Spin asymmetries and their accuracy in electron-nucleus collisions

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May, 2022

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Contents:

1a. Definition of spin asymmetries

Elastic scattering of polarized electrons (ζ_i)
into helicity $\pm$ eigenstates $(\zeta_f = \pm\hat{\boldsymbol{k}}_f)$:

$$\frac{d\sigma}{d\Omega}(\zeta_i, \pm) \;=\; \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_{unpol} \Big[1 \;+\; S\,(\zeta_i \cdot \boldsymbol{n})$$

$$+\;\Big\{L\,(\zeta_i \cdot \hat{\boldsymbol{k}}_i) \;-\; R\,(\zeta_i \cdot (\boldsymbol{n} \times \hat{\boldsymbol{k}}_i))\Big\}\,(\pm)\Big]$$

$\boldsymbol{n} \upuparrows \boldsymbol{k}_i \times \boldsymbol{k}_f$ normal to scattering plane

$$L^{\mathrm{PWBA}} = 1 - \frac{2m^2c^4\sin^2\vartheta_f/2}{E_i^2\cos^2\vartheta_f/2 + m^2c^2\sin^2\vartheta/2} = 1 \;\text{ if } m = 0$$

longitudinal spin asymmetry

S = Sherman function (perpendicular spin asymmetry)

1b. Motivation:

Parity violation experiments

Scattering potential $V(r) = V_{\rm coul}(r) + \gamma_5\, V_1(r)$

$$\gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Longrightarrow \gamma_5 \begin{pmatrix} g \\ f \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix}$$

large component: $g \sim \chi_{1/2} = \binom{1}{0}$ even parity
small component: $f \sim (\boldsymbol{\sigma k})\, \chi_{1/2}$ odd parity $(\boldsymbol{k} \mapsto -\boldsymbol{k})$

Decomposition into helicity eigenstates: $\psi = \psi_+ + \psi_-$
$\psi_\pm = \frac{1}{2}\,(1 \pm \gamma_5)\,\psi$

$m = 0 \implies \psi_\pm$ eigenstates to $V_{\rm coul} \pm V_1$
$\Longrightarrow$ helicity conservation $(L = 1)$:

$$\frac{d\sigma}{d\Omega}(\zeta_i = +, \pm) = \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_{unpol} [1 \pm L\,(\zeta_i \cdot \hat{\boldsymbol{k}}_i)]$$

$= 0$ for spin flip

Parity violating asymmetry

$$A_L = \frac{d\sigma/d\Omega(\psi_+) - d\sigma/d\Omega(\psi_-)}{d\sigma/d\Omega(\psi)} \qquad \text{measure of } V_1$$

In fact $m \neq 0 \implies L < 1$ no helicity conservation
Sum rule: $S^2 + L^2 + R^2 = 1 \implies S \neq 0$

Background asymmetry arises from:
beam admixture δ of perpendicularly polarized e^-:

$$\frac{d\sigma}{d\Omega}(\delta\,\boldsymbol{\zeta}_{i\perp}, \pm) = \frac{1}{2}\left(\frac{d\sigma}{d\Omega}\right)_{unpol}[1 + \delta\, S\,(\boldsymbol{\zeta}_{i\perp}\cdot\boldsymbol{n})]$$

$\implies$ Coulomb asymmetry

$$\delta\, S = \frac{d\sigma/d\Omega(+\delta) - d\sigma/d\Omega(-\delta)}{(d\sigma/d\Omega)_{unpol}}$$

adds to A_L

2. Asymmetry in elastic scattering from spin-0 nuclei

Scattering operator for potential scattering

$$\hat{f}_{\rm coul}(k_i, \vartheta_f) \;=\; A \;+\; B\; \boldsymbol{n} \cdot \boldsymbol{\sigma} \qquad\qquad \boldsymbol{n} \;\upuparrows\; \boldsymbol{k}_i \times \boldsymbol{k}_f$$

Perpendicularly polarized electrons $(\zeta_i = \pm \boldsymbol{n})$:

$$\frac{d\sigma_{\rm coul}}{d\Omega}(\zeta_i) = \left(\frac{d\sigma_{\rm coul}}{d\Omega}\right)_{unpol} [1 \;+\; S_{\rm coul}\,(\zeta_i \cdot \boldsymbol{n})]$$

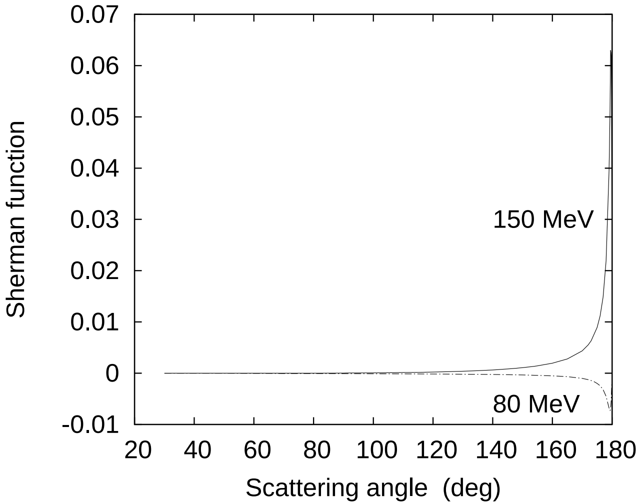
with

$$\left(\frac{d\sigma_{\rm coul}}{d\Omega}\right)_{unpol} \;=\; \frac{k_f}{k_i}\;\frac{1}{f_{\rm rec}}\;\left[|A|^2 \;+\; |B|^2\right] \qquad \text{(recoil prefactor)}$$

$$S_{\rm coul} \;=\; \frac{2\;{\rm Re}\;(\,A\,B^*)}{|A|^2 + |B|^2} \qquad \text{relates to spin-flip}$$

(purely relativistic effect)

$S^{\rm PWBA} \;=\; 0$ spin asymmetry sensitive to higher-order effects



Sherman function
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
150 MeV
80 MeV
20
40
60
80
100
120
140
160
180
Scattering angle (deg)

3. Asymmetry from excitation of ^{12}C

Excitation cross section of spin-zero nucleus to $L^\pi(\omega_L)$:

$$\frac{d\sigma}{d\Omega}(\zeta_i) \;=\; \frac{k_f}{k_i}\,\frac{4\pi^3 E_i E_f}{f_{\rm rec}\;\;c^2}\sum_{\sigma_f}\sum_{M_L}\left|A_{fi}^{\rm coul}\;+\;A_{fi}^{\rm mag}\right|^2$$

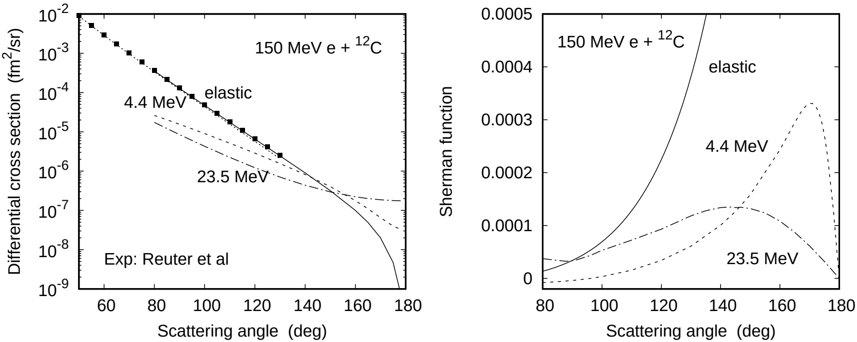
$$\begin{pmatrix} A_{fi}^{\rm coul} \\ A_{fi}^{\rm mag}\end{pmatrix} \;=\; \frac{1}{c}\int d\boldsymbol{r}_N d\boldsymbol{r}_e\;(\psi_{k_f}^{(\sigma_f)+}(\boldsymbol{r}_e)\begin{pmatrix} -1 \\ \boldsymbol{\alpha}\end{pmatrix}\psi_{k_i}^{(\sigma_i)}(\boldsymbol{r}_e))$$
$$\times\;\frac{e^{i\omega_L/c|\boldsymbol{r}_e-\boldsymbol{r}_N|}}{|\boldsymbol{r}_e-\boldsymbol{r}_N|}\begin{pmatrix} \varrho_L(r_N)\;Y^*_{LM_L}(\Omega_N) \\ -i\sum_\lambda J_{L\lambda}(r_N)\;\boldsymbol{Y}^{M_L*}_{L\lambda}(\Omega_N)\end{pmatrix}$$

For parity $\pi=(-1)^L$: $\varrho_L,\;J_{L,L+1},\;J_{L,L-1}$
nuclear transition densities calculated from QRPA, QPM models (Ponomarev)

For perpendicular beam polarization ($\zeta_i=\pm\boldsymbol{n}$):

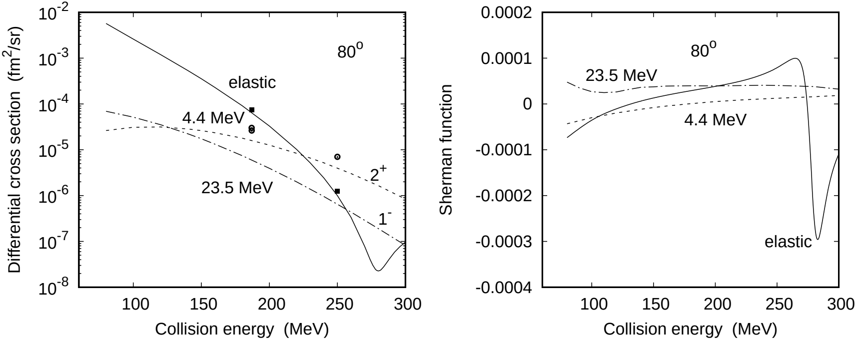
$$S \;=\; \frac{d\sigma/d\Omega(\zeta_i) - d\sigma/d\Omega(-\zeta_i)}{(d\sigma/d\Omega)_{\rm unpol}}$$

Angular distribution of the electron-impact excitation cross section and Sherman function S for ^{12}C, $L = 1, 2$:



Elastic scattering dominant at small angles
Excitation dominant at large angles

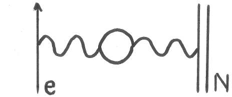
Energy distribution of the electron-impact excitation cross section and Sherman function S for ^{12}C:



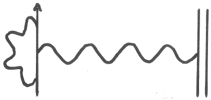
Experiment: Fregeau, Crannell

Cross section minimum corresponds to extrema of S

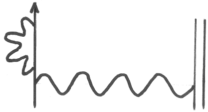
4a. Influence of QED effects in elastic scattering



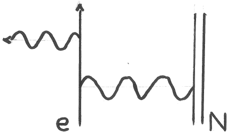
vacuum polarization



vertex correction



self energy



bremsstrahlung

Methods of calculation:

$$f_{\rm coul}^{\rm vac} \;=\; f_{\rm coul}(V_{\rm coul} \;+\; V_{\rm Uehling}) \qquad\qquad \text{phase-shift method}$$

$$A_{fi}^{vs},\; d\sigma_{\rm softbr}/d\Omega \quad \text{in Born} \qquad\qquad \text{(Tsai, Maximon)}$$

hard bremsstrahlung:

Sommerfeld-Maue theory (weak relativistic)

$\omega > \omega_0 \approx 1$ MeV

$\omega_{\rm max} = \Delta E$ detector resolution (4 MeV)

(smaller than 2_1^+-excitation energy)

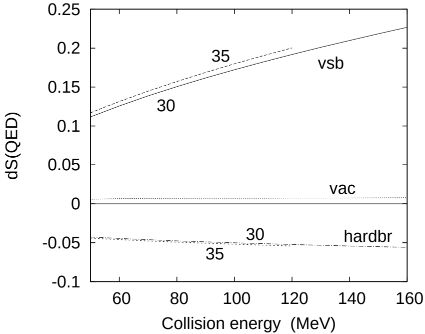
Elastic scattering with QED effects:

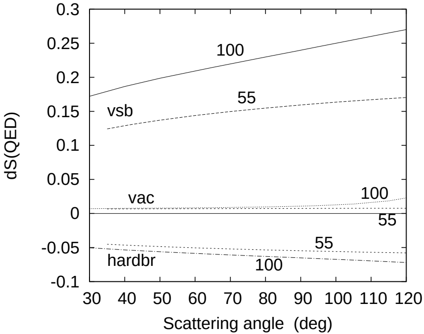
$$\frac{d\sigma^{\rm QED}}{d\Omega}(\zeta_i) \;=\; \frac{k_f}{k_i}\,\frac{1}{f_{\rm rec}}\sum_{\sigma_f}\left[|f_{\rm coul}^{\rm vac}|^2 \;+\; 2\;{\rm Re}\;\left\{f_{\rm coul}^{Born*}\cdot A_{fi}^{vs}\right\}\right.$$

$$\left. +\;\frac{d\sigma_{\rm softbr}}{d\Omega}(\omega \le \omega_0)\right] \;+\; \frac{d\sigma_{\rm hardbr}}{d\Omega}(\omega_0 < \omega \le \omega_{\rm max})$$

$$S \;=\; \frac{d\sigma^{QED}/d\Omega(\zeta_i)\;-\;d\sigma^{QED}/d\Omega(-\zeta_i)}{(d\sigma^{QED}/d\Omega)_{unpol}}$$

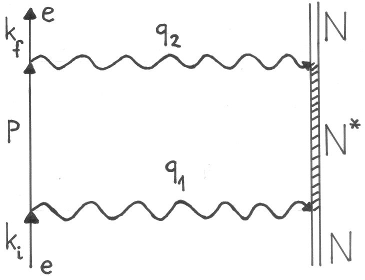
$$S \;=\; S_{\mathrm{coul}}\,(1+\Delta S) \qquad \text{with} \quad \Delta S \;=\; S/S_{coul}\;-1$$





4b. Dispersion in elastic scattering

Box diagram : second Born (Friar, Rosen)



Virtual nuclear excitation to state L, M

Nuclear transition matrix element $T_{00} \sim (F_L^c\, Y_{LM})(\boldsymbol{q}_2)\cdot(F_L^c\, Y_{LM}^*)(\boldsymbol{q}_1)$

$$T_{0m} \sim (F_L^c\, Y_{LM})(\boldsymbol{q}_2) \cdot \sum_{\lambda=L\pm1} (F_{L\lambda}^{te}\, \boldsymbol{Y}_{L\lambda}^{M*})_m(\boldsymbol{q}_1)$$

Charge form factor

$$F_L^c(q) = \int r_N^2 dr_N\, \varrho_L(r_N)\, j_L(qr_N)$$

Transverse form factor

$$F_{L\lambda}^{te}(q) = \int r_N^2 dr_N\, J_{L\lambda}(r_N)\, j_\lambda(qr_N)$$

$$T_{mn} \sim \sum_{\lambda=L\pm1} (F^{te}_{L\lambda}\, \boldsymbol{Y}^{M}_{L\lambda})_m(\boldsymbol{q}_2) \cdot \sum_{\lambda'=L\pm1} (F^{te}_{L\lambda'}\, \boldsymbol{Y}^{M*}_{L\lambda'})_n(\boldsymbol{q}_1)$$

Dispersion amplitude

$$A^{\mathrm{box}}_{fi} = \frac{\sqrt{E_i E_f}}{\pi^2\, c^3} \sum_{LM,\omega_L} \int d\boldsymbol{p} \frac{1}{(q_2^2 + i\epsilon)(q_1^2 + i\epsilon)} \sum_{\mu,\nu=0}^{3} t_{\mu\nu}\ T^{\mu\nu}$$

Electron transition matrix element

$$t_{\mu\nu} = c\, u^{(\sigma_f)+}_{k_f} \gamma_0\gamma_\mu \frac{E_p + c\boldsymbol{\alpha p} + \beta mc^2}{E_p^2 - \boldsymbol{p}^2c^2 - m^2c^4 + i\epsilon} \gamma_0\gamma_\nu\ u^{(\sigma_i)}_{k_i}$$

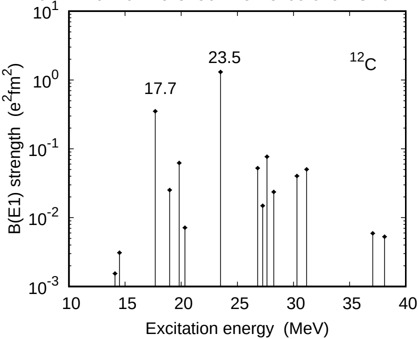
intermediate electron energy $E_p \approx E_i - \omega_L$

Photon momenta $\boldsymbol{q}_1 = \boldsymbol{k}_i - \boldsymbol{p},\ \boldsymbol{q}_2 = \boldsymbol{p} - \boldsymbol{k}_f$

Cross section including dispersion

$$\frac{d\sigma^{\text{box}}}{d\Omega}(\zeta_i) = \frac{d\sigma_{\text{coul}}}{d\Omega}(\zeta_i) + \frac{k_f}{k_i}\frac{1}{f_{\text{rec}}}\sum_{\sigma_f} 2\ \mathrm{Re}\ \left\{ f^*_{\text{coul}} \cdot A^{\text{box}}_{fi} \right\}$$

Dominant nuclear excitations of ^{12}C:



Giant dipole resonance: (1^-)

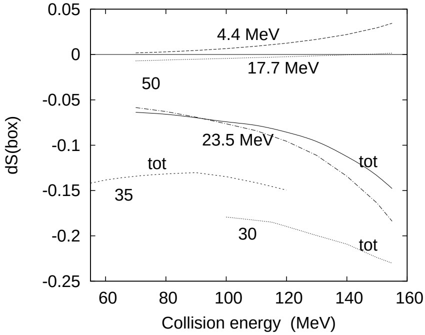
$\omega_L = 23.5$ MeV
$\omega_L = 17.7$ MeV

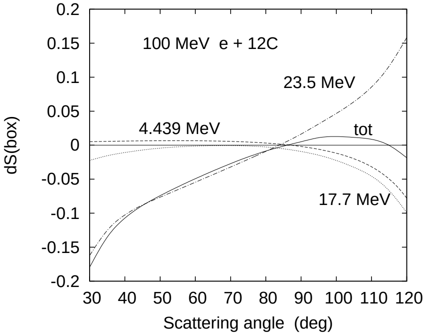
Quadrupole: (2^+_1) at $\omega_L = 4.439$ MeV

$$S^{\text{box}} = \frac{d\sigma^{\text{box}}/d\Omega(\zeta_i) - d\sigma^{\text{box}}/d\Omega(-\zeta_i)}{(d\sigma^{\text{box}}/d\Omega)_{unpol}}$$

Asymmetry change by dispersion

$$\Delta S_{\text{tot}} = \frac{S^{\text{box}}}{S_{\text{coul}}} - 1 \approx \sum_{L,\omega_L} \Delta S(L,\omega_L)$$



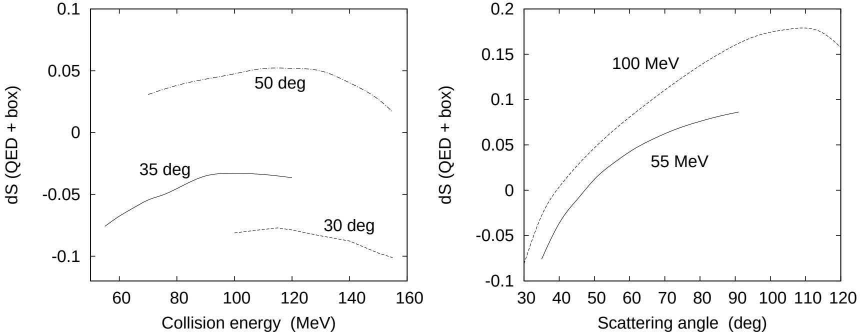


Dominant contribution to ΔS_{tot} from 23.5 MeV state for small ϑ_f

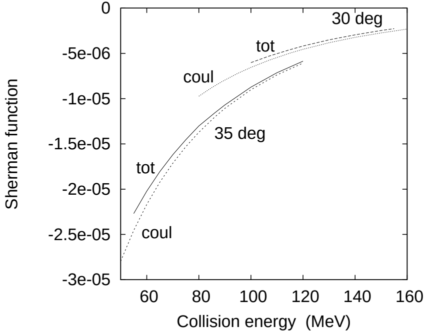
Increase of asymmetry with decreasing angle

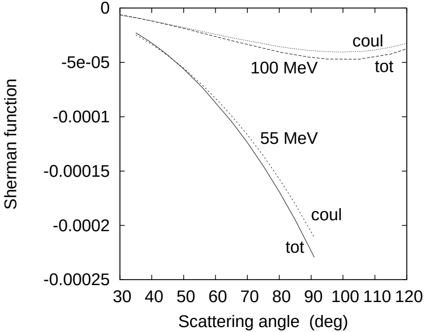
4c. Combined QED and dispersion effects

$$\frac{d\sigma^{\mathrm{QED+box}}}{d\Omega}(\zeta_i) = \frac{k_f}{k_i}\frac{1}{f_{\mathrm{rec}}}\sum_{\sigma_f}\Big[\,|f^{\mathrm{vac}}_{\mathrm{coul}}|^2 + 2\,\mathrm{Re}\,\Big\{f^{\mathrm{Born}*}_{\mathrm{coul}}\cdot(A^{vs}_{fi}+A^{\mathrm{box}}_{fi})\Big\} + \frac{d\sigma_{\mathrm{softbr}}}{d\Omega}\Big] + \frac{d\sigma_{\mathrm{hardbr}}}{d\Omega}$$



Visibility of the QED + box correction





4d. Results for the MESA experiments

55 + 155 MeV $e+^{12}$C

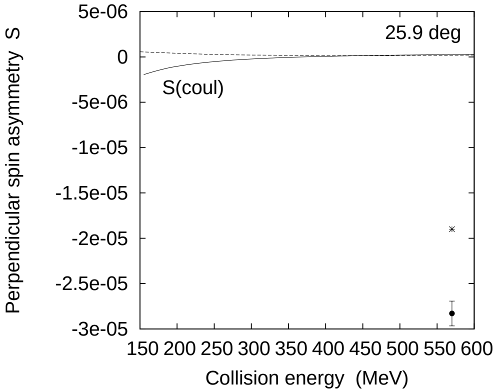
	55 MeV /35°	55 MeV /145°	155 MeV /35°
$S_{\rm coul}$	-2.46e-5	-8.58e-4	-3.13e-6
$\Delta S_{\rm vacpol}$	6.74e-3	7.89e-3	8.18e-3
ΔS_{vsb}	0.124	0.177	0.233
$\Delta S_{\rm hardbr}$	-4.52e-2	-5.99e-2	-5.76e-2
$\Delta S_{\rm box}$	-0.142	-1.39e-2	-0.195
$\Delta S_{\rm QED+box}$	-7.59e-2	8.52e-2	-5.87e-2
$S_{\rm tot}$	-2.27e-5	-9.32e-4	-2.95e-6

Accuracy:

hard bremsstrahlung: $\omega_{\rm max} \equiv \Delta E = 4$ MeV (detector resolution)
$|\Delta S_{\rm hardbr}|$ decreases for higher detector resolution
$\Delta S_{\rm box}$: numerics $\lesssim 5\%$
transient excited states: higher multipoles??

Thank you!

Perpendicular spin asymmetry for ^{12}C at 25.9°



Box correction to S_{coul}

Theory: Gorshteyn

Exp: Esser et al.