“Sommerfeld effect in the MSSM with non-relativistic EFT”

MITP Programme “Effective Theory and Dark Matter”, 16-27 March 2015, Mainz, Germany

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(1) The Sommerfeld effect

(2) Non-relativistic MSSM - matching and Schrödinger equation

(3) Relic abundance in benchmark models

(4) Outlook (into the MSSM parameter space)

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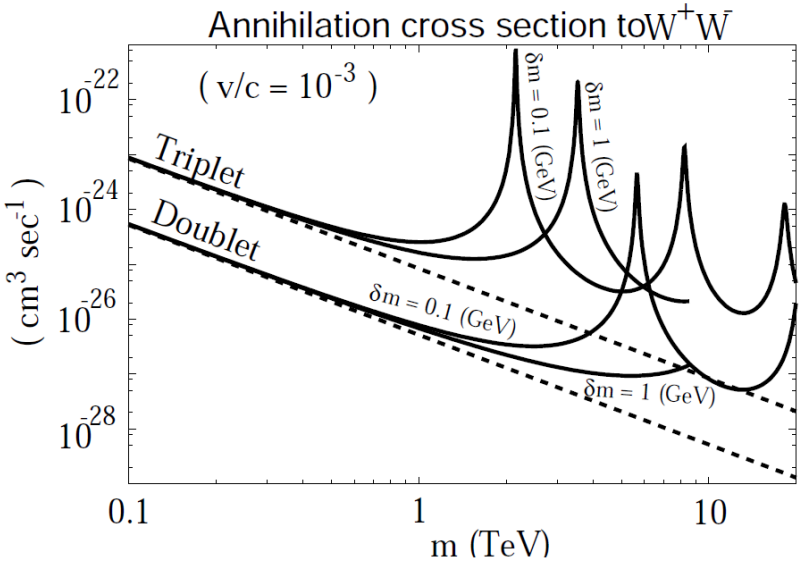
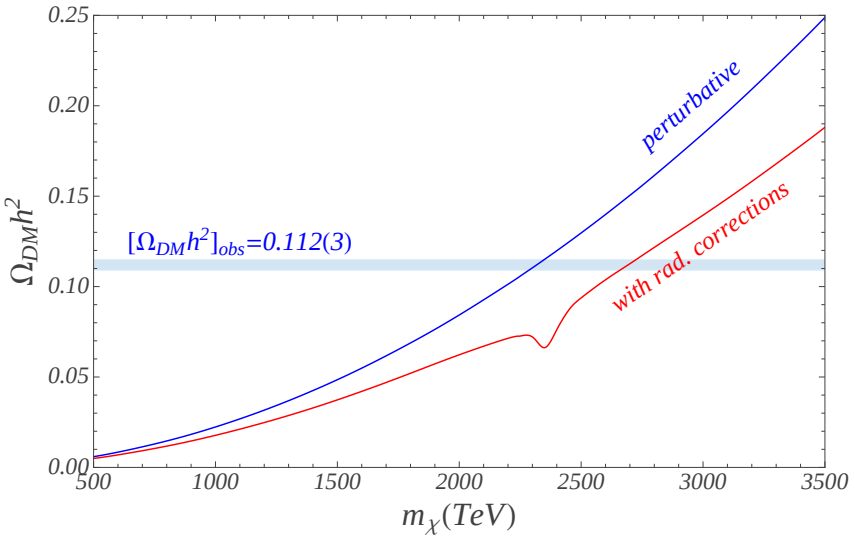
MB, C. Hellmann, P. Ruiz-Femenia, [arXiv:1210.7928, 1411.6924, 1411.6930];
C. Hellmann, P. Ruiz-Femenia [arXiv:1302.0200];
MB, A. Bharucha, F. Dighera, C. Hellmann, A. Hryczuk, S. Recksiegel, P. Ruiz-Femenia, work in progress

Motivation and Aims

Large enhancements of annihilation cross section for $m_\chi \gg M_{\mathrm{EW}}$ possible due to non-relativistic scattering prior to annihilation [Hisano et al. (2004,2006)]



[Hisano et al., 2004]



Pure Wino, following [Hisano et al., 2006]

Aim

- General MSSM with mixed (wino-higgsino-bino) neutralino DM
- Systematic approximations within NREFT (partial-wave separation, potentials, expansion in mass splittings for off-diagonal annihilation)

- Dark matter pair annihilation occurs at small non-relativistic velocities

  - $v \sim 10^{-3}$ in galaxies (up to 10^{-6} in dwarf galaxies)
  - In relic density calculations

$$\langle \sigma v \rangle = \frac{M_\chi}{4\pi T} \int dv\, 4\pi v\, \sigma(v) v\, \mathrm{e}^{-\frac{M_\chi v^2}{4T}}$$

  and freeze-out occurs at $T \approx M/25$, i.e. $v \approx 1/5$.

- Large quantum corrections from the potential region, if

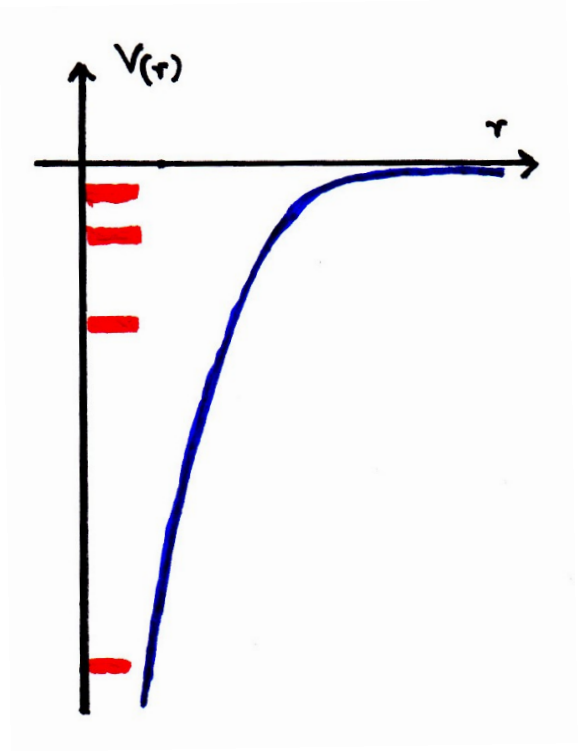
$$v \lesssim \pi\alpha \qquad \text{and} \qquad m \lesssim M_\chi \alpha$$

  - Unknown light exchange particle (speculative) [Arkani-Hamed et al., 2008]
  - For WIMPS with electroweak interactions when $M_\chi \gtrsim 2\,\text{TeV}$. Yukawa potential [Hisano et al., 2006]

$$V(r) = -\frac{\alpha}{r}\, e^{-M_{W,Z} r}$$

- Motivated by the non-observation of light SUSY particles at LHC.
  Natural mass range for thermal relic.

Resonance effect for the Yukawa potential



Range $r \sim 1/m$ cuts off Rydberg states [$r_{\text{Ryd}} \sim n/(M_\chi \alpha)$]

Finite number of levels

$$n \lesssim \frac{M_\chi \alpha}{m}$$

Increasing M_χ adds levels from above. Zero-energy bound states for certain M_χ. Then

$$S \propto \frac{1}{E - E_{\text{bind}}} \sim \frac{1}{v^2}$$

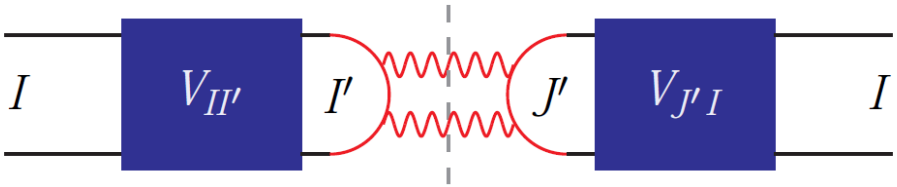
stronger than $1/v$ Coulomb enhancement.

Resonant enhancement at certain values of M_χ starting in TeV range.

Sommerfeld enhancement in the general MSSM

MSSM with $M_\chi \gg M_Z$: degeneracies are natural (electroweak multiplets) → coannihilation
14 $\chi_i^0\chi_j^0$, $\chi_i^+\chi_i^-$ charge-0, 8 $\chi_i^0\chi_j^+$ charge +1, 3 $\chi_i^+\chi_j^+$ charge +2 states + conjugates.

Scatter into one another through Yukawa interaction. Each annihilates into a multitude of SM final states.



$$\sigma_I(v) = \sum_i \Gamma_i({}^{2S+1}L_J)_{I'J'} \langle[\chi\chi]_I|\mathcal{O}({}^{2S+1}L_J)_{I'J'}|[\chi\chi]_I\rangle \overset{\text{Born}}{=} a_I + b_I v^2$$

I Compute the potentials from Z, W, Higgs exchange

II Compute the tree-level coefficients of *off-diagonal* partial wave forward-amplitudes

III Solve Schrödinger equation for operator matrix elements (wave-functions + derivatives at origin)

IV Solve Boltzmann equation for relic density

Previous work [Hisano et al. (2004, 2006); Cirelli et al. (2007, 2008, 2009), Hryczuk et al. (2010, 2014)]: pure-Wino and/or -Higgsino LSP limit; no off-diagonals away from pure-W/H limits; no partial-wave separation.

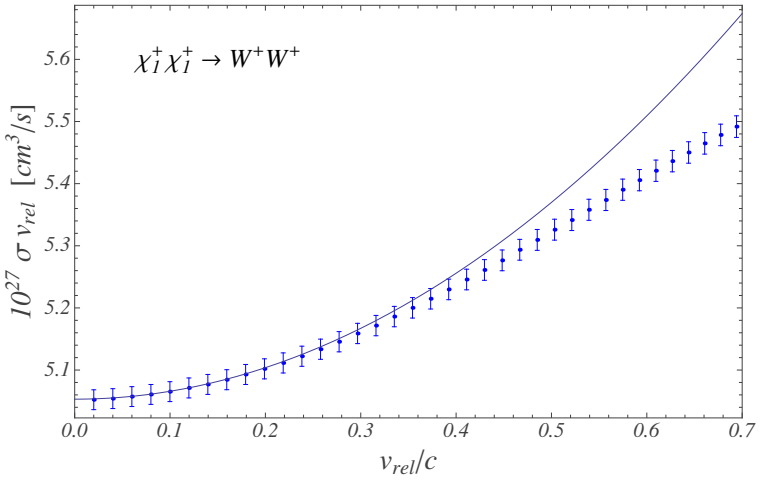
Short-distance matching

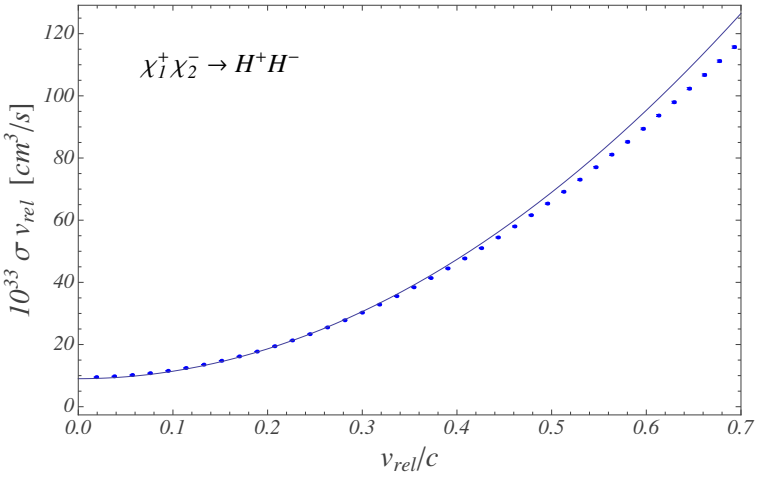
$\mathcal{O}^{\chi\chi\to\chi\chi}(^1P_1)$	$\xi^\dagger_{e_4}\left(-\frac{i}{2}\overleftrightarrow{\partial}\right)\xi^c_{e_3}\ \cdot\ \xi^{c\dagger}_{e_2}\left(-\frac{i}{2}\overleftrightarrow{\partial}\right)\xi_{e_1}$
$\mathcal{O}^{\chi\chi\to\chi\chi}(^3P_0)$	$\frac{1}{3}\,\xi^\dagger_{e_4}\left(-\frac{i}{2}\overleftrightarrow{\partial}\cdot\boldsymbol{\sigma}\right)\xi^c_{e_3}\ \cdot\ \xi^{c\dagger}_{e_2}\left(-\frac{i}{2}\overleftrightarrow{\partial}\cdot\boldsymbol{\sigma}\right)\xi_{e_1}$
$\mathcal{O}^{\chi\chi\to\chi\chi}(^3P_1)$	$\frac{1}{2}\,\xi^\dagger_{e_4}\left(-\frac{i}{2}\overleftrightarrow{\partial}\times\boldsymbol{\sigma}\right)\xi^c_{e_3}\ \cdot\ \xi^{c\dagger}_{e_2}\left(-\frac{i}{2}\overleftrightarrow{\partial}\times\boldsymbol{\sigma}\right)\xi_{e_1}$
$\mathcal{O}^{\chi\chi\to\chi\chi}(^3P_2)$	$\xi^\dagger_{e_4}\left(-\frac{i}{2}\overleftrightarrow{\partial}^{(i}\boldsymbol{\sigma}^{j)}\right)\xi^c_{e_3}\ \cdot\ \xi^{c\dagger}_{e_2}\left(-\frac{i}{2}\overleftrightarrow{\partial}^{(i}\boldsymbol{\sigma}^{j)}\right)\xi_{e_1}$
$\mathcal{P}^{\chi\chi\to\chi\chi}(^1S_0)$	$\frac{1}{2}\left[\xi^\dagger_{e_4}\xi^c_{e_3}\ \cdot\ \xi^{c\dagger}_{e_2}\left(-\frac{i}{2}\overleftrightarrow{\partial}\right)^2\xi_{e_1}+\xi^\dagger_{e_4}\left(-\frac{i}{2}\overleftrightarrow{\partial}\right)^2\xi^c_{e_3}\ \cdot\ \xi^{c\dagger}_{e_2}\xi_{e_1}\right]$
$\mathcal{P}^{\chi\chi\to\chi\chi}(^3S_1)$	$\frac{1}{2}\left[\xi^\dagger_{e_4}\boldsymbol{\sigma}\,\xi^c_{e_3}\ \cdot\ \xi^{c\dagger}_{e_2}\boldsymbol{\sigma}\left(-\frac{i}{2}\overleftrightarrow{\partial}\right)^2\xi_{e_1}+\xi^\dagger_{e_4}\boldsymbol{\sigma}\left(-\frac{i}{2}\overleftrightarrow{\partial}\right)^2\xi^c_{e_3}\ \cdot\ \xi^{c\dagger}_{e_2}\boldsymbol{\sigma}\,\xi_{e_1}\right]$
$\mathcal{Q}_1^{\chi\chi\to\chi\chi}(^1S_0)$	$(\delta m\,M)\,\xi^\dagger_{e_4}\xi^c_{e_3}\ \cdot\ \xi^{c\dagger}_{e_2}\xi_{e_1}$
$\mathcal{Q}_1^{\chi\chi\to\chi\chi}(^3S_1)$	$(\delta m\,M)\,\xi^\dagger_{e_4}\boldsymbol{\sigma}\,\xi^c_{e_3}\ \cdot\ \xi^{c\dagger}_{e_2}\boldsymbol{\sigma}\,\xi_{e_1}$
$\mathcal{Q}_2^{\chi\chi\to\chi\chi}(^1S_0)$	$(\delta\overline{m}\,M)\,\xi^\dagger_{e_4}\xi^c_{e_3}\ \cdot\ \xi^{c\dagger}_{e_2}\xi_{e_1}$
$\mathcal{Q}_2^{\chi\chi\to\chi\chi}(^3S_1)$	$(\delta\overline{m}\,M)\,\xi^\dagger_{e_4}\boldsymbol{\sigma}\,\xi^c_{e_3}\ \cdot\ \xi^{c\dagger}_{e_2}\boldsymbol{\sigma}\,\xi_{e_1}$

$$m_1 + m_2 + \frac{\vec{p}^{\,2}}{\mu} + \ldots$$
$$= m_3 + m_4 + \frac{\vec{p}'^{\,2}}{\mu} + \ldots$$

Mass splitting must be formally smaller than $\mathcal{O}(m_\chi v^2)$ for consistent NR expansion.

- Off-diagonal annihilation matching coefficients cannot be obtained from existing codes (DARKSUSY, micrOmega, ...)
- Analytic computation of all annihilation channels.
  $8 \times (14 \times 14 + 2 \times 8 \times 8 + 2 \times 3 \times 3) = 2736$ matching coefficients,
  83456 exclusive channels (Feynman gauge, MSSM)





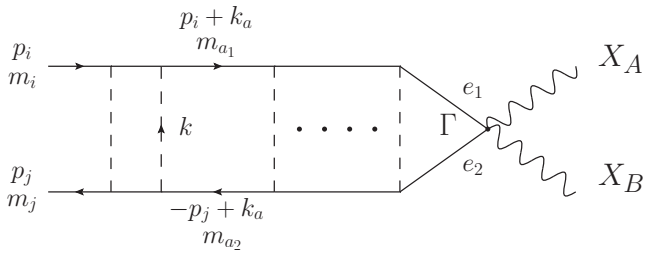
- $\mathcal{O}(v^2)$ accuracy sufficient for relic density computations (and, of course, annihlation in present Universe)

NRMSSM matrix element calculation

$$\mathcal{L}^{\text{NRMSSM}} = \sum_i \chi_i^\dagger \left(i\partial_t - (m_i - m_{\text{LSP}}) + \frac{\vec{\partial}^{\,2}}{2m_{\text{LSP}}} \right) \chi_i$$
$$- \sum_{\chi\chi\to\chi\chi} \int d^3\vec{r}\, V^{\chi\chi\to\chi\chi}_{\{e_1e_2\}\{e_4e_3\}}(r)\, \chi^\dagger_{e_4}(t,\vec{x})\chi^\dagger_{e_3}(t,\vec{x}+\vec{r})\chi_{e_1}(t,\vec{x})\chi_{e_2}(t,\vec{x}+\vec{r}) + \dots$$

$$V^{\chi\chi\to\chi\chi}_{\{e_1e_2\}\{e_4e_3\}}(r) = \left[A\,\mathbf{1}\otimes\mathbf{1} + B\big(\vec{\sigma}\otimes\mathbf{1} + \mathbf{1}\otimes\vec{\sigma}\big)\right]_{e_1e_2e_4e_3} \frac{e^{-m_\phi r}}{r}\,,$$

For $v \ll 1, m_\phi \ll m_\chi$ leading contributions from ladder diagrams. Summation equivalent to solving a multi-channel (matrix) Schrödinger equation.



For the S-wave operators define the ratio to the tree-level matrix element in the $[\chi\chi]_I$ state

$$S_I = \sum_{J,K} \frac{\psi_E(0)_{I\to J}}{\psi_E(0)_{I,\text{free}}} \times \frac{\Gamma_{JK}}{\Gamma_{II}} \times \frac{\psi^*_E(0)_{K\to I}}{\psi^*_E(0)_{I,\text{free}}}$$

Scattering wave function from numerical solution of a matrix Schrödinger equation.

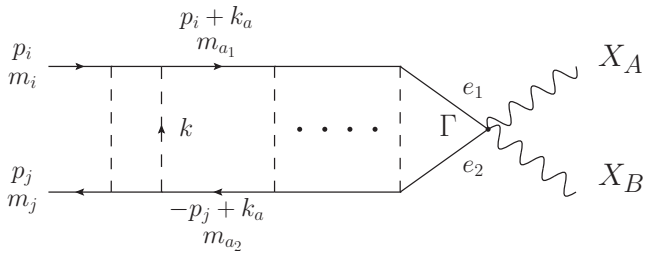
- Sommerfeld-corrected cross section

$$\sigma_{ij}|\vec{v}_i - \vec{v}_j| = \sum_{{}^1S_0,\,{}^3S_1} S_I({}^{2s+1}L_J)\,\Gamma_{II}({}^{2s+1}L_J) + \vec{p}_i^{\,2}\Big[\sum_{{}^1P_1,\,{}^3P_J} S_I({}^{2s+1}L_J)\,\Gamma_{II}({}^{2s+1}L_J) + \sum_{{}^1S_0,\,{}^3S_1} S_I^{p^2}({}^{2s+1}L_J)\,\Gamma_{II}^{p^2}({}^{2s+1}L_J)\Big]$$

S_I must be computed for each (relative) velocity.

- $\Gamma_{IJ}^{p^2}({}^{2s+1}L_J)$ can be related to $\Gamma_{IJ}({}^{2s+1}L_J)$ $(L = S)$ by equation of motion.

- Heavy two-particle channels have a small effect, and the dominant contribution comes from the last loop. Include analytically in an effective annihilation matrix for the lower-mass channels to reduce the CPU time.



Solving the Schrödinger equation [Slatyer (2009)]

$$\left(\left[-\frac{d^2}{dr^2}+\frac{l(l+1)}{r^2}-m_{\rm LSP}E\right]\delta^{ab}+m_{\rm LSP}V^{ab}(r)\right)[u_l(r)]_{bi}=0$$

Solve for regular solutions

$$[u_l(r_0)]_{ai}=\frac{1}{2l+1}\,\hat{r}_0^{l+1}\,\delta_{ai}\,,\qquad [u_l'(r_0)]_{ai}=\frac{l+1}{2l+1}\,\hat{r}_0^{l}\,\delta_{ai}\,,$$

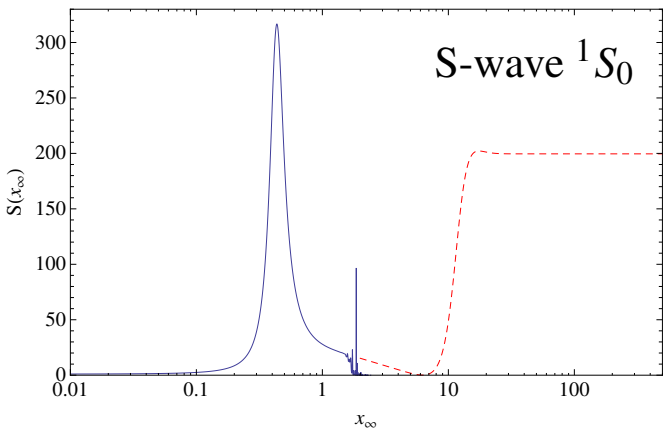
$$U_{aj}(r_\infty)=e^{ik_a r_\infty}\,\left([u_l'(r_\infty)]_{aj}-ik_a[u_l(r_\infty)]_{aj}\right)\,,$$

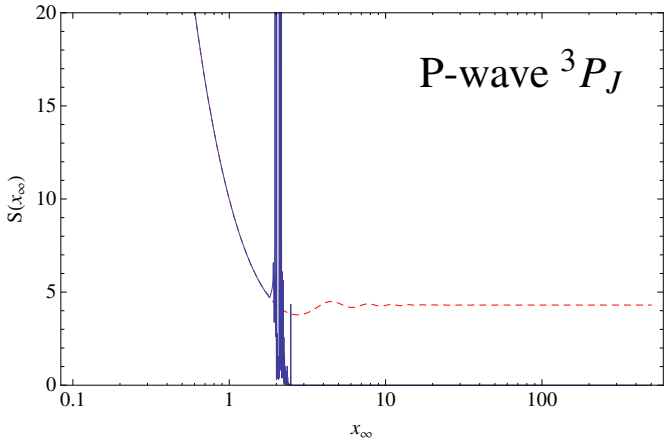
Relation to Sommerfeld factor

$$S_i[^{2S+1}L_J]=\left(\frac{(2L-1)!!}{k_i^L}\right)^2\,\frac{[T^\dagger]_{ie'}\,\Gamma_{ee'}(^{2S+1}L_J)\,T_{ei}}{\Gamma_{ii}(^{2S+1}L_J)|_{\rm LO}}$$

$$T=U^{-1}$$

Matrix inversion practically impossible due to mixing with kinematically closed channels with $M_{\chi\chi}-[2m_{\rm LSP}+E]>M_{\rm EW}^2/m_{\rm LSP}\approx$ few GeV.





Solution is a modification of the modification [Ershov (2011)] of the variable phase method (originally developed for nuclear physics problems)

$$[u_l(x)]_{ai} = f_a(x)\alpha_{ai}(x) - g_a(x)\beta_{ai}(x) \qquad \text{with} \qquad f_a(x)\alpha'_{ai}(x) - g_a(x)\beta'_{ai}(x) = 0$$

$$f_a(x) = \sqrt{\frac{\pi x}{2}}\, J_{l+\frac{1}{2}}(\hat{k}_a x) \qquad g_a(x) = -\sqrt{\frac{\pi x}{2}}\left[Y_{l+\frac{1}{2}}(\hat{k}_a x) - iJ_{l+\frac{1}{2}}(\hat{k}_a x)\right]$$

$$N'_{ab} = \delta_{ab} + \left(\frac{g'_a}{g_a} + \frac{g'_b}{g_b}\right) N_{ab} - N_{ac}\,\frac{\hat{V}_{cd}}{E}\,N_{db}\,,$$

$$\tilde{\alpha}^{-1\,\prime}_{ia} = \tilde{\alpha}^{-1}_{ib} Z_{ba} \qquad \text{with} \qquad Z_{ab} \equiv -\frac{g'_a}{g_a}\,\delta_{ab} + \frac{\hat{V}_{ac}}{E}\,N_{cb}\,.$$

$$T_{ia}(x_\infty) \overset{x_\infty \to \infty}{=} e^{-i\hat{k}_a x_\infty}\,\tilde{\alpha}^{-1}_{ai}(x_\infty)$$

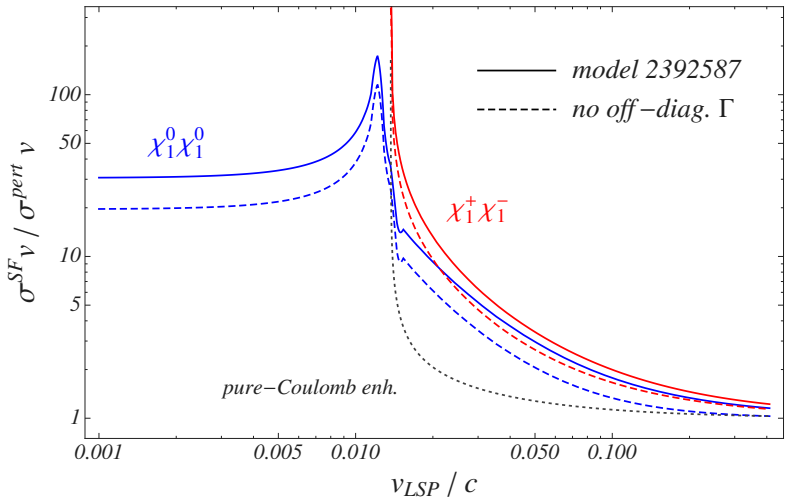
Wino-like χ_1^0 DM

- pMSSM benchmark model "2392587" [Cahill-Rowley et al. (2013)]
- (Approximate) $SU(2)_L$ triplet states χ_1^0, $\chi_1^\pm$. $m_{\chi_1^0} = 1650.664\,\text{GeV}$, $|Z_{N\,21}|^2 = 0.999$
  Mass-splitting due to radiative corrections: $\delta m_{\text{rad}} = m_{\chi^+} - m_{\chi^0} \approx 0.155\,\text{GeV}$
- (co-)annihilation sectors

neutral	$\chi_1^0\,\chi_1^0,\ \chi_1^+\,\chi_1^-$
single charged	$\chi_1^0\,\chi_1^+\ (\chi_1^0\,\chi_1^-)$
double charged	$\chi_1^+\,\chi_1^+\ (\chi_1^-\,\chi_1^-)$

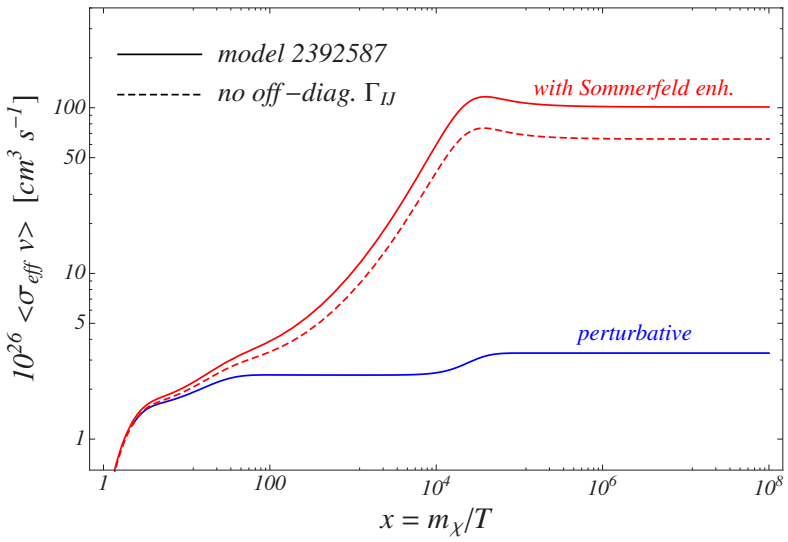
$\rightarrow$ coupled system in neutral $^1S_0$ reactions:
matrix-valued potential V_{IJ}

- Approximate treatment of all other states (CPU time)



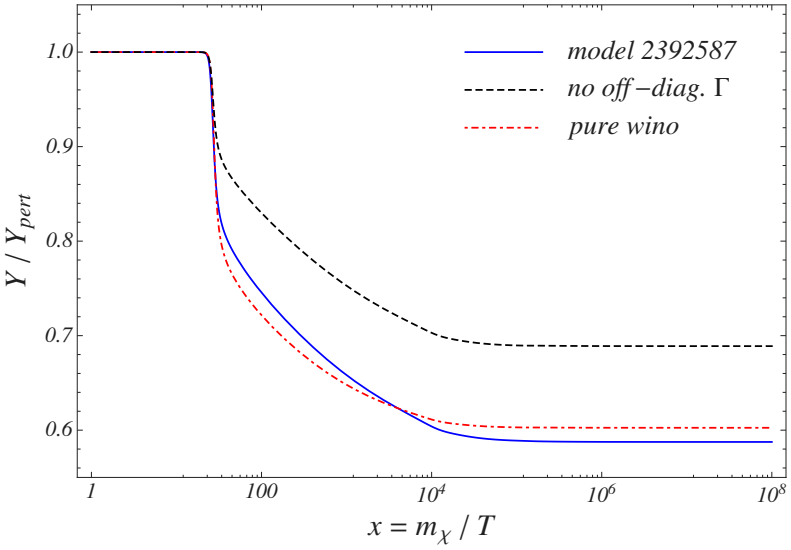
- $\chi_1^0\chi_1^0$:
  v_{LSP}-independent enh. well below,
  resonance region at $\chi_1^+\chi_1^-$ thresh.
- $\chi_1^+\chi_1^-$: Coulomb enh. above threshold
- no off-diagonal Γ:
  $\lesssim 30\%$ reduction in $\sigma^{\text{SF}}v/\sigma^{\text{pert}}v$

Thermally averaged cross section summed over all initial and final states and relic abundance
$\Omega_{\chi_1^0} h^2 = \rho^0_{\chi_1^0}/\rho_{crit}\, h^2 = m_\chi s_0 Y_0/\rho_{crit}\, h^2$



$$\langle \sigma_{\rm eff}\, v\rangle = \sum_{i,j} \langle \sigma_{ij}\, v_{ij}\rangle\, n_i^{\rm eq} n_j^{\rm eq}/n_{\rm eq}^2$$

- $x \simeq 20$:
  χ_1^0, $\chi_1^\pm$ (chem.) freeze-out off thermal bath
- $x \simeq 10^4$: $\chi_1^\pm$ decouple
  $\left[n_{\chi_1^+}/n_{\chi_1^0} \propto \exp(-\delta m/m_{\chi_1^0}\, x)\right]$



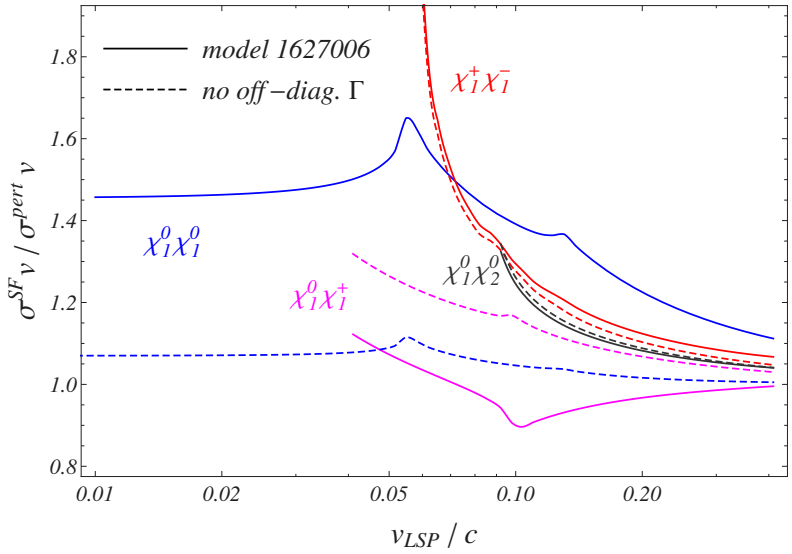
$$Y = \sum_i n_i/s$$

- $\Omega_\chi^{\rm pert}\, h^2 = 0.109(3)$
- $\Omega_\chi^{\rm SF}\, h^2 = 0.064(5) \rightarrow$ 40% reduction
- $\sim$ 15% error on $\Omega_\chi^{\rm SF}\, h^2$ if no off-diag. Γ
- pure-wino: $\Omega_\chi^{\rm SF}\, h^2 = 0.033$,
  $\Omega_\chi^{\rm pert}\, h^2 = 0.055$

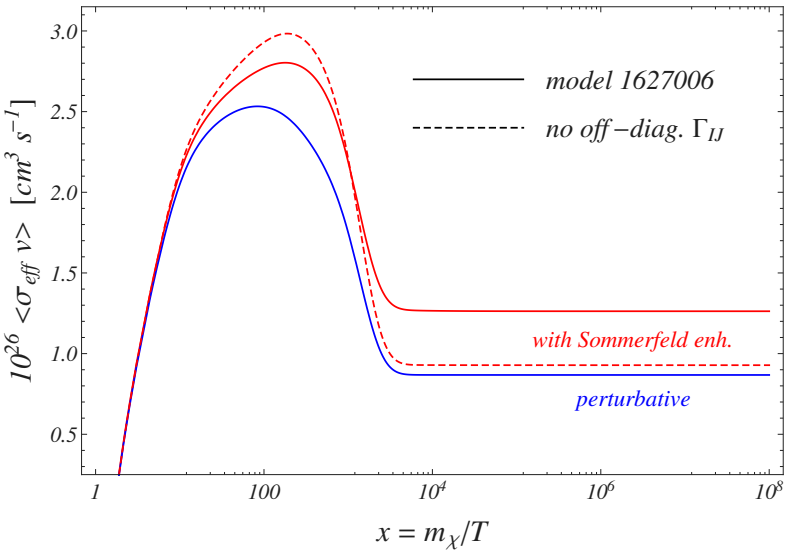
Higgsino-like χ_1^0 DM

- pMSSM benchmark model "1627006" [Cahill-Rowley et al. (2013)]
- Two approximate $SU(2)_L$ doublet states $\chi_{1,2}^0$, $\chi_1^\pm$. $|Z_{N\,31}|^2 + |Z_{N\,41}|^2 = 0.98$
  $m_{\chi_1^0} = 1172.31\,\text{GeV}$
  Tree-level mass-splittings: $\delta m_{\chi_1^+} = 1.8\,\text{GeV}$, $\delta m_{\chi_2^0} = 9.5\,\text{GeV}$
- (co-)annihilation sectors

neutral	$\chi_1^0\,\chi_1^0, \chi_2^0\,\chi_1^0, \chi_2^0\,\chi_2^0,\ \chi_1^+\,\chi_1^-$
single charged	$\chi_1^0\,\chi_1^+, \chi_2^0\,\chi_1^+\ (\chi_1^0\,\chi_1^-, \chi_2^0\,\chi_1^+)$
double charged	suppressed by $\mathcal{O}(M_{\text{EW}}/m_\chi)$

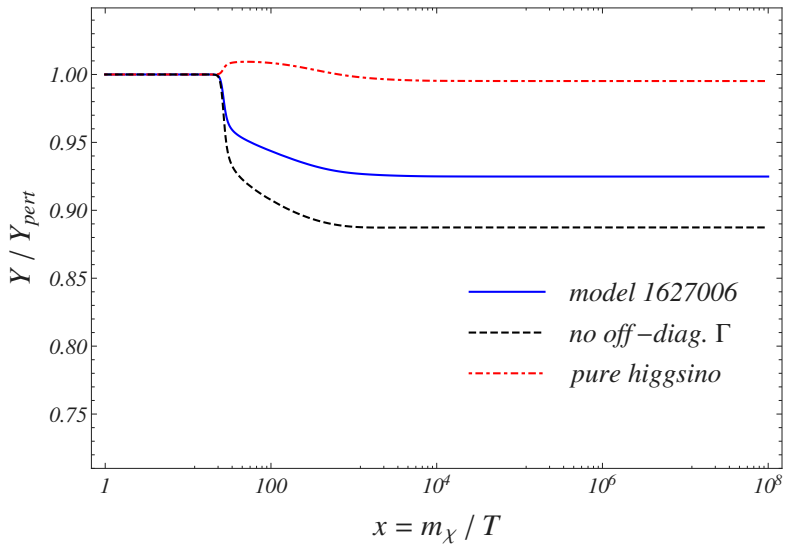


- Smaller effect (larger mass splittngs, smaller couplings)
- Destructive interference for $\chi_1^0\chi_1^\pm$ due to off-diagonal annihilation (solid magenta)



$\langle \sigma_{\rm eff}\, v \rangle$

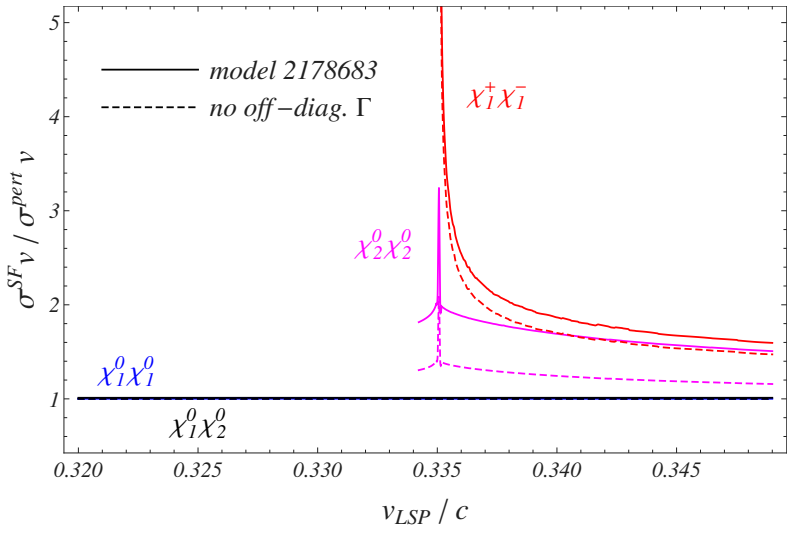
- Note destructive interference from single-charged sector near $x \simeq 20$.



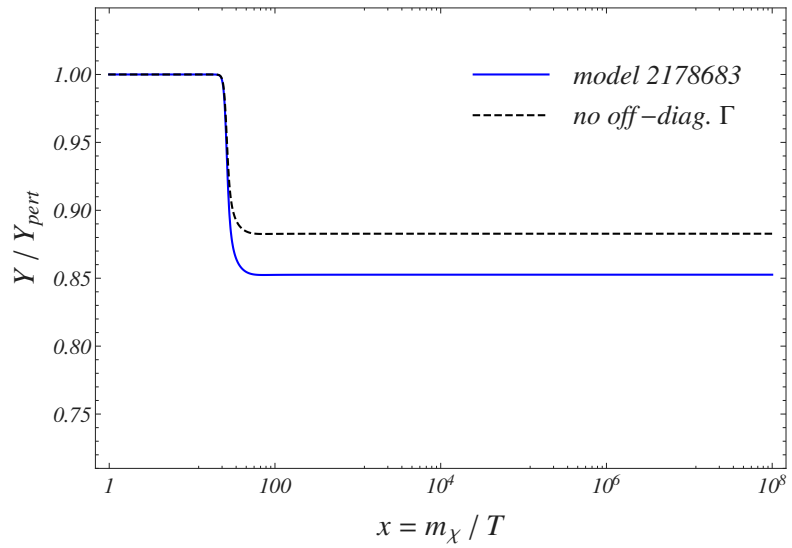
$$Y \;=\; \sum_i n_i/s$$

- $\Omega_\chi^{\rm pert}\, h^2 \;=\; 0.108$
- $\Omega_\chi^{\rm SF}\, h^2 \;=\; 0.100 \;\rightarrow\;$ 8% reduction
- pure-Higgsino: almost no effect due to cancellation between enhancement in charge-neutral and suppression in single-charged (co-)annihilation sector.

Bino-like χ_1^0 DM



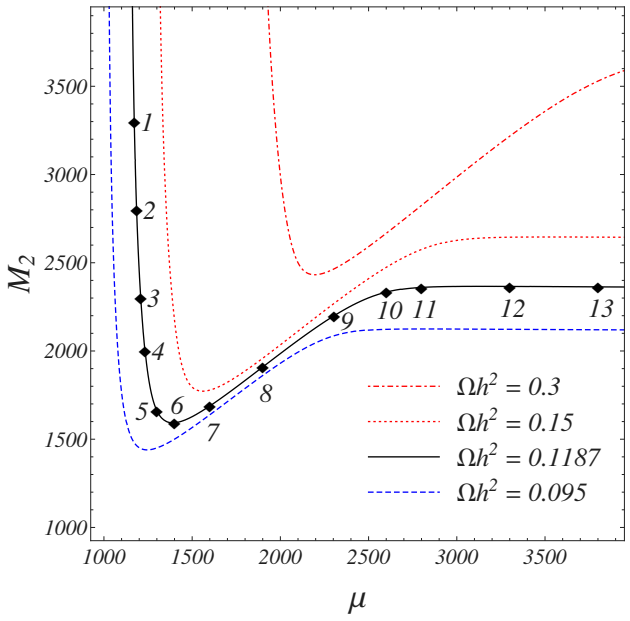
- pMSSM benchmark model "2178683"
  [Cahill-Rowley et al. (2013)]
- Bino-LSP $m_{\chi_1^0} = 488.8\,\text{GeV}$
- Co-annihilating wino sector with $m_{\chi_2^0} = 516.0\,\text{GeV}$, $m_{\chi_1^\pm} = 516.2\,\text{GeV}$.
  Annihilation cross sections 10^3 times larger than bino annihilation.

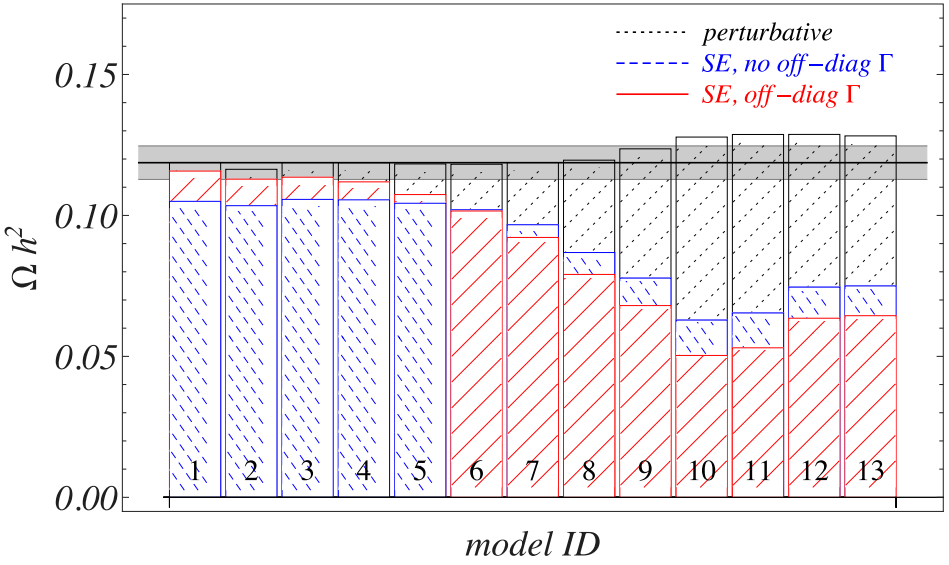


$$Y = \sum_i n_i / s$$

- $\Omega_\chi^{\text{pert}} h^2 = 0.120$
- $\Omega_\chi^{\text{SF}} h^2 = 0.102 \rightarrow$ 15% reduction

Higgsino-to-wino trajectory





- 13 models with $\Omega^{\text{DarkSUSY}} h^2 = 0.1187$
- trajectory in $\mu - M_2$ plane
  - models $1-6$: higgsino-like χ_1^0: $\Omega^{\text{SF}} h^2 / \Omega^{\text{pert}} h^2 \sim 0.97 - 0.86$
  - models $7-9$: mixed wino-higgsino χ_1^0: $\Omega^{\text{SF}} h^2 / \Omega^{\text{pert}} h^2 \sim (0.78,\ 0.66,\ 0.55)$
  - models $10-13$: wino-like χ_1^0: $\Omega^{\text{SF}} h^2 / \Omega^{\text{pert}} h^2 \sim 0.39 - 0.50$
    [note resonance]
- Off-diagonal terms strongly enhance the variation of the SE along the trajectory.

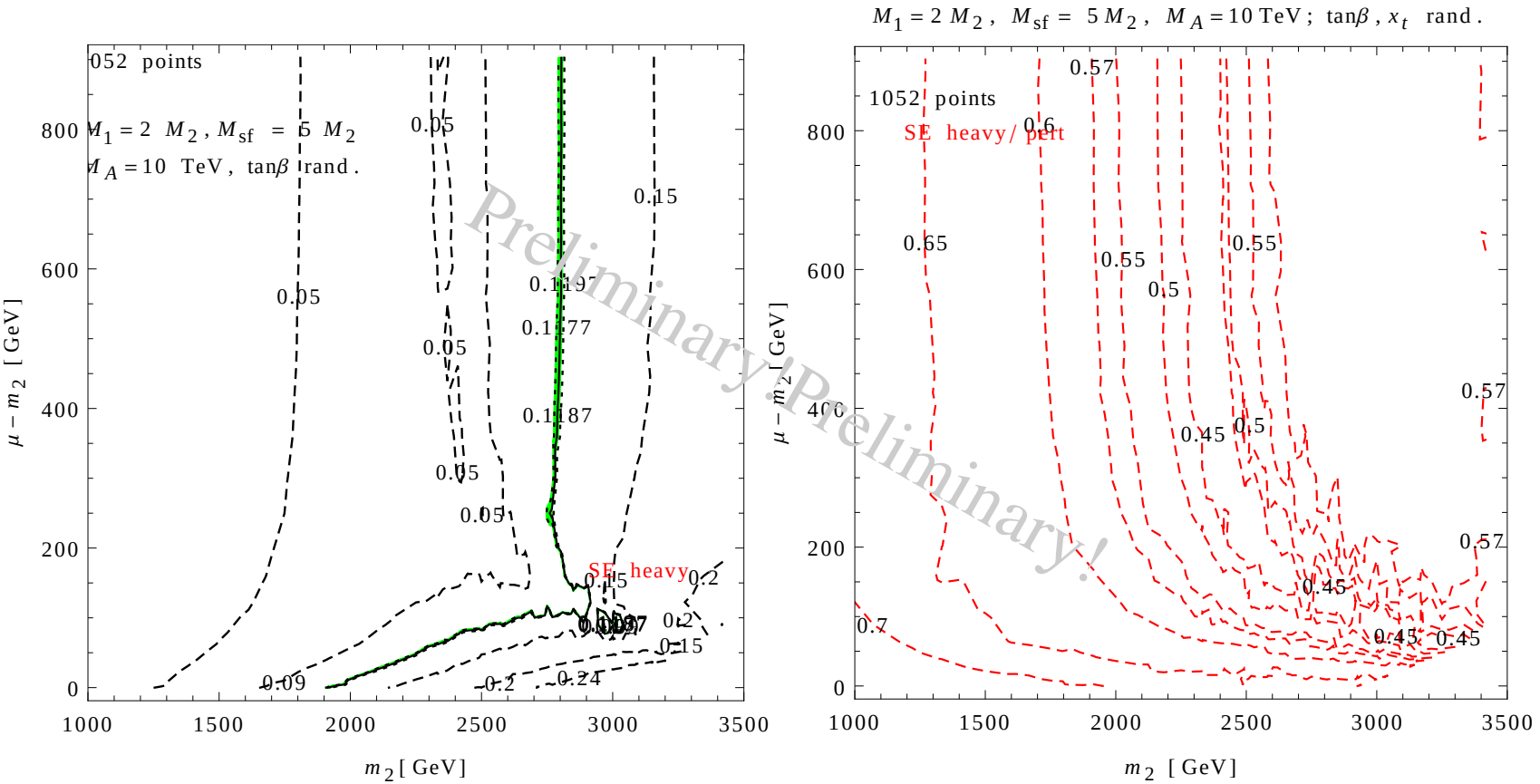
General investigation [MB, A. Bharucha, F. Dighera, C. Hellmann, A. Hryczuk, S. Recksiegel, P. Ruiz-Femenia, work in progress]

- Identify MSSM parameter space where SE is the dominant radiative correction (not necessarily $\mathcal{O}(1)$).
- Include experimental constraints on MSSM parameter space, including direct detection of DM.
- Include exact 1-loop on-shell mass splittings.
- Analyse thermal corrections.
- Separate exclusive final states.
- Strong constraints on pure wino from indirect detection [Cohen et al. (2013); Fan, Reece (2013); Hryczuk et al. (2014)]
  In MSSM dependence on sfermion masses, Higgsino-, bino-fraction. Allowed parameter space?

Also a CPU problem. Aim at 10min/parameter point.

Example: Higgsino fraction

Wino-like LSP,
sfermions and heavy Higgses (practically) decoupled



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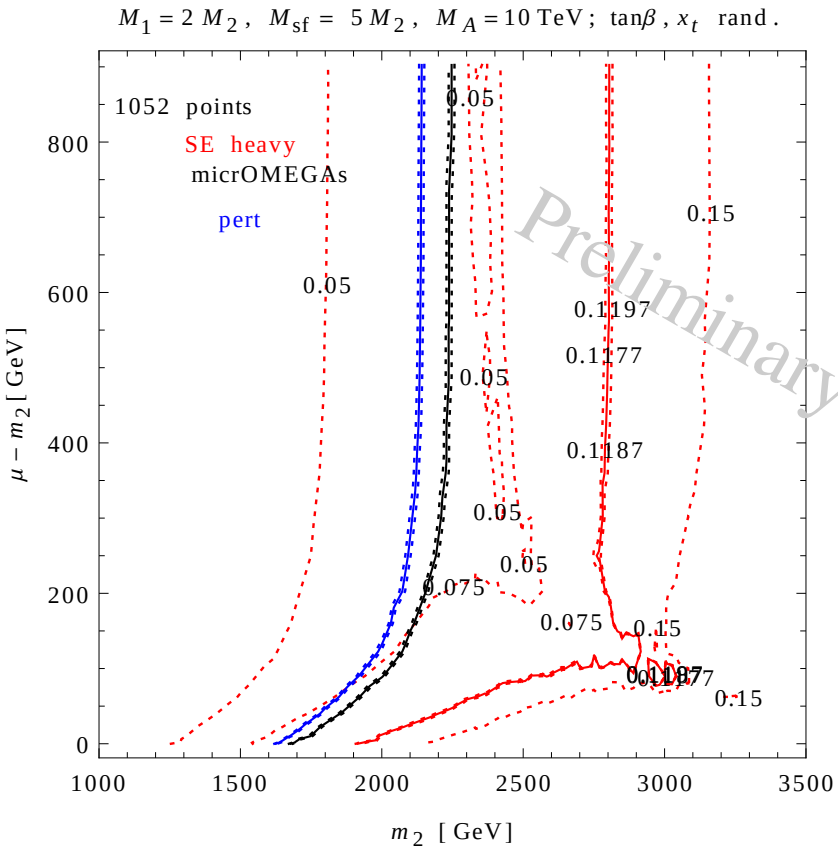

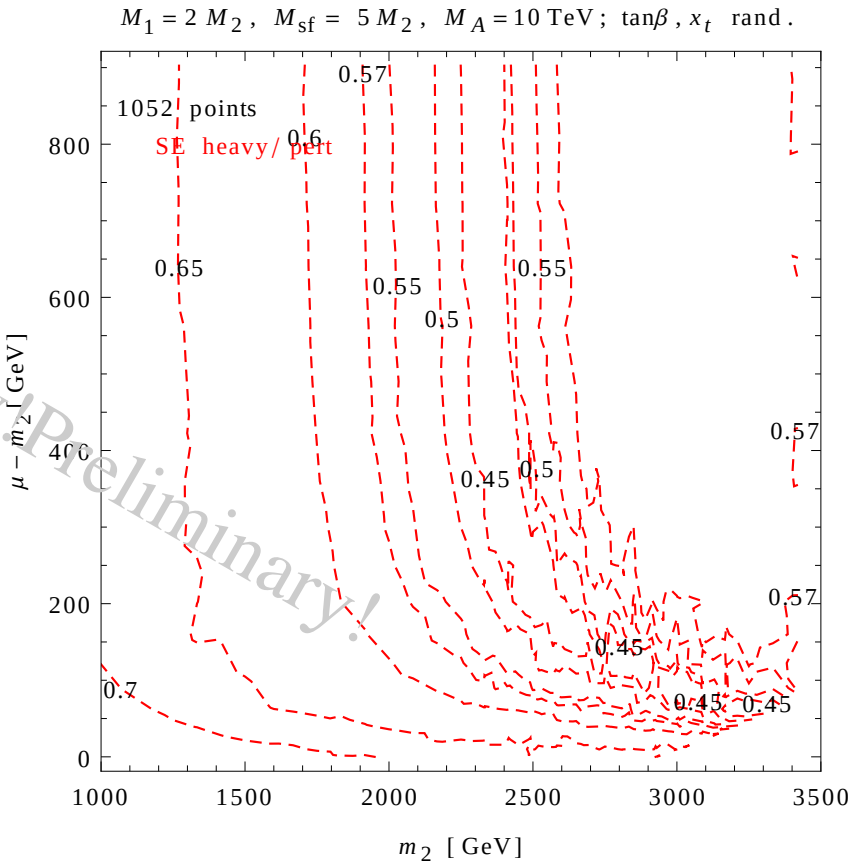

Summary

- Sommerfeld enhancement often relevant in still viable MSSM parameter regions.
- Provide method for χ_1^0 relic abundance calculation including $\sigma^{\mathrm{SF}} v$ for
  - general MSSM (generic χ_1^0 composition)
  - leading order potential interactions in coupled two-particle systems
  - up to $\mathcal{O}(v^2)$ effects in the short-distance annihilation
  - including partial-wave separation and off-diagonal annihilation
  - New method to solve the multi-channel Schrödinger equation without numerical instabilities.
  - Approximate treatment of heavy channels.
- largest effect for wino-like χ_1^0: $\sim 40 - 60\%$ reduction of $\Omega_\chi h^2$
- still 10% or so for higgsino-like χ_1^0 model and some bino-like cases.
- In preparation: *Scan of MSSM parameter space*
  Identify regions where the Sommerfeld enhancement is the dominant radiative correction (above $\sim 15\%$ correction on $\sigma_{\chi\chi} v$)
  Relic density and cosmic-ray signatures.