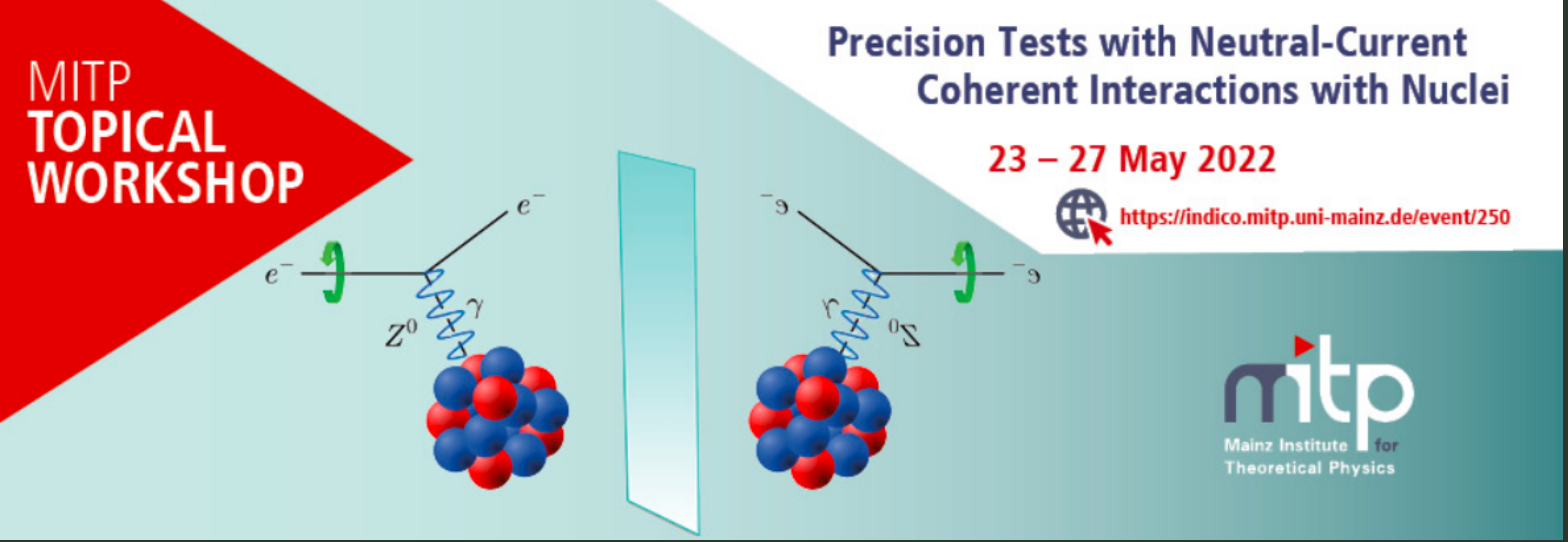
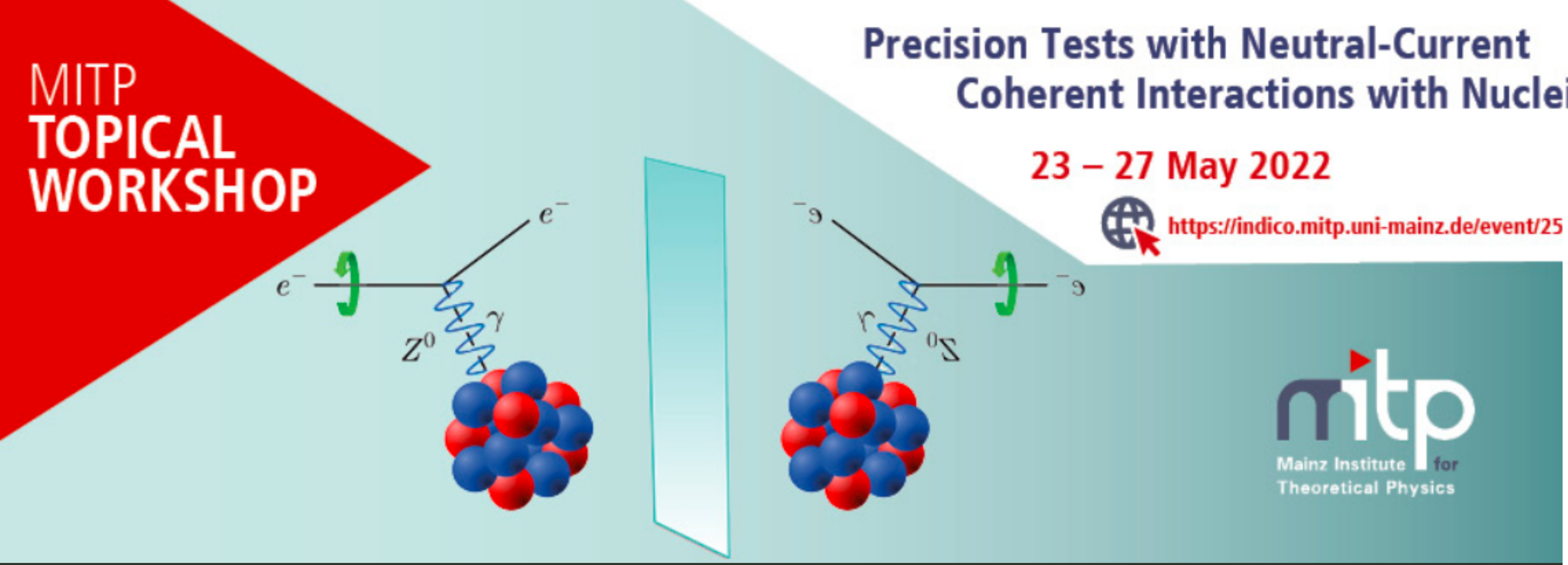
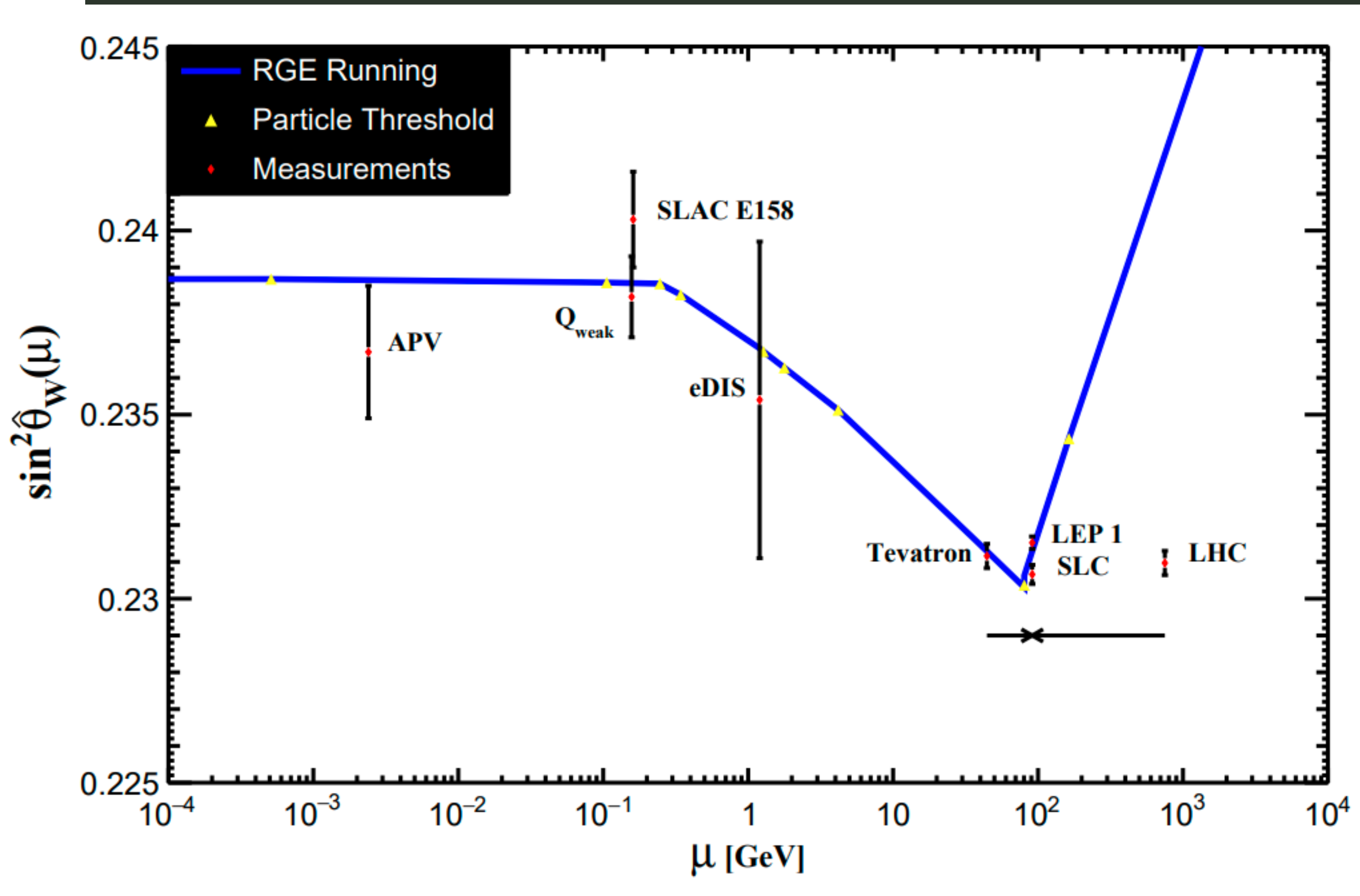


Precise running of the electroweak mixing angle

Rodolfo Ferro Hernández.





Precise running of the electroweak mixing angle

Rodolfo Ferro Hernández.

1. Motivation
2. Definition(s) of the weak mixing angle
3. RGE
4. Uncertainties
5. Results
6. Conclusions and future work

Motivation

- Possible light new physics. For example Z´s.

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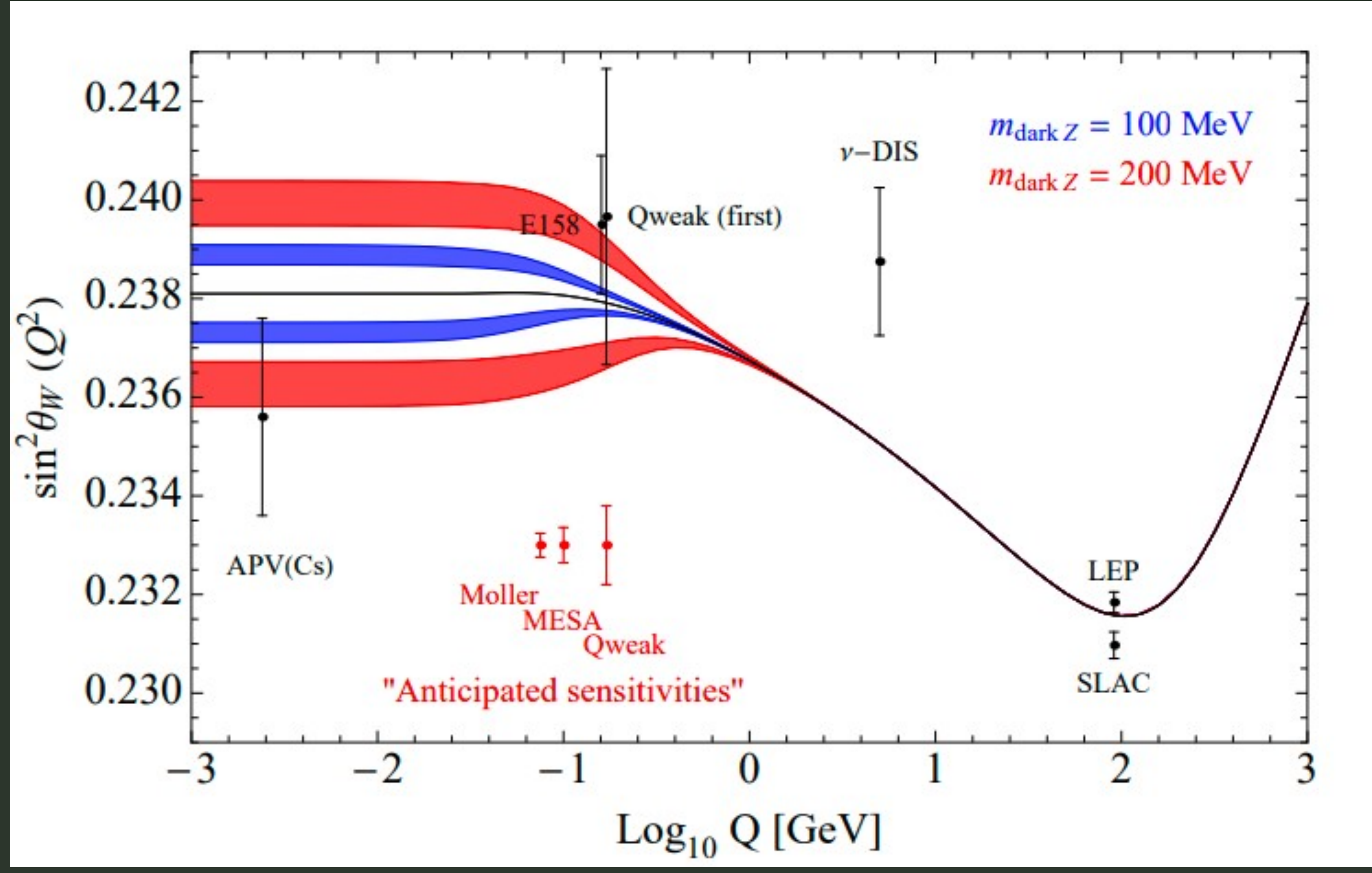
$$\epsilon B_{\mu\nu} B'^{\mu\nu} \qquad \delta M_Z M'_Z$$

Marciano, Davoudiasl, Lee (2014)



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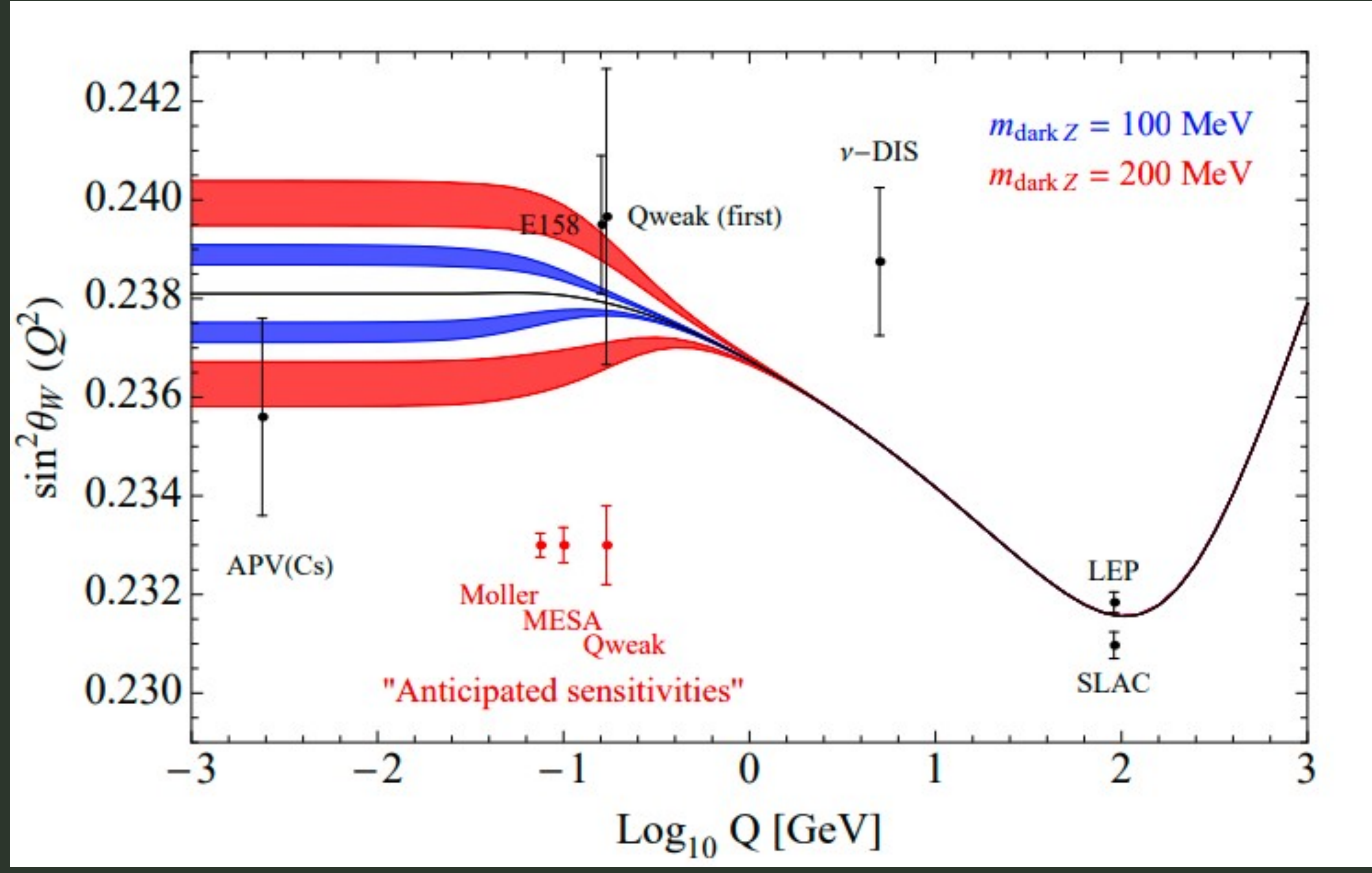

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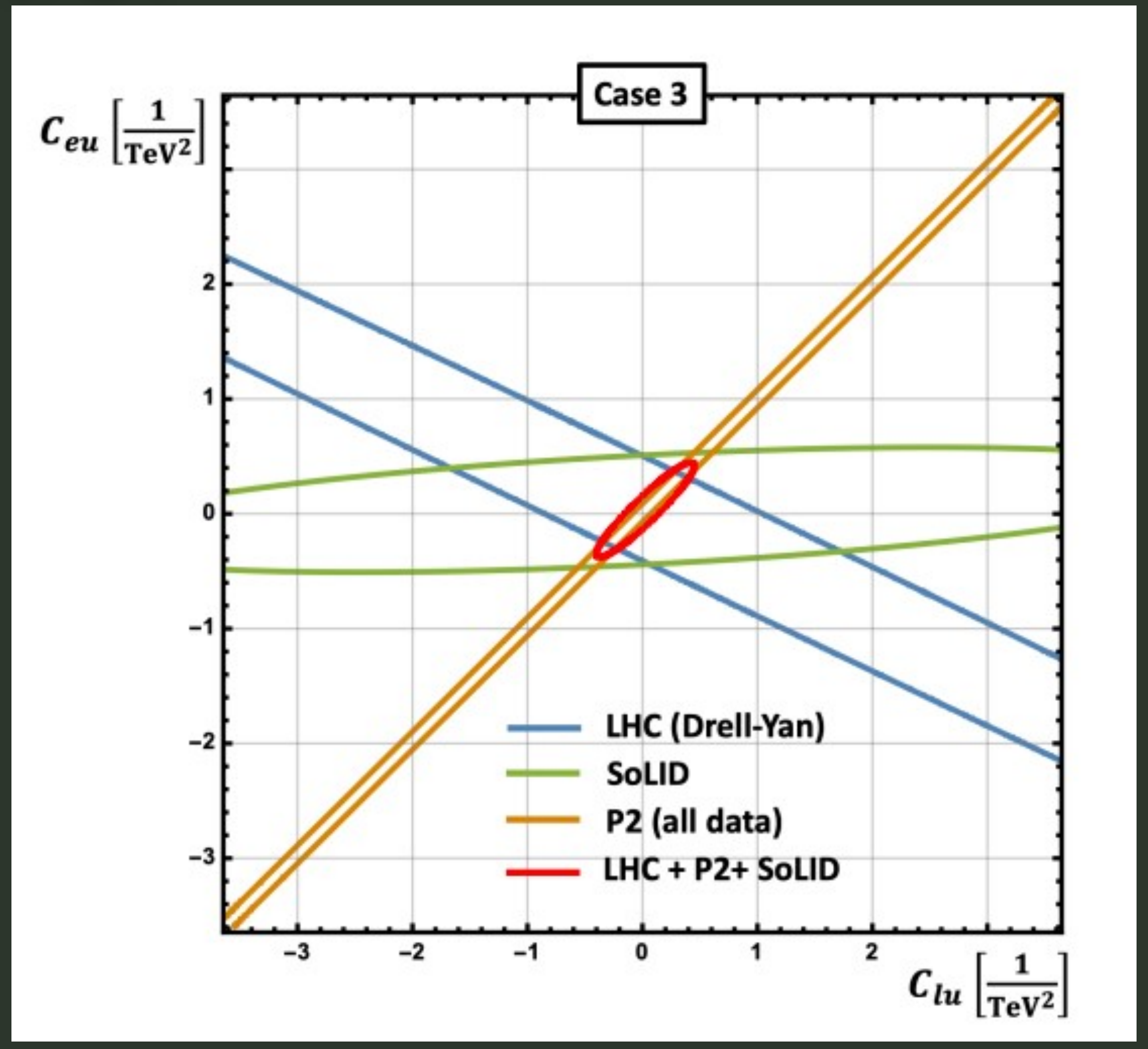

Boughezal, Petriello, and Daniel Wiegand (2021)

$$\mathcal{L} = \mathcal{L}_{SM} + \sum_{n=5}^{\infty} \frac{\mathcal{O}^{(n)}}{\Lambda^n}$$

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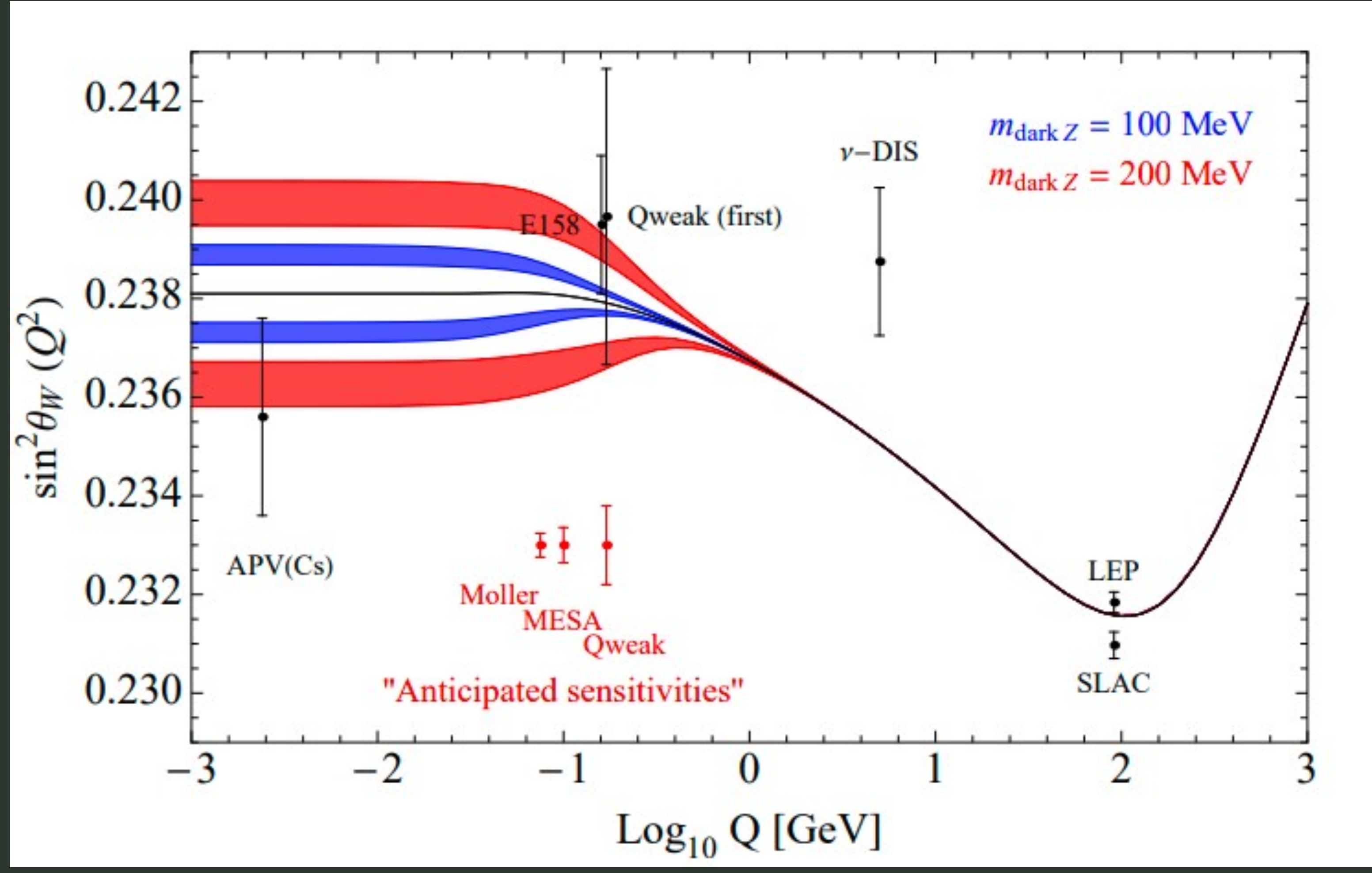
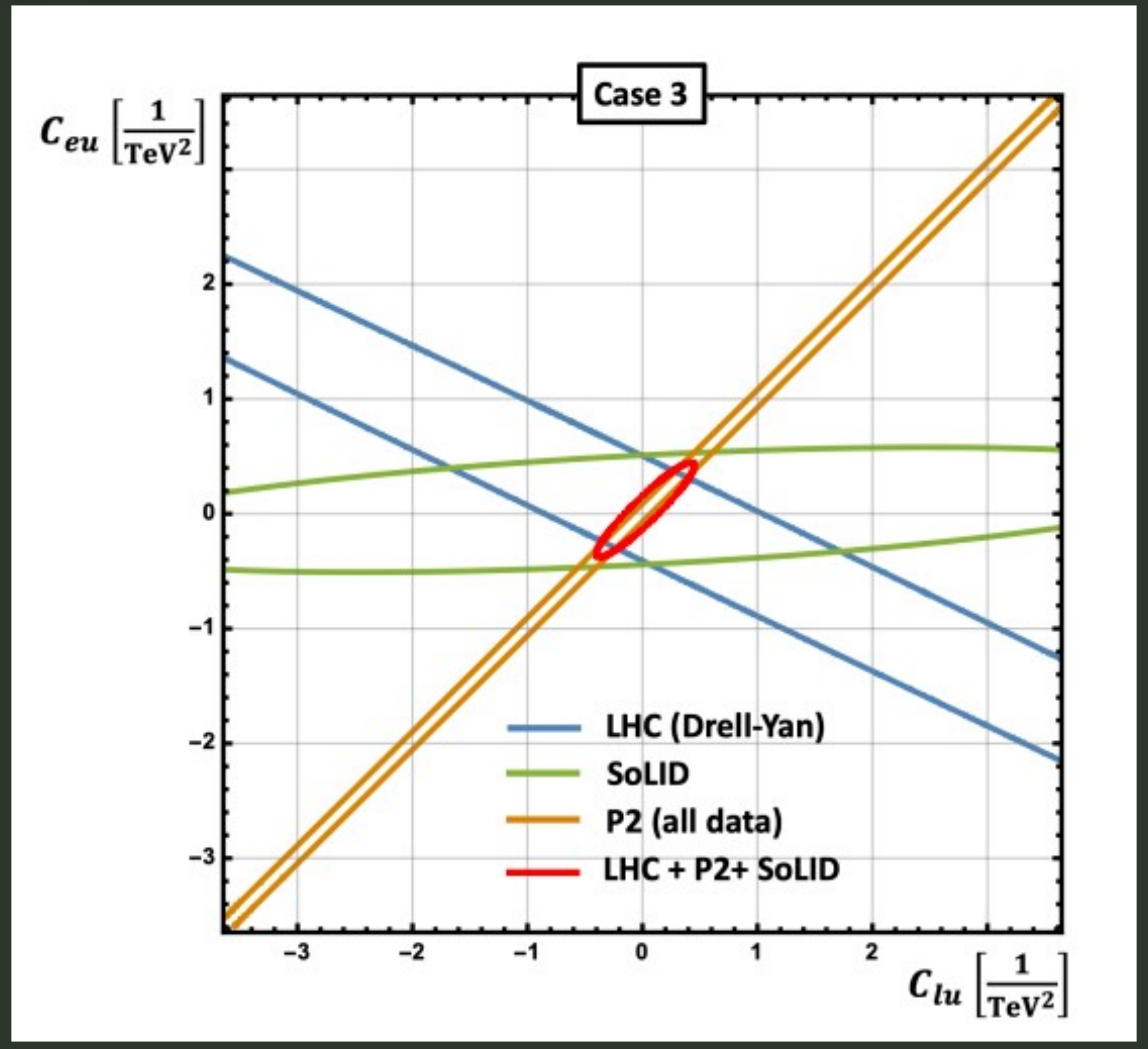


Marciano, Davoudiasl, Lee (2014)

- Possible light new physics. For example Z´s.
- Complementarity between the energy and precision frontier
- Future low energy experiments will be able to measure the weak mixing angle with tiny error ~0.0003.



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Definitions

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$$\sin^2\theta = 1 - \frac{M_W^2}{M_Z^2}$$

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$$\sin^2\hat{\theta} \equiv \frac{g'^2}{g'^2 + g^2}$$

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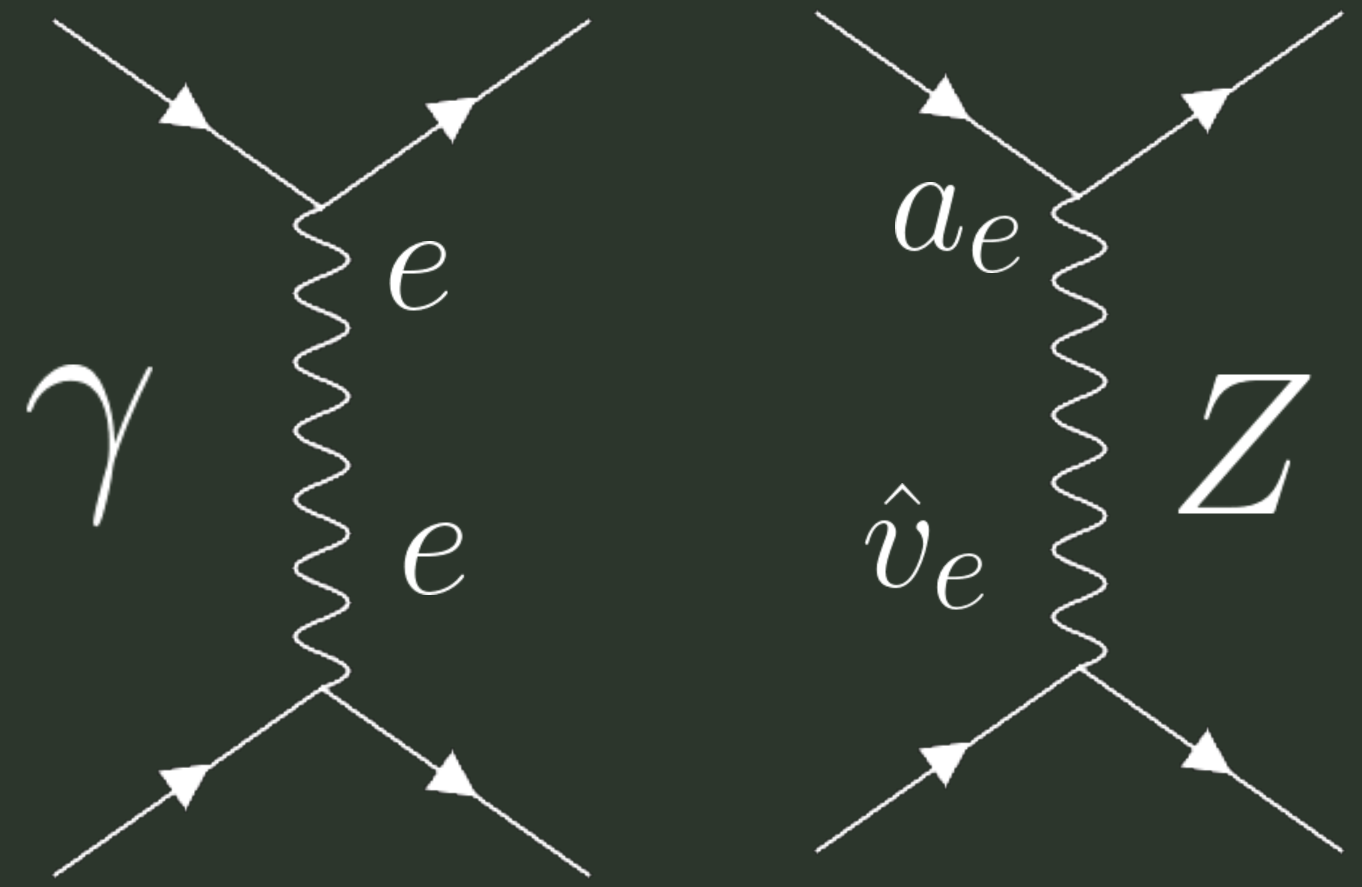
At leading order ⟶ $\sin^2\hat{\theta} = \sin^2\theta$

Including loop corrections ⟶ $\rho\cos^2\hat{\theta} = \cos^2\theta$

Polarized electron scattering Asymmetry at NLO

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$



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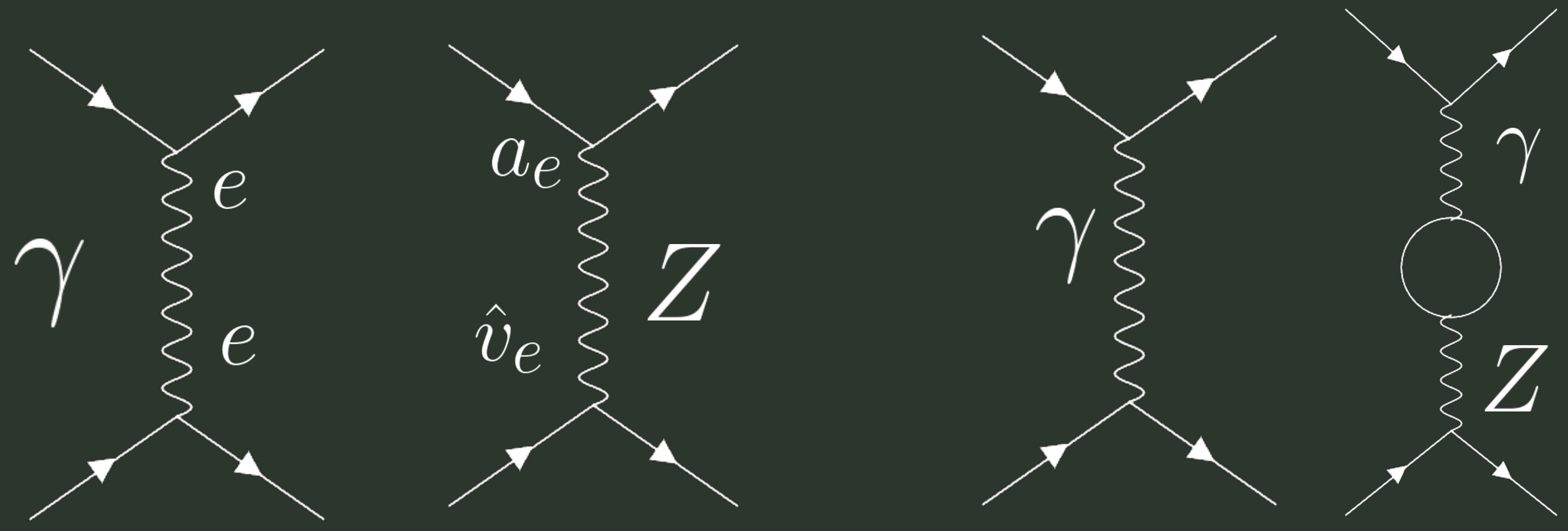

+ Vertex+box

RGE

From the γZ bubble we obtain,

$$\mu^2 \frac{d\hat{v}_f}{d\mu^2} = \frac{\hat{\alpha} Q_f}{24\pi} \left[\sum_i K_i \gamma_i \hat{v}_i Q_i + 12\sigma \left(\sum_q Q_q \right) \left(\sum_q \hat{v}_q \right) \right]$$

while the RGE for α is

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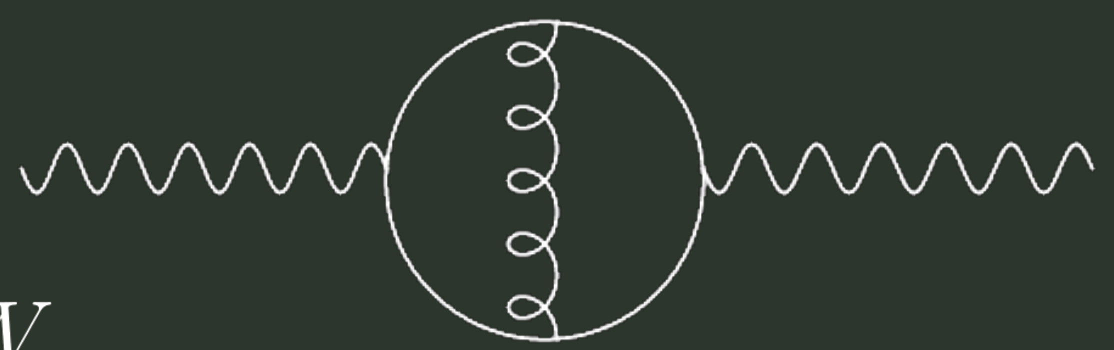

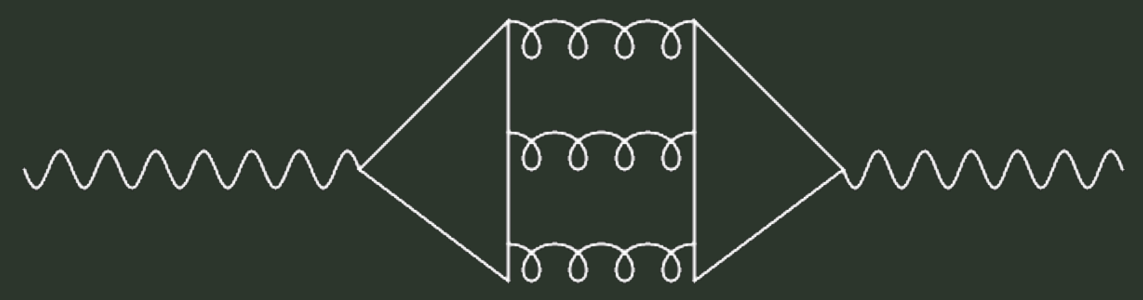

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$$\hat{s}^2(\mu) = \hat{s}^2(\mu_0)\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \lambda_1\left[1 - \frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)}\right] + \frac{\hat{\alpha}(\mu)}{\pi}\left[\frac{\lambda_2}{3}\ln\frac{\mu^2}{\mu_0^2} + \frac{3\lambda_3}{4}\ln\frac{\hat{\alpha}(\mu)}{\hat{\alpha}(\mu_0)} + \tilde{\sigma}(\mu_0) - \tilde{\sigma}(\mu)\right]$$

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Numerical constants, depend on the number of particles

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Disconnected contribution

Solve for $\hat{\alpha}$ to get $\hat{s}^2$!

RGE down to zero

Below the particle mass, such particle is removed from the RGE.

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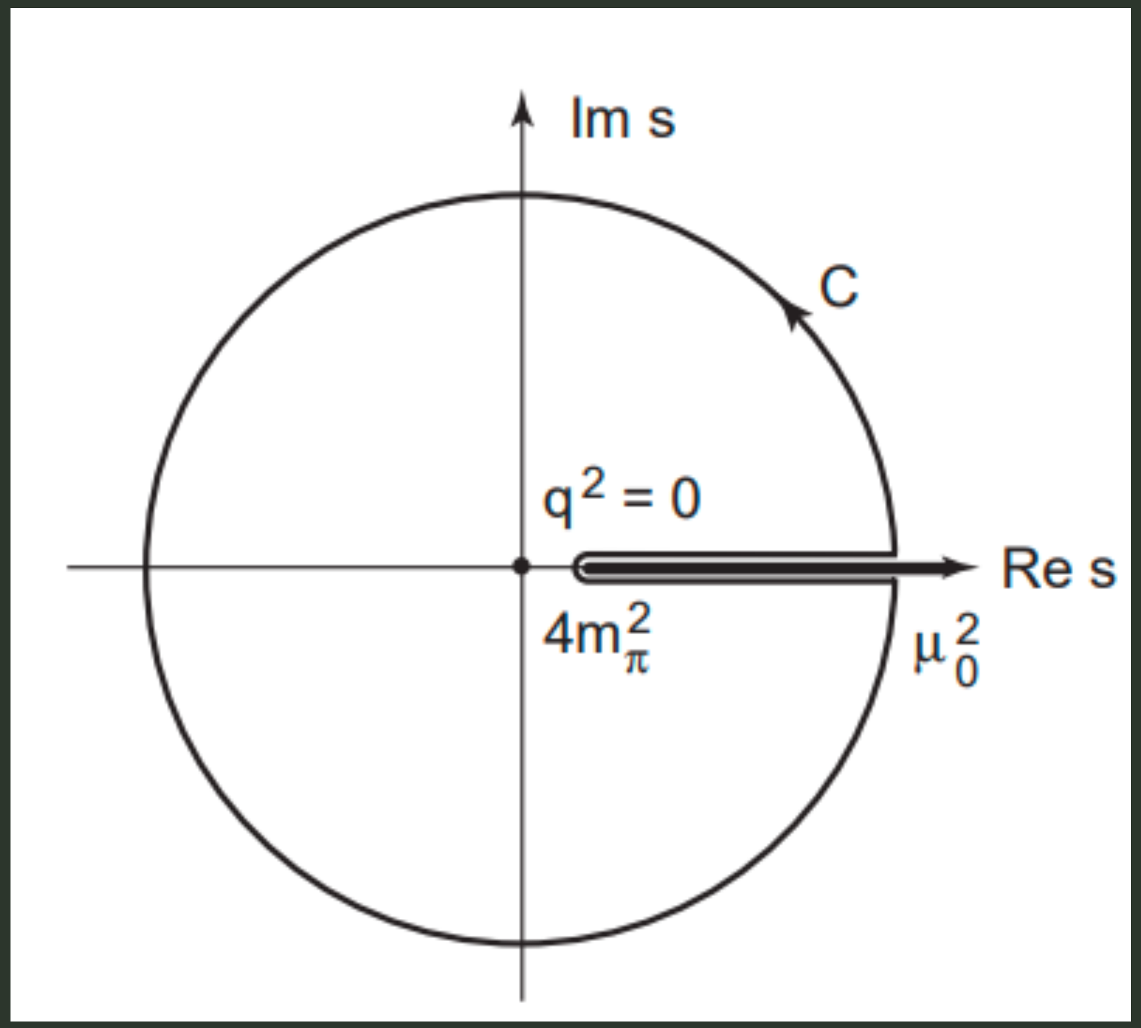
Need to estimate the disconnected contributions

The light quarks

$$\hat{\alpha}(\mu) = \frac{\alpha_0}{1 - 4\pi\alpha_0 \hat{\Pi}(0,\mu)}$$

$$\hat{\Pi}(0) = \frac{1}{\pi}\int_{4m_\pi^2}^{\mu_0^2} \frac{ds}{s - i\epsilon} \,\mathrm{Im}\,\hat{\Pi}(s) + \frac{1}{2\pi i}\oint_{|s|=\mu_0^2} \frac{ds}{s}\hat{\Pi}(s)$$



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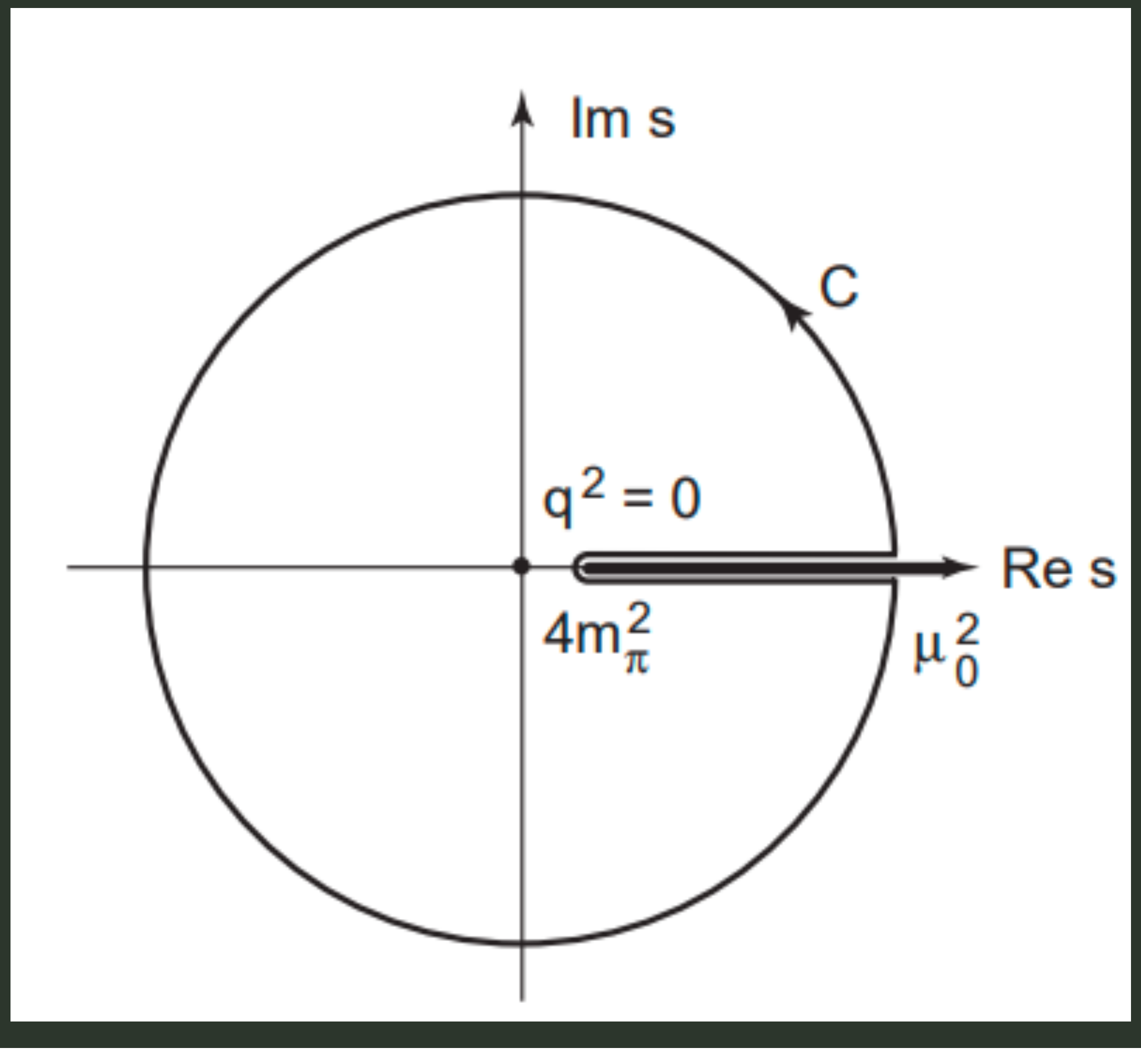
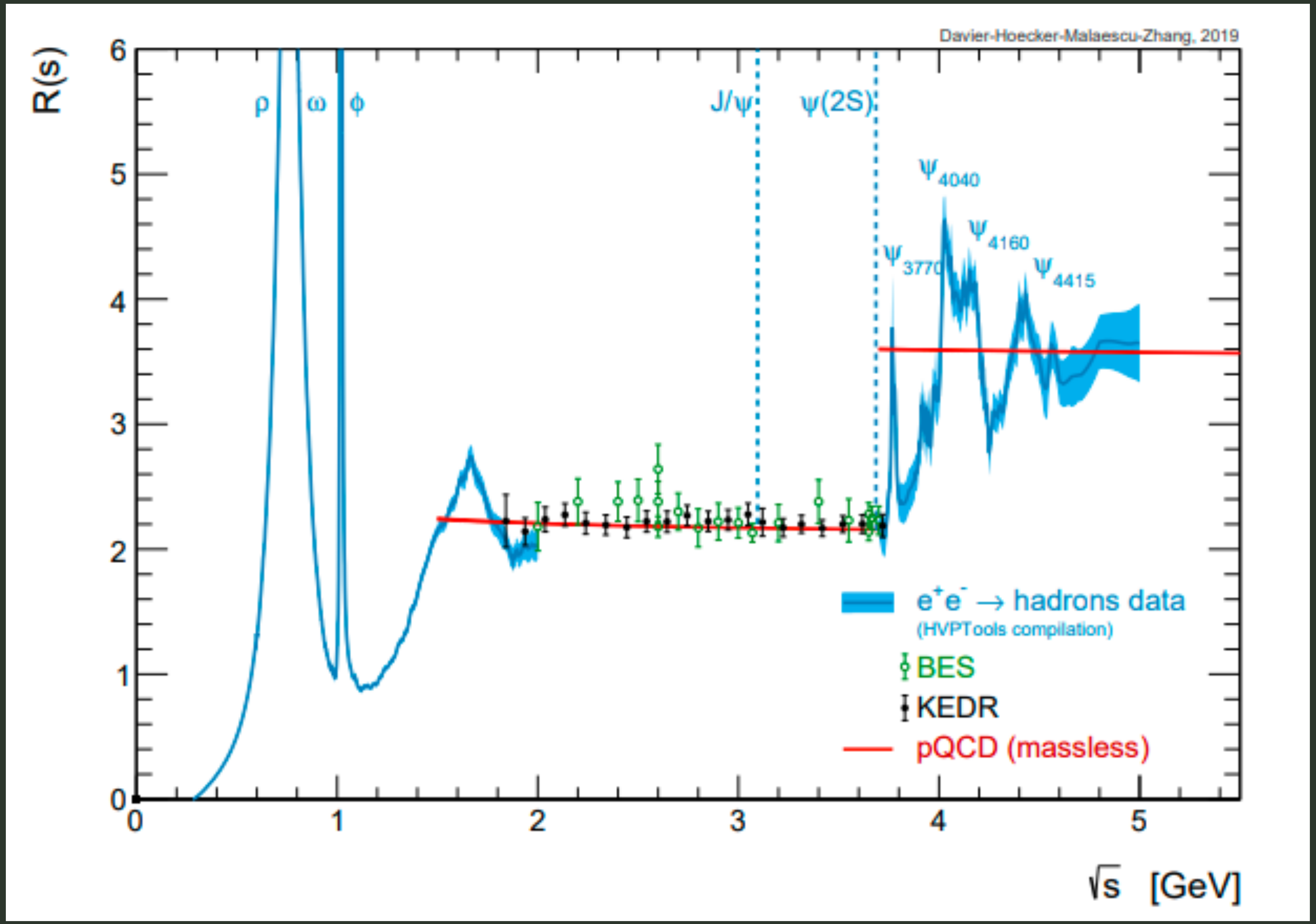
$$\Delta\hat{\alpha}^{(3)}(\mu_0) = \frac{\alpha}{3\pi}\int_{4m_\pi^2}^{\mu_0^2} ds\,\frac{R(s)}{s - i\epsilon} + 4\pi I^{(3)},$$

Data

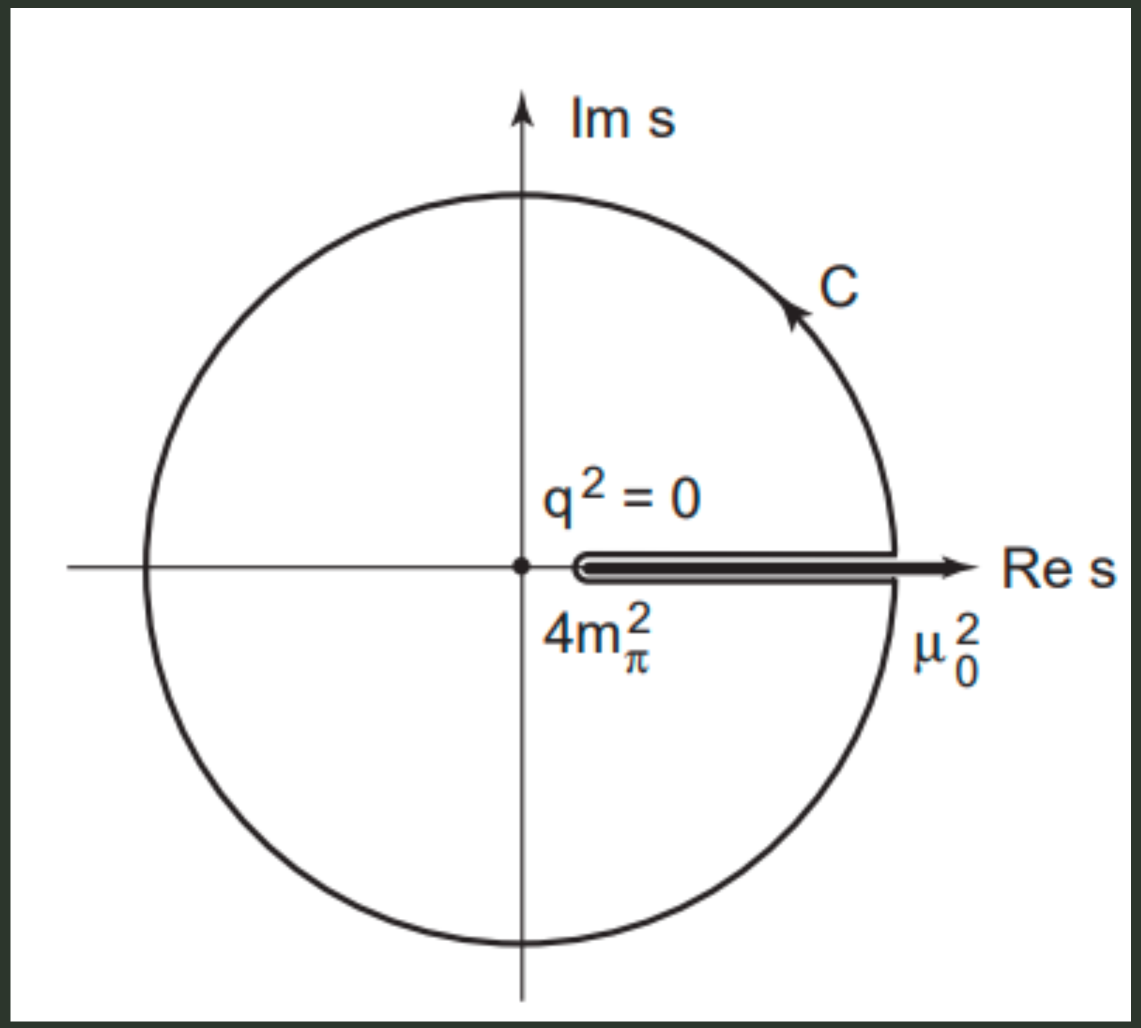
pQCD





M. Davier A. Hoecker, Malaescu, Zhang 2017

The light quarks



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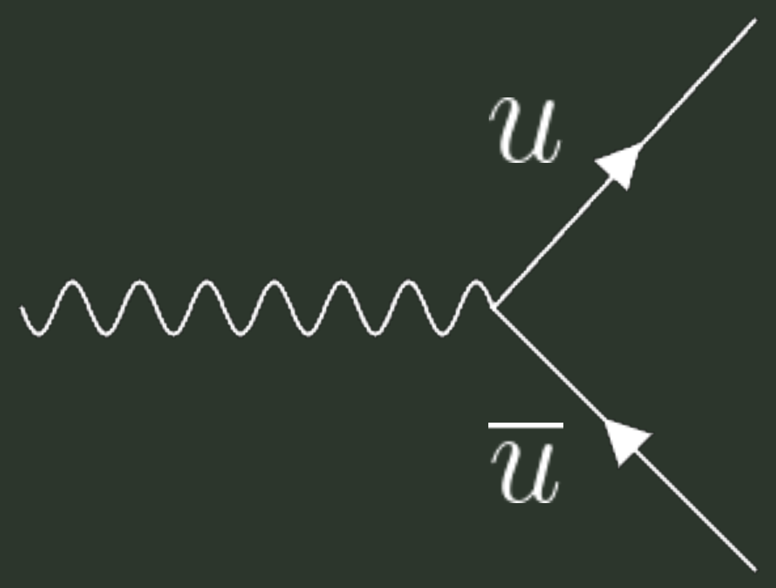
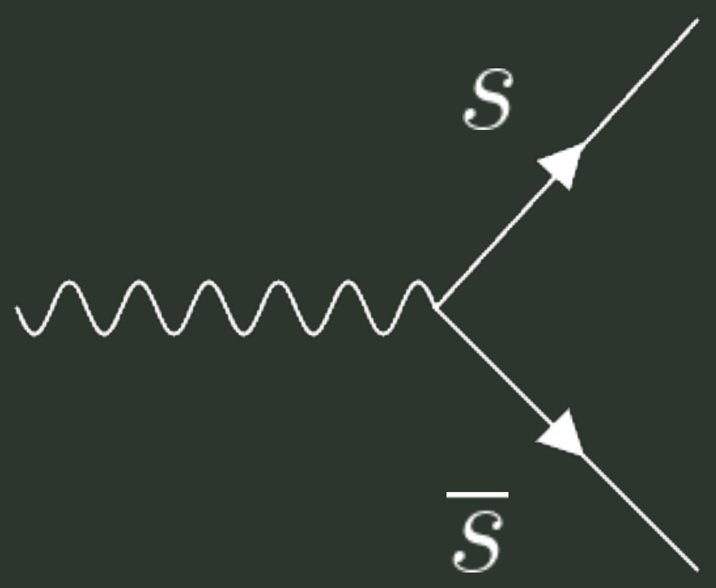
$$\delta\hat{s}^2(0) = \left[\frac{1}{2} - \hat{s}^2\right] \delta\Delta\hat{\alpha}^{(3)}(2\ \mathrm{GeV}) = \mp 1.2 \times 10^{-5},$$

Strange quark contribution





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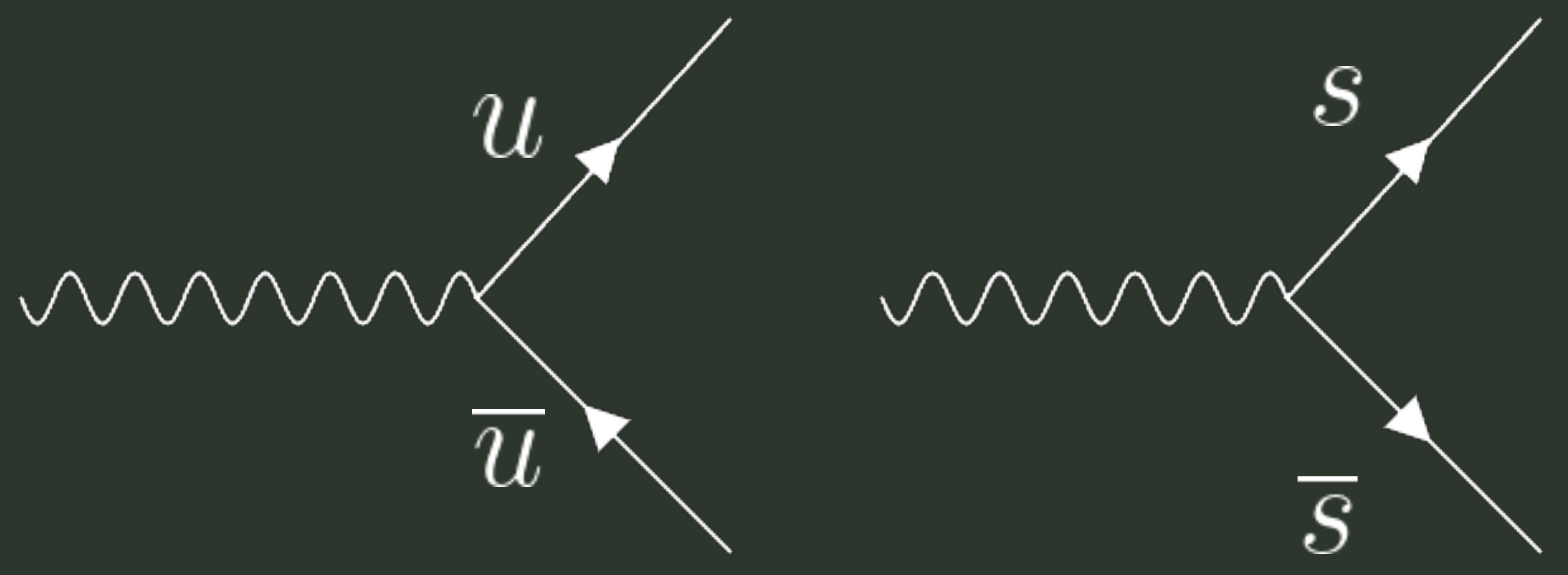

Use channels that correspond to strange quark current: $\phi(1020), \;\; \phi(1680)$

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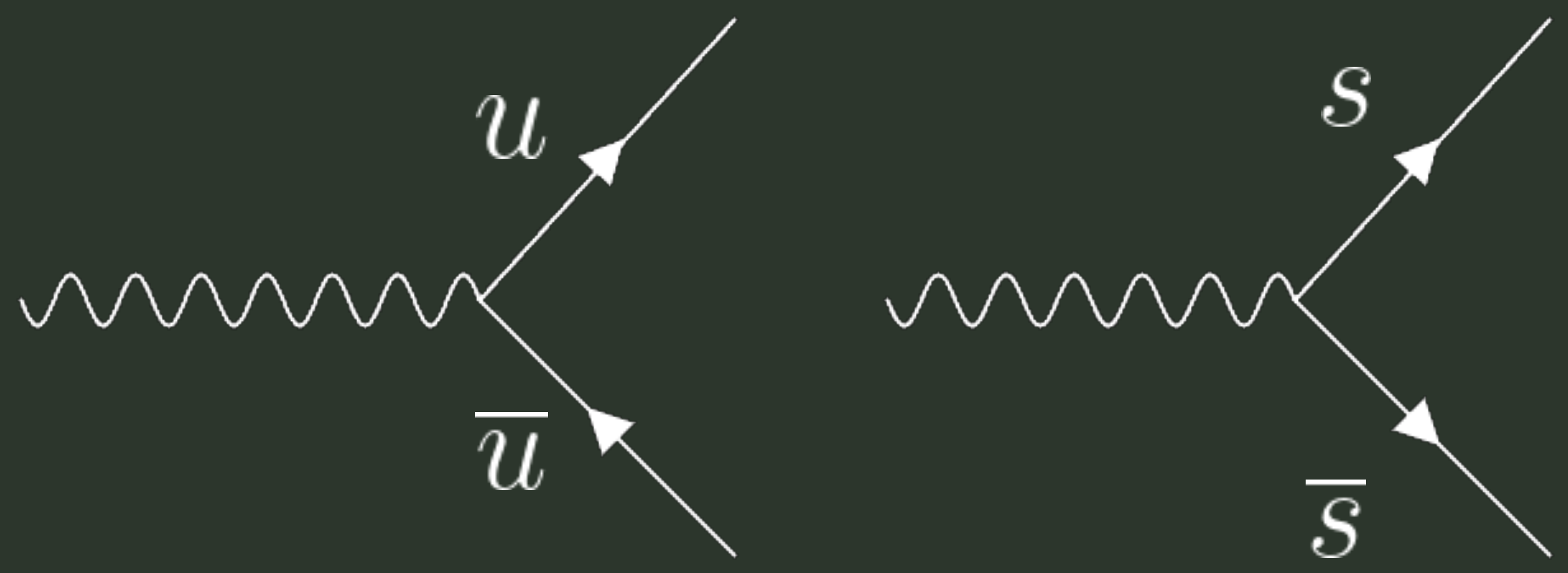

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For kaon channels we take a 50% contribution $\pm$50%.

channel	$a_\mu \times 10^{10}$	$\Delta\alpha \times 10^4$
$K\bar{K}$ (non $-\ \phi$)	3.62	0.76
$K\bar{K}2\pi$	0.85	0.30
$K\bar{K}3\pi$	-0.03	-0.01
$K\bar{K}\eta$	0.01	0.00
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Total	4.46	1.05

arXiv:1712.09146

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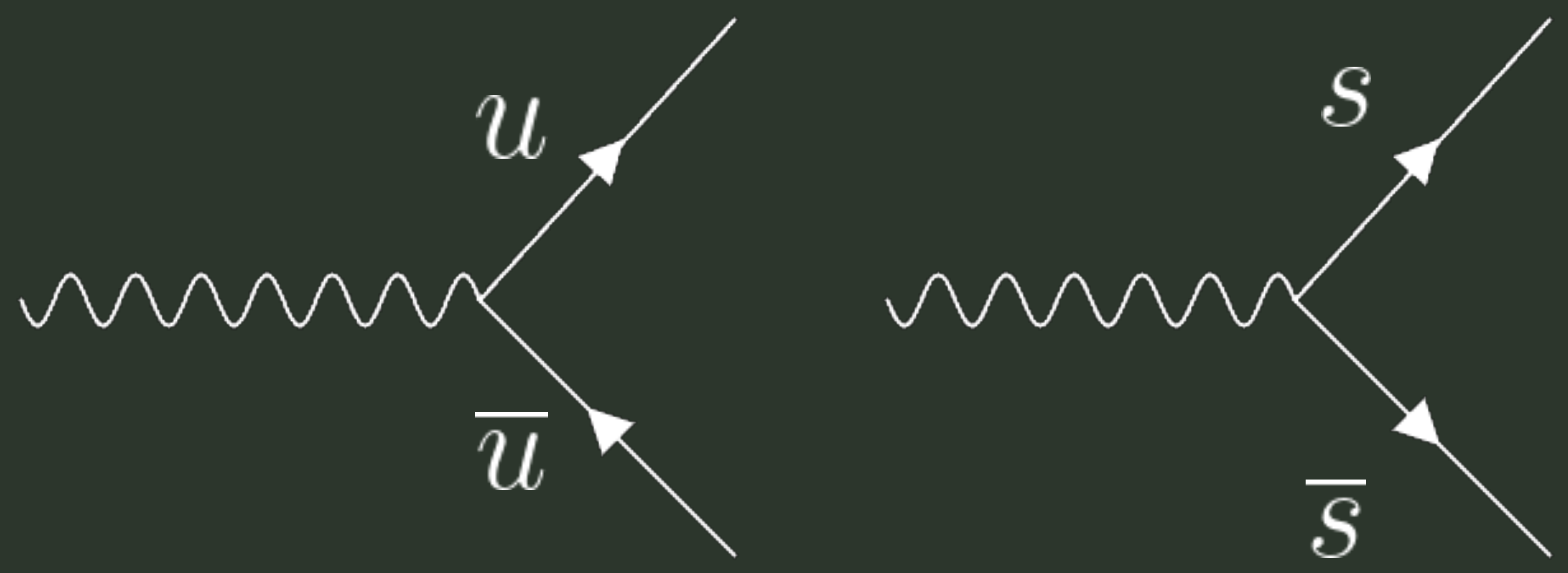

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$$\Delta_s\hat{\alpha}(\bar{m}_c) = Q_s^2 \frac{\alpha}{\pi} K^s_{\rm QCD}(\bar{m}_c) \ln \frac{\bar{m}_c^2}{\bar{m}_s^2}$$

After combination with lattice (RBC and UKQCD 2016):

$$\Delta_s\hat{\alpha}(\bar{m}_c) = (8.71 \pm 0.32) \times 10^{-4}$$

$$\delta\hat{s}^2(0) \simeq \frac{1}{20}\,\delta\Delta\hat{\alpha}^{(2)}(\bar{m}_c) = \pm 1.0 \times 10^{-5}$$

Error budget

source	$\delta \sin^2 \hat{\theta}_W(0) \times 10^5$
$\Delta\hat{\alpha}^{(3)}(2\ \text{GeV})$	1.2
flavor separation	1.0
isospin breaking	0.7
singlet contribution	0.3
PQCD	0.6
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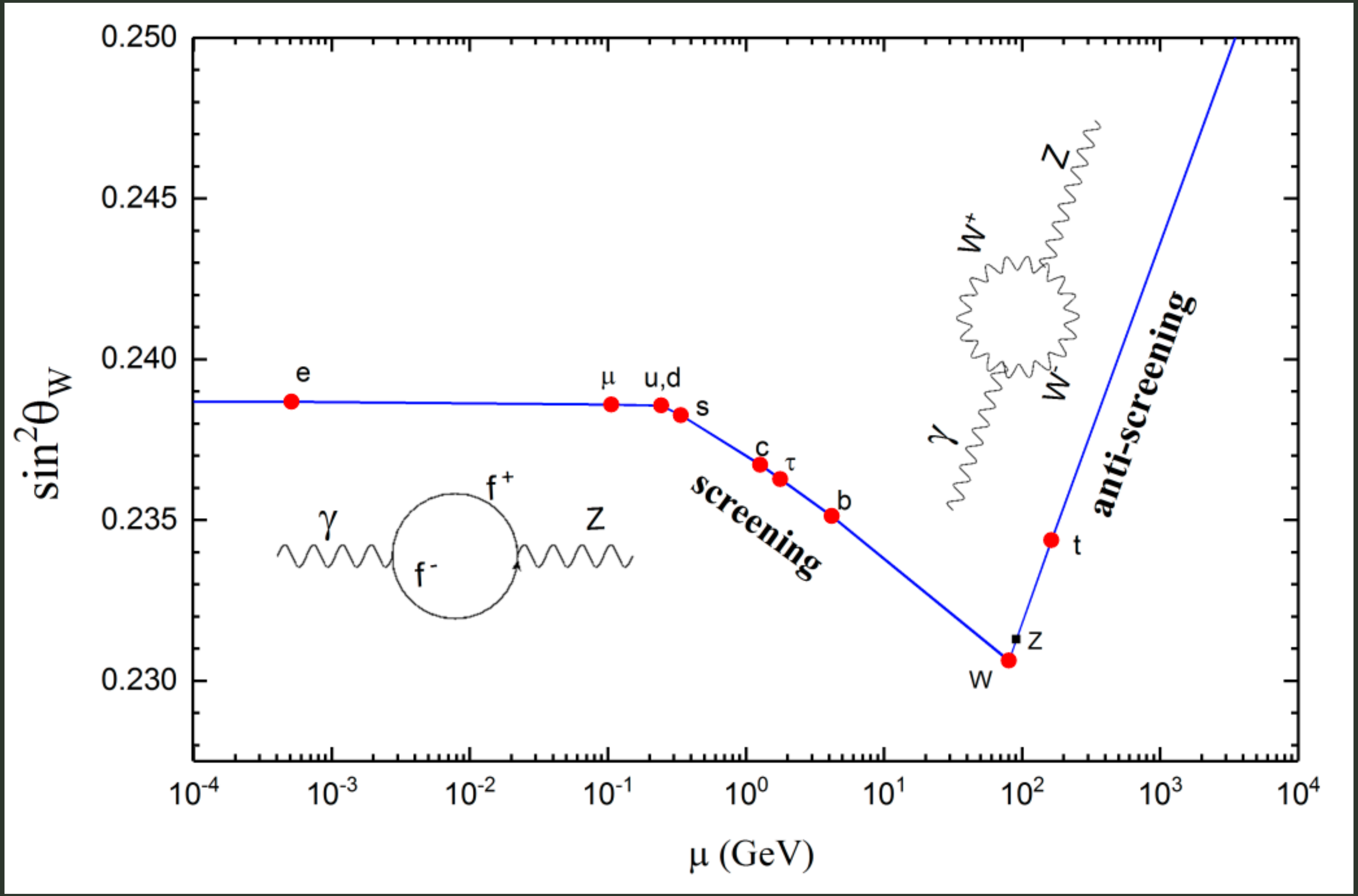
RBC/UKQCD 1512.09054



$$\sin^2 \hat{\theta}_W (0) = 0.23868 \pm 0.00005 \pm 0.00002$$

Polarized electron scattering asymmetry at NLO

In the $\overline{\text{MS}}$ scheme at $\mu = M_Z$ (Marciano and Czarnecki 1995).

$$A_{\rm LR}^{1-\rm loop} = \frac{\rho G_\mu Q^2}{\sqrt{2}\pi\alpha} \frac{1-y}{1+y^4+(1-y)^4} \Bigg[1 - 4\kappa(0)\hat{s}_Z^2 + \frac{\alpha}{4\pi \hat{s}_Z^2} - \frac{3\alpha}{32\pi \hat{s}_Z^2 \hat{c}_Z^2}(1-4\hat{s}_Z^2)\left(1+(1-4\hat{s}_Z^2)^2\right) - \frac{\alpha}{4\pi}(1-4\hat{s}_Z^2)\left\{\frac{22}{3}\ln\frac{y m_Z^2}{Q^2} + \frac{85}{9} + f(y)\right\} + F_2(y,Q^2)\Bigg]$$

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$$+ \frac{\alpha}{4\pi\hat{s}_Z^2} - \frac{3\alpha}{32\pi\hat{s}_Z^2\hat{c}_Z^2}(1-4\hat{s}_Z^2)\left(1+(1-4\hat{s}_Z^2)^2\right)$$

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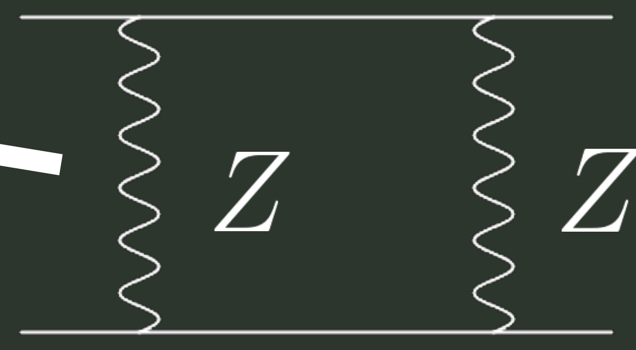

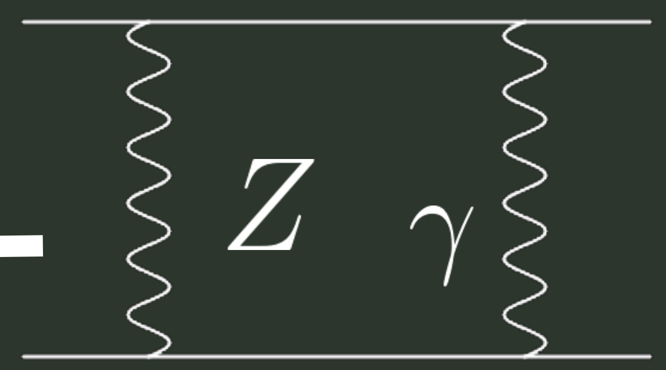

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$$\hat{s}_Z^2 - \hat{s}_0^2 = \frac{\alpha}{\pi}\left[\frac{1}{6}\sum_f \left(T_{3f}Q_f - 2\hat{s}_Z^2 Q_f^2\right)\ln\frac{m_f^2}{m_Z^2} - \left(\frac{7}{4}\hat{c}_Z^2 + \frac{1}{24}\right)\ln\frac{m_W^2}{m_Z^2} + \frac{1}{6} - \frac{\hat{s}_Z^2}{6}\right]$$

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same logs!

$$\kappa(0)\hat{s}_Z^2 = \hat{s}_0^2 - \frac{2\alpha}{9\pi} + \mathcal{O}(\alpha^2)$$

Asymmetry at NNLO

$$A_{\rm LR} = \frac{G_\mu Q^2}{\sqrt{2}\pi\alpha} \frac{1-y}{1+y^4+(1-y)^4}(1 - 4\sin^2\theta_W + \Delta Q_W^e)$$ (arXiv:1912.08220, arXiv:2202.11976)

	$\hat{s}(m_Z)$–α scheme* [5] ($X{=}\alpha$)	$\hat{s}(0)$–α scheme ($X{=}\alpha$)	$\hat{s}(0)$–G_μ scheme ($X{=}G_\mu$)
$1-4\hat{s}^2$	74.40	45.56	45.56
$X\,\Delta Q^{e,X}_{W(1,1)}$	−29.04	+ 0.39	+ 0.43
$X\,\Delta Q^{e,X}_{W(1,0)}$	+ 3.06	+ 0.77	+ 0.84
$X^2\,\Delta Q^{e,X}_{W(2,2)}$	− 0.18	+ 0.07	+ 0.05
$X^2\,\Delta Q^{e,X}_{W(2,1)}$	+ 1.18	− 1.15	− 1.30
$X\,\Delta Q^{e,X}_{W,\Delta\rho}$	—	− 0.05	− 0.06
Sum	49.42	45.60	45.52

*no QCD corrections

$$Q_W^e = (45.83 \pm 0.08_{\,\hat{s}(0)} \pm 0.06_{\,\Delta Q^{e,X}_{W(2,1)}(\text{had})} \pm 0.13_{\,\Delta Q^{e,X}_{W(2,0)}(\text{missing})} \pm 0.23_{\,\text{scheme}}) \times 10^{-3}$$

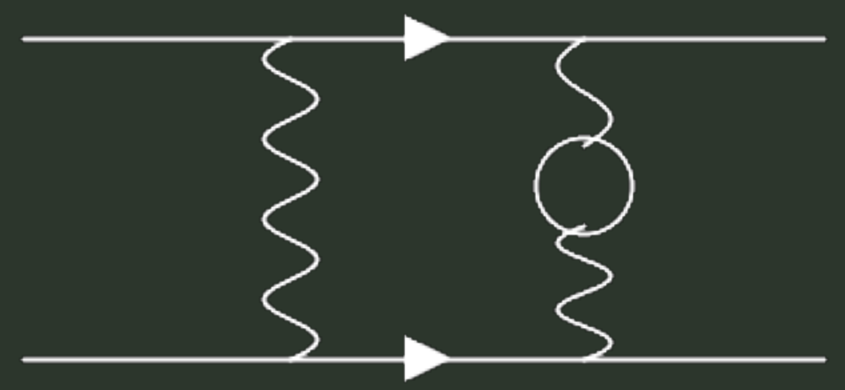
$$Q_W^e = (45.83 \pm 0.28)\times 10^{-3}$$ ⟶ 0.6% theoretical uncertainty

Asymmetry at NNLO

$$A_{\rm LR} = \frac{G_\mu Q^2}{\sqrt{2}\pi\alpha}\frac{1-y}{1+y^4+(1-y)^4}(1-4\sin^2\theta_W+\Delta Q_W^e)$$ (arXiv:1912.08220, arXiv:2202.11976)

	$\hat{s}(m_Z)$–α scheme* [5] (X=α)	$\hat{s}(0)$–α scheme (X=α)	$\hat{s}(0)$–G_μ scheme (X=G_μ)
$1-4\hat{s}^2$	74.40	45.56	45.56
$X\,\Delta Q^{e,X}_{W(1,1)}$	−29.04	+ 0.39	+ 0.43
$X\,\Delta Q^{e,X}_{W(1,0)}$	+ 3.06	+ 0.77	+ 0.84
$X^2\,\Delta Q^{e,X}_{W(2,2)}$	− 0.18	+ 0.07	+ 0.05
$X^2\,\Delta Q^{e,X}_{W(2,1)}$	+ 1.18	− 1.15	− 1.30
$X\,\Delta Q^{e,X}_{W,\Delta\rho}$	—	− 0.05	− 0.06
Sum	49.42	45.60	45.52

*no QCD corrections



$$Q_W^e = (45.83 \pm 0.08_{\,\hat{s}(0)} \pm 0.06_{\,\Delta Q^{e,X}_{W(2,1)}(\text{had})} \pm 0.13_{\,\Delta Q^{e,X}_{W(2,0)}(\text{missing})} \pm 0.23_{\,\text{scheme}}) \times 10^{-3}$$

$$Q_W^e = (45.83 \pm 0.28)\times 10^{-3}$$ → 0.6% theoretical uncertainty

Outlook.

Combine with lattice results to the γZ vacuum polarisation here at Mainz (arXiv:2203.08676)

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Use q dependent vacuum polarization to improve the results on box diagram (Möller)

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Find a way to systematically re-sum contribution from the boxes, LEFT?