Modern Nuclear Theory & Neutron Skins

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Precision Tests with neutral-current coherent interactions with nuclei

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Collaborators

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The Hamiltonian knows best



- An "ab initio revolution" is changing how we compute atomic nuclei
- What was long believed to be the exclusive domain of nuclear energy density functionals, is becoming accessible by Hamiltonians
 - Link nuclear structure to forces between few nucleons
 - Compute bulk properties (radii, BEs), spectra, and transitions
 - Perform symmetry projection
- Ideas from EFT and the RG, and algorithms with an affordable scaling are paving the way

Coupled-cluster method (CCSD approximation)

$$\Psi = e^{T} |\Phi\rangle$$

$$T = T_{1} + T_{2} + \dots$$

$$T_{1} = \sum_{ia} t_{i}^{a} a_{a}^{\dagger} a_{i} T_{2} = \frac{1}{4} \sum_{ijab} t_{ij}^{ab} a_{a}^{\dagger} a_{b}^{\dagger} a_{j} a_{i}$$

- © Scales gently (polynomial) with increasing system size
- Truncation is only approximation (\mathbf{U})
- A lot of freedom in the choice of reference state (spherical, deformed, pairing,...)

Correlations are *exponentiated* 1p-1h and 2p-2h excitations. Part of Ap-Ah excitations included!



 $\langle \Phi | H | \Phi \rangle$

 $0 = \langle \Phi_i^a | \overline{H} | \Phi \rangle$

E =

Coupled cluster equations

ia

Normal-ordering effectively reduces the

A-body to a few-body problem

Alternative view: CCSD generates similarity transformed Hamiltonian with no 1p-1h and no 2p-2h excitations. $0 = \langle \Phi_{ii}^{ab} | \overline{H} | \Phi \rangle$

$$\overline{H} \equiv e^{-T}He^{T} = \left(He^{T}\right)_{c} = \left(H + HT_{1} + HT_{2} + \frac{1}{2}HT_{1}^{2} + \dots\right)_{c}$$

Nuclear forces from chiral effective field theory

[Weinberg; van Kolck; Epelbaum *et al.*; Entem & Machleidt; ...] NN 3N 4NLO \mathcal{O} NLO $\mathcal{O}\left(\frac{Q^2}{\Lambda^2}\right)$ $\mathrm{N^{2}LO}\ \mathcal{O}$ $\mathrm{N}^{3}\mathrm{LO}\ \mathcal{O}$

- Developing higher orders and higher rank (3NF, 4NF) [Epelbaum 2006; Bernard et al 2007; Krebs et al 2012; Hebeler et al 2015; Entem et al 2017, Reinert et al 2017...]
- Propagation of uncertainties on the horizon [Navarro Perez 2014, Carlsson et al 2015, Ekström & Hagen 2019, Drischler et al 2020]
- Different optimization protocols [Ekström et al 2013, Carlsson et al 2016]
- Improved understanding/handling via SRG [Bogner et al 2003; Bogner et al 2007]
- local / semi-local / non-local formulations
 [Epelbaum et al 2015, Gezerlis et al 2013/2014, Binder et al 2018]
- Chiral EFT's with explicit Delta isobars [Krebs et al 2018, Piarulli et al 2017, Ekström et al 2017, Jiang et al (2020)]

Some chiral potentials (models) work better than others



NNLO_{sat}: Accurate radii and BEs

Simultaneous optimization of NN and 3NFs Include radii and BEs of ³H, ^{3,4}He, ¹⁴C, ¹⁶O in the optimization Harder interaction: difficult to converge beyond ⁵⁶Ni

A. Ekström *et al*, Phys. Rev. C **91**, 051301(R) (2015)

$\Delta NNLO_{GO}$: Accurate radii, BEs, and symmetry energy

Chiral EFT with explicit delta isobars Include charge radii and binding energies of ³H, ^{3,4}He, and nuclear matter saturation in the optimization

W. G. Jiang, et al., Phys. Rev. C 102, 054301 (2020)



1.8/2.0(EM): Accurate BEs and spectra

Soft interaction: SRG N3LO from Entem & Machleidt with 3NF at NNLO from chiral EFT

K. Hebeler *et al* PRC (2011). T. Morris *et al*, PRL (2018).

Charge radius and dipole polarizability of 8He



Towards island of inversion with ab initio methods



Coupled-cluster computations of deformed nuclei – natural orbitals



- Coupled-cluster calculations from axially symmetric reference states
- Natural orbitals from many-body perturbation theory [A. Tichai, et al PRC (2019)] yields rapid convergence with respect 3p3h excitations in CCSDT-1
- Hartree-Fock with projection after variation (PAV) gives upper bound on the energy gain from symmetry restoration

S. J. Novario, G. Hagen, G. R. Jansen, T. Papenbrock, Phys. Rev. C 102, 051303 (2020)

Computations of neon isotopes

- Dripline correctly predicted at ³⁴Ne
- Charge radii predicts shell closures at N = 8, N = 14, and at N = 20



N = 12, hw = 16MeV

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S. J. Novario, G. Hagen, G. R. Jansen, T. Papenbrock, Phys. Rev. C 102, 051303 (2020)

Computations of magnesium isotopes

- Dripline predicted at ⁴⁰Mg continuum may impact the location of the dripline
- Charge radii predicts shell closures at N = 8, N = 14, and at N = 20
- The bands indicate uncertainties from model-space truncations



S. J. Novario, G. Hagen, G. R. Jansen, T. Papenbrock, Phys. Rev. C 102, 051303 (2020)

Symmetry restored coupled-cluster theory

Projection after variation (PAV):

$$E_J = \frac{\langle \Psi | P_J H | \Psi \rangle}{\langle \widetilde{\Psi} | P_J | \Psi \rangle}$$

Right state is parametrized: $|\Psi
angle=e^T|\Phi_0
angle$

Left state is parametrized as:

$$\langle \widetilde{\Psi} | = \langle \Phi_0 | (1 + \Lambda) e^{-T} \text{ or } \langle \widetilde{\Psi} | = \langle \Phi_0 |$$

Coupled-cluster bi-variational energy expression



Naïve energy expression

Dynamic correlation (large contribution and requires size-extensive methods)

Static correlation (can use non size-extensive methods)





Benchmark computation for 8Be and 20Ne

G. Hagen et al, arXiv:2201.07298 (2022)



CCD spectra a bit too compressed, but we are getting there ...

Computation of ³⁴Mg



Summary of results

- Symmetry projection has little impact on radii
- Matched low-energy constants to the rigid rotor model (leading order EFT for rotations)
- The EFT accurately reproduces rotational bands

	Unprojected			Projected					EFT	
²⁰ Ne	E	$\langle J^2 \rangle$	$R_{\mathrm ch}~(\mathrm{fm})$	δE	$E^{(0)} = E + \delta E$	$R_{\mathrm ch}~(\mathrm{fm})$	$E^{(2)} - E^{(0)}$	$E^{(4)} - E^{(0)}$	$E^{(2)} - E^{(0)}$	$E^{(4)} - E^{(0)}$
HF	-59.442	22.778	2.623	-5.760	-65.202	2.619	1.26	4.34	1.5 ± 0.1	5.1 ± 1.3
SLD	-122.467	19.059	2.601	-4.332	-126.799	2.598	1.13	3.90	1.4 ± 0.1	4.5 ± 1.2
CCD	-142.666	16.128	2.621	-3.627	-146.293	2.620	1.19	3.68	1.3 ± 0.1	4.5 ± 1.2

	Unprojected			Projected					EFT	
$^{34}\mathrm{Mg}$	E	$\langle J^2 \rangle$	$R_{\mathrm ch}~(\mathrm{fm})$	δE	$E^{(0)} = E + \delta E$	$R_{\mathrm ch}~(\mathrm{fm})$	$E^{(2)} - E^{(0)}$	$E^{(4)} - E^{(0)}$	$E^{(2)} - E^{(0)}$	$E^{(4)} - E^{(0)}$
HF	-85.687	24.740	2.727	-3.184	-88.87	2.724	0.67	2.29	0.77 ± 0.06	2.6 ± 0.7
SLD	-177.938	22.790	2.707	-2.479	-180.42	2.704	0.60	2.05	0.65 ± 0.05	2.2 ± 0.6
CCD	-221.315	20.213	2.725	-1.893	-223.21	2.722	0.53	1.69	0.56 ± 0.04	1.9 ± 0.5

G. Hagen et al, arXiv:2201.07298 (2022)

Neon isotopes: Inclusion of three-body forces and more accurate left state

Rotational structure of neutron-rich neon isotopes in good agreement with data



Interaction 1.8/2.0(EM) from Hebeler et al (2012) over-emphasizes N=20 shell closure 32,34 Ne are as rotational as 34 Mg

Charge radii of neutron-rich potassium isotopes



- First high precision measurement of ⁵²K charge radius by CRIS @ ISOLDE/CERN
- Steep increase in charge radii beyond N = 28 challenges theory
- No signature of N = 32 shell closure
- Isotope shifts not sensitive to details of NNLO chiral Hamiltonians

A. Koszorus, et al, Nature Physics, Open Access (2021)

Coupled-cluster computations of even-even Ca-Zn nuclei

- We construct natural orbitals from a Hartree-Fock calculation using Nmax = 14.
- The normal-ordered Hamiltonian in natural orbitals is truncated to a smaller modelspace (See J. Hoppe et al. Phys. Rev. C 103, 014321 (2021))
- We achieve rapid convergence for energies and radii

	$\hbar\omega = 12$	MeV	$\hbar\omega = 16 \text{ MeV}$		
$N_{ m max}^{ m nat}$	$E({ m MeV})$	$R_{ m ch}({ m fm})$	$E({ m MeV})$	$R_{ m ch}({ m fm})$	
6	-473.731	3.857	-474.445	3.848	
8	-513.502	3.882	-515.685	3.869	
10	-520.787	3.896	-523.355	3.882	
12	-521.746	3.900	-524.384	3.886	



M. Kortelainen, Z. H. Sun, G. Hagen, W. Nazarewicz, T. Papenbrock, P-G. Reinhard, Phys. Rev. C 105, L021303 (2022)

Coupled-cluster computations of even-even Ca-Zn nuclei



M. Kortelainen, Z. H. Sun, G. Hagen, W. Nazarewicz, T. Papenbrock, P-G. Reinhard, Phys. Rev. C 105, L021303 (2022)

Element independent increase in radii beyond N = 28 for Ca-Zn isotopes The trend is explained by fitting the Z averaged isotope shift to a parabolic expression from generalized seniority picture

$$\delta \langle r_c^2 \rangle^{A_{\rm m}, A_{\rm m}+n} = an + bn^2$$



Neutron skin and dipole polarizability of ⁴⁸Ca



- Neutron skin significantly smaller than in DFT
- Results for ⁴⁸Ca agrees with CREX $R_{skin} = 0.121 \pm 0.035 \text{fm}$

CREX: $F_w(q = 0.873 \text{ fm}^{-1}) = 0.1304 \pm 0.0052$ Coupled-Cluster: $0.102 \le F_w(q = 0.873 \text{ fm}^{-1}) \le 0.161$



Neutron skin and dipole polarizability of ⁴⁸Ca



J. Simonis, S. Bacca1, and G. Hagen Eur. Phys. J. A 55 (2019)

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Coherent elastic neutrino scattering (CEvNS) on ⁴⁰Ar

Coherent cross section
$$\frac{d\sigma}{dT}(E_{\nu},T) \simeq \frac{G_F^2}{4\pi} M \left[1 - \frac{MT}{2E_{\nu}^2}\right] Q_W^2 F_W^2(q^2)$$

- Good agreement with data for charge form-factor in ⁴⁰Ar
- Mild sensitivity to employed interaction in energy region relevant to coherent scattering
- Need higher-precision experiments in order to inform/constrain nuclear models



C. G. Payne, S. Bacca, G. Hagen, W. Jiang, T. Papenbrock, et al Phys. Rev. C 100, 061304(R) (2019)

Coherent elastic neutrino scattering (CEvNS) on ⁴⁰Ar

The neutron radius and skin of ⁴⁰Ar from coupled cluster with interactions from chiral EFTs are consistent with DFT predictions – This is contrary to the case of ⁴⁸Ca



C. G. Payne, S. Bacca, G. Hagen, W. Jiang, T. Papenbrock, et al Phys. Rev. C 100, 061304(R) (2019)

Radii, skins, and dipole polarizability of nickel isotopes



S. Malbrunot-Ettenauer, et al, Phys. Rev. Lett. 128, 022502 (2022)

Kaufmann et al, Phys. Rev. Lett. 124, 132502 (2020)

Hamilt	tonian	α_D	R_p	R_n	R _{skin}	R_c			
1.8/2.0	0 (EM)	3.58(18)	3.62(1)	3.82(1)	0.201(1)	3.70(1)			
2.0/2.0	0 (EM)	3.83(23)	3.69(2)	3.89(2)	0.202(3)	3.77(1)			
2.2/2.0	0 (EM)	4.04(28)	3.74(2)	3.94(2)	0.203(4)	3.82(2)			
2.0/2.0	0 (PWA)	4.87(40)	3.97(2)	4.17(3)	0.204(8)	4.05(2)			
NNLO	sat	4.65(49)	3.93(4)	4.11(5)	0.183(8)	4.00(4)			
6.5 –	2.0/2 2.0/2 1.8/2	2.0 (EM) 2.0 (PWA) 2.0 (EM)	2.2/2. NNLC	0 (EM) D _{sat}	Exp.: Ro Exp.: Th	ossi <i>et al.</i> nis work			
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) 0 0 4.0	In the second								
3.5									
3.0 3.6	3.	.7 3	.8	3.9	4.0	4.1			
	$R_c(^{68}Ni)$ [fm]								

Trends of neutron skins of mirror nuclei





Different methods and interactions give a linear relation between neutron skin and isospin asymmetry

 $\delta(^{208}\text{Pb}) = 44/208 = 0.212$ $\Delta R_{np}(^{208}\text{Pb}) = 0.241 \pm 0.028\text{fm}$ Neutron skin of sodium isotopes as a function of isospin asymmetry. Data used available matter radii and charge radii from isotope shift measurements using two different values of the atomic parameter K_{SMS}

B. Ohayon, R. F. Garcia Ruiz, Z. H. Sun, G. Hagen, T. Papenbrock, B. K. Sahoo, Phys. Rev. C **105**, L031305 (2022)

Why do some interaction models work better than others?



To answer this we need predictions with rigorous **uncertainty quantification** and **sensitivity analyses** that are grounded in the description of the underlying nuclear Hamiltonian

Andreas Ekström, Gaute Hagen PRL 123, 252501 (2019)

Global sensitivity analysis of the radius and binding energy of 16-0

<u>Sensitivity analysis</u> addresses the question 'How much does each model parameter contribute to the uncertainty in the prediction?'

<u>Global</u> methods deal with the uncertainties of the outputs due to input variations over the whole domain.

Computational bottleneck

A global sensitivity analyses of the binding energy and charge radius of a nucleus like 16-O requires more than one million model evaluations



Andreas Ekström, Gaute Hagen PRL 123, 252501 (2019)

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- Generalization of the eigenvector continuation method [Frame D. et al., Phys. Rev. Lett. 121, 032501 (2018)]
- Write the Hamiltonian in a linearized form

$$H(\vec{\alpha}) = \sum_{i=0}^{N_{\rm LECs}=16} \alpha_i h_i$$

- Select "training points" where we solve exact CCSD
- Project the target Hamiltonian onto sub-space of training vectors and diagonalize the generalized eigen value problem

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Sub-space projected coupled-cluster – cross validation in 16 dimensions

- Select 64 and 128 sub-space vectors in the 16 dimensional space of LECs using a space-filling latin hypercube design
- Select 200 randomly exact CCSD calculations in a 20% domain around NNLO_{sat}
- With 64 subspace vectors we achieve a 1% accuracy relative to exact CCSD solutions

A global sensitivity analysis of the radius and binding energy of ¹⁶O

- Compute the binding energy and charge radius at one million different values of the 16 LECs in one hour on a standard laptop (would require 20 years of equivalent exact CCSD computations)
- About 60% of the variance in the energy can is attributed to the 3S1-wave, whereas the radius depends sensitively on several LECs and their higher-order correlations

- The capability of performing billions of ab-initio simulations of selected nuclei opens up entirely new possibilities for making predictions and addressing uncertainties
- History matching identify regions of parameter space that give results consistent with data (NN phase-shifts, A = 2,3,4 observables and 16-O BE and radius

Using E/A, E(2+) and R_p of Ca-48 as calibration data we are left with 34 interactions (out of 5x10⁸ parametrizations)

Posterior predictive distribution for the neutron skin in ²⁰⁸Pb compared to experiments using electroweak (purple), hadronic (red), electromagnetic (green), and gravitational waves (blue) probes.

 $R_{skin}(208Pb) = 0.14 - 0.20$ fm (68% credible interval) exhibits a mild tension with the value extracted from PREX-2

Baishan Hu, Weiguang Jiang, Takayuki Miyagi, Zhonghao Sun, et al, arXiv:2112.01125 (2022)

Baishan Hu, Weiguang Jiang, Takayuki Miyagi, Zhonghao Sun, et al, arXiv:2112.01125 (2022)

- Different models predict similar correlation between the neutron-skin and the slope of symmetry energy (L)
- The neutron skin of ²⁰⁸Pb is (weakly) correlated with the ¹S₀ nucleon-nucleon scattering phase-shift
- A realistic description of the ¹S₀ scattering phase shift implies a neutron skin in tension with PREX-2

Baishan Hu, Weiguang Jiang, Takayuki Miyagi, Zhonghao Sun, et al, arXiv:2112.01125 (2022)

D. Adhikari et al, arXiv:2205.11593 (2022)

Summary

- Towards mass-table computations based on Hamiltonian methods
 - most even-even nuclei now possible with symmetry projection
 - Interactions with "good" saturation properties yield accurate description of BEs, radii and skins in light, medium-mass and heavy nuclei
 - shell closures predicted at N = 8, 14 in neon and magnesium and no signature of N = 32 shell closure in potassium
 - Universal trend of radii beyond N = 28 for even-even Ca-Zn isotopes
 - Odd-odd and odd-even nuclei are more challenging
 - Predicted N = 20 shell closure is not supported by data in isotopes of neon and magnesium
 - Steep increase in radii beyond N = 28 in potassium challenges theory
- Prediction of small neutron skin in ⁴⁸Ca confirmed by CREX
- Coherent neutrino scattering on ⁴⁰Ar a stepping stone for neutrino response (see talk by Asia Sobczyk)

Summary

- Developed emulators that allows us to sample ~10⁸ different Hamiltonians in a short time for medium mass nuclei
 - A global sensitivity analysis revealed the role of various LECs in the binding energy and radius of ¹⁶O
- Combining accurate emulators, novel statistical tools, and Bayesian inference allowed us to make accurate predictions for the neutron skin and related observables in ²⁰⁸Pb (see talk by Weiguang Jiang)
- Neutron skin of ²⁰⁸Pb in mild tension with PREX-2
- Confirmed correlations (seen in mean-field approaches) between the neutron skin of ²⁰⁸Pb and the symmetry energy and its slope in nuclear matter

Thank you for your attention!