Theoretical uncertainties count

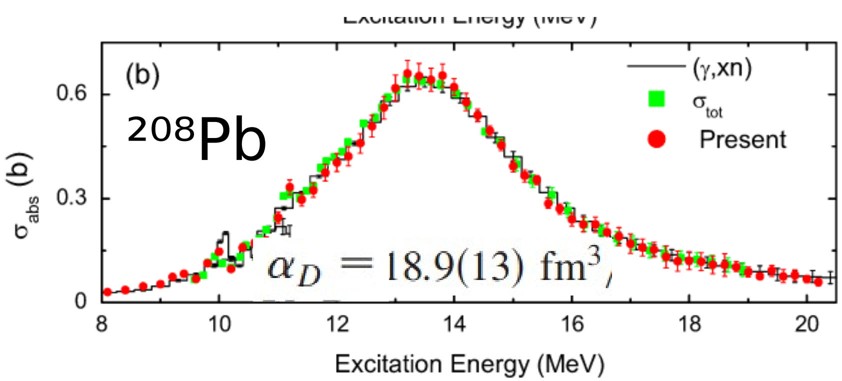
Xavier Roca Maza
Università degli studi di Milano & INFN

Precision Tests with Neutral-Current Coherent Interactions with Nuclei
Mainz Institute for Theoretical Physics, Johannes Gutenberg University
May 23 – 27, 2022

Table of contents

- **Recent experimental results:** PREx and CREx (JLab), α_D in ^{48}Ca and ^{208}Pb (RCNP)
- **Simple models for A_{pv} and α_D:** hints for physical correlations
- **Previous analysis** based on **correlations** shown within different **EDFs**
- **New analysis** taking into account also **theoretical error** quantification within each **EDF**
- **Conclusions**

Recent experimental results



Tamii et al. PRL107 062502 (2011)

PREx: D. Adhikari et al. (PREX Collaboration) Phys. Rev. Lett. 126, 172502 (2021)

Our final results for A_{PV}^{meas} and F_W with the acceptance described by $\epsilon(\theta)$ and $\langle Q^2 \rangle = 0.00616\ \mathrm{GeV}^2$ are

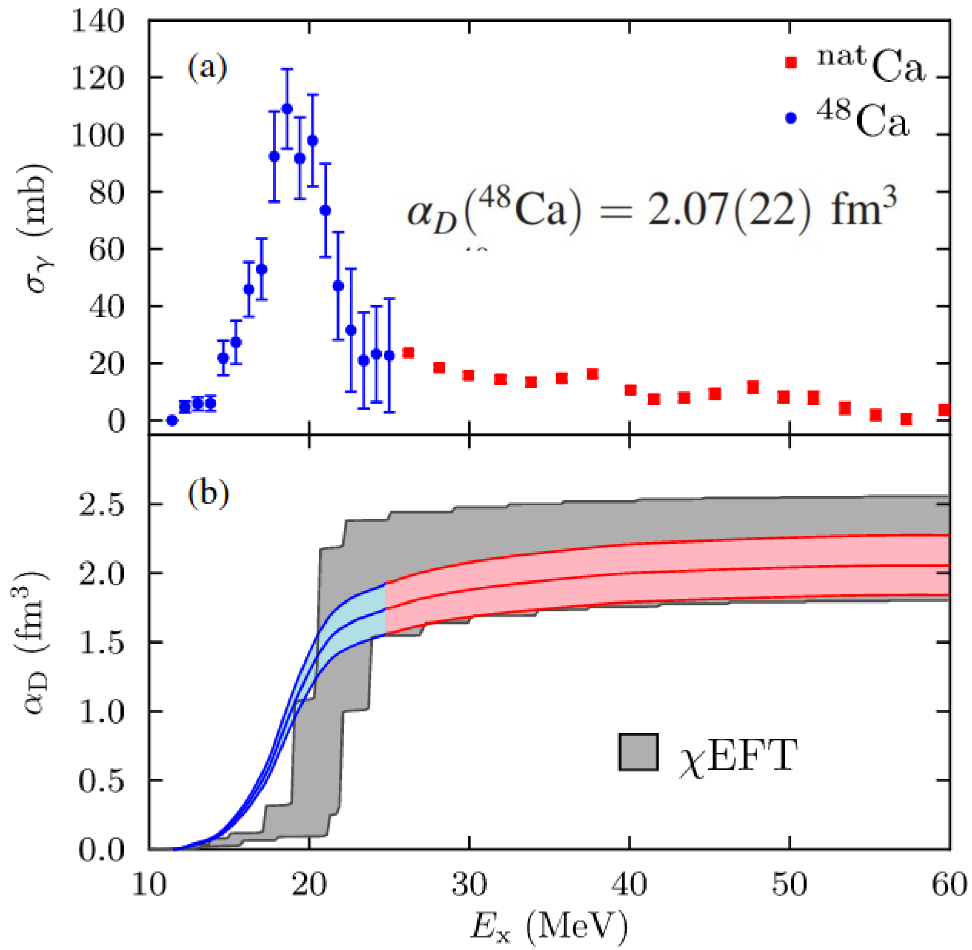
$$A_{PV}^{meas} = 550 \pm 16\,(\mathrm{stat}) \pm 8\,(\mathrm{syst})\ \mathrm{ppb}$$

$$F_W(\langle Q^2 \rangle) = 0.368 \pm 0.013\,(\mathrm{exp}) \pm 0.001\,(\mathrm{theo}),$$

CREx: publication expected soon. Data below preliminary.

mean scattering angle:	$\bar{\theta}_{Ca}$	4.51 ± 0.02
transferred momentum:	$\langle Q^2 \rangle$	$0.0297 \pm 0.0002\ \mathrm{GeV}^2$
	q	$0.873 \pm 0.006\ \mathrm{fm}^{-1}$
beam energy:	E_{beam}	2182.5 ± 1.5 MeV
weak charge:	Q_W	26.073 (26.074?)
parity viol. asymmetry:	$A_{PV}^{(Ca)}$	2658.6 ± 113.2 ppb
weak form factor at Q^2:	$F_W^{(Ca)}$	$0.1297 \pm 4.3\%$

Final value slightly different: see KK's talk or check arXiv today



J. Birkhan et al. Phys. Rev. Lett. 118, 252501 (2017)

Parity Violating Asymmetry:
definition and simple model (PWBA; $F_W \approx F_n$; and $F_{ch} \approx F_p$)

$$A_{pv} = \frac{d\sigma_+/d\Omega - d\sigma_-/d\Omega}{d\sigma_+/d\Omega + d\sigma_-/d\Omega}$$

$$A_{pv} = \frac{G_F q^2}{4\pi\alpha\sqrt{2}}\left[4\sin^2\theta_W + \frac{F_n(q) - F_p(q)}{F_p(q)}\right]$$

... which depends on $F_n(q) - F_p(q)$. For $q \to 0$, it is approximately,

$$\begin{aligned} -\frac{q^2}{6}\left(\langle r_n^2\rangle - \langle r_p^2\rangle\right) &= -\frac{q^2}{6}\left[\Delta r_{np}\left(\langle r_n^2\rangle^{1/2} + \langle r_p^2\rangle^{1/2}\right)\right] \\ &= -\frac{q^2}{6}\left(2\langle r_p^2\rangle^{1/2}\Delta r_{np} + \Delta r_{np}^2\right) \end{aligned}$$

variation of A_{pv} at a fixed q dominated by the variation of Δr_{np}. $F_p(q)$ well fixed by experiment

Dipole Polarizability: definition

The **linear response** or dynamic polarizability of a **nuclear system excited** from its g.s., $|0\rangle$, to an excited state, $|\nu\rangle$, due to the **action of an external isovector oscillating field** (dipolar in our case) of the form $(Fe^{iwt} + F^{\dagger}e^{-iwt})$:

$$F_{JM} = \sum_{i}^{A} r^J Y_{JM}(\hat{r})\tau_z(i) \ (\Delta L = 1 \rightarrow \text{Dipole})$$

is proportional to the **static polarizability** for small oscillations

$\alpha = (8\pi/9)e^2 m_{-1} = (8\pi/9)e^2 \sum_{\nu} |\langle\nu|F|0\rangle|^2/E$ where m_{-1} **is the inverse energy weighted moment** of the **strength function**, defined as, $S(E) = \sum_{\nu} |\langle\nu|F|0\rangle|^2 \delta(E - E_{\nu})$

Dipole Polarizability: simple model

electric polarizability measures tendency of the nuclear charge distribution to be distorted $\left(\alpha \sim \frac{\text{electric dipole moment}}{\text{external electric field}}\right)$

- The dielectric theorem establishes that the m_{-1} moment can be computed from the expectation value of the Hamiltonian in the constrained ground state $\mathcal{H}' = \mathcal{H} + \lambda \mathcal{D}$.

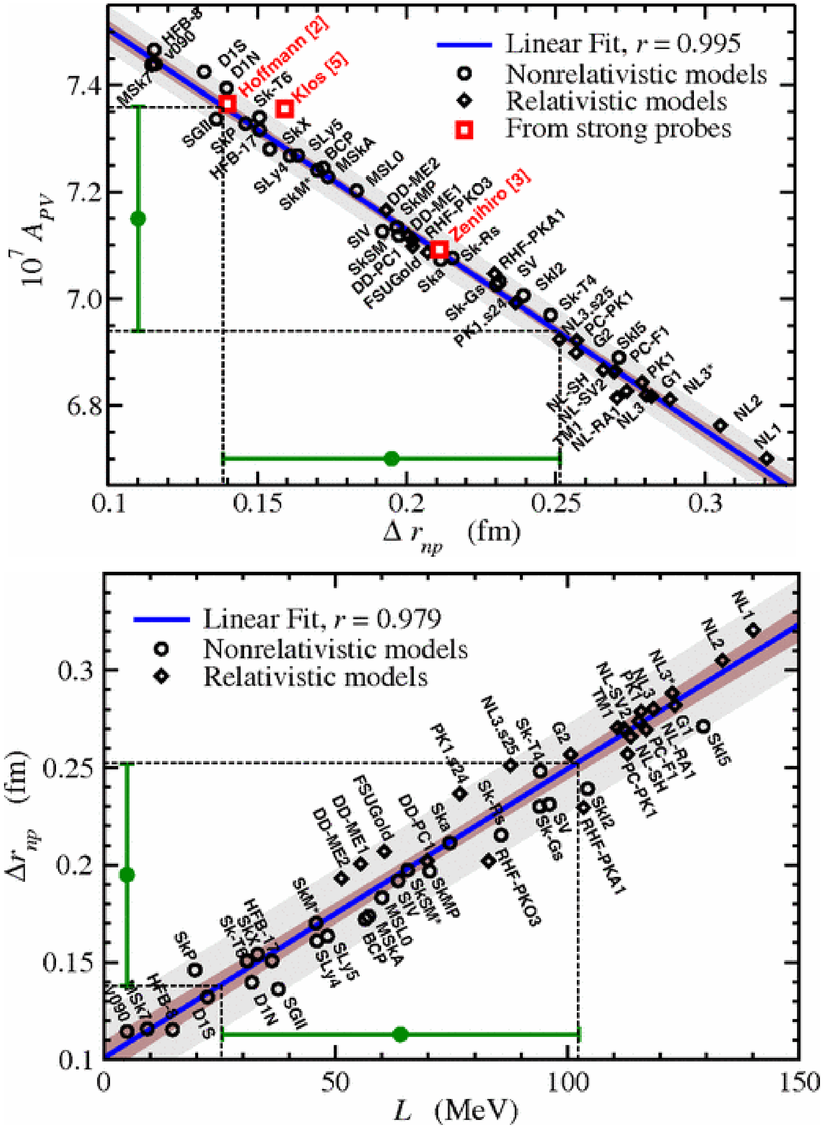
Adopting the Droplet Model:

$$m_{-1} \approx \frac{A\langle r^2\rangle^{1/2}}{48J}\left(1 + \frac{15}{4}\frac{J}{Q}A^{-1/3}\right)$$

within the same model, connection with the neutron skin thickness:

$$\alpha_D \approx \frac{A\langle r^2\rangle}{12J}\left[1 + \frac{5}{2}\frac{\Delta r_{np} + \sqrt{\frac{3}{5}}\frac{e^2 Z}{70J} - \Delta r_{np}^{\text{surface}}}{\langle r^2\rangle^{1/2}(I - I_C)}\right]$$

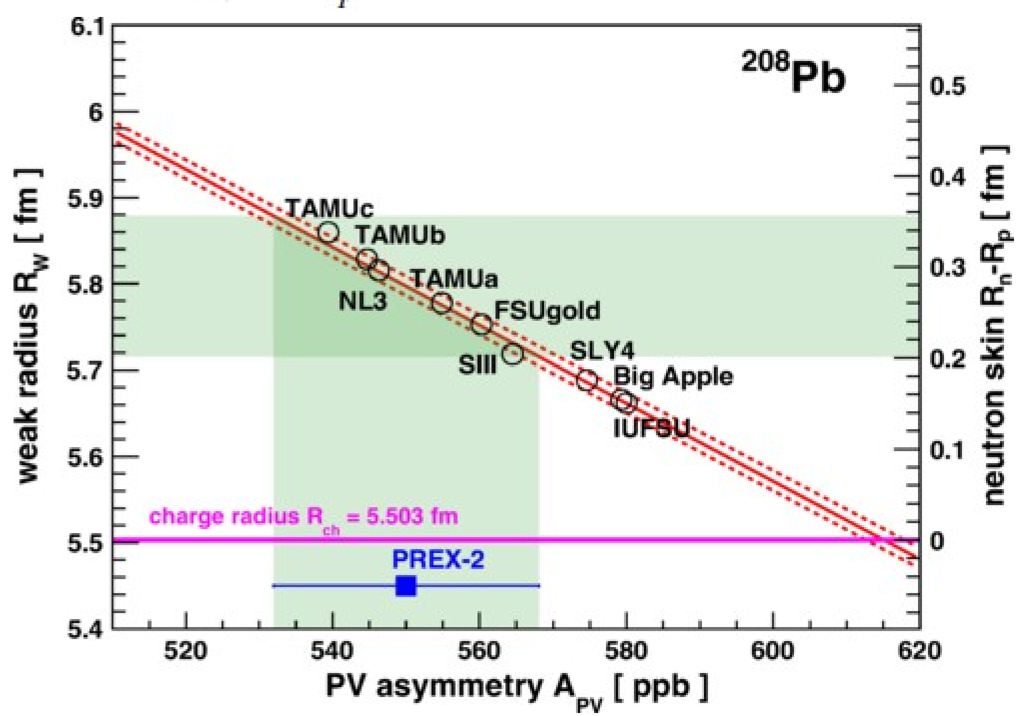
Analysis based on EDFs correlations between A_{pv}, ΔR_{ch} and L



X. Roca-Maza, et al. Phys. Rev. Lett. 106, 252501 (2011)

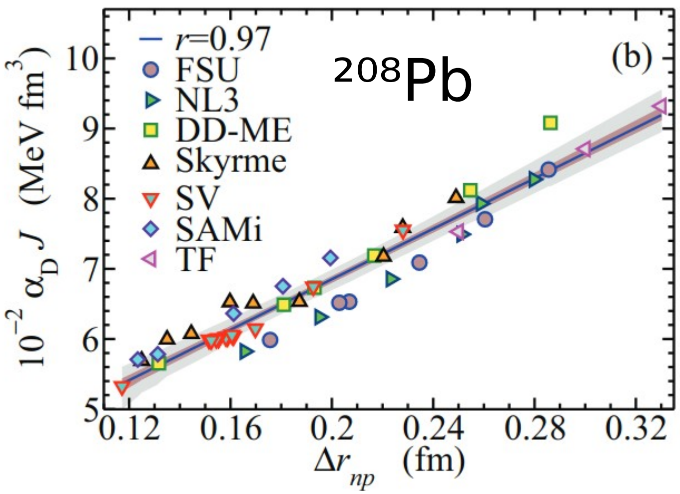
EDFs callibrated to **reproduce** the **binding energy** and **charge radii** of some **selected nuclei**. In some cases pseudo-observables related to the EoS are also used

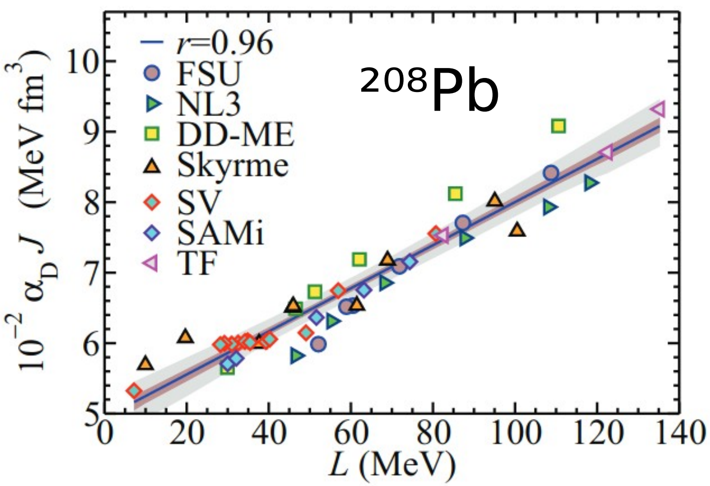
$$R_n - R_p = 0.283 \pm 0.071 \text{ fm.}$$



D. Adhikari et al. Phys. Rev. Lett. 126, 172502 (2021)

Analysis based on EDFs correlations between α_D, ΔR_{ch} and L





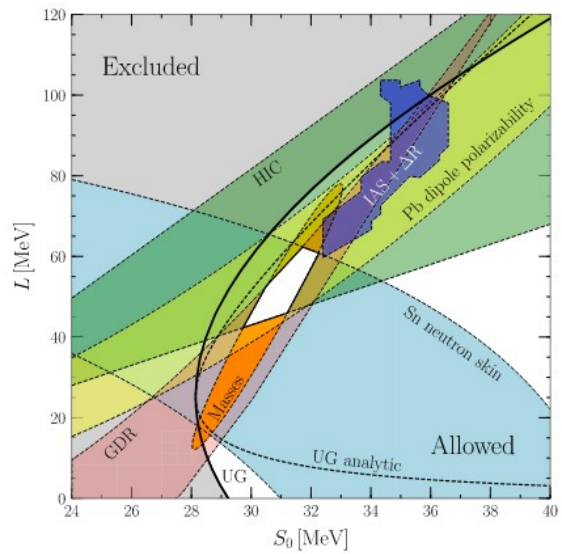
$$\Delta r_{np} = 0.165 \pm (0.009)_{\rm expt} \pm (0.013)_{\rm theor} \pm (0.021)_{\rm est}\ \text{fm}.$$

X. Roca-Maza,et al. Phys. Rev. C 88, 024316 (2013)

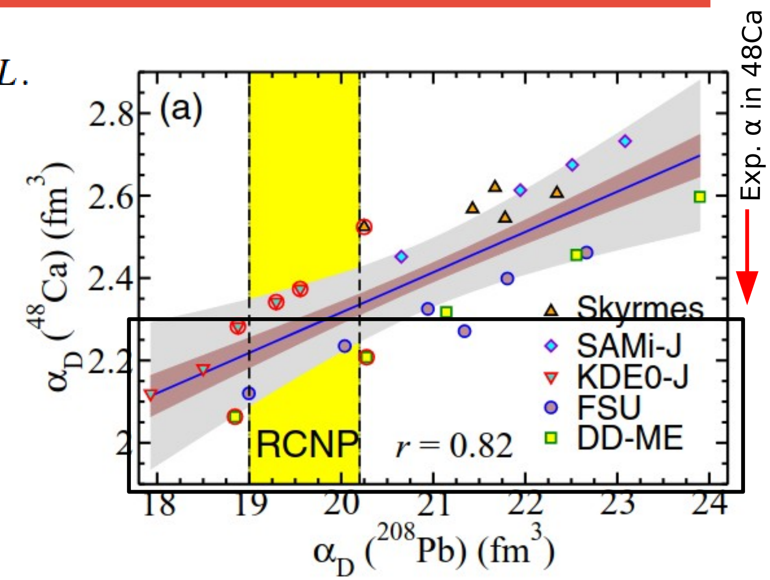
X. Roca-Maza,et al. Phys. Rev. C 92, 064304 (2015)

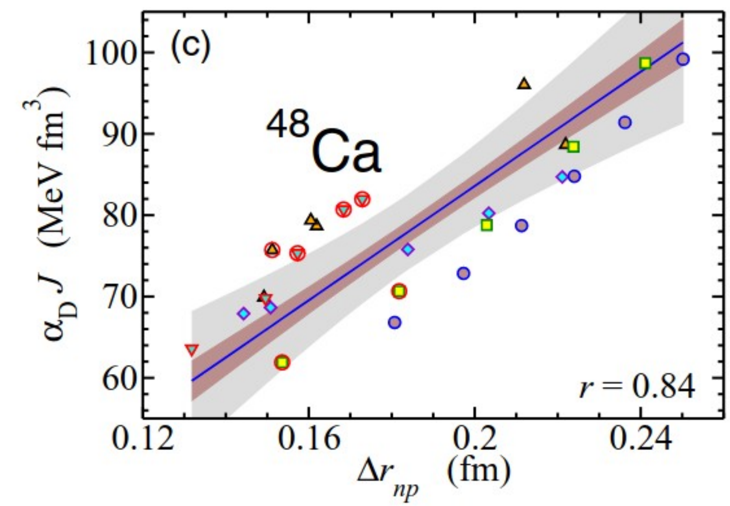
$$J = (24.5 \pm 0.8) + (0.168 \pm 0.007)L.$$

Ingo Tews et al 2017 ApJ 848 105



Typical plot containing different constraints from different analysis performed in different ways on two non-observable quantities





X. Roca-Maza, et al. Phys. Rev. C 92, 064304 (2015)

New analysis taking into account model error quantification

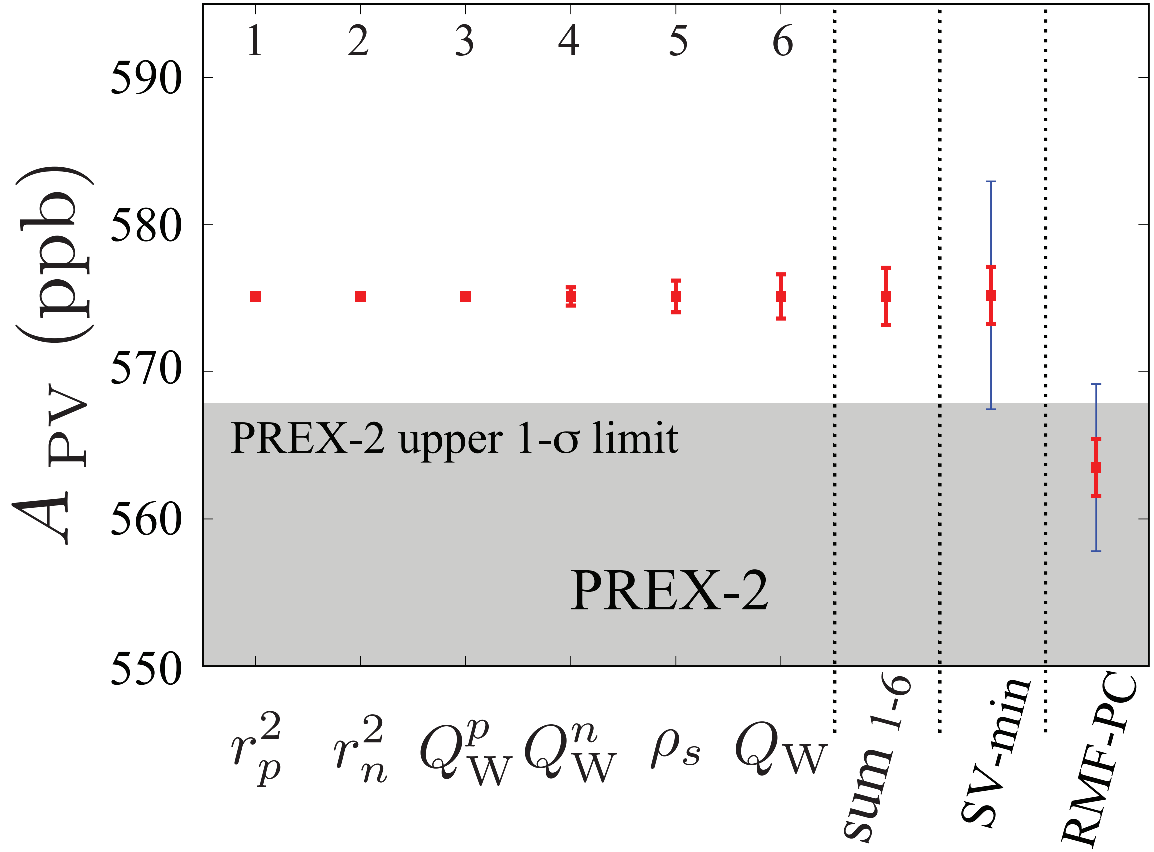
How to improve current analysis?

- Include theoretic **statistical errors** and **correlations** within a given **EDF** parametrization
- **Fitting** procedure including experimental data not only on **B** and $\mathbf{R_{ch}}$ but also on $\mathbf{A_{pv}}$ and/or $\boldsymbol{\alpha_D}$ (“informed” EDFs)
- **Extension** of available **EDFs** to account for missing systematic uncertainties and more flexibility

  ...
- Other issues related to theory?

PREx: theo. uncertainty budget for A_{PV}

Uncertainties in the determination of the Form Factors is smaller than typical EDFs statistical uncertainties



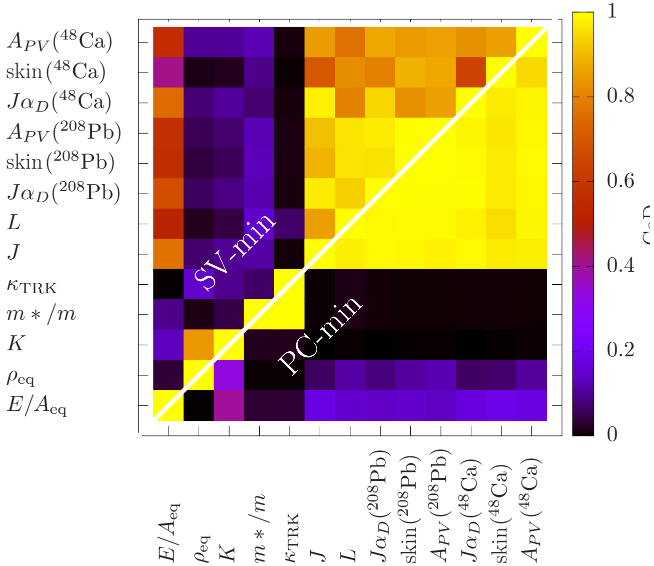
Parameter	Value
$\langle r_p^2 \rangle$ (fm^2)	0.726 ± 0.019
$\langle r_n^2 \rangle$ (fm^2)	-0.1161 ± 0.0022
μ_p	2.792 847
μ_n	-1.9130
$Q_p^{(W)}$	0.0713 ± 0.0001
$Q_n^{(W)}$	-0.9888 ± 0.0011
ρ_s	-0.24 ± 0.70
κ_s	-0.017 ± 0.004
$Q_{126,82}^{(W)}$	-117.9 ± 0.3

Thin blue bars: statistical model uncertainties (related to neutron and proton densities)

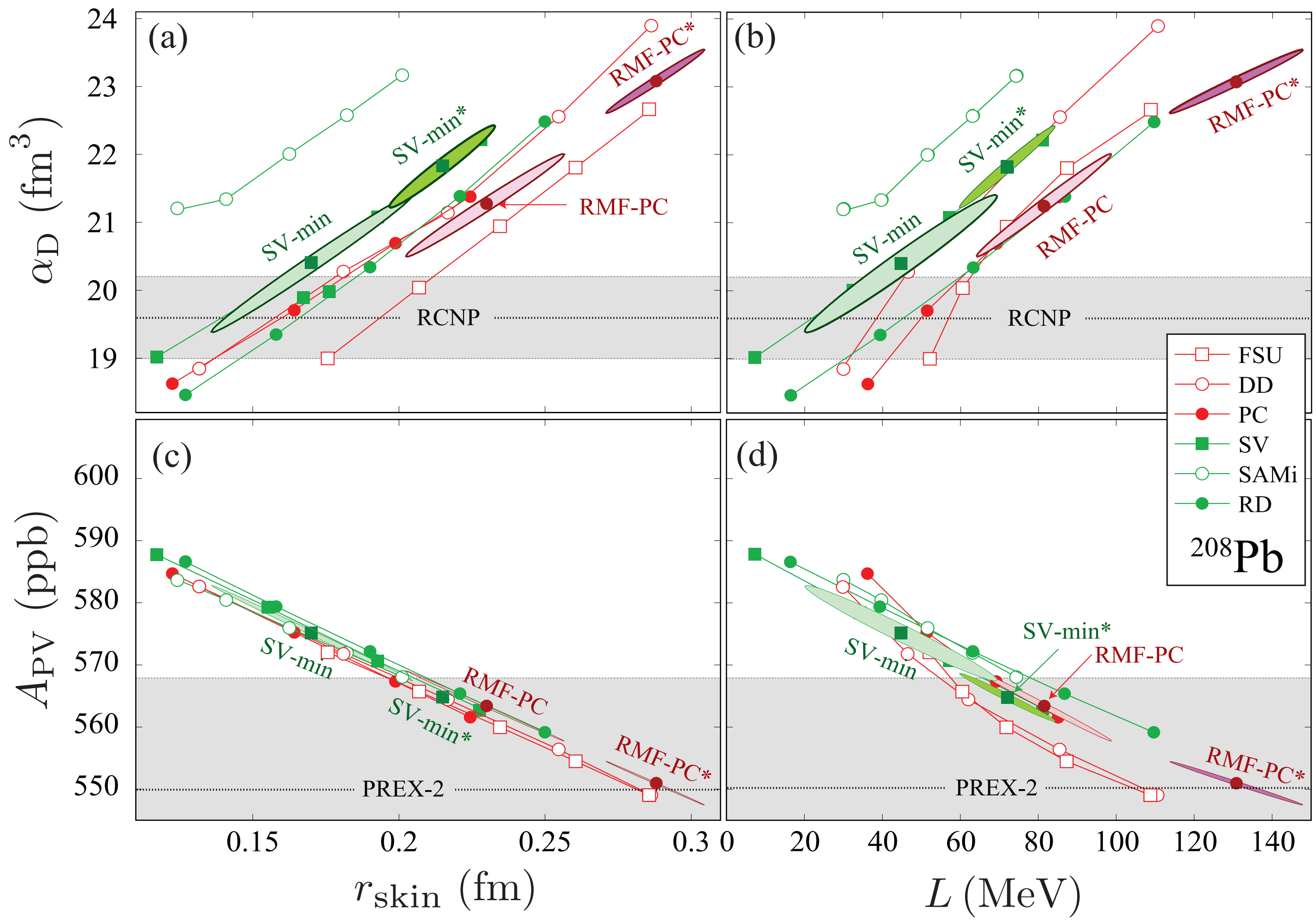
Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz Phys. Rev. Lett. 127, 232501 (2021)

EDF predictions with error ellipsoids for A_{PV} and α_D in ^{208}Pb

Correlation ellipsoids within each **EDF** show **similar correlations** than the **systematic** study with many **EDFs ↔ have we learnt something?**



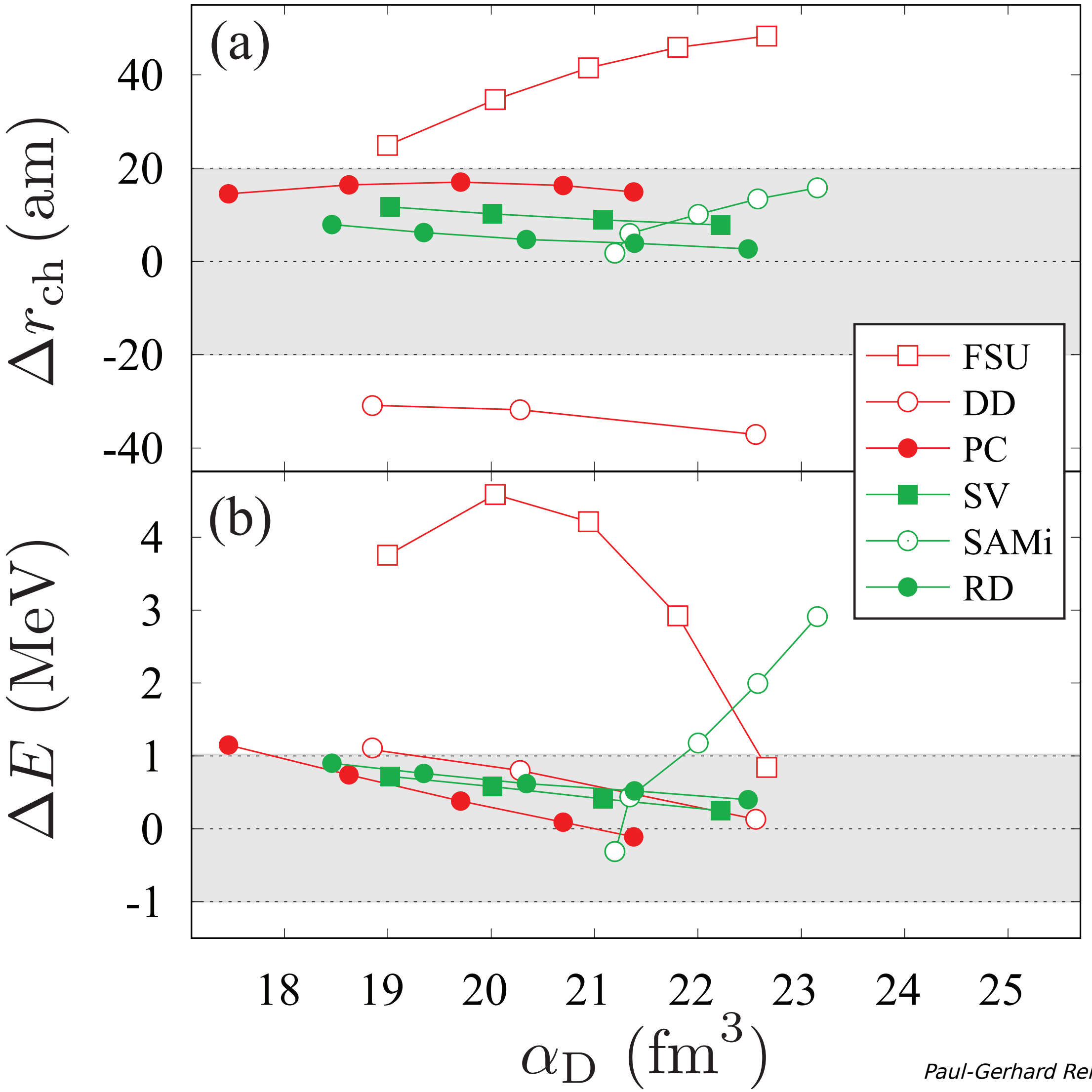
Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz (to be submitted 2022)



FSU→ Non linear Walecka model (Rel)
DD→ Meson-exchange with density dependent couplings (Rel)
PC→ zero-range density dependent couplings (Rel)
SV&SAMi→ Skyrme (zero-range, Non-rel)
RD→ Skyrme with modified density dependence (ρ^integer powers)

Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz Phys. Rev. Lett. 127, 232501 (2021)

How these models perform for *B* and R_{ch} in ^{208}Pb?

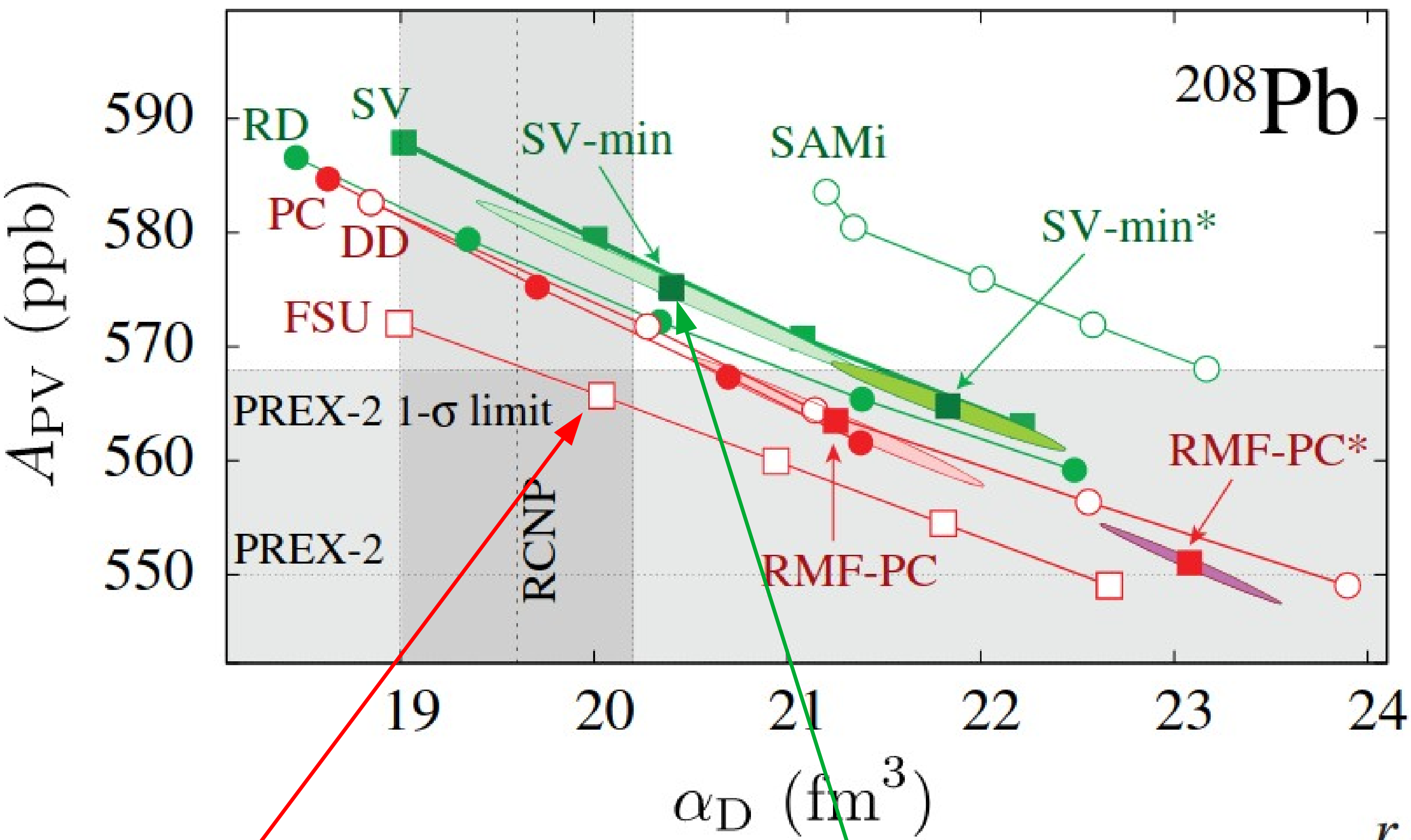


→ The **residuals** of the charge **radius** (a) and binding **energy** (b) of ^{208}**Pb** for the theoretical models.

→ The **grey bands** around the perfect match indicate the typical performance of EDFs (i.e. **typical** r.m.s. **deviation** taken over all nuclei where correlation effects are small)

Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz Phys. Rev. Lett. 127, 232501 (2021)

A_{pv} versus α_D in well callibarted EDFs



B and **Rch** away from "expected" EDF accuracy

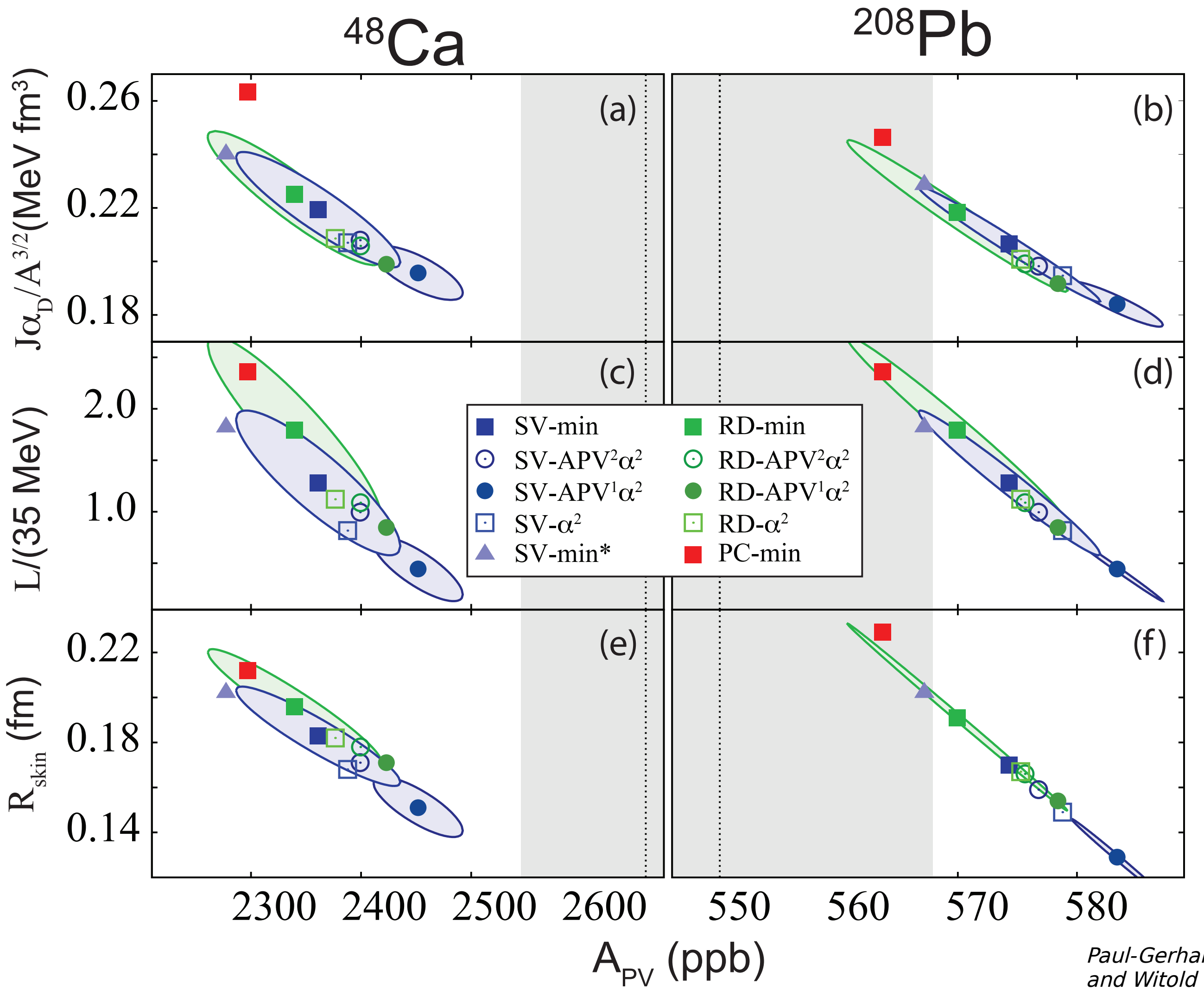
Theoretical and experimental 1σ errors overlap for SV-min

SV-min, suitable model to predict EoS around ρ_0 and Δr_{np}

$$r_{skin} = 0.19 \pm 0.02 \text{ fm}$$

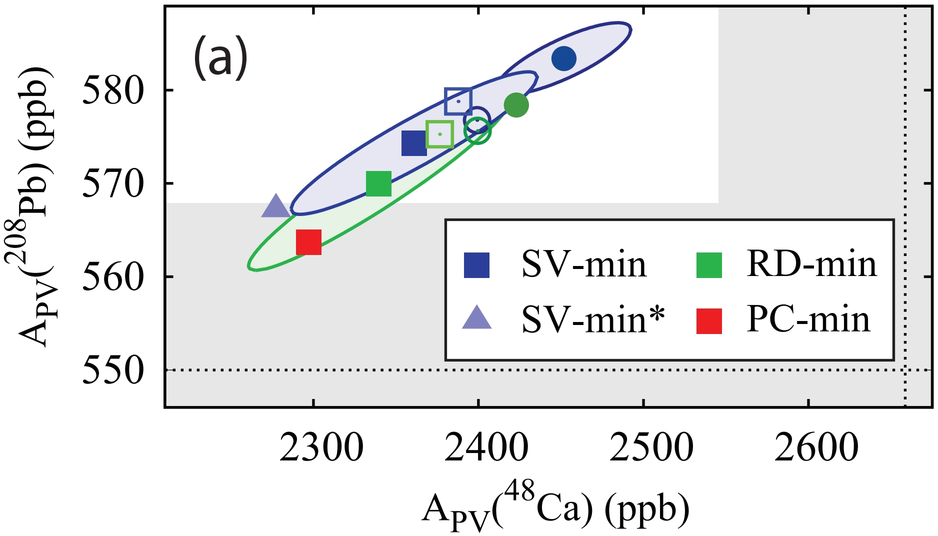
$$L = 54 \pm 8 \text{ MeV}$$

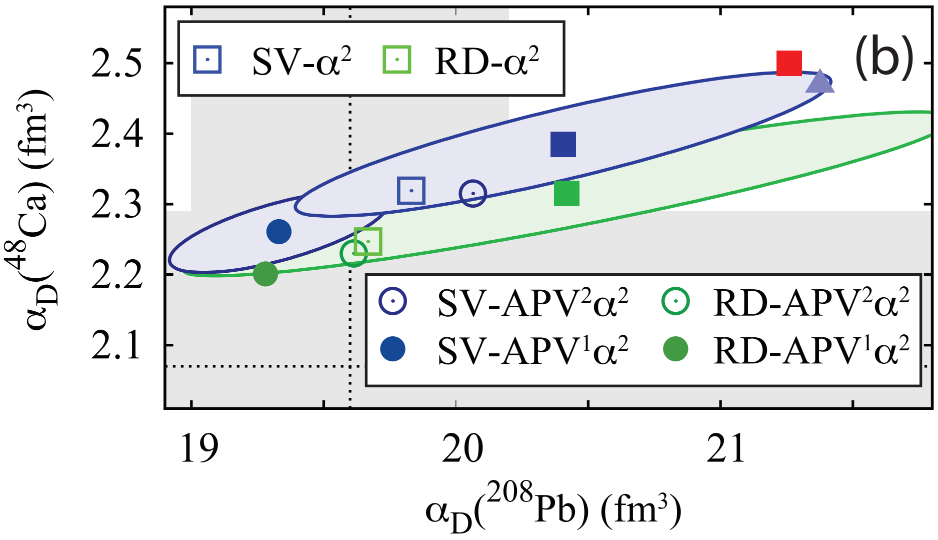
Fitting A_{PV} and α_D



Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz (to be submitted 2022)

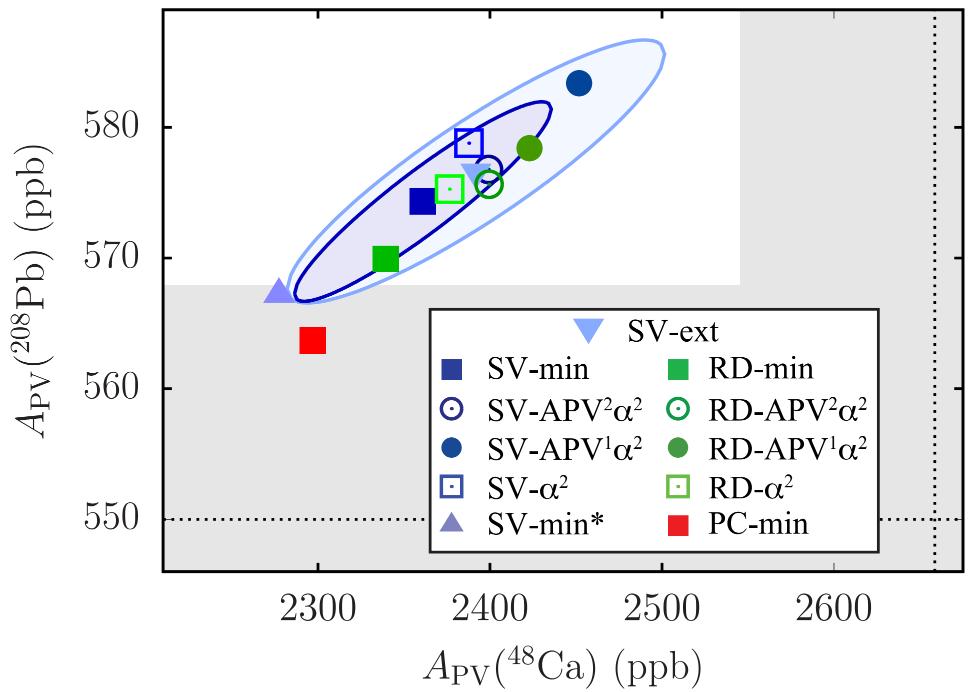
Are EDFs incompatible with experimental data?



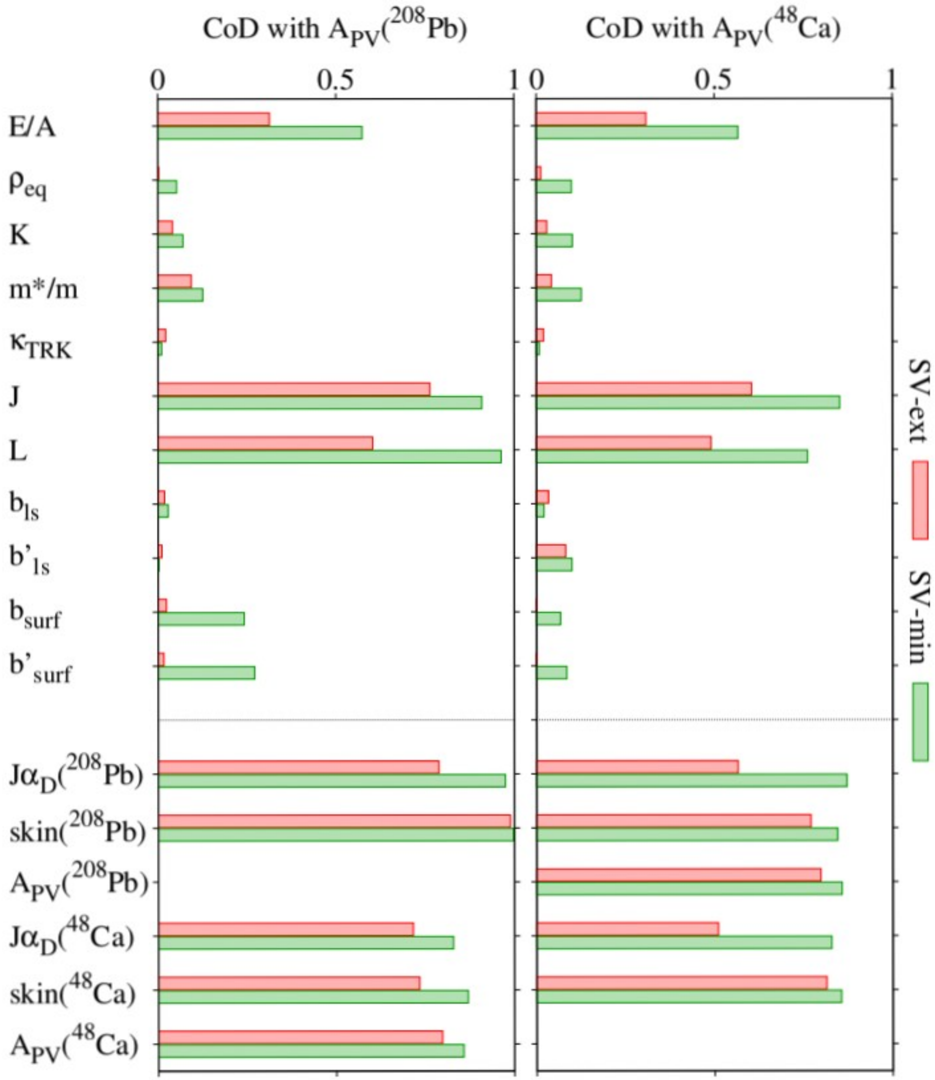


Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz (to be submitted 2022)

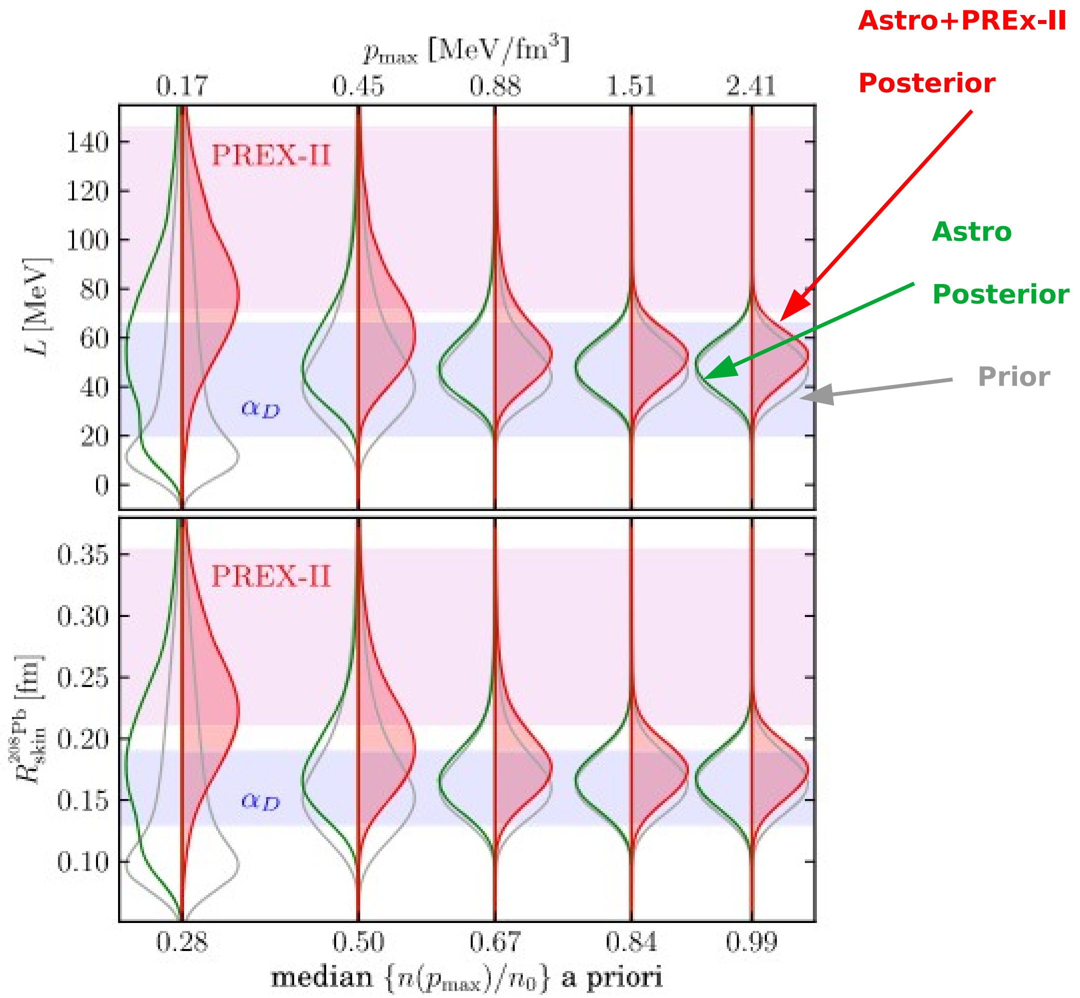
Does a richer EDF improve?



Paul-Gerhard Reinhard, Xavier Roca-Maza, and Witold Nazarewicz (to be submitted 2022)



Other example: minimal model assumptions and statistical analysis



Reed Essick, et al. Phys. Rev. Lett. 127, 192701 (2021)

Non-parametric **equation of state** representation derived from **observations of neutron stars** with minimal modeling assumptions.

The resulting **astrophysical constraints** from heavy pulsar masses, LIGO/Virgo, and NICER clearly **favor "small"** values of the **neutron skin** and **L**.

Combining astrophysical data with **PREX-II** and **chiral effective field theory** constraints yields

J=33.0 ± 2.0 MeV

L=53 ± 15MeV

R_{skin}=0.17 ± 0.04 fm

Conclusions

- **Current EDFs** show strong ***systematic*** and ***statistic*** **correlations** between $\mathbf{A_{pv}}$ and $\mathbf{\Delta R_{np}}$ or $\mathbf{\alpha_D J}$ and $\mathbf{\Delta R_{np}}$
- **Fitting masses and radii** do not give enough **information** on $\mathbf{A_{pv}}$**,** $\mathbf{\alpha_D J}$ and $\mathbf{\Delta R_{np}}$ (additional reason for the strong correlation)
- **Extending** the **fitting protocol** to include $\mathbf{A_{pv}}$ and $\mathbf{\alpha_D}$ do **not change** the **correlations** above since **experimental errors** on these observables **are** still **large → model predictions** remain **biased** by **masses** and **radii**
- More **accurate measurements** on $\mathbf{A_{pv}}$ could point to **model deficiencies,** while **EDFs** seem to **accommodate** better $\mathbf{\alpha_D}$
- **Current EDFs** are able to overlap within **~1σ** all experimental **data** (except A_{pv} in ^{48}Ca where SV-ext gives the best result being away ~1.5σ from experiment)
- **Ideally:** measure $\mathbf{A_{pv}}$ at **different kinematics** or $\mathbf{A_{pv}}$ on **different nuclei** with same (or better) accuracy → better **constraints** to **models**

Collaborators

B. K. Agrawal[1]
P. F. Bortignon, M. Brenna,
G. Colò[2,3]
W. Nazarewicz[4,5,6]
N. Paar, D. Vretenar[7]
J. Piekarewicz[8]

P.-G. Reinhard[9]
Michal Warda[10]
Mario Centelles, Xavier
Viñas[11]