Nuclear Polarizability Corrections to the Lamb-Shift

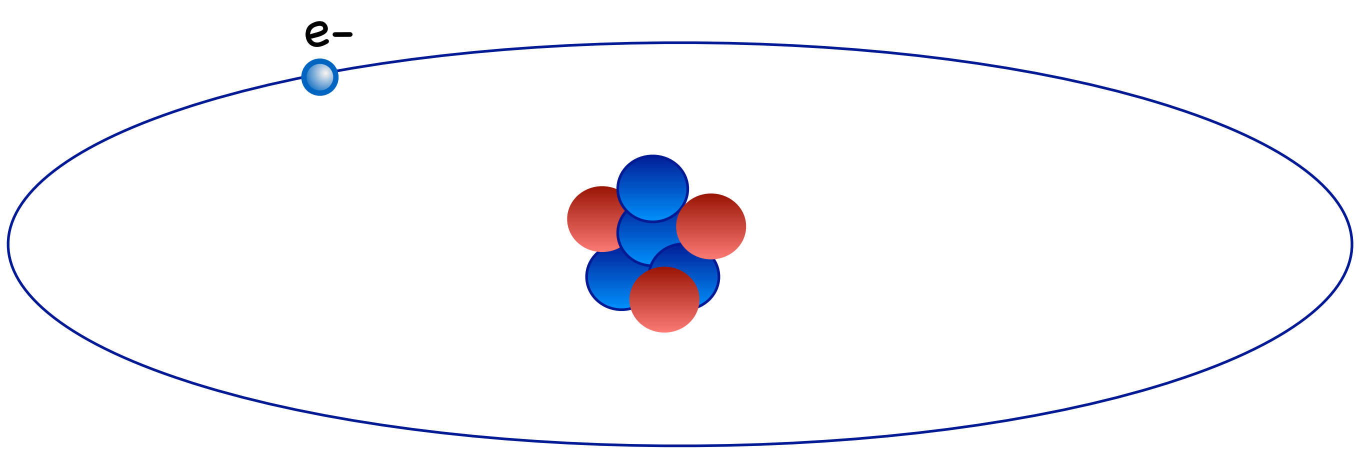
— Muonic atoms beyond μH —

Sonia Bacca

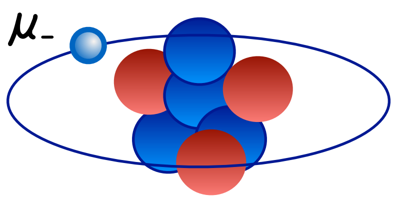
Johannes Gutenberg University, Mainz

Hydrogen-like muonic atoms

Ordinary atoms



Muonic atoms



muon more sensitive to the nucleus

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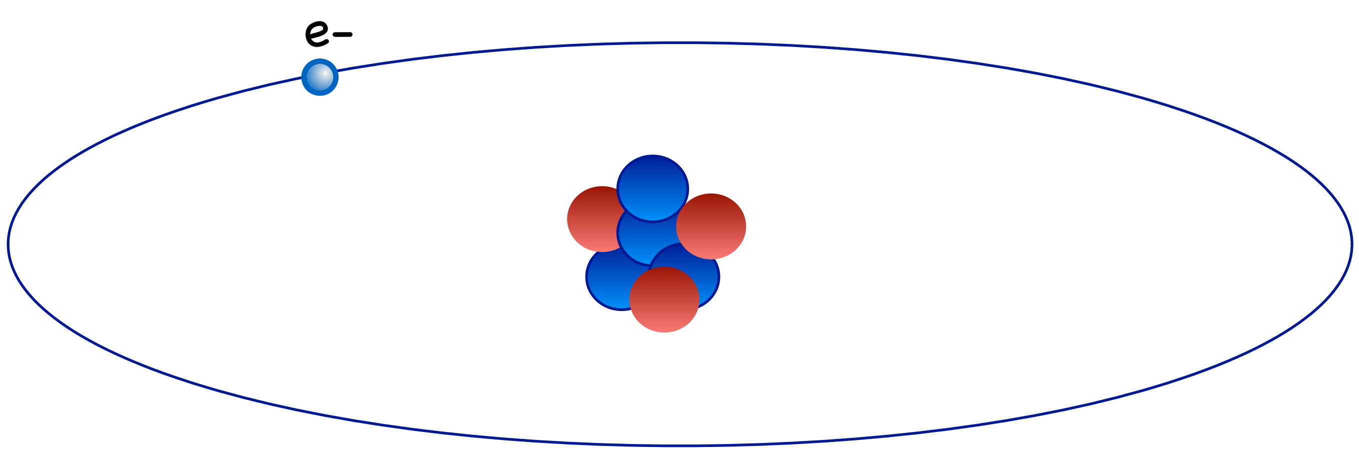

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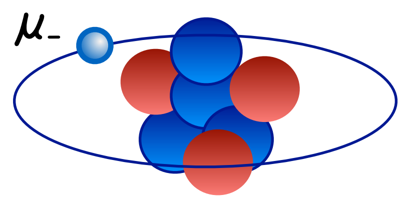
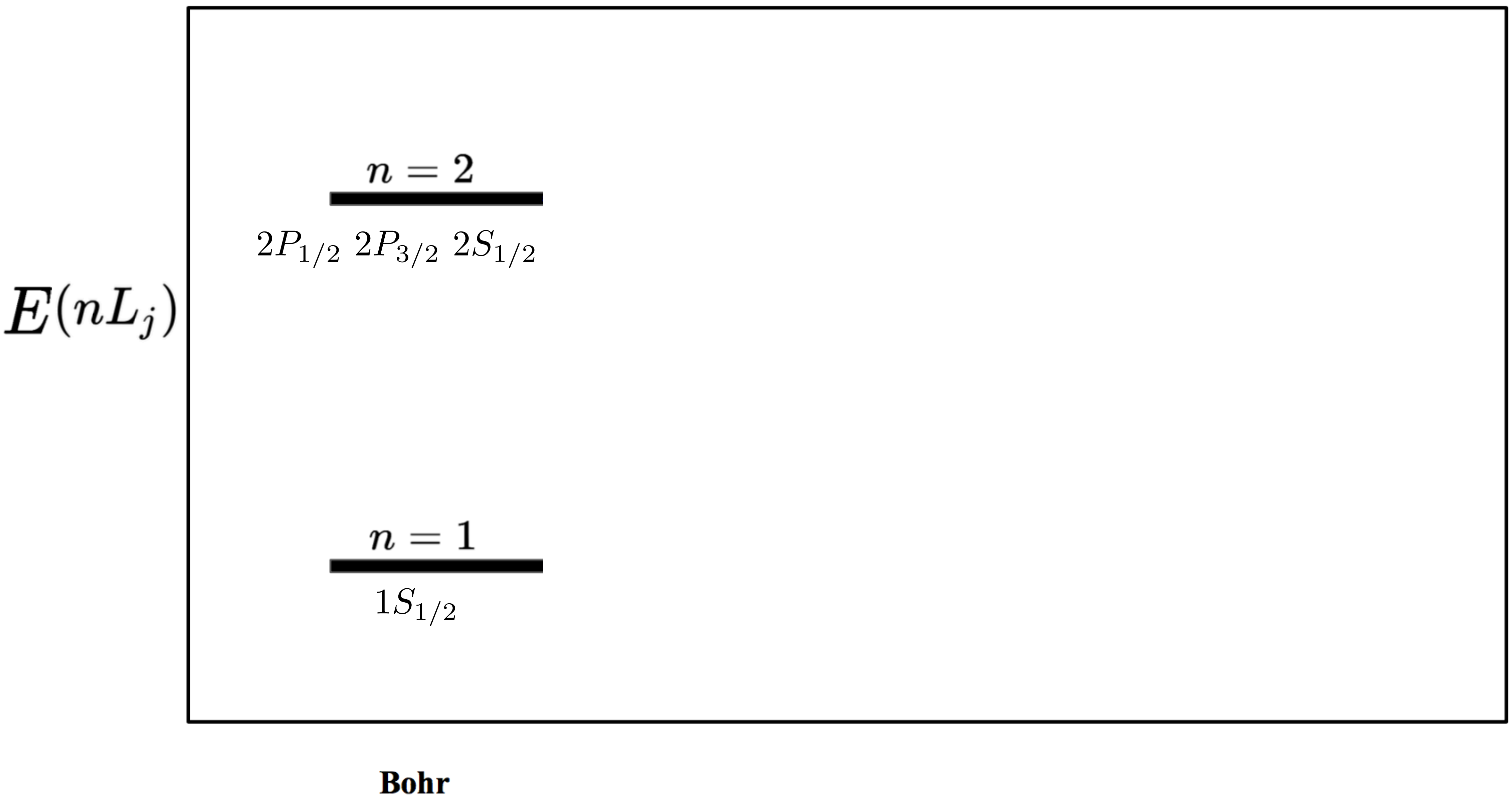


muon more sensitive to the nucleus

Can be used as a precision probe for the nucleus

Lamb Shift



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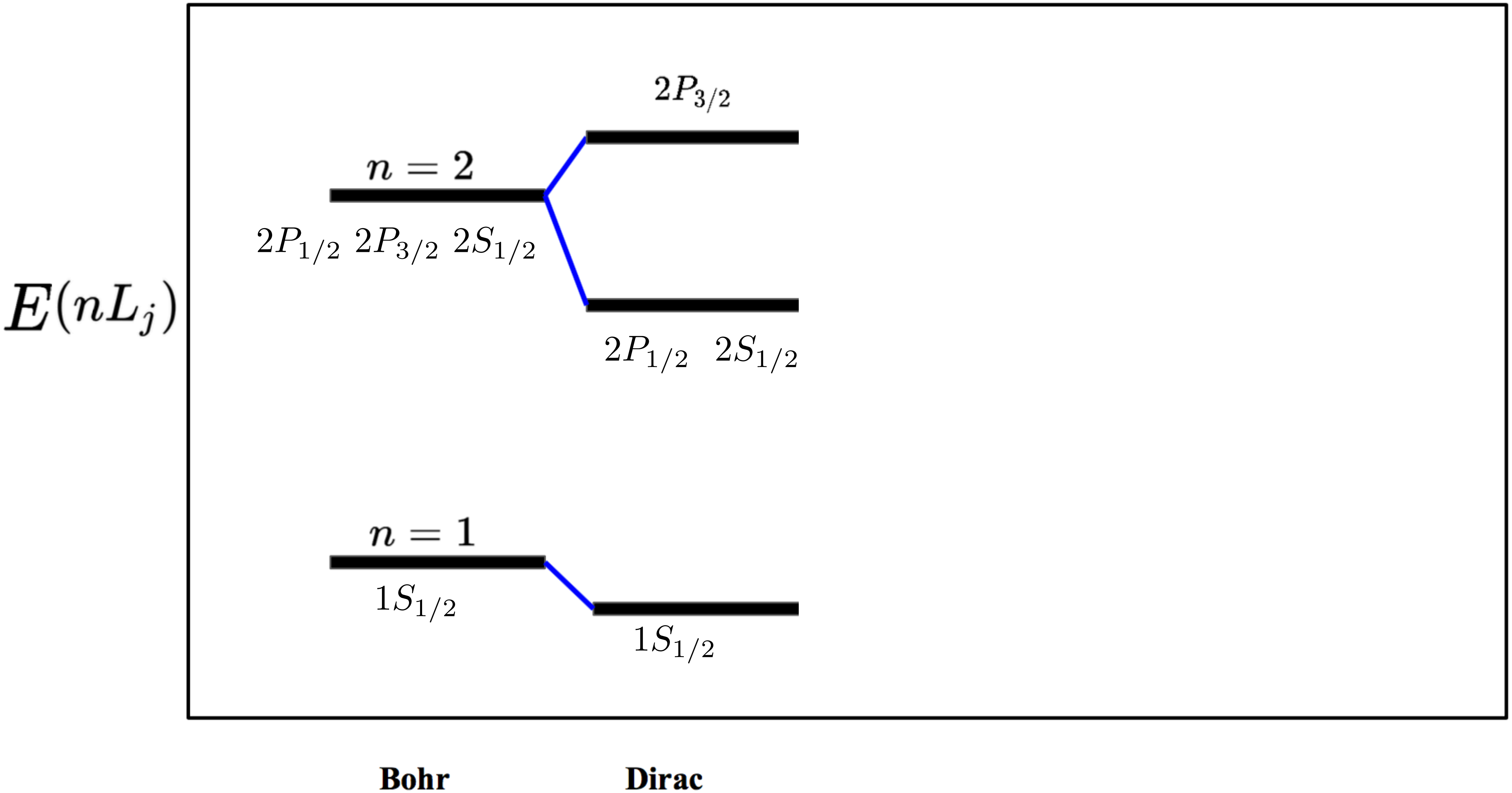

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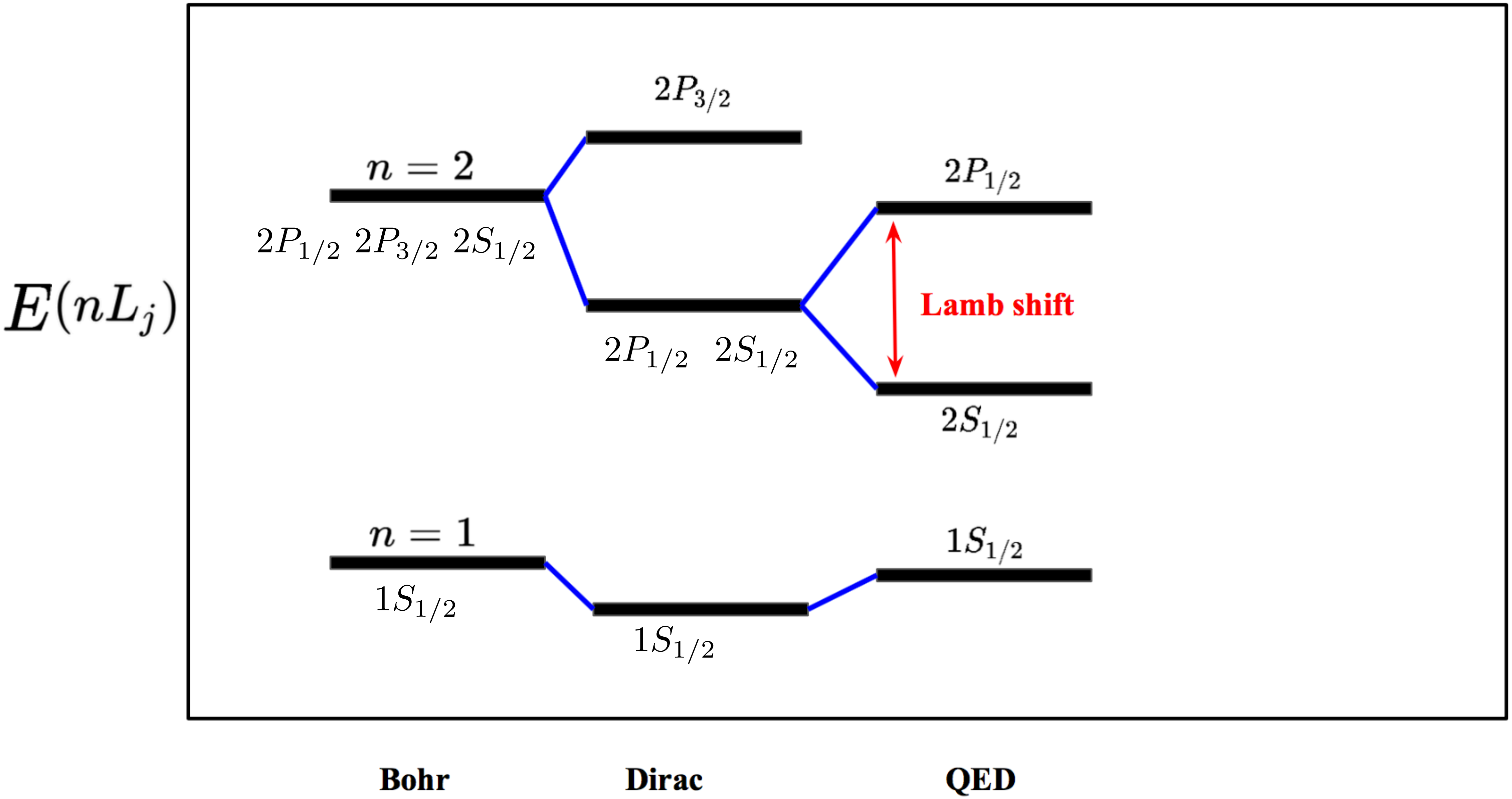
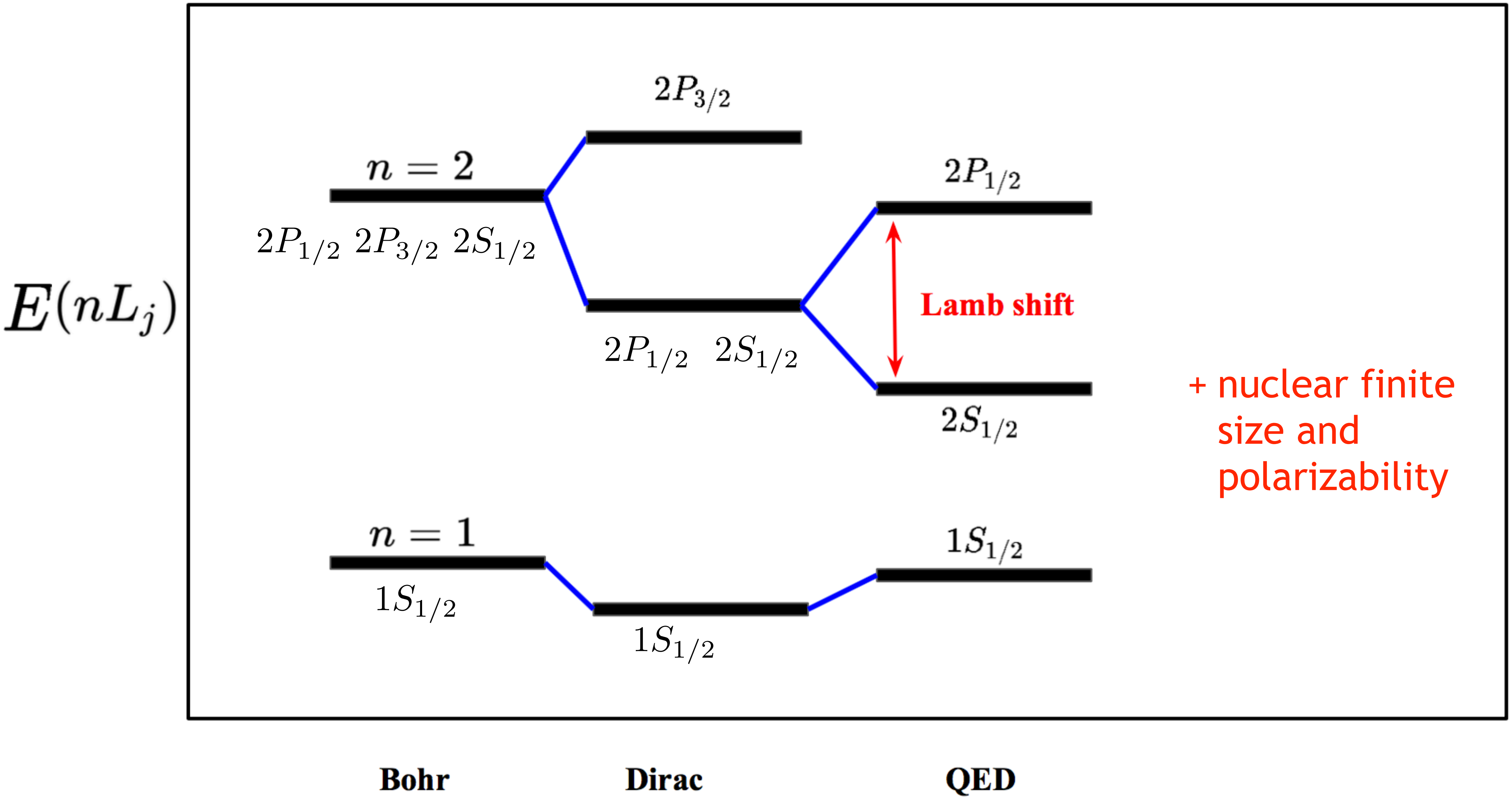


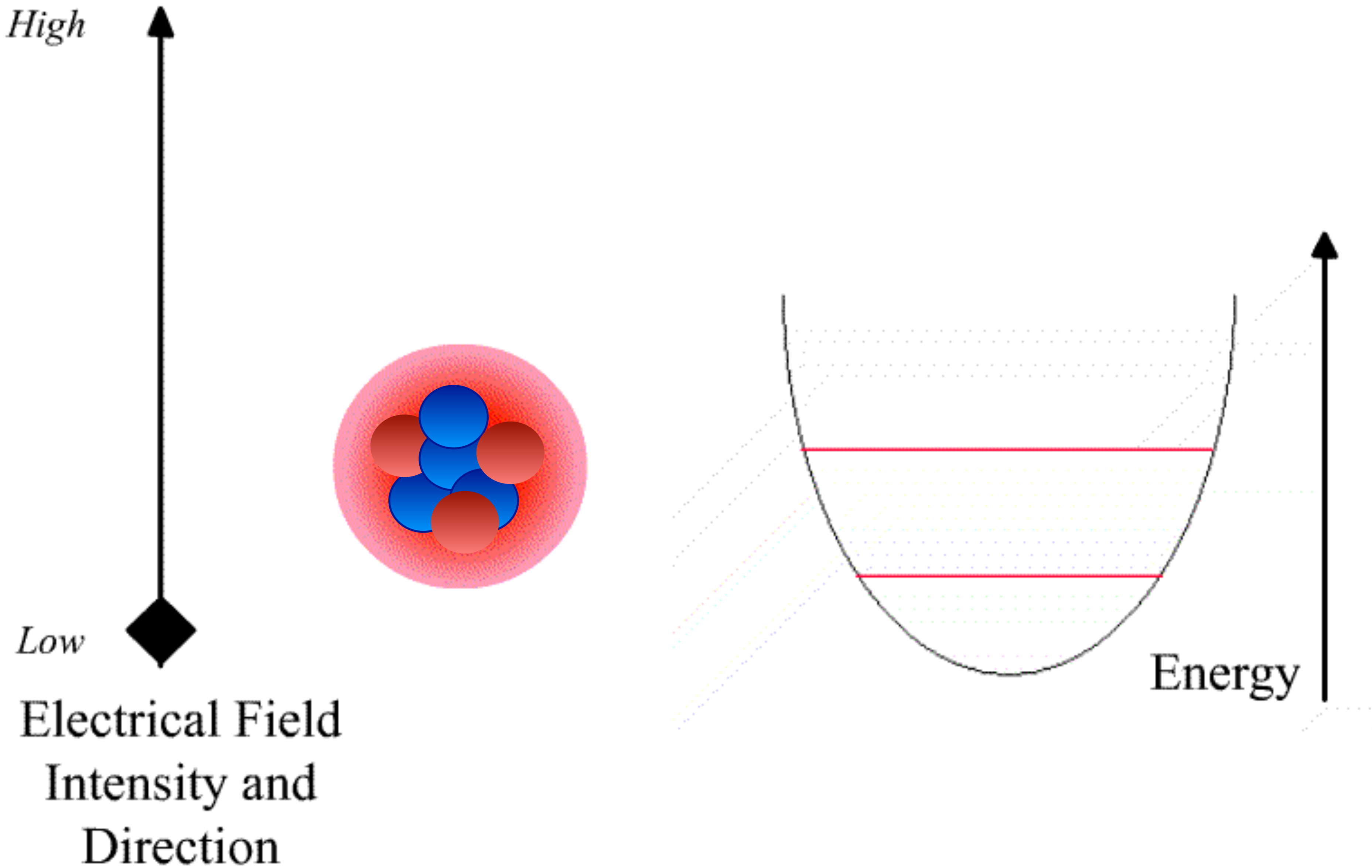
$2P_{3/2}$
$n = 2$
$2P_{1/2}$
$2P_{1/2}\ 2P_{3/2}\ 2S_{1/2}$
$E(nL_j)$
Lamb shift
$2P_{1/2}\ \ 2S_{1/2}$
$2S_{1/2}$
$n = 1$
$1S_{1/2}$
$1S_{1/2}$
$1S_{1/2}$
Bohr
Dirac
QED

Lamb Shift



+ nuclear finite size and polarizability

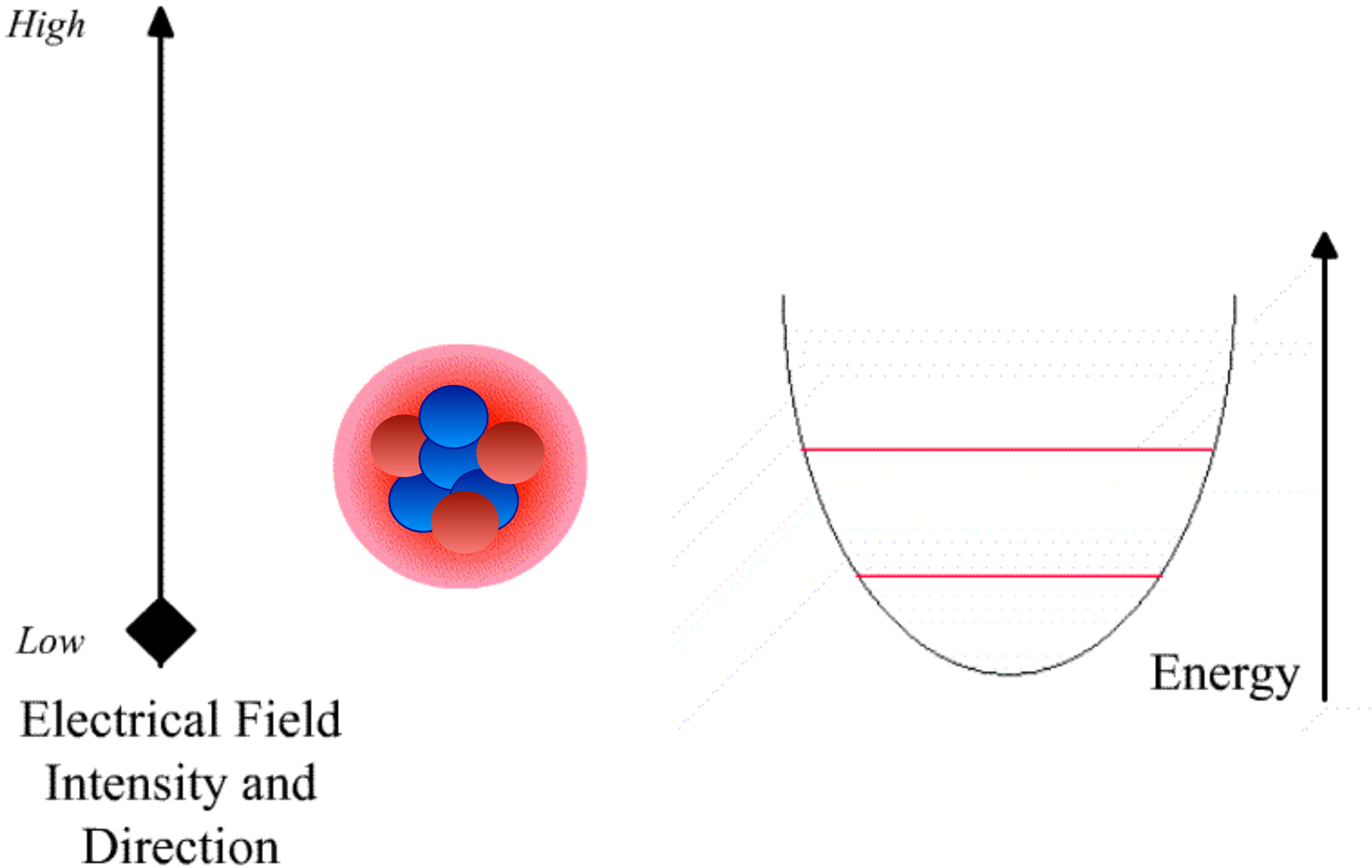
Concept of polarizability



$$\mathbf{P} = \alpha_{\mathbf{E1}} \mathbf{E}$$

Courtesy of V. Pascalutsa

Concept of polarizability



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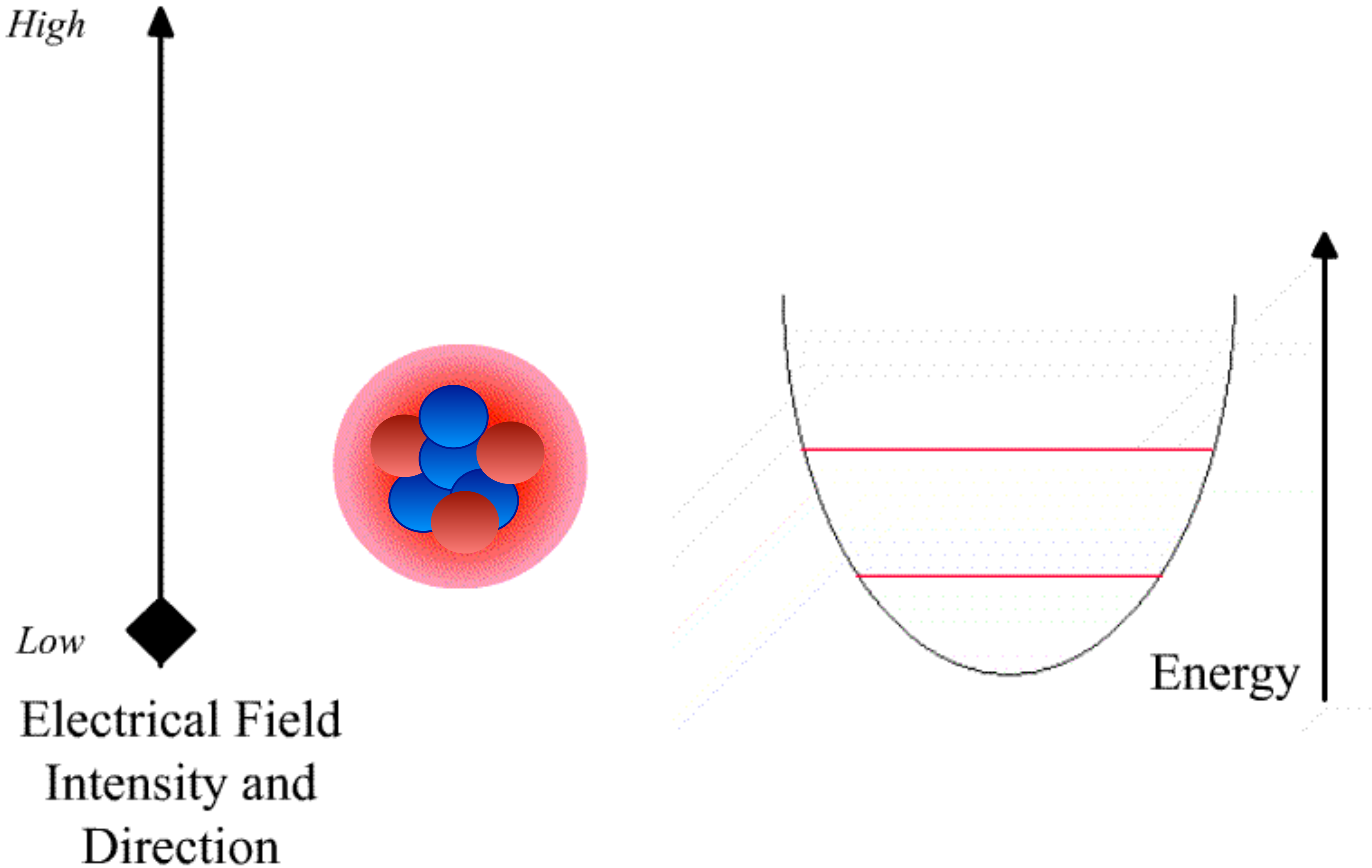

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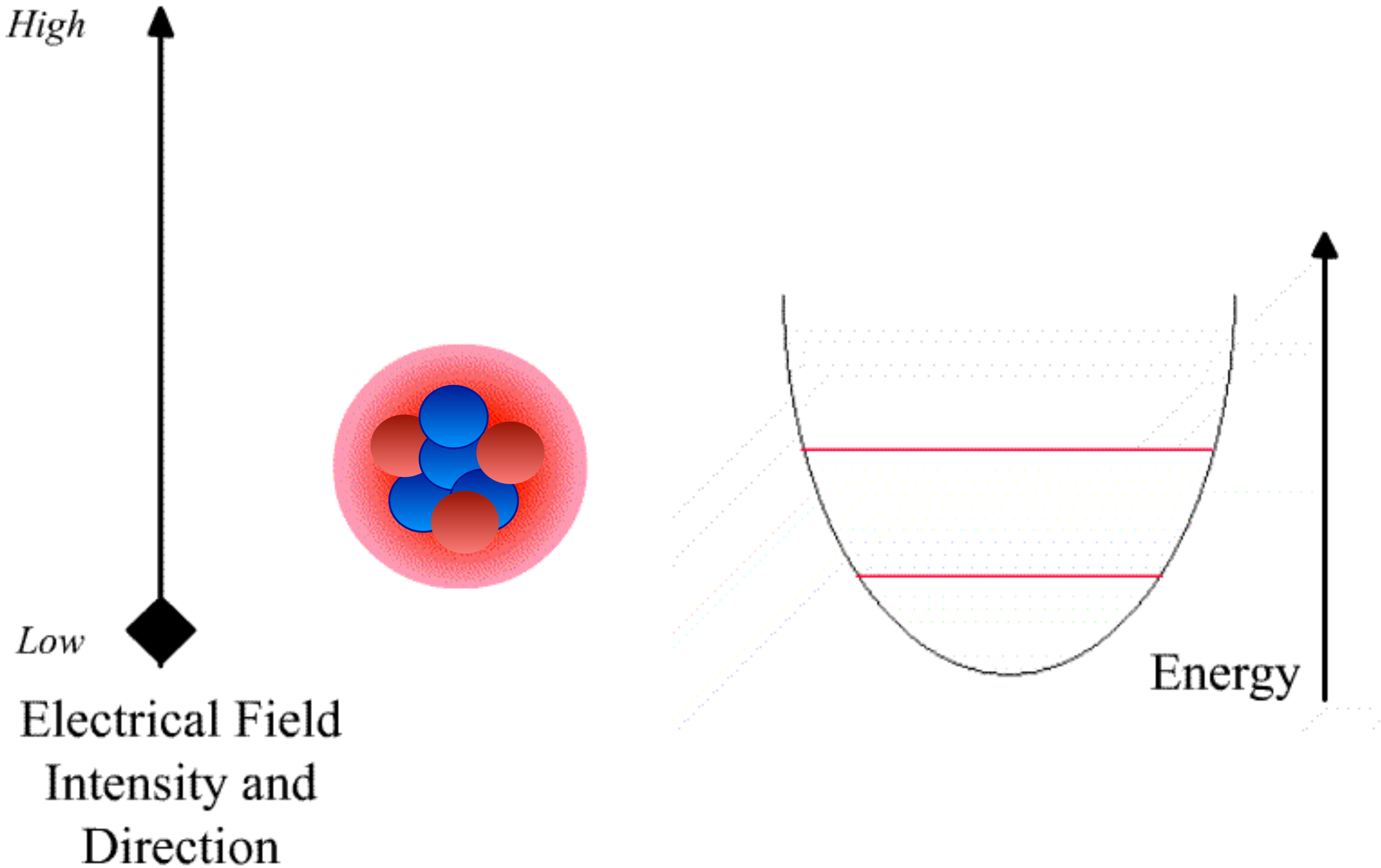

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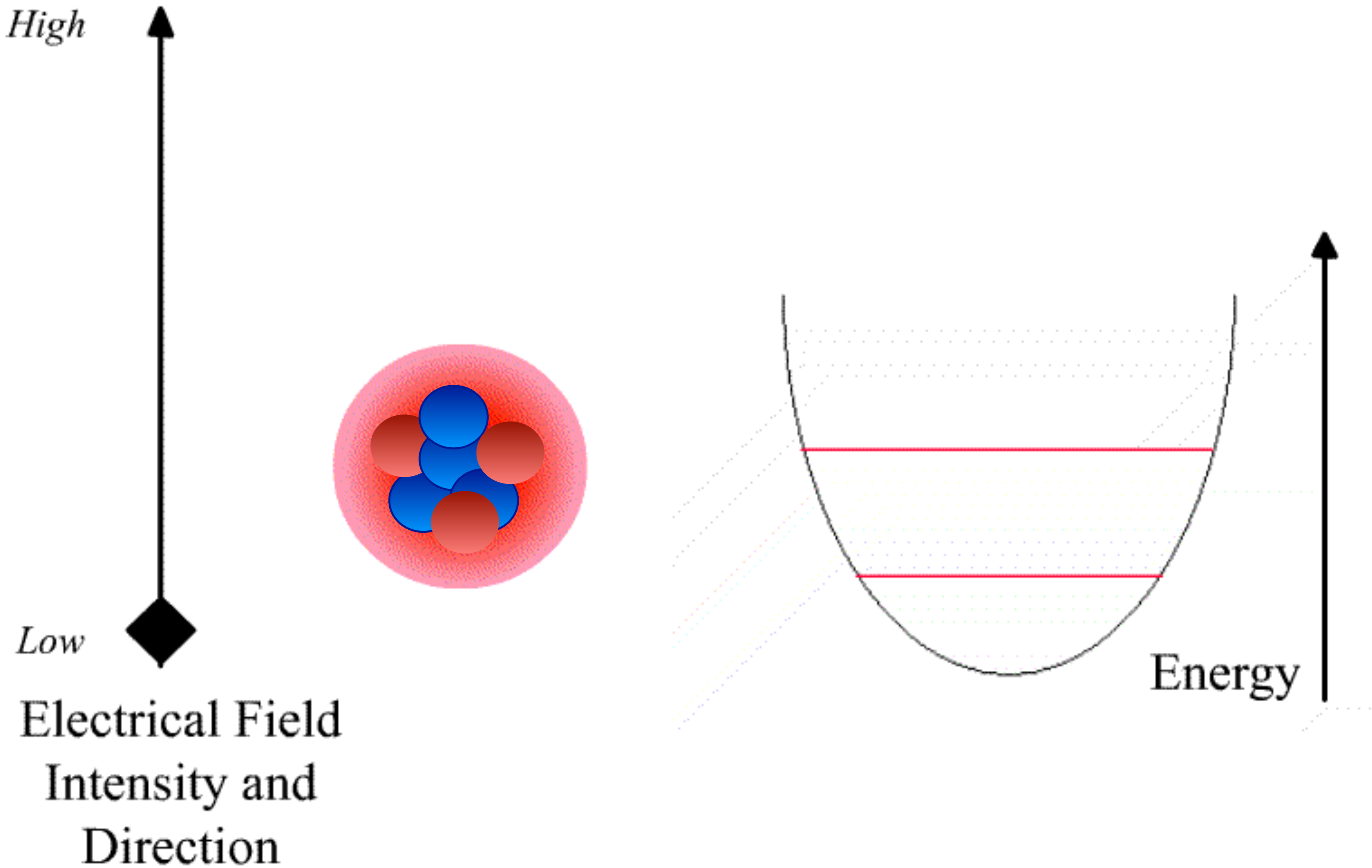

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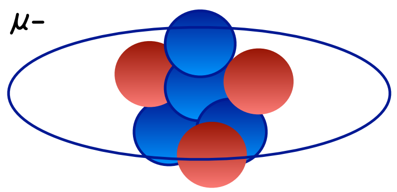
Beyond μH

Strong experimental program at PSI (Switzerland) from the CREMA collaboration to unravel the mystery by studying the **Lamb shift in other muonic atoms than μH:**



$$\Delta E_{2S-2P} = \delta_{\mathrm{QED}} + \frac{m_r^4 (Z\alpha)^4}{12} \langle r_c^2 \rangle + \delta_{\mathrm{TPE}}$$

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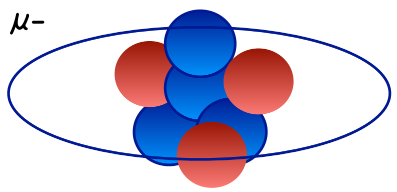

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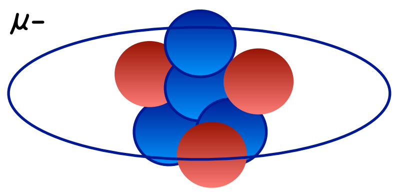

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what you want to extract

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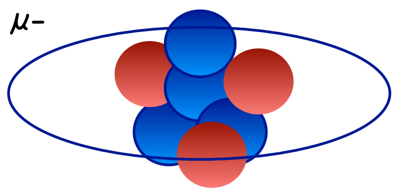

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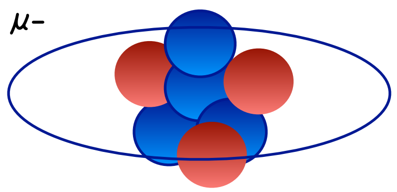

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well known

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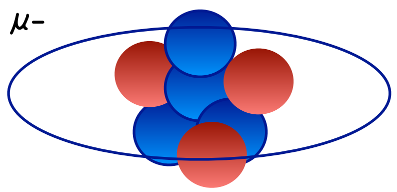

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not well known

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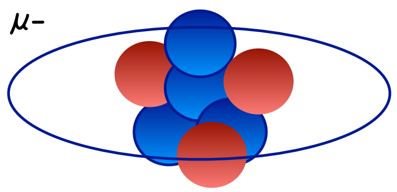

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well known

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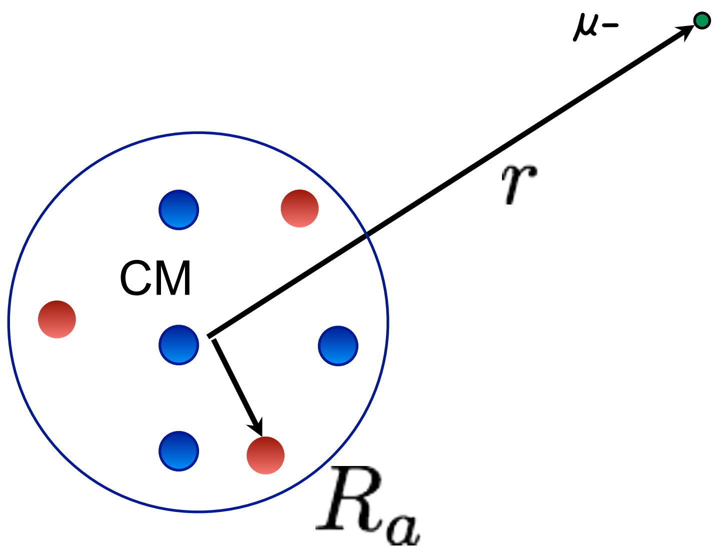
Experimental Campaign

- μD $\longrightarrow$ Science **353**, 669 (2016)
- μ^4He^+ $\longrightarrow$ Nature **589**, 527 (2021)
- μ^3He^+ $\longrightarrow$ measured
- μ^3H $\longrightarrow$ impossible because triton is radioactive
- μ^6Li^{2+} $\longrightarrow$ future plan
- μ^7Li^{2+} $\longrightarrow$ future plan

Theoretical derivation of TPE

$$H = H_N + H_\mu + \Delta V$$

$$H_\mu = \frac{p^2}{2m_r} - \frac{Z\alpha}{r}$$



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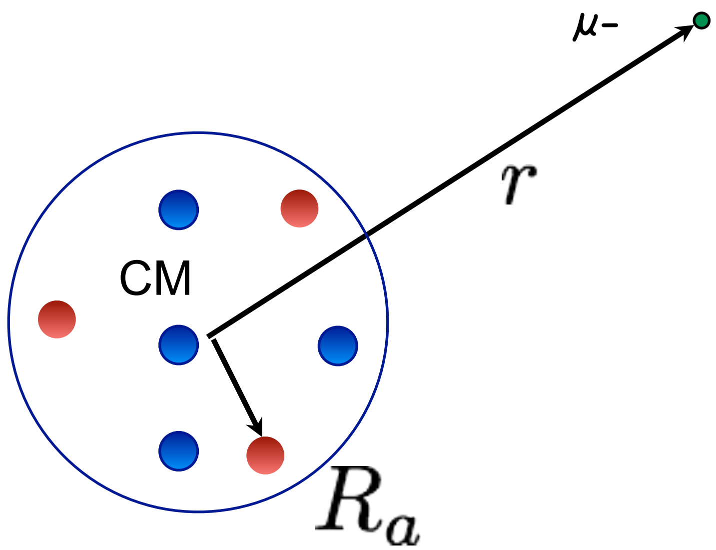

Perturbative potential:
correction to the bulk Coulomb

$$\Delta V = \sum_a^Z \alpha \left(\frac{1}{r} - \frac{1}{|\vec{r} - \vec{R}_a|} \right)$$

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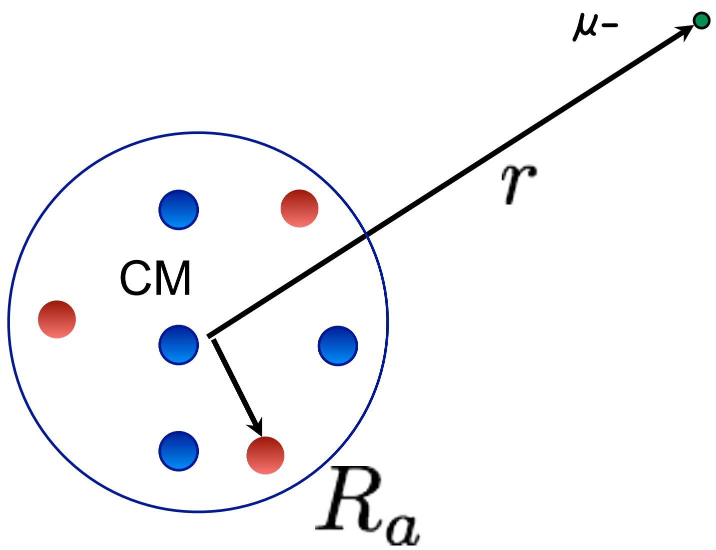

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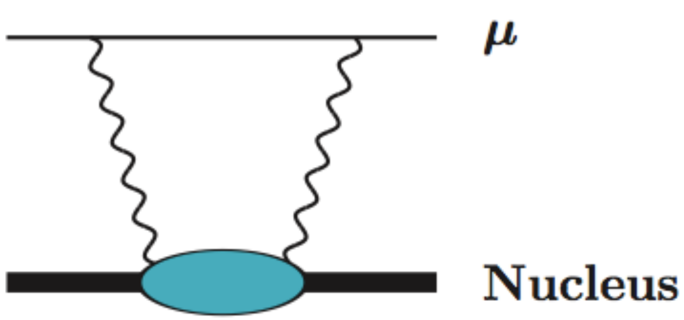
$$\Delta V = \sum_a^Z \alpha \left(\frac{1}{r} - \frac{1}{|\vec{r} - \vec{R}_a|} \right)$$

Using perturbation theory at second order
one obtains the expression for TPE
up to order $(Z\alpha)^5$

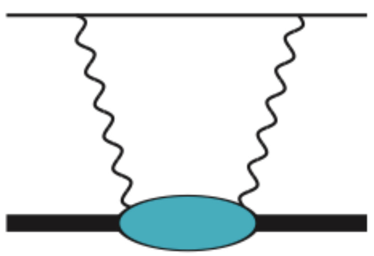


$$P = \langle N_0 \mu | \Delta V G \Delta V | N_0 \mu \rangle$$

Theoretical derivation of TPE

C.Ji et al., JPG: Part. Nucl. **45**, 093002 (2018)

$$\delta_{\rm TPE} = \boxed{\delta^A_{\rm Zem}} + \delta^N_{\rm Zem} + \boxed{\delta^A_{\rm pol}} + \delta^N_{\rm pol}$$



- Non relativistic term

$$\delta^A_{\rm pol} = \sum_{N \neq N_0} \int d^3R\, d^3R' \rho^p_N(\mathbf{R}) W(\mathbf{R}, \mathbf{R}', \omega_N) \rho^p_N(\mathbf{R}')$$

$$\rho^p_N(\mathbf{R}) = \langle N | \frac{1}{Z} \sum_{i=1}^{A} \delta(\mathbf{R} - \mathbf{R_i})\ \hat{e}^p_i\ | N_0 \rangle$$

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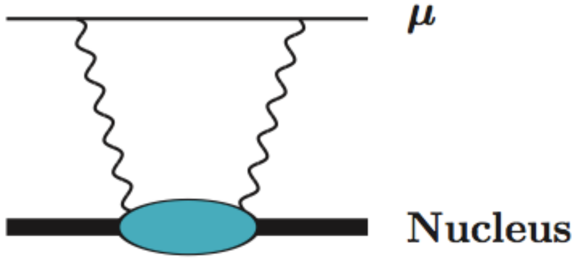

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$$W(\mathbf{R},\mathbf{R}',\omega_N) = \frac{\pi}{6m_r}\ (Z\alpha)^2\ \phi^2(0)\ \left(\frac{2m_r}{\omega_N}\right)^{\frac{3}{2}} \left[\ \eta^2 - \frac{1}{4}\ \eta^3 + \frac{1}{20}\ \eta^4 + \ \dots\ \right]$$

$$\eta = \sqrt{2m_r\omega} |\boldsymbol{R}-\boldsymbol{R}'| \sim \sqrt{\frac{m_r}{m_N}} = 0.33$$

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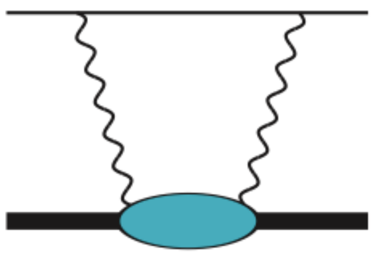

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$$\delta^A_{\rm pol} \to \sum_i C_i \int d\omega f(\omega/m_r) R_{O_i}(\omega)$$

Special sum rules of electromagnetic response functions

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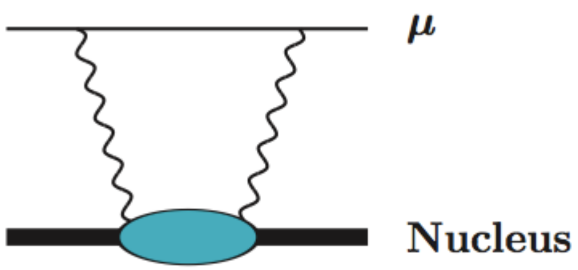


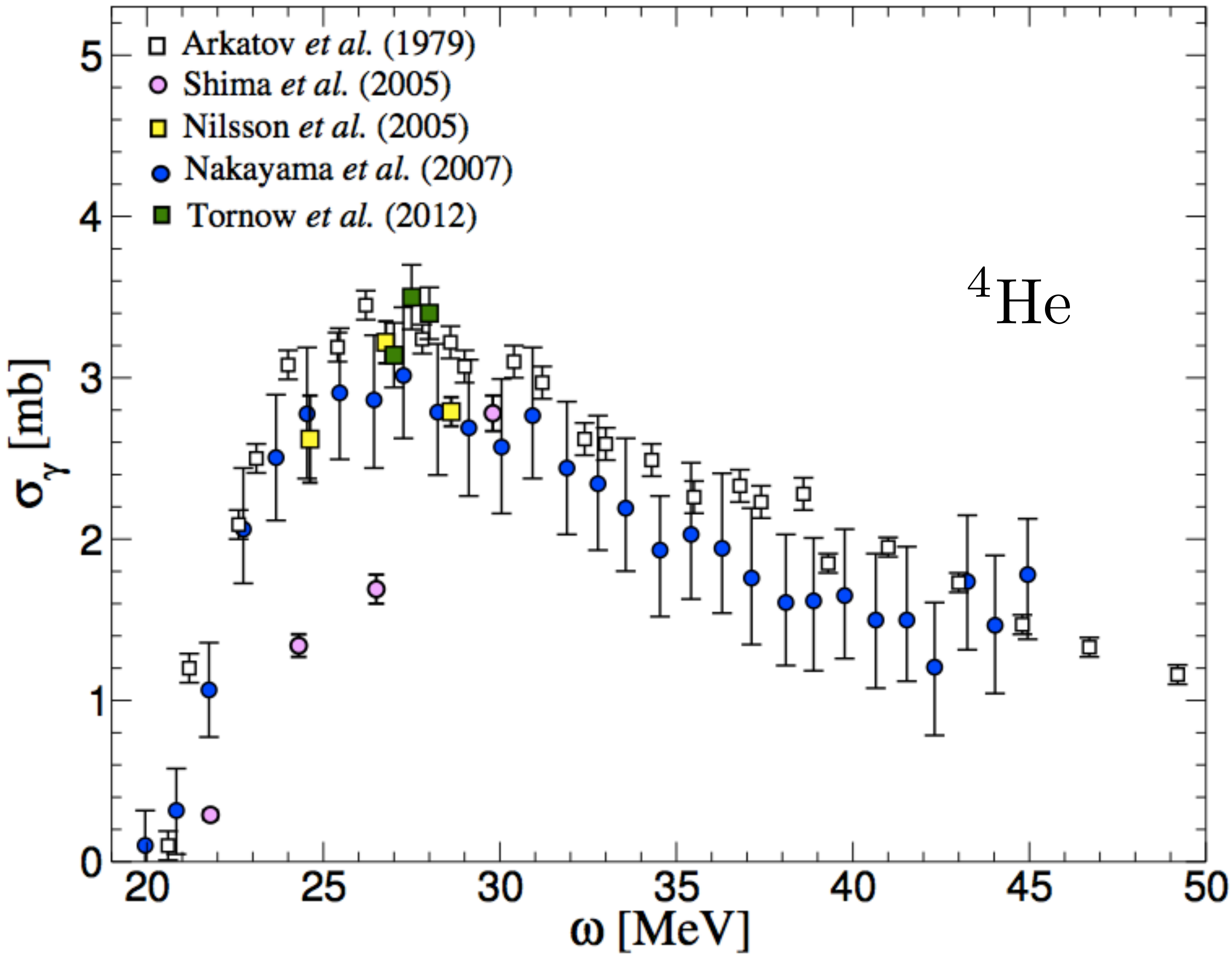
Dominant Term

$$\delta^{A}_{\rm pol} = \boxed{\delta^{(0)}_{D1}} + \delta^{(1)}_{R3} + \cancel{\delta^{(1)}_{Z3}} + \delta^{(2)}_{R^2} + \delta^{(2)}_{Q} + \delta^{(2)}_{D1D3} + \delta^{(0)}_{C}$$
$$+\delta^{(0)}_{L} + \delta^{(0)}_{T} + \delta^{(0)}_{M} + \delta^{(1)}_{R1} + \cancel{\delta^{(1)}_{Z1}} + \delta^{(2)}_{NS}$$

$$\delta^{A}_{\rm Zem} = -\cancel{\delta^{(1)}_{Z3}} - \cancel{\delta^{(1)}_{Z1}}$$

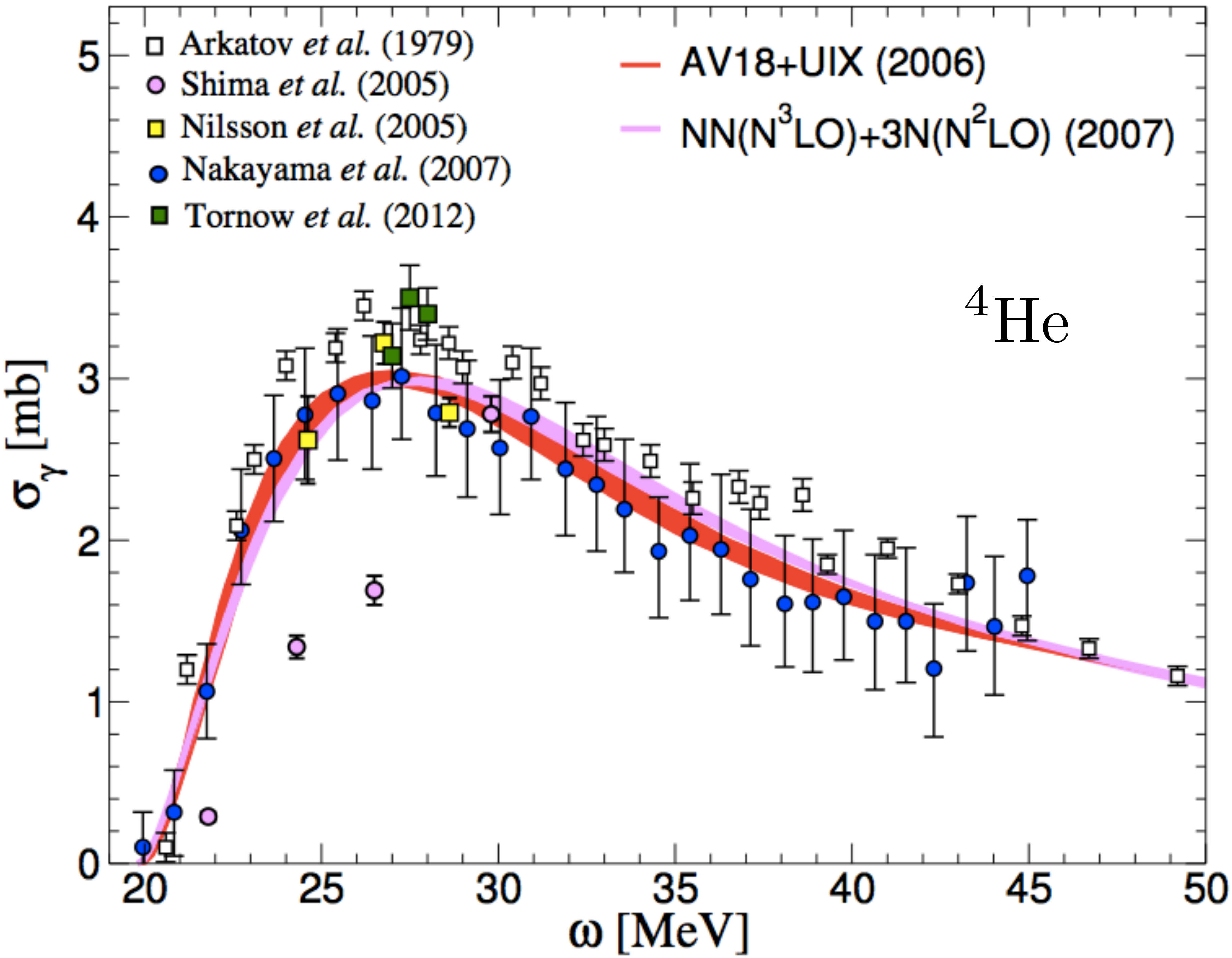
Friar an Payne ('97)

Extraction from experimental data



S.B. and Saori Pastore, Journal of Physics G.: Nucl. Part. Phys. **41**, 123002 (2014)

Theory is more precise



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A matter of precision

The uncertainty of the extracted radius depends on the precision of the TPE

$$\Delta E_{2S-2P} = \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{OPE}} \langle r_c^2 \rangle + \delta_{\mathrm{TPE}}$$

Roughly:	95%	4%	1%

TPE needs to be know precisely, in order to exploit the experimental precision.

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Uncertainties comparison

Atom	Exp uncertainty on ΔE_{2S-2P}	Uncertainty on TPE prior to the discovery of the proton radius puzzle
μ^2H	0.003 meV	0.03 meV*
μ^3He$^+$	0.08 meV	1 meV
μ^4He$^+$	0.06 meV	0.6 meV
$\mu\,^{6,7}$Li^{++}	0.7 meV	4 meV

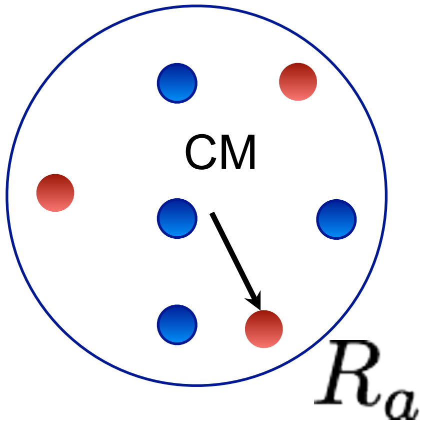
*Leidemann, Rosenfelder ’95 using few-body methods

Ab Initio Nuclear Theory

- Solve the Schrödinger equation for few-nucleons

$$H_N|\psi_i\rangle = E_i|\psi_i\rangle$$

$$H_N = T + V_{NN}(\Lambda) + V_{3N}(\Lambda) + \ldots$$



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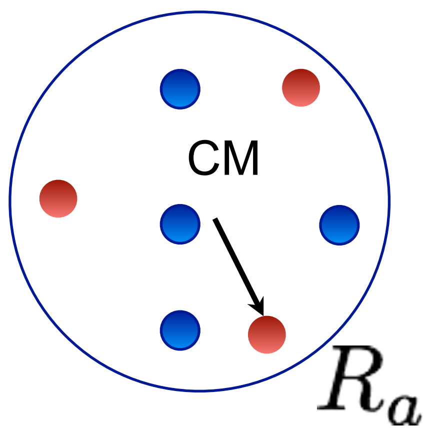

Hyper-spherical harmonics expansions for A=3,4,6,7 ⇒ Barnea, Leidemann, Orlandini PRC **61** (2000) 054001

For A=2 we use an harmonic oscillator expansion

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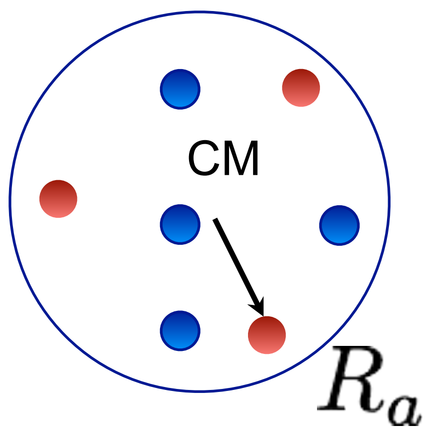

Hyper-spherical harmonics expansions for A=3,4,6,7 $\Rightarrow$ Barnea, Leidemann, Orlandini PRC **61** (2000) 054001

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  Nevo-Dinur, Ji, SB, Barnea, Phys.Rev.C 89 (2014) 6, 064317

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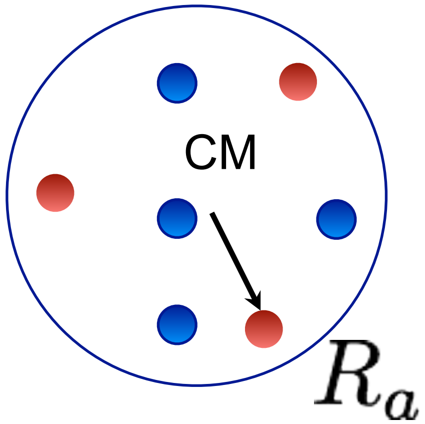

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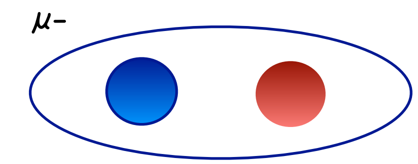
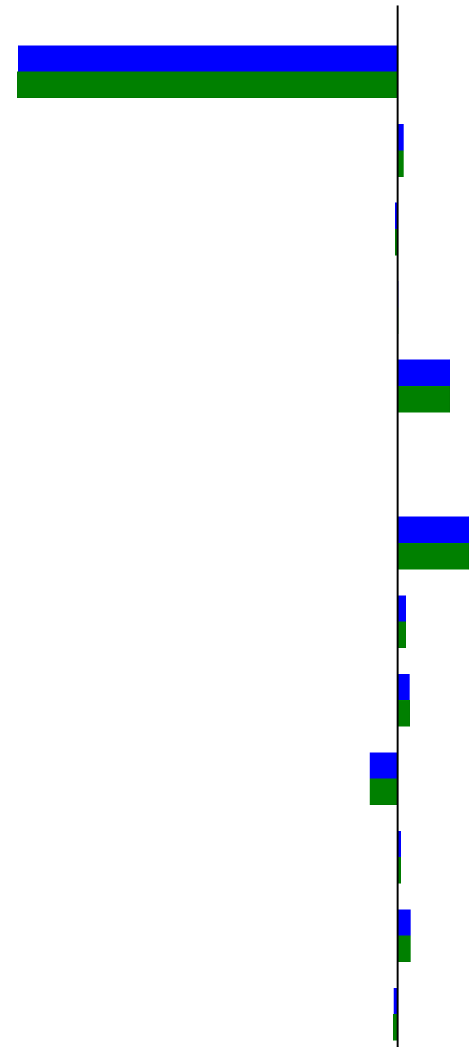
- We will use traditional potentials (AV18/UIX) or interactions derived from chiral effective filed theory (at various orders)

Muonic Deuterium





	AV18	χEFT
$\delta^{(0)}_{D1}$	-1.907	-1.912
$\delta^{(0)}_{L}$	0.029	0.029
$\delta^{(0)}_{T}$	-0.012	-0.012
$\delta^{(0)}_{M}$	0.003	0.003
$\delta^{(0)}_{C}$	0.262	0.262
$\delta^{(1)}_{R3}$	-	-
$\delta^{(1)}_{Z3}$	0.357	0.359
$\delta^{(2)}_{R^2}$	0.042	0.041
$\delta^{(2)}_{Q}$	0.061	0.061
$\delta^{(2)}_{D1D3}$	-0.139	-0.139
$\delta^{(1)}_{R1}$	0.017	0.017
$\delta^{(1)}_{Z1}$	0.064	0.064
$\delta^{(2)}_{NS}$	-0.020	-0.021

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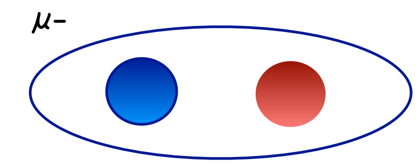
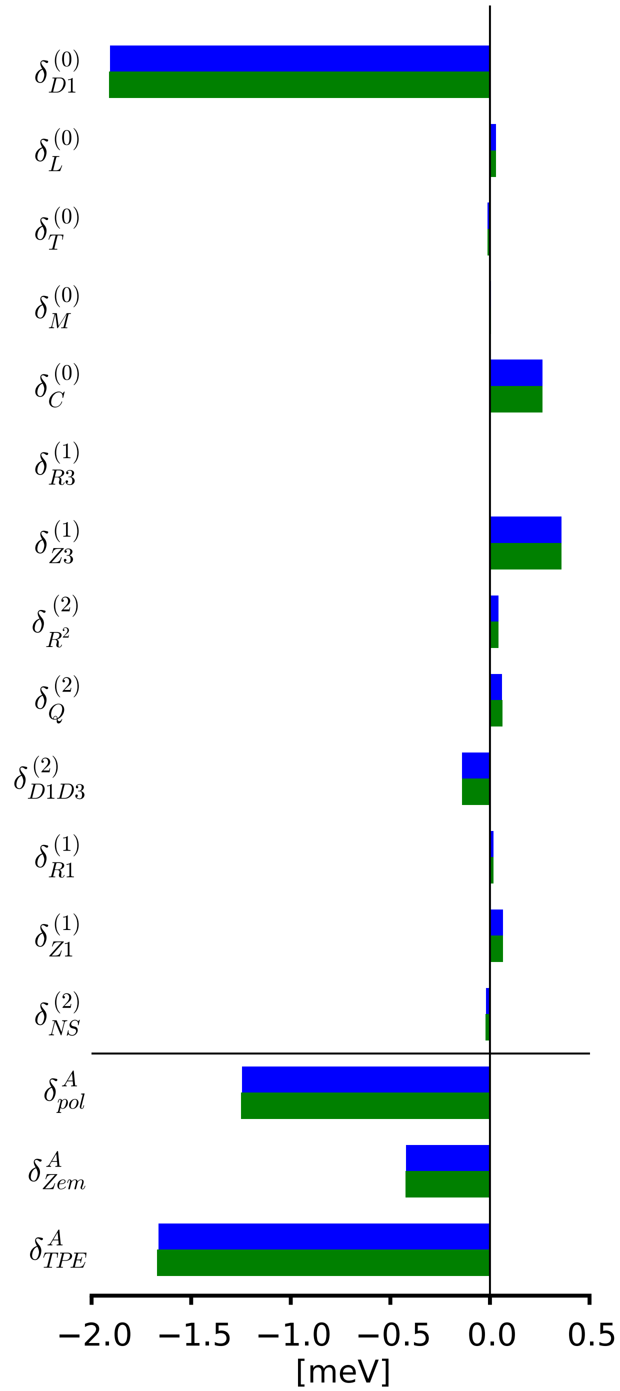


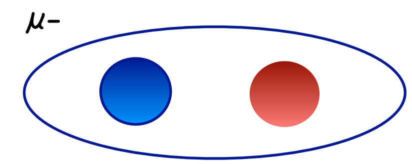


	AV18	χEFT
$\delta_{D1}^{(0)}$	-1.907	-1.912
$\delta_{L}^{(0)}$	0.029	0.029
$\delta_{T}^{(0)}$	-0.012	-0.012
$\delta_{M}^{(0)}$	0.003	0.003
$\delta_{C}^{(0)}$	0.262	0.262
$\delta_{R3}^{(1)}$	-	-
$\delta_{Z3}^{(1)}$	0.357	0.359
$\delta_{R^2}^{(2)}$	0.042	0.041
$\delta_{Q}^{(2)}$	0.061	0.061
$\delta_{D1D3}^{(2)}$	-0.139	-0.139
$\delta_{R1}^{(1)}$	0.017	0.017
$\delta_{Z1}^{(1)}$	0.064	0.064
$\delta_{NS}^{(2)}$	-0.020	-0.021
δ_{pol}^{A}	-1.243	-1.248
δ_{Zem}^{A}	-0.421	-0.423
δ_{TPE}^{A}	-1.664	-1.671

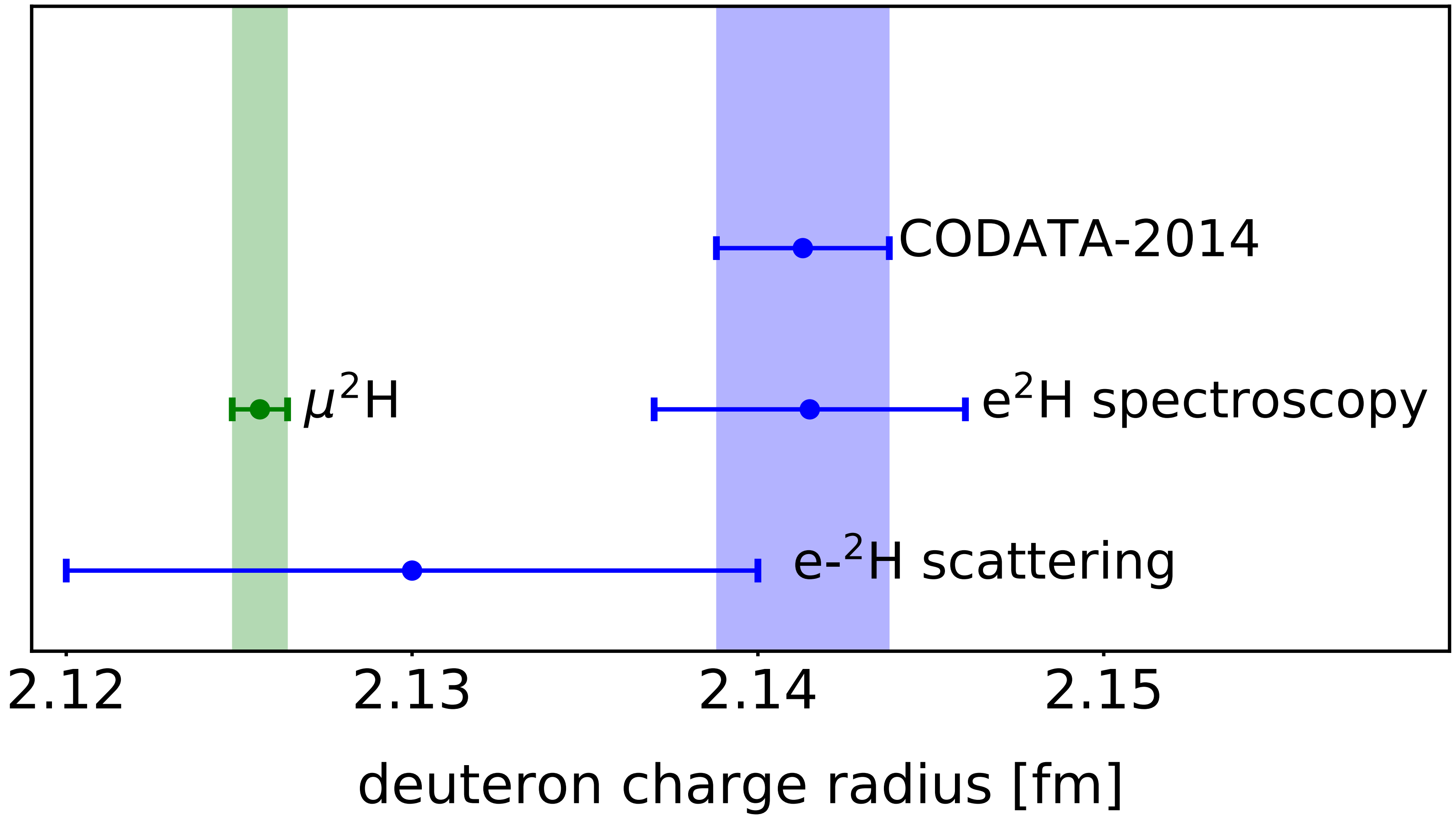
J. Hernandez et al, Phys. Lett. B **736**, 344 (2014)

AV18 in agreement with Pachucki (2011)+ Pachucki, Wienczek (2015)

Deuteron charge radius



Pohl et al, Science **353**, 669 (2016)

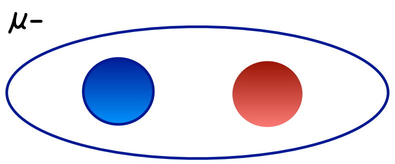


$$\Delta E_{2S-2P} = \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{OPE}}\langle r_c^2 \rangle + \delta_{\mathrm{TPE}}$$

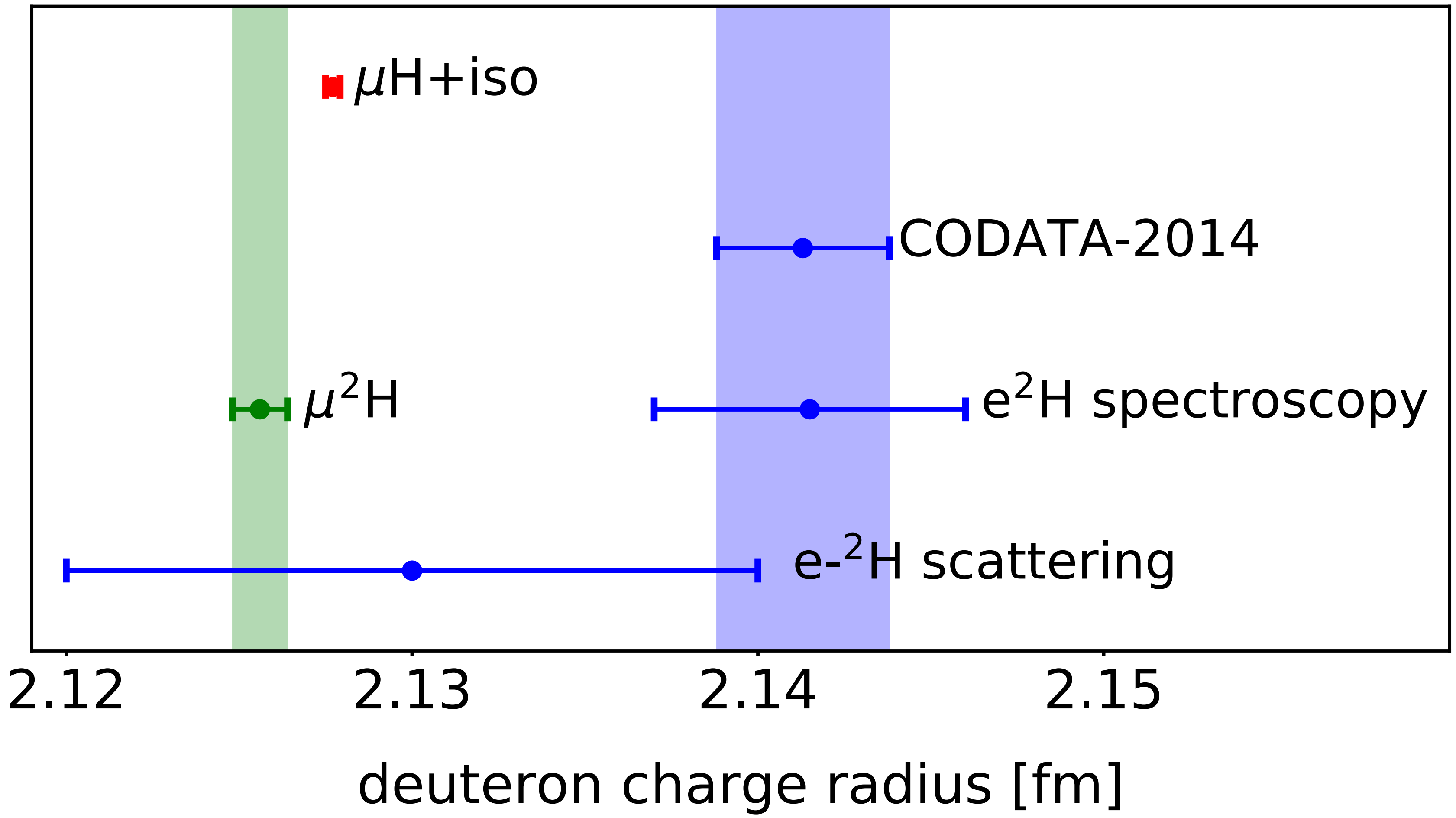
Hernandez et al., PLB **736**, 334 (2014)

Pachucki (2011)+ Pachucki, Wienczek (2015)

Deuteron charge radius



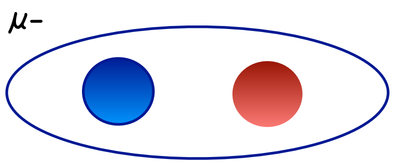
Pohl et al, Science **353**, 669 (2016)



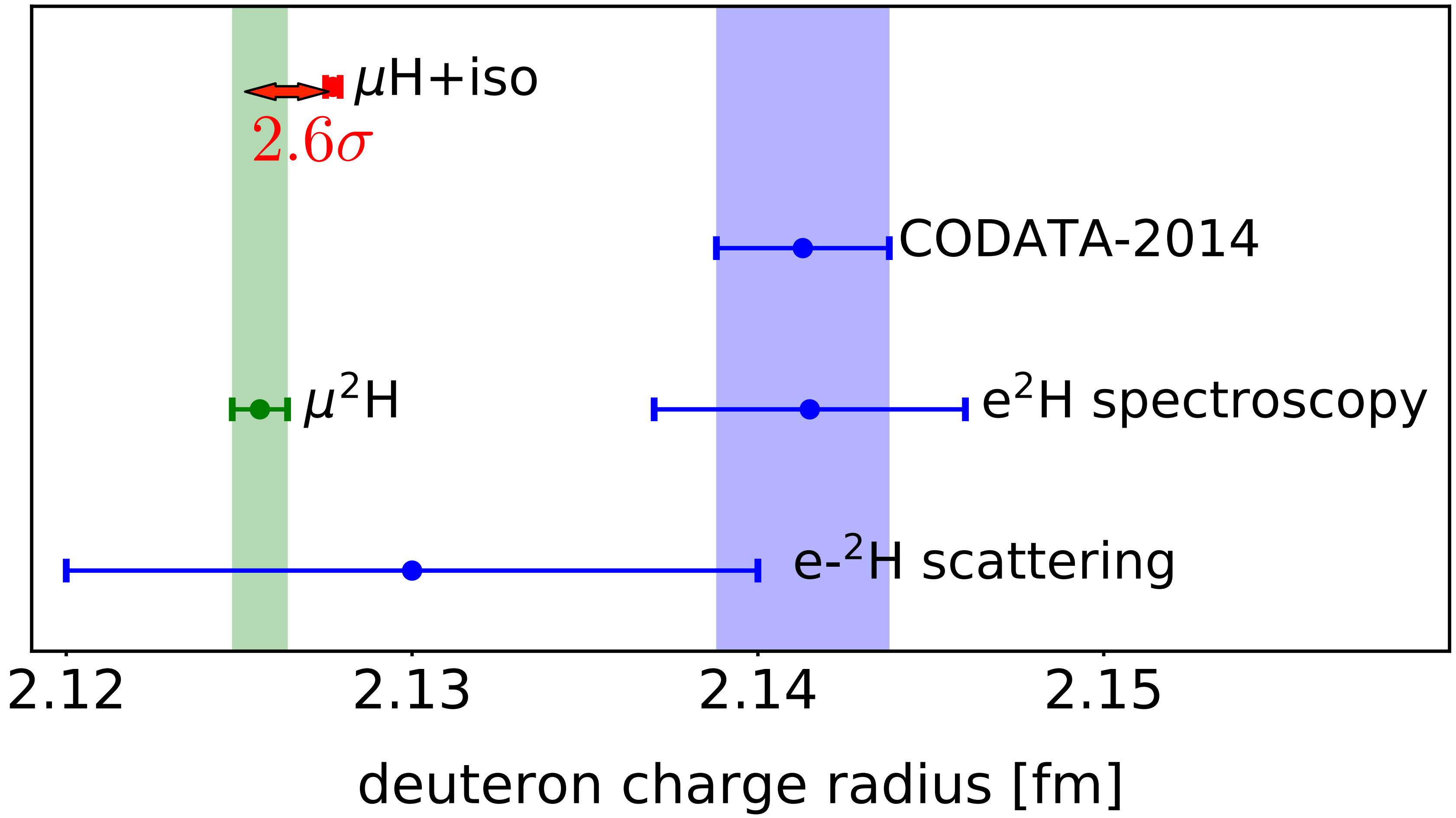
$$\Delta E_{2S-2P} = \delta_{\rm QED} + \mathcal{A}_{\rm OPE}\langle r_c^2\rangle + \delta_{\rm TPE}$$

μH+iso: r_p from μH and deuterium isotopic shift $r^2_d - r^2_p$: Parthey et al., PRL **104** 233001 (2010)

Deuteron charge radius



Pohl et al, Science **353**, 669 (2016)

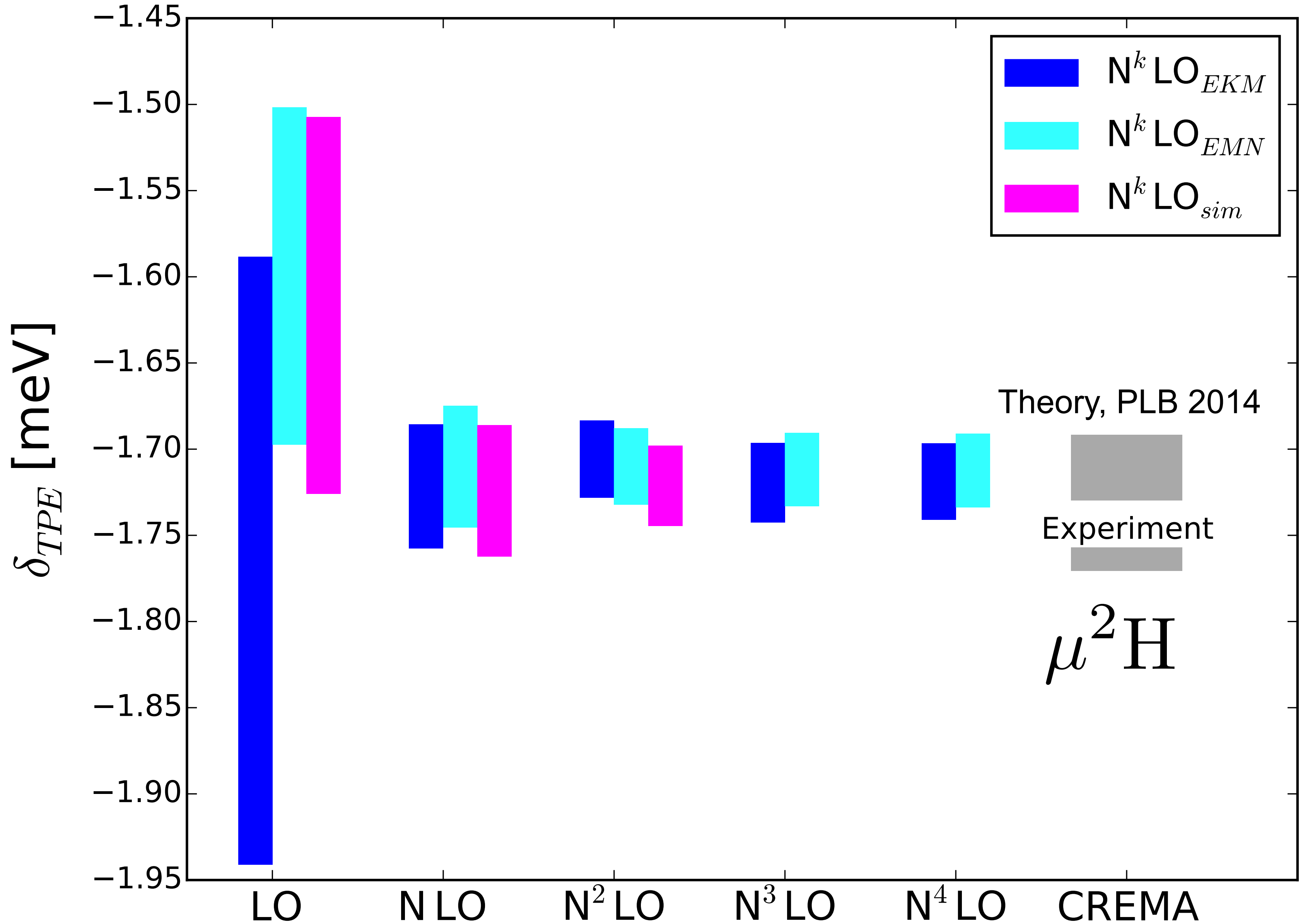


$$\Delta E_{2S-2P} = \delta_{\mathrm{QED}} + \mathcal{A}_{\mathrm{OPE}} \langle r_c^2 \rangle + \delta_{\mathrm{TPE}}$$

μH+iso: r_p from μH and deuterium isotopic shift r^2_d -r^2_p: Parthey et al., PRL **104** 233001 (2010)

Order-by-order chiral expansion

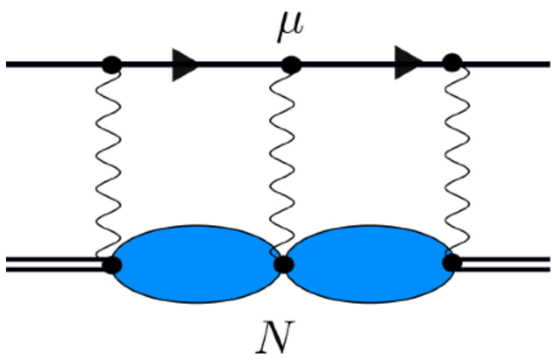
J. Hernandez et al, Phys. Lett. B **778**, 377 (2018)



Only sightly mitigate the “small” proton radius puzzle (2.6 to 2 σ)

Higher order corrections in α

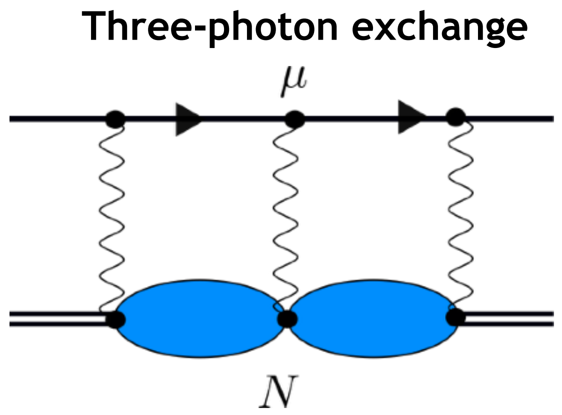
Three-photon exchange



Pachucki et al., Phys. Rev. A **97** 062511 (2018)

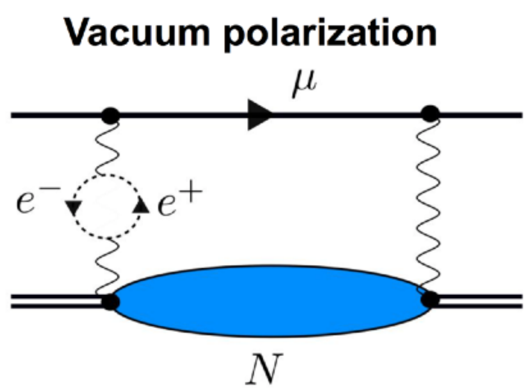
$(Z\alpha)^6$ correction

Higher order corrections in α



Pachucki et al., Phys. Rev. A **97** 062511 (2018)

$(Z\alpha)^6$ correction



One the many α^6 corrections, supposedly the largest

Kalinowski, Phys. Rev. A **99** 030501 (2019)

$\delta_{\mathrm{TPE}} = -1.750^{+14}_{-16}$ meV Theory

$\delta_{\mathrm{TPE}} = -1.7638(68)$ meV Exp

Consistent within 1σ

solves the deuteron-radius puzzle

Options for estimation of TPE

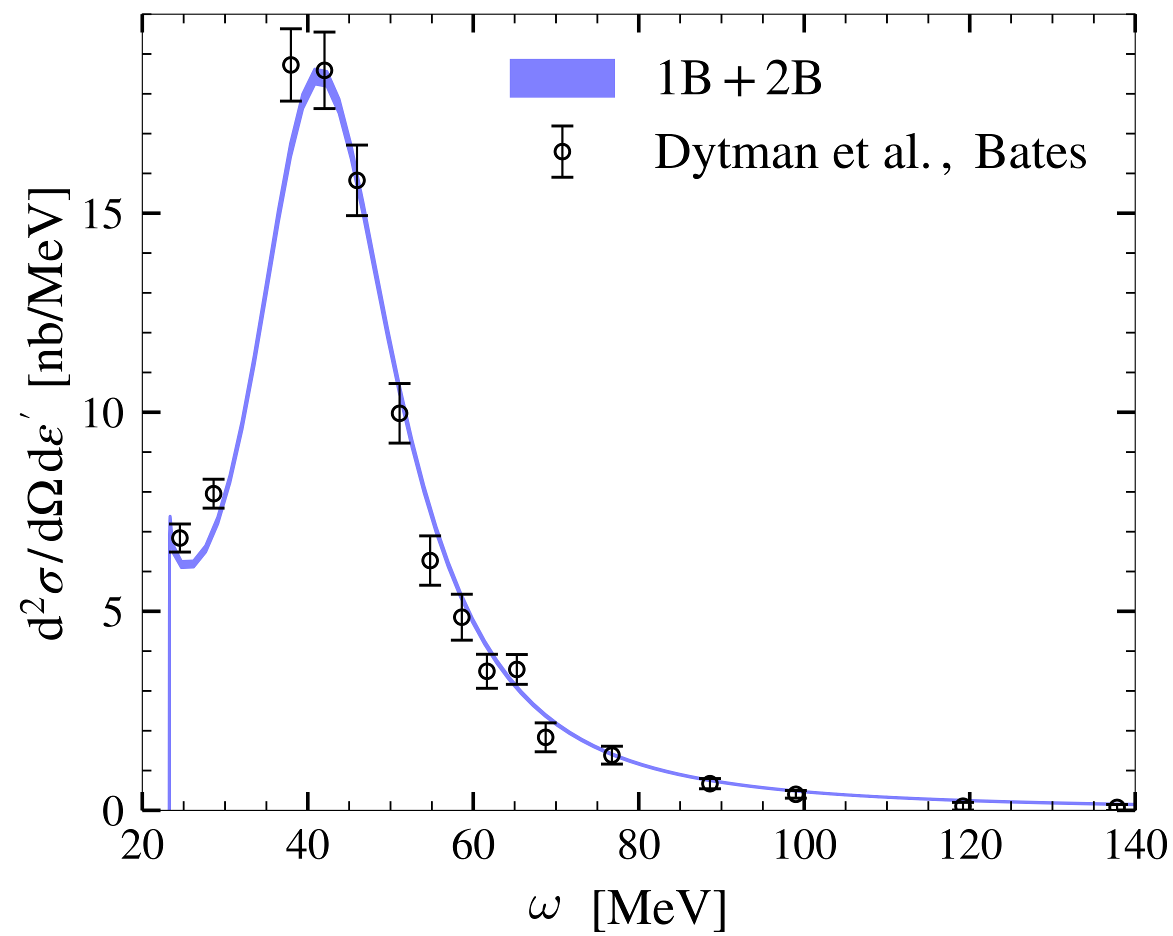
- From experimental data
- From theoretical calculations
- Dispersion relations (see F. Hagelstein's talk)

Hybrid Approach

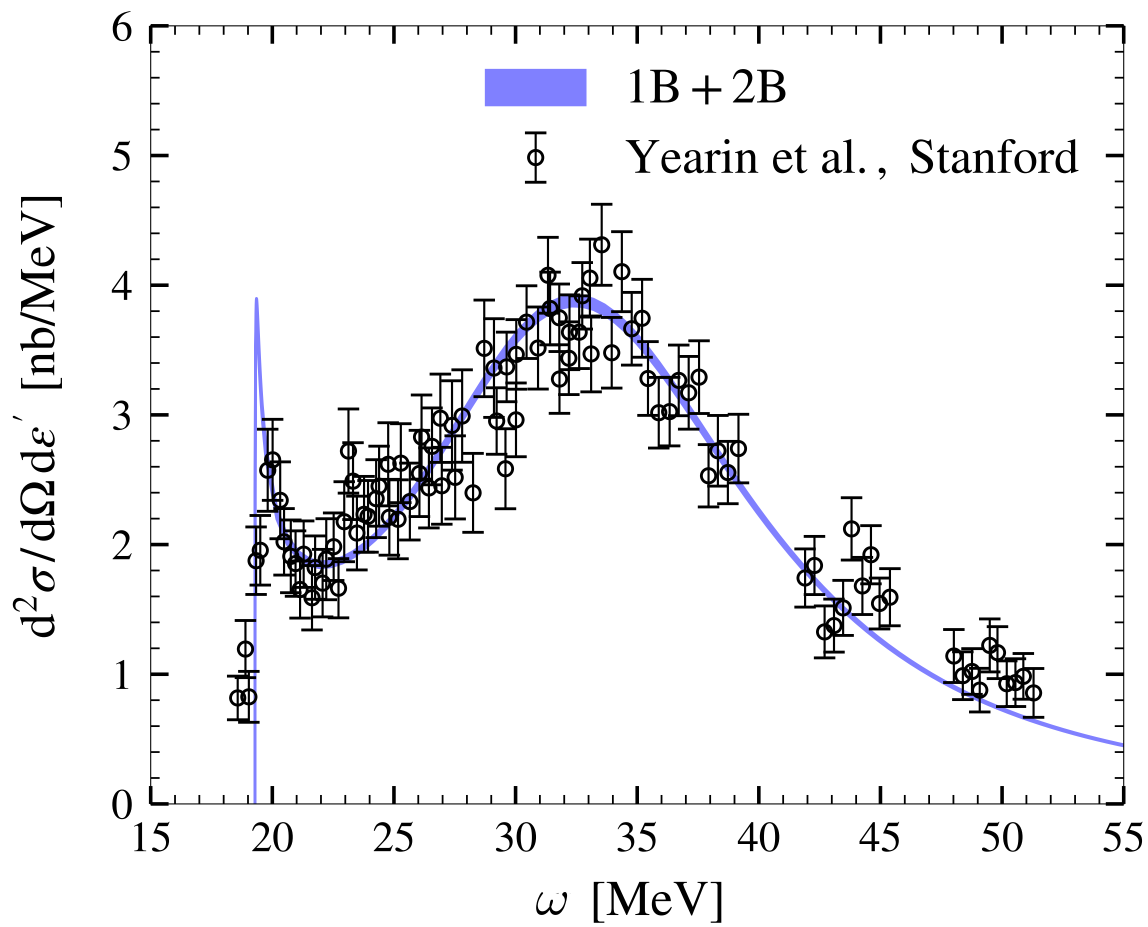
B. Acharya

Use chiral EFT to calculate response functions and feed dispersion relations

B. Acharya, V. Lensky, M.Gorchtein, SB, M.Vanderhaghen, PRC 103 (2021) 2, 024001



$\varepsilon = 292.8\ \mathrm{MeV}, \theta = 60^\circ$



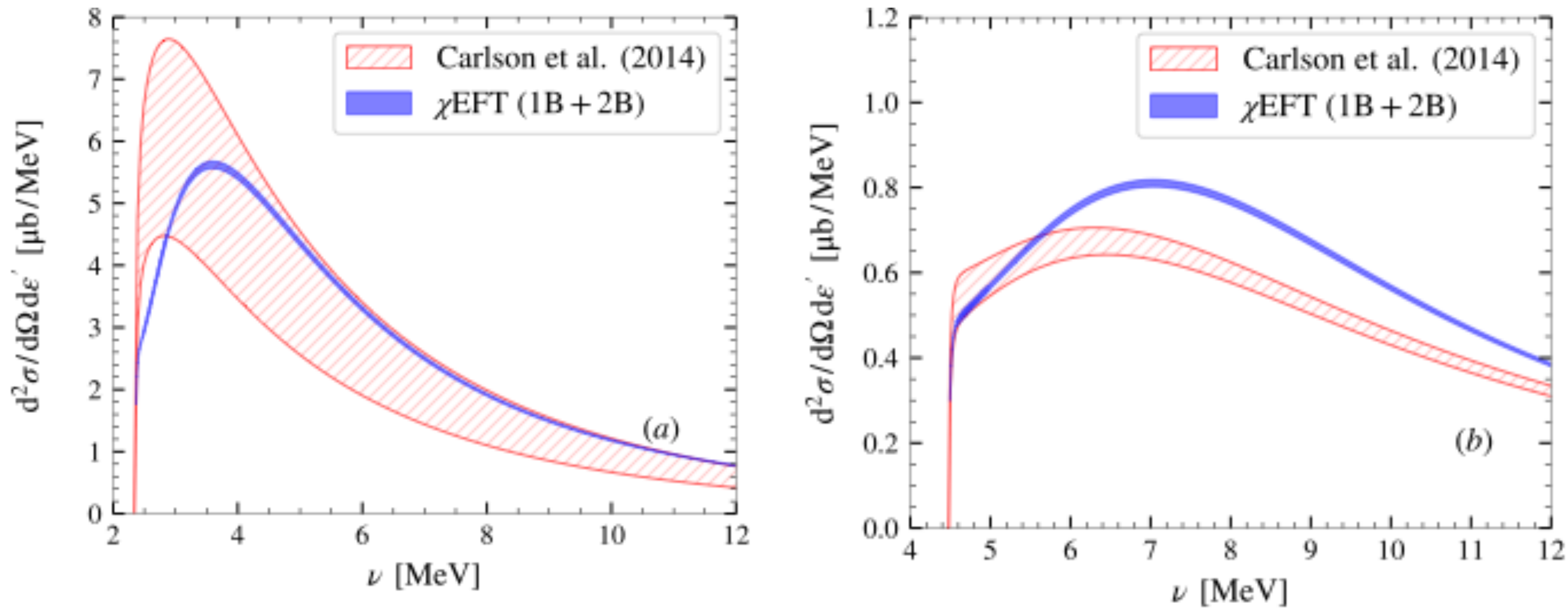
$\varepsilon = 146.9\ \mathrm{MeV}, \theta = 135^\circ$

Hybrid Approach

B. Acharya

Use chiral EFT to calculate response functions and feed dispersion relations

B. Acharya, V. Lensky, M.Gorchtein, SB, M.Vanderhaghen, PRC 103 (2021) 2, 024001

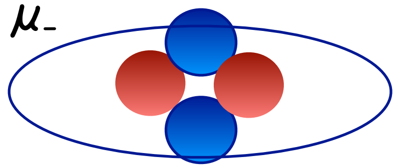


ω ω

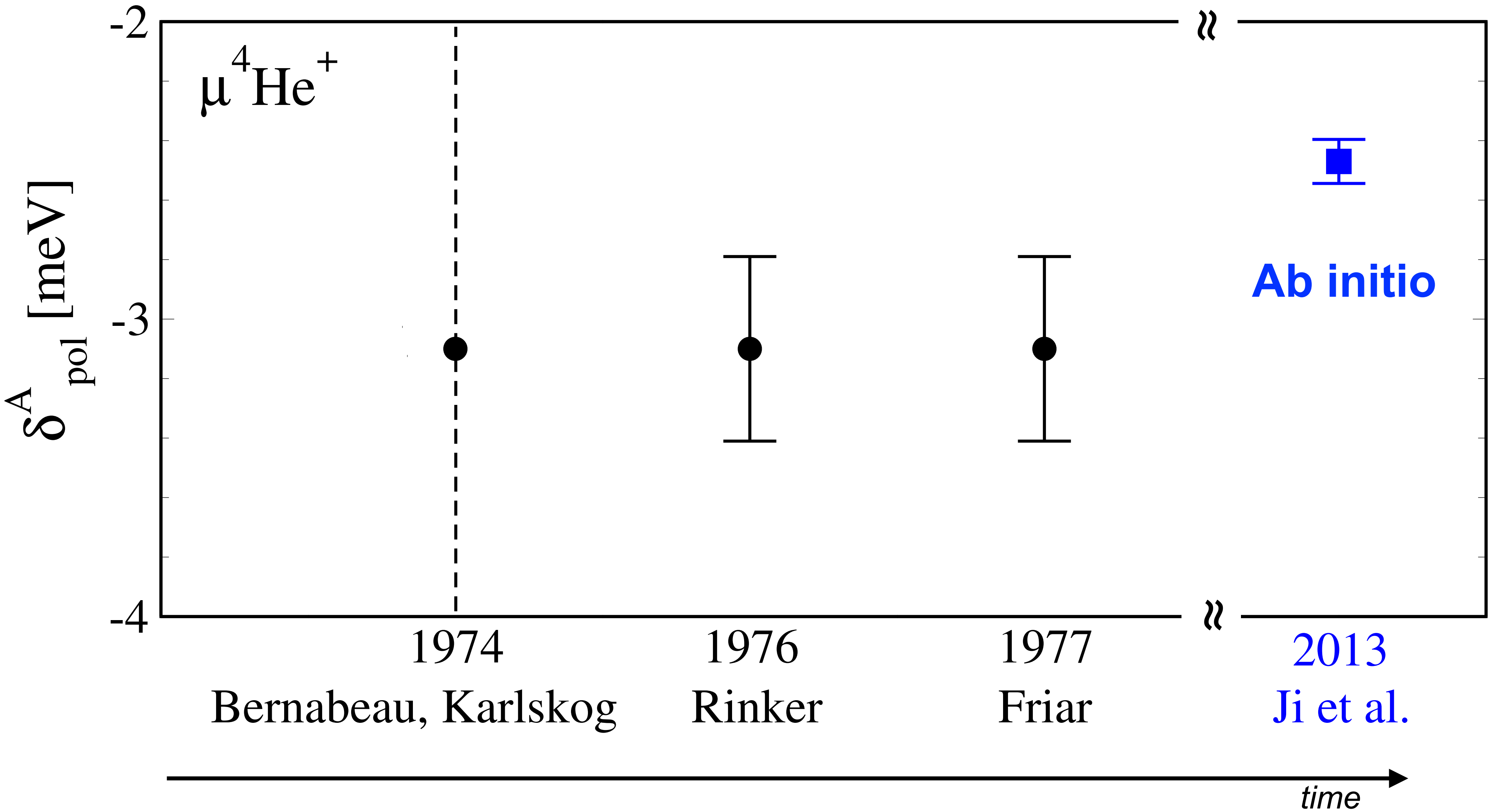
muonic deuterium	δ_{TPE} [meV]
This work — 1B+2B	-1.695(13)
Carlson, Gorchtein, Vanderhaeghen, Phys. Rev. A 89, 022504 (2014)	-2.011(740)

Precision is dramatically improved

Muonic $^4He^+$



C.Ji et al., JPG: Part. Nucl. **45,** 093002 (2018)



- **Reduction of uncertainties in polarizability**
- **Results used in experiment, no puzzle Nature 589, 527 (2021)**

Reduction of Uncertainties

Atom	Exp uncertainty on ΔE_{2S-2P}	Uncertainty on TPE prior to the discovery of the puzzle	Uncertainty on TPE: ab initio
μ^2H	0.003 meV	0.03 meV	0.02 meV
μ^3He^+	0.08 meV	1 meV	0.3 meV
μ^4He^+	0.06 meV	0.6 meV	0.4 meV
$\mu\,^{6,7}Li^{++}$	0.7 meV	4 meV	Lower bound 0.4 meV*

* Li Muli, Poggialini, SB, SciPost Phys.Proc. 3 (2020) 028

Uncertainties quantifications

Sources
- Numerical
- Nuclear model
- Isospin symmetry breaking
- Nucleon-size
- Truncation of multiples
- η-expansion
- expansion in $Z\alpha$

C.Ji et al., JPG: Part. Nucl. **45**, 093002 (2018)

In %

	μ^2H			μ^3H			μ^3He$^+$			μ^4He$^+$		
	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$
Numerical	0.0	0.0	0.0	0.1	0.0	0.1	0.4	0.1	0.1	0.4	0.3	0.4
Nuclear-model	0.3	0.5	0.4	1.3	2.4	1.7	0.7	1.8	1.5	3.9	4.6	4.4
ISB	0.2	0.2	0.2	0.7	0.2	0.5	1.8	0.2	0.5	2.2	0.5	0.5
Nucleon-size	0.3	0.8	0.0	0.6	0.9	0.2	1.2	1.3	0.9	2.7	2.0	1.2
Relativistic	0.0	—	0.0	0.1	—	0.1	0.4	—	0.1	0.1	—	0.0
Coulomb	0.4	—	0.3	0.5	—	0.3	3.0	—	0.9	0.4	—	0.1
η-expansion	0.4	—	0.3	1.3	—	0.9	1.1	—	0.3	0.8	—	0.2
Higher $Z\alpha$	0.7	—	0.5	0.7	—	0.5	1.5	—	0.4	1.5	—	0.4
Total	1.0	0.9	0.8	2.3	2.2	2.0	4.2	2.2	2.1	5.5	5.1	4.6

Uncertainties quantifications

Sources

- Numerical
- Nuclear model
- Isospin symmetry breaking
- Nucleon-size
- Truncation of multiples
- η-expansion
- expansion in $Z\alpha$

C.Ji et al., JPG: Part. Nucl. **45,** 093002 (2018)

In %

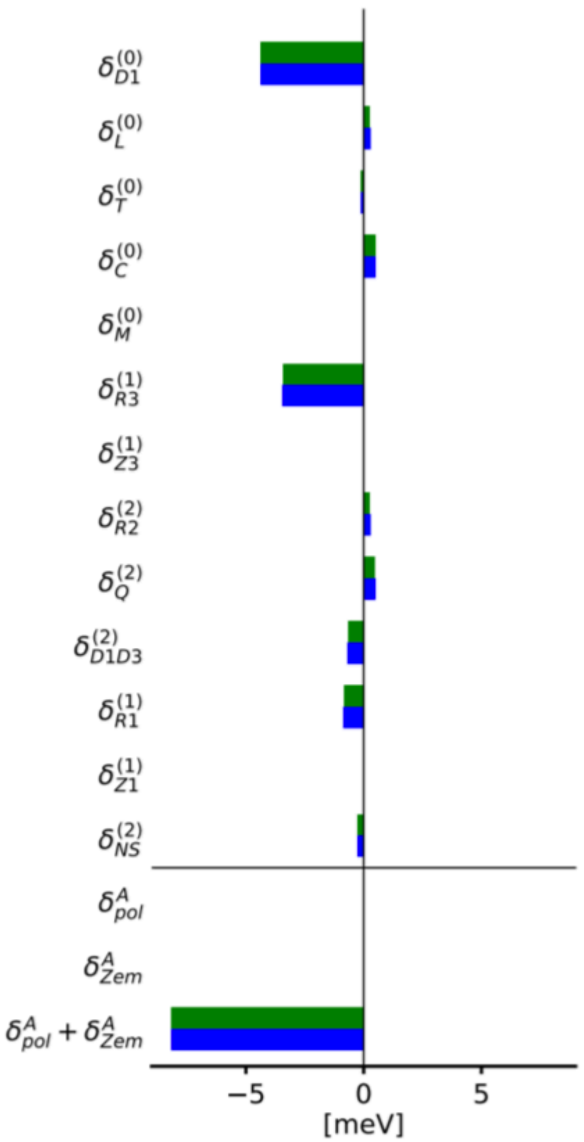
	μ^2H			μ^3H			μ^3He$^+$			μ^4He$^+$		
	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$	$\delta^A_{\rm pol}$	$\delta^A_{\rm Zem}$	$\delta^A_{\rm TPE}$
Numerical	0.0	0.0	0.0	0.1	0.0	0.1	0.4	0.1	0.1	0.4	0.3	0.4
Nuclear-model	0.3	0.5	0.4	1.3	2.4	1.7	0.7	1.8	1.5	3.9	4.6	4.4
ISB	0.2	0.2	0.2	0.7	0.2	0.5	1.8	0.2	0.5	2.2	0.5	0.5
Nucleon-size	0.3	0.8	0.0	0.6	0.9	0.2	1.2	1.3	0.9	2.7	2.0	1.2
Relativistic	0.0	—	0.0	0.1	—	0.1	0.4	—	0.1	0.1	—	0.0
Coulomb	0.4	—	0.3	0.5	—	0.3	3.0	—	0.9	0.4	—	0.1
η-expansion	0.4	—	0.3	1.3	—	0.9	1.1	—	0.3	0.8	—	0.2
Higher $Z\alpha$	0.7	—	0.5	0.7	—	0.5	1.5	—	0.4	1.5	—	0.4
Total	1.0	0.9	0.8	2.3	2.2	2.0	4.2	2.2	2.1	5.5	5.1	4.6

μ^4He$^+$ order-by-order chiral expansion

Using local chiral forces (Gezerlis et al.)

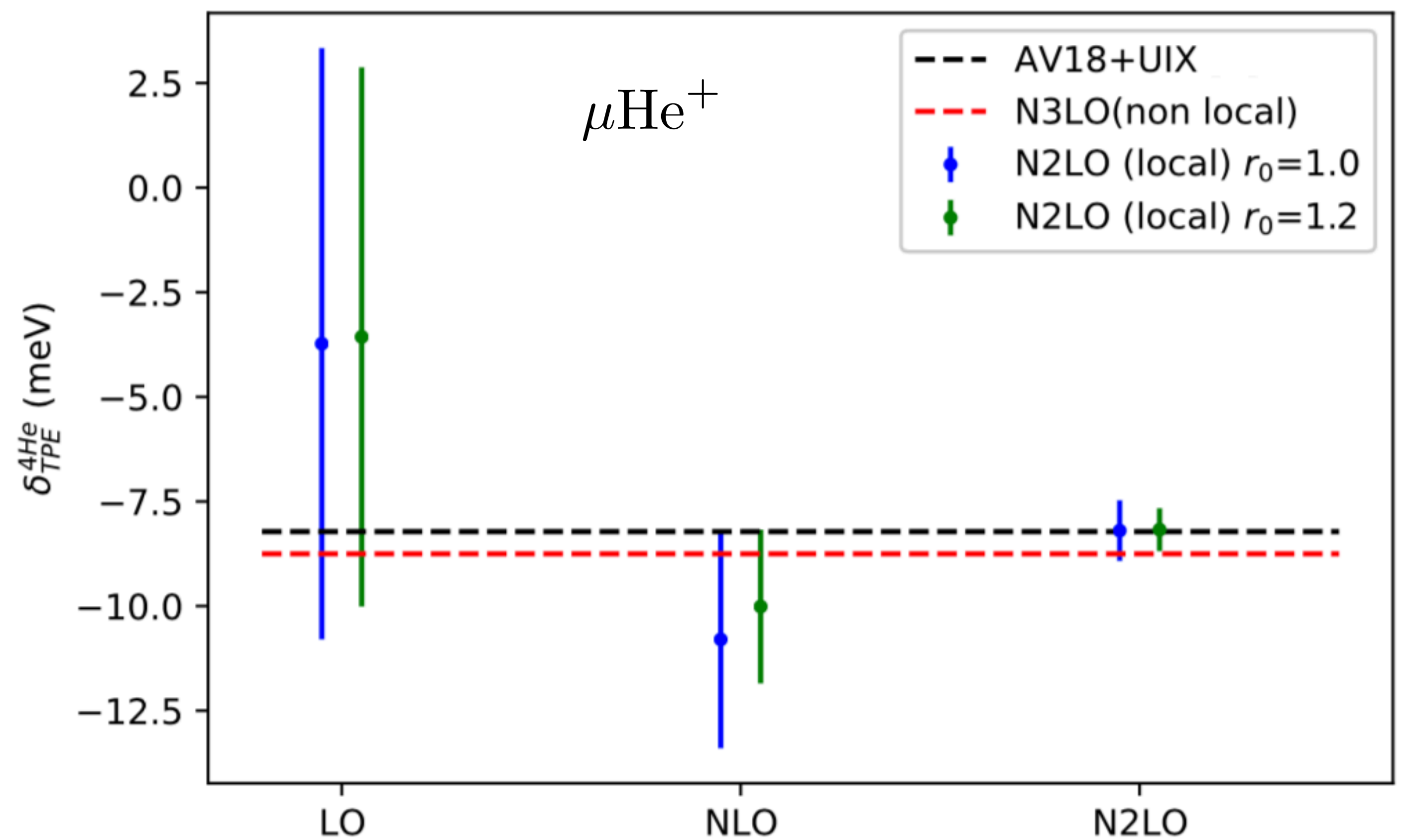
Simone Li Muli

SS Li Muli, SB, N Barnea, Front.in Phys. 9 (2021) 214→Benchmark of HH with GFMC



	$r_0 = 1.2$	$r_0 = 1.0$
$\delta^{(0)}_{D1}$	-4.386	-4.373
$\delta^{(0)}_{L}$	0.272	0.287
$\delta^{(0)}_{T}$	-0.124	-0.125
$\delta^{(0)}_{C}$	0.517	0.514
$\delta^{(0)}_{M}$	0.011	0.011
$\delta^{(1)}_{R3}$	-3.422	-3.477
$\delta^{(1)}_{Z3}$	-	-
$\delta^{(2)}_{R2}$	0.267	0.285
$\delta^{(2)}_{Q}$	0.484	0.505
$\delta^{(2)}_{D1D3}$	-0.668	-0.69
$\delta^{(1)}_{R1}$	-0.846	-0.856
$\delta^{(1)}_{Z1}$	-	-
$\delta^{(2)}_{NS}$	-0.272	-0.277
δ^{A}_{pol}	-	-
δ^{A}_{Zem}	-	-
$\delta^{A}_{pol} + \delta^{A}_{Zem}$	-8.174	-8.196

SS Li Muli, SB, N Barnea, in preparation (2022)

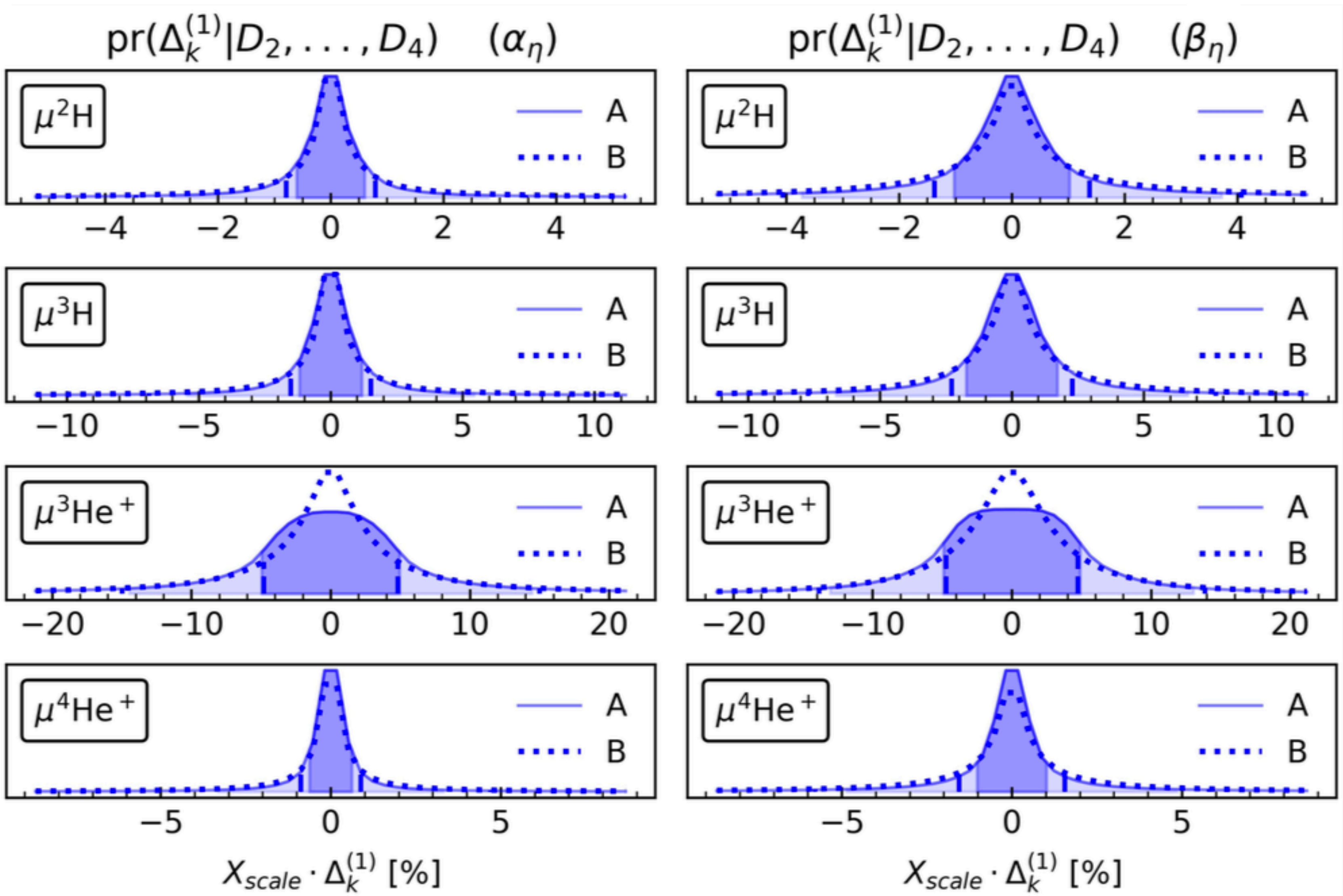


Uncertainties become slightly larger
Need to go to N3LO to improve uncertainties

Bayesian estimates of the η-expansion uncertainty

SS Li Muli, B Acharya, OJ Hernandez, SB, arXiv:2203.10792

Starting from reasonable choices of the Bayesian priors for the η distribution, we derive the Bayesian posteriors for the values of η, and estimate the uncertainties due to truncation of the η–expansion.



	μ^2H	μ^3H	μ^3He$^+$	μ^4He$^+$
[1]	0.4%	1.3%	1.1%	0.8%
This work	0.8%	1.5%	4.8%	0.9%

[1] C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)

Summary and Outlook

- Ab initio calculations have allowed to substantially reduce uncertainties in TPE
- These calculations are needed to support any spectroscopic measurement with muonic atoms
- Future perspectives: hyperfine splitting; three-photon exchange

Thanks to my collaborators

N.Barnea, **B. Acharya, M.Gorchteyn, V. Lensky, O.J. Hernandez**, **C.Ji**, **S.Li Muli, N.Nevo Dinur**, M.Vanderhaghen,...

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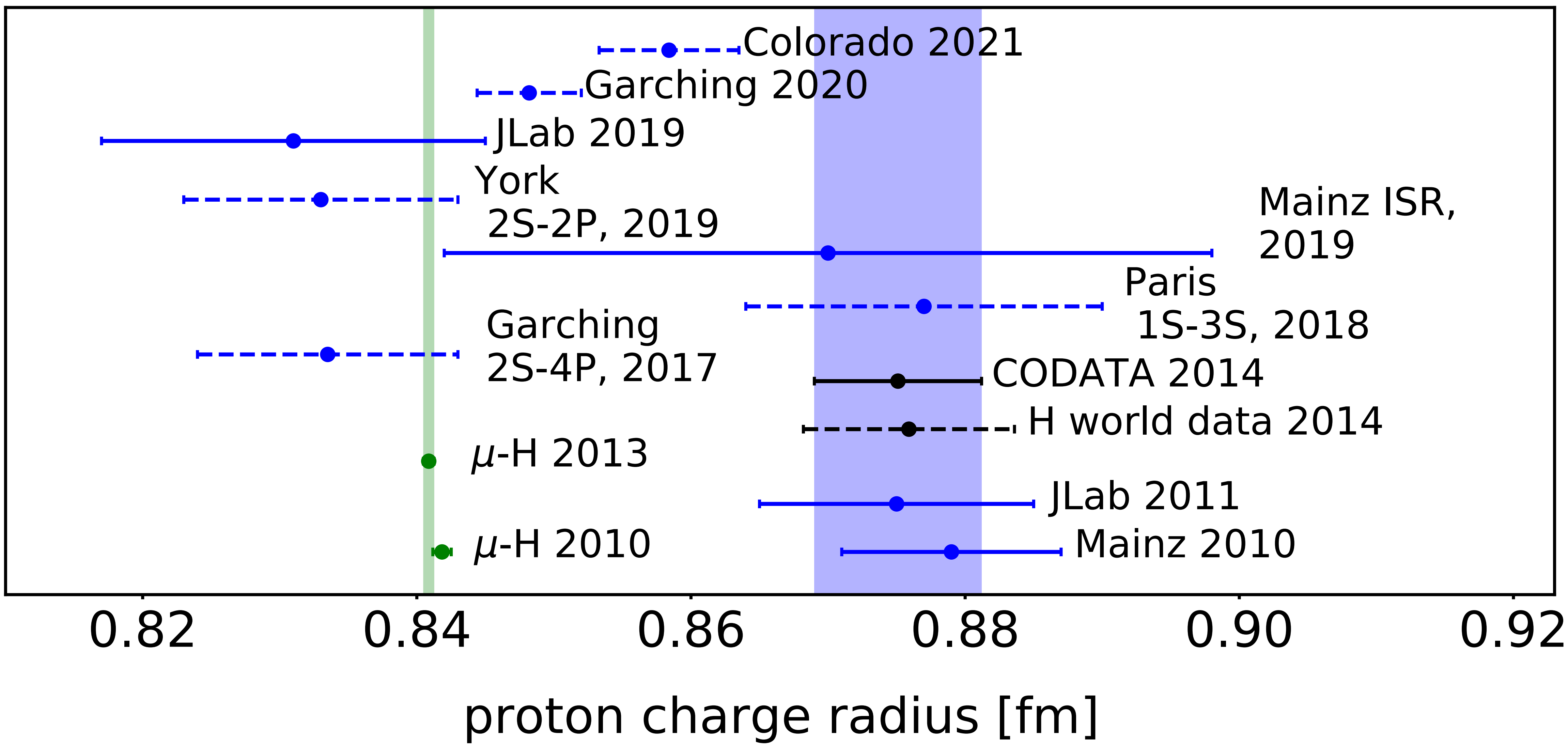
Thank you for your attention!

Backup Slides

Proton Radius Puzzle

— Today's situation —

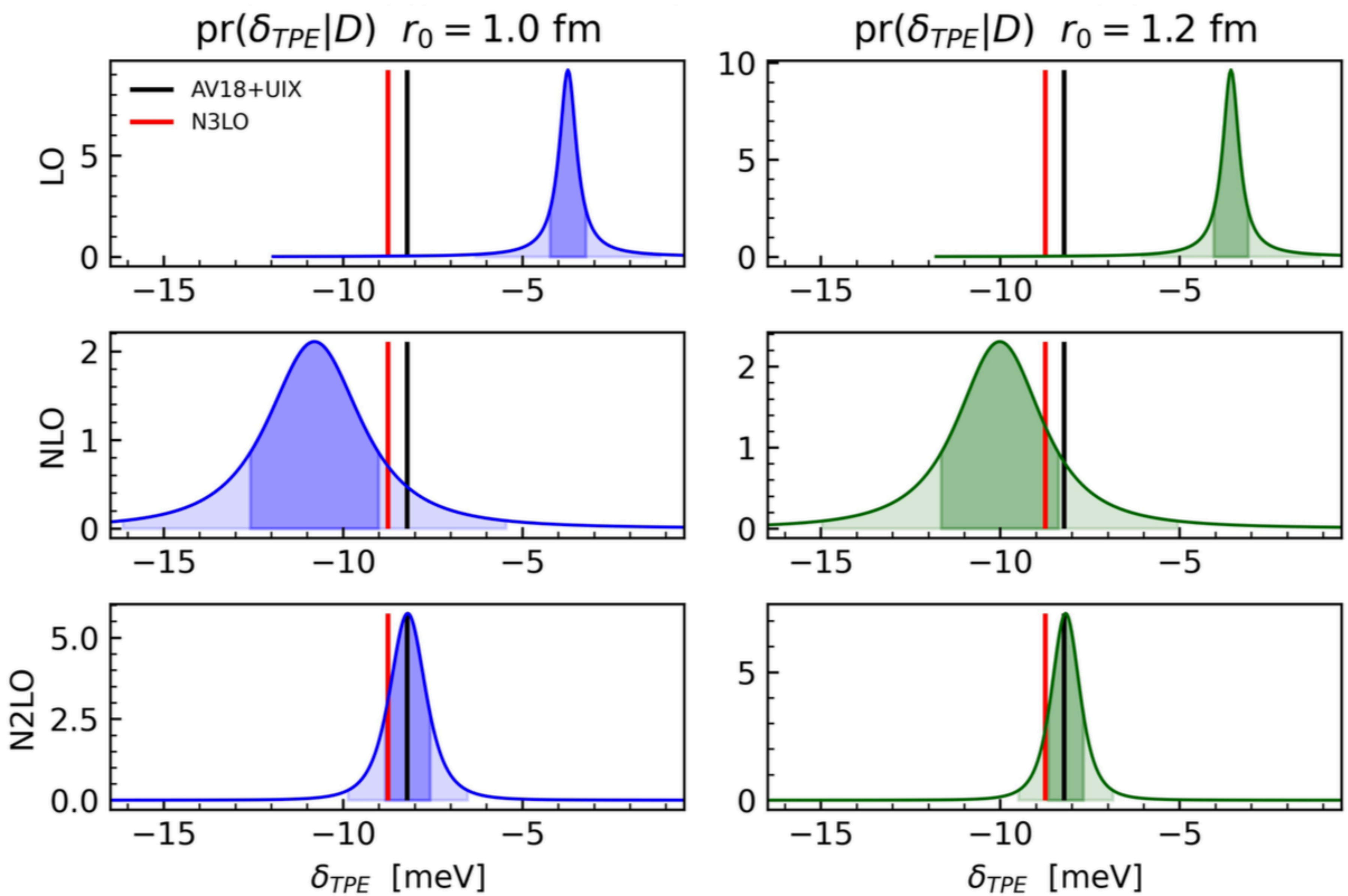


Possibly explained by unaccounted systematic uncertainties…

μ^4He^+ order-by-order chiral expansion

—Bayesian Analysis—

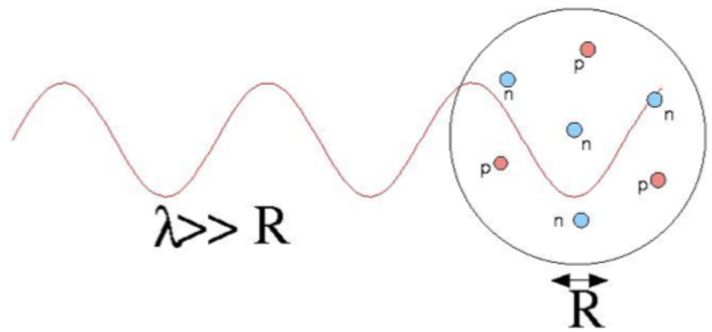
SS Li Muli, SB, N Barnea, in preparation (2022)



C. Ji, et al. J. Phys. G: Nucl. Part. Phys. 45 (2018)

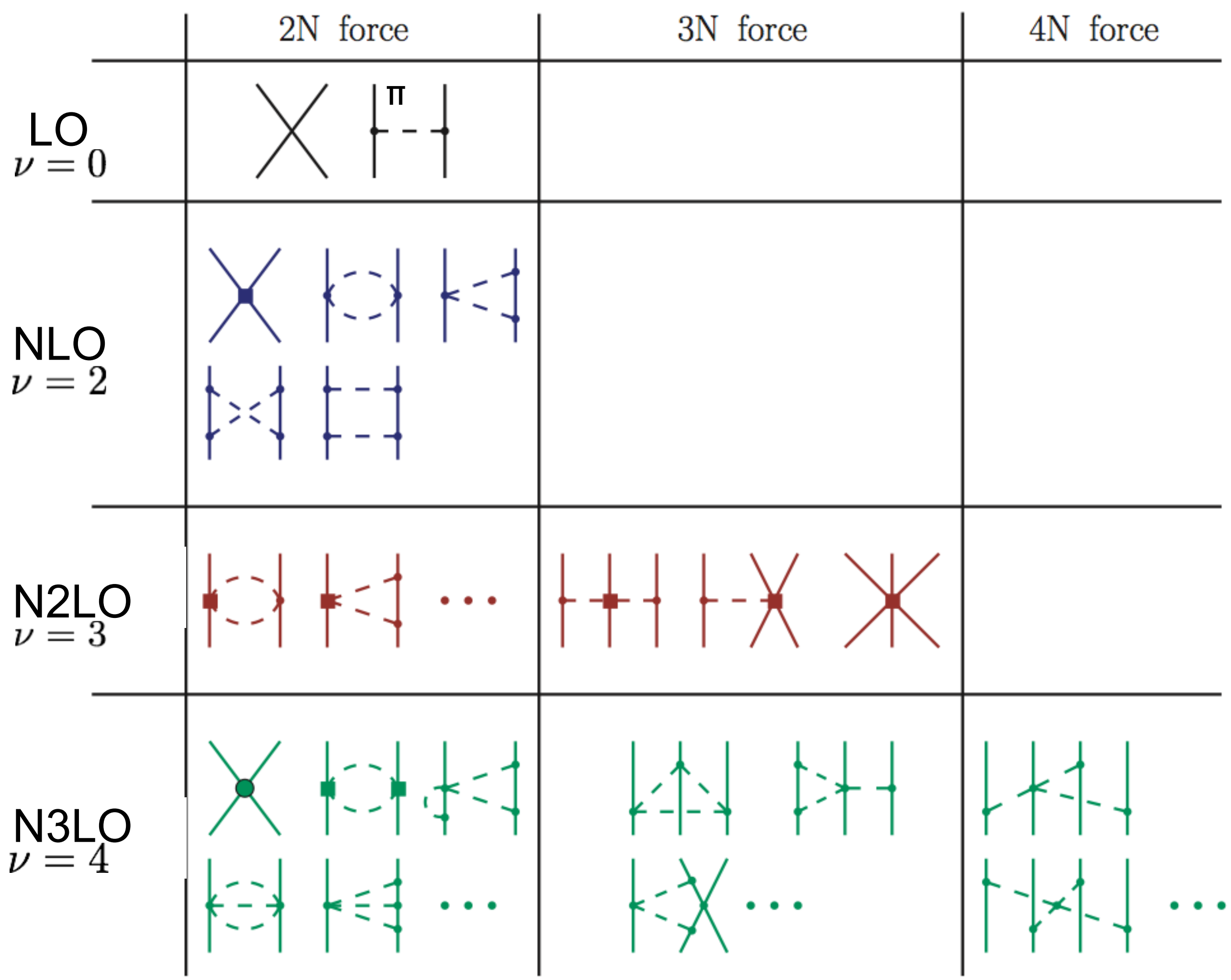
15

Chiral Effective Field Theory



Separation of scales

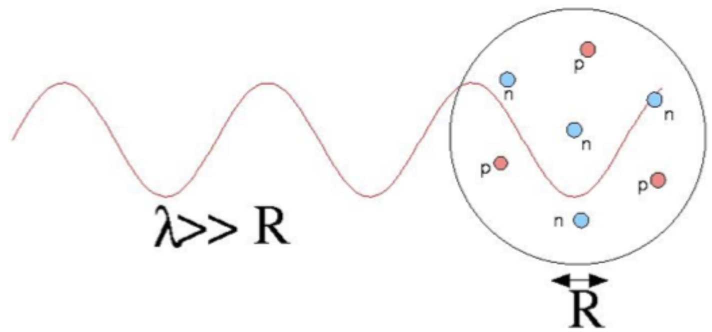
$$\frac{1}{\lambda} = Q \ll \Lambda_{\rm b} = \frac{1}{R}$$



Allows for a systematic expansion of nuclear forces.

Details of short distance physics not resolved, but captured in low energy constants (LEC)

Chiral Effective Field Theory



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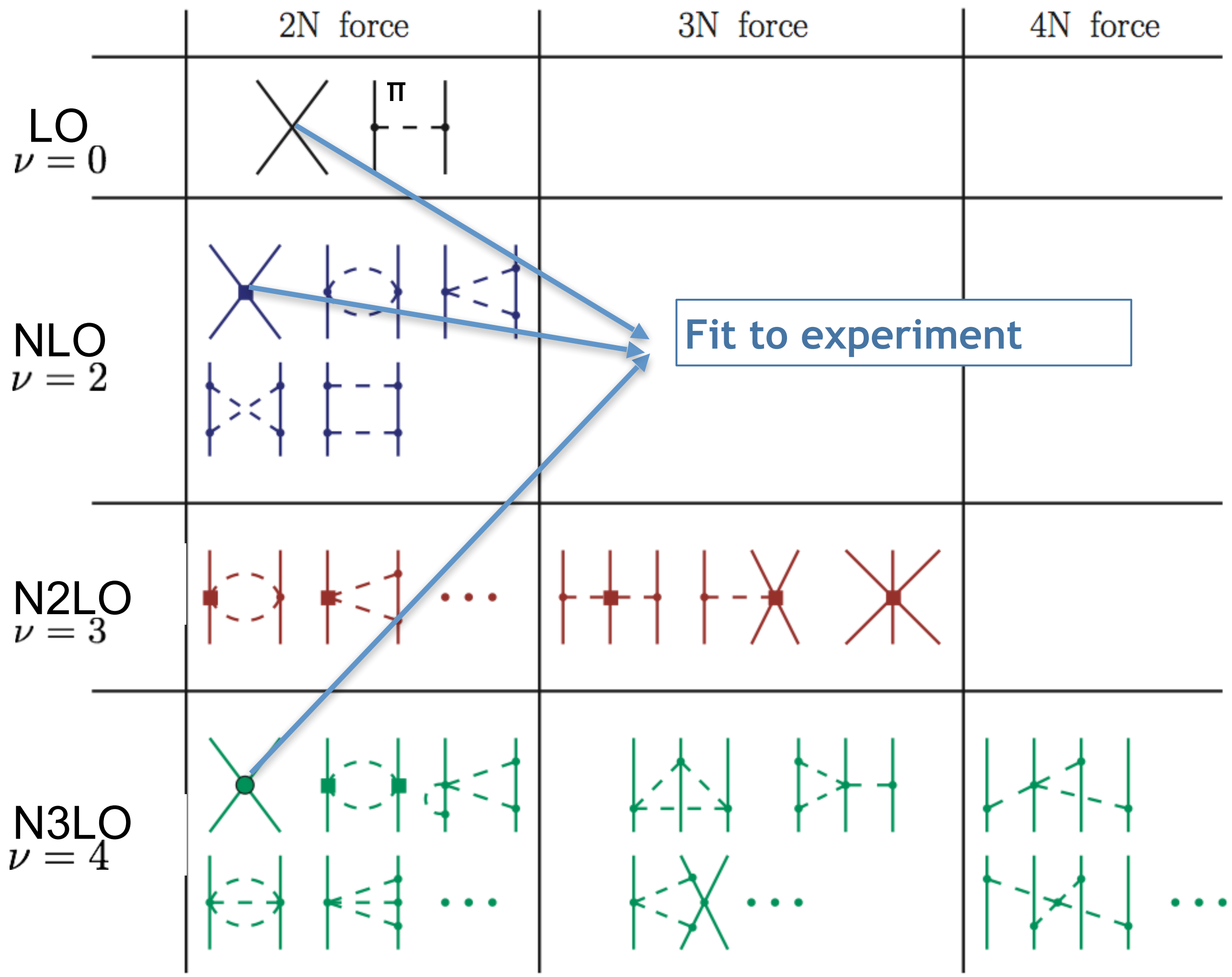

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