phys.Rev.A 102 (2020) 2, 022409
JES, A. Roggero arXiv:2110.02108 (accepted in Phys.Rev.E)

ChEK method

Chebyshev Expansion of integral Kernel





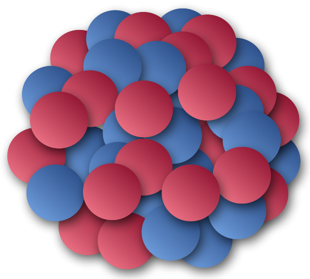
S. Bacca, N. Barnea, G. Hagen, G. Orlandini; Phys.Rev.C 90 (2014) 6

➡ No assumption about the shape of the response

➡ Rigorous error estimation

➡ Convenient when the response has a complicated structure

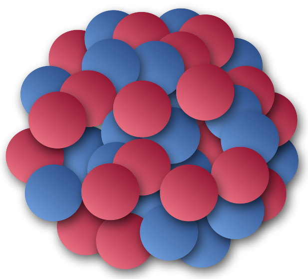
Low/high energies



$$\hat{H}|\psi_A\rangle = E|\psi_A\rangle$$

Many-body problem

Low/high energies



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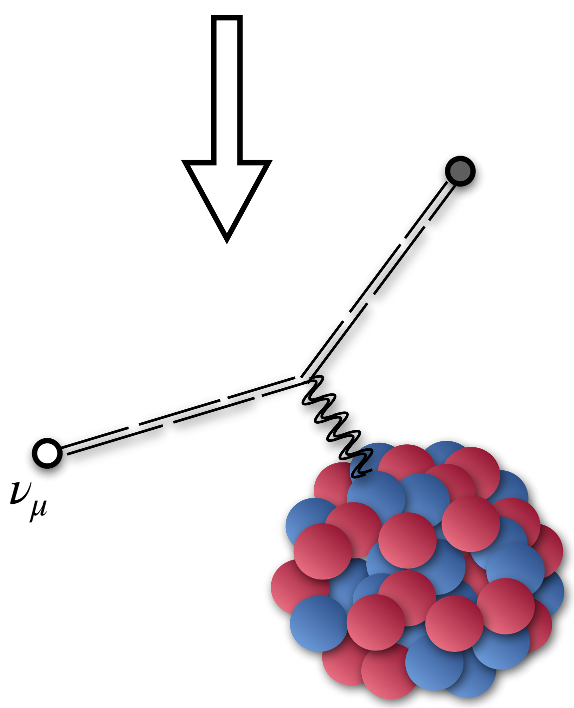
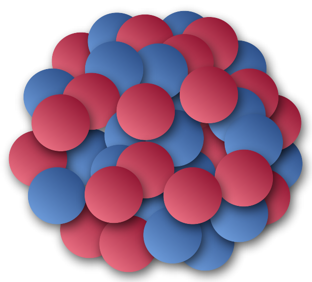


$$\langle\psi_f\,|\,\hat{j}\,|\,\psi_A\rangle$$

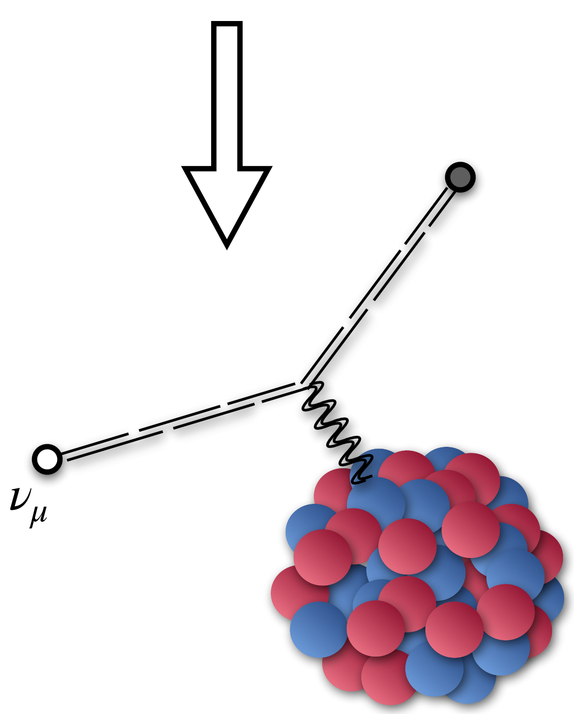
Electroweak responses

Low/high energies



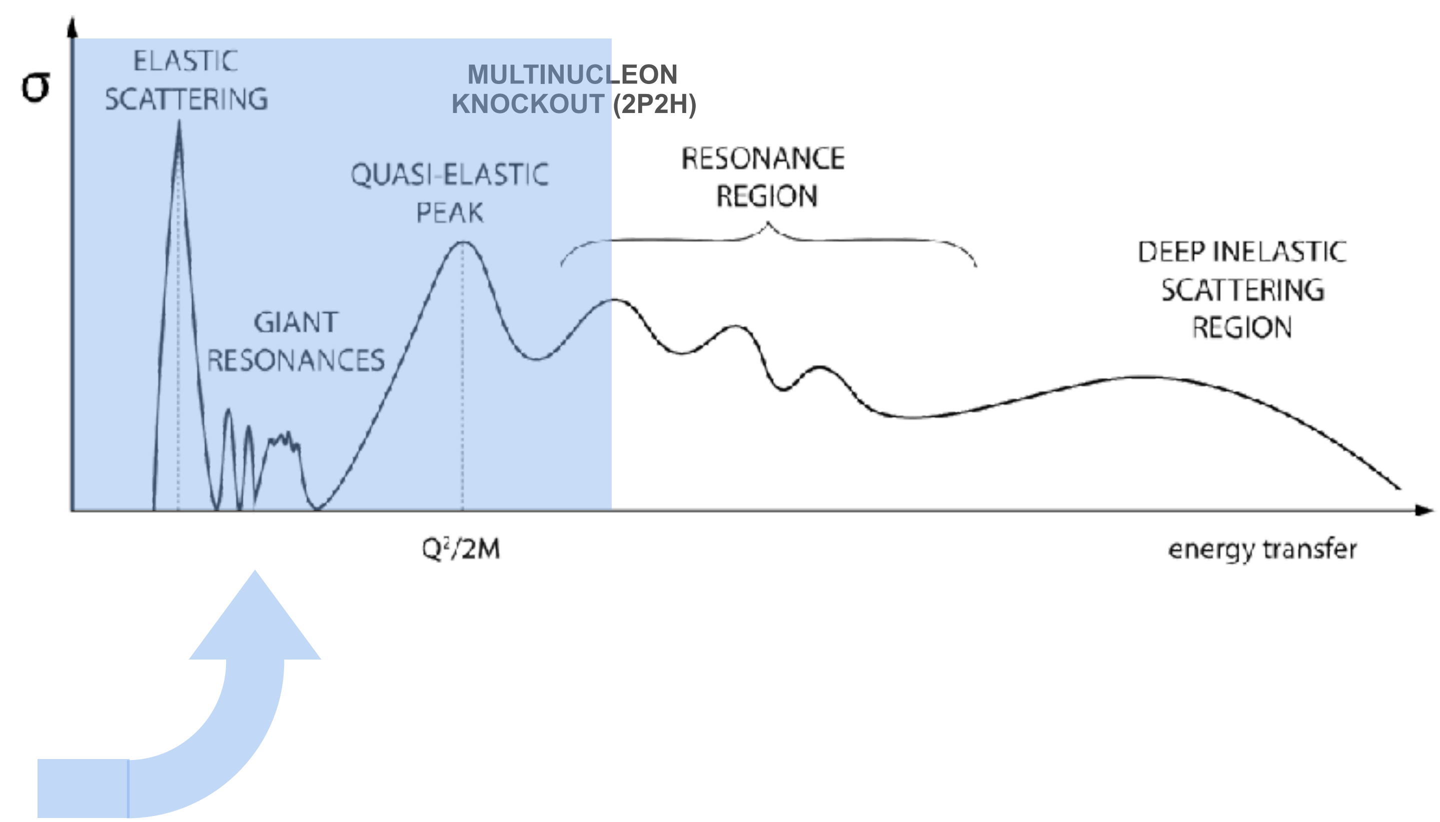
$\hat{H}|\psi_A\rangle = E|\psi_A\rangle$

Many-body problem

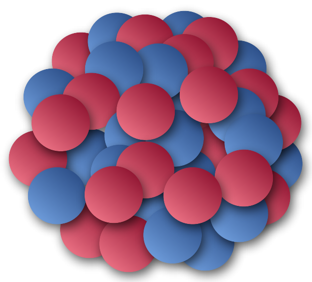


$\langle\psi_f|\hat{j}|\psi_A\rangle$

Electroweak responses

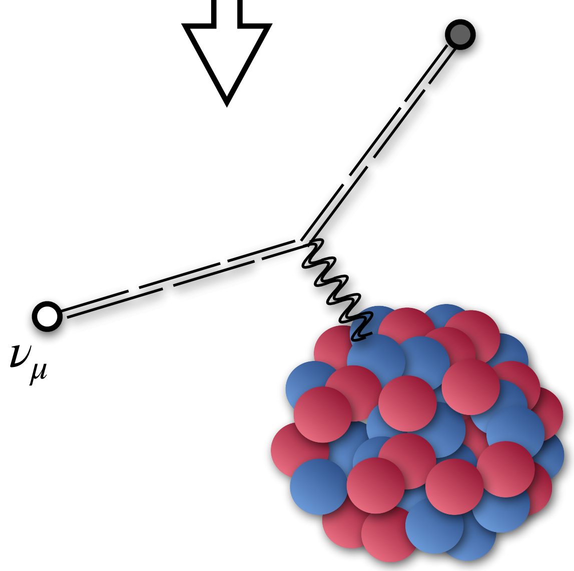


Low/high energies



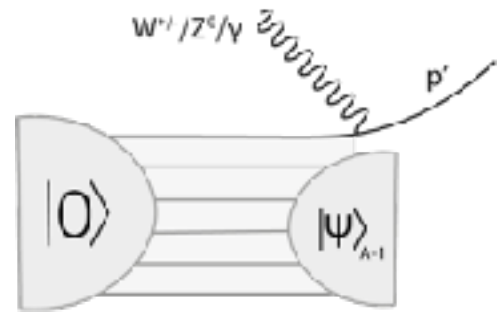
$$\hat{H}\,|\psi_A\rangle = E\,|\psi_A\rangle$$

Many-body problem

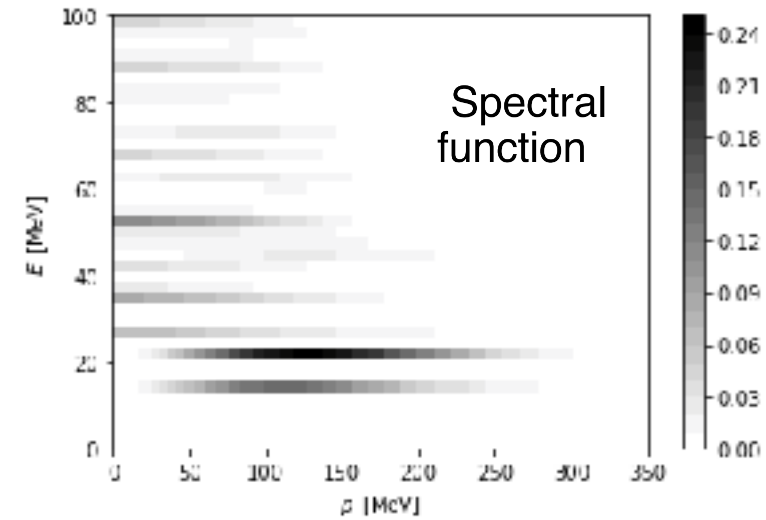


$$\langle\psi_f|\,\hat{j}\,|\psi_A\rangle$$

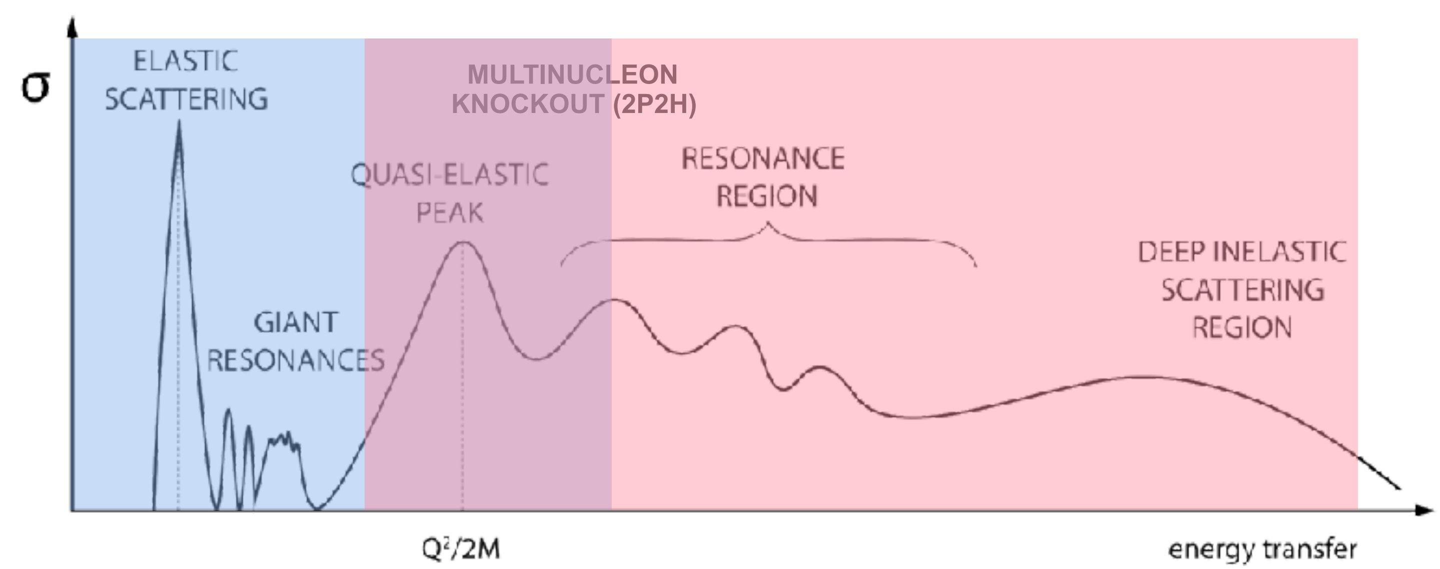
Electroweak responses



Impulse Approximation

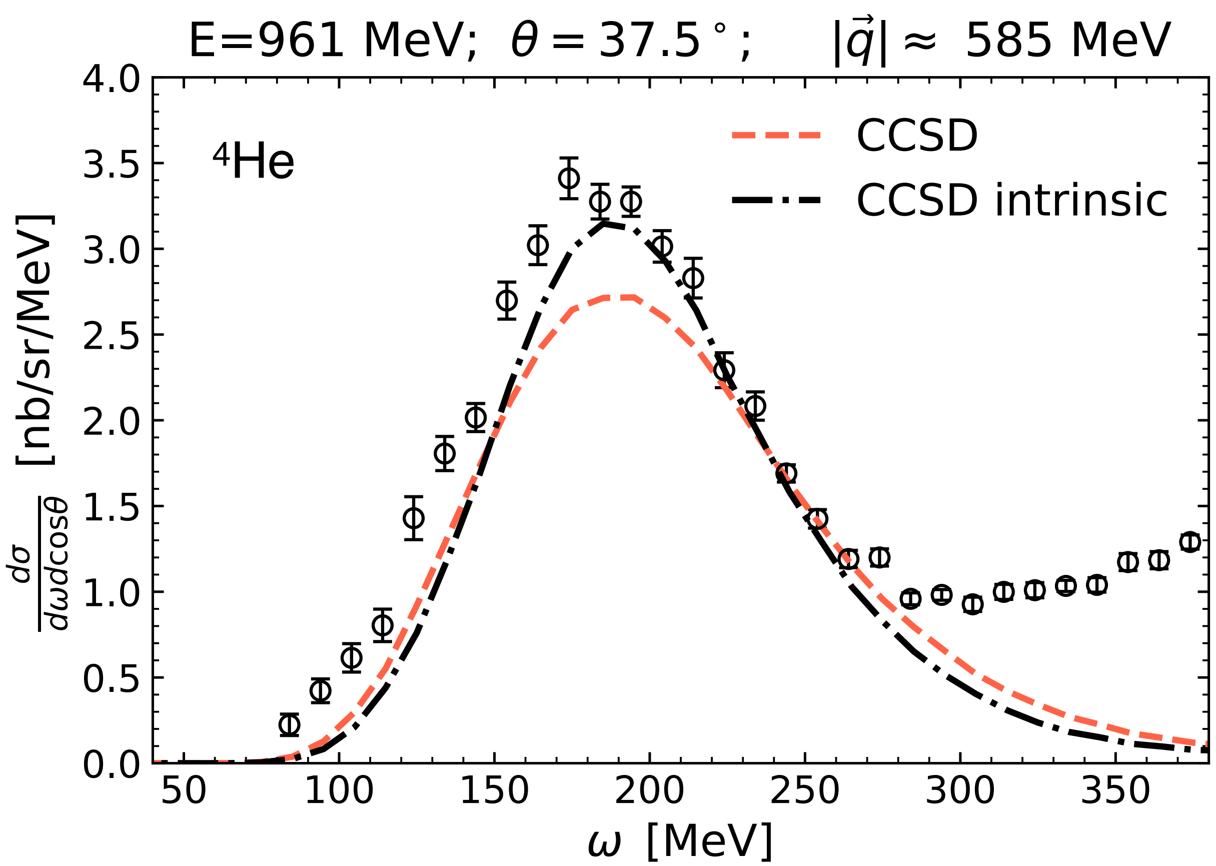


Probability density of finding nucleon $(E, \mathbf{p})$ in ground state nucleus

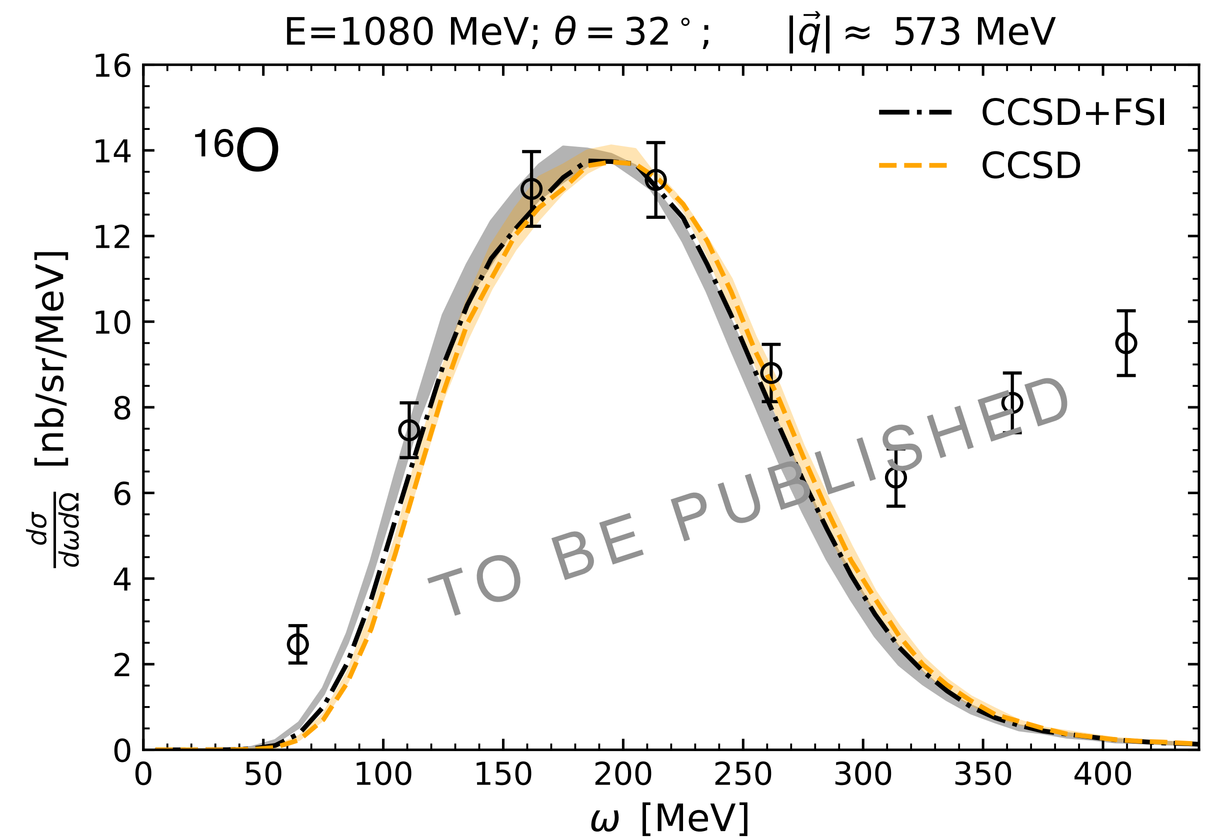


Spectral function

Coupled Cluster + ChEK method



JES, S. Bacca, G. Hagen, T. Papenbrock arXiv: 2205.03592



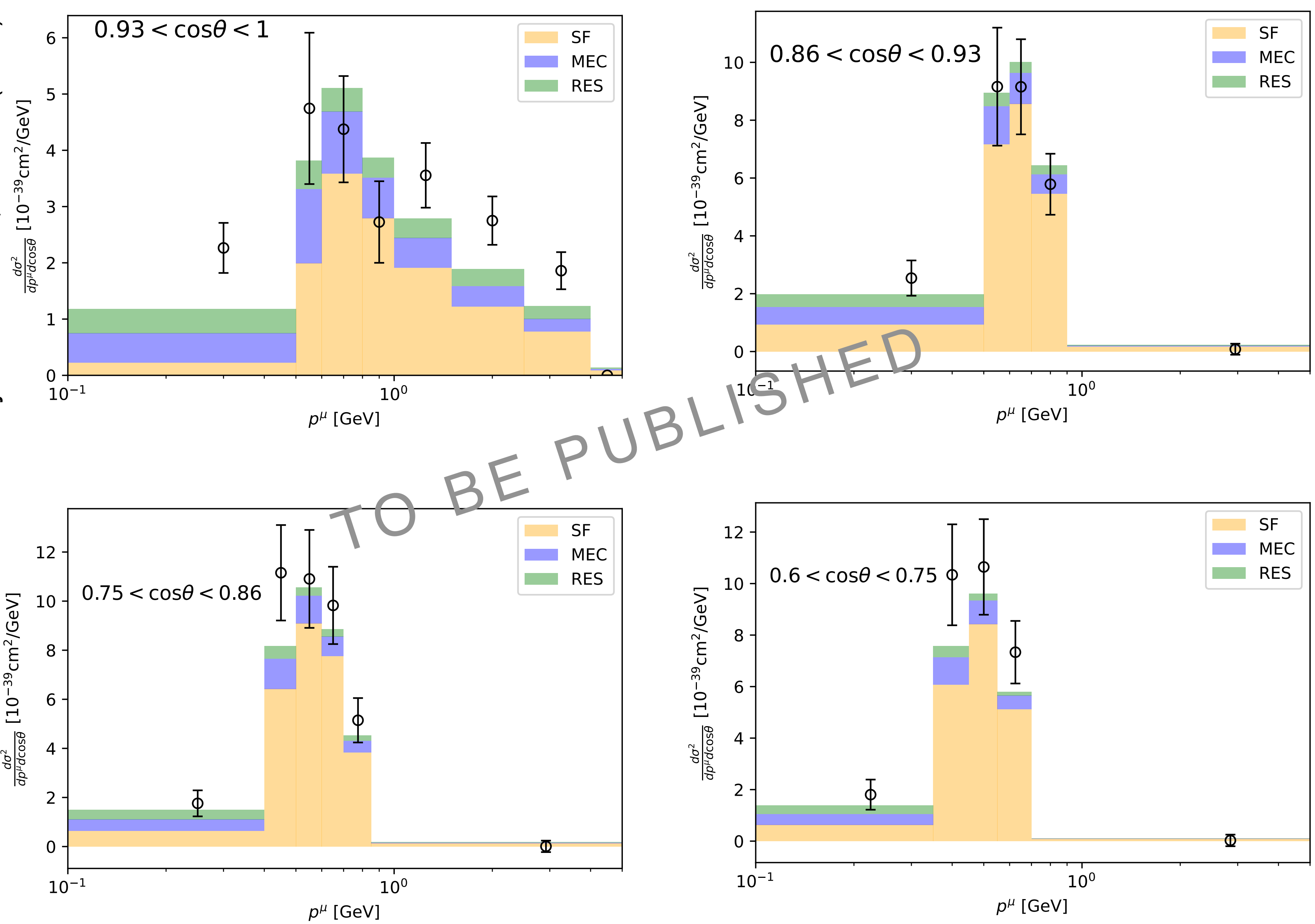
JES et al, in preparation (2022)

Spectral function for neutrinos

$$\nu_\mu + {}^{16}\mathrm{O} \rightarrow \mu^- + X$$

- Comparison with T2K long baseline ν oscillation experiment
- $\mathrm{CC}0\pi$ events
- Spectral function implemented into NuWro Monte Carlo generator

Data: Phys. Rev. D 101, 112004 (2020)



JES et al, in preparation (2022)

Outlook

- LIT-CC results for electron scattering → we are ready to address electroweak processes
- Various sources of theoretical uncertainty taken into account
- Reconstruction of the nuclear response introduces an additional source of error
  - Inversion procedure gives stable results for smooth responses
  - ChEK → way to go with complicated responses
- Spectral function → relativistic regime, semi-inclusive reactions

Thank you for attention

BACKUP

Nuclear hamiltonian

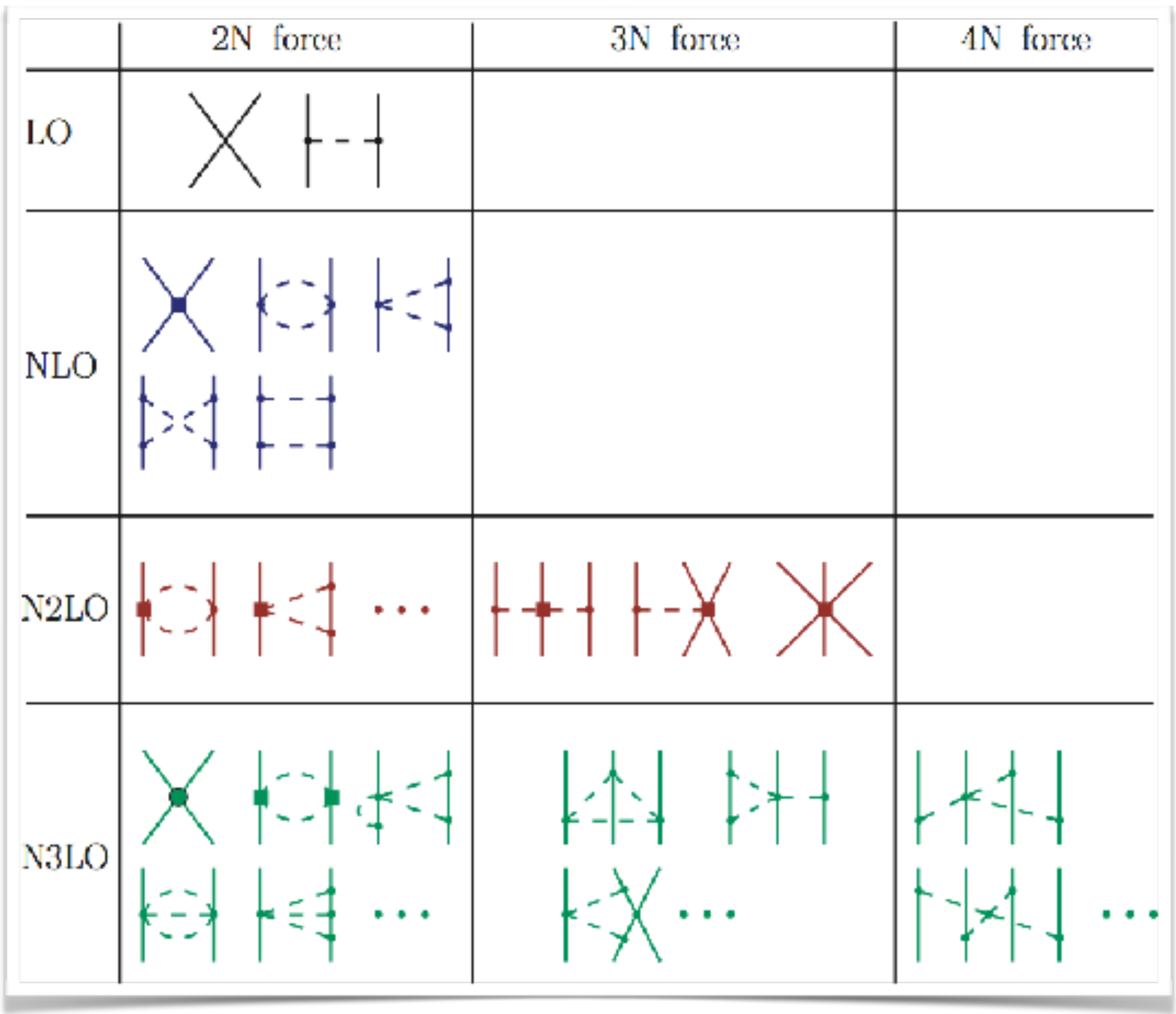
$$\mathscr{H} = \sum_i \frac{p_i^2}{2m} + \sum_{i<j} v_{ij} + \sum_{i<j<k} V_{ijk} + \ldots$$

$n = 0$

$n = 2$

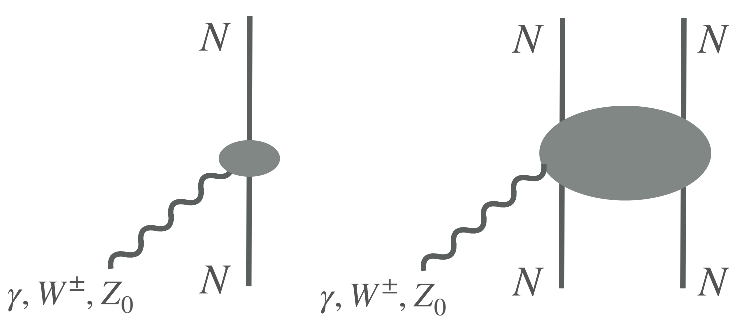
$n = 3$

$n = 4$

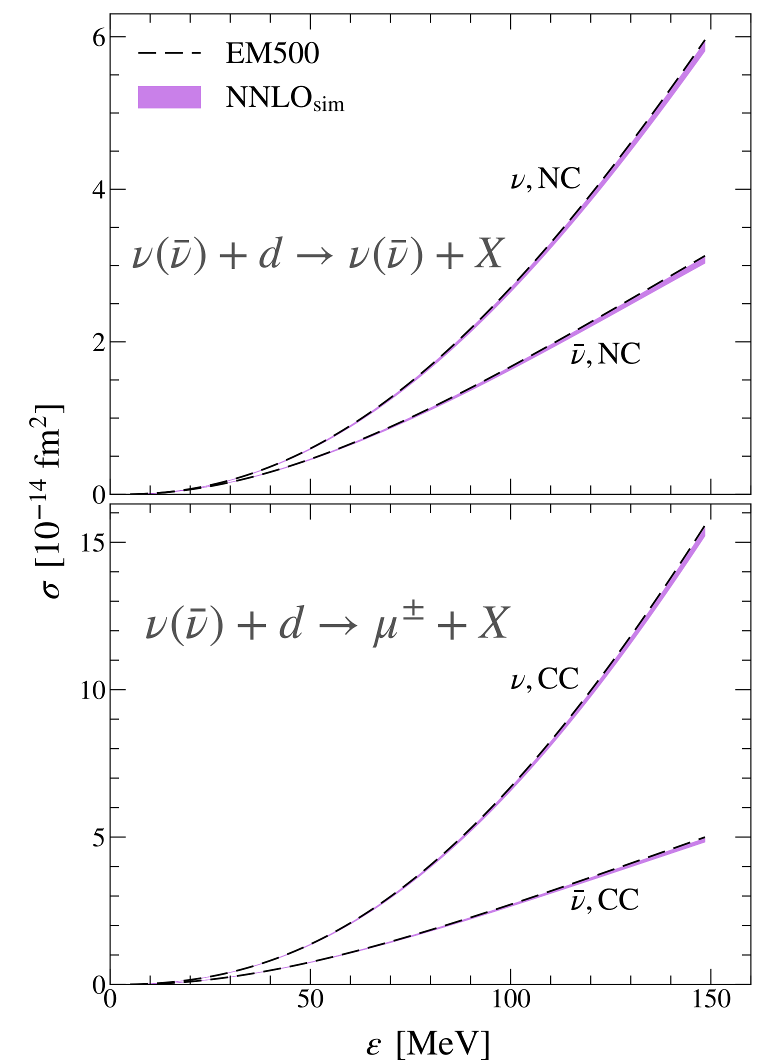


Electroweak currents

$$J = \sum_i j_i + \sum_{i<j} j_{ij} + \dots$$



known to give significant contribution for neutrino-nucleus scattering



Current decomposition into multipoles needed for various ab initio methods: CC, No Core Shell Model, In-Medium Similarity Renormalization Group

Multipole decomposition for 1- and 2-body EW currents

B. Acharya, S. Bacca
Phys.Rev.C 101 (2020) 1, 015505

Coupled cluster method

Reference state (Hartree-Fock): $|\Psi\rangle$

Include correlations through e^T operator

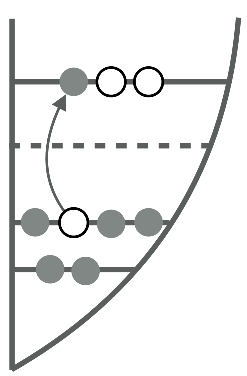
similarity transformed Hamiltonian (non-Hermitian)

$$e^{-T}\mathcal{H}e^{T}|\Psi\rangle \equiv \bar{\mathcal{H}}|\Psi\rangle = E|\Psi\rangle$$

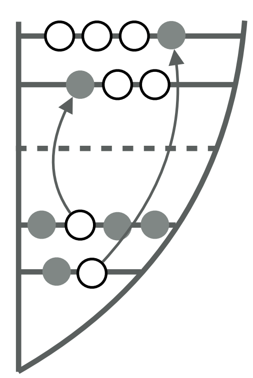
Expansion: $T = \sum t_a^i a_a^\dagger a_i + \sum t_{ab}^{ij} a_a^\dagger a_b^\dagger a_i a_j + \ldots$

←coefficients obtained through coupled cluster equations

singles



doubles



Details on inversion procedure

- Basis functions

$$R_L(\omega) = \sum_{i=1}^{N} c_i \omega^{n_0} e^{-\frac{\omega}{\beta_i}}$$

- Stability of the inversion procedure:
  - Vary the parameters n_0, β_i and number of basis functions N (6-9)
  - Use LITs of various width Γ (5, 10, 20 MeV)

ChEK method

$$S_{\mu\nu}(\sigma, q) = \int d\omega K(\omega, \sigma) R_{\mu\nu}(\omega, q) = \langle \Psi | J_\mu^\dagger \; K(\mathscr{H}, \sigma) \; J_\nu | \Psi \rangle$$

- Expansion in Chebyshev polynomials

$$K(\mathscr{H}, \sigma) = \sum_{k=0}^{N} c_k(\sigma) T_k(\mathscr{H})$$

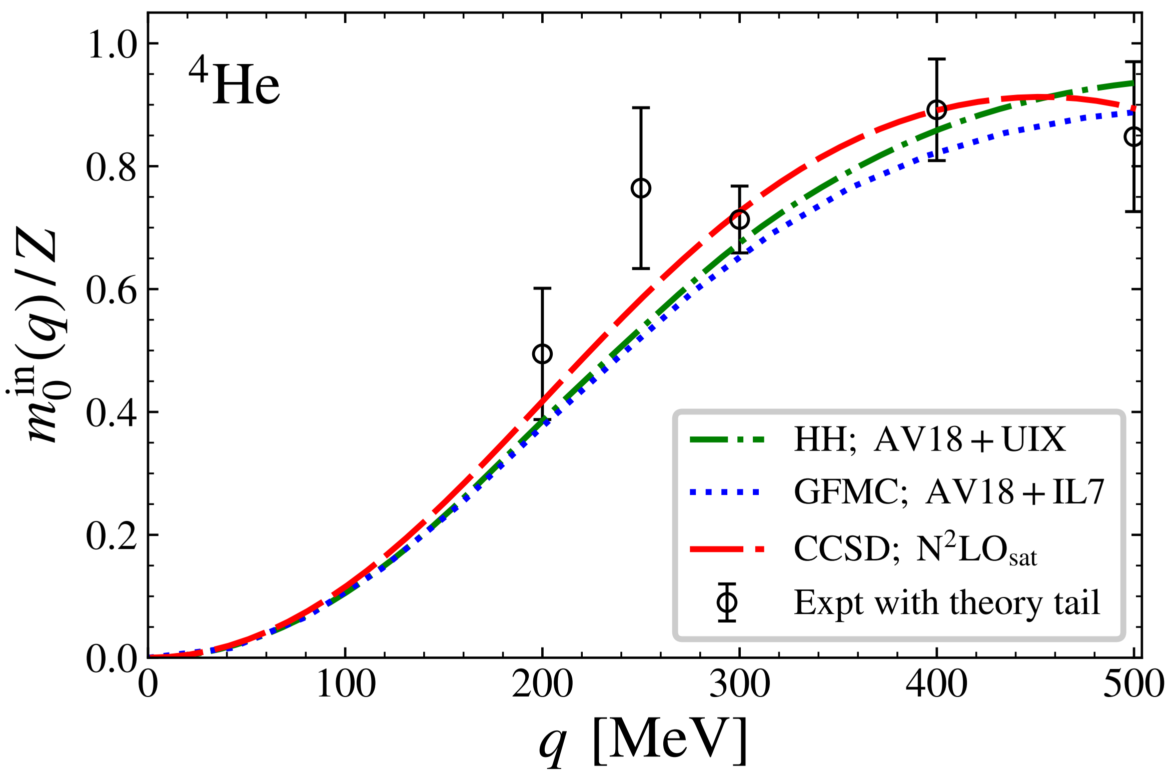
- Recursive relations of Chebyshev polynomials

$$T_0(x) = 1; \quad T_{-1}(x) = T_1(x) = x$$

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$

Coulomb sum rule

$$m_0(q) = \int d\omega R_L(\omega, q) = \sum_{f \neq 0} |\langle \Psi_f | \hat{\rho} | \Psi \rangle|^2 = \langle \Psi | \hat{\rho}^\dagger \hat{\rho} | \Psi \rangle - |F_{el}(q)|^2$$



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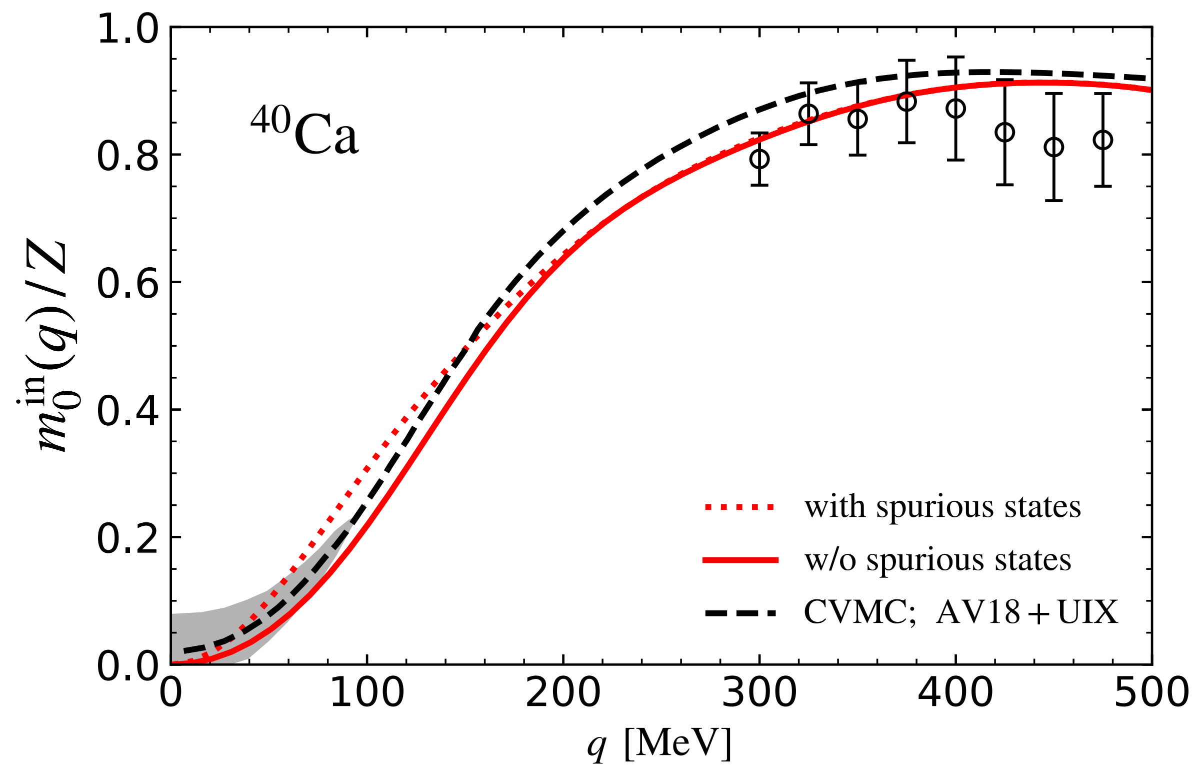

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