

Chiral power counting of one- and two-body currents, isospin violation, and the pion–nucleon σ -term

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MH, P. Klos, A. Schwenk 1503.04811

C. Ditsche, MH, B. Kubis, U.-G. Meißner, JHEP 1206 (2012) 043, JHEP 1206 (2012) 063

A. Crivellin, MH, M. Procura, PRD 89 (2014) 054021

MH, J. Ruiz de Elvira, B. Kubis, U.-G. Meißner, in preparation

Outline

1 Chiral power counting of one- and two-body currents

- Chiral counting
- Matching to NREFT

2 Scalar couplings for u - and d -quark

3 Phenomenological determination of the pion–nucleon σ -term

- Cheng–Dashen theorem, scalar form factor
- Scattering lengths
- Roy–Steiner equations for πN scattering

4 Conclusions

Nuclear theory for direct detection

- **Non-relativistic EFT**: NR nucleon and WIMP fields [Fan et al. 2010](#), [Fitzpatrick et al. 2012](#)

- Scales $\mathcal{O}(M_\pi)$ integrated out (spontaneous breaking of chiral symmetry of QCD)
- Additional matching step to derive limits on **WIMP parameter space**

- **Chiral EFT**: nucleon, pion, and WIMP fields

[Prézeau et al. 2003](#), [Cirigliano et al. 2012, 2013](#), [Menéndez et al. 2012](#), [Klos et al. 2013](#)

Talks by Menéndez (this morning), Cirigliano, Klos (next Tu)

- **Chiral symmetry** included by construction
- Predicts hierarchy of WIMP–nucleon currents, **one-body** (χN) and **two-body** (χNN)
- Here: vector, axial-vector, scalar, pseudoscalar terms, matching to NREFT

Chiral counting

- Starting point: **effective WIMP Lagrangian** Goodman et al. 2010

$$\begin{aligned}\mathcal{L}_\chi = & \frac{1}{\Lambda^3} \sum_q \left[C_q^{SS} \bar{\chi} \chi m_q \bar{q} q + C_q^{PS} \bar{\chi} i \gamma_5 \chi m_q \bar{q} q + C_q^{SP} \bar{\chi} \chi m_q \bar{q} i \gamma_5 q + C_q^{PP} \bar{\chi} i \gamma_5 \chi m_q \bar{q} i \gamma_5 q \right] \\ & + \frac{1}{\Lambda^2} \sum_q \left[C_q^{VV} \bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu q + C_q^{AV} \bar{\chi} \gamma^\mu \gamma_5 \chi \bar{q} \gamma_\mu q + C_q^{VA} \bar{\chi} \gamma^\mu \chi \bar{q} \gamma_\mu \gamma_5 q + C_q^{AA} \bar{\chi} \gamma^\mu \gamma_5 \chi \bar{q} \gamma_\mu \gamma_5 q \right] \\ & + \frac{1}{\Lambda^3} \left[C_g^S \bar{\chi} \chi \alpha_s G_{\mu\nu}^a G_a^{\mu\nu} \right]\end{aligned}$$

- Chiral power counting**

$$\partial = \mathcal{O}(p), \quad m_q = \mathcal{O}(p^2) = \mathcal{O}(M_\pi^2), \quad a_\mu, v_\mu = \mathcal{O}(p), \quad \frac{\partial}{m_N} = \mathcal{O}(p^2)$$

→ construction of effective Lagrangian for nucleon and pion fields

→ organize in terms of **chiral order ν** , $\mathcal{M} = \mathcal{O}(p^\nu)$

Chiral counting: summary

	Nucleon	V		A			Nucleon	S	P
WIMP		t	x	t	x		WIMP		
V	1b	0	$1 + 2$	2	$0 + 2$		1b	2	1
	2b	4	$2 + 2$	2	$4 + 2$		2b	3	5
	2b NLO	—	—	5	$3 + 2$		2b NLO	—	4
A	1b	$0 + 2$	1	$2 + 2$	0		1b	$2 + 2$	$1 + 2$
	2b	$4 + 2$	2	$2 + 2$	4		2b	$3 + 2$	$5 + 2$
	2b NLO	—	—	$5 + 2$	3		2b NLO	—	$4 + 2$

- +2 from NR expansion of WIMP spinors, terms can be dropped if $m_\chi \gg m_N$
- Red: all terms up to $\nu = 3$
- Two-body currents: AA Menéndez et al. 2012, Klos et al. 2013, SS Cirigliano et al. 2012, but new currents in AV and VA channel 1503.04811

2b currents: examples

- Scalar current Cirigliano et al. 2012

$$\mathcal{M}_{2,\text{NR}}^{SS} = -\chi_{r'}^\dagger \chi_r \left(\frac{g_A}{2F_\pi} \right)^2 f_\pi M_\pi^2 \chi_{s_1'}^\dagger \chi_{s_2'}^\dagger \tau_1 \cdot \tau_2 X_{12}^\pi \chi_{s_1} \chi_{s_2}$$

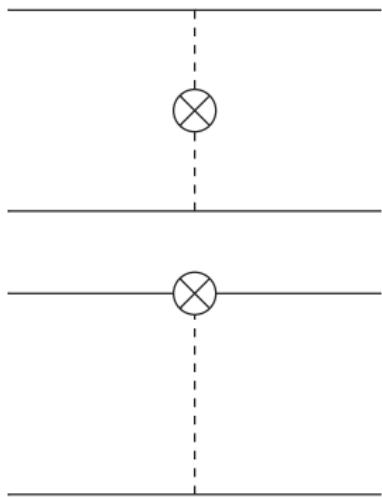
$$X_{12}^\pi = \frac{\sigma_1 \cdot \mathbf{q}_1 \sigma_2 \cdot \mathbf{q}_2}{(\mathbf{q}_1^2 + M_\pi^2)(\mathbf{q}_2^2 + M_\pi^2)} \quad f_\pi = \frac{1}{\Lambda^3} \sum_{q=u,d} C_q^{SS} f_q^\pi$$

- Vector current

$$\mathcal{M}_{2,\text{NR}}^{AV} = -\frac{C_u^{AV} - C_d^{AV}}{\Lambda^2} \left(\frac{g_A}{2F_\pi} \right)^2 \chi_{r'}^\dagger \sigma \chi_r \cdot \chi_{s'_1}^\dagger \chi_{s'_2}^\dagger i [\tau_1 \times \tau_2]^3 \left[\frac{\sigma_1 \cdot \mathbf{q}_1 \sigma_2}{\mathbf{q}_1^2 + M_\pi^2} - \frac{\sigma_2 \cdot \mathbf{q}_2 \sigma_1}{\mathbf{q}_2^2 + M_\pi^2} + (\mathbf{q}_1 - \mathbf{q}_2) X_{12}^\pi \right] \chi_{s_1} \chi_{s_2}$$

- Axial current

$$\mathcal{M}_{2,\text{NR}}^{\text{VA}} = -\frac{C_u^{\text{VA}} - C_d^{\text{VA}}}{\Lambda^2} \frac{g_A}{4F_\pi^2} \chi_{r'}^\dagger \chi_r \chi_{s'_1}^\dagger \chi_{s'_2}^\dagger \left\{ i[\tau_1 \times \tau_2]^3 \frac{\sigma_2 \cdot \mathbf{q}_2}{\mathbf{q}_2^2 + M_\pi^2} + (1 \leftrightarrow 2) \right\} \chi_{s_1} \chi_{s_2}$$



Matching to NREFT

Operator basis in NREFT

$$\begin{aligned}\mathcal{O}_1 &= \mathbb{1}, & \mathcal{O}_2 &= (\mathbf{v}^\perp)^2, & \mathcal{O}_3 &= i\mathbf{S}_N \cdot (\mathbf{q} \times \mathbf{v}^\perp), & \mathcal{O}_4 &= \mathbf{S}_X \cdot \mathbf{S}_N \\ \mathcal{O}_5 &= i\mathbf{S}_X \cdot (\mathbf{q} \times \mathbf{v}^\perp), & \mathcal{O}_6 &= \mathbf{S}_X \cdot \mathbf{q} \mathbf{S}_N \cdot \mathbf{q}, & \mathcal{O}_7 &= \mathbf{S}_N \cdot \mathbf{v}^\perp, & \mathcal{O}_8 &= \mathbf{S}_X \cdot \mathbf{v}^\perp \\ \mathcal{O}_9 &= i\mathbf{S}_X \cdot (\mathbf{S}_N \times \mathbf{q}), & \mathcal{O}_{10} &= i\mathbf{S}_N \cdot \mathbf{q}, & \mathcal{O}_{11} &= i\mathbf{S}_X \cdot \mathbf{q}\end{aligned}$$

Matching to ChEFT

$$\begin{aligned}\mathcal{M}_{1,\text{NR}}^{\text{SS}} &= \chi_{r'}^\dagger \chi_{s'}^\dagger \mathcal{O}_1 f_N(t) \chi_r \chi_s, & \mathcal{M}_{1,\text{NR}}^{\text{SP}} &= \chi_{r'}^\dagger \chi_{s'}^\dagger \mathcal{O}_{10} g_5^N(t) \chi_r \chi_s, & \mathcal{M}_{1,\text{NR}}^{\text{PP}} &= \frac{1}{m_\chi} \chi_{r'}^\dagger \chi_{s'}^\dagger \mathcal{O}_6 h_5^N(t) \chi_r \chi_s \\ \mathcal{M}_{1,\text{NR}}^{\text{VV}} &= \chi_{r'}^\dagger \chi_{s'}^\dagger \left\{ \mathcal{O}_1 \left(f_1^{V,N}(t) + \frac{t}{4m_N^2} f_2^{V,N}(t) \right) + \frac{1}{m_N} \mathcal{O}_3 f_2^{V,N}(t) + \frac{1}{m_N m_\chi} (t \mathcal{O}_4 + \mathcal{O}_6) f_2^{V,N}(t) \right\} \chi_r \chi_s \\ \mathcal{M}_{1,\text{NR}}^{\text{AV}} &= \chi_{r'}^\dagger \chi_{s'}^\dagger \left\{ 2\mathcal{O}_8 f_1^{V,N}(t) + \frac{2}{m_N} \mathcal{O}_9 (f_1^{V,N}(t) + f_2^{V,N}(t)) \right\} \chi_r \chi_s \\ \mathcal{M}_{1,\text{NR}}^{\text{AA}} &= \chi_{r'}^\dagger \chi_{s'}^\dagger \left\{ -4\mathcal{O}_4 g_A^N(t) + \frac{1}{m_N^2} \mathcal{O}_6 g_P^N(t) \right\} \chi_r \chi_s, & \mathcal{M}_{1,\text{NR}}^{\text{VA}} &= \chi_{r'}^\dagger \chi_{s'}^\dagger \left\{ -2\mathcal{O}_7 + \frac{2}{m_\chi} \mathcal{O}_9 \right\} h_A^N(t) \chi_r \chi_s\end{aligned}$$

Conclusions

- \mathcal{O}_2 , \mathcal{O}_5 , and \mathcal{O}_{11} do not appear at $\nu = 3$, not all \mathcal{O}_i independent
- 2b operators of similar or even greater importance than some of the 1b operators
- But: does not include summation over nucleus, i.e. **coherence effects**

Nuclear matrix elements: scalar couplings

- Scalar nuclear matrix elements: $f_N = \frac{m_N}{\Lambda^3} \sum_{q=u,d,s} C_q^{SS} f_q^N + \dots$

→ scalar couplings $\langle N | m_q \bar{q} q | N \rangle = f_q^N m_N \quad N \in \{p, n\}$

- Determination of two-flavor couplings Ellis et al. 2000, micrOMEGAs

$$y = \frac{2\langle N | \bar{s}s | N \rangle}{\langle N | \bar{u}u + \bar{d}d | N \rangle} \quad z = \frac{\langle N | \bar{u}u - \bar{s}s | N \rangle}{\langle N | \bar{d}d - \bar{s}s | N \rangle} \quad \sigma_{\pi N} = \langle N | \hat{m}(\bar{u}u + \bar{d}d) | N \rangle$$
$$f_d^p = \frac{2\sigma_{\pi N}}{(1 + \frac{m_u}{m_d}) m_p (1 + \alpha)} \quad \alpha = \frac{2z - (z-1)y}{2 + (z-1)y} \quad \hat{m} = \frac{m_u + m_d}{2}$$

→ Two-flavor couplings from **SU(3)** quantities!

- Even worse

- z from **LO fits** to baryon masses Cheng 1989
- Isospin violation** within this framework

→ do the calculation based on **SU(2) ChPT** Crivellin, MH, Procura 2014

Scalar couplings for u - and d -quark

- Numbers ($\xi = \frac{m_d - m_u}{m_d + m_u}$)

$$f_u^N = \frac{\sigma_{\pi N}(1 - \xi)}{2m_N} + \Delta f_u^N$$

$$\Delta f_u^P = (1.0 \pm 0.2) \times 10^{-3}$$

$$\Delta f_d^P = (-2.1 \pm 0.4) \times 10^{-3}$$

$$f_d^N = \frac{\sigma_{\pi N}(1 + \xi)}{2m_N} + \Delta f_d^N$$

$$\Delta f_u^n = (-1.0 \pm 0.2) \times 10^{-3}$$

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- Isospin violation

$$f_{u,d}^P - f_{u,d}^N = 2B_{C5}(m_d - m_u) \left(1 \mp \frac{1}{\xi}\right) \quad (m_p - m_n)^{\text{str}} = 4B_{C5}(m_d - m_u)$$

- Compare with $SU(3)$ approach micrOMEGAs 2013

$$f_u^P - f_u^N = (1.9 \pm 0.4) \times 10^{-3}$$

$$f_u^P - f_u^N \Big|_{SU(3)} = 4.3 \times 10^{-3}$$

$$f_d^P - f_d^N = (-4.1 \pm 0.7) \times 10^{-3}$$

$$f_d^P - f_d^N \Big|_{SU(3)} = -8.2 \times 10^{-3}$$

↪ **Isospin violation** overestimated by a **factor 2**

- Becomes relevant in case of cancellations (“blind spots”) Crivellin et al. 1503.03478
- Central value completely fixed by $\sigma_{\pi N}$

Status of the phenomenological determination of $\sigma_{\pi N}$

- **Karlsruhe/Helsinki** partial-wave analysis KH80 Höhler et al. 1980s
 - comprehensive analyticity constraints, old data
- Formalism for the extraction of $\sigma_{\pi N}$ via the **Cheng–Dashen low-energy theorem**
Gasser, Leutwyler, Locher, Sainio 1988, Gasser, Leutwyler, Sainio 1991
 - “canonical value” $\sigma_{\pi N} \sim 45$ MeV, based on KH80 input
- **GWU/SAID** partial-wave analysis Pavan, Strakovsky, Workman, Arndt 2002
 - much larger value $\sigma_{\pi N} = (64 \pm 8)$ MeV
- ChPT fits vary according to PWA input Fettes, Meißner 2000
 - (same problem in different regularizations (w/ and w/o Δ) Alarcón et al. 2012)

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 - (same problem in different regularizations (w/ and w/o Δ) Alarcón et al. 2012)
- This talk: two new sources of information on low-energy πN scattering
 - Precision extraction of **πN scattering lengths** from **hadronic atoms**
 - **Roy-equation constraints**: analyticity, unitarity, crossing symmetry

Extraction of $\sigma_{\pi N}$ from πN scattering

- Scalar form factor of the nucleon

$$\sigma(t) = \langle N(p') | \hat{m}(\bar{u}u + \bar{d}d) | N(p) \rangle \quad t = (p' - p)^2 \quad \sigma_{\pi N} = \sigma(0)$$

- Low-energy theorem Cheng, Dashen 1971

$$F_\pi^2 \bar{D}^+ (\nu = 0, t = 2M_\pi^2) = \sigma(2M_\pi^2) + \Delta_R$$

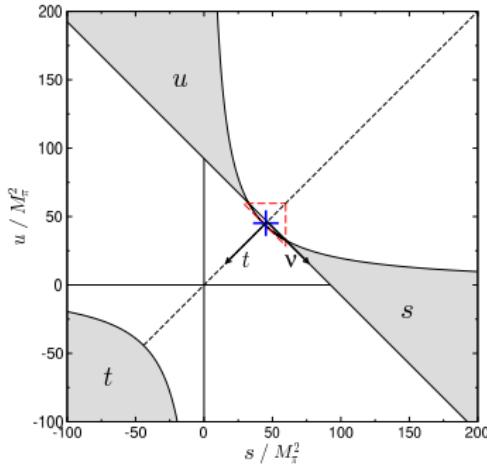
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$$\underbrace{F_\pi^2 \bar{D}^+ (\nu = 0, t = 2M_\pi^2)}_{F_\pi^2 (d_{00}^+ + 2M_\pi^2 d_{01}^+) + \Delta_D} = \sigma(2M_\pi^2) + \Delta_R$$



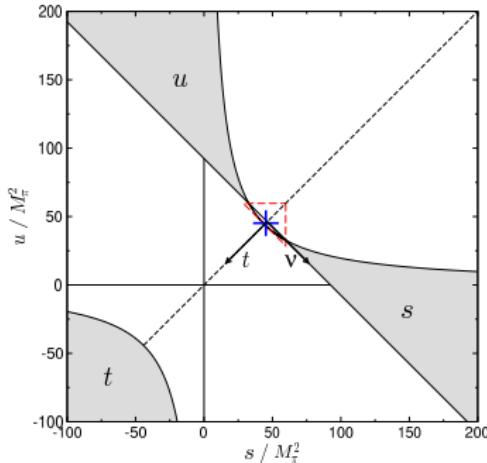
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Extraction of $\sigma_{\pi N}$ from πN scattering

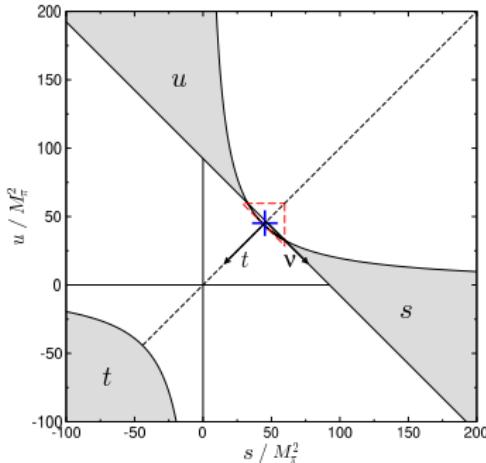
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- Remainder $|\Delta_R| \lesssim 2$ MeV small Bernard, Kaiser, Meißen 1996



Extraction of $\sigma_{\pi N}$ from πN scattering

- Scalar form factor of the nucleon

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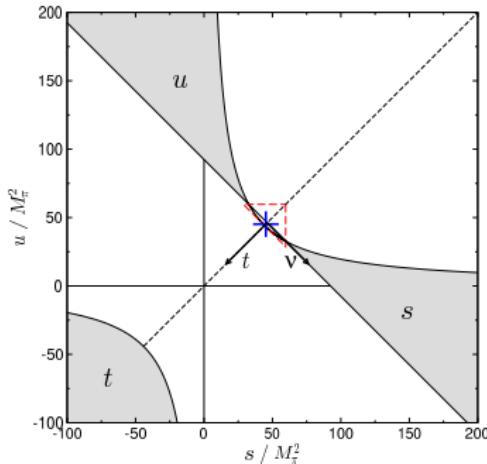
$$\underbrace{F_\pi^2 \bar{D}^+ (\nu = 0, t = 2M_\pi^2)}_{F_\pi^2 (d_{00}^+ + 2M_\pi^2 d_{01}^+) + \Delta_D} = \underbrace{\sigma(2M_\pi^2)}_{\sigma_{\pi N} + \Delta_R} + \Delta_R$$

- Remainder $|\Delta_R| \lesssim 2$ MeV small Bernard, Kaiser, Mei&Bddot;nner 1996

- Dispersive approach Gasser, Leutwyler, Sainio 1991

$$\Delta_D - \Delta_\sigma = (-3.3 \pm 0.2) \text{ MeV}$$

but error only covers $\pi\pi$ phase shifts



Improvements

- Modern $\pi\pi$ phase shifts, $K\bar{K}$ intermediate states, sensitivity to πN parameters

$$\Delta_D - \Delta_\sigma = (-1.8 \pm 0.2) \text{ MeV}$$

- Isospin violation: define “**isoscalar**” as

$$X^+ \rightarrow X^p \equiv \frac{1}{2}(X_{\pi^+ p \rightarrow \pi^+ p} + X_{\pi^- p \rightarrow \pi^- p}) \quad X = D, d_{00}, d_{01}, a_{0+}, \dots$$

and “**isospin limit**” by proton and charged pion

- Calculate isospin-violating corrections in $SU(2)$ ChPT

$$\sigma_p = F_\pi^2 (d_{00}^p + 2M_\pi^2 d_{01}^p) + \underbrace{\Delta_D - \Delta_\sigma}_{(-1.8 \pm 0.2) \text{ MeV}} + \underbrace{\Delta_R}_{\lesssim 2 \text{ MeV}} + \underbrace{\frac{81g_A^2 M_\pi \Delta_\pi}{256\pi F_\pi^2}}_{+3.4 \text{ MeV}} + \underbrace{\frac{e^2}{2} F_\pi^2 (4f_1 + f_2)}_{(-0.4 \pm 2.2) \text{ MeV}}$$

→ sizable correction from $\Delta_\pi = M_\pi^2 - M_{\pi^0}^2$, but “wrong” direction

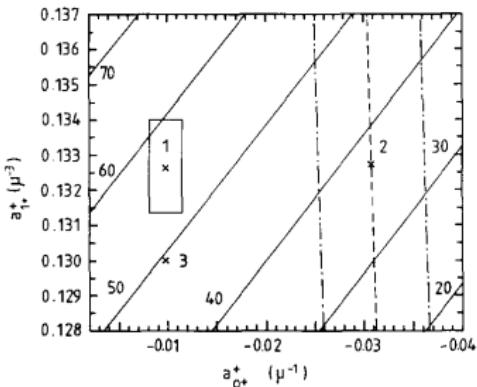
How to get d_{00}^+ and d_{01}^+ ?

- Standard approach Gasser, Leutwyler, Locher, Sainio 1988:

replace d_{00}^+ and d_{01}^+ in favor of threshold

parameters: a_{0+}^+ and a_{1+}^+ (notation: $a_{\ell\pm 1/2}^{l=\pm}$)

↪ corrections from PWA via DRs (D^+ and E^+)



- Coupling constant:

$$g^2/4\pi = 14.28/13.75$$

- S-wave:

$$a_{0+}^+ = -10/0 \times 10^{-3} M_\pi^{-1}$$

- P-wave: a_{1+}^+ needs to be known very precisely

	Born	a_{0+}^+	a_{1+}^+	D^+	E^+	Σ_d
KH80	-133	-7	+352	-91	-72	50
FA01	-127	0	+351	-88	-69	67
diff.	+6	+7	-1	+3	+3	17

$$\Sigma_d = F_\pi^2 (d_{00}^+ + 2M_\pi^2 d_{01}^+)$$

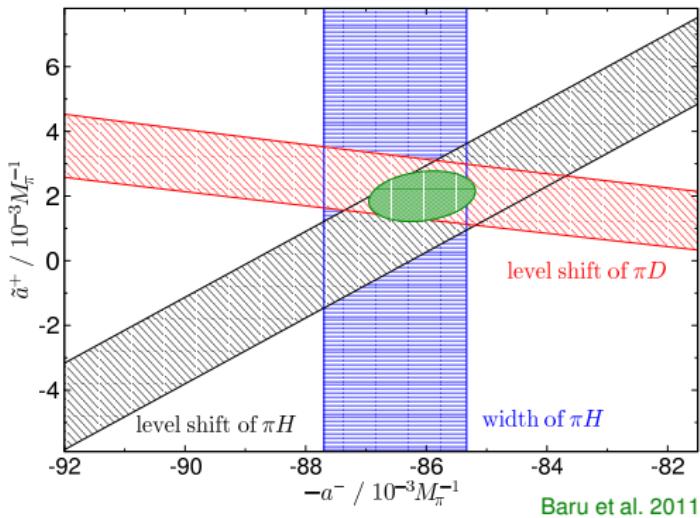
Hadronic atoms: constraints for πN

$$\tilde{a}^+ = a^+ + \frac{1}{4\pi(1+M_\pi/m_p)} \left\{ \frac{4(M_\pi^2 - M_{\pi^0}^2)}{F_\pi^2} c_1 - 2e^2 f_1 \right\}$$

- $\pi H/\pi D$: bound state of π^- and p/d , spectrum sensitive to threshold πN amplitude
- **Combined analysis** of πH and πD

$$a^+ \equiv a_{0+}^+ = (7.5 \pm 3.1) \times 10^{-3} M_\pi^{-1}$$

$$a^- \equiv a_{0+}^- = (86.0 \pm 0.9) \times 10^{-3} M_\pi^{-1}$$

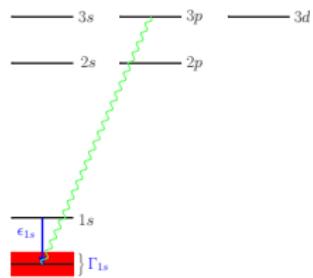


Baru et al. 2011

- In the following, determine d_{00}^p and d_{01}^p by solving
Roy–Steiner equations
- Constraints: scattering lengths in isospin basis

$$a_{0+}^{1/2} = (169.8 \pm 2.0) \times 10^{-3} M_\pi^{-1}$$

$$a_{0+}^{3/2} = (-86.3 \pm 1.8) \times 10^{-3} M_\pi^{-1}$$



Roy(-Steiner) equations

Roy(-Steiner) equations = Dispersion relations + partial-wave expansion
+ crossing symmetry + unitarity

- Coupled system of integral equations for partial waves
- Self-consistency condition for phase shifts
- Equations rigorously valid for a finite energy range
 - introduce matching point s_m
- Consider only partial waves for $J \leq J_{\max}$
- Input
 - High-energy region: $\text{Im } t_J^I(s)$ for $s \geq s_m$ and all J
 - Higher partial waves: $\text{Im } t_J^I(s)$ for $J > J_{\max}$ and all s
 - Inelasticities: $\eta_J^I(s)$ for $J \leq J_{\max}$ and $4M_\pi^2 \leq s \leq s_m$
- Output
 - Self-consistent solution for phase shifts: $\delta_J^I(s)$ for $J \leq J_{\max}$ and $4M_\pi^2 \leq s \leq s_m$
 - Constraints on subtraction constants

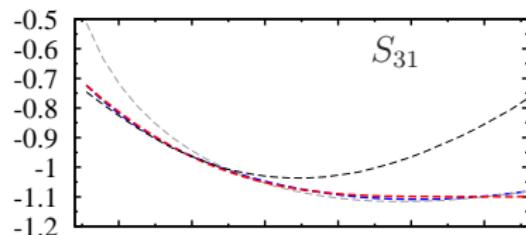
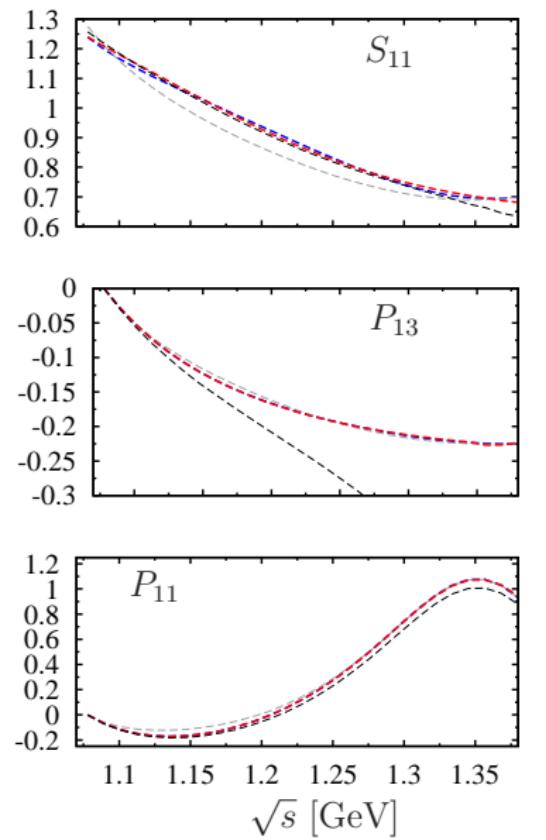
Roy–Steiner solution: strategy

- Introduce as many **subtractions** as necessary to match dof Gasser, Wanders 1999
- Minimize difference between LHS and RHS on a grid of points W_j

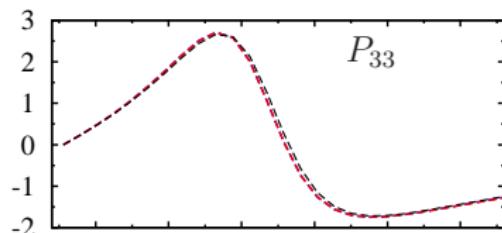
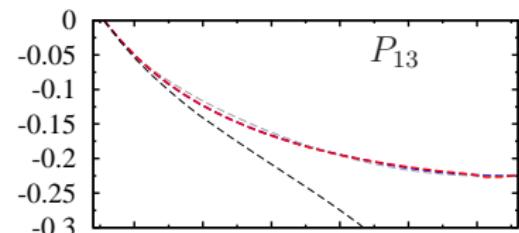
$$\chi_{\text{RS}}^2 = \sum_{\ell, l_s, \pm} \sum_{j=1}^N \left(\frac{\text{Re } f_{\ell \pm}^{l_s}(W_j) - F[f_{\ell \pm}^{l_s}](W_j)}{\text{Re } f_{\ell \pm}^{l_s}(W_j)} \right)^2$$

- Impose scattering lengths as constraints in the fit

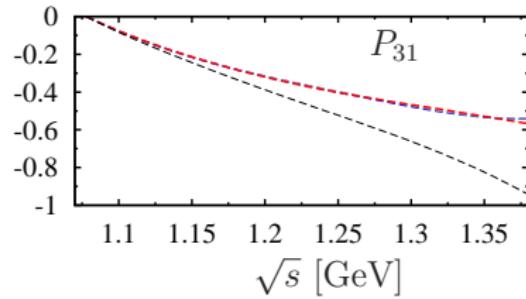
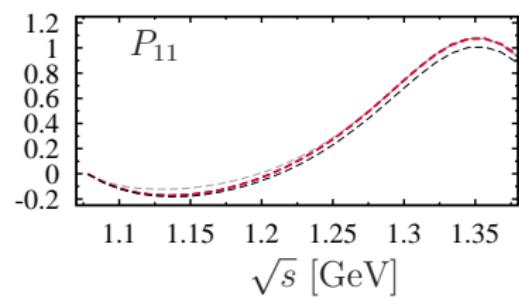
Roy–Steiner solution: reproducing KH80



blue/red
↔
LHS/RHS
after fit



gray/black
↔
LHS/RHS
before fit



notation: $L_{2I_1S_2J}$

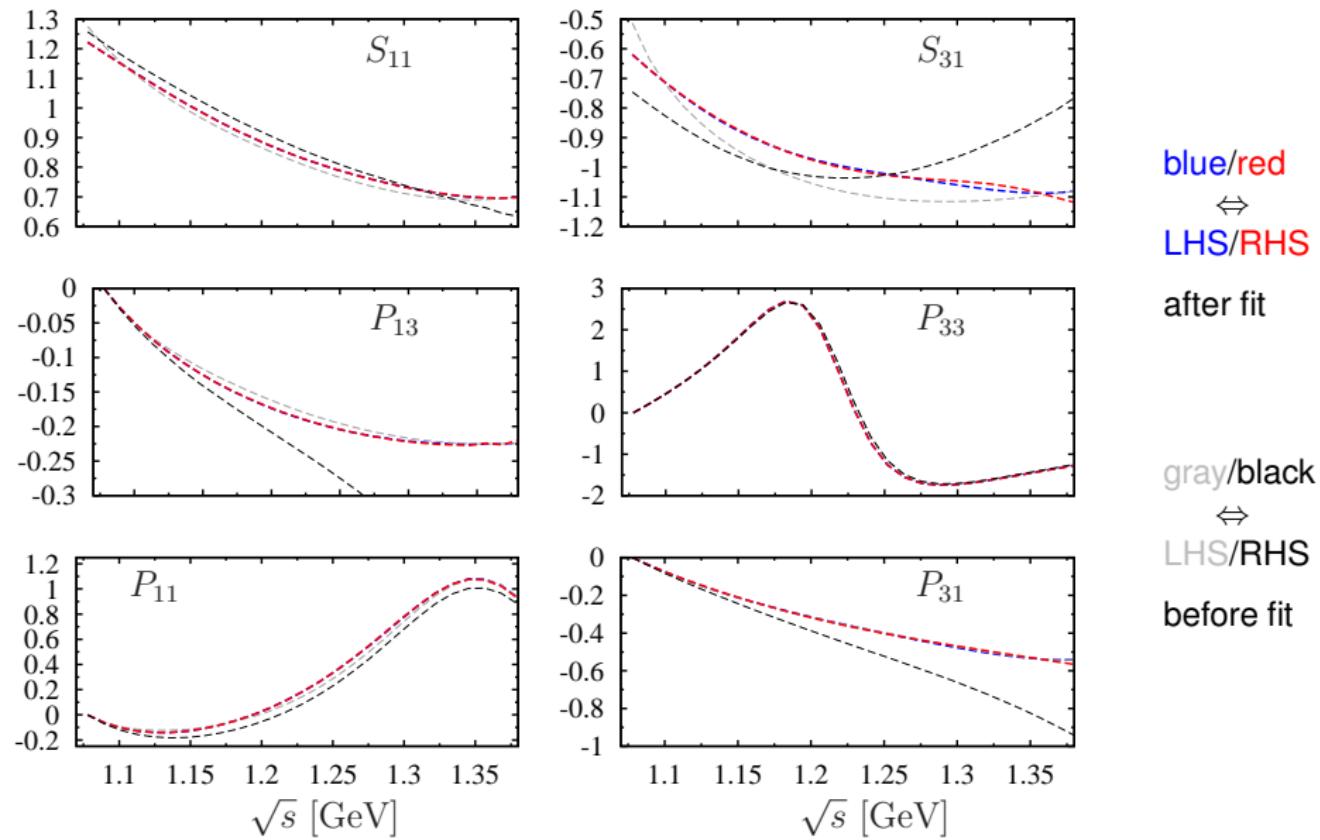
Roy–Steiner solution: reproducing KH80

- Resulting parameters with KH80 scattering lengths as constraint:

	$d_{00}^+ [M_\pi^{-1}]$	$d_{01}^+ [M_\pi^{-3}]$	$\Sigma_d = F_\pi^2 (d_{00}^+ + 2M_\pi^2 d_{01}^+) [\text{MeV}]$
KH80	-1.46(10)	1.14(2)	50(7)
KH80 fit	-1.54	1.16	48

- KH80 **internally consistent**

Roy–Steiner solution: hadronic-atom fit



Roy–Steiner solution: lesson for the σ -term

- Resulting parameters:

	$d_{00}^+ [M_\pi^{-1}]$	$d_{01}^+ [M_\pi^{-3}]$	$\Sigma_d = F_\pi^2 (d_{00}^+ + 2M_\pi^2 d_{01}^+) [\text{MeV}]$
KH80	−1.46(10)	1.14(2)	50(7)
KH80 fit	−1.54	1.163	48
hadronic-atom fit	−1.36	1.155	58

- Modern input for the scattering lengths does increase Σ_d , but by just about half the amount that Pavan, Strakovsky, Workman, Arndt 2002 found
- Compared to Gasser, Leutwyler, Locher, Sainio 1988: necessity for a_{1+}^+ eliminated by Roy–Steiner self-consistency condition

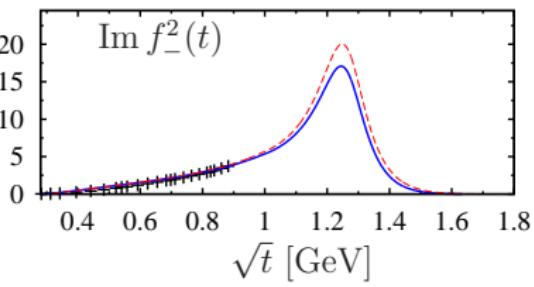
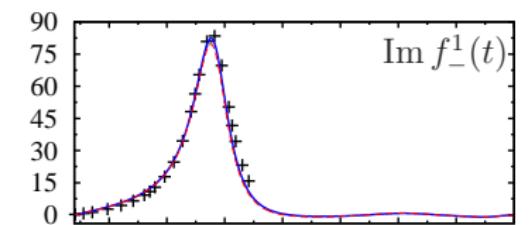
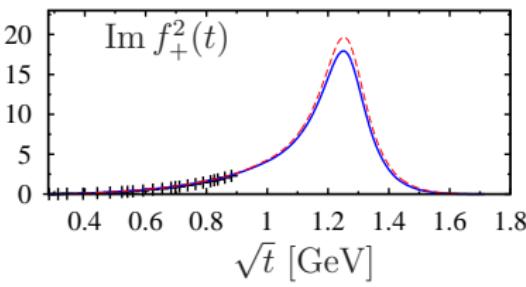
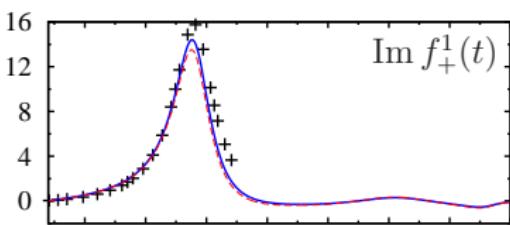
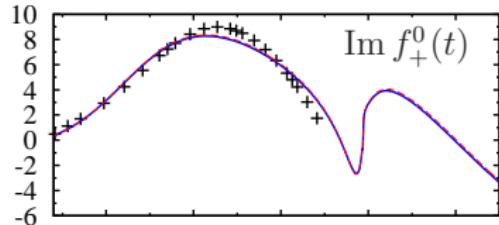
Error analysis (preliminary)

$$\sigma_{\pi N} = 59.1 \pm \underbrace{0.7}_{\text{flat directions}} \pm \underbrace{0.3}_{\text{matching}} \pm \underbrace{0.5}_{\text{systematics}} \pm \underbrace{1.7}_{\text{scattering lengths}} \pm \underbrace{3.0}_{\text{low-energy theorem}} \text{ MeV}$$
$$= 59.1 \pm 3.5 \text{ MeV}$$

Conclusions

- **Chiral counting** of one- and two-body currents
 - Predicts hierarchy among \mathcal{O}_i from NREFT
 - New 2b currents found
 - Coherence to be analyzed
- **Scalar couplings**: isospin violation and σ -term
 - Review of **standard procedure** to extract $\sigma_{\pi N}$ from πN scattering
 - KH80 PWA self-consistent, but at odds with **hadronic-atom phenomenology**
 - Roy–Steiner formalism **reproduces KH80** results with KH80 input
 - With modern input for scattering lengths $\sigma_{\pi N}$ **increases**, but now reliably extracted with data-driven method

Roy–Steiner solution: t -channel for KH80 fit



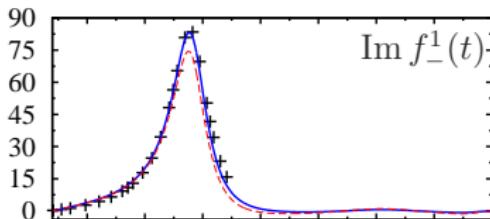
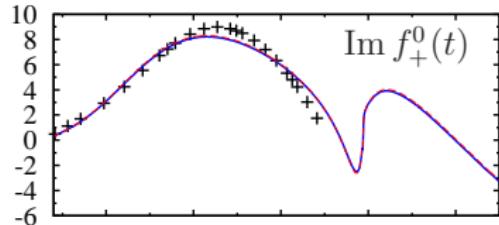
black: KH80

blue/red

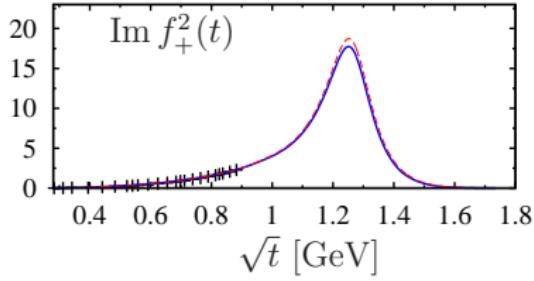
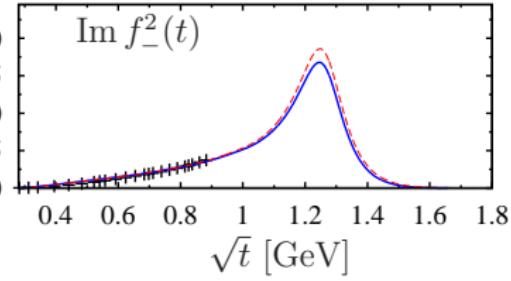
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before/after fit

Roy–Steiner solution: t -channel for hadronic-atom fit

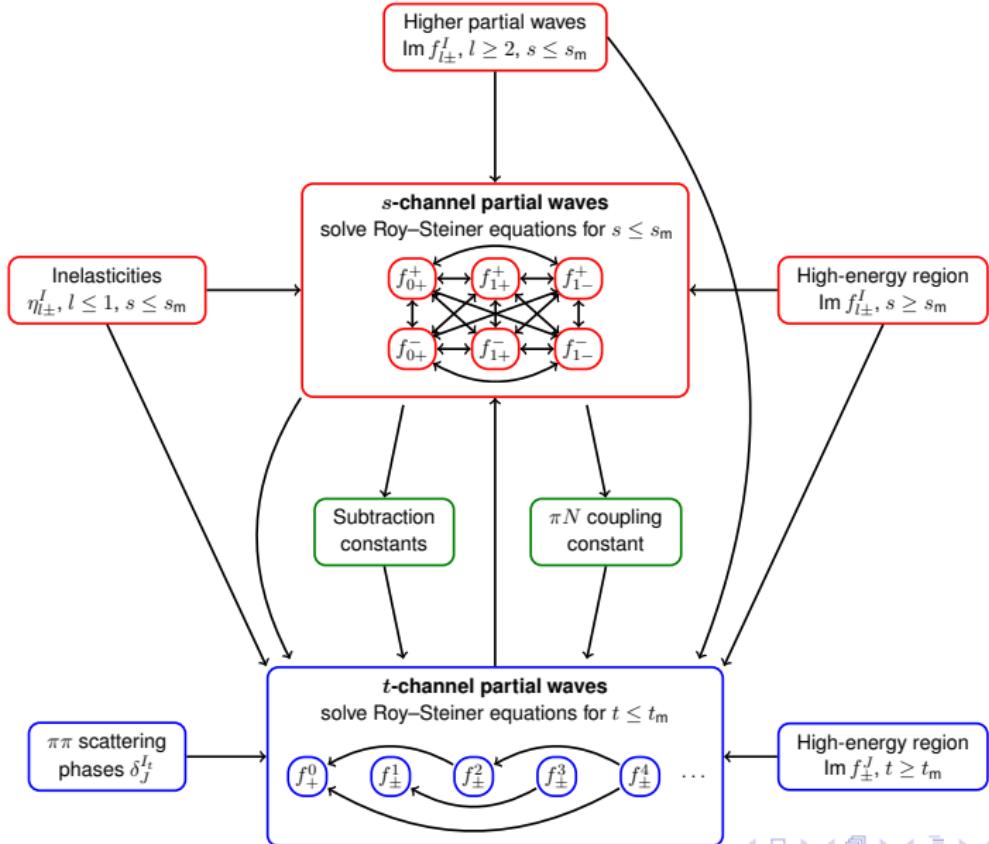


black: KH80



blue/red
↔
before/after fit

Roy–Steiner equations for πN scattering: schematics



Scalar couplings for u - and d -quark

- Expansion of the **nucleon mass** including **isospin violation** Meißner, Steininger 1998

$$m_{\text{p/n}} = m_0 - 4c_1 M_{\pi^0}^2 \pm 2Bc_5(m_d - m_u) - \frac{e^2 F_\pi^2}{2}(f_1 \pm f_2 + f_3) - \frac{g_A^2(2M_{\pi^\pm}^3 + M_{\pi^0}^3)}{32\pi F_\pi^2} + \mathcal{O}(M_\pi^4)$$

- Feynman–Hellmann + Gell-Mann–Oakes–Renner

$$f_u^N = \frac{m_u}{m_N} \frac{\partial m_N}{\partial m_u} = B \frac{m_u}{m_N} \frac{\partial m_N}{\partial M_{\pi^0}^2} = -\frac{2B}{m_N} m_u \left[2c_1 \pm c_5 + \frac{3g_A^2(2M_{\pi^\pm} + M_{\pi^0})}{128\pi F_\pi^2} \right]$$

$$f_d^N = \frac{m_d}{m_N} \frac{\partial m_N}{\partial m_d} = B \frac{m_d}{m_N} \frac{\partial m_N}{\partial M_{\pi^0}^2} = -\frac{2B}{m_N} m_d \left[2c_1 \mp c_5 + \frac{3g_A^2(2M_{\pi^\pm} + M_{\pi^0})}{128\pi F_\pi^2} \right]$$

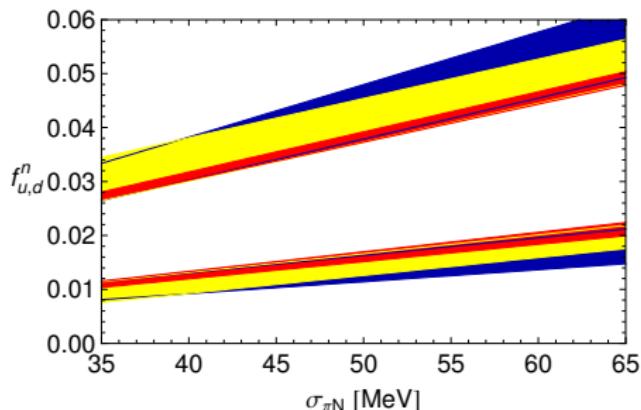
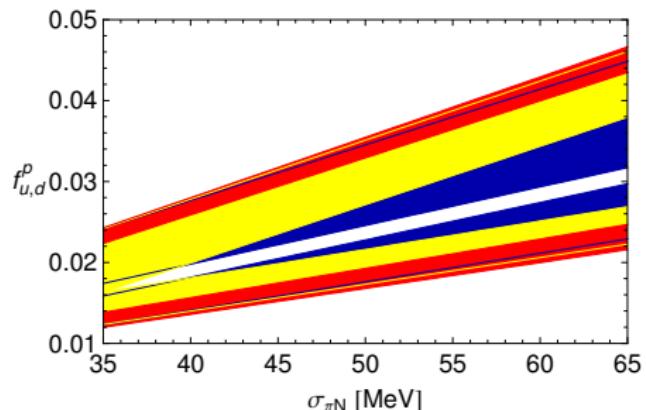
- Expressed in terms of $\sigma_{\pi N}$

$$m_N f_u^N = \frac{\sigma_{\pi N}}{2} (1 - \xi) \pm B c_5 (m_d - m_u) \left(1 - \frac{1}{\xi} \right)$$

$$m_N f_d^N = \frac{\sigma_{\pi N}}{2} (1 + \xi) \pm B c_5 (m_d - m_u) \left(1 + \frac{1}{\xi} \right)$$

$$\sigma_{\pi N} = \frac{1}{2} \left(\langle p | \hat{m}(\bar{u}u + \bar{d}d) | p \rangle + \langle n | \hat{m}(\bar{u}u + \bar{d}d) | n \rangle \right) \quad \xi = \frac{m_d - m_u}{m_d + m_u} = 0.36 \pm 0.04 \quad (\text{FLAG})$$

Scalar couplings for u - and d -quark



- Upper/lower \Leftrightarrow down-/up-coupling
- Color coding
 - ① Red: $SU(2)$ approach
 - ② Yellow: $SU(3)$ approach, y from $\sigma_{\pi N}$
 - ③ Blue: $SU(3)$ approach, y from lattice

Strangeness coupling

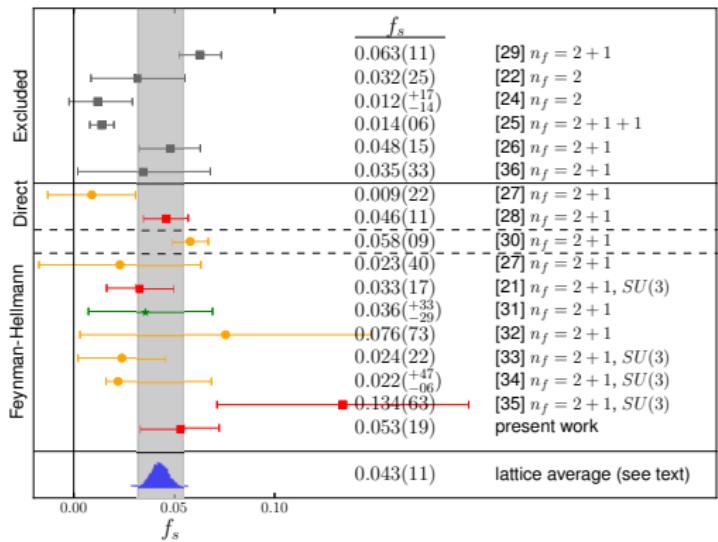
- f_s^N from $\sigma_{\pi N}$ via $SU(3)$ ChPT,
but **large uncertainties**

$$m_N f_s^N = \frac{m_s}{2\hat{m}} (\sigma_{\pi N} - \sigma_0)$$

- **Lattice** average

$$f_s^N = 0.043 \pm 0.011$$

→ large strangeness content
seems unlikely



Junnarkar, Walker-Loud 2013

Heavy quarks

- **Trace anomaly** of the energy-momentum tensor

$$m_N = \langle N | \theta_\mu^\mu | N \rangle = \left\langle N \left| \sum_{q \leq n_f} m_q \bar{q} q + \frac{\beta_{QCD}^{n_f}}{2g_s} G_a^a G_a^{\mu\nu} \right| N \right\rangle \quad \beta_{QCD}^{n_f} = - \left(11 - \frac{2n_f}{3} \right) g_s \frac{\alpha_s}{4\pi}$$

- Integrating out the heavy quarks: $n_f = 3$

$$m_N = \left\langle N \left| \sum_{q=u,d,s} m_q \bar{q} q - \frac{9}{8\pi} \alpha_s G_a^a G_a^{\mu\nu} \right| N \right\rangle$$

- **Heavy-quark** contribution Shifman, Vainshtein, Zakharov 1978

$$f_Q^N = \frac{1}{m_N} \langle N | m_Q \bar{Q} Q | N \rangle = - \frac{1}{m_N} \frac{2}{3} g_s \frac{\alpha_s}{4\pi} \frac{1}{2g_s} \langle N | G_a^a G_a^{\mu\nu} | N \rangle = - \frac{\alpha_s}{12\pi m_N} \langle N | G_a^a G_a^{\mu\nu} | N \rangle$$

$$f_Q^N = \frac{2}{27} \left(1 - \sum_{q=u,d,s} f_q^N \right)$$

↪ fixed in terms of **light flavors**

Dispersion relation for the scalar form factor of the nucleon

- Unitarity relation

$$\text{Im } \otimes = \text{Im } \otimes + \text{Im } \otimes$$
$$\text{Im } \sigma(t) = \frac{2}{4m^2 - t} \left\{ \frac{3}{4} \sigma_t^\pi (F_\pi^S(t))^* f_+^0(t) + \sigma_t^K (F_K^S(t))^* h_+^0(t) \right\}$$

Dispersion relation for the scalar form factor of the nucleon

- Unitarity relation

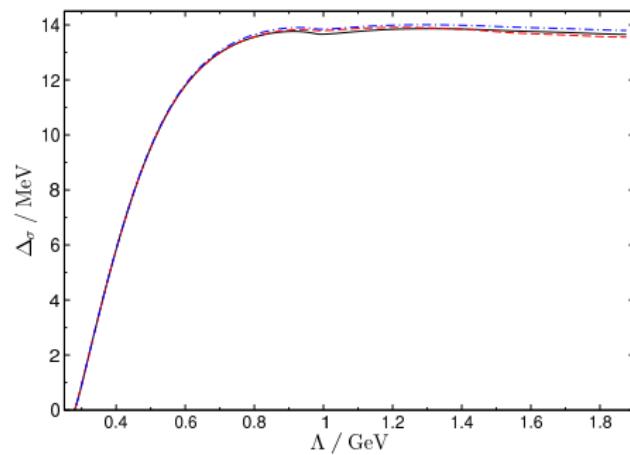
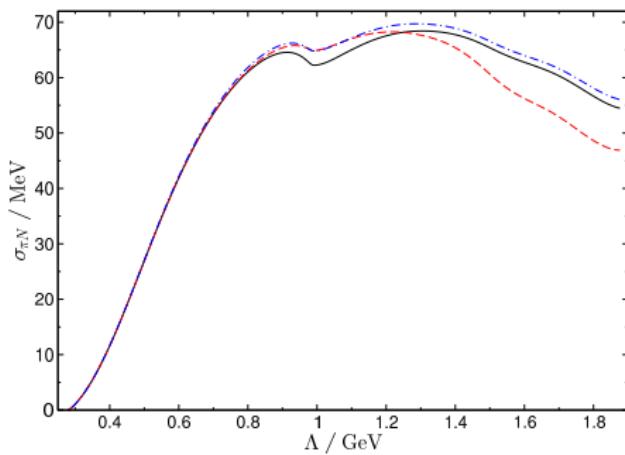
$$\text{Im} \otimes = \text{Im} \otimes \text{---} + \text{Im} \otimes \text{---}$$
$$\text{Im } \sigma(t) = \frac{2}{4m^2 - t} \left\{ \frac{3}{4} \sigma_t^\pi (F_\pi^S(t))^* f_+^0(t) + \sigma_t^K (F_K^S(t))^* h_+^0(t) \right\}$$

- Dispersion relation

$$\sigma(t) = \frac{1}{\pi} \int_{4M_\pi^2}^{\infty} dt' \frac{\text{Im } \sigma(t')}{t' - t} = \sigma_{\pi N} + \frac{t}{\pi} \int_{4M_\pi^2}^{\infty} dt' \frac{\text{Im } \sigma(t')}{t'(t' - t)}$$

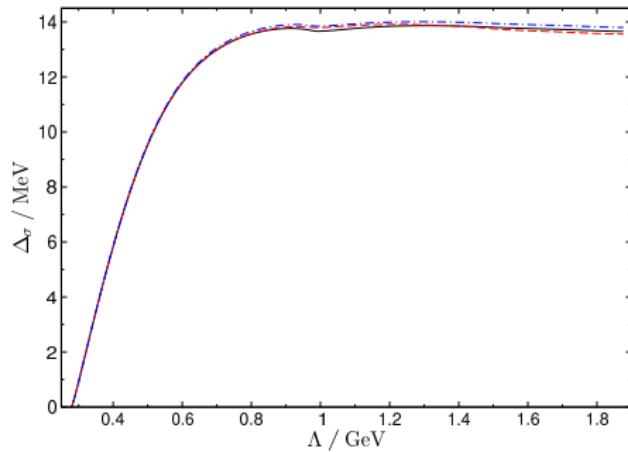
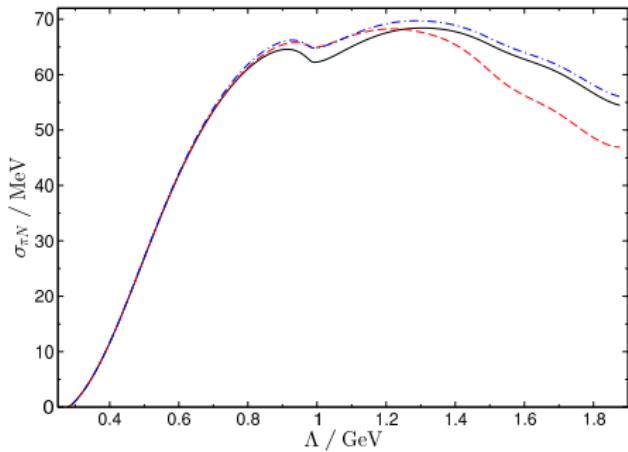
- Unsubtracted:** $\sigma_{\pi N} = \sigma(0)$
- Once-subtracted:** $\Delta_\sigma = \sigma(2M_\pi^2) - \sigma_{\pi N}$

Convergence of the dispersive integral



- **Unsubtracted:** slow convergence
- **Once-subtracted:** stable result for $\Lambda \gtrsim 1$ GeV

Convergence of the dispersive integral



- **Unsubtracted:** slow convergence
- **Once-subtracted:** stable result for $\Lambda \gtrsim 1$ GeV
- Result for Δ_σ depends on pion–nucleon parameters (notation: $\bar{X}(\nu, t) = \sum_{n,m=0}^{\infty} x_{nm} \nu^{2n} t^m$)
$$\Delta_\sigma = (13.9 \pm 0.3) \text{ MeV}$$

$$+ Z_1 \left(\frac{g^2}{4\pi} - 14.28 \right) + Z_2 \left(d_{00}^+ M_\pi + 1.46 \right) + Z_3 \left(d_{01}^+ M_\pi^3 - 1.14 \right) + Z_4 \left(b_{00}^+ M_\pi^3 + 3.54 \right)$$

$$Z_1 = 0.36 \text{ MeV} \quad Z_2 = 0.57 \text{ MeV} \quad Z_3 = 12.0 \text{ MeV} \quad Z_4 = -0.81 \text{ MeV}$$

Summary: σ -term corrections

• Scalar form factor

$$\Delta_{\sigma} = (13.9 \pm 0.3) \text{ MeV}$$

$$+ Z_1 \left(\frac{g^2}{4\pi} - 14.28 \right) + Z_2 \left(d_{00}^+ M_\pi + 1.46 \right) + Z_3 \left(d_{01}^+ M_\pi^3 - 1.14 \right) + Z_4 \left(b_{00}^+ M_\pi^3 + 3.54 \right)$$

$$Z_1 = 0.36 \text{ MeV} \quad Z_2 = 0.57 \text{ MeV} \quad Z_3 = 12.0 \text{ MeV} \quad Z_4 = -0.81 \text{ MeV}$$

• πN amplitude

$$\Delta_D = (12.1 \pm 0.3) \text{ MeV}$$

$$+ \tilde{Z}_1 \left(\frac{g^2}{4\pi} - 14.28 \right) + \tilde{Z}_2 \left(d_{00}^+ M_\pi + 1.46 \right) + \tilde{Z}_3 \left(d_{01}^+ M_\pi^3 - 1.14 \right) + \tilde{Z}_4 \left(b_{00}^+ M_\pi^3 + 3.54 \right)$$

$$\tilde{Z}_1 = 0.42 \text{ MeV} \quad \tilde{Z}_2 = 0.67 \text{ MeV} \quad \tilde{Z}_3 = 12.0 \text{ MeV} \quad \tilde{Z}_4 = -0.77 \text{ MeV}$$

→ most of the dependence on the πN parameters cancels in the difference!

Full correction

$$\Delta_D - \Delta_{\sigma} = (-1.8 \pm 0.2) \text{ MeV}$$

Second correction: Δ_D

t-channel expansion

$$\bar{D}^+(\nu = 0, t) = 4\pi \left\{ -\frac{1}{p_t^2} \bar{f}_+^0(t) + \frac{5}{2} q_t^2 \bar{f}_+^2(t) - \frac{27}{8} p_t^2 q_t^4 \bar{f}_+^4(t) + \frac{65}{16} p_t^4 q_t^6 \bar{f}_+^6(t) + \dots \right\}$$

- Insert t-channel RS equations for Born-term-subtracted amplitudes $\bar{f}_+^J(t)$

$$\bar{D}^+(\nu = 0, t) = d_{00}^+ + d_{01}^+ t - 16t^2 \int_{4M_\pi^2}^\infty dt' \frac{\text{Im } f_+^0(t')}{t'^2(t' - 4m^2)(t' - t)} + \{J \geq 2\} + \{s\text{-channel integrals}\}$$

- $\Delta_D = F_\pi^2 (\bar{D}^+(\nu = 0, t = 2M_\pi^2) - d_{00}^+ - 2M_\pi^2 d_{01}^+)$ from evaluation at $t = 2M_\pi^2$

$$\Delta_D = (12.1 \pm 0.3) \text{ MeV}$$

$$+ \tilde{Z}_1 \left(\frac{g^2}{4\pi} - 14.28 \right) + \tilde{Z}_2 \left(d_{00}^+ M_\pi + 1.46 \right) + \tilde{Z}_3 \left(d_{01}^+ M_\pi^3 - 1.14 \right) + \tilde{Z}_4 \left(b_{00}^+ M_\pi^3 + 3.54 \right)$$

$$\tilde{Z}_1 = 0.42 \text{ MeV} \quad \tilde{Z}_2 = 0.67 \text{ MeV} \quad \tilde{Z}_3 = 12.0 \text{ MeV} \quad \tilde{Z}_4 = -0.77 \text{ MeV}$$

Origin of the cancellation

- Dominant contribution from dispersive integral over $f_+^0(t)$

$$\Delta_\sigma = \frac{3M_\pi^2}{\pi} \int_{4M_\pi^2}^\infty dt' \frac{\sigma_{t'}^\pi(F_\pi^S(t'))^* f_+^0(t')}{t'(t'-2M_\pi^2)(4m^2-t')} + \dots$$

$$\Delta_D = 64F_\pi^2 M_\pi^4 \int_{4M_\pi^2}^\infty dt' \frac{\text{Im } f_+^0(t')}{t'^2(t'-2M_\pi^2)(4m^2-t')} + \dots = 64F_\pi^2 M_\pi^4 \int_{4M_\pi^2}^\infty dt' \frac{\sigma_{t'}^\pi(t_0^0(t'))^* f_+^0(t')}{t'^2(t'-2M_\pi^2)(4m^2-t')} + \dots$$

- Largest contribution around $t' = 4M_\pi^2$

$$\frac{\Delta_\sigma}{\Delta_D} \rightarrow \frac{3M_\pi^2}{\pi} \frac{(F_\pi^S(t'))^* t'}{64F_\pi^2 M_\pi^4 (f_0^0(t'))^*} \rightarrow \frac{3M_\pi^4}{\pi} \frac{32\pi F_\pi^2 t'}{64F_\pi^2 M_\pi^4 (2t' - M_\pi^2)} \rightarrow \frac{6}{7} = \frac{18}{21} \quad \text{ChPT: } \frac{\Delta_\sigma}{\Delta_D} = \frac{18}{23} + \mathcal{O}(M_\pi)$$

- This explains

- Δ_σ and Δ_D of similar size
- Strong curvature generated by $\pi\pi$ rescattering
 - ↪ sensitivity to $\pi\pi$ phase shift reduced in the difference Gasser, Leutwyler, Sainio 1991
- Spectral functions depend similarly on $f_+^0(t)$ ↪ sensitivity to πN parameters reduced

Cheng–Dashen theorem in the presence of isospin breaking

- Define “**isoscalar**” as

$$X^+ \rightarrow X^p \equiv \frac{1}{2}(X_{\pi^+ p \rightarrow \pi^+ p} + X_{\pi^- p \rightarrow \pi^- p}) \quad X = D, d_{00}, d_{01}, a_{0+}, \dots$$

and “**isospin limit**” by proton and charged pion

- Assume virtual photons to be removed
→ scenario closest to actual πN PWAs
- Calculate **IV corrections** in $SU(2)$ ChPT, mainly due to $\Delta_\pi = M_\pi^2 - M_{\pi^0}^2$
 - For the σ -term, no difference at $\mathcal{O}(p^3)$

$$\sigma_{\pi N} = \sigma_p = \sigma_n = -4c_1 M_{\pi^0}^2 - \frac{3g_A^2 M_{\pi^0}^2}{64\pi F_\pi^2} (2M_\pi + M_{\pi^0}) + \mathcal{O}(M_\pi^4)$$

- Slope of the scalar form factor

$$\Delta_\sigma^p = \sigma_p(2M_\pi^2) - \sigma_p = \frac{3g_A^2 M_\pi^3}{64\pi F_\pi^2} + \frac{g_A^2 M_\pi \Delta_\pi}{128\pi F_\pi^2} \left(-7 + \sqrt{2} \log(3 + 2\sqrt{2}) \right) + \mathcal{O}(M_\pi^4)$$

and similarly for Δ_D^p

Cheng–Dashen theorem in the presence of isospin breaking

- Putting things together

$$\begin{aligned}\sigma_p &= F_\pi^2 (d_{00}^p + 2M_\pi^2 d_{01}^p) + \Delta_D - \Delta_\sigma + (\Delta_D^p - \Delta_D) - (\Delta_\sigma^p - \Delta_\sigma) \\ &\quad + \sigma_p(2M_\pi^2) - F_\pi^2 \bar{D}_p(0, 2M_\pi^2) \\ &= F_\pi^2 (d_{00}^p + 2M_\pi^2 d_{01}^p) + \underbrace{\Delta_D - \Delta_\sigma}_{(-1.8 \pm 0.2) \text{ MeV}} + \underbrace{\Delta_R}_{\lesssim 2 \text{ MeV}} + \underbrace{\frac{81g_A^2 M_\pi \Delta_\pi}{256\pi F_\pi^2}}_{+3.4 \text{ MeV}} + \underbrace{\frac{e^2}{2} F_\pi^2 (4f_1 + f_2)}_{(-0.4 \pm 2.2) \text{ MeV}}\end{aligned}$$

→ indeed sizable correction from Δ_π , but “wrong” direction

- In the following, determine d_{00}^p and d_{01}^p by solving **Roy–Steiner equations**
- Constraints: scattering lengths from hadronic atoms (virtual-photon subtracted)

$$a_{0+}^{1/2} = (169.8 \pm 2.0) \times 10^{-3} M_\pi^{-1} \quad a_{0+}^{3/2} = (-86.3 \pm 1.8) \times 10^{-3} M_\pi^{-1}$$

- First step: πN coupling constant

Goldberger–Miyazawa–Oehme sum rule

- Fixed- t dispersion relations at threshold \Rightarrow **GMO sum rule**

$$\frac{g^2}{4\pi} = \left(\left(\frac{m_p + m_n}{M_\pi} \right)^2 - 1 \right) \left\{ \left(1 + \frac{M_\pi}{m_p} \right) \frac{M_\pi}{4} (\mathbf{a}_{\pi^- p} - \mathbf{a}_{\pi^+ p}) - \frac{M_\pi^2}{2} J^- \right\}$$
$$= 13.66 \pm 0.12 \pm 0.15$$

$$J^- = \frac{1}{4\pi^2} \int_0^\infty dk \frac{\sigma_{\pi^- p}^{\text{tot}}(k) - \sigma_{\pi^+ p}^{\text{tot}}(k)}{\sqrt{M_\pi^2 + k^2}}$$

- J^- known quite accurately Ericson et al. 2002, Abaev et al. 2007

- Other determinations:

	de Swart et al. 97	Arndt et al. 94	Ericson et al. 02	Bugg et al. 73	KH80
method	NN	πN	GMO	πN	πN
$g^2/4\pi$	13.54 ± 0.05	13.75 ± 0.15	14.11 ± 0.20	14.30 ± 0.18	14.28

- With KH80 scattering lengths $g^2/4\pi = 14.28$ is reproduced exactly!
→ discrepancy for $g^2/4\pi$ related to wrong scattering lengths

Roy equations = Dispersion relations + partial-wave expansion
+ crossing symmetry + unitarity

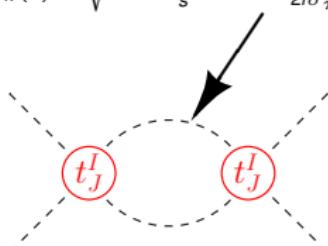
- Coupled system of integral equations for partial waves $t_J^l(s)$ Roy 1971

$$t_J^l(s) = k_J^l(s) + \sum_{l'=0}^2 \sum_{J'=0}^{\infty} \int_{4M_{\pi}^2}^{\infty} ds' K_{JJ'}^{ll'}(s, s') \text{Im } t_{J'}^{l'}(s')$$

$\pi\pi$ Roy equations

Roy equations = Dispersion relations + partial-wave expansion
+ crossing symmetry + unitarity

- Coupled system of integral equations for partial waves $t_J^I(s)$ Roy 1971

$$\underbrace{t_J^I(s)}_{\frac{e^{2i\delta_J^I(s)}}{2i\sigma_\pi(s)} - 1} = k_J^I(s) + \sum_{I'=0}^2 \sum_{J'=0}^{\infty} \int_{4M_\pi^2}^{\infty} ds' K_{JJ'}^{II'}(s, s') \underbrace{\text{Im } t_{J'}^{I'}(s')}_{\frac{1}{\sigma_\pi(s)} \sin^2 \delta_{J'}^{I'}(s')}$$


$\pi\pi$ Roy equations

Roy equations = Dispersion relations + partial-wave expansion
+ crossing symmetry + unitarity

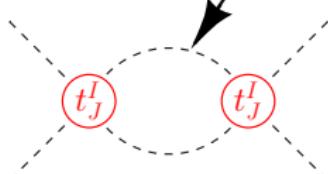
- Coupled system of integral equations for partial waves $t_J^I(s)$ Roy 1971

$$t_J^I(s) = \underbrace{k_J^I(s)}_{\delta_0^I a_0^I + \dots} + \sum_{I'=0}^2 \sum_{J'=0}^{\infty} \int_{4M_\pi^2}^{\infty} ds' K_{JJ'}^{II'}(s, s') \underbrace{\text{Im } t_{J'}^{I'}(s')}_{\frac{1}{\sigma_\pi(s)} \sin^2 \delta_{J'}^{I'}(s')}$$

$\sigma_\pi(s) = \sqrt{1 - \frac{4M_\pi^2}{s}}$

$\frac{e^{2i\delta_J^I(s)}}{2i\sigma_\pi(s)} - 1$

free parameters a_0^0, a_0^2



$\pi\pi$ Roy equations

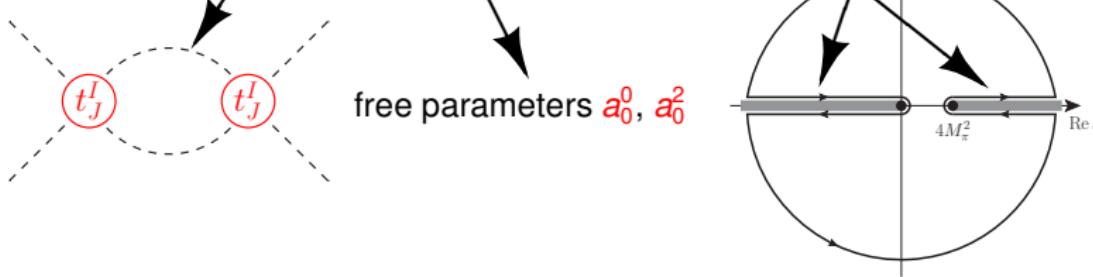
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- Coupled system of integral equations for partial waves $t_J^I(s)$ Roy 1971

$$t_J^I(s) = \underbrace{k_J^I(s)}_{\delta_0^I a_0^I + \dots} + \sum_{I'=0}^2 \sum_{J'=0}^{\infty} \int_{4M_\pi^2}^{\infty} ds' \underbrace{K_{JJ'}^{II'}(s, s')}_{\frac{1}{\pi} \frac{\delta_{JJ'} \delta_{II'}}{s' - s - i\epsilon} + \bar{K}_{JJ'}^{II'}(s, s')} \underbrace{\text{Im } t_{J'}^{I'}(s')}_{\frac{1}{\sigma_\pi(s)} \sin^2 \delta_{J'}^{I'}(s')}$$

$\sigma_\pi(s) = \sqrt{1 - \frac{4M_\pi^2 e^{-2i\delta_J^I(s)}}{s - 2i\sigma_\pi(s)} - 1}$

free parameters a_0^0, a_0^2



→ Self-consistency condition for phase shifts

Removing Coulomb effects: the pp scattering length

- Consider first a more familiar example: **pp scattering**

- Split total phase shift into **pure Coulomb** σ^C + **remainder** δ_{pp}^C
- δ_{pp}^C related to strong amplitude $T_{pp}(k)$ by

$$k(\cot \delta_{pp}^C - i) = -\frac{4\pi}{m} \frac{e^{2i\sigma^C}}{T_{pp}(k)} \quad k = |\mathbf{k}|$$

- Modified effective range expansion** Bethe 1949

$$k \left[C_\eta^2 (\cot \delta_{pp}^C - i) + 2\eta H(\eta) \right] = -\frac{1}{a_{pp}^C} + \frac{1}{2} r_0 k^2 + \dots$$

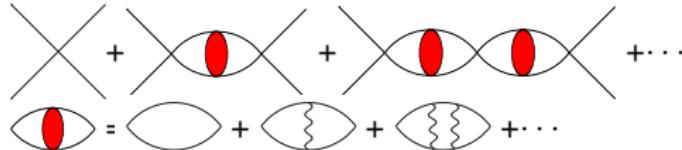
$$C_\eta^2 = \frac{2\pi\eta}{e^{2\pi\eta} - 1} \quad \eta = \frac{\alpha m}{2k} \quad H(\eta) = \psi(i\eta) + \frac{1}{2i\eta} - \log(i\eta) \quad \psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}$$

- Removal of **residual Coulomb** effects **scale-dependent**

$$\frac{1}{a_{pp}} = \frac{1}{a_{pp}^C} + \alpha m \left[\log \frac{1}{\alpha M r_0} - 0.33 \right] \quad \text{Jackson, Blatt 1950}$$

$$\frac{1}{a_{pp}} = \frac{1}{a_{pp}^C} + \alpha m \left[\log \frac{\mu\sqrt{\pi}}{\alpha M} + 1 - \frac{3}{2}\gamma_E \right] \quad \text{Kong, Ravndal 1999}$$

Removing Coulomb effects: the pp scattering length



- Difference due to **Coulomb-dressed bubble sum**

$$\frac{1}{a_{pp}} = \frac{1}{a_{pp}^C} + \alpha m \left[\log \frac{1}{\alpha M r_0} - 0.33 \right] \quad \text{Jackson, Blatt 1950}$$

$$\frac{1}{a_{pp}} = \frac{1}{a_{pp}^C} + \alpha m \left[\log \frac{\mu \sqrt{\pi}}{\alpha M} + 1 - \frac{3}{2} \gamma_E \right] \quad \text{Kong, Ravndal 1999}$$

- a_{pp} supposed to correspond to strong part of the potential, but **Coulomb-nuclear interference** depends on short-distance part of the nuclear force
- Numbers for **singlet channel**

- $a_{pp}^C = (-7.8063 \pm 0.0026) \text{ fm}$ Bergervoet et al. 1988
- $a_{np} = (-23.749 \pm 0.008) \text{ fm}$ Koester, Nistler 1975
- $a_{nn} = (-18.8 \pm 0.5) \text{ fm}$ González et al. 2006
- $a_{pp} = (-17.3 \pm 0.4) \text{ fm}$ Miller et al. 1990

→ $a_{pp}^C - a_{pp}$ **huge effect!**

Back to πN scattering

- **Deser formula:** shift and $a_{\pi-p}$ in NREFT

$$\epsilon_{1s} = -2\alpha^3 \mu_H^2 a_{\pi-p} (1 + 2\alpha(1 - \log \alpha) \mu_H a_{\pi-p} + \dots)$$



- **ChPT convention** for scattering length Lyubovitskij, Rusetsky 2000

$$e^{-2i\sigma C} T_{\pi-p} = \frac{\pi \alpha \mu_H a_{\pi-p}}{k} - 2\alpha \mu_H (a_{\pi-p})^2 \log \frac{k}{\mu_H} + a_{\pi-p} + \mathcal{O}(k, \alpha^2)$$

- **Compare with mERE:** expand first in α , then in k

$$e^{-2i\sigma C} T_{\pi-p} = \frac{\pi \alpha \mu_H a_{\pi-p}^C}{k} - 2\alpha \mu_H (a_{\pi-p}^C)^2 \left(\gamma_E + \log \frac{k}{\alpha \mu_H} \right) + a_{\pi-p}^C + \mathcal{O}(k, \alpha^2)$$

↪ same $\log \alpha$ as in Deser formula!

ChPT vs. mERE scattering length

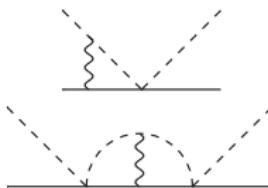
$$\underbrace{a_{\pi-p}}_{86.1 \pm 1.8} = a_{\pi-p}^C + \underbrace{2\alpha \mu_H (a_{\pi-p}^C)^2 (\log \alpha - \gamma_E)}_{-0.5} + \mathcal{O}(\alpha^2)$$

Subtraction of virtual-photon effects

- Application in **dispersion relations** \hookrightarrow analytic properties

- Effects calculable in ChPT, e.g.

- Coulomb pole $\sim 1/k$ at NLO, $\mathcal{O}(p^3)$
- $\log k$ first at two loops, $\mathcal{O}(p^5)$



- Subtract virtual-photon contributions

- Finite terms** \Rightarrow fine
- UV divergent photon loops** \Rightarrow need to separate mass-difference and virtual-photon contributions to LECs \Rightarrow scale dependence

- How large are these effects?

- Full: $a_{\pi^- p} - a_{\pi^+ p} = (172.8 \pm 1.6) \times 10^{-3} M_\pi^{-1}$
- Virtual photons: $a_{\pi^- p}^\gamma - a_{\pi^+ p}^\gamma = (2.1 \pm 1.8) \times 10^{-3} M_\pi^{-1}$
- Virtual-photon subtracted: $a_{\pi^- p}^\chi - a_{\pi^+ p}^\chi = (170.7 \pm 2.4) \times 10^{-3} M_\pi^{-1}$

\hookrightarrow much smaller than in a_{pp}