TECHNISCHE UNIVERSITÄT DARMSTADT

Nuclear structure factors, inelastic WIMP-nucleus scattering, and chiral power counting

Philipp Klos

with M. Hoferichter, J. Menéndez and A. Schwenk

Effective Theories and Dark Matter

MITP, March 24, 2015

European Research Council

Established by the European Commission

- WIMP-nucleus interaction
- Nuclear structure factors
- Inelastic WIMP-nucleus scattering
- Chiral power counting for other WIMP responses
- Conclusion

Introduction
WIMP-nucleus interaction

TECHNISCHE UNIVERSITÄT DARMSTADT

Transition amplitude of WIMP-nucleus scattering

$$\sigma \propto \left| \langle \text{final} | H_{\chi-\text{nucleus}} | \text{inital} \rangle \right|^2$$

Two tasks:

Description of initial and final nuclear states

$\rightarrow$ Large-scale shell model calculations (Javier's talk last week)

Description of WIMP-nucleus interaction

WIMP-nucleus interaction

TECHNISCHE UNIVERSITÄT DARMSTADT

Cross section of WIMP-nucleus interaction depends on structure factor $S_A(q)$.

$$\frac{d\sigma}{dq^2} = \frac{2}{\pi v^2}\frac{1}{2}\sum_{s_i,s_f}\frac{1}{2J_i+1}\sum_{M_i,M_f}\left|\langle f|\sum_A H_\chi^{SD}|i\rangle\right|^2 = \frac{8G_F^2}{(2J_i+1)v^2}S_A(q),$$

Spin-dependent (SD) WIMP-nucleus interaction:

$$H_\chi^{SD} = \sqrt{2}G_F\int d^3r\ \underbrace{A_{N\mu}(\mathbf{r})}_{\text{nucleon current}}\ \underbrace{A_\chi^\mu(\mathbf{r})}_{\text{WIMP current}}$$

Axial-vector–axial-vector couping

Chiral effective field theory

TECHNISCHE UNIVERSITÄT DARMSTADT

- Chiral EFT describes consistently both nuclear forces and currents

  Epelbaum, Hammer, and Meißner, RMP **81**, 1773 (2009)

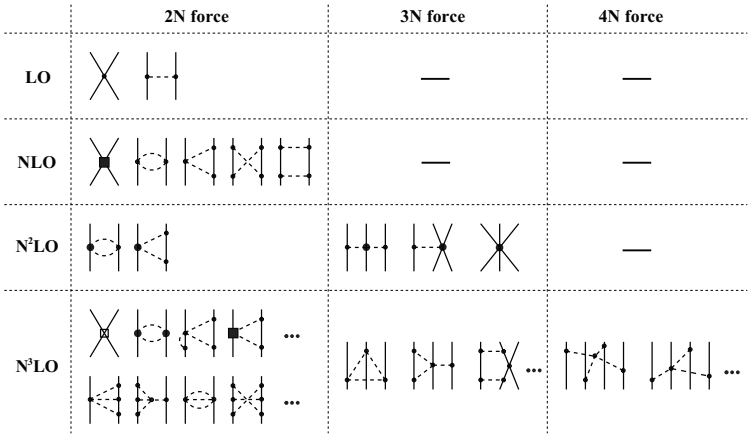
- Same low-energy constants appear in nuclear forces and currents

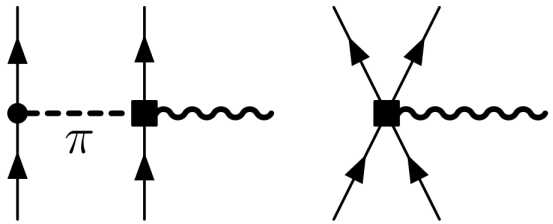
- Leading axial-vector two-body currents completely determined

  Park *et al.*, PRC **67**, 055206 (2003)

  Gårdestig and Phillips, PRL **96**, 232301 (2006)

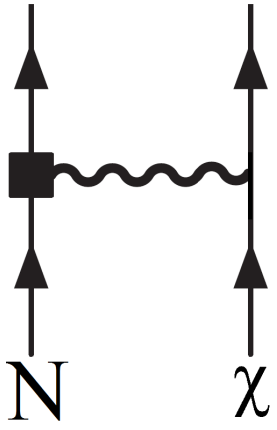
  Gazit, Quaglioni, and Navrátil, PRL **103**, 102502 (2009)





c_1, c_3, c_4, c_6 $\quad$ c_D

Nuclear currents from chiral EFT
One-body current



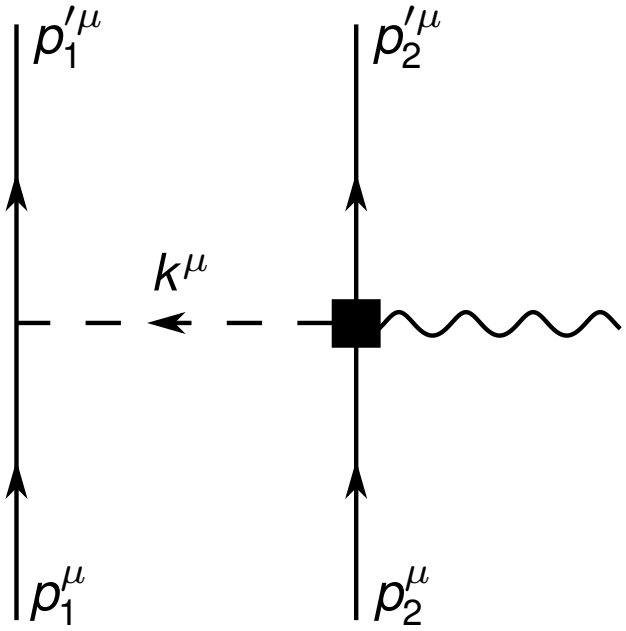
$$\sum_{i=1}^{A} \mathbf{A}_{i,1\mathrm{b}} = \sum_{i=1}^{A} \left(\mathbf{A}^{0}_{i,1\mathrm{b}} + \mathbf{A}^{3}_{i,1\mathrm{b}}\right) = \sum_{i=1}^{A} \frac{1}{2} \left[\underbrace{a_0 \boldsymbol{\sigma}_i}_{\text{isoscalar}} + \underbrace{a_1 \tau_i^3 \left(\frac{g_A(q^2)}{g_A} \boldsymbol{\sigma}_i - \frac{g_P(q^2)}{2mg_A} (\mathbf{q} \cdot \boldsymbol{\sigma}_i)\, \mathbf{q} \right)}_{\text{isovector}} \right]$$

a_0 and a_1 are isoscalar and isovector coupling constants

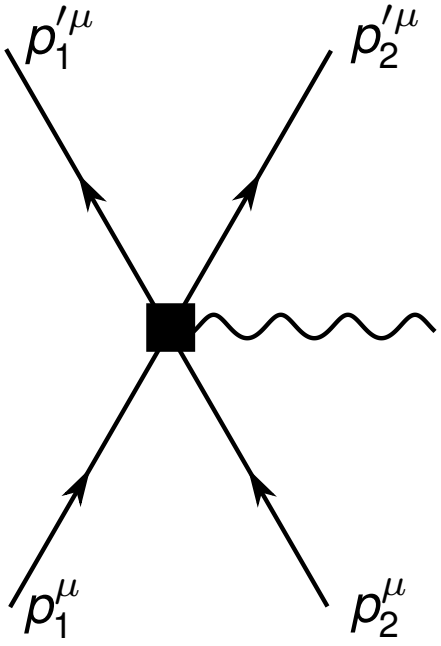
Nuclear currents from chiral EFT
Two-body currents

TECHNISCHE UNIVERSITÄT DARMSTADT

At order Q^3, 2b currents enter:



Pion exchange currents



Contact currents

- Finite **q** contributions to 2b currents derived recently

  Hoferichter, PK, Schwenk, arXiv 1503.04811

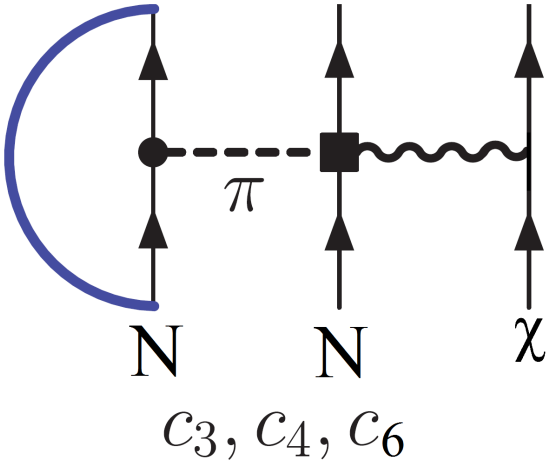
Normal ordered one-body current

TECHNISCHE UNIVERSITÄT DARMSTADT

Summing over Fermi distribution assuming nuclear density ρ yields the normal ordered one-body current:

$$\mathbf{A}^{\text{eff}}_{i,2\text{b}} = \sum_j \left(1 - P_{ij}\right) \mathbf{A}^3_{ij}$$

with exchange operator P_{ij}



Combining 1b and normal-ordered 1b part of 2b currents yields

$$\mathbf{A}^a_{i,1\text{b}+2\text{b}} = \frac{1}{2} a_1 \tau^a_i \left[\left(\frac{g_A(q^2)}{g_A} + \delta a_1(q) \right) \boldsymbol{\sigma}_i + \left(-\frac{g_P(q^2) q^2}{2 M g_A} + \delta a_1^P(q) \right) \frac{(\mathbf{q} \cdot \boldsymbol{\sigma}_i)\mathbf{q}}{q^2} \right]$$

with contributions from 2b currents δa_1 and δa_1^P

Structure factors: Elastic scattering

TECHNISCHE UNIVERSITÄT DARMSTADT

Recall the structure factor:

$$S_A(q) \propto |\langle \text{initial}| H_{\chi N} |\text{initial}\rangle|^2$$

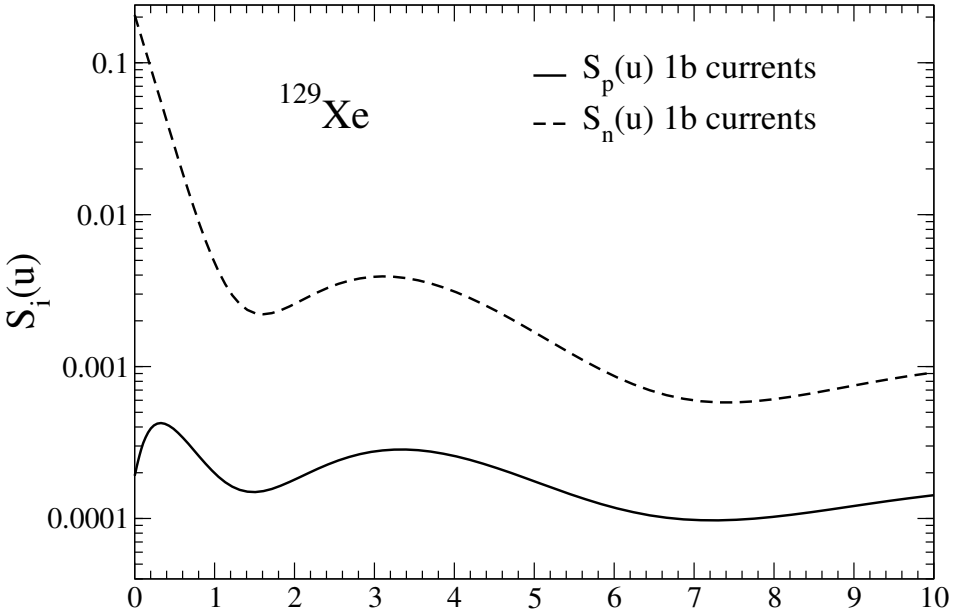
For $q = 0$ the structure factor is a function of the spin expectation values of protons $\langle \mathbf{S}_p \rangle$ and neutrons $\langle \mathbf{S}_n \rangle$:

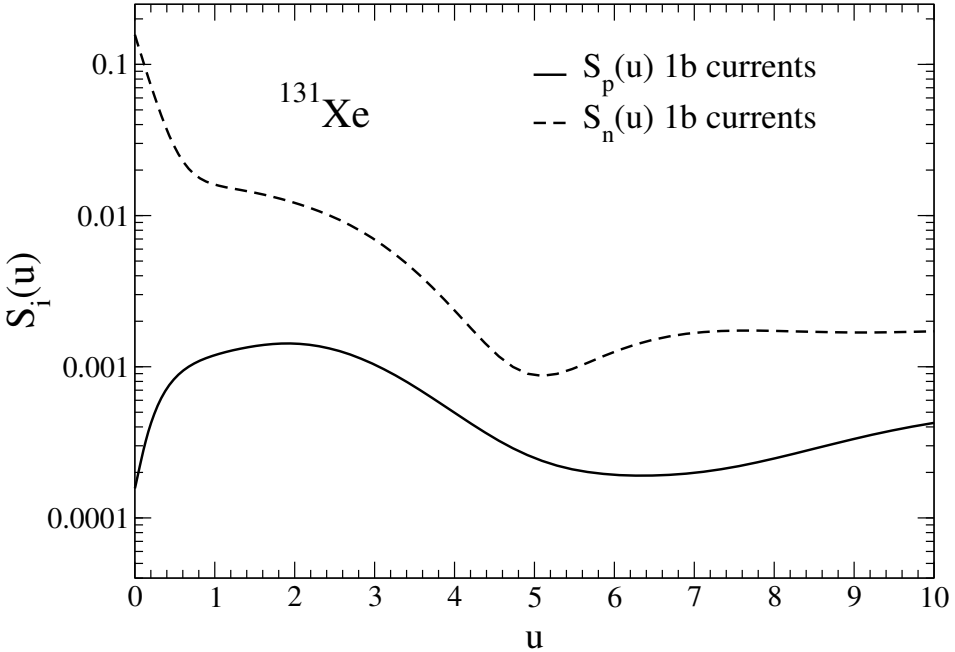
$$S_A(0) \propto \left|(a_0 + a_1)\langle \mathbf{S}_p \rangle + (a_0 - a_1)\langle \mathbf{S}_n \rangle\right|^2$$

Define

$a_0 = a_1 = 1$	proton-only structure factor	$S_p(q)$
$a_0 = -a_1 = 1$	neutron-only structure factor	$S_n(q)$

Structure factors: Elastic scattering



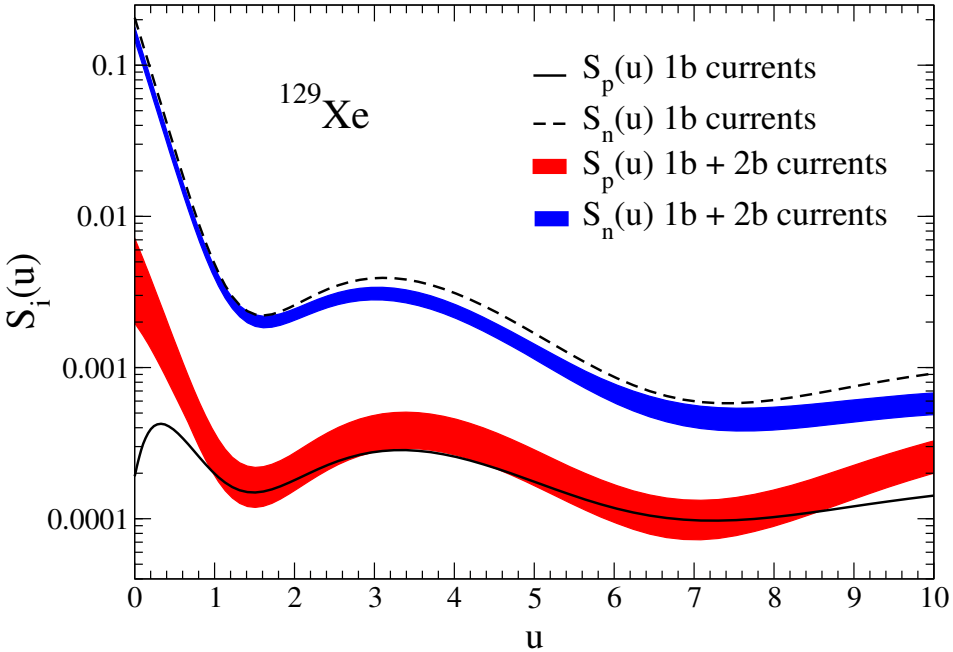


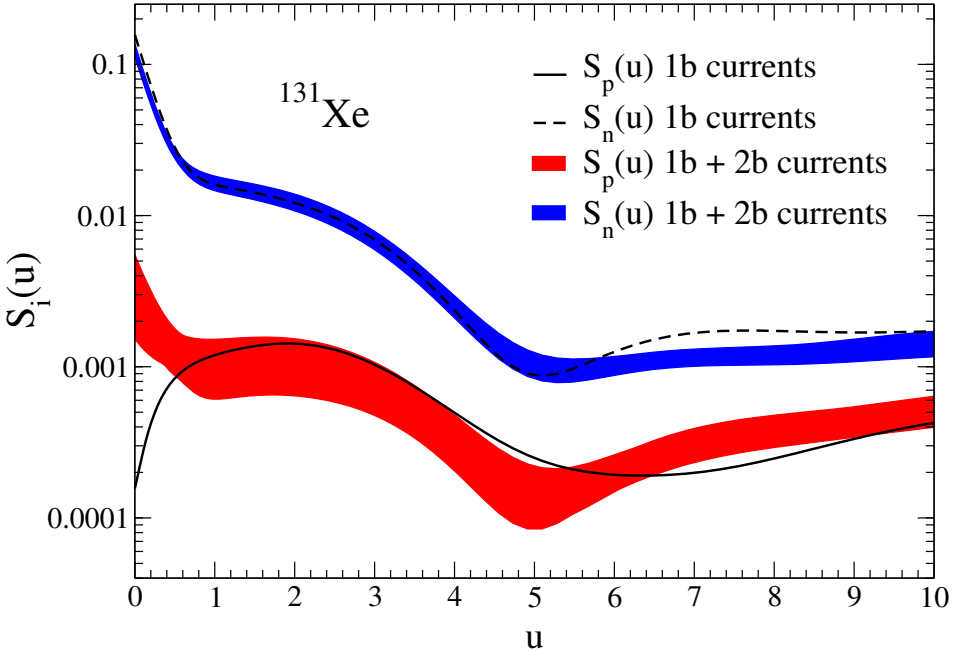
$u = q^2 b^2/2$ with harmonic oscillator length b

^{129}Xe		^{131}Xe	
$\langle \mathbf{S}_p \rangle$	$\langle \mathbf{S}_n \rangle$	$\langle \mathbf{S}_p \rangle$	$\langle \mathbf{S}_n \rangle$
0.010	0.329	−0.009	−0.272

Structure factors: Elastic scattering

TECHNISCHE UNIVERSITÄT DARMSTADT

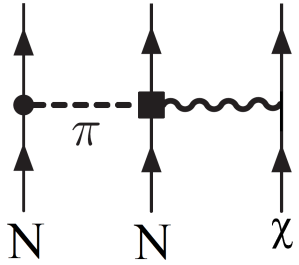




$u = q^2 b^2/2$ with harmonic oscillator length b

- 2b currents $\rightarrow$ at low momentum transfer neutrons contribute to proton structure factor $S_p(u)$
- $S_n(u)$ reduced by 20% for low momentum transfers

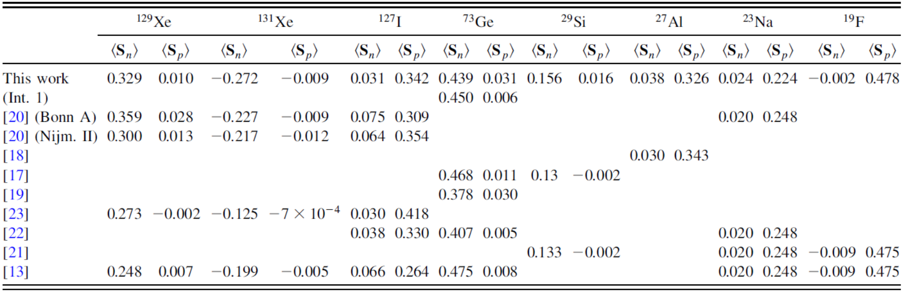
PK, Menéndez, Gazit, Schwenk, PRD **88**, 083516 (2013)



c_3, c_4, c_6

Spin-expectation values for different isotopes

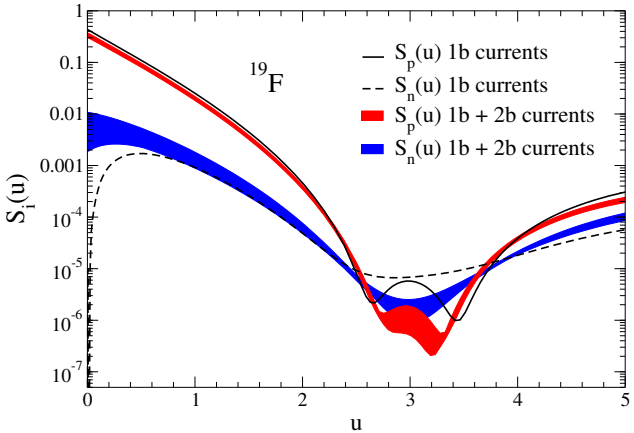
TABLE III. Calculated spin expectation values for protons $\langle \mathbf{S}_p \rangle$ and neutrons $\langle \mathbf{S}_n \rangle$ of ^{129,131}Xe, ^{127}I, ^{73}Ge, ^{29}Si, ^{27}Al, ^{23}Na, and ^{19}F, compared to the previous calculations of Refs. [13,17–23].



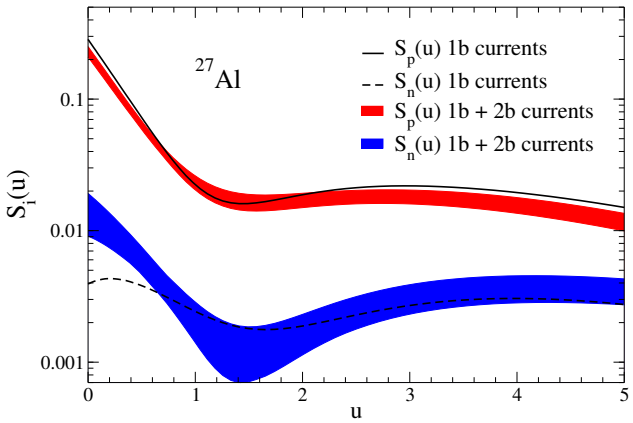
	^{129}Xe		^{131}Xe		^{127}I		^{73}Ge		^{29}Si		^{27}Al		^{23}Na		^{19}F	
	$\langle \mathbf{S}_n \rangle$	$\langle \mathbf{S}_p \rangle$	$\langle \mathbf{S}_n \rangle$	$\langle \mathbf{S}_p \rangle$	$\langle \mathbf{S}_n \rangle$	$\langle \mathbf{S}_p \rangle$	$\langle \mathbf{S}_n \rangle$	$\langle \mathbf{S}_p \rangle$	$\langle \mathbf{S}_n \rangle$	$\langle \mathbf{S}_p \rangle$	$\langle \mathbf{S}_n \rangle$	$\langle \mathbf{S}_p \rangle$	$\langle \mathbf{S}_n \rangle$	$\langle \mathbf{S}_p \rangle$	$\langle \mathbf{S}_n \rangle$	$\langle \mathbf{S}_p \rangle$
This work	0.329	0.010	−0.272	−0.009	0.031	0.342	0.439	0.031	0.156	0.016	0.038	0.326	0.024	0.224	−0.002	0.478
(Int. 1)							0.450	0.006								
[20] (Bonn A)	0.359	0.028	−0.227	−0.009	0.075	0.309							0.020	0.248		
[20] (Nijm. II)	0.300	0.013	−0.217	−0.012	0.064	0.354										
[18]											0.030	0.343				
[17]							0.468	0.011	0.13	−0.002						
[19]							0.378	0.030								
[23]	0.273	−0.002	−0.125	-7×10^{-4}	0.030	0.418										
[22]					0.038	0.330	0.407	0.005					0.020	0.248		
[21]									0.133	−0.002			0.020	0.248	−0.009	0.475
[13]	0.248	0.007	−0.199	−0.005	0.066	0.264	0.475	0.008					0.020	0.248	−0.009	0.475

PK, Menéndez, Gazit, Schwenk, PRD **88**, 083516 (2013)

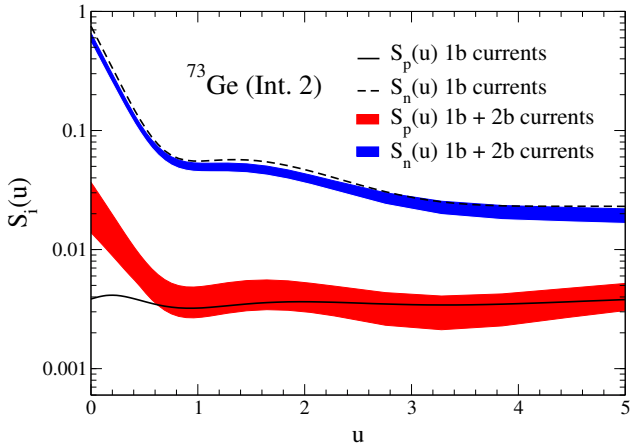
Structure factors for different isotopes: Elastic scattering



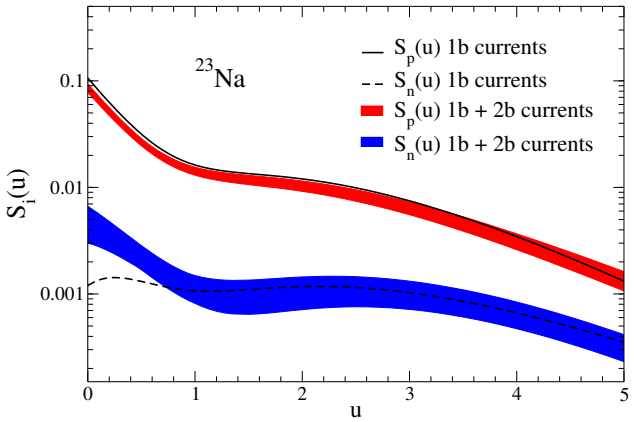
PICASSO, COUPP, SIMPLE



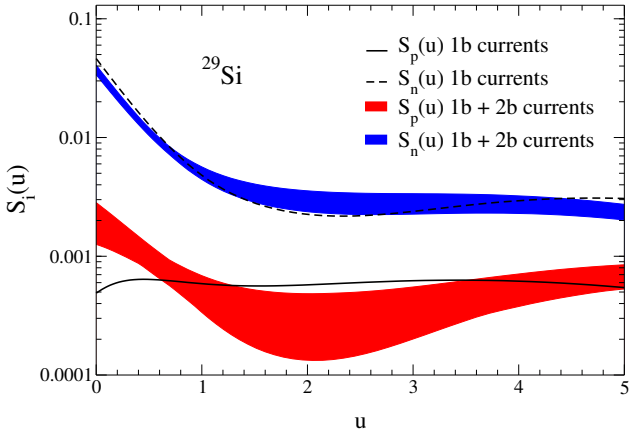
CRESST



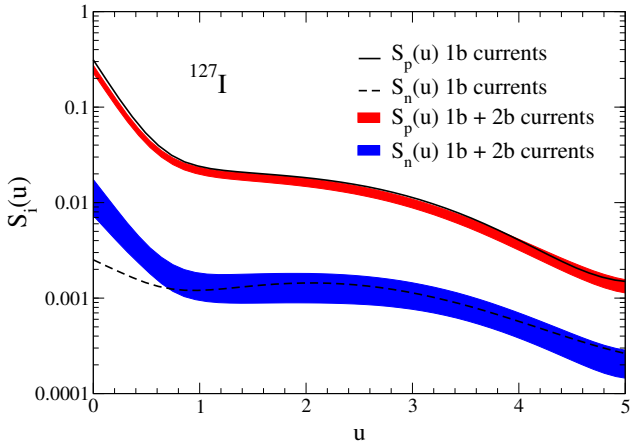
CDMS, EDELWEISS, EURECA



DAMA, ANAIS, DM-Ice



CDMS-II

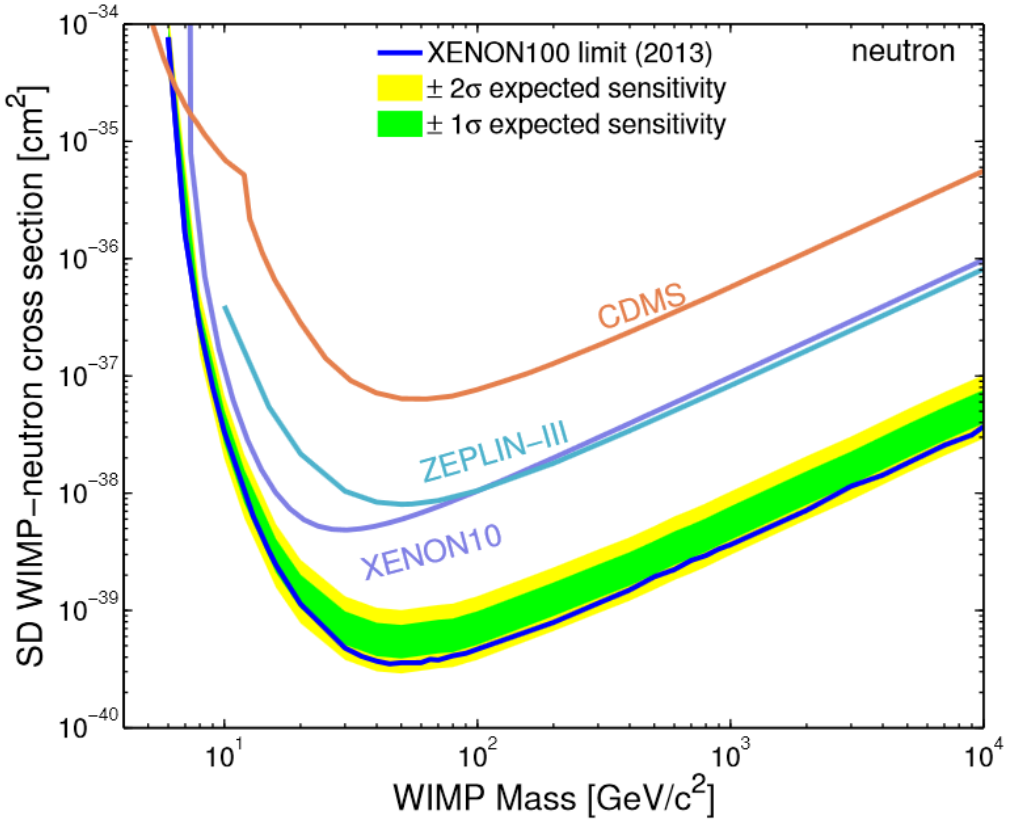


DAMA, ANAIS, DM-Ice

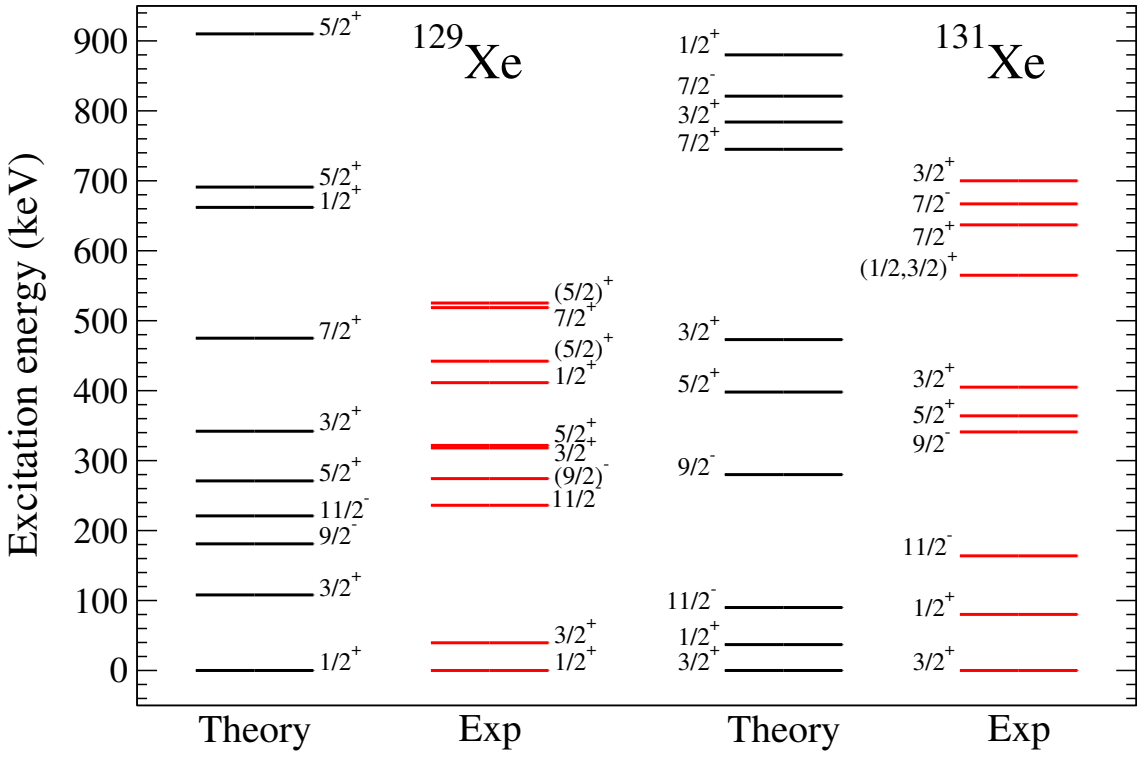
XENON100 spin-dependent limit

TECHNISCHE UNIVERSITÄT DARMSTADT

Structure factors and uncertainties in currents used in XENON100 spin-dependent analysis: XENON100, PRL **111**, 021301 (2013)



Inelastic scattering
Xenon spectra



- Excitation to low-lying first excited state (40 keV / 80 keV) possible
- Nuclear recoil + prompt deexcitation gamma can be observed

Types of WIMP-nucleon interactions
Spin-independent (SI)

TECHNISCHE UNIVERSITÄT DARMSTADT

Elastic scattering:

- All nucleons contribute (coherent)

$$\langle \text{initial} | \sum_{i}^{A} H_{\chi N}^{\text{SI}} | \text{initial} \rangle \propto A$$

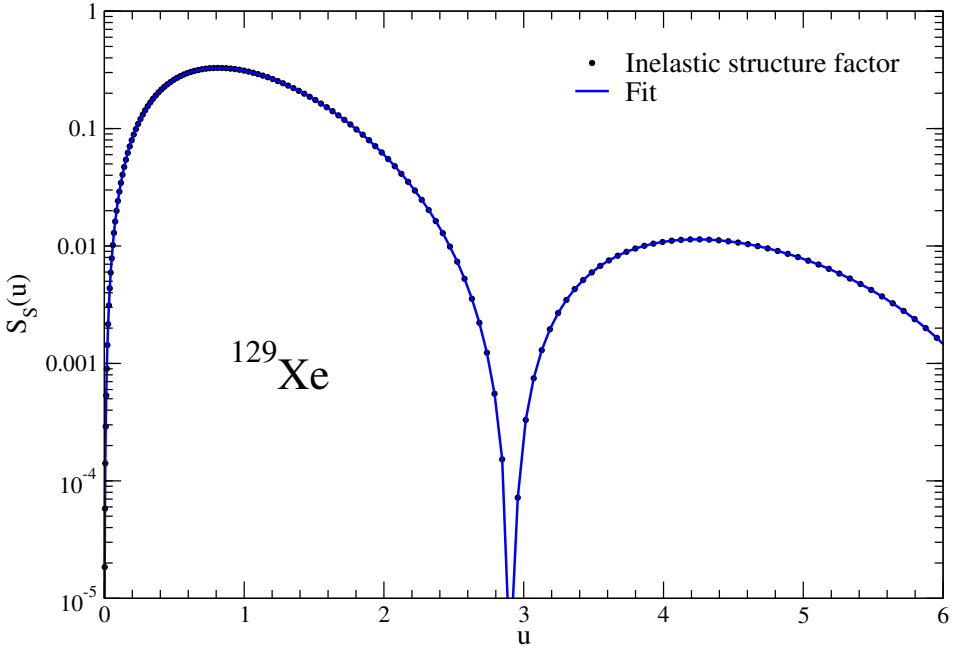
Inelastic scattering:

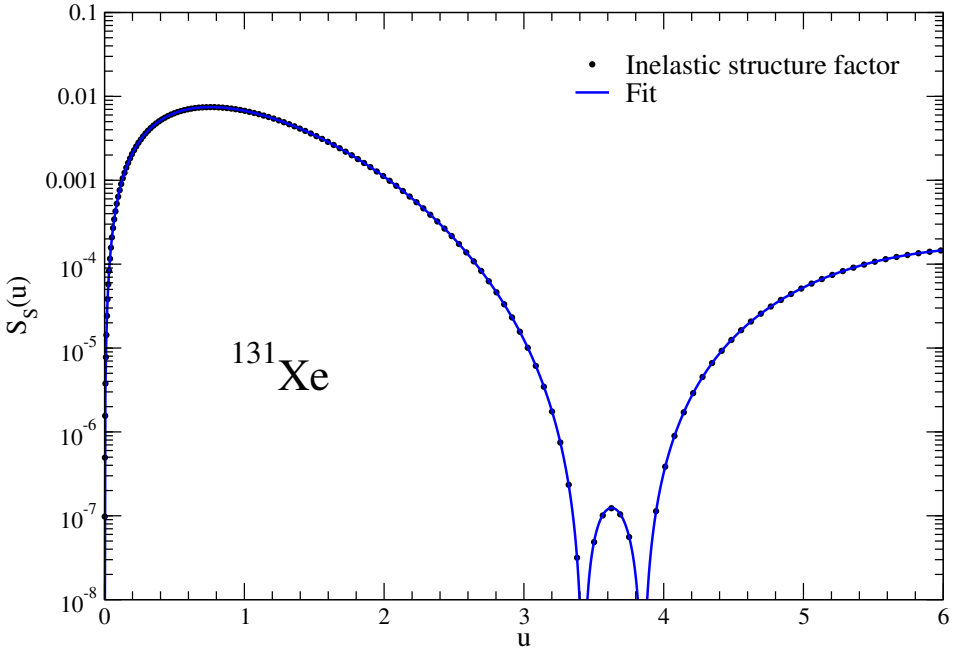
- For experimentally relevant isotopes transitions between ground state and first excited state are of single-particle nature

$$\langle \text{final} | \sum_{i}^{A} H_{\chi N}^{\text{SI}} | \text{initial} \rangle \propto 1$$

- **For SI interaction, inelastic scattering strongly suppressed**

Structure factors: SI inelastic scattering





$u = q^2 b^2/2$ with harmonic oscillator length b

Vietze, PK, Menéndez, Haxton, Schwenk, PRD **91**, 043520 (2015)

- Suppressed by $A^{-2} \sim 10^{-4}$ compared to elastic

Types of WIMP-nucleus interactions
Spin-dependent (SD)

TECHNISCHE UNIVERSITÄT DARMSTADT

Elastic scattering

- Spin carried mostly by unpaired nucleons

$$\langle \text{initial} | \sum_{i}^{A} H_{\chi N}^{\text{SD}} | \text{initial} \rangle \propto 1$$

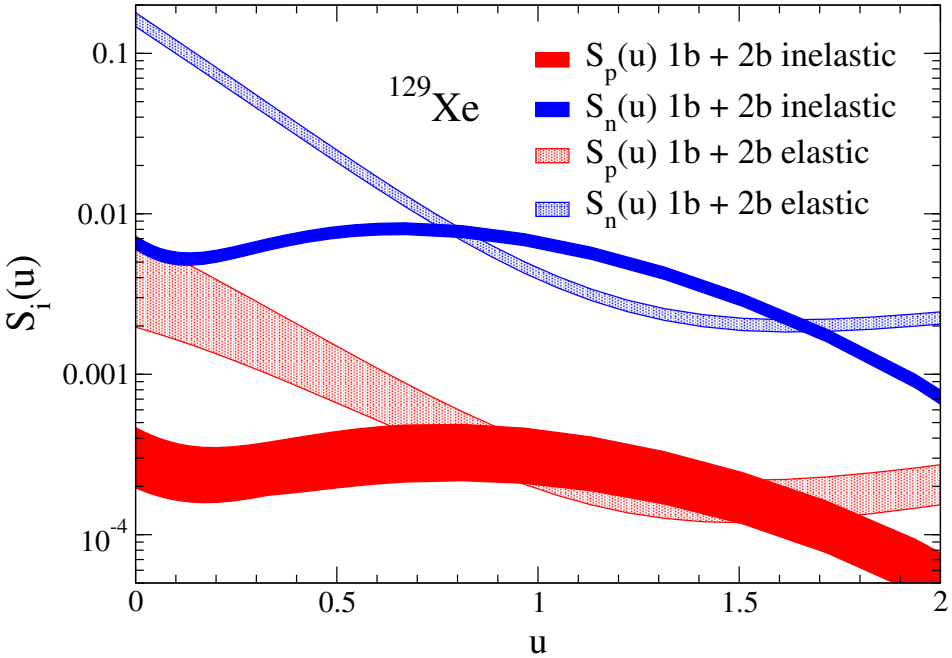
Inelastic scattering

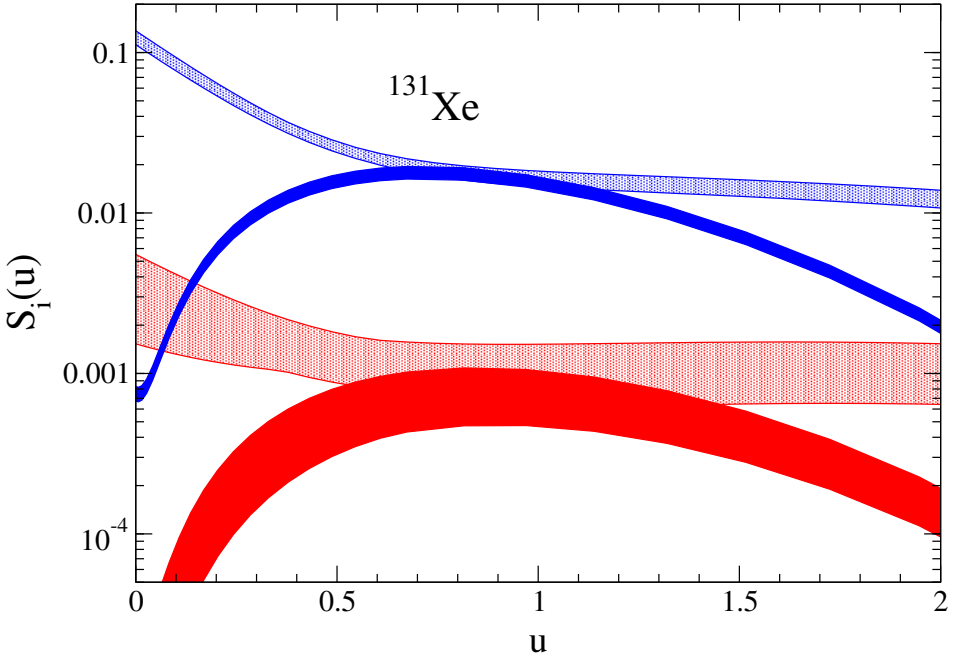
- Transition also of single-particle nature

$$\langle \text{final} | \sum_{i}^{A} H_{\chi N}^{\text{SD}} | \text{initial} \rangle \propto 1$$

- **SD channel sensitive to both elastic and inelastic scattering**

Structure factors: SD inelastic scattering



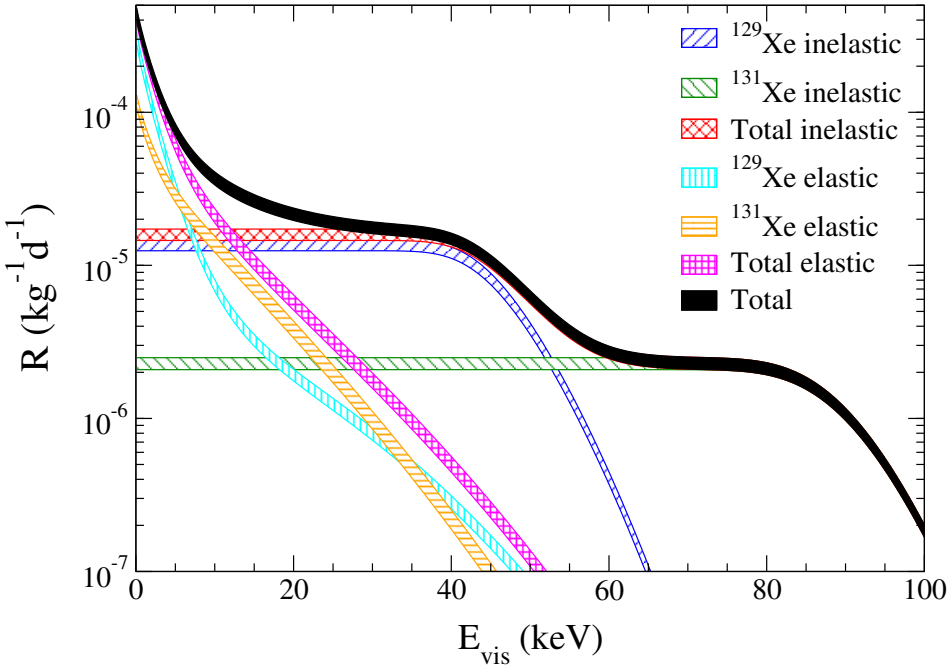


$u = q^2 b^2/2$ with harmonic oscillator length b

Baudis, Kessler, PK, Lang, Menéndez, Reichard, Schwenk, PRD **88**, 115014 (2013)

- Inelastic comparable to elastic scattering at $u \approx 1$ ($q \approx 125$ MeV)

Inelastic scattering
Integrated recoil spectra



Mass [GeV]	^{129}Xe	^{131}Xe	Total
10	–	–	–
25	5	–	5
50	7	17	9
100	7	24	12
250	9	32	19
500	11	35	24

TABLE II. Minimum energy E_{vis} in keV above which the observed inelastic spectrum for ^{129}Xe, ^{131}Xe and for the total spectrum starts to dominate the elastic one for various WIMP masses.

- One plateau per excited state
- Combined information from elastic and inelastic channel will allow to **determine dominant interaction channel** in one experiment
- **Inelastic excitation sensitive to WIMP mass**

Other WIMP responses

TECHNISCHE UNIVERSITÄT DARMSTADT

Spin-dependent interaction:

Axial-vector–axial-vector coupling: $H_{\chi N} = (A_\chi)_\mu (A_N)^\mu$

WIMP current: $A^\mu_\chi = \bar{\chi}\gamma^\mu\gamma_5\chi$

nucleon current: $A^\mu_N = A^\mu_{1b} + A^\mu_{2b}$

Spin-independent interaction:

Scalar-scalar coupling: $H_{\chi N} = S_\chi S_N$

General WIMP-nucleon interaction Hamiltonian:

$$H_{\chi N} = (V_\chi + A_\chi)_\mu (V_N + A_N)^\mu + (S_\chi + P_\chi)(S_N + P_N) + \ldots$$

V	vector	S	scalar
A	axial-vector	P	pseudoscalar
		...	tensor, spin-2, ...

Other WIMP responses
Chiral power counting

TECHNISCHE UNIVERSITÄT DARMSTADT

Combined counting of WIMP-Nucleon scattering amplitude

WIMP	Nucleon	V t	V $\mathbf{x}$	A t	A $\mathbf{x}$
V	1b	0	1 + 2	2	0 + 2
	2b	4	2 + 2	2	4 + 2
	2b NLO	—	—	5	3 + 2
A	1b	0 + 2	1	2 + 2	0
	2b	4 + 2	2	2 + 2	4
	2b NLO	—	—	5 + 2	3

WIMP	Nucleon	S	P
S	1b	2	1
	2b	3	5
	2b NLO	—	4
P	1b	2 + 2	1 + 2
	2b	3 + 2	5 + 2
	2b NLO	—	4 + 2

Hoferichter, PK, Schwenk, arXiv 1503.04811

- WIMP mass counted like nucleon mass
  $\rightarrow$ "+2" due to non-rel. expansion of WIMP fields
- More than only scalar-scalar and axial-vector–axial-vector up to Q^3
- Coherence effects not included

Matching to NREFT

Fitzpatrick, *et al.*, JCAP (2013), Fan, Reece and Wang, JCAP (2010)

Matching of NREFT operators $\mathcal{O}_i$ to chiral currents:

$$\mathcal{M}^{SS}_{1,\mathrm{NR}} = \chi^\dagger_{r'}\chi^\dagger_{s'}\mathcal{O}_1 f_N(t)\chi_r\chi_s,$$

$$\mathcal{M}^{SP}_{1,\mathrm{NR}} = \chi^\dagger_{r'}\chi^\dagger_{s'}\mathcal{O}_{10} g^N_5(t)\chi_r\chi_s,$$

$$\mathcal{M}^{PP}_{1,\mathrm{NR}} = \frac{1}{m_\chi}\chi^\dagger_{r'}\chi^\dagger_{s'}\mathcal{O}_6 h^N_5(t)\chi_r\chi_s,$$

$$\mathcal{M}^{VV}_{1,\mathrm{NR}} = \chi^\dagger_{r'}\chi^\dagger_{s'}\left[\mathcal{O}_1\left(f_1^{V,N}(t) + \frac{t}{4m_N^2}f_2^{V,N}(t)\right) + \frac{1}{m_N}\mathcal{O}_3 f_2^{V,N}(t) + \frac{1}{m_N m_\chi}\left(t\mathcal{O}_4 + \mathcal{O}_6\right)f_2^{V,N}(t)\right]\chi_r\chi_s,$$

$$\mathcal{M}^{AV}_{1,\mathrm{NR}} = \chi^\dagger_{r'}\chi^\dagger_{s'}\left[2\mathcal{O}_8 f_1^{V,N}(t) + \frac{2}{m_N}\mathcal{O}_9\left(f_1^{V,N}(t) + f_2^{V,N}(t)\right)\right]\chi_r\chi_s,$$

$$\mathcal{M}^{AA}_{1,\mathrm{NR}} = \chi^\dagger_{r'}\chi^\dagger_{s'}\left[-4\mathcal{O}_4 g_A^N(t) + \frac{1}{m_N^2}\mathcal{O}_6 g_P^N(t)\right]\chi_r\chi_s,$$

$$\mathcal{M}^{VA}_{1,\mathrm{NR}} = \chi^\dagger_{r'}\chi^\dagger_{s'}\left[-2\mathcal{O}_7 + \frac{2}{m_\chi}\mathcal{O}_9\right]h_A^N(t)\chi_r\chi_s.$$

- Only 8 $\mathcal{O}_i$ appear at Q^3, $\mathcal{O}_i$ not independent

Conclusion

TECHNISCHE UNIVERSITÄT DARMSTADT

- State-of-the-art large-scale shell-model calculations used to predict spin-independent / spin-dependent WIMP responses
- Chiral EFT two-body currents especially relevant at low-momentum transfer
- Results used by XENON100 collaboration for spin-dependent limits
- Inelastic scattering allows to distinguish between SI and SD interactions and provides new sensitivity to WIMP mass
- Other WIMP responses beyond standard SI/SD relevant at Q^3
- Chiral counting predicts hierarchy amongst different channels