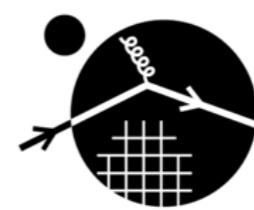


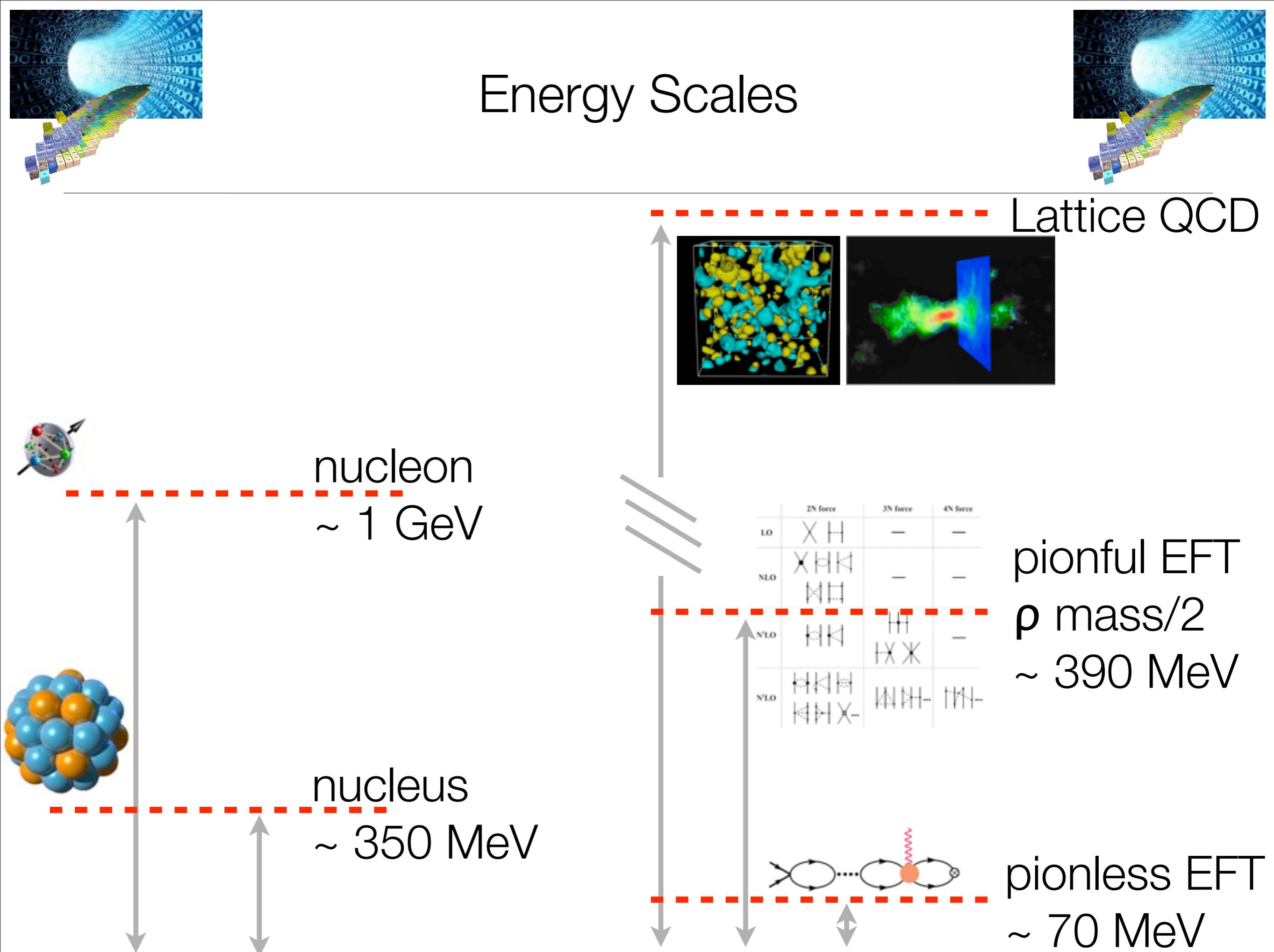
Lattice QCD Calculations for Dark Matter Exploration

Martin J Savage

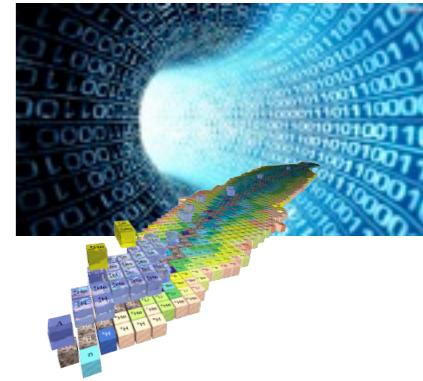
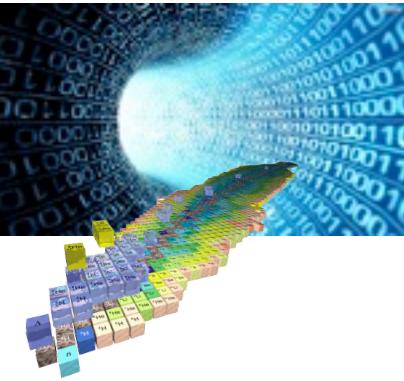


INSTITUTE for
NUCLEAR THEORY

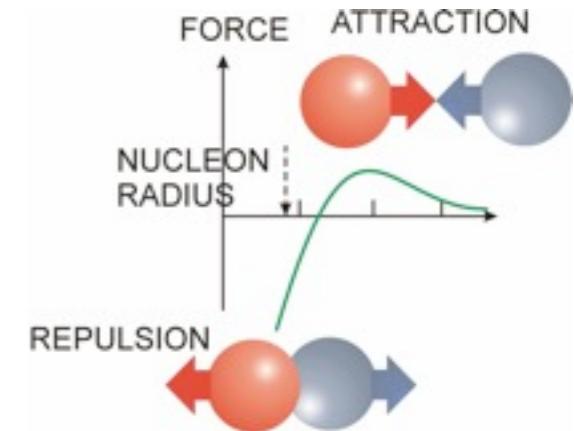
Energy Scales



Why Lattice QCD ?



- Nuclear Models provide a QM ***interpolation*** of experimental data



- EFTs for Nuclei

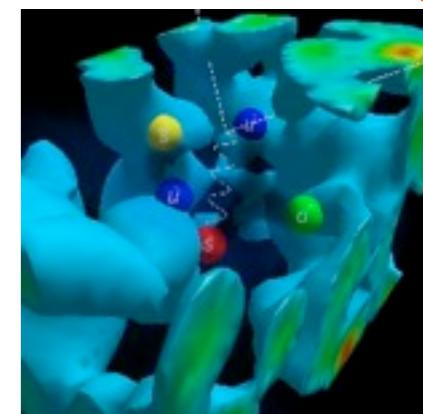
- model dependence largely pushed to higher orders - truncation
- proliferation of counterterms for higher precision - run out of constraints
- momentum and quark-mass expansions below $p \sim 1$ GeV (350 MeV)
 - pionful - chiral symmetries of QCD
 - required to rigorously describe nuclei at physical light-quark masses
- momentum expansion below $p \sim 70$ MeV
 - pionless - ERE (Bethe 1930's), EFT generalization in 1990's (SNO analysis, etc)
 - not rigorously applicable to nuclei, but numerically seems to not be too bad (why?)



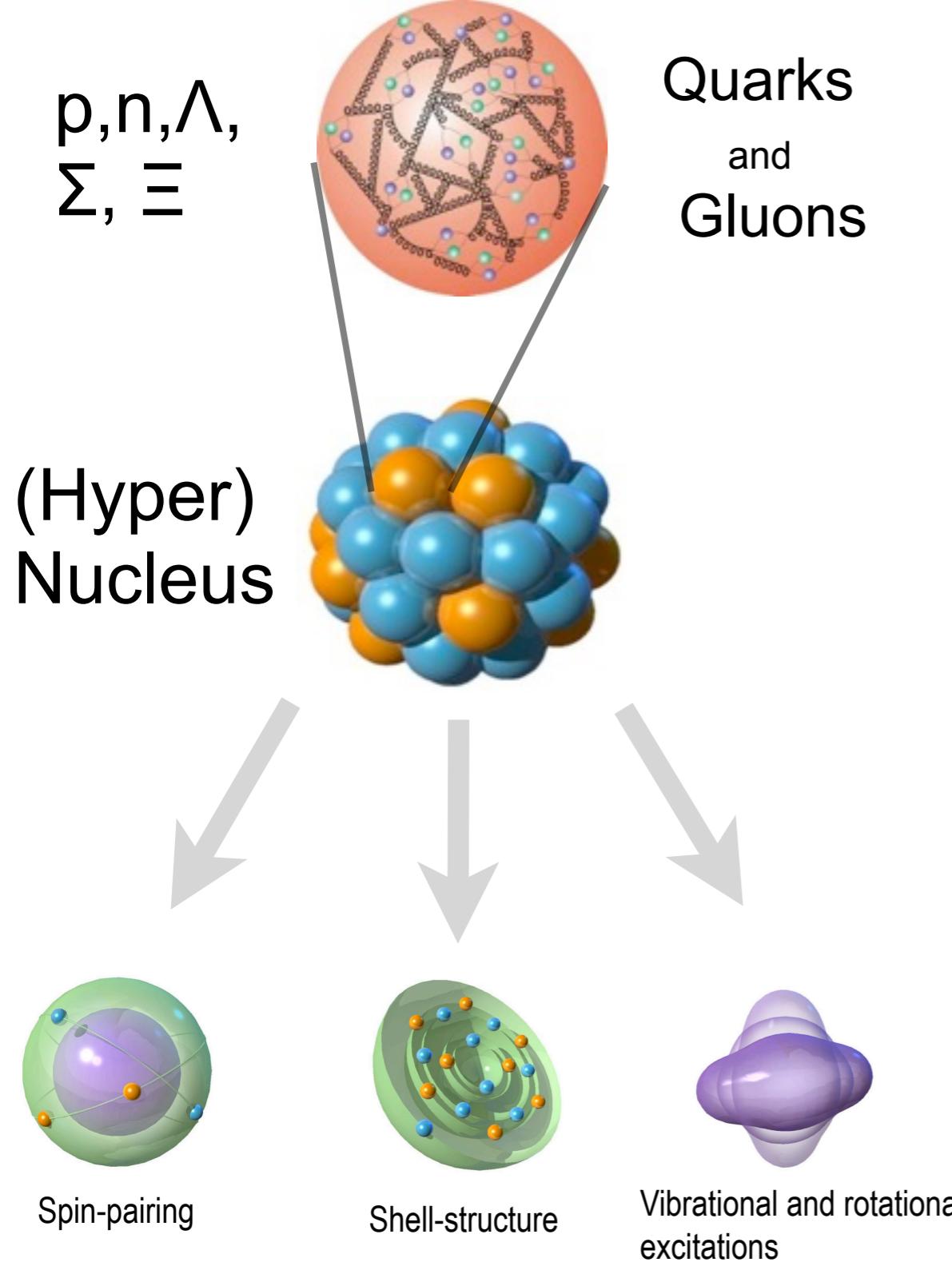
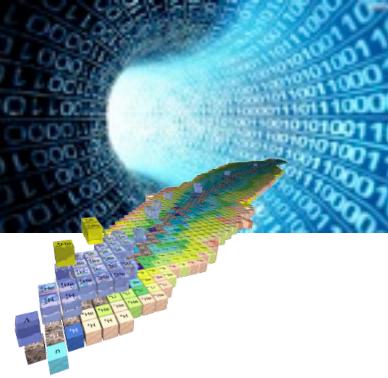
(DM context, see Hoferichter, Klos, Schwenk)

- Lattice QCD

- valid below $p \sim \pi/a$ - corrections to QCD $\sim (a p)^n$ - no truncation
 - $a \sim 0.1$ fm $\sim (2 \text{ GeV})^{-1}$
 - systematically remove by extrapolating calcs with different a



The Structure and Interactions of Matter from Quantum Chromodynamics

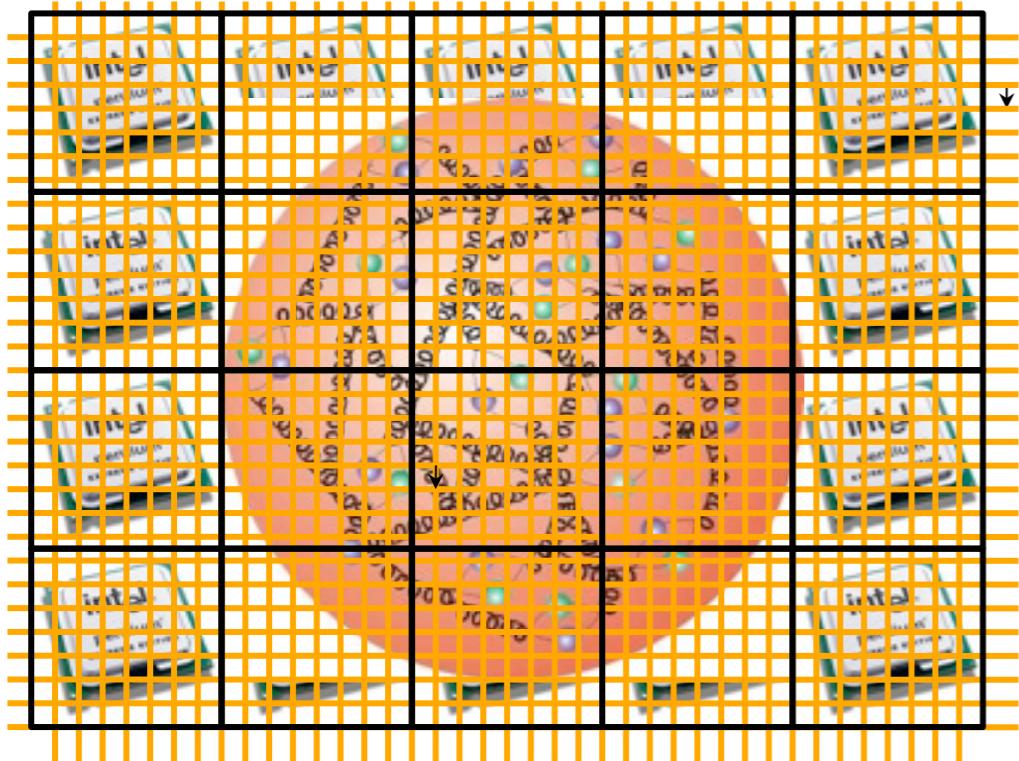


$$\frac{m_u}{\Lambda_{\text{QCD}}} \quad \frac{m_d}{\Lambda_{\text{QCD}}} \quad \frac{m_s}{\Lambda_{\text{QCD}}} \quad \alpha_e$$

Small number of input parameters responsible for all of strongly interacting matter

Lattice QCD

Using a Discretized Spacetime



Lattice Spacing :
 $a \ll 1/\Lambda\chi$
(Nearly Continuum)

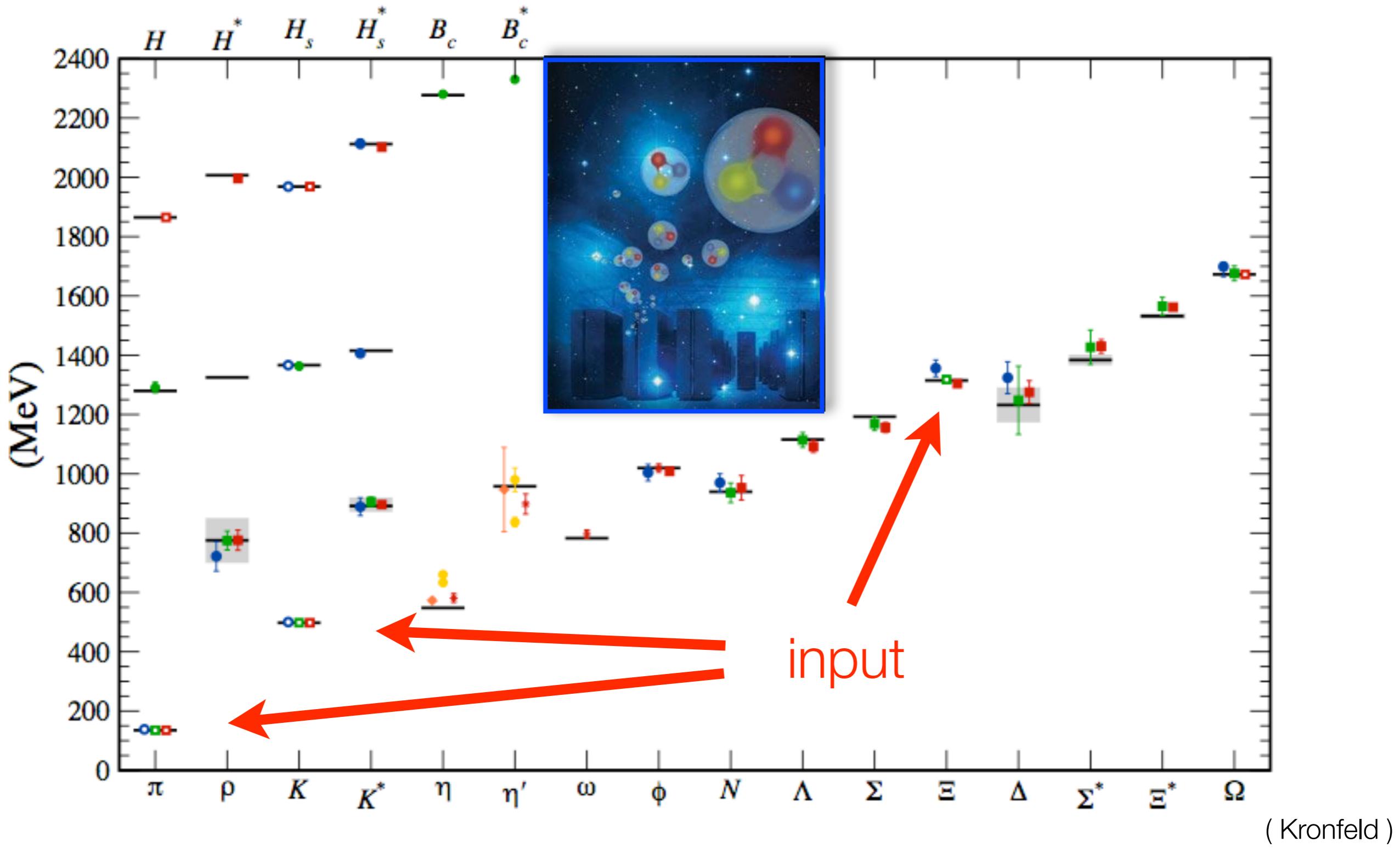
Lattice Volume :
 $m_\pi L \gg 2\pi$
(Nearly Infinite Volume)

Extrapolation to $a = 0$ and $L = \infty$

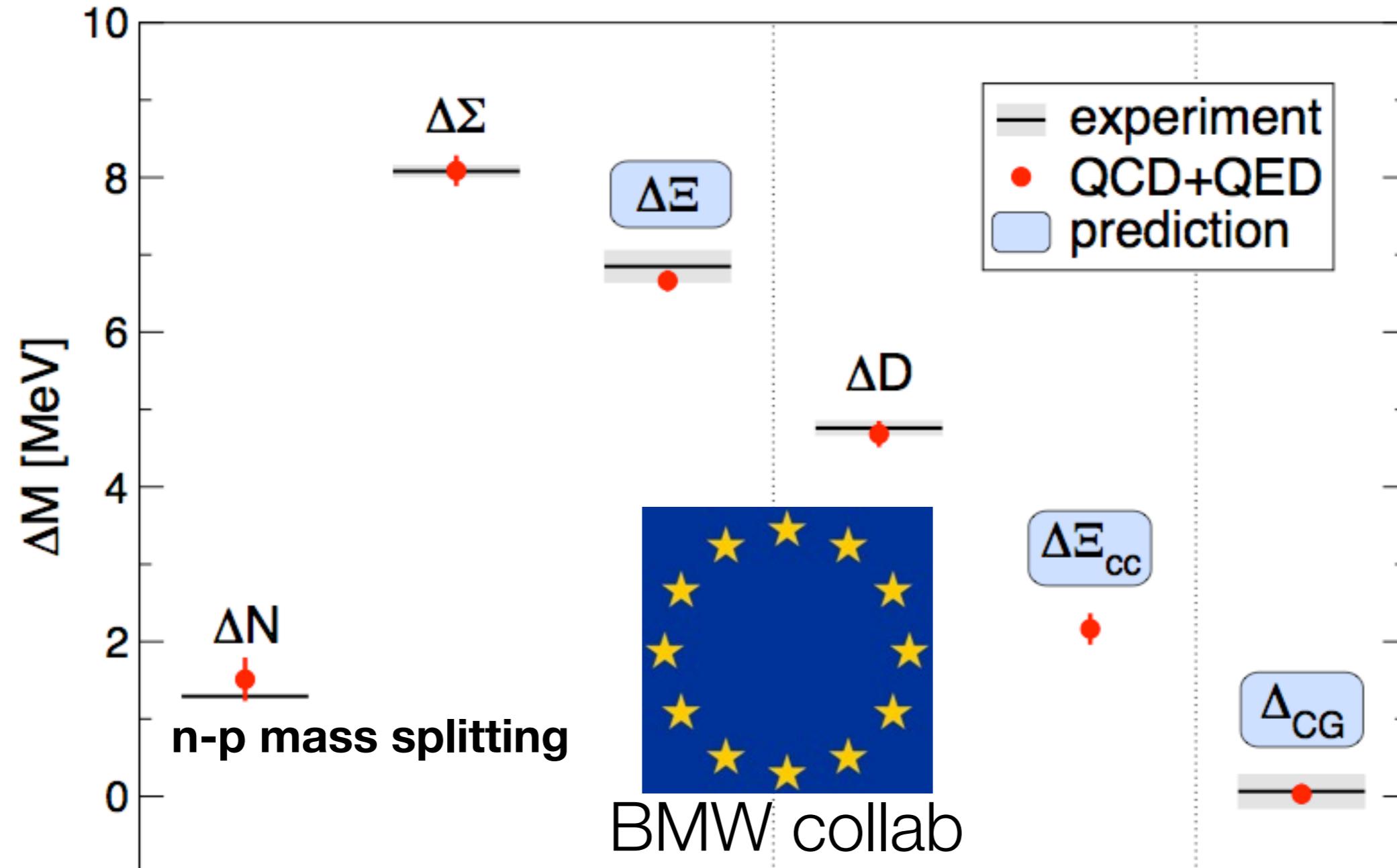
Systematically remove non-QCD parts of calculation

$$\langle \hat{\theta} \rangle \sim \int \mathcal{D}\mathcal{U}_\mu \hat{\theta}[\mathcal{U}_\mu] \det[\kappa[\mathcal{U}_\mu]] e^{-S_{YM}} \rightarrow \frac{1}{N} \sum_{\text{gluon cfgs}}^N \hat{\theta}[\mathcal{U}_\mu]$$

Hadron Masses

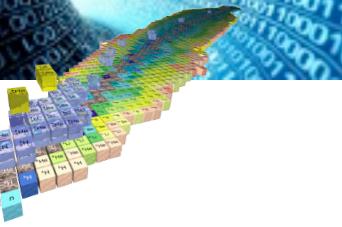


The Bleeding Edge of Lattice QCD

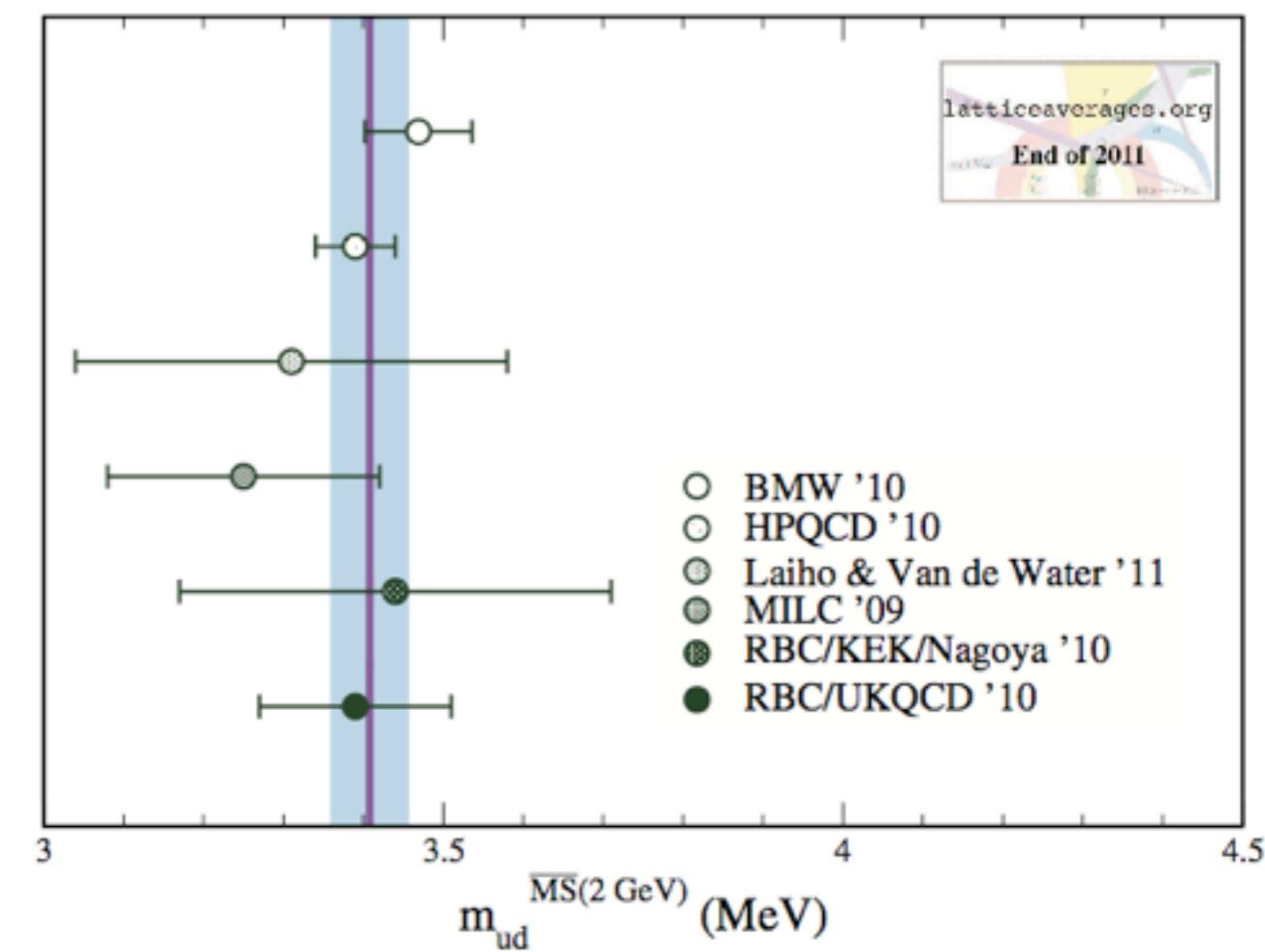
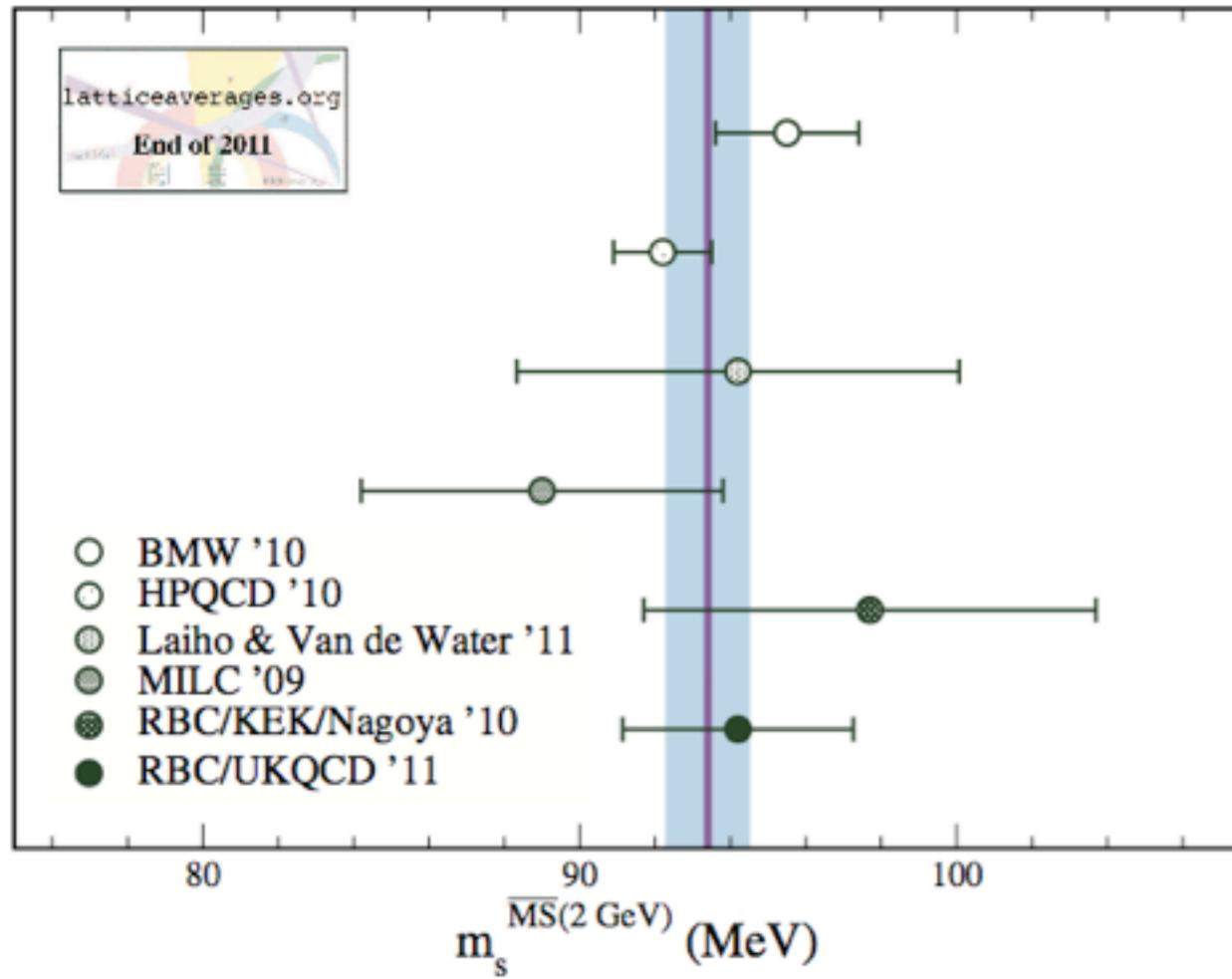


- Physical up, down, strange and charm quark masses
- Fully dynamical QCD+QED

Quark Masses

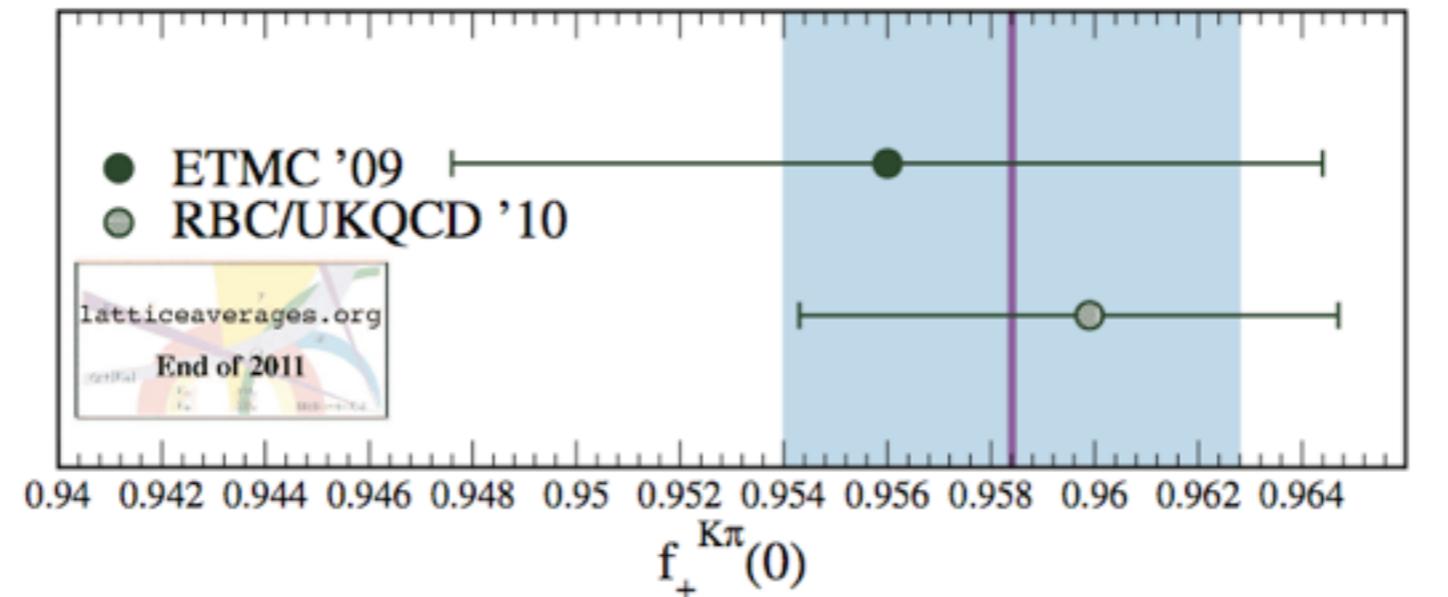


(Laiho, 2013)

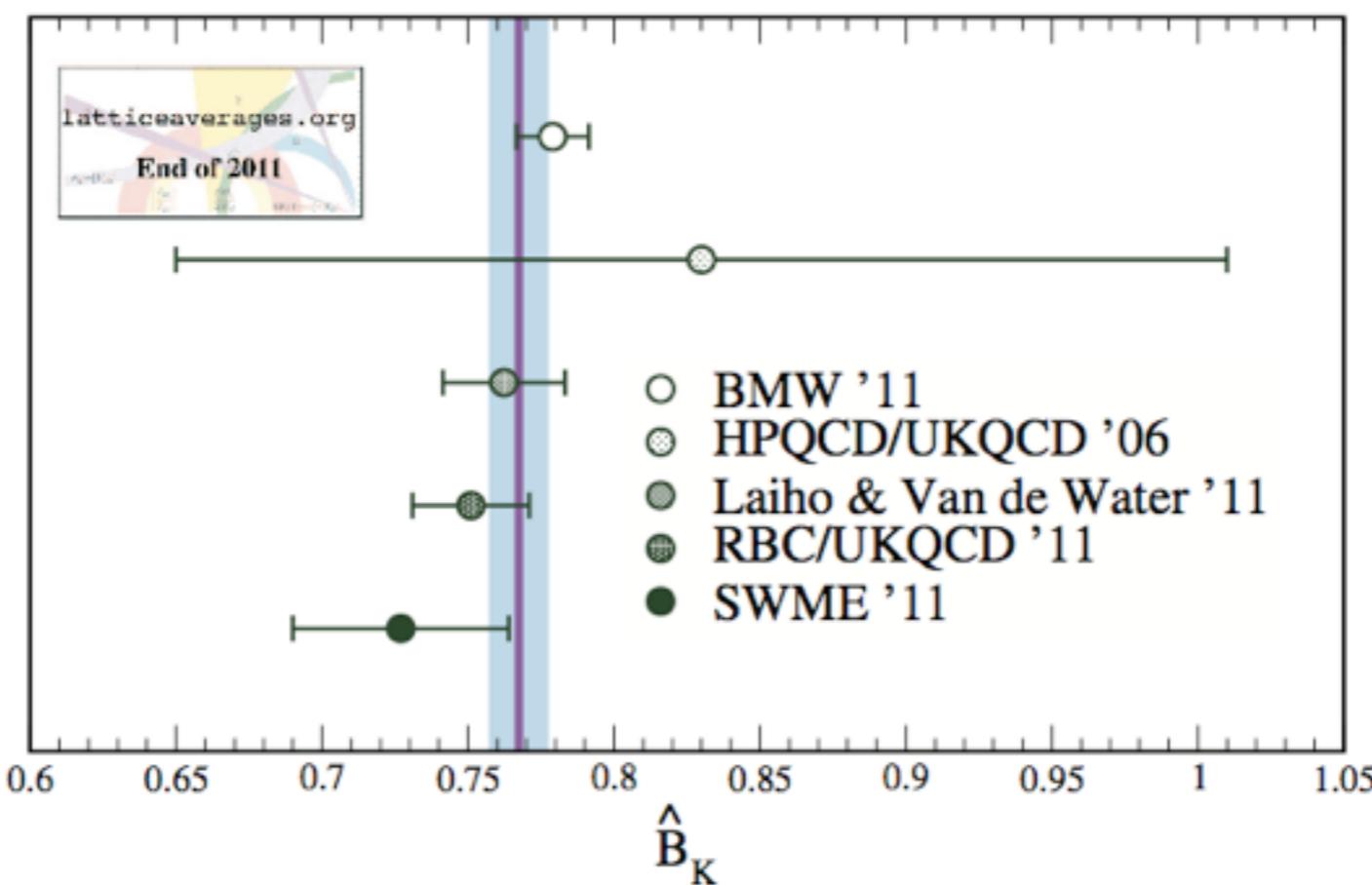


Form Factors and the “Vacuum Insertion”

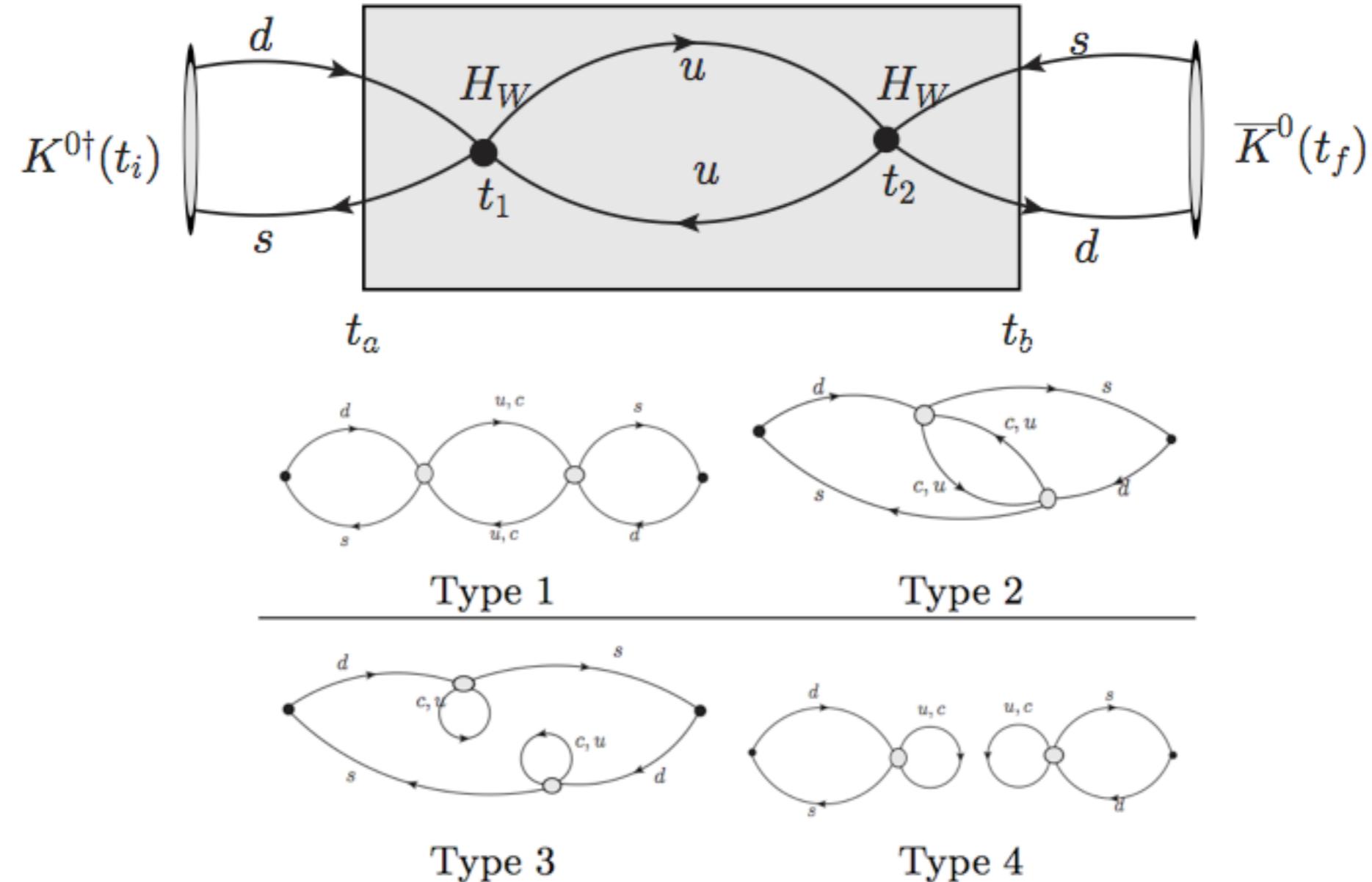
(Laiho, 2013)



$K \rightarrow \pi \ell \nu$



K_L-K_S Mass Difference



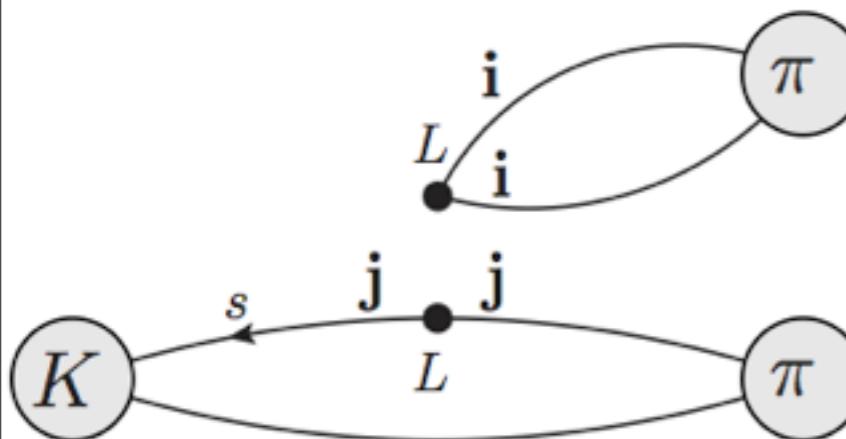
$m_\pi \sim 330$ MeV **RBC/UKQCD**

$$\Delta M_K = 3.19(41)(96) \times 10^{-12} \text{ MeV}$$

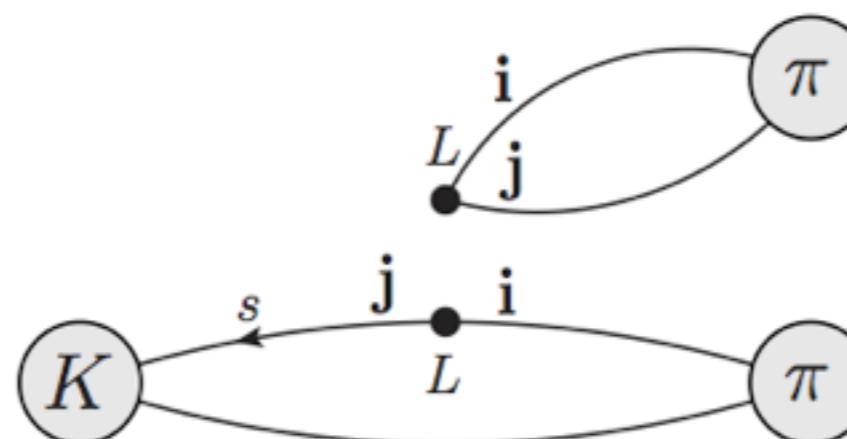
Expt:
 $3.483(6) \times 10^{-12} \text{ MeV}$

Weak Decays and the $\Delta I = 1/2$ rule

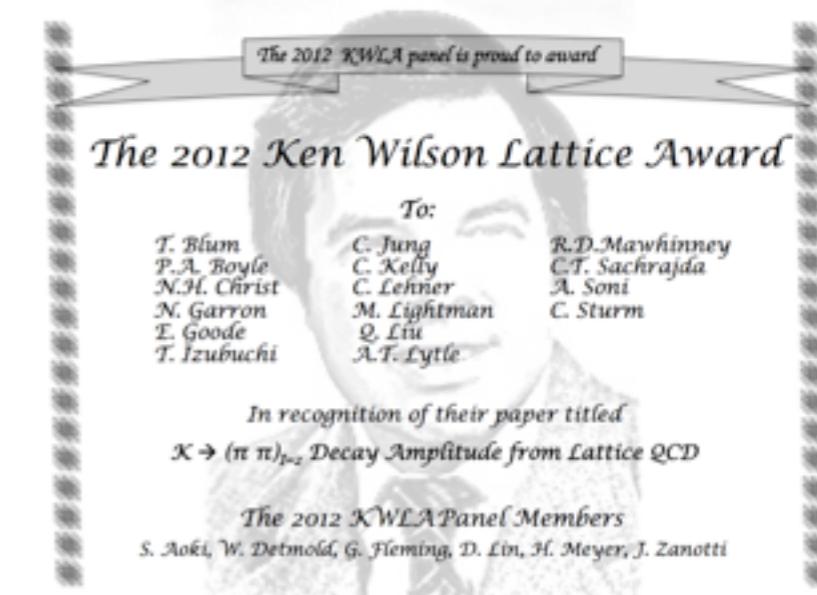
$$K \rightarrow \pi\pi$$



Contraction ①.



Contraction ②.



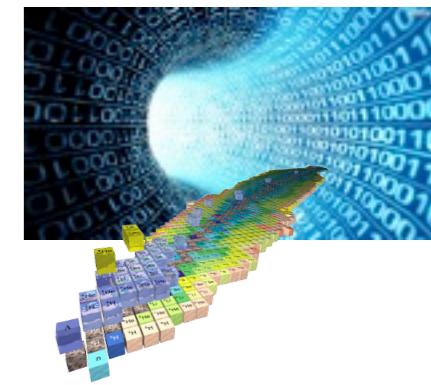
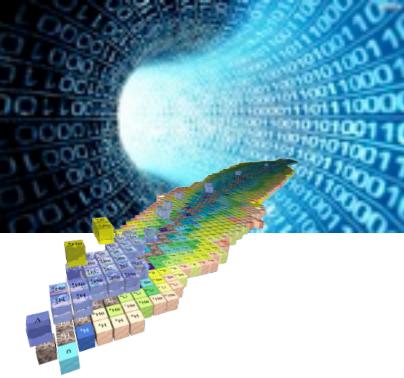
RBC/UKQCD

$$\frac{\text{Re}A_0}{\text{Re}A_2} = \begin{cases} 9.1(2.1) & \text{for } m_K = 878 \text{ MeV}, m_\pi = 422 \text{ MeV} \\ 12.0(1.7) & \text{for } m_K = 662 \text{ MeV}, m_\pi = 329 \text{ MeV} \end{cases}$$

$$\Delta I = 1/2 \text{ rule}$$

$$\text{Re}A_0/\text{Re}A_2 \simeq 22.5$$

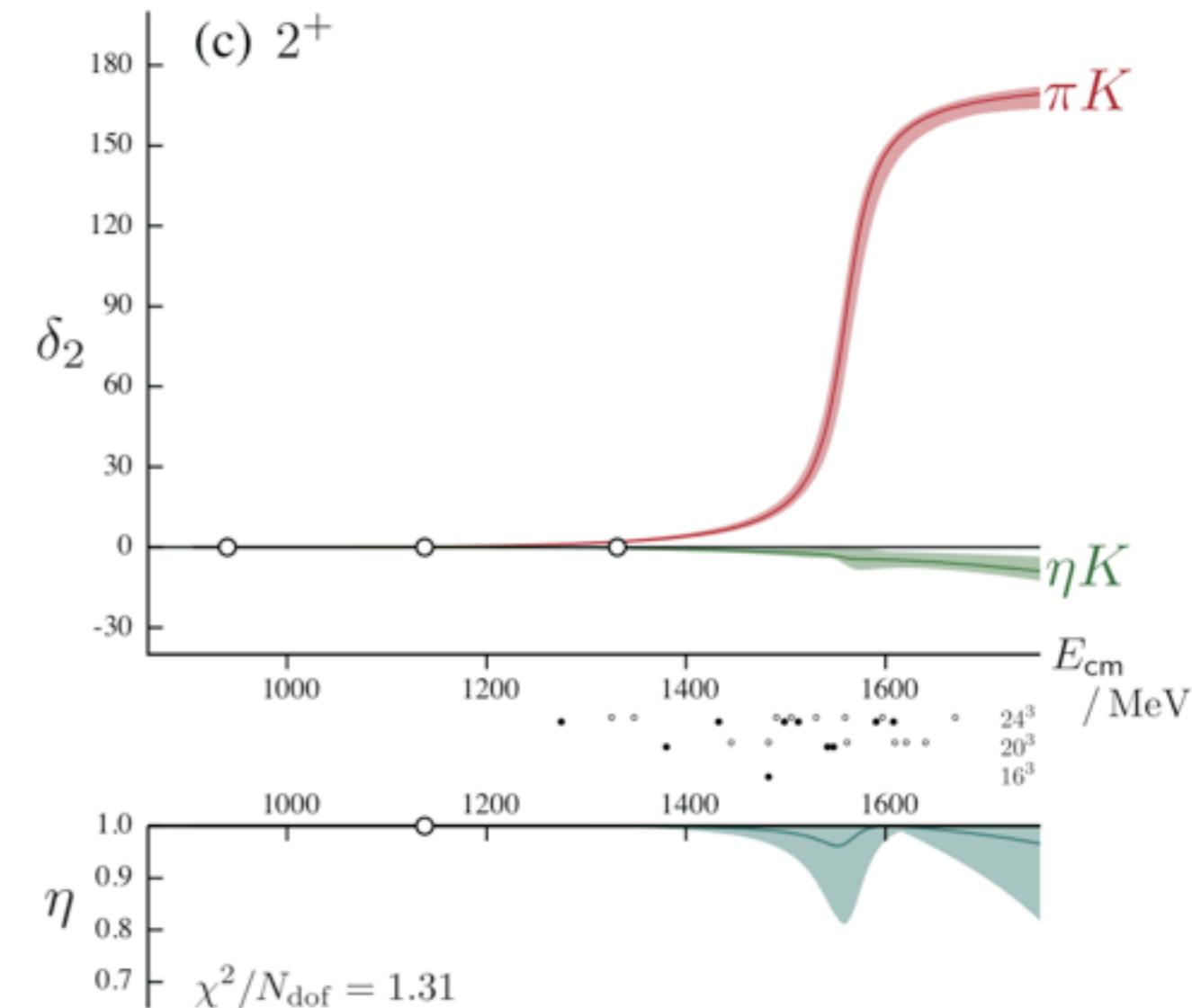
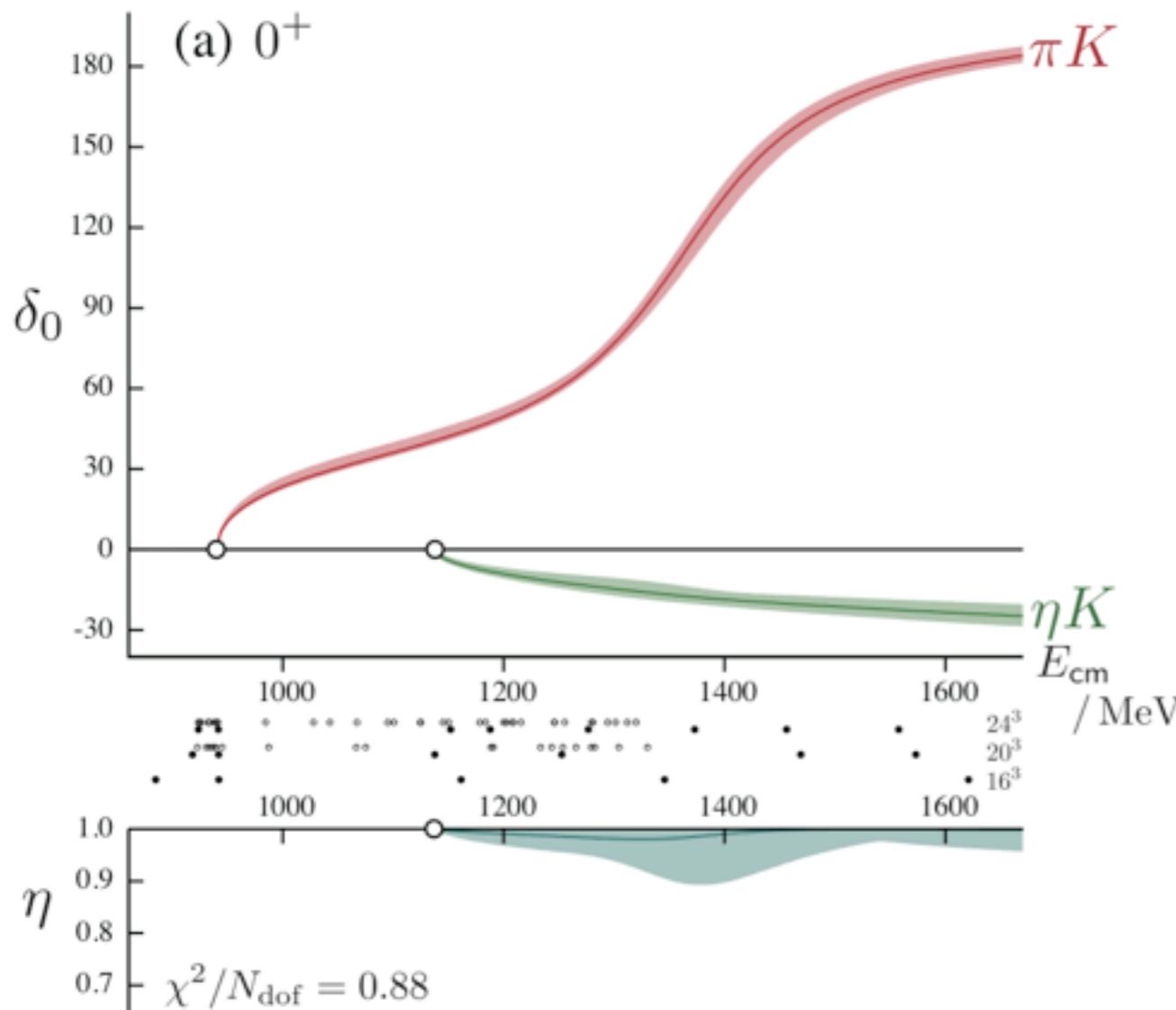
Scattering: Coupled Channels Calculations



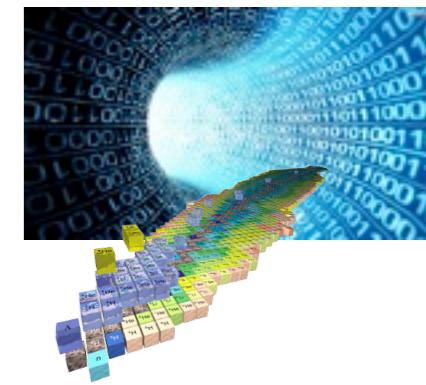
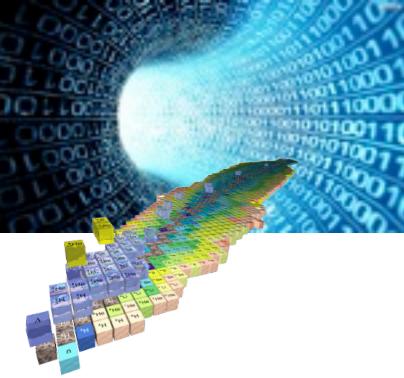
Dudek et al

$m_\pi \sim 390$ MeV

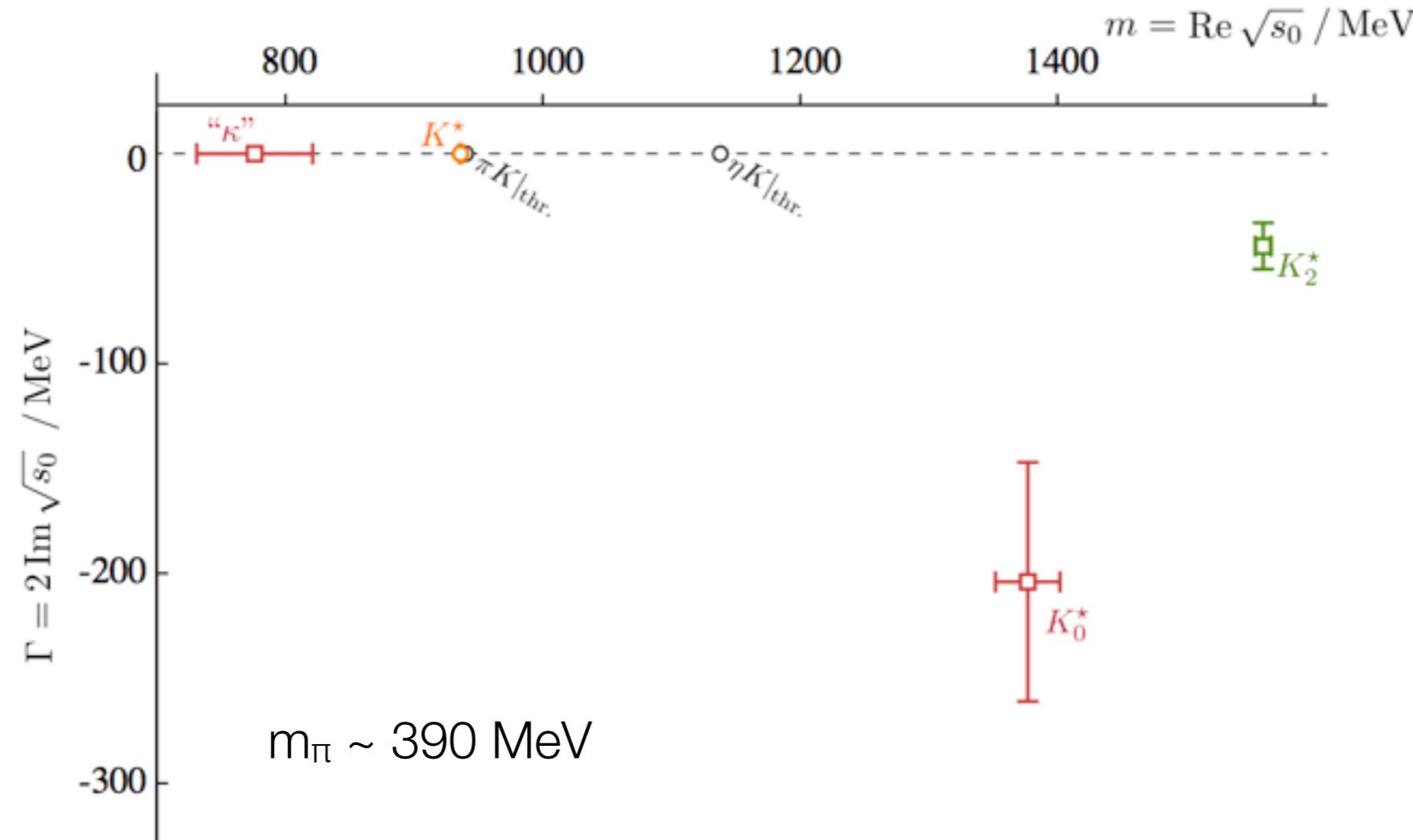
Jefferson Lab



Scattering: Coupled Channels Calculations



Dudek et



Jefferson Lab

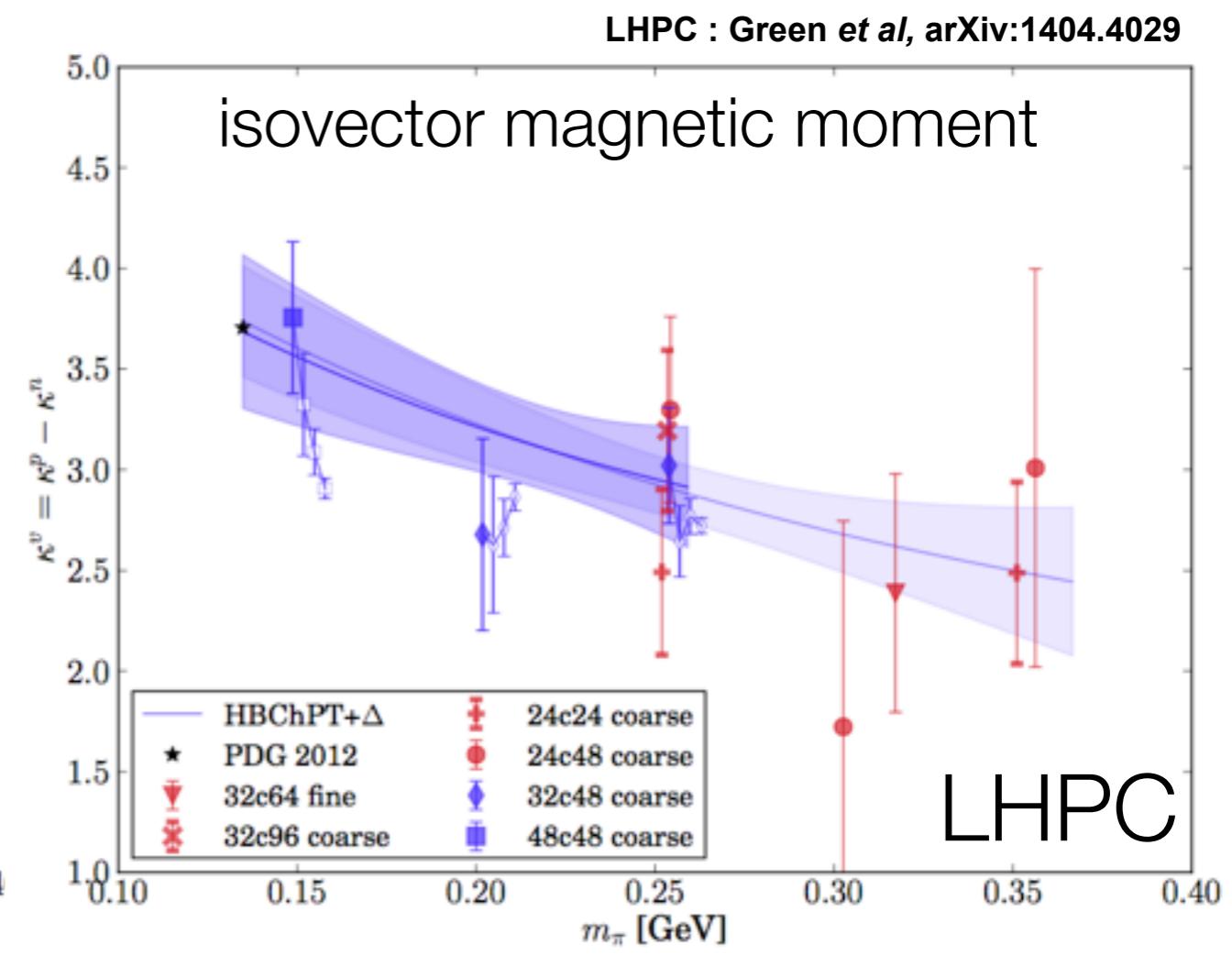
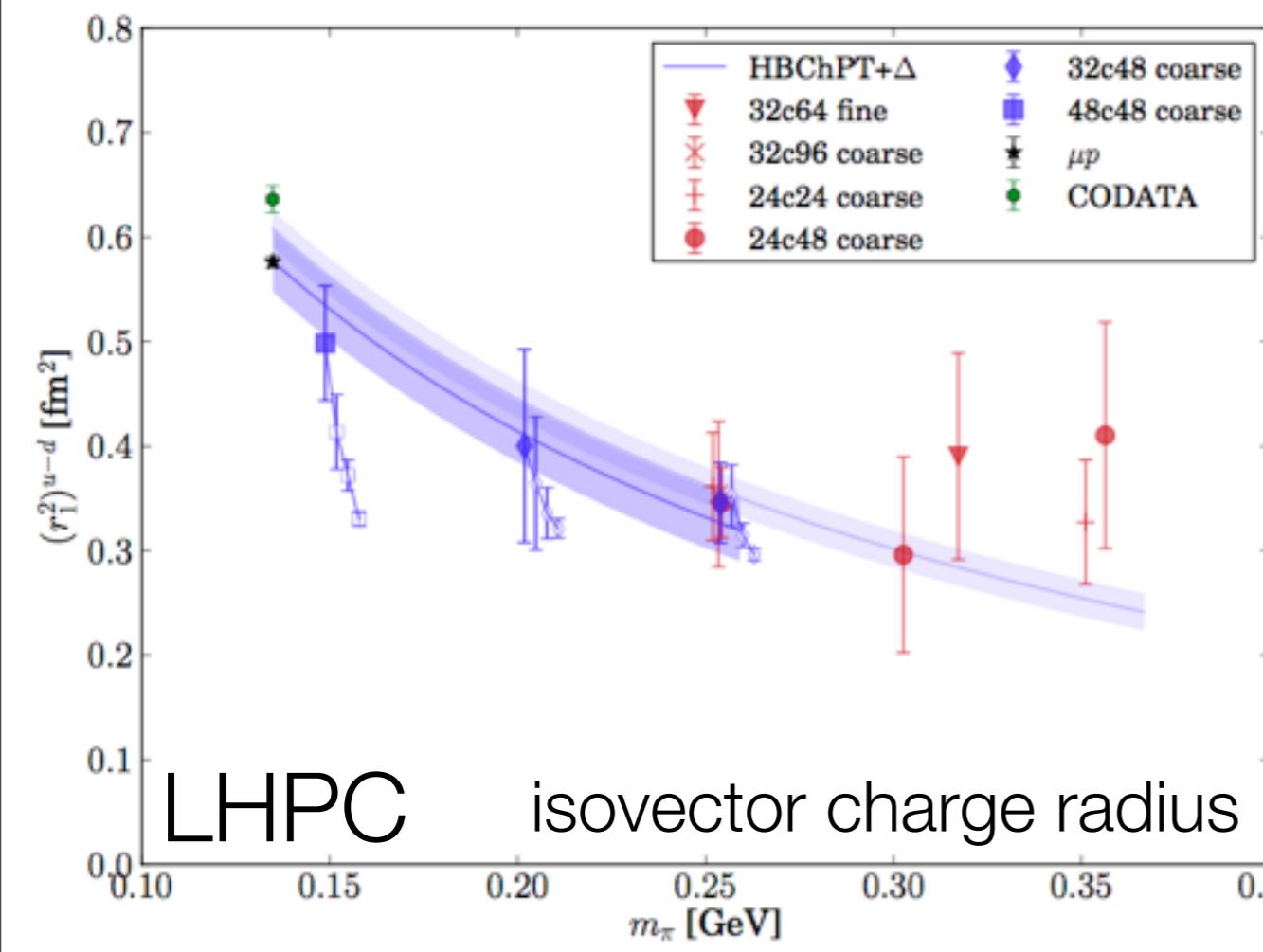
- Bound states and Resonances from T-matrix
- QCD predictions at these quark masses
- Efforts to extend to multi-hadron coupled channels

Nucleon Structure

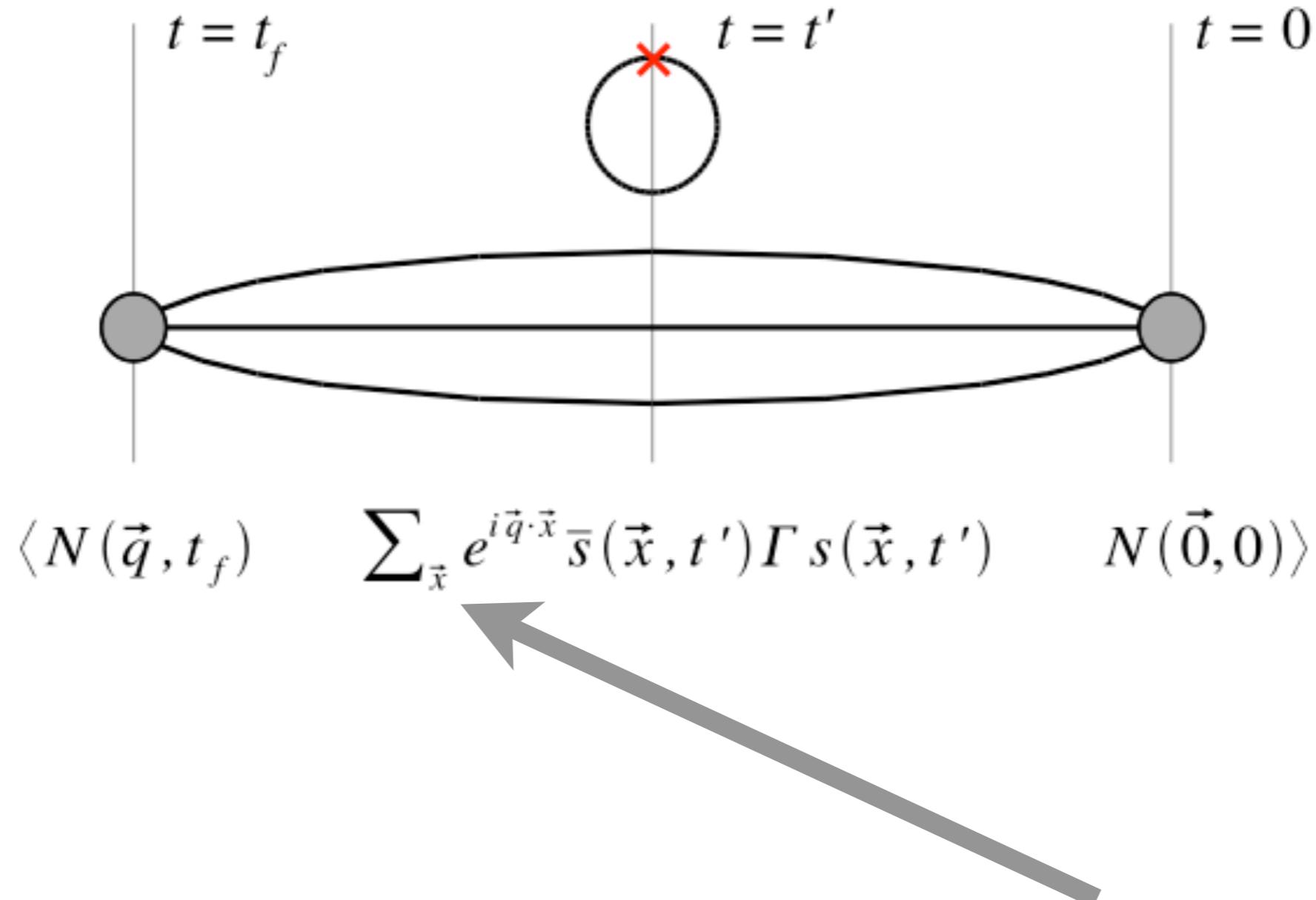
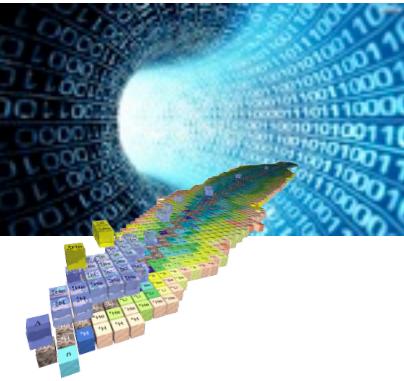


First LQCD calculations at physical pion mass during 2012

PDG value of neutron properties

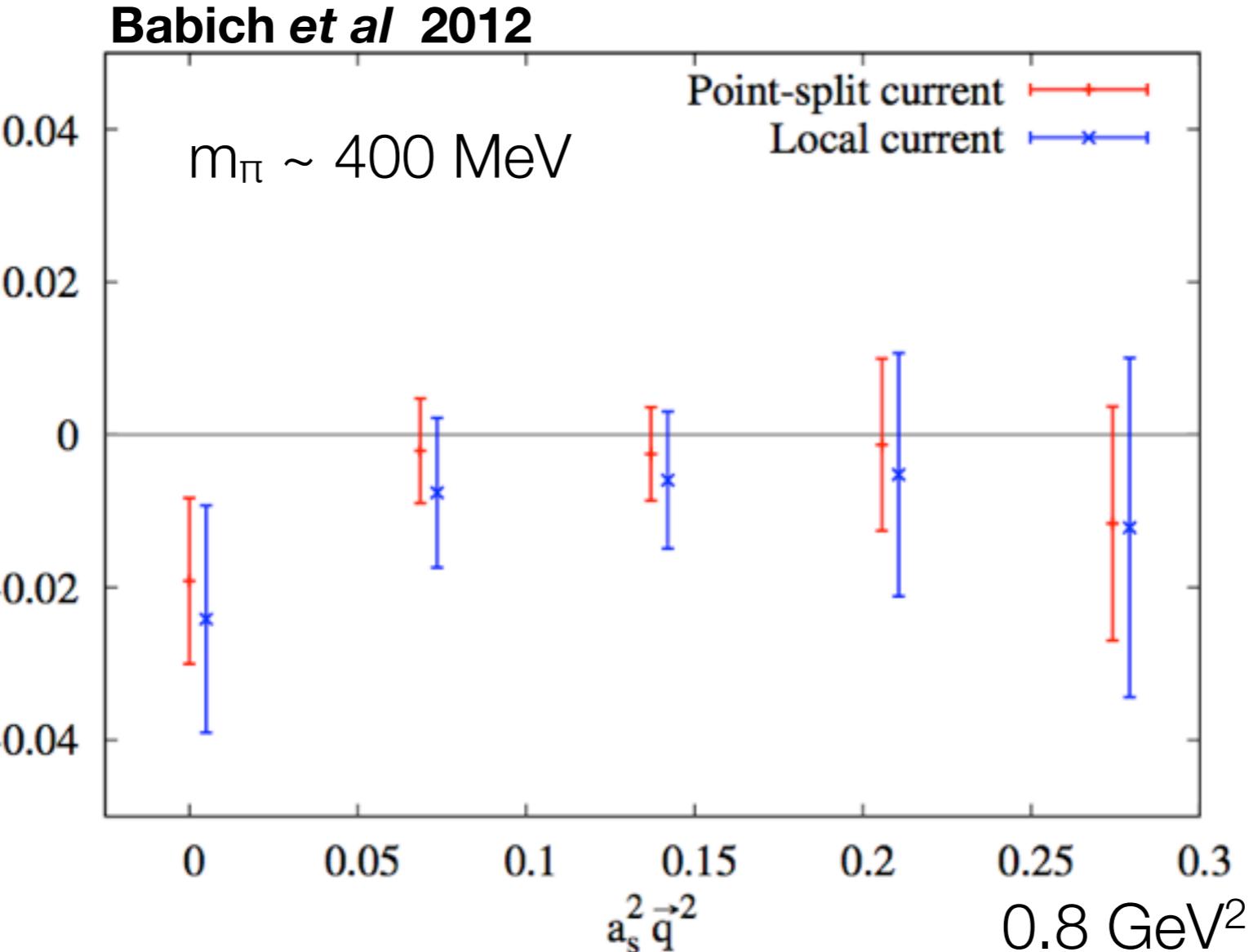


Strange Form Factors of the Nucleon



- Disconnected diagram only
- Need propagator at each point in the lattice volume

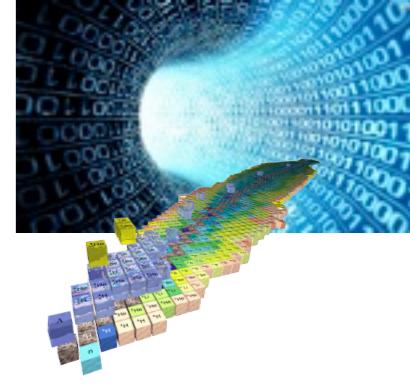
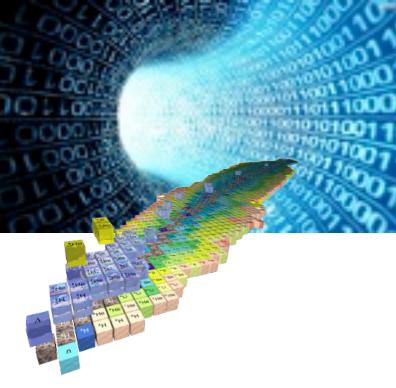
Strange Form Factors of the Nucleon



Engelhardt (2012)

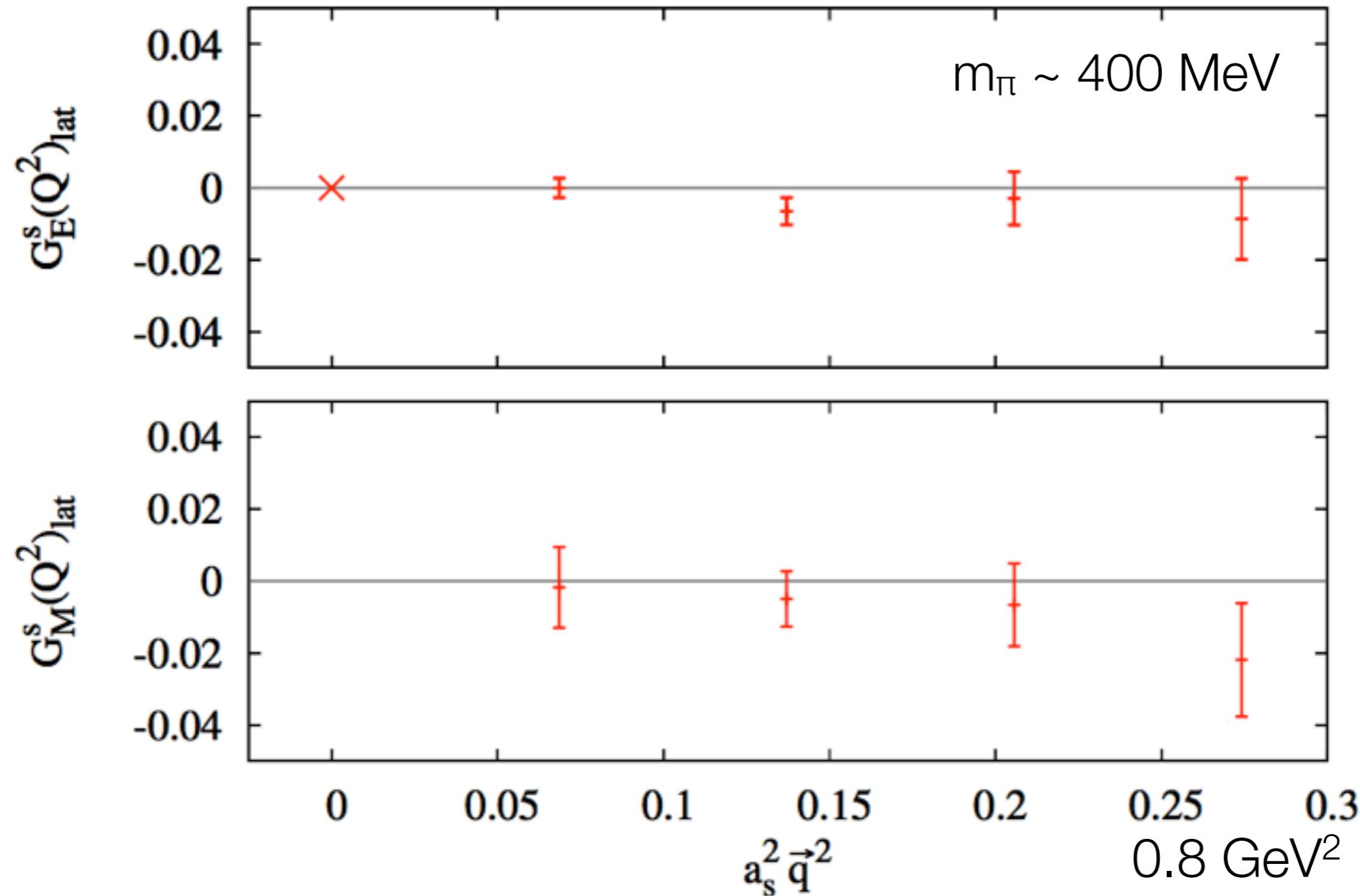
$$\Delta s = -0.031(16)(^{+3}_{-1})(1)(4)(3)(4)$$

$m_\pi \sim 140 \text{ MeV}$



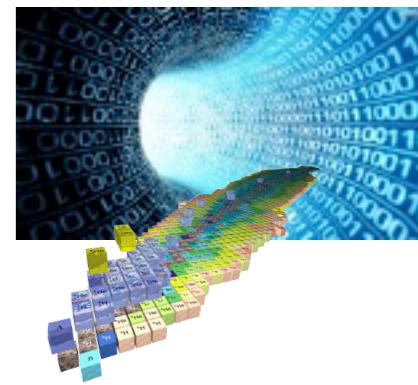
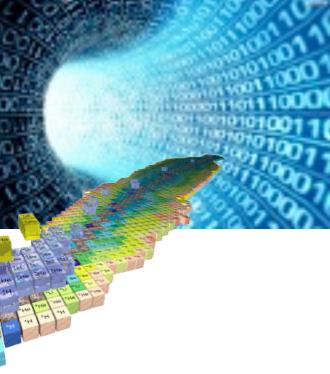
Strange Form Factors of the Nucleon

Babich et al 2012



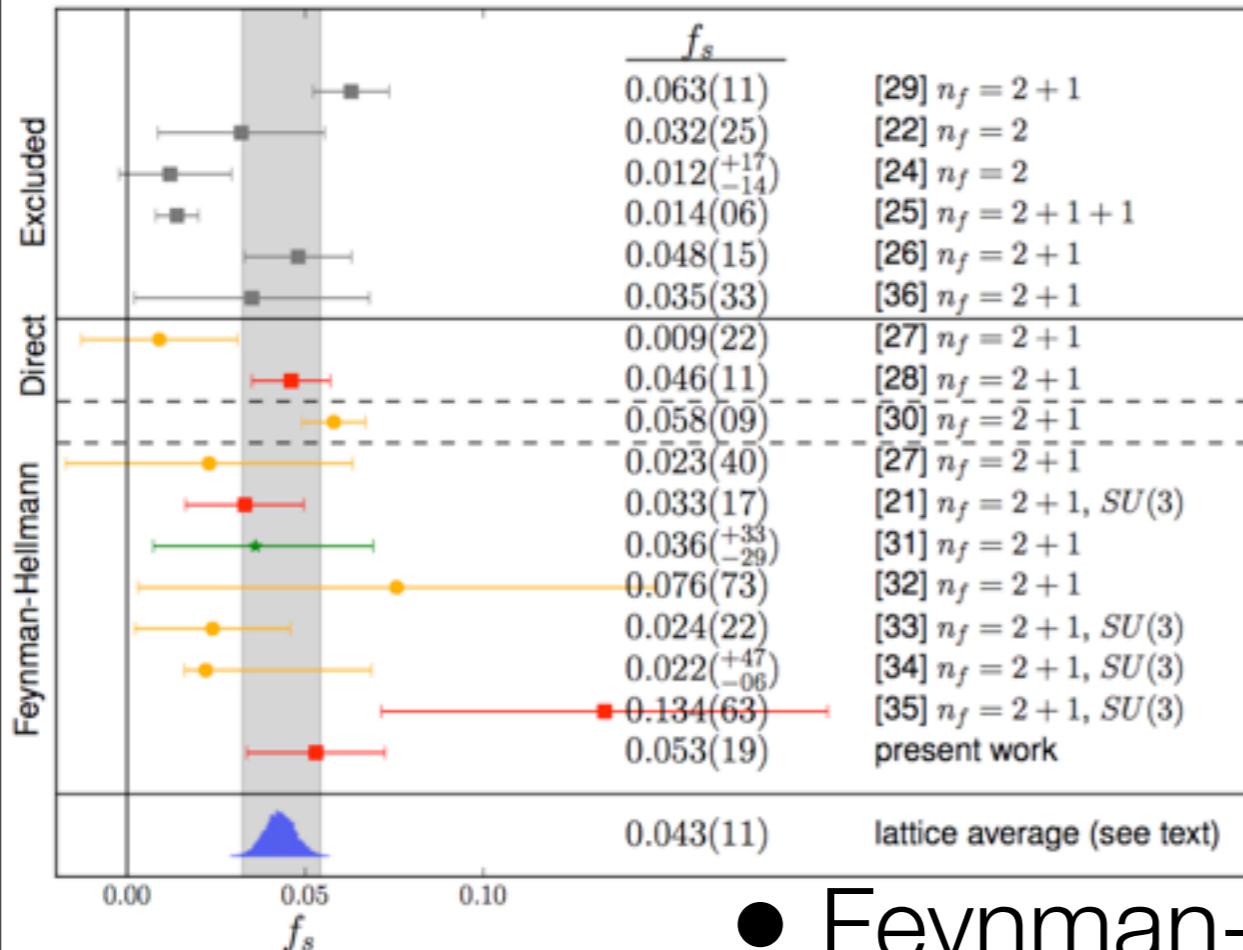
- Recent analysis : Shanahan et al, [arXiv:1403.6537](https://arxiv.org/abs/1403.6537)

Nucleon Matrix Elements



Pari Junnarkar's PhD thesis @ UNH (Beane)

Junnarkar and Walker-Loud, Phys.Rev. D87 (2013) 11, 114510

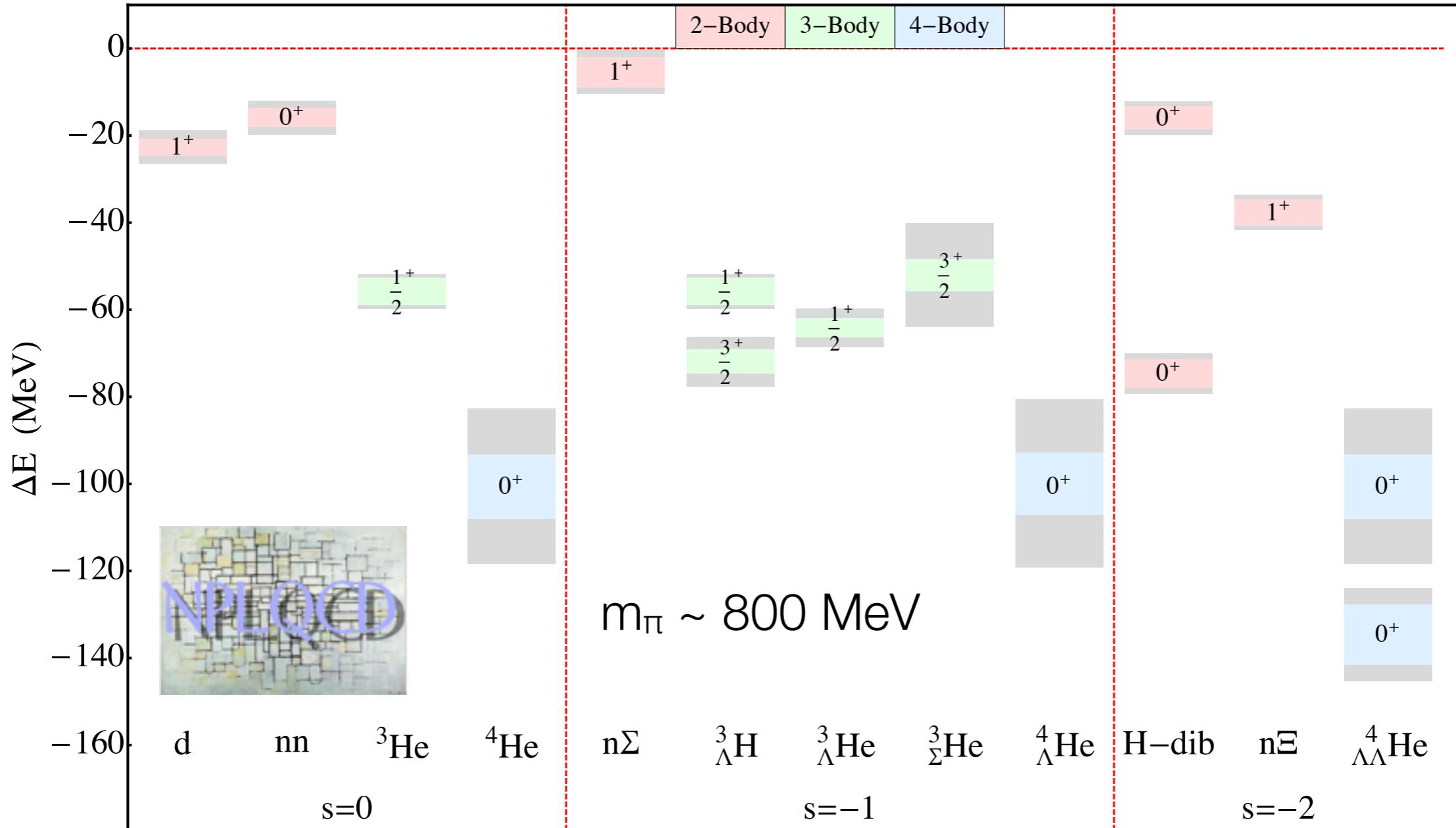


$$m_s \langle N | \bar{s}s | N \rangle = 48 \pm 10 \pm 15 \text{ MeV},$$
$$f_s = 0.051 \pm 0.011 \pm 0.016$$

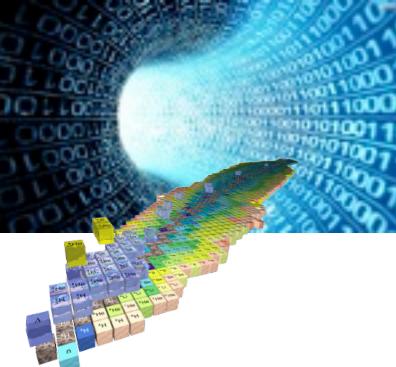
- Feynman-Hellman - 2pt function
 - M_N as a function of m_s
 - multiple ensembles of lattices
- Operator Insertion
 - multiple propagator calculations

(Hyper)Nuclei from QCD

Beane et al, Phys.Rev. D87 (2013) 3, 034506, Phys.Rev. C88 (2013) 2, 024003



Extensive study of s-shell nuclei and hypernuclei, and baryon-baryon interactions at SU(3) symmetric point



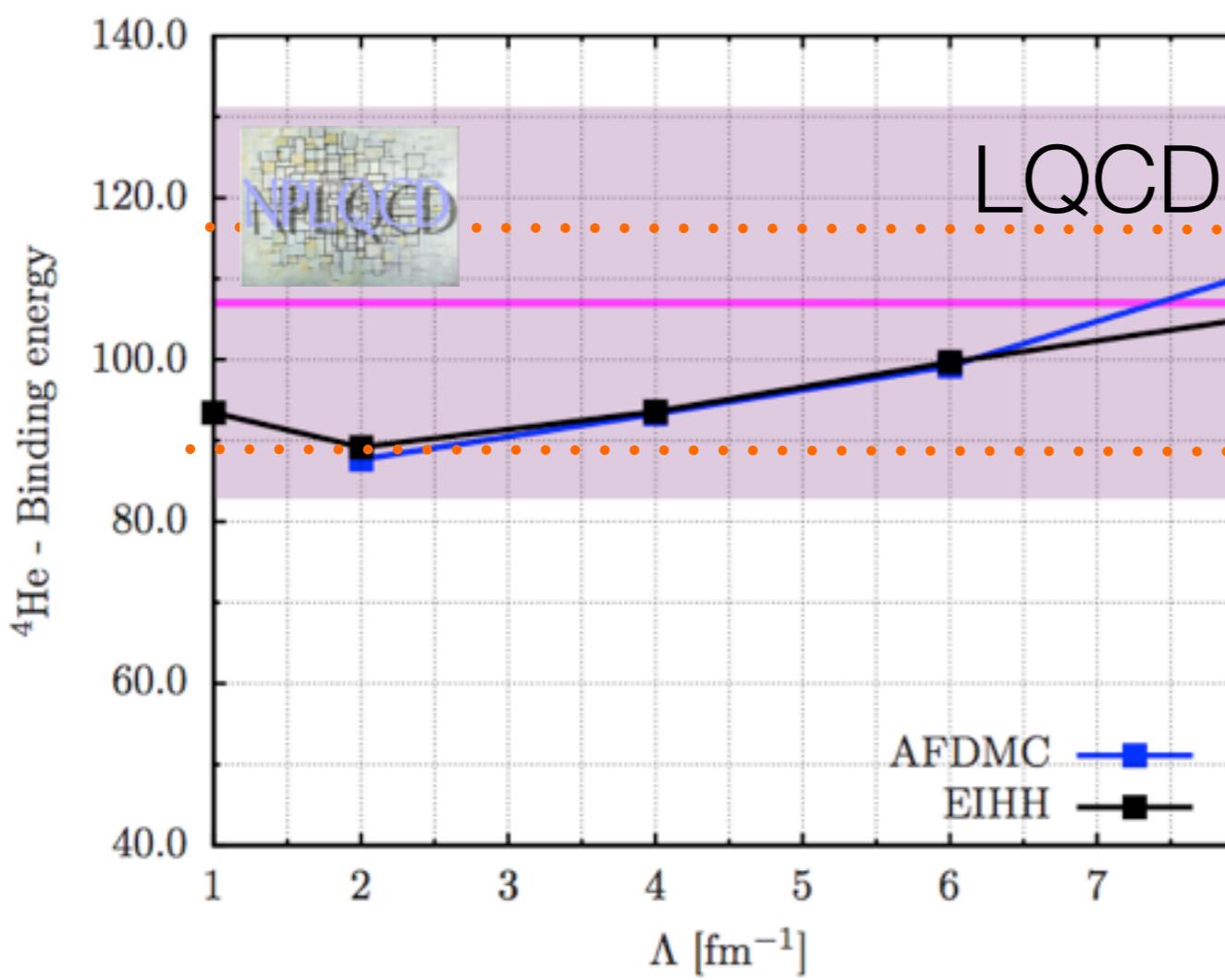
LQCD to Pionless EFT to Nuclei



LQCD Nuclei for 800 MeV pions

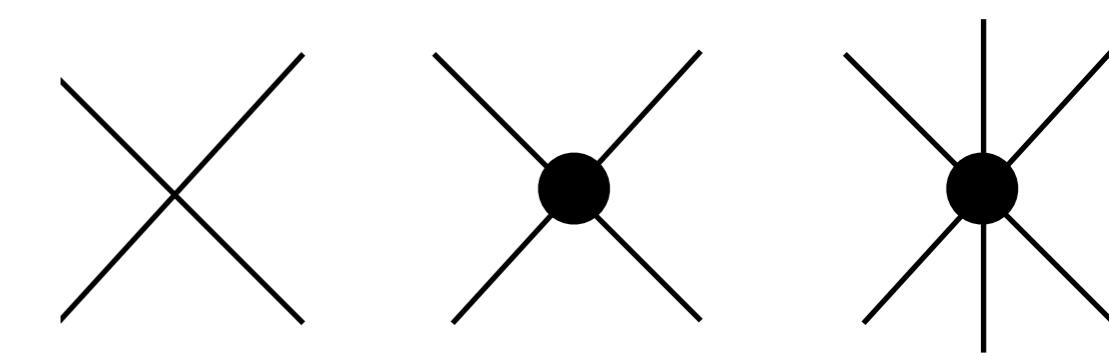
- Fit 2-body and 3-body LQCD bindings
- Predict 4-body, c/w LQCD prediction

Barnea, Congressi, Gazit,
Pederiva and van Kolck
arXiv:1311.4966

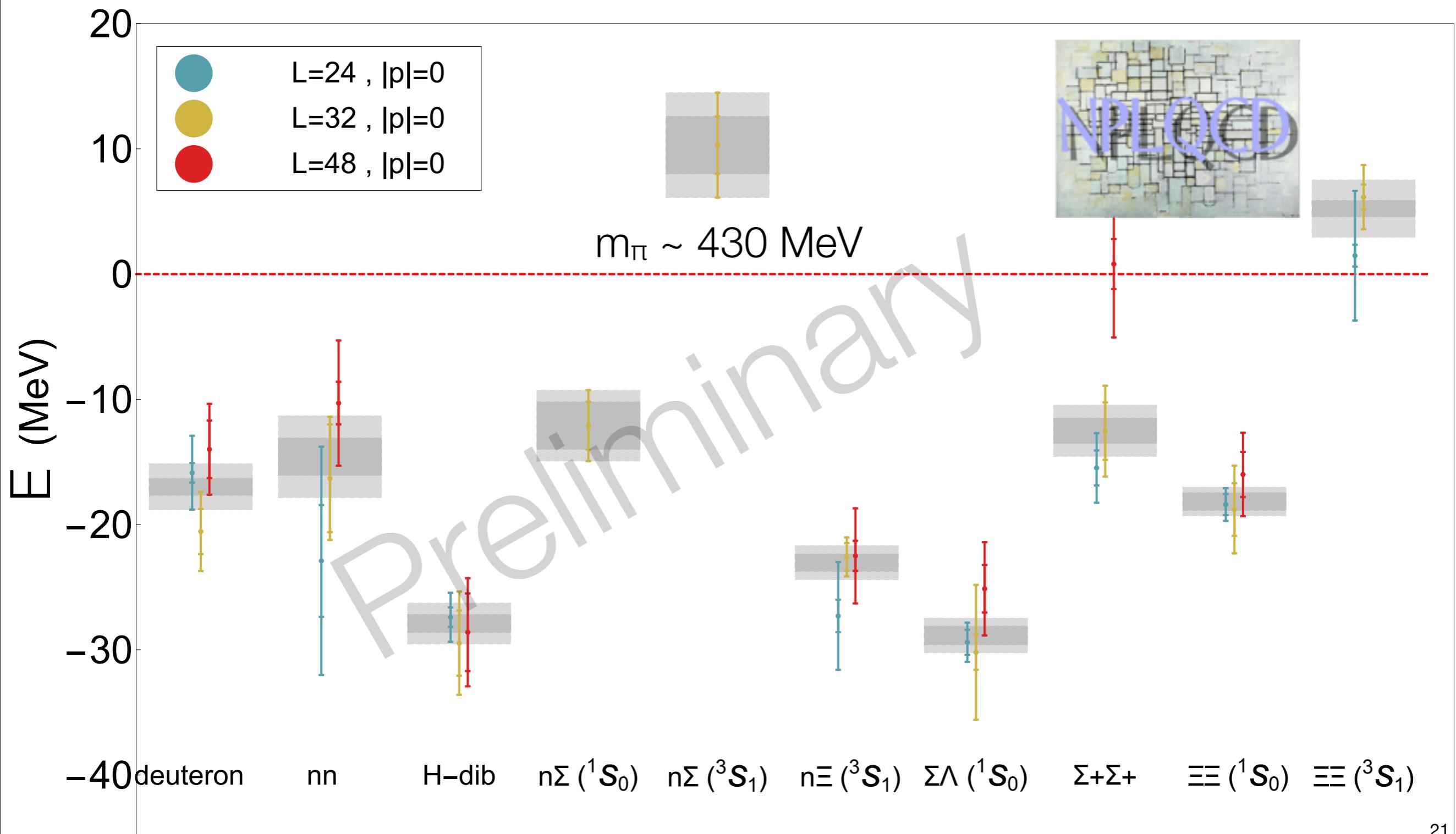
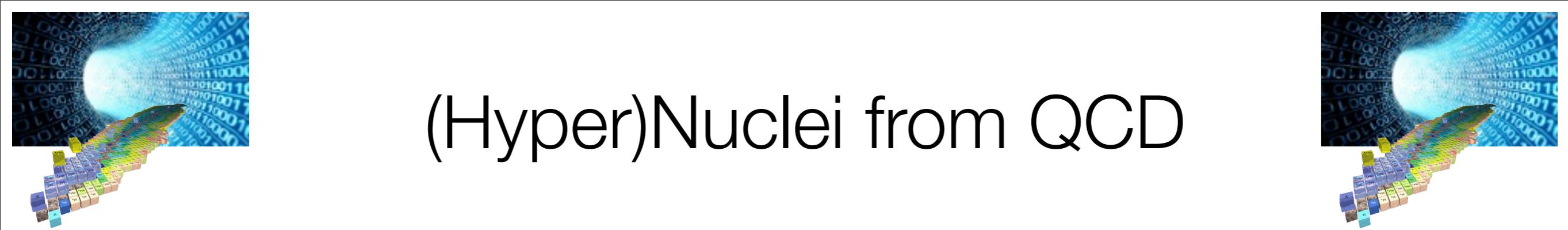


pionless EFT valid
for nuclei

“First Contact”

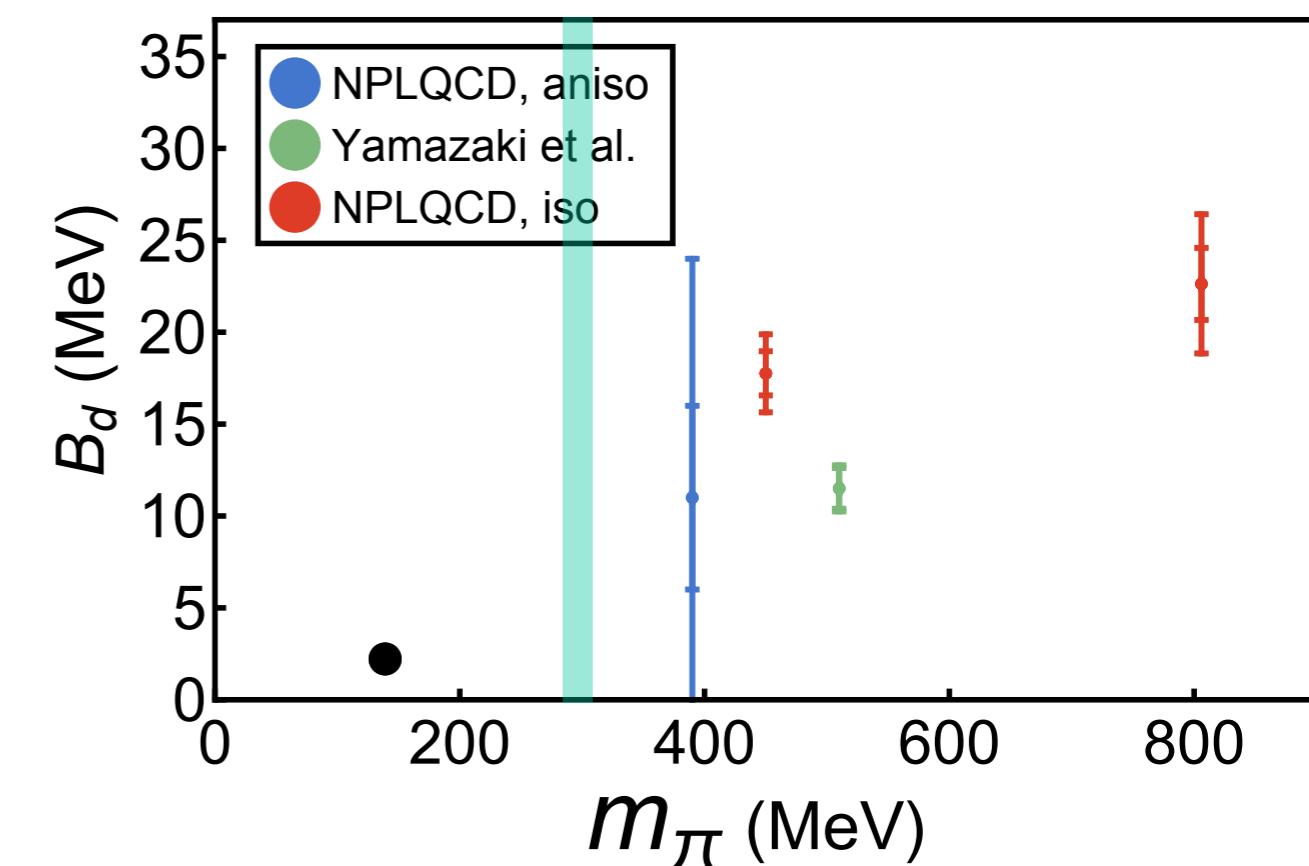
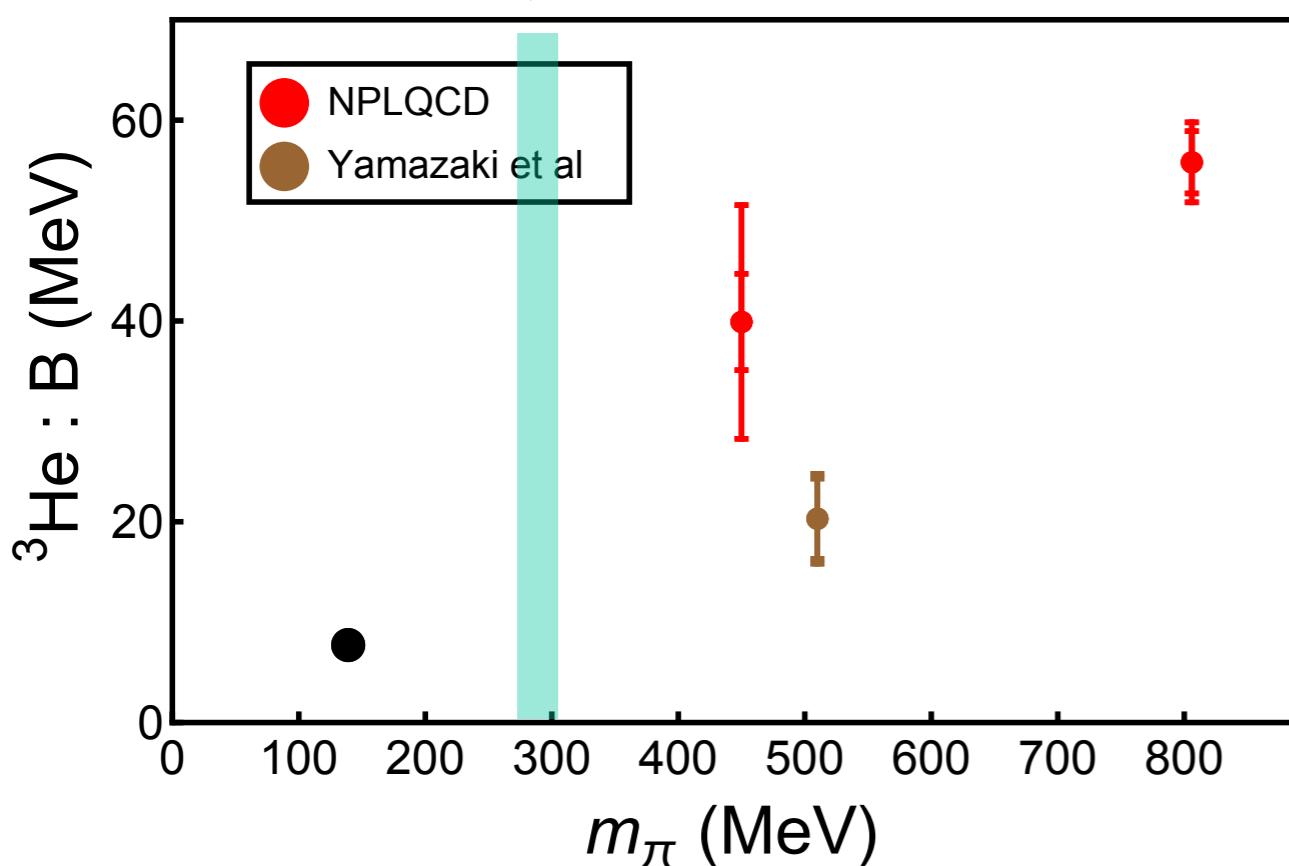


(Hyper)Nuclei from QCD

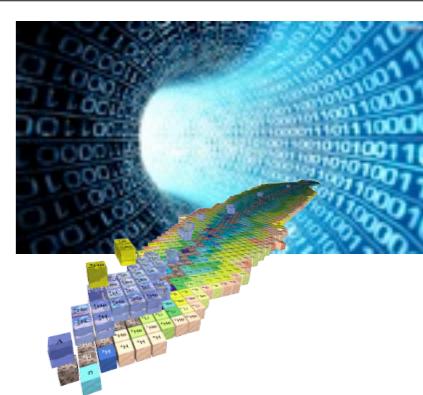


Light Nuclei : Quark Mass Effects

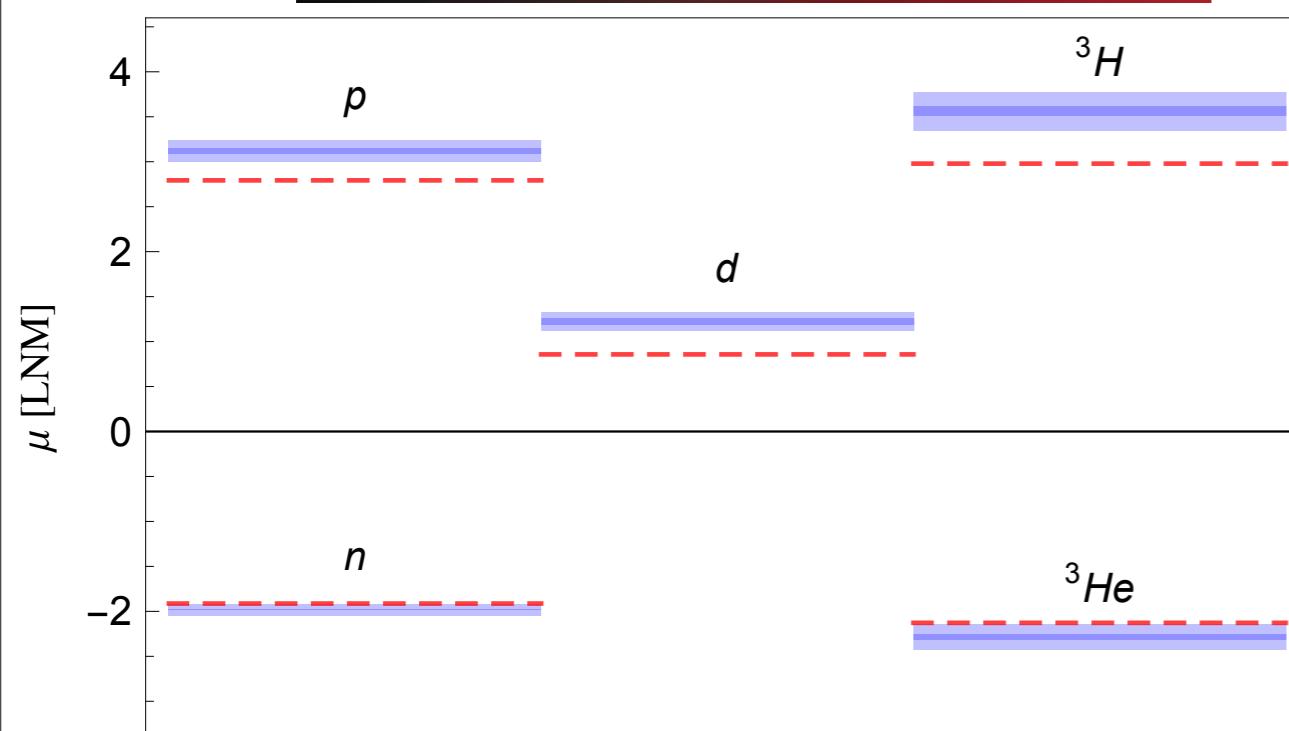
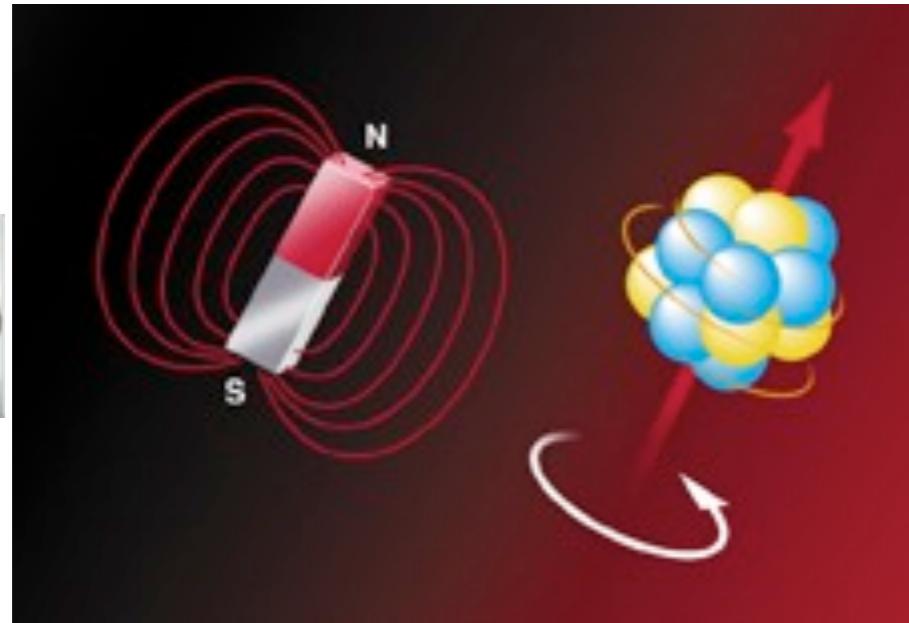
current production



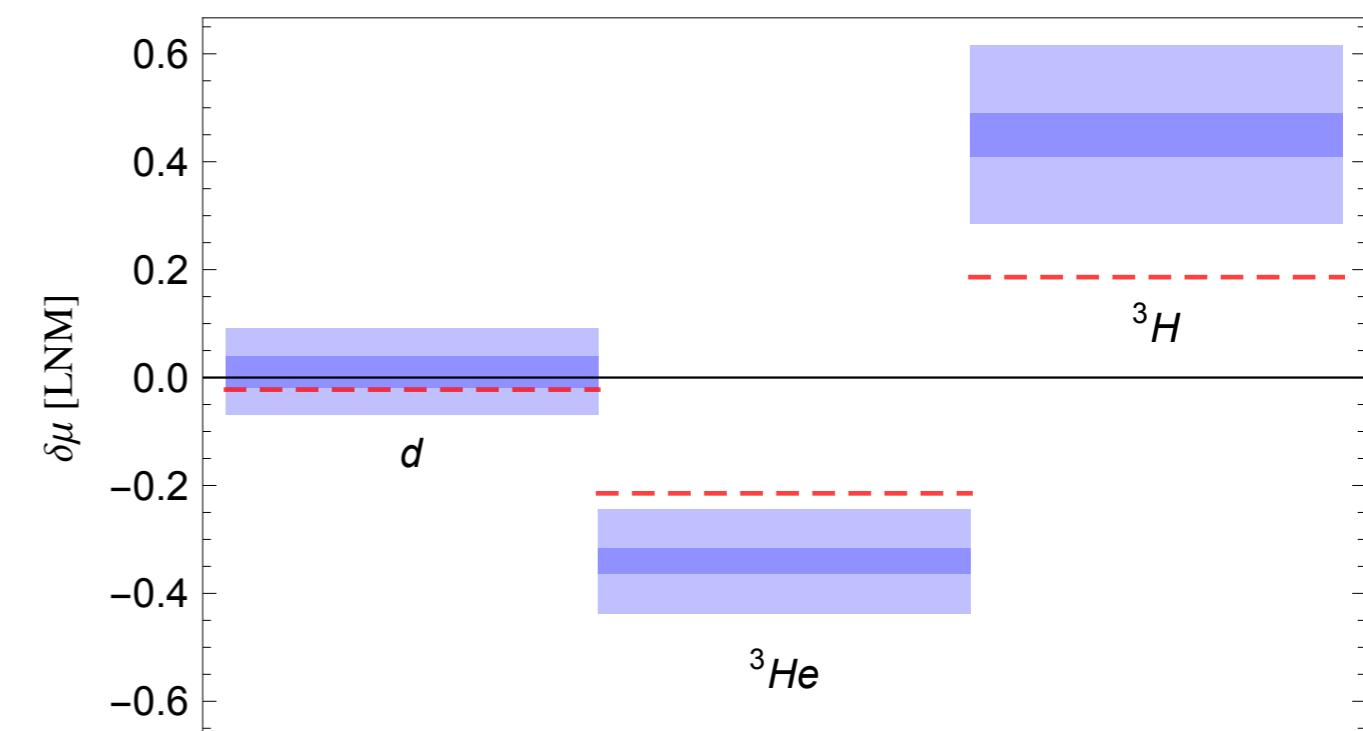
The Structure of Nuclei : Magnetic Moments



NPLQCD

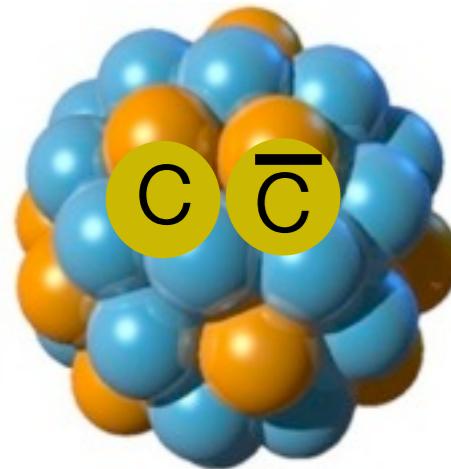
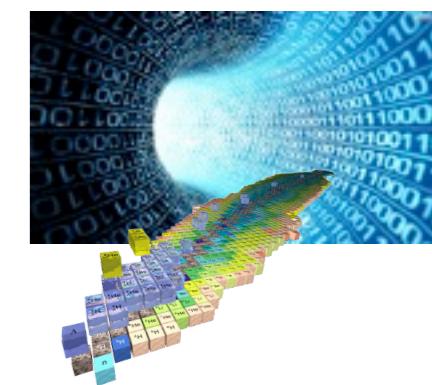


$m_\pi \sim 800$ MeV Vs Nature



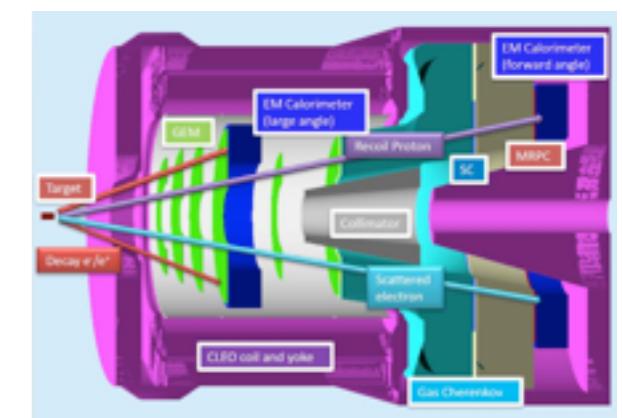
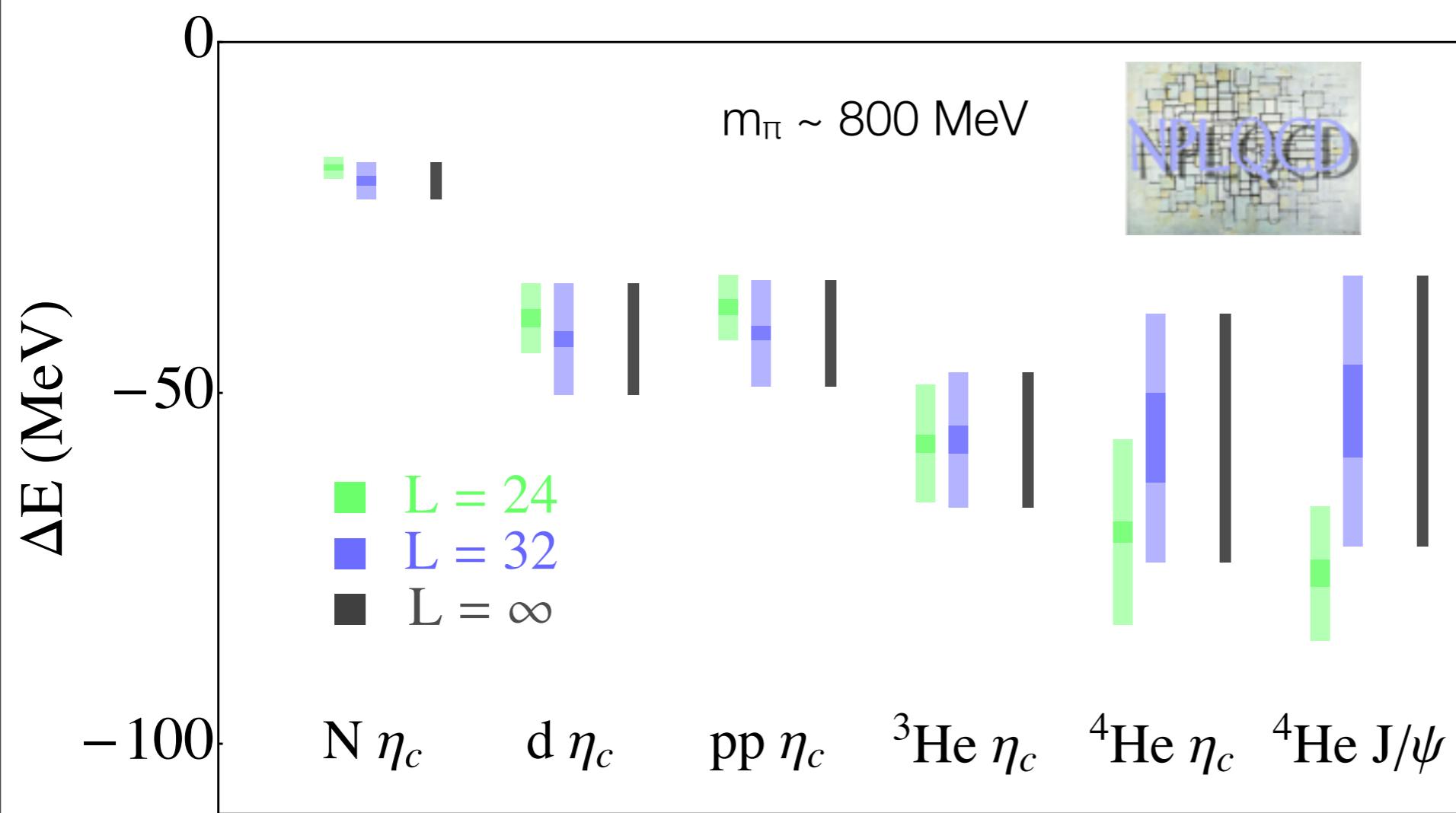
Nuclei are collections of nucleons
- shell model phenomenology!

Exotic Nuclei - Charmonium



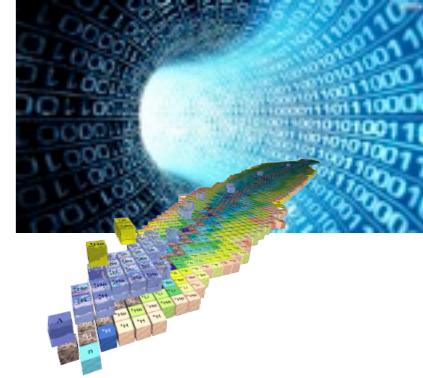
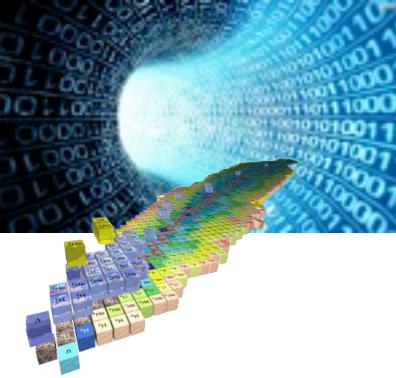
(techni-baryon DM interactions)

$$\mathcal{L} \sim r_\chi^3 \chi_v^\dagger \chi_v G_{\mu\nu} G^{\mu\nu}, \quad r_\chi^3 \chi_v^\dagger \chi_v v_\alpha v_\beta G_\mu^\alpha G^{\mu\beta}$$



Athenna

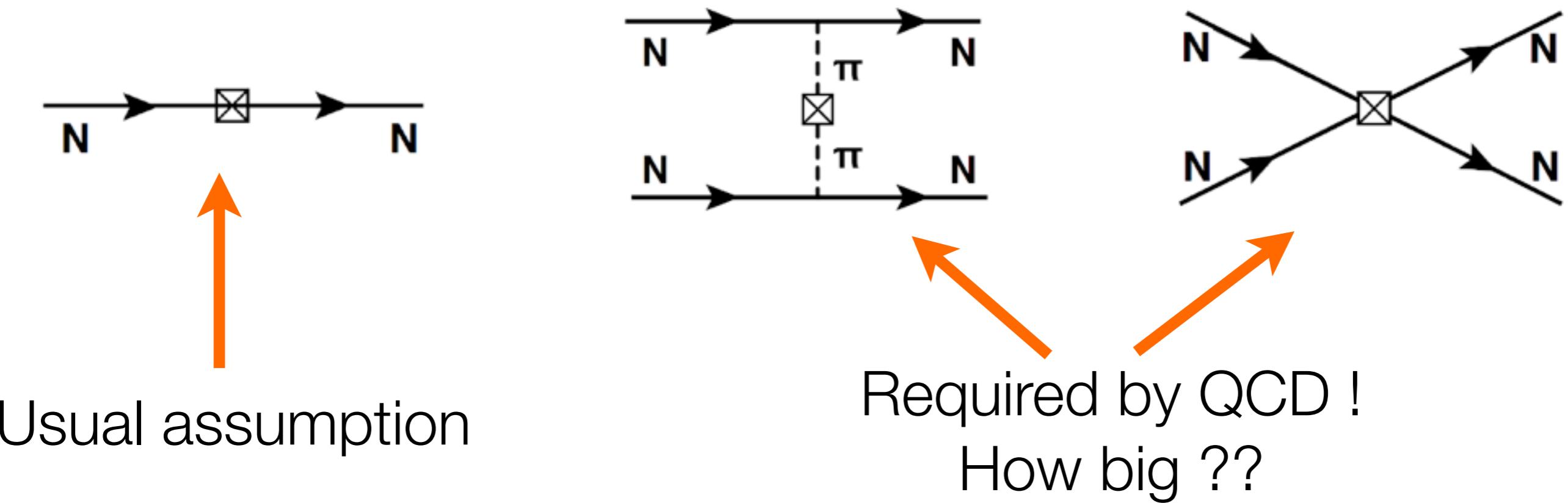


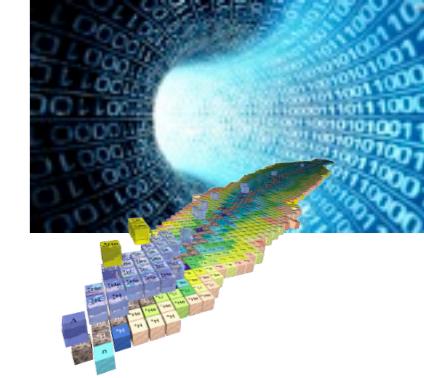
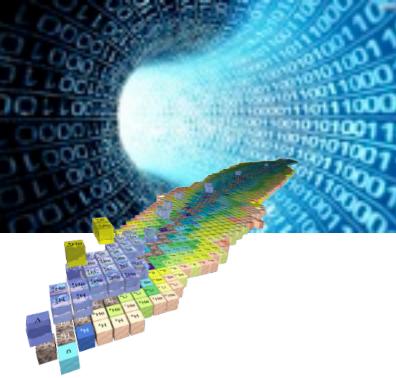


Nuclear σ -Terms and Dark Matter Interactions

NPLQCD : arXiv:1306.6939 (2013)

$$\begin{aligned}\mathcal{L} &= G_F \bar{\chi}\chi \sum_q a_S^{(q)} \bar{q}q = G_F \bar{\chi}\chi \bar{q}a_S q \\ &= \frac{G_F}{2} \bar{\chi}\chi \left[(a_S^{(u)} + a_S^{(d)})\bar{q}q + (a_S^{(u)} - a_S^{(d)})\bar{q}\tau^3 q + 2 a_S^{(s)}\bar{s}s + \dots \right]\end{aligned}$$





Nuclear σ -Terms and Dark Matter Interactions

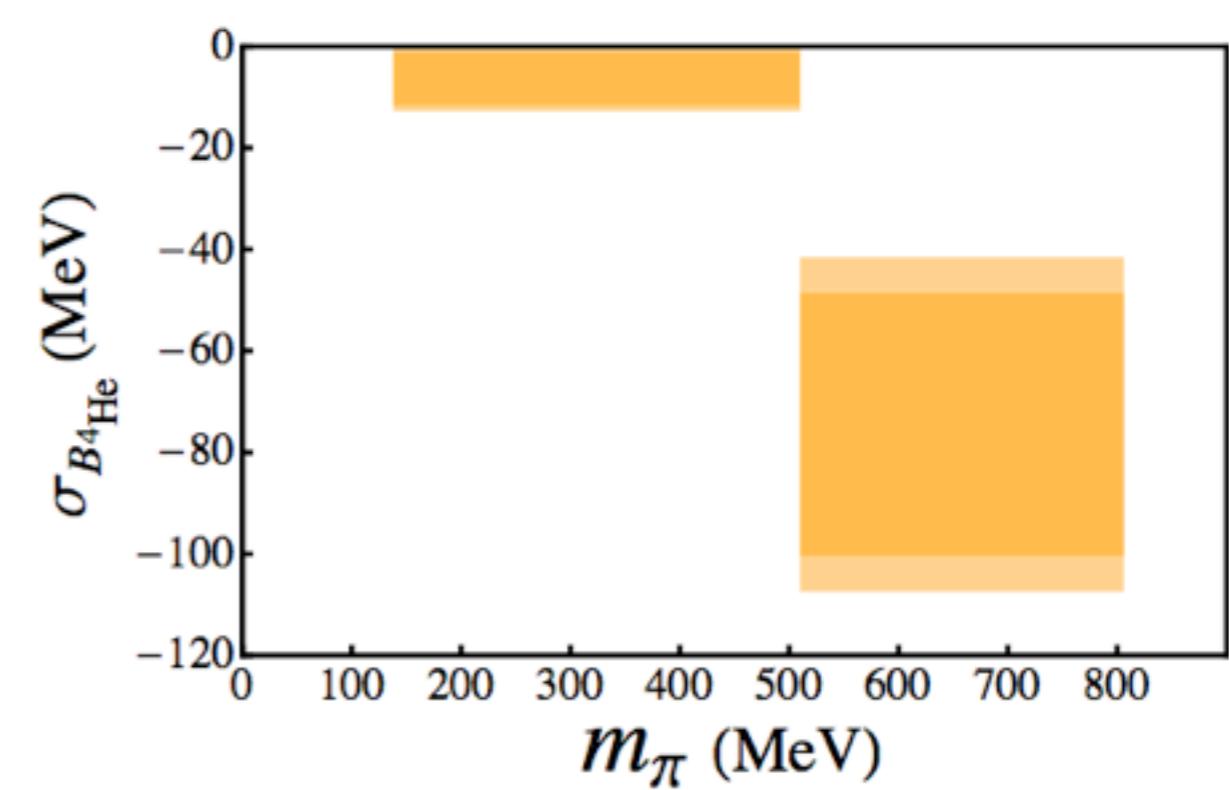
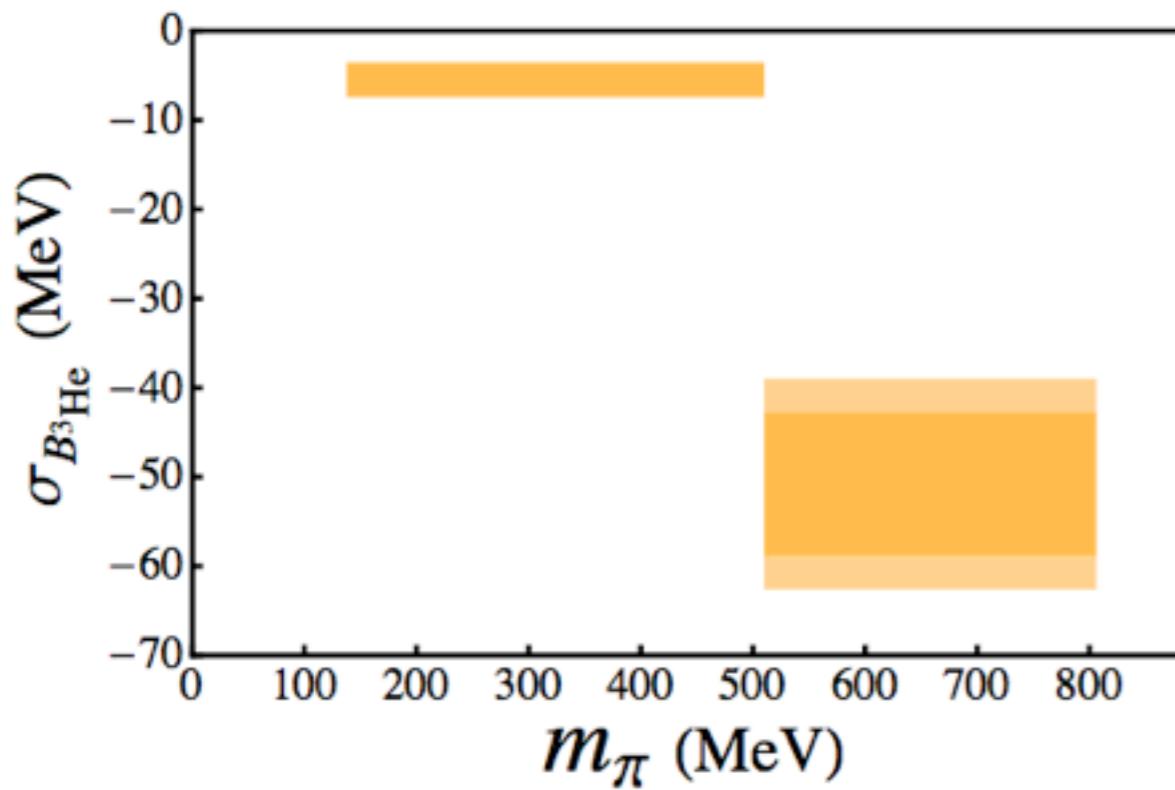
NPLQCD : arXiv:1306.6939 (2013)

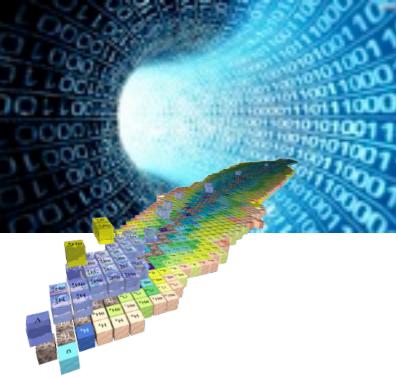
Nuclear σ -terms

$$\begin{aligned}\sigma_{Z,N} &= \overline{m} \langle Z, N(\text{gs}) | \bar{u}u + \bar{d}d | Z, N(\text{gs}) \rangle = \overline{m} \frac{d}{d\overline{m}} E_{Z,N}^{(\text{gs})} \\ &= [1 + \mathcal{O}(m_\pi^2)] \frac{m_\pi}{2} \frac{d}{dm_\pi} E_{Z,N}^{(\text{gs})}\end{aligned}$$

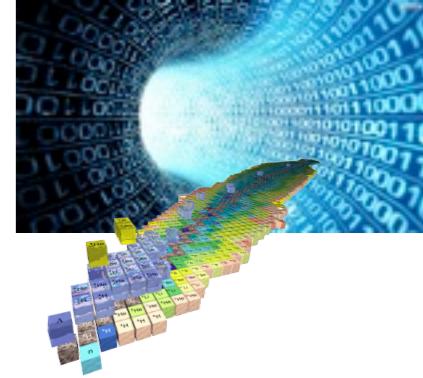


σ -terms from the binding energy only





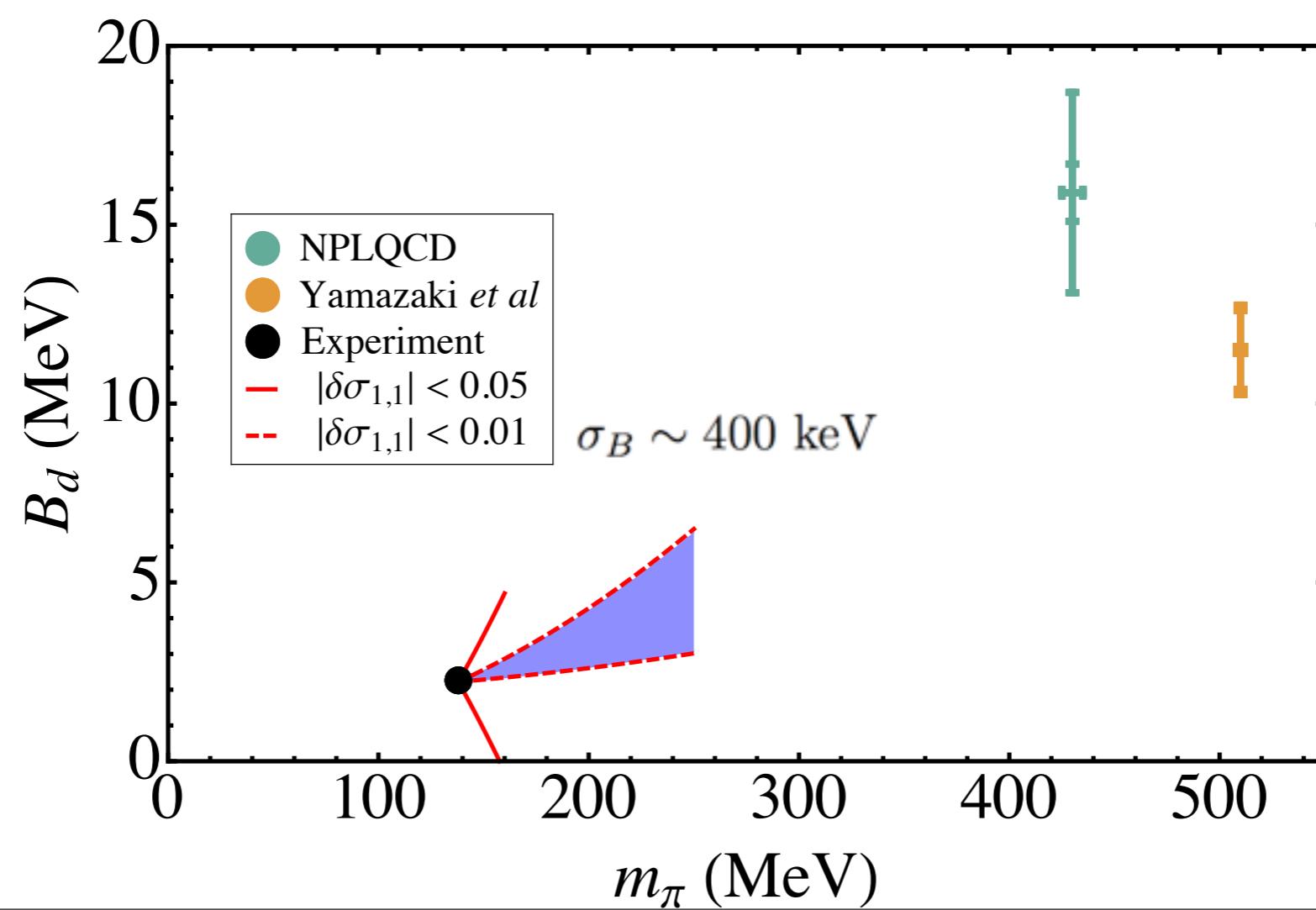
Nuclear σ -Terms and Dark Matter Interactions



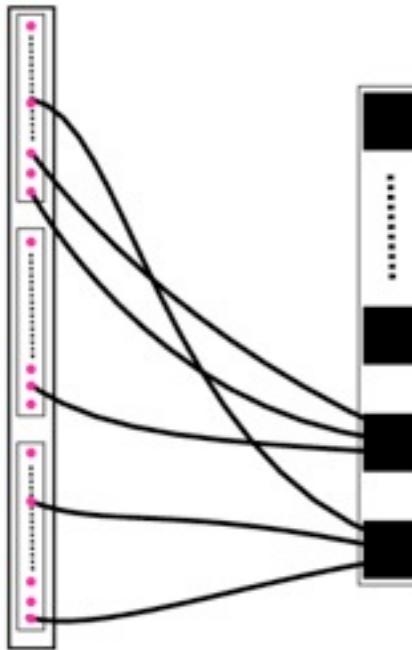
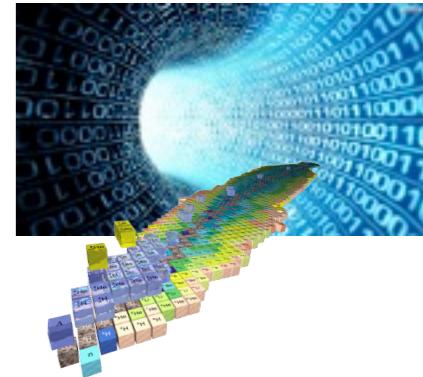
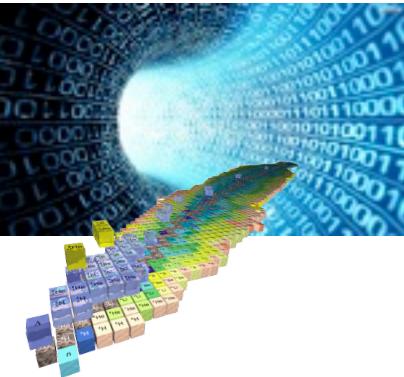
NPLQCD : arXiv:1306.6939 (2013)

Nuclear σ -terms

$$\delta\sigma_{Z,N} = \frac{\langle Z, N(\text{gs}) | \bar{u}u + \bar{d}d | Z, N(\text{gs}) \rangle}{A \langle N | \bar{u}u + \bar{d}d | N \rangle} - 1$$

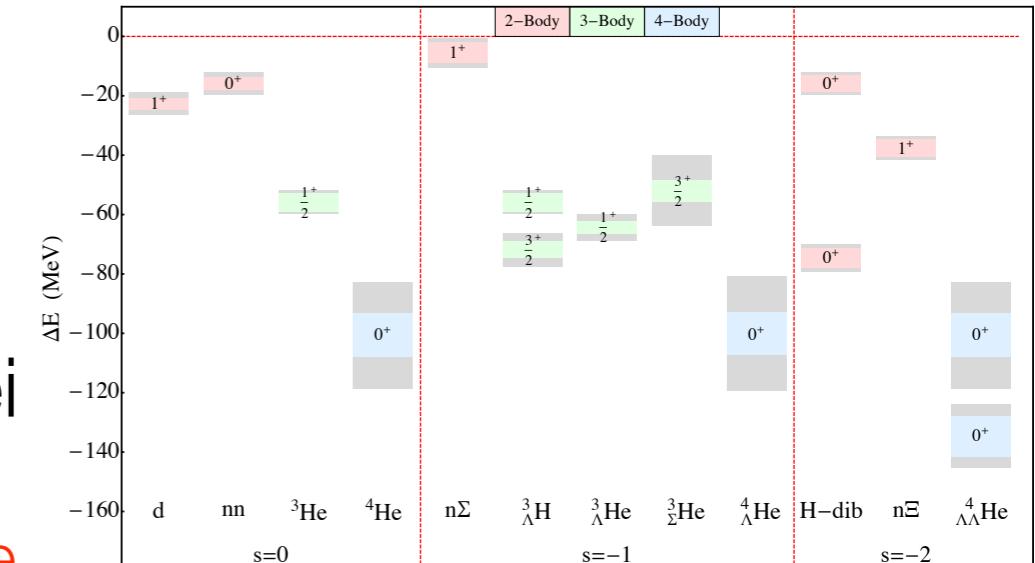


Roadblocks of the Past



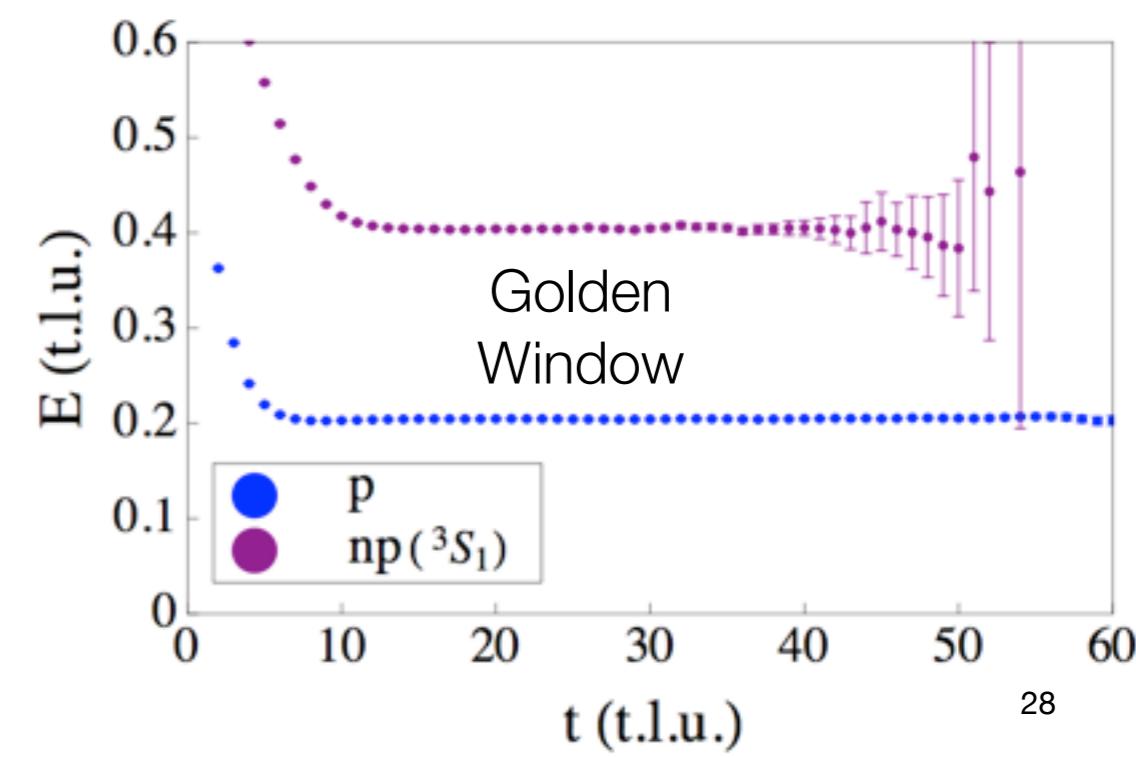
Contractions - 2012
no longer an issue for light nuclei

e.g. ${}^4\text{He}$: 0.8 core-seconds per time-slice
(Orginos+Detmold)



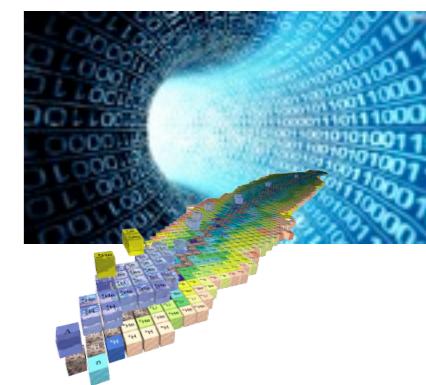
Signal to Noise

Large numbers of measurements
Source and Sink structure is critical



Future Computational Needs

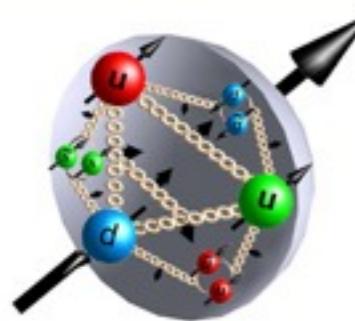
Physics Objectives



2007-2014 ...

Structure of the Baryons

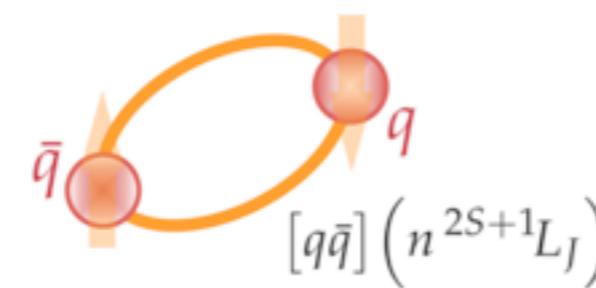
multiple L,T, lattice spacings
multiple discretizations
N predictions for mq(phys)



140 MeV

Meson and Baryon Spectroscopy

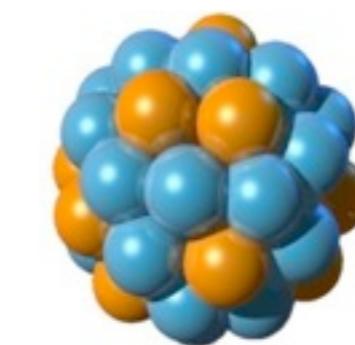
multiple L,T
one lattice spacing
resolved spectrum
mapped out resonances



300 MeV

Nuclei and Nuclear Forces

multiple L,T
one lattice spacing
light (hyper-)nuclei, scattering
simple properties of nuclei



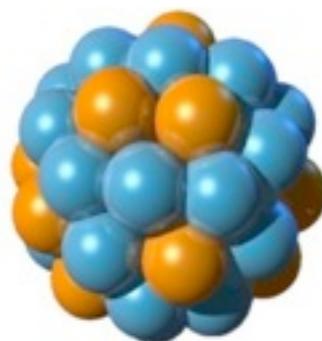
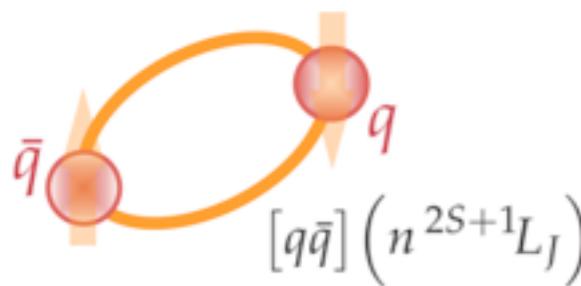
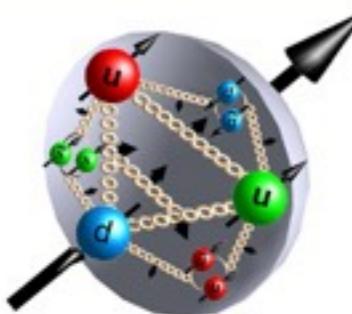
800 MeV

Pion Mass

Future Computational Needs

Physics Objectives

Before 2022 ...



140 MeV

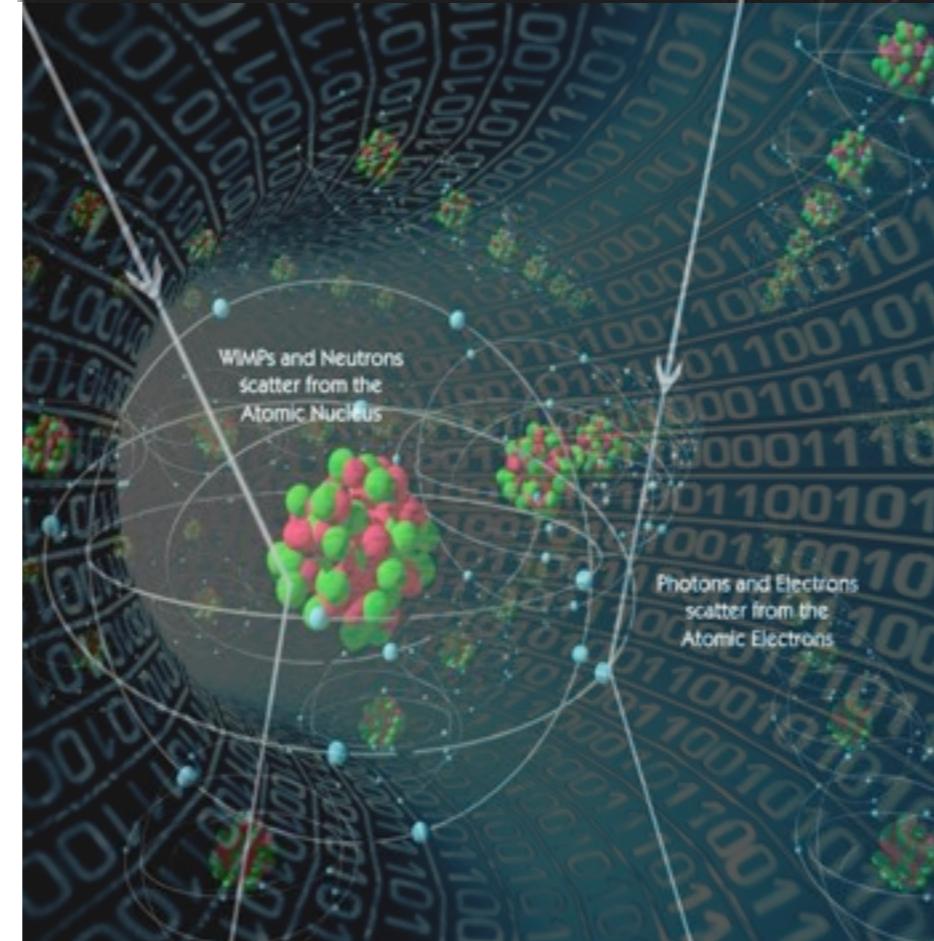
300 MeV

800 MeV

Pion Mass

- physical pion mass with $n_f = 1 + 1 + 1 + 1$
- electromagnetism
- precision calculations
- multiple lattices volumes with large T
- multiple lattice spacings
- multiple discretizations
- fully quantified uncertainties
- complement experimental program
- guide future experimental program
- provide critical inputs for theory

Summary



Lattice QCD is coming of age - precision nucleon physics and is starting to determine the properties of simple nuclear systems.

Matching to EFTs extends reach of LQCD calculations

LQCD + Pionful theory calculations required for reliable exploration of DM using direct detection results

END
