QCD anatomy of WIMP-nucleon interactions

Mikhail Solon
UCB/LBNL

MITP workshop on Effective Theories and Dark Matter
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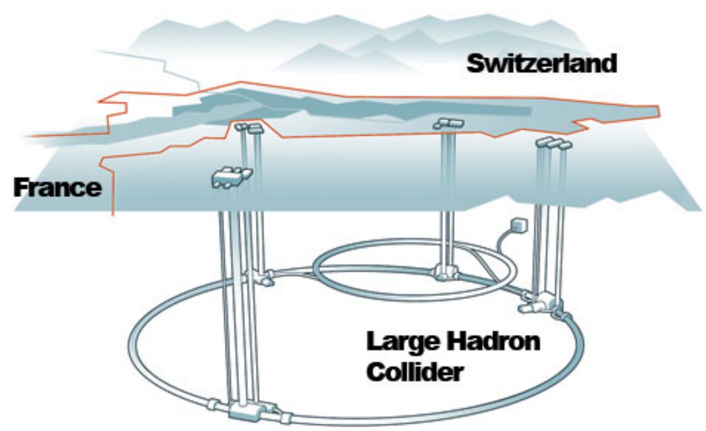
based on work with R. Hill: 1409.8290
see also 1111.0016, 1309.4092, 1401.3339.

$$\Omega_M h^2 \neq \Omega_B h^2$$

0.1423 ± 0.0029 $\qquad$ 0.02207 ± 0.00033



theory dreamscape $\longleftrightarrow$ experimental searches

signals, backgrounds

model-dependent uncertainties

model-independent uncertainties

Scrutiny of underlying astrophysics is important, but we'll stick to Standard Model physics here.

M annihilation: sommerfeld enhancement, bound states, thermal bath effects, Sudakov logs

production: complementarity

m_W

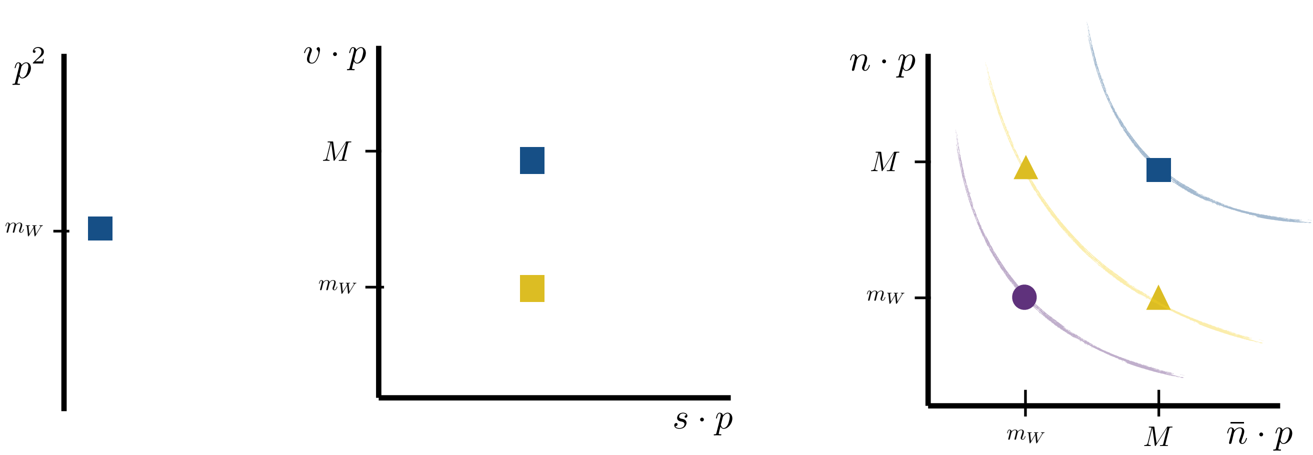
QCD and EW running

$m_b\,, m_c$

m_N scattering: nucleon matrix elements, DM-nucleon EFT, multinucleon effects

Develop an effective theory framework to put a handle on model-dependent and -independent uncertainties

	calculability	universality	precision
QCD	brown muck, simple	factorization, heavy quark symmetry	O(1 - 10 %), control uncertainties
DM	unknown	SM anatomy	O(10^2 - 10^4 %)

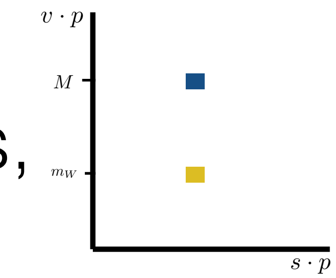


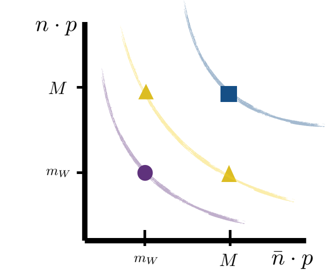
LHC is carving out parameter space, pushing to regions requiring precision

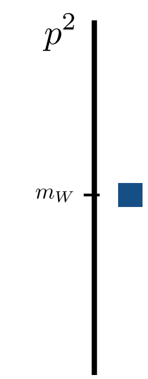
Heavy electroweak charged WIMPs

M

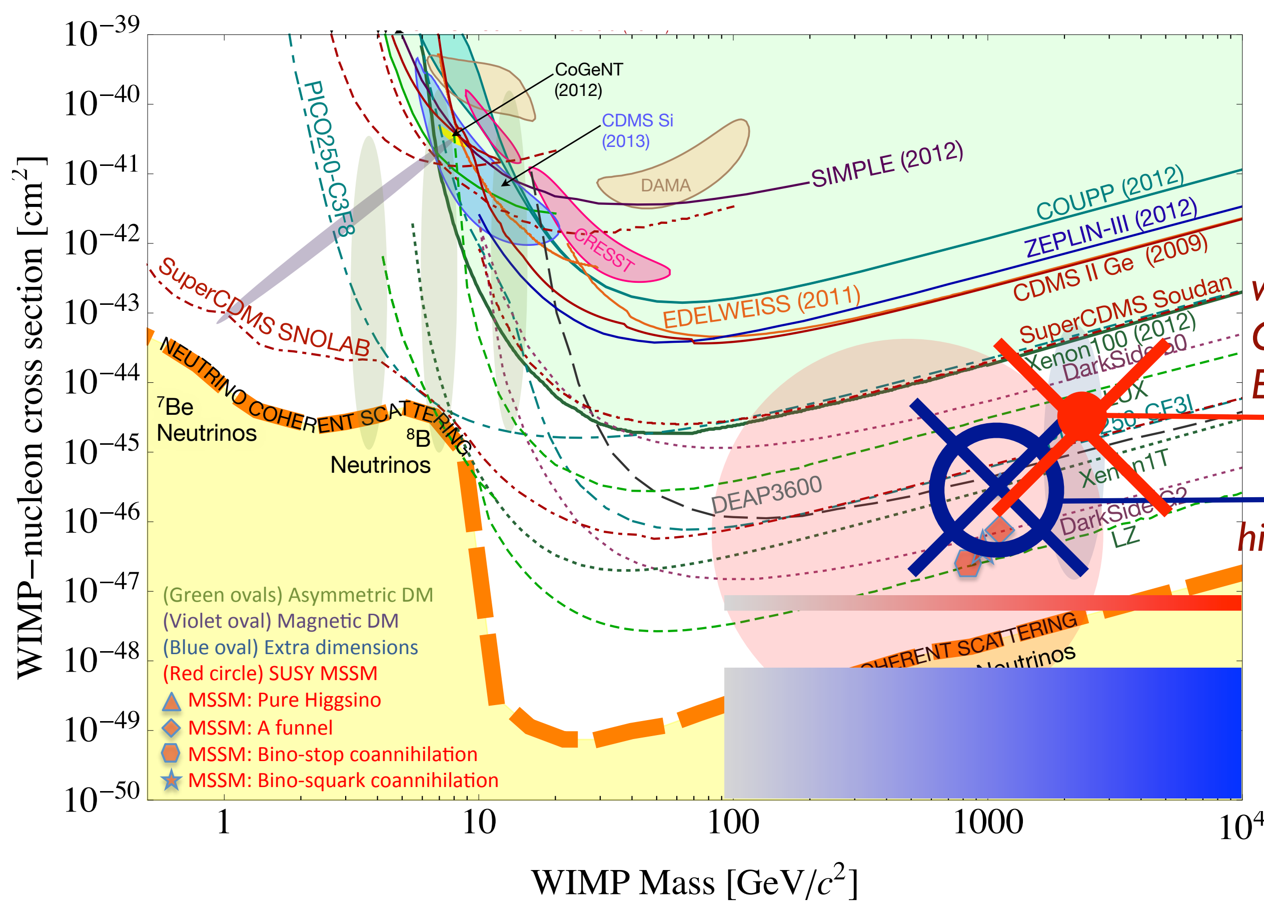
annihilation: thermal, theoretical control of Sudakov logs,
production: null results pushing to higher limits







m_W



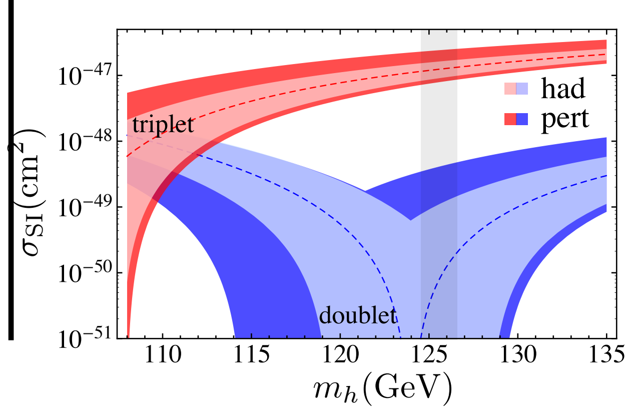
wino: dimensional estimate
Cirelli, Fornengo, Strumia (2005)
Essig (2009)

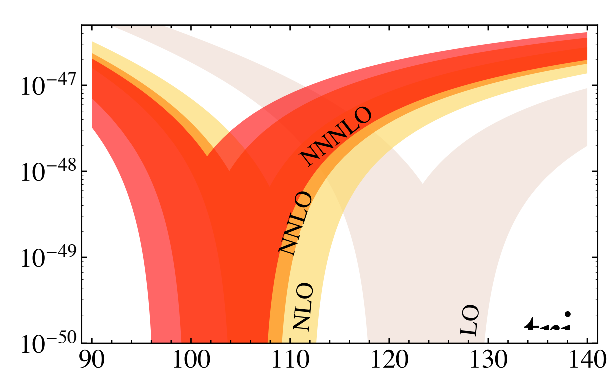
higgsino: Snowmass CF1 (2013)
(MicrOMEGAs)

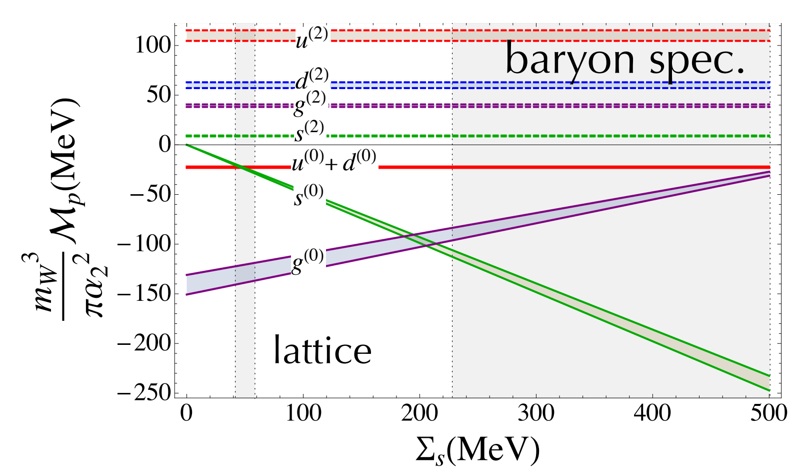
this work

$m_b\,, m_c$

m_N

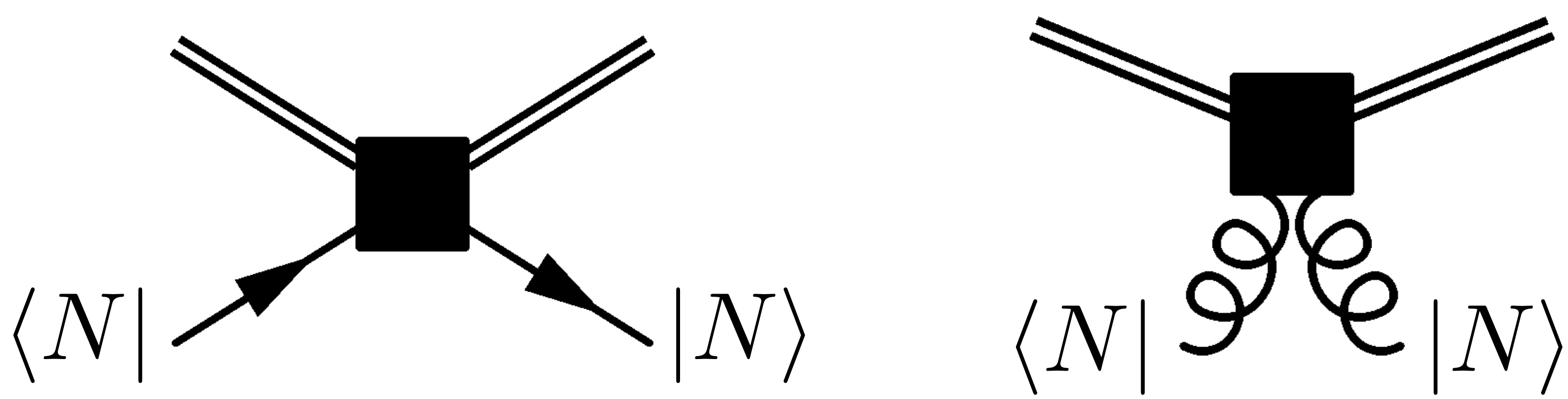






In the rest of the talk,

$$\mathcal{L}_{\mathrm{DM}} + \mathcal{L}_{\mathrm{SM}}$$



and illustrate with phenomenological examples.

Zeroth order question: why bother with radiative corrections?

$$\mathcal{L} = \sum_i c_i(\mu) O_i(\mu) \qquad \mathcal{M}_{\text{phys}} = \sum_i c_i(\mu) \langle O_i(\mu) \rangle \qquad \frac{d\mathcal{M}_{\text{phys}}}{d\mu} = 0$$

μ_1

$$\sim \alpha \log \frac{\mu_1}{\mu_2}$$

- get the LO (LL) result
- some matrix elements acessible only at a certain scale
- use complementarity
- (avoid certain uncertainties)

μ_2

Currents: relativistic scalar or fermion

$$\mathcal{L}_{\phi,\mathrm{SM}} = \sum_{q=u,d,s,c,b} \left\{ \frac{c_{\phi 1,q}}{m_W^2} |\phi|^2 m_q \bar{q} q + \frac{c_{\phi 2,q}}{m_W^2} |\phi|^2 m_q \bar{q} i\gamma_5 q + \frac{c_{\phi 3,q}}{m_W^2} \phi^* i\partial_-^\mu \phi \bar{q}\gamma_\mu q + \frac{c_{\phi 4,q}}{m_W^2} \phi^* i\partial_-^\mu \phi \bar{q}\gamma_\mu \gamma_5 q \right\}$$
$$+ \frac{c_{\phi 5}}{m_W^2} |\phi|^2 G^A_{\alpha\beta} G^{A\alpha\beta} + \frac{c_{\phi 6}}{m_W^2} |\phi|^2 G^A_{\alpha\beta} \tilde{G}^{A\alpha\beta} + \cdots . \qquad n = 3, 4.$$

$$\mathcal{L}_{\psi,\mathrm{SM}} = \frac{c_{\psi 1}}{m_W} \bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu} + \frac{c_{\psi 2}}{m_W} \bar{\psi}\sigma^{\mu\nu}\psi \tilde{F}_{\mu\nu} + \sum_{q=u,d,s,c,b} \left\{ \frac{c_{\psi 3,q}}{m_W^2} \bar{\psi}\gamma^\mu\gamma_5\psi \bar{q}\gamma_\mu q + \frac{c_{\psi 4,q}}{m_W^2} \bar{\psi}\gamma^\mu\gamma_5\psi \bar{q}\gamma_\mu\gamma_5 q + \frac{c_{\psi 5,q}}{m_W^2} \bar{\psi}\gamma^\mu\psi \bar{q}\gamma_\mu q \right.$$
$$+ \frac{c_{\psi 6,q}}{m_W^2} \bar{\psi}\gamma^\mu\psi \bar{q}\gamma_\mu\gamma_5 q + \frac{c_{\psi 7,q}}{m_W^3} \bar{\psi}\psi m_q \bar{q} q + \frac{c_{\psi 8,q}}{m_W^3} \bar{\psi} i\gamma_5 \psi m_q \bar{q} q + \frac{c_{\psi 9,q}}{m_W^3} \bar{\psi}\psi m_q \bar{q} i\gamma_5 q + \frac{c_{\psi 10,q}}{m_W^3} \bar{\psi} i\gamma_5\psi m_q \bar{q} i\gamma_5 q$$
$$+ \frac{c_{\psi 11,q}}{m_W^3} \bar{\psi} i\partial_-^\mu \psi \bar{q}\gamma_\mu q + \frac{c_{\psi 12,q}}{m_W^3} \bar{\psi}\gamma_5 \partial_-^\mu \psi \bar{q}\gamma_\mu q + \frac{c_{\psi 13,q}}{m_W^3} \bar{\psi} i\partial_-^\mu \psi \bar{q}\gamma_\mu\gamma_5 q + \frac{c_{\psi 14,q}}{m_W^3} \bar{\psi}\gamma_5 \partial_-^\mu \psi \bar{q}\gamma_\mu\gamma_5 q$$
$$\left. + \frac{c_{\psi 15,q}}{m_W^3} \bar{\psi}\sigma_{\mu\nu}\psi m_q \bar{q}\sigma^{\mu\nu} q + \frac{c_{\psi 16,q}}{m_W^3} \epsilon_{\mu\nu\rho\sigma} \bar{\psi}\sigma^{\mu\nu}\psi m_q \bar{q}\sigma^{\rho\sigma} q \right\} + \frac{c_{\psi 17}}{m_W^3} \bar{\psi}\psi G^A_{\alpha\beta} G^{A\alpha\beta}$$
$$+ \frac{c_{\psi 18}}{m_W^3} \bar{\psi} i\gamma_5\psi G^A_{\alpha\beta} G^{A\alpha\beta} + \frac{c_{\psi 19}}{m_W^3} \bar{\psi}\psi G^A_{\alpha\beta} \tilde{G}^{A\alpha\beta} + \frac{c_{\psi 20}}{m_W^3} \bar{\psi} i\gamma_5\psi G^A_{\alpha\beta} \tilde{G}^{A\alpha\beta} + \cdots , \qquad n = 1, 2, 5, 6, 11, 12, 13, 14, 15, 16$$

Currents: heavy particle field

$$\mathcal{L}_{\chi_v,\mathrm{SM}} = \frac{c_{\chi 1}}{m_W}\bar\chi_v\sigma_\perp^{\mu\nu}\chi_v F_{\mu\nu} + \frac{c_{\chi 2}}{m_W}\bar\chi_v\sigma_\perp^{\mu\nu}\chi_v\tilde F_{\mu\nu} + \sum_{q=u,d,s,c,b}\Big\{\frac{c_{\chi 3,q}}{m_W^2}\epsilon_{\mu\nu\rho\sigma}v^\mu\bar\chi_v\sigma_\perp^{\nu\rho}\chi_v\bar q\gamma^\sigma q$$
$$+\frac{c_{\chi 4,q}}{m_W^2}\epsilon_{\mu\nu\rho\sigma}v^\mu\bar\chi_v\sigma_\perp^{\nu\rho}\chi_v\bar q\gamma^\sigma\gamma_5 q + \frac{c_{\chi 5,q}}{m_W^2}\bar\chi_v\chi_v\bar q \not{v} q + \frac{c_{\chi 6,q}}{m_W^2}\bar\chi_v\chi_v\bar q\not{v}\gamma_5 q + \frac{c_{\chi 7,q}}{m_W^3}\bar\chi_v\chi_v m_q\bar q q$$
$$+\frac{c_{\chi 8,q}}{m_W^3}\bar\chi_v\chi_v\bar q\not{v} iv\cdot D_- q + \frac{c_{\chi 9,q}}{m_W^3}\bar\chi_v\chi_v m_q\bar q i\gamma_5 q + \frac{c_{\chi 10,q}}{m_W^3}\bar\chi_v\chi_v\bar q\not{v}\gamma_5 iv\cdot D_- q$$
$$+\frac{c_{\chi 11,q}}{m_W^3}\bar\chi_v\sigma_\perp^{\mu\nu}i\partial^\perp_{-\mu}\chi_v\bar q\gamma_\nu q + \frac{c_{\chi 12,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma}\bar\chi_v\sigma_\perp^{\mu\nu}i\partial_-^{\perp\rho}\chi_v\bar q\gamma^\sigma q + \frac{c_{\chi 13,q}}{m_W^3}\bar\chi_v\sigma_\perp^{\mu\nu}i\partial^\perp_{-\mu}\chi_v\bar q\gamma_\nu\gamma_5 q$$
$$+\frac{c_{\chi 14,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma}\bar\chi_v\sigma_\perp^{\mu\nu}i\partial_-^{\perp\rho}\chi_v\bar q\gamma^\sigma\gamma_5 q + \frac{c_{\chi 15,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma}v^\mu\bar\chi_v\sigma_\perp^{\nu\rho}\chi_v\bar q(\not{v} iD_-^\sigma + \gamma^\sigma iv\cdot D_-)q$$
$$+\frac{c_{\chi 16,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma}v^\mu\bar\chi_v\sigma_\perp^{\nu\rho}\chi_v\bar q(\not{v} iD_-^\sigma + \gamma^\sigma iv\cdot D_-)\gamma_5 q + \frac{c_{\chi 17,q}}{m_W^3}\bar\chi_v i\partial_-^{\perp\mu}\chi_v\bar q\gamma_\mu q$$
$$+\frac{c_{\chi 18,q}}{m_W^3}\bar\chi_v\sigma_\perp^{\mu\nu}\partial^\perp_{+\mu}\chi_v\bar q\gamma_\nu q + \frac{c_{\chi 18,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma}\bar\chi_v\sigma_\perp^{\mu\nu}\partial_+^{\perp\rho}\chi_v\bar q\gamma^\sigma q + \frac{c_{\chi 20,q}}{m_W^3}\bar\chi_v i\partial_-^{\perp\mu}\chi_v\bar q\gamma_\mu\gamma_5 q$$
$$+\frac{c_{\chi 21,q}}{m_W^3}\bar\chi_v\sigma_\perp^{\mu\nu}\partial^\perp_{+\mu}\chi_v\bar q\gamma_\nu\gamma_5 q + \frac{c_{\chi 22,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma}\bar\chi_v\sigma_\perp^{\mu\nu}\partial_+^{\perp\rho}\chi_v\bar q\gamma^\sigma\gamma_5 q + \frac{c_{\chi 23,q}}{m_W^3}\bar\chi_v\sigma_\perp^{\mu\nu}\chi_v m_q\bar q\sigma_{\mu\nu}q$$
$$+\frac{c_{\chi 24,q}}{m_W^3}\epsilon_{\mu\nu\rho\sigma}\bar\chi_v\sigma_\perp^{\mu\nu}\chi_v m_q\bar q\sigma^{\rho\sigma}q\Big\} + \frac{c_{\chi 25}}{m_W^3}\bar\chi_v\chi_v G^A_{\alpha\beta}G^{A\alpha\beta} + \frac{c_{\chi 26}}{m_W^3}\bar\chi_v\chi_v G^A_{\alpha\beta}\tilde G^{A\alpha\beta}$$
$$+\frac{c_{\chi 27}}{m_W^3}\bar\chi_v\chi_v v_\mu v_\nu G^{A\mu}{}_\alpha G^{A\nu\alpha} + \frac{c_{\chi 28}}{m_W^3}\bar\chi_v\sigma_\perp^{\mu\nu}\chi_v\epsilon_{\mu\nu\alpha\beta}v^\alpha v^\gamma G^{A\beta\delta}G^A_{\gamma\delta} + \cdots,$$

$n = 1, 2, 5, 6,\ 15, 16, 17, 18, 19, 20, 21, 22, \ddot{23}, 24$

$$\chi_v(x) \to e^{iq\cdot x}\left[1 + \frac{iq\cdot D_\perp}{2M^2} + \frac{1}{4M^2}\sigma_{\alpha\beta}q^\alpha D_\perp^\beta + \ldots\right]\chi_v(\mathcal{B}^{-1}x)$$

$$\frac{m_W}{M}c_{\chi 3} + 2c_{\chi 12} = \frac{m_W}{M}c_{\chi 4} + 2c_{\chi 14} = \frac{m_W}{M}c_{\chi 5} - 2c_{\chi 17}$$
$$= \frac{m_W}{M}c_{\chi 6} - 2c_{\chi 20} = c_{\chi 11} = c_{\chi 13} = 0,$$

Heinonen, Hill, Solon 2012

Through dimension seven, there are seven operator classes closed under renormalization and transforming irreducibly under continuous and discrete Lorentz transformations.

QCD operator basis
$V_q^\mu = \bar{q}\gamma^\mu q$
$A_q^\mu = \bar{q}\gamma^\mu \gamma_5 q$
$T_q^{\mu\nu} = i m_q \bar{q}\sigma^{\mu\nu}\gamma_5 q$
$O_q^{(0)} = m_q \bar{q} q,\ O_g^{(0)} = G^A_{\mu\nu} G^{A\mu\nu}$
$O_{5q}^{(0)} = m_q \bar{q} i\gamma_5 q,\ O_{5g}^{(0)} = \epsilon^{\mu\nu\rho\sigma} G^A_{\mu\nu} G^A_{\rho\sigma}$
$O_q^{(2)\mu\nu} = \frac{1}{2}\bar{q}(\gamma^{\{\mu} iD_-^{\nu\}} - \frac{g^{\mu\nu}}{4} i\not{D}_-)q,$
$O_g^{(2)\mu\nu} = -G^{A\mu\lambda} G^{A\nu}{}_\lambda + \frac{g^{\mu\nu}}{4}(G^A_{\alpha\beta})^2$
$O_{5q}^{(2)\mu\nu} = \frac{1}{2}\bar{q}\gamma^{\{\mu} iD_-^{\nu\}}\gamma_5 q$

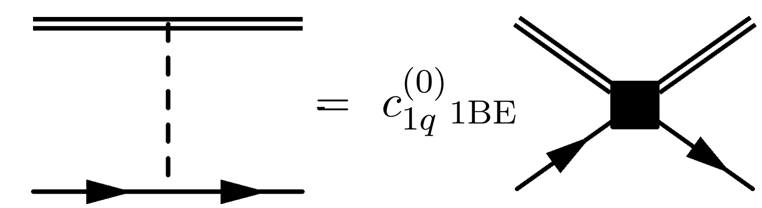
Example: Weak-scale matching

$$\mathcal{L}_{\rm DM} + \mathcal{L}_{\rm SM}$$

$$m_W \sim m_Z \sim m_h \sim m_t \qquad \downarrow$$

$$\mathcal{L}_{\phi,\rm SM} + \mathcal{L}_{n_f=5\ \rm QCD}$$

$$\mathcal{L}_{\psi,\rm SM} = \frac{1}{2}\bar{\psi}(i\partial\!\!\!/ - M')\psi - \frac{1}{\Lambda}\bar{\psi}(c'_{\psi 1} + i c'_{\psi 2}\gamma_5)\psi H^\dagger H + \cdots$$



$$\downarrow$$

$$\mathcal{L}_{\psi,\rm SM} = \frac{1}{2}\bar{\psi}(i\partial\!\!\!/ - M)\psi + \frac{1}{m_W^3}\Big[\bar{\psi}(c_{\psi 7} + i c_{\psi 8}\gamma_5)\psi \sum_q m_q \bar{q} q$$

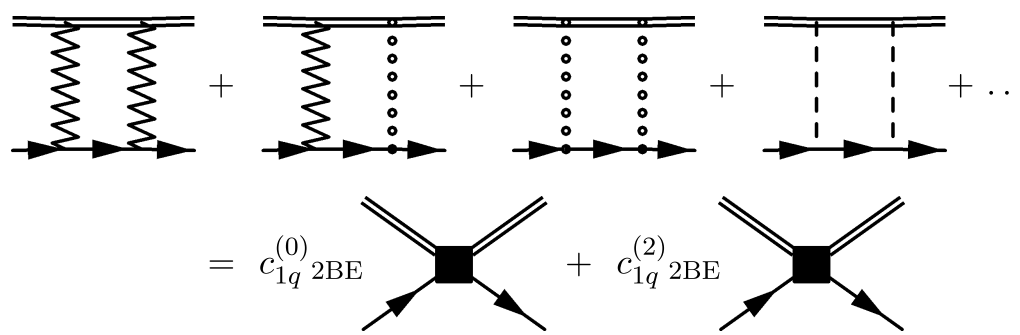
$$+ \bar{\psi}(c_{\psi 17} + i c_{\psi 18}\gamma_5)\psi G^A_{\mu\nu} G^{A\mu\nu}\Big] + \cdots, \qquad \psi \to e^{-i\phi\gamma_5}\psi, \qquad \tan 2\phi = \frac{c'_{\psi 2} v^2}{c'_{\psi 1} v^2 + M'\Lambda}$$

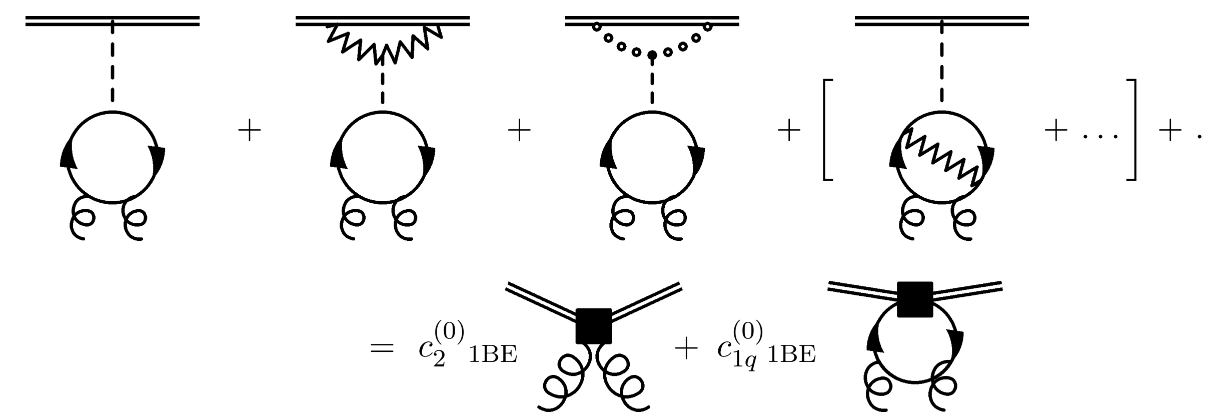
$$M = \sqrt{\left(M' + \frac{c'_{\psi 1} v^2}{\Lambda}\right)^2 + \left(\frac{c'_{\psi 2} v^2}{\Lambda}\right)^2},$$

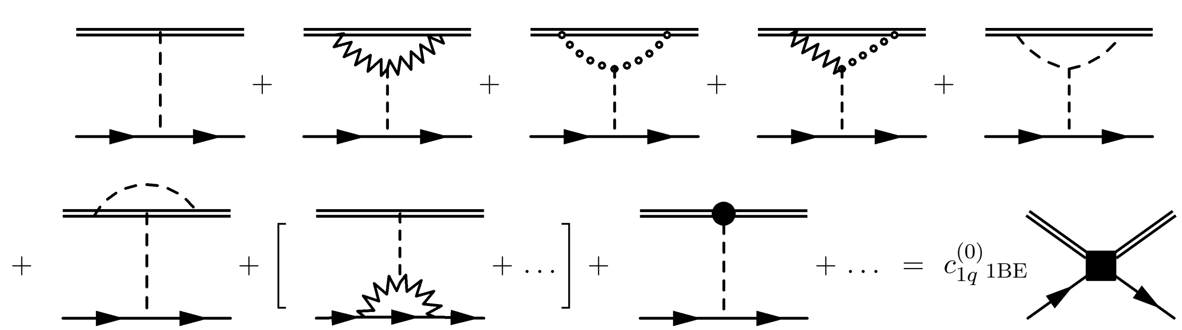
$$\{c_{\psi 7}, c_{\psi 8}\} = \frac{m_W^3 M'}{m_h^2 \Lambda M}\left\{c'_{\psi 1} + \frac{v^2}{M'\Lambda}[c'^2_{\psi 1} + c'^2_{\psi 2}], c'_{\psi 2}\right\}$$

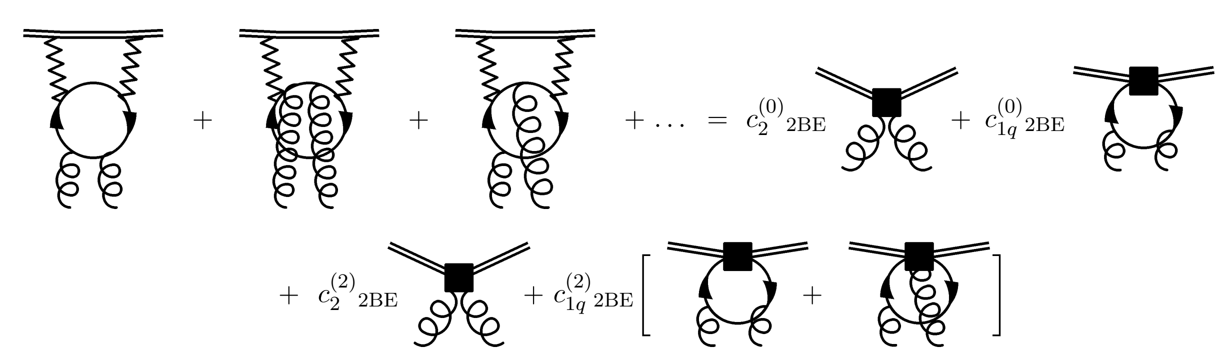
$$\{c_{\psi 17}, c_{\psi 18}\} = -\frac{\alpha_s(m_W)}{12\pi}\{c_{\psi 7}, c_{\psi 8}\}.$$

Weak-scale matching for electroweak charged DM done completely in 1401.3339



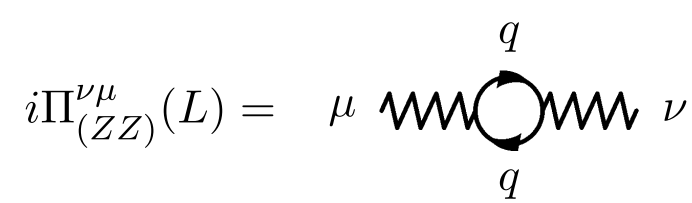






extend on-shell scheme

reduces to five integrals



EW pol. tensors

Renormalization constants, anomalous dimensions, and RGE solutions

$$O_i^{\text{bare}} = Z_{ij}(\mu) O_j^{\text{ren}}(\mu),$$

$$c_i^{\text{ren}}(\mu) = Z_{ji}(\mu) c_j^{\text{bare}}$$

$$\frac{d}{d\log\mu} O_i = -\gamma_{ij} O_j,$$

$$\frac{d}{d\log\mu} c_i = \gamma_{ji} c_j,$$

$$\gamma_{ij} \equiv Z_{ik}^{-1} \frac{d}{d\log\mu} Z_{kj}$$

$$c_i(\mu_l) = R_{ij}(\mu_l, \mu_h) c_j(\mu_h)$$

$$r(t) = \left(\frac{\alpha_s(\mu_l)}{\alpha_s(\mu_h)}\right)^{-\frac{1}{2\beta_0}(\frac{64}{9}+\frac{4}{3}t)}.$$

Operator	Solution to coefficient running
V_q	$R_V = 1$
A_q	$R_A^{(\text{singlet})} = \exp\{\frac{2n_f}{\pi\beta_0}[\alpha_s(\mu_h) - \alpha_s(\mu_l)] + \mathcal{O}(\alpha_s^2)\},$
	$R_A^{(\text{nonsinglet})} = 1$
T_q	$R_T = (\frac{\alpha_s(\mu_l)}{\alpha_s(\mu_h)})^{-\frac{16}{3\beta_0}}[1 + \mathcal{O}(\alpha_s)]$
$O_q^{(0)}, O_g^{(0)}$	$R_{qq}^{(0)} = 1,\ R_{qg}^{(0)} = 2[\gamma_m(\mu_h) - \gamma_m(\mu_l)]/\tilde{\beta}(\mu_h),$
	$R_{gq}^{(0)} = 0,\ R_{gg}^{(0)} = \tilde{\beta}(\mu_l)/\tilde{\beta}(\mu_h)$
$O_{5q}^{(0)}, O_{5g}^{(0)}$	$R_{5,qq}^{(0)} = 1,\ R_{5,qg}^{(0)} = \frac{16}{\beta_0}(\frac{\alpha_s(\mu_l)}{\alpha_s(\mu_h)} - 1) + \mathcal{O}(\alpha_s),$
	$R_{5,gq}^{(0)} = 0,\ R_{5,gg}^{(0)} = \frac{\alpha_s(\mu_l)}{\alpha_s(\mu_h)} + \mathcal{O}(\alpha_s)$
$O_q^{(2)}, O_g^{(2)}$	$R_{qq}^{(2)} - R_{qq'}^{(2)} = r(0) + \mathcal{O}(\alpha_s),$
	$R_{qq'}^{(2)} = \frac{1}{n_f}[\frac{16r(n_f)+3n_f}{16+3n_f} - r(0)] + \mathcal{O}(\alpha_s),$
	$R_{qg}^{(2)} = \frac{16[1-r(n_f)]}{16+3n_f} + \mathcal{O}(\alpha_s),$
	$R_{gq}^{(2)} = \frac{3[1-r(n_f)]}{16+3n_f} + \mathcal{O}(\alpha_s),\ R_{gg}^{(2)} = \frac{16+3n_f r(n_f)}{16+3n_f} + \mathcal{O}(\alpha_s)$
$O_{5q}^{(2)}$	$R_5^{(2)} = (\frac{\alpha_s(\mu_l)}{\alpha_s(\mu_h)})^{-\frac{32}{9\beta_0}}[1 + \mathcal{O}(\alpha_s)]$

Wilson coefficient renormalization

$$c_q^{(0)}(\mu) = \sum_{q'} Z_{q'q}^{(0)}(\mu) c_{q'}^{(0)\text{bare}} + Z_{gq}^{(0)}(\mu) c_g^{(0)\text{bare}} = c_q^{(0)\text{bare}} + \mathcal{O}(\alpha_s^2)$$

$$c_g^{(0)}(\mu) = \sum_{q'} Z_{q'g}^{(0)}(\mu) c_{q'}^{(0)\text{bare}} + Z_{gg}^{(0)}(\mu) c_g^{(0)\text{bare}} = c_g^{(0)\text{bare}} + \mathcal{O}(\alpha_s^2)$$

$$c_q^{(2)}(\mu) = \sum_{q'} Z_{q'q}^{(2)}(\mu) c_{q'}^{(2)\text{bare}} + Z_{gq}^{(2)}(\mu) c_g^{(2)\text{bare}} = c_q^{(2)\text{bare}} + \mathcal{O}(\alpha_s),$$

$$c_g^{(2)}(\mu) = \sum_{q'} Z_{q'g}^{(2)}(\mu) c_{q'}^{(2)\text{bare}} + Z_{gg}^{(2)}(\mu) c_g^{(2)\text{bare}} = \sum_q \frac{1}{\epsilon} \frac{\alpha_s}{6\pi} c_q^{(2)\text{bare}} + c_g^{(2)\text{bare}} + \mathcal{O}(\alpha_s^2)$$

Heavy quark thresholds

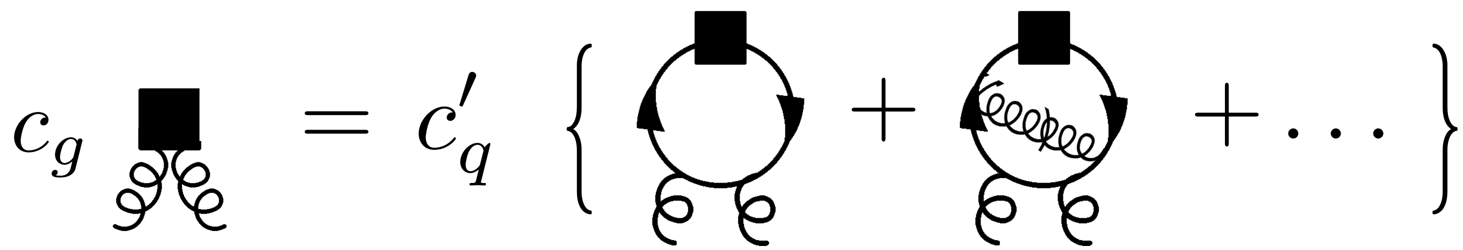
$$m_b \quad \begin{array}{c} \mathcal{L}_{n_f=4\,\mathrm{QCD}} \\ \downarrow \\ \mathcal{L}_{n_f=5\,\mathrm{QCD}} \end{array}$$

$$c_i(\mu_Q) = M_{ij}(\mu_Q) c'_j(\mu_Q).$$

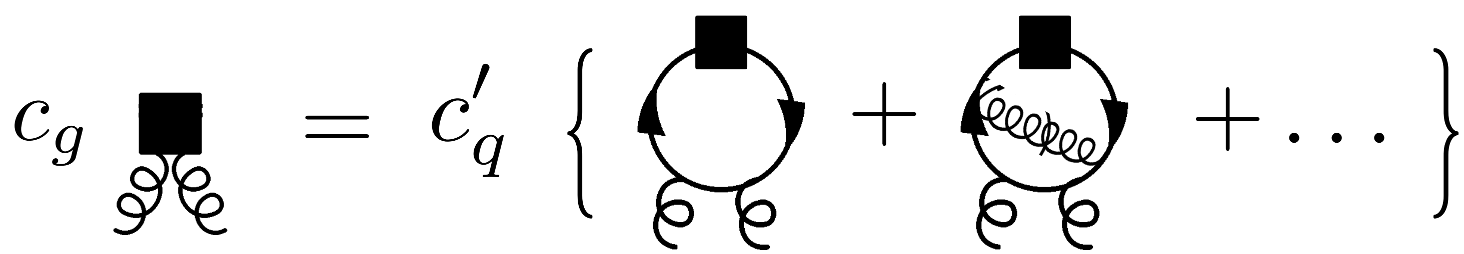
Operator	Solution to matching condition
V_q	$M_V = 1$
A_q	$M_A = 1 + \mathcal{O}(\alpha_s^2)$
T_q	$M_T = 1 + \mathcal{O}(\alpha_s^2)$
$O_q^{(0)}, O_g^{(0)}$	$M_{gQ}^{(0)} = -\frac{\alpha'_s(\mu_Q)}{12\pi}\{1 + \frac{\alpha'_s(\mu_Q)}{4\pi}[11 - \frac{4}{3}\log\frac{\mu_Q}{m_Q}] + \mathcal{O}(\alpha_s^2)\}$, $M_{gg}^{(0)} = 1 - \frac{\alpha'_s(\mu_Q)}{3\pi}\log\frac{\mu_Q}{m_Q} + \mathcal{O}(\alpha_s^2)$
$O_{5q}^{(0)}, O_{5g}^{(0)}$	$M_{5,gQ}^{(0)} = \frac{\alpha'_s(\mu_Q)}{8\pi} + \mathcal{O}(\alpha_s^2)$, $M_{5,gg}^{(0)} = 1 + \mathcal{O}(\alpha_s)$
$O_q^{(2)}, O_g^{(2)}$	$M_{gQ}^{(2)} = \frac{\alpha'_s}{3\pi}\log\frac{\mu_Q}{m_Q} + \mathcal{O}(\alpha_s^2)$, $M_{gg}^{(2)} = 1 + \mathcal{O}(\alpha_s)$
$O_{5q}^{(2)}$	$M_5^{(2)} = 1 + \mathcal{O}(\alpha_s^2)$

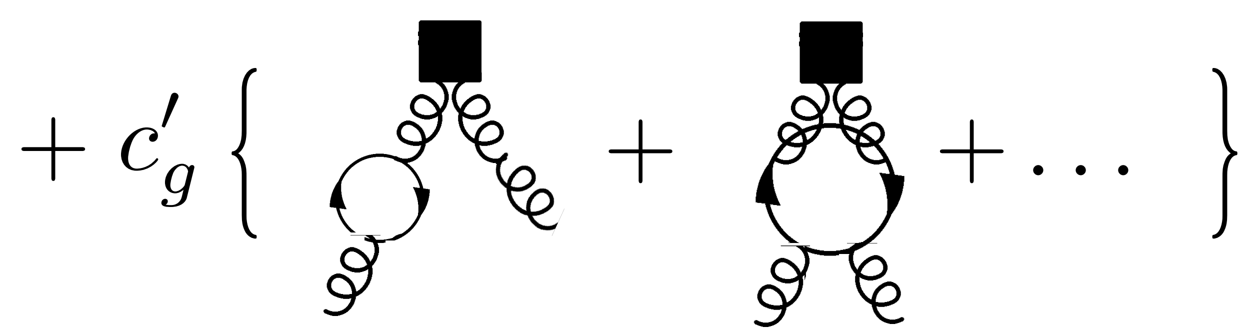
Sum rule constraints on scalar matrix elements

$$\bar{\chi}\chi\left\{\bar{q}q\,, G_{\mu\nu}G^{\mu\nu}\right\} \qquad h\left\{\bar{q}q\,, G_{\mu\nu}G^{\mu\nu}\right\}$$



low energy theorems





Sum rule constraints on scalar matrix elements

$$\langle \theta^\mu_\mu \rangle = m_N = (1-\gamma_m) \sum_{q=u,d,s,\ldots}^{n_f} \langle O_q^{(0)} \rangle + \frac{\tilde{\beta}}{2} \langle O_g^{(0)} \rangle$$

$$\langle O_i'^{(S)} \rangle(\mu_h) = R_{ji}^{(S)}(\mu,\mu_h) \langle O_j^{(S)} \rangle(\mu),$$

$$\langle O_i'^{(S)} \rangle(\mu_b) = M_{ji}^{(S)}(\mu_b) \langle O_j^{(S)} \rangle(\mu_b) + \mathcal{O}(1/m_b)$$

$$R(\mu,\mu_h) = \left(\begin{array}{ccc|c} 1 & & & R_{qg} \\ & \ddots & & \vdots \\ & & 1 & R_{qg} \\ \hline 0 & \cdots & 0 & R_{gg} \end{array}\right) \qquad M(\mu_Q) = \left(\begin{array}{ccc|c|c} & & & M_{qQ} & M_{qg} \\ & \mathbb{1}(M_{qq} - M_{qq'}) + \mathbb{J} M_{qq'} & & \vdots & \vdots \\ & & & M_{qQ} & M_{qg} \\ \hline M_{gq} & \cdots & M_{gq} & M_{gQ} & M_{gg} \end{array}\right)$$

$$\frac{2}{\tilde{\beta}(\mu)} R_{gg} = \frac{2}{\tilde{\beta}(\mu_h)},$$

$$R_{qg} - \frac{2}{\tilde{\beta}(\mu)}[1-\gamma_m(\mu)] R_{gg} = -\frac{2}{\tilde{\beta}(\mu_h)}[1-\gamma_m(\mu_h)]$$

$$M_{qq} \equiv 1, \qquad M_{qq'} \equiv 0, \qquad M_{gq} \equiv 0,$$

$$M_{gg} = \frac{\tilde{\beta}^{(n_f)}}{\tilde{\beta}^{(n_f+1)}} - \frac{2}{\tilde{\beta}^{(n_f+1)}}[1-\gamma_m^{(n_f+1)}] M_{gQ},$$

$$M_{gq} = \frac{2}{\tilde{\beta}^{(n_f+1)}}[\gamma_m^{(n_f+1)} - \gamma_m^{(n_f)}] - \frac{2}{\tilde{\beta}^{(n_f+1)}}[1-\gamma_m^{(n_f+1)}] M_{qQ}$$

Sum rule constraints on scalar matrix elements

Reduces dominant theoretical uncertainty, which comes from $\alpha_s(\mu_c)$

For heavy WIMP scattering this is an O(50-70%) reductions,
and the remaining uncertainty comes from $\alpha_s(\mu_t)$,
requiring higher order matching at the weak scale.

Equivalently, we have the best perturbative QCD
estimate of the charm scalar matrix element.

$$
\begin{aligned}
f_{c,N}^{(0)\prime} &= 0.083 - 0.103\lambda + \mathcal{O}(\alpha_s^4, 1/m_c) \\
&= 0.073(3) + \mathcal{O}(\alpha_s^4, 1/m_c), \\
f_{q,N}^{(0)\prime} &= f_{q,N}^{(0)} + \mathcal{O}(1/m_c),
\end{aligned}
$$

$$
f_{c,N}^{(0)\prime} = \begin{cases} 0.10(3) \\ 0.07(3) \end{cases}
$$

Hadronic matrix elements: vector, axial-vector, antisymmetric tensor

$$\langle N(k')|V_\mu^{(q)}|N(k)\rangle \equiv \bar{u}(k')\left[F_1^{(N,q)}(q^2)\gamma_\mu + \frac{i}{2m_N}F_2^{(N,q)}(q^2)\sigma_{\mu\nu}q^\nu\right]u(k)$$

q	$F_1^{(p,q)}(0)$	$F_2^{(p,q)}(0)$	$F_2^{(p,q)}(0)$
u	2	1.62(2)	1.65(7)
d	1	−2.08(2)	−2.05(7)
s	0	−0.046(19)	−0.017(74)

quark content magnetic moment

$$\langle N(k')|A_\mu^{(q)}|N(k)\rangle \equiv \bar{u}^{(N)}(k')\left[F_A^{(N,q)}(q^2)\gamma_\mu\gamma_5 + \frac{1}{2m_N}F_{P'}^{(N,q)}(q^2)\gamma_5 q_\mu\right]u^{(N)}(k)$$

μ (GeV)	$F_A^{(p,u)}(0)$	$F_A^{(p,d)}(0)$	$F_A^{(p,s)}(0)$	Reference
1–2	0.75(8)	−0.51(8)	−0.15(8)	[59]
1	0.80(3)	−0.46(4)	−0.12(8)	[60]
2	0.79(5)	−0.46(5)	−0.13(10)	[60]

semileptonic decay and νp scattering
polarized DIS

$$\frac{E_k}{m_N}\langle N(k)|T_{\mu\nu}^{(q)}|N(k)\rangle \equiv \frac{2}{m_N}s^{[\mu}k^{\nu]}m_q(\mu)t_{q,N}(\mu)$$

μ (GeV)	$t_{u,p}(\mu)$	$t_{d,p}(\mu)$	$t_{s,p}(\mu)$	Reference
…	4/3	−1/3	0	…
1	0.88(6)	−0.24(5)	−0.05(3)	…
1.4	0.84(6)	−0.23(5)	−0.05(3)	[63]
2	0.81(6)	−0.22(5)	−0.05(3)	…

(polarized DIS), NR quark model, lattice

Hadronic matrix elements: scalar and pseudoscalar

$$\frac{E_{\boldsymbol{k}}}{m_N}\langle N(k)|O_q^{(0)}|N(k)\rangle \equiv m_N f_{q,N}^{(0)},$$

$$\frac{-9\alpha_s(\mu)}{8\pi}\frac{E_{\boldsymbol{k}}}{m_N}\langle N(k)|O_g^{(0)}(\mu)|N(k)\rangle \equiv m_N f_{g,N}^{(0)}(\mu).$$

$$f_{u,N}^{(0)} = \frac{R_{ud}}{1+R_{ud}}\frac{\Sigma_{\pi N}}{m_N}(1+\xi),$$

$$f_{d,N}^{(0)} = \frac{1}{1+R_{ud}}\frac{\Sigma_{\pi N}}{m_N}(1-\xi), \qquad \xi = \frac{1+R_{ud}}{1-R_{ud}}\frac{\Sigma_-}{2\Sigma_{\pi N}},$$

$$\Sigma_{\pi N} = \frac{m_u+m_d}{2}\langle N|(\bar{u}u+\bar{d}d)|N\rangle = 44(13)\ \text{MeV},$$

$$\Sigma_- = (m_d-m_u)\langle N|(\bar{u}u-\bar{d}d)|N\rangle = \pm 2(2)\ \text{MeV},$$

$$\left[\Sigma_- = \pm 2(1)\,\text{MeV}\right]$$

q	$f_{q,p}^{(0)}$	$f_{q,n}^{(0)}$
u	0.016(5)(3)(1)	$0.014(5)\binom{+2}{-3}(1)$
d	0.029(9)(3)(2)	$0.034(9)\binom{+3}{-2}(2)$
s	0.043(21)	0.043(21)

lattice

Lattice determination of charm is interesting, and would assess impact of power corrections

$$\langle N(k')|O_{5q}^{(0)}|N(k)\rangle \equiv m_N f_{5q,N}^{(0)}(q^2)\bar{u}(k')i\gamma_5 u(k),$$

$$\langle N(k')|O_{5g}^{(0)}|N(k)\rangle \equiv m_N f_{5g,N}^{(0)}(q^2,\mu)\bar{u}(k')i\gamma_5 u(k),$$

$$\sum_q \partial_\mu A_q^\mu = \sum_q 2im_q\bar{q}\gamma_5 q - \frac{g^2 n_f}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}G^a_{\mu\nu}G^a_{\rho\sigma},$$

$$\sum_{q=u,d,s}\langle N(k')|\bar{q}i\gamma_5 q|N(k)\rangle \equiv \kappa(q^2,\mu)\bar{u}(k')i\gamma_5 u(k)$$

q	$f_{5q,p}^{(0)}$	Reference [79]	$f_{5q,n}^{(0)}$	Reference [79]
u	0.42(8)(1)	0.43	−0.41(8)(1)	−0.42
d	−0.84(8)(3)	−0.84	0.85(8)(3)	0.85
s	−0.48(8)(1)(3)	−0.50	−0.06(8)(1)(3)	−0.08

recent confusion in the literature studying simplified models for the galactic excess: 1406.5542, 1404.0022, …

Hadronic matrix elements: CP-even and CP-odd tensors

$$\frac{E_{\boldsymbol{k}}}{m_N}\langle N(k)|O_q^{(2)\mu\nu}(\mu)|N(k)\rangle \equiv \frac{1}{m_N}\left(k^\mu k^\nu - \frac{g^{\mu\nu}}{4}m_N^2\right) f_{q,N}^{(2)}(\mu)$$

$$\frac{E_{\boldsymbol{k}}}{m_N}\langle N(k)|O_g^{(2)\mu\nu}(\mu)|N(k)\rangle \equiv \frac{1}{m_N}\left(k^\mu k^\nu - \frac{g^{\mu\nu}}{4}m_N^2\right) f_{g,N}^{(2)}(\mu)$$

μ (GeV)	$f_{u,p}^{(2)}(\mu)$	$f_{d,p}^{(2)}(\mu)$	$f_{s,p}^{(2)}(\mu)$	$f_{c,p}^{(2)}(\mu)$	$f_{b,p}^{(2)}(\mu)$	$f_{g,p}^{(2)}(\mu)$
1	0.404(9)	0.217(8)	0.024(4)	…	…	0.356(29)
1.2	0.383(8)	0.208(8)	0.027(4)	…	…	0.381(25)
1.4	0.370(8)	0.202(7)	0.030(4)	…	…	0.398(23)
2	0.346(7)	0.192(6)	0.034(3)	…	…	0.419(19)
$80.4/\sqrt{2}$	0.260(4)	0.158(4)	0.053(2)	0.036(1)	0.0219(4)	0.470(8)
100	0.253(4)	0.156(4)	0.055(2)	0.038(1)	0.0246(5)	0.472(8)
$172\sqrt{2}$	0.244(4)	0.152(3)	0.057(2)	0.042(1)	0.028(1)	0.476(7)

PDFs from unpolarized DIS

$$\frac{E_{\boldsymbol{k}}}{m_N}\langle N(k)|O_{5q}^{(2)\mu\nu}(\mu)|N(k)\rangle \equiv s^{\{\mu}k^{\nu\}} f_{5q,N}^{(2)}(\mu)$$

μ (GeV)	$f_{5u,p}^{(2)}(\mu)$	$f_{5d,p}^{(2)}(\mu)$	$f_{5s,p}^{(2)}(\mu)$
1	0.186(7)	−0.069(8)	−0.007(6)
1.2	0.175(6)	−0.065(7)	−0.006(6)
1.4	0.167(6)	−0.062(7)	−0.006(5)
2	0.154(5)	−0.056(6)	−0.005(5)

PDFs from polarized DIS

Nucleon level effective theory and relativistic invariance

$$
\begin{aligned}
\mathcal{L}_{N\chi,PT} = {} & \frac{1}{m_N^2}\{d_1 N^\dagger \sigma^i N \chi^\dagger \sigma^i \chi + d_2 N^\dagger N \chi^\dagger \chi\} + \frac{1}{m_N^4}\{d_3 N^\dagger \partial_+^i N \chi^\dagger \partial_+^i \chi + d_4 N^\dagger \partial_-^i N \chi^\dagger \partial_-^i \chi \\
& + d_5 N^\dagger (\boldsymbol{\partial}^2 + \overleftarrow{\boldsymbol{\partial}}^2) N \chi^\dagger \chi + d_6 N^\dagger N \chi^\dagger (\boldsymbol{\partial}^2 + \overleftarrow{\boldsymbol{\partial}}^2)\chi + i d_8 \epsilon^{ijk} N^\dagger \sigma^i \partial_-^j N \chi^\dagger \partial_+^k \chi \\
& + i d_9 \epsilon^{ijk} N^\dagger \sigma^i \partial_+^j N \chi^\dagger \partial_-^k \chi + i d_{11} \epsilon^{ijk} N^\dagger \partial_+^k N \chi^\dagger \sigma^i \partial_-^j \chi + i d_{12} \epsilon^{ijk} N^\dagger \partial_-^k N \chi^\dagger \sigma^i \partial_+^j \chi \\
& + d_{13} N^\dagger \sigma^i \partial_+^j N \chi^\dagger \sigma^i \partial_+^j \chi + d_{14} N^\dagger \sigma^i \partial_-^j N \chi^\dagger \sigma^i \partial_-^j \chi + d_{15} N^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\partial}_+ N \chi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\partial}_+ \chi \\
& + d_{16} N^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\partial}_- N \chi^\dagger \boldsymbol{\sigma} \cdot \boldsymbol{\partial}_- \chi + d_{17} N^\dagger \sigma^i \partial_-^j N \chi^\dagger \sigma^j \partial_-^i \chi \\
& + d_{18} N^\dagger \sigma^i (\boldsymbol{\partial}^2 + \overleftarrow{\boldsymbol{\partial}}^2) N \chi^\dagger \sigma^i \chi + d_{19} N^\dagger \sigma^i (\partial^i \partial^j + \overleftarrow{\partial}^j \overleftarrow{\partial}^i) N \chi^\dagger \sigma^j \chi \\
& + d_{20} N^\dagger \sigma^i N \chi^\dagger \sigma^i (\boldsymbol{\partial}^2 + \overleftarrow{\boldsymbol{\partial}}^2)\chi + d_{21} N^\dagger \sigma^i N \chi^\dagger \sigma^j (\partial^i \partial^j + \overleftarrow{\partial}^j \overleftarrow{\partial}^i)\chi\} + \mathcal{O}(1/m_N^6),
\end{aligned}
$$

d's can be matched from NR limit of form factors

$$
\begin{aligned}
u_\mu V_q^\mu = {} & [F_1^{(q)}(0)]\bar{N}_u N_u + \frac{1}{m_N^2}\left\{\left[-\frac{1}{8}F_1^{(q)}(0) - m_N^2 F_1^{(q)\prime}(0) - \frac{1}{4}F_2^{(q)}(0)\right]\partial_\perp^2(\bar{N}_u N_u)\right. \\
& \left. + \left[-\frac{1}{4}F_1^{(q)}(0) - \frac{1}{2}F_2^{(q)}(0)\right] i\bar{N}_u \partial_\perp^\mu \overleftarrow{\partial}_\perp^\nu \sigma_{\perp\mu\nu} N_u\right\} + \mathcal{O}(1/m_N^4), \\
V_{q\perp}^\mu = {} & \frac{1}{m_N}\left\{\left[\frac{1}{2}F_1^{(q)}(0)\right] i\bar{N}_u \overleftrightarrow{\partial}_\perp^\mu N_u + \left[\frac{1}{2}F_1^{(q)}(0) + \frac{1}{2}F_2^{(q)}(0)\right]\partial_{\perp\nu}(\bar{N}_u \sigma_\perp^{\mu\nu} N_u)\right\} + \mathcal{O}(1/m_N^3), \\
u_\mu A_q^\mu = {} & \frac{1}{m_N}\left\{\left[-\frac{1}{4}F_A^{(q)}(0)\right] i\epsilon^{\mu\nu\rho\sigma} u_\mu \bar{N}_u \overleftrightarrow{\partial}_{\perp\nu} \sigma_{\perp\rho\sigma} N_u\right\} + \mathcal{O}(1/m_N^3), \\
A_{q\perp}^\mu = {} & \left[-\frac{1}{2}F_A^{(q)}(0)\right]\epsilon^{\mu\nu\rho\sigma} u_\nu \bar{N}_u \sigma_{\perp\rho\sigma} N_u \\
& + \frac{1}{m_N^2}\left\{\left[\frac{1}{8}F_A^{(q)}(0) + m_N^2 F_A^{(q)\prime}(0)\right]\epsilon^{\mu\nu\rho\sigma} u_\nu \bar{N}_u \overleftarrow{\partial}_\perp^\alpha \partial_{\perp\alpha} \sigma_{\perp\rho\sigma} N_u\right. \\
& + \left[-\frac{1}{16}F_A^{(q)}(0) + \frac{1}{2}m_N^2 F_A^{(q)\prime}(0)\right]\epsilon^{\mu\nu\rho\sigma} u_\nu \bar{N}_u (\overleftarrow{\partial}^2 + \partial_\perp^2)\sigma_{\perp\rho\sigma} N_u \\
& + \left[-\frac{1}{8}F_{P'}^{(q)}(0)\right]\epsilon_{\alpha\beta\gamma\delta} u^\gamma \bar{N}_u (\partial_\perp^\mu \partial_\perp^\alpha + \overleftarrow{\partial}_\perp^\mu \overleftarrow{\partial}_\perp^\alpha)\sigma_\perp^{\beta\delta} N_u \\
& + \left[-\frac{1}{8}F_A^{(q)}(0) - \frac{1}{8}F_{P'}^{(q)}(0)\right]\epsilon_{\alpha\beta\gamma\delta} u^\gamma \bar{N}_u (\partial_\perp^\mu \overleftarrow{\partial}_\perp^\alpha + \overleftarrow{\partial}_\perp^\mu \partial_\perp^\alpha)\sigma_\perp^{\beta\delta} N_u \\
& \left. + \left[-\frac{1}{4}F_A^{(q)}(0)\right] i\epsilon^{\mu\nu\alpha\beta} u_\nu \bar{N}_u \partial_{\perp\alpha} \overleftarrow{\partial}_{\perp\beta} N_u\right\} + \mathcal{O}(1/m_N^4),
\end{aligned}
$$

$$
\begin{aligned}
T_q^{\mu\nu} &= m_N\left[\left(\frac{m_q t_q}{m_N}\right)\epsilon^{\alpha\beta\gamma[\mu}u^{\nu]}u_\alpha \bar{N}\sigma_{\beta\gamma}^\perp N + \mathcal{O}(1/m_N^2)\right] \\
O_q^{(0)} &= m_N[f_q^{(0)}\bar{N}_u N_u + \mathcal{O}(1/m_N^2)], \\
O_g^{(0)} &= m_N\left[\left(\frac{-8\pi}{9\alpha_s}\right)f_g^{(0)}\bar{N}_u N_u + \mathcal{O}(1/m_N^2)\right], \\
O_{5q,5g}^{(0)} &= \frac{1}{4}f_{5q,5g}^{(0)}\epsilon^{\mu\nu\rho\sigma}u_\mu \partial_{\perp\nu}(\bar{N}\sigma_{\rho\sigma}^\perp N) + \mathcal{O}(1/m_N^2), \\
u_\mu u_\nu O_{q,g}^{(2)\mu\nu} &= m_N\left[\frac{3}{4}f_{q,g}^{(2)}\bar{N}_u N_u + \mathcal{O}(1/m_N^2)\right], \\
O_{5q}^{(2)\mu\nu} &= m_N\left[\frac{1}{2}f_{5q}^{(2)}\epsilon^{\alpha\beta\gamma\{\mu}u^{\nu\}}u_\alpha \bar{N}\sigma_{\beta\gamma}^\perp N + \mathcal{O}(1/m_N^2)\right],
\end{aligned}
$$

Nucleon level effective theory and relativistic invariance

$$\begin{aligned}\mathcal{L}_{N\chi,PT} = \frac{1}{m_N^2}\{d_1 N^\dagger \sigma^i N \chi^\dagger \sigma^i \chi + d_2 N^\dagger N \chi^\dagger \chi\} + \frac{1}{m_N^4}\{d_3 N^\dagger \partial_+^i N \chi^\dagger \partial_+^i \chi + d_4 N^\dagger \partial_-^i N \chi^\dagger \partial_-^i \chi \\ + d_5 N^\dagger (\boldsymbol{\partial}^2 + \overleftarrow{\boldsymbol{\partial}}^2) N \chi^\dagger \chi + d_6 N^\dagger N \chi^\dagger (\boldsymbol{\partial}^2 + \overleftarrow{\boldsymbol{\partial}}^2)\chi + i d_8 \epsilon^{ijk} N^\dagger \sigma^i \partial_-^j N \chi^\dagger \partial_+^k \chi \\ + i d_9 \epsilon^{ijk} N^\dagger \sigma^i \partial_+^j N \chi^\dagger \partial_-^k \chi + i d_{11} \epsilon^{ijk} N^\dagger \partial_+^k N \chi^\dagger \sigma^i \partial_-^j \chi + i d_{12} \epsilon^{ijk} N^\dagger \partial_-^k N \chi^\dagger \sigma^i \partial_+^j \chi \\ + d_{13} N^\dagger \sigma^i \partial_+^j N \chi^\dagger \sigma^i \partial_+^j \chi + d_{14} N^\dagger \sigma^i \partial_-^j N \chi^\dagger \sigma^i \partial_-^j \chi + d_{15} N^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\partial}_+ N \chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\partial}_+ \chi \\ + d_{16} N^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\partial}_- N \chi^\dagger \boldsymbol{\sigma}\cdot\boldsymbol{\partial}_- \chi + d_{17} N^\dagger \sigma^i \partial_-^j N \chi^\dagger \sigma^j \partial_-^i \chi \\ + d_{18} N^\dagger \sigma^i (\boldsymbol{\partial}^2 + \overleftarrow{\boldsymbol{\partial}}^2) N \chi^\dagger \sigma^i \chi + d_{19} N^\dagger \sigma^i (\partial^i \partial^j + \overleftarrow{\partial}^j \overleftarrow{\partial}^i) N \chi^\dagger \sigma^j \chi \\ + d_{20} N^\dagger \sigma^i N \chi^\dagger \sigma^i (\boldsymbol{\partial}^2 + \overleftarrow{\boldsymbol{\partial}}^2)\chi + d_{21} N^\dagger \sigma^i N \chi^\dagger \sigma^j (\partial^i \partial^j + \overleftarrow{\partial}^j \overleftarrow{\partial}^i)\chi\} + \mathcal{O}(1/m_N^6),\end{aligned}$$

d's can be matched from NR limit of form factors

impose Lorentz symmetry

$$N \to e^{i m_N \boldsymbol{\eta}\cdot\boldsymbol{x}}\left[1 - \frac{i\boldsymbol{\eta}\cdot\boldsymbol{\partial}}{2m_N} + \frac{\boldsymbol{\sigma}\times\boldsymbol{\eta}\cdot\boldsymbol{\partial}}{4m_N} + \dots\right]N, \qquad \chi \to e^{i m_\chi \boldsymbol{\eta}\cdot\boldsymbol{x}}\left[1 - \frac{i\boldsymbol{\eta}\cdot\boldsymbol{\partial}}{2m_\chi} + \frac{\boldsymbol{\sigma}\times\boldsymbol{\eta}\cdot\boldsymbol{\partial}}{4m_\chi} + \dots\right]\chi$$

$$\partial_t \to \partial_t - \boldsymbol{\eta}\cdot\boldsymbol{\partial}, \qquad \boldsymbol{\partial} \to \boldsymbol{\partial} - \boldsymbol{\eta}\partial_t.$$

$$r d_4 + d_5 = \frac{d_2}{4}, \qquad d_5 = r^2 d_6, \qquad 8r(d_8 + r d_9) = -r d_2 + d_1, \qquad 8r(r d_{11} + d_{12}) = -d_2 + r d_1,$$

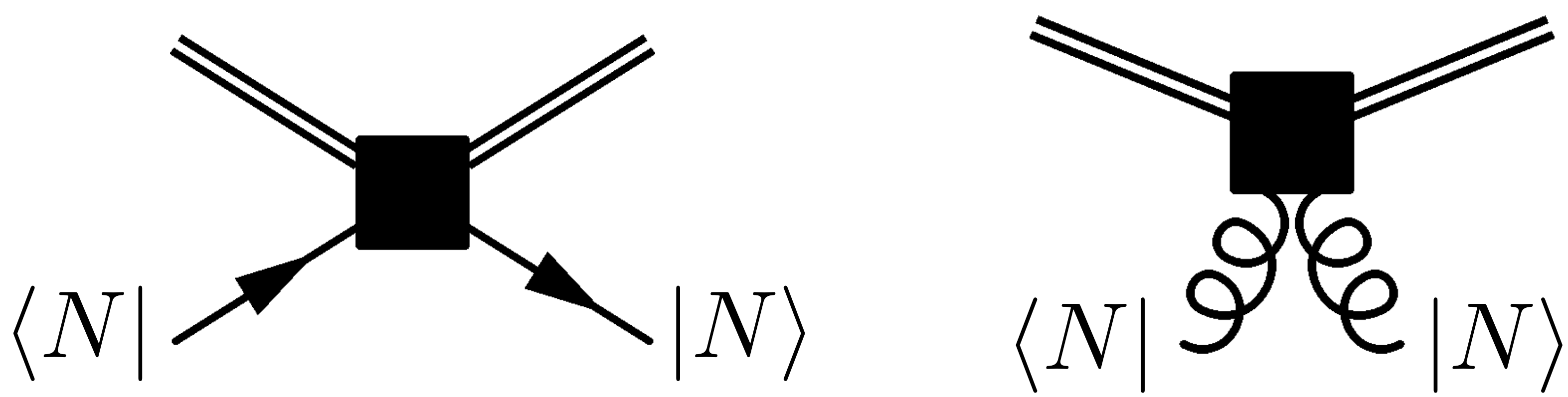
$$r d_{14} + d_{18} = \frac{d_1}{4}, \qquad d_{18} = r^2 d_{20}, \qquad 2r d_{16} + d_{19} = \frac{d_1}{4}, \qquad r(d_{16} + d_{17}) + d_{19} = 0, \qquad d_{19} = r^2 d_{21}$$

or Galilean?

$$N \to e^{i m_N \boldsymbol{\eta}\cdot\boldsymbol{x}} N, \qquad \chi \to e^{i m_\chi \boldsymbol{\eta}\cdot\boldsymbol{x}}\chi, \qquad \mathbf{v}_{\rm rel} \equiv \frac{1}{2}\left[\frac{\boldsymbol{p}+\boldsymbol{p}'}{m_N} - \frac{\boldsymbol{k}+\boldsymbol{k}'}{m_\chi}\right], \qquad \boldsymbol{q} \equiv \boldsymbol{p}' - \boldsymbol{p} = \boldsymbol{k} - \boldsymbol{k}', \qquad \boldsymbol{P} \equiv \boldsymbol{p} + \boldsymbol{k} = \boldsymbol{p}' + \boldsymbol{k}'.$$

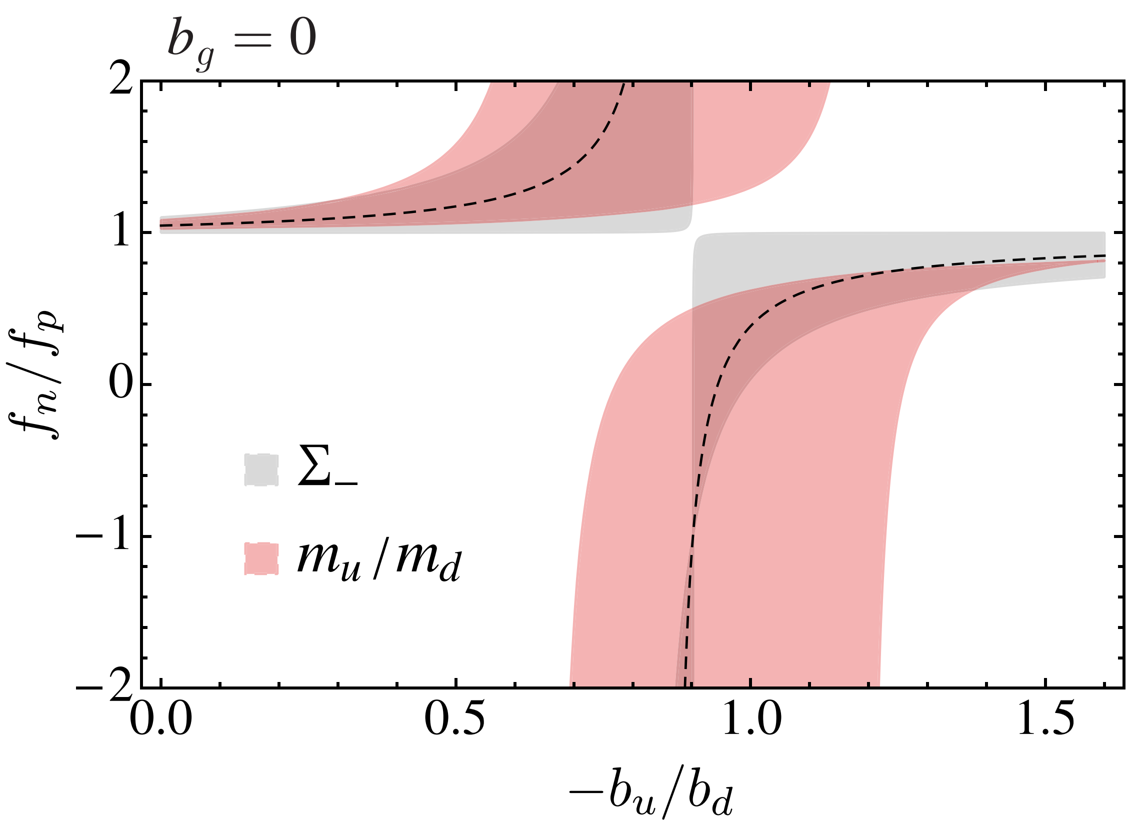
$$\partial_t \to \partial_t - \boldsymbol{\eta}\cdot\boldsymbol{\partial}, \qquad \boldsymbol{\partial} \to \boldsymbol{\partial},$$

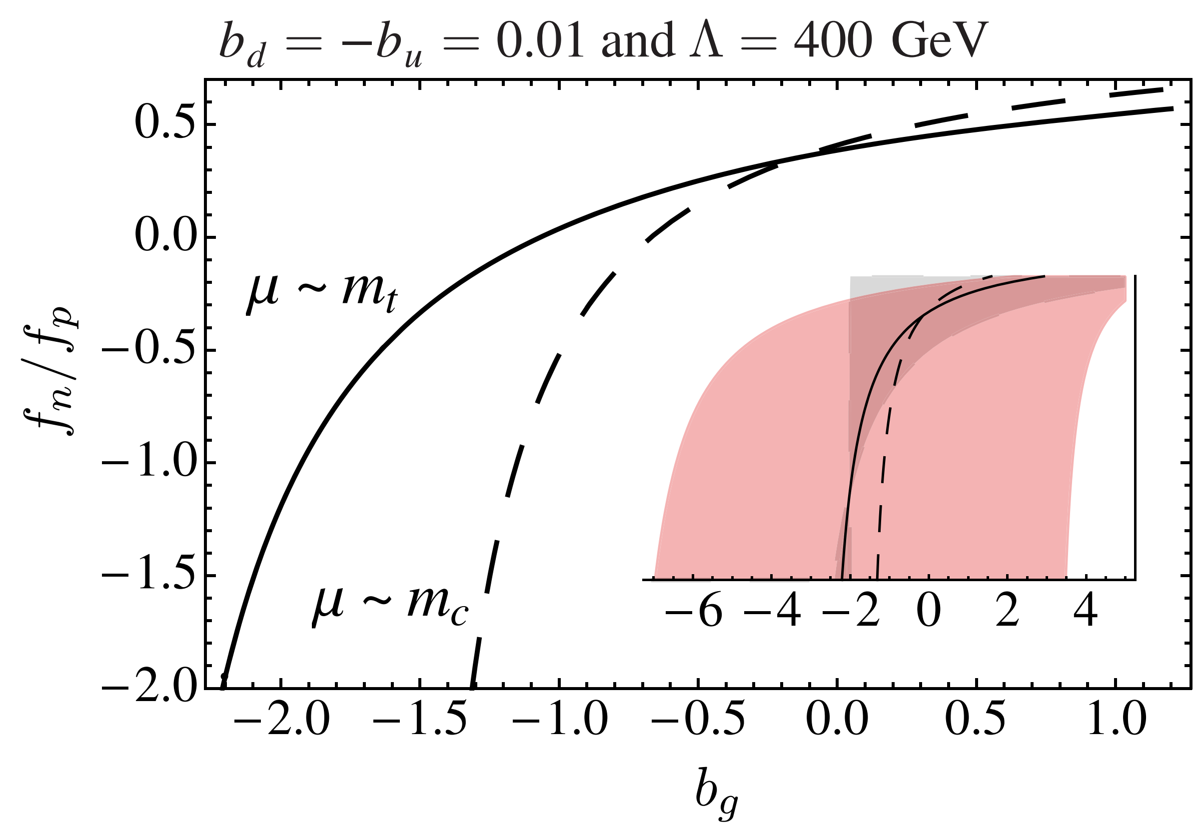
$$\mathcal{L}_{\mathrm{DM}} + \mathcal{L}_{\mathrm{SM}}$$



Example: Isospin violating dark matter

$$\mathcal{L}_{\chi,\mathrm{SM}} = \frac{1}{\Lambda^2}\bar{\chi}\chi\left[b_u\bar{u}u + b_d\bar{d}d + \frac{b_g}{\Lambda}(G^a_{\mu\nu})^2\right]$$





Meaningful predictions require both a precise knowledge of hadronic inputs and a careful treatment of renormalization effects.

Example: Heavy WIMP scattering

r

$$PV = RT\left(1+\ldots\right)$$

μ

$$\frac{\partial^2\chi}{\partial x^2} = \frac{1}{c_s^2}\frac{\partial^2\chi}{\partial t^2}$$

λ

$$c_s = c_{s,0} + c_{s,1}\frac{V_p}{V} + \ldots$$

universal gas law

M

$$\mathcal{L} = \bar{h}_v\left\{iv\cdot D + \ldots\right\}h_v$$

$iD_\mu = i\partial_\mu + g_1 Y B_\mu + g_2 W^a t^a$

m_W

$$\mathcal{L} = \bar{\chi}_v\chi_v\sum_{i,S} c_i^{(S)}\mathcal{O}_i^{(S)}$$

$$\mathcal{O}_q^{(0)} = m_q\bar{q}q \qquad \mathcal{O}_g^{(0)} = (G^A_{\mu\nu})^2$$

m_b

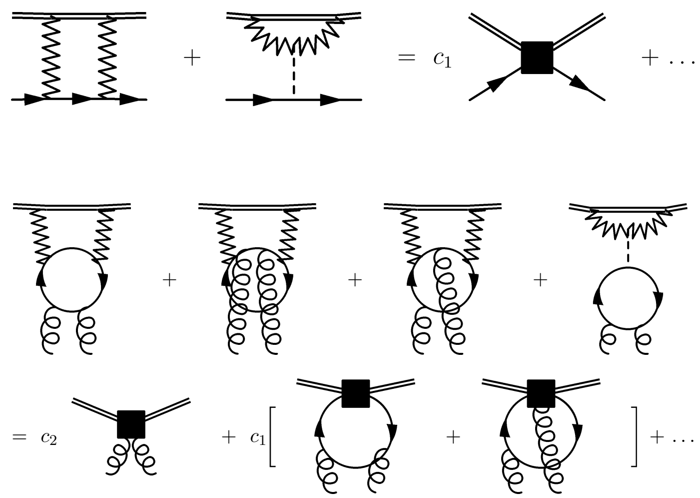
$$\mathcal{O}_q^{(2)} = v_\mu v_\nu \frac{1}{2}\bar{q}\left(\gamma^{\{\mu}iD_-^{\nu\}} - g^{\mu\nu}\mathrm{tr}\right)q$$

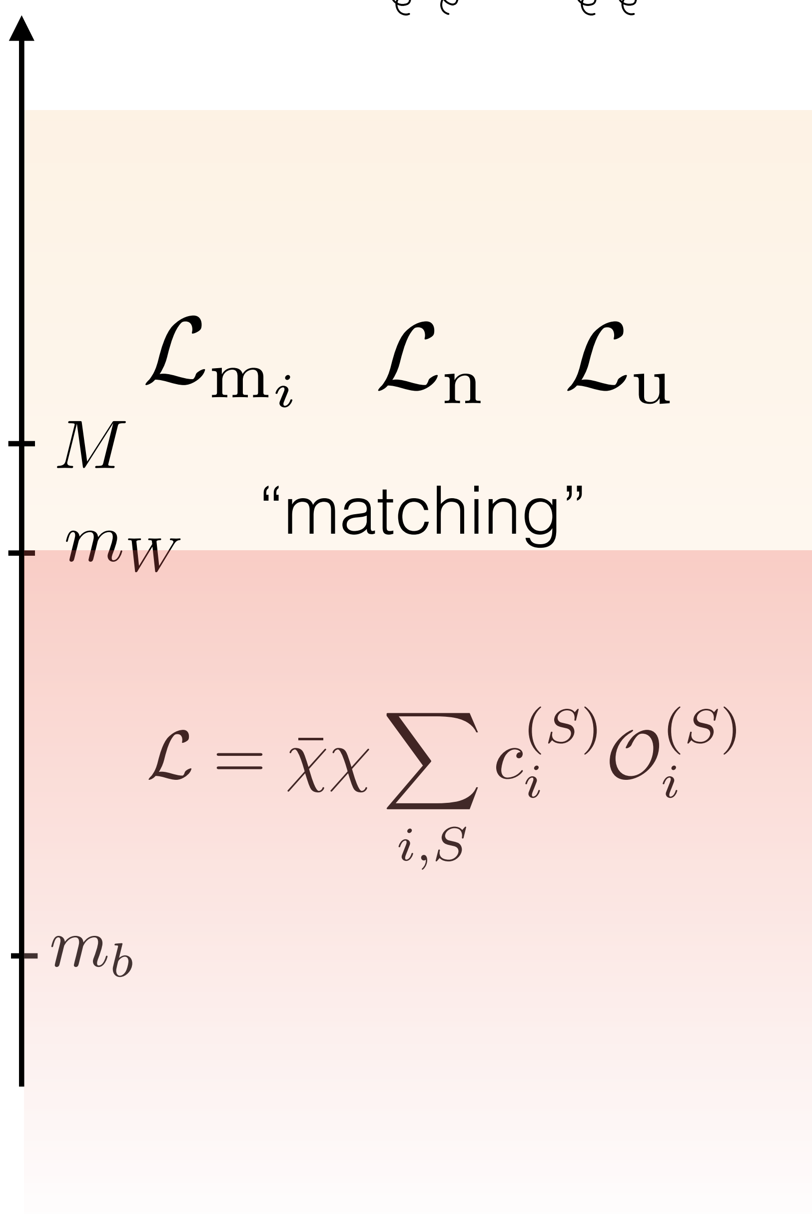
$$\mathcal{O}_g^{(2)} = v_\mu v_\nu\left(-G^{A\mu\lambda}G^{A\nu}{}_\lambda - g^{\mu\nu}\mathrm{tr}\right)$$

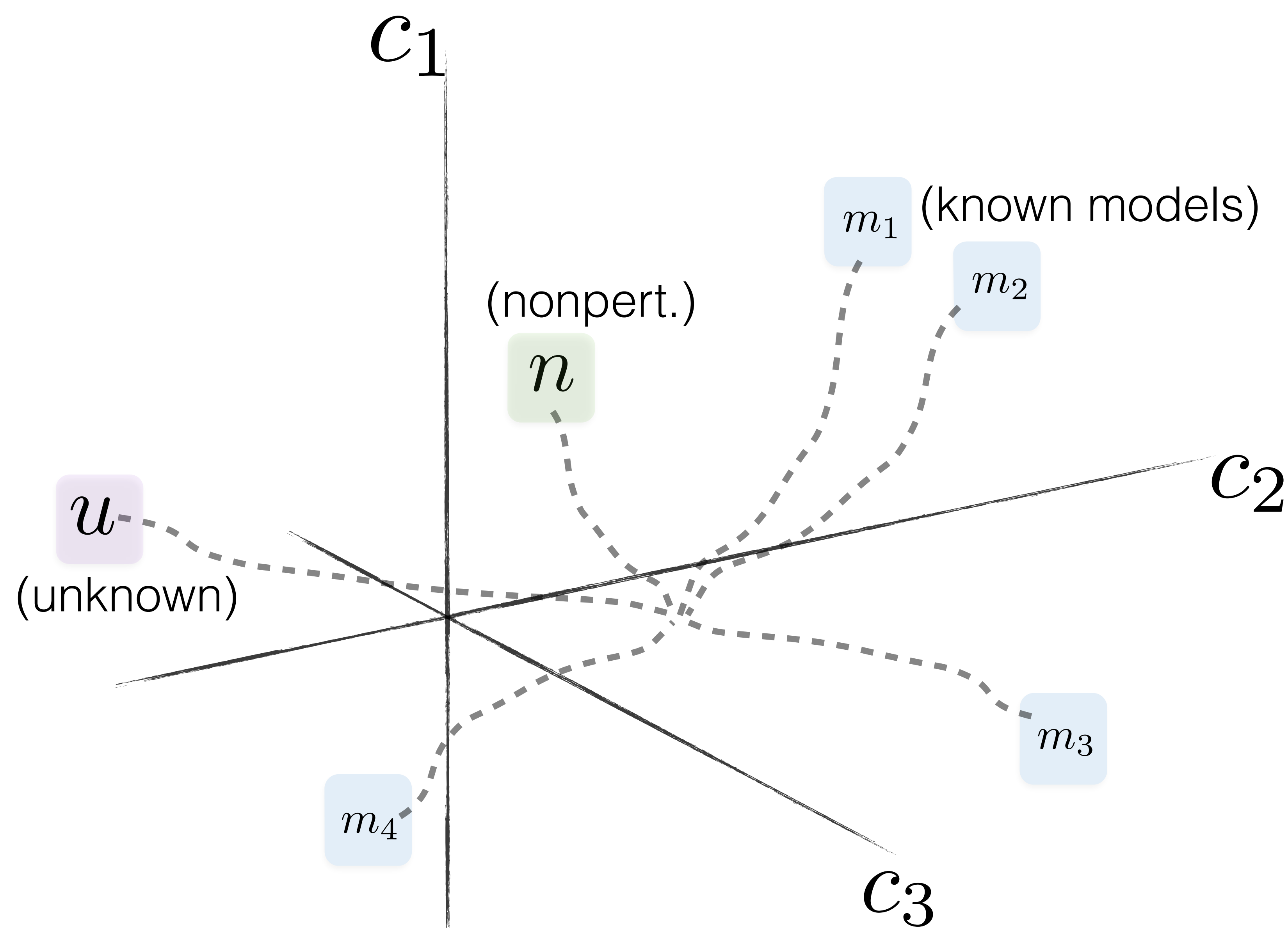
$$c_i = c_{i,0} + c_{i,1}\frac{m_W}{M} + \ldots$$

universal heavy WIMP limit

Universal heavy WIMP limit







$$c_i = c_{i,0} + c_{i,1}\frac{m_W}{M} + \dots$$

μ_t μ_b μ_c μ_0

$$\vec{c}^{(S)}_{(3)}(\mu_0) = R^{(S)}_{(3)}(\mu_0,\mu_c) M^{(S)}_{(3,4)}(\mu_c) R^{(S)}_{(4)}(\mu_c,\mu_b) M^{(S)}_{(4,5)}(\mu_b) R^{(S)}_{(5)}(\mu_b,\mu_t) \vec{c}^{(S)}_{(5)}(\mu_t)$$

J=1, Y=0

	u	d	s	c	b	g
$c^{(0)}(\mu_t,5)$	-0.407	-0.407	-0.407	-0.407	-0.424	0.004
$c^{(0)}(\mu_b,5)$	-0.418	-0.418	-0.418	-0.418	-0.436	0.009
$c^{(0)}(\mu_b,4)$	-0.418	-0.418	-0.418	-0.418	-	0.012
$c^{(0)}(\mu_c,4)$	-0.443	-0.443	-0.443	-0.443	-	0.022
$c^{(0)}(\mu_c,3)$	-0.443	-0.443	-0.443	-	-	0.028
$c^{(0)}(\mu_0,3)$	-0.458	-0.458	-0.458	-	-	0.033
$\langle N\vert c^{(0)}(\mu_0,3)O^{(0)}\vert N\rangle$ (MeV)	-8	-13	-18	-	-	-128

$$\mathcal{M}_p^{(0)} = -167\left(^{+1}_{-1}\right)\left(^{+0}_{-1}\right)\boxed{\left(^{+5}_{-14}\right)}(2)(3)(5)\ \text{MeV}$$

	u	d	s	c	b	g
$c^{(2)}(\mu_t,5)$	0.667	0.667	0.667	0.667	0.091	-0.050
$c^{(2)}(\mu_b,5)$	0.498	0.498	0.498	0.498	0.073	0.080
$c^{(2)}(\mu_b,4)$	0.498	0.498	0.498	0.498	-	0.080
$c^{(2)}(\mu_c,4)$	0.418	0.418	0.418	0.418	-	0.140
$c^{(2)}(\mu_c,3)$	0.418	0.418	0.418	-	-	0.140
$c^{(2)}(\mu_0,3)$	0.405	0.405	0.405	-	-	0.147
$\langle N\vert c^{(2)}(70,5)O^{(2)}\vert N\rangle$ (MeV)	116	71	24	17	1	-9
$\langle N\vert c^{(2)}(\mu_0,3)O^{(2)}\vert N\rangle$ (MeV)	109	59	8	-	-	40

$$\mathcal{M}_p^{(2)} = 216\boxed{\left(^{+11}_{-7}\right)}(2)(2)(1)(2)\ \text{MeV}$$

Transparency of WIMPs to nucleons

$$\sigma \sim |\mathcal{M}^{(0)} + \mathcal{M}^{(2)}|^2$$

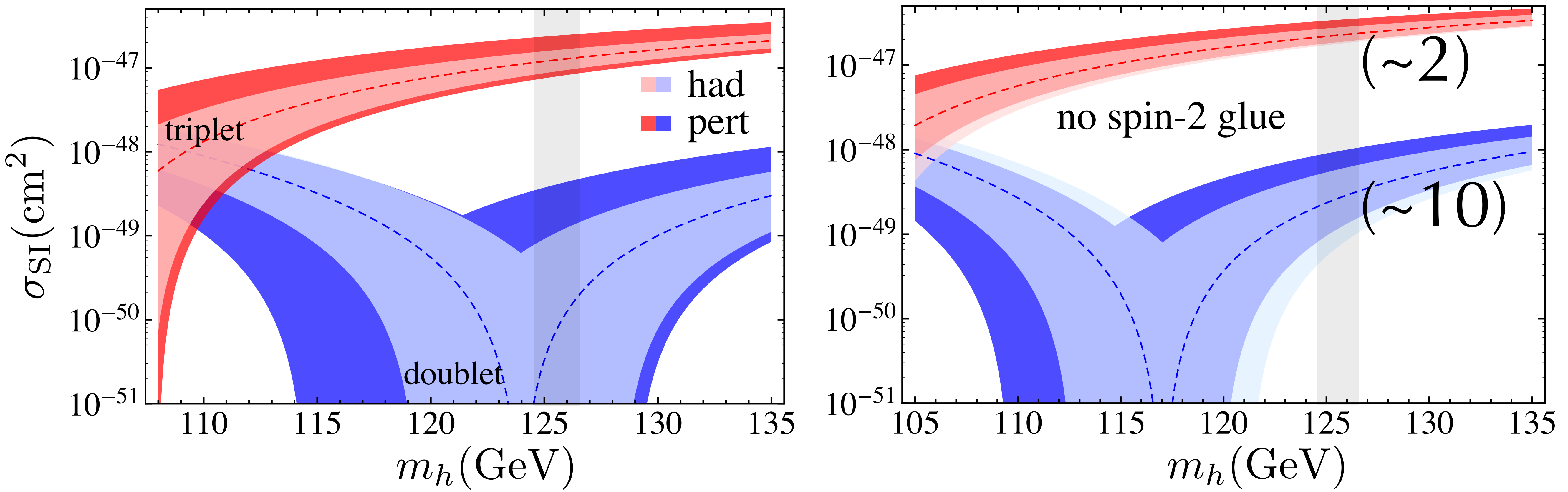
$$\mathcal{M}_p^{(0)} = -167\left({}^{+1}_{-1}\right)\left({}^{+0}_{-1}\right)\left({}^{+5}_{-14}\right)(2)(3)(5)\ \mathrm{MeV}$$

$$\mathcal{M}_p^{(2)} = 216\left({}^{+11}_{-7}\right)(2)(2)(1)(2)\ \mathrm{MeV}$$

J=1,Y=0: $\mathcal{M}_p^{(2)} + \mathcal{M}_p^{(0)} = 49\left({}^{+19}_{-10}\right)(7)\ \mathrm{MeV}$

J=1/2,Y=1/2: $\mathcal{M}_p^{(2)} + \mathcal{M}_p^{(0)} = 1.5\left({}^{+7}_{-4}\right)(3)\ \mathrm{MeV}$

Model-independent uncertainties



$$\alpha_s(\mu_t), m_W/M, m_b/m_W, \Lambda^2_{\rm QCD}/m_c^2$$

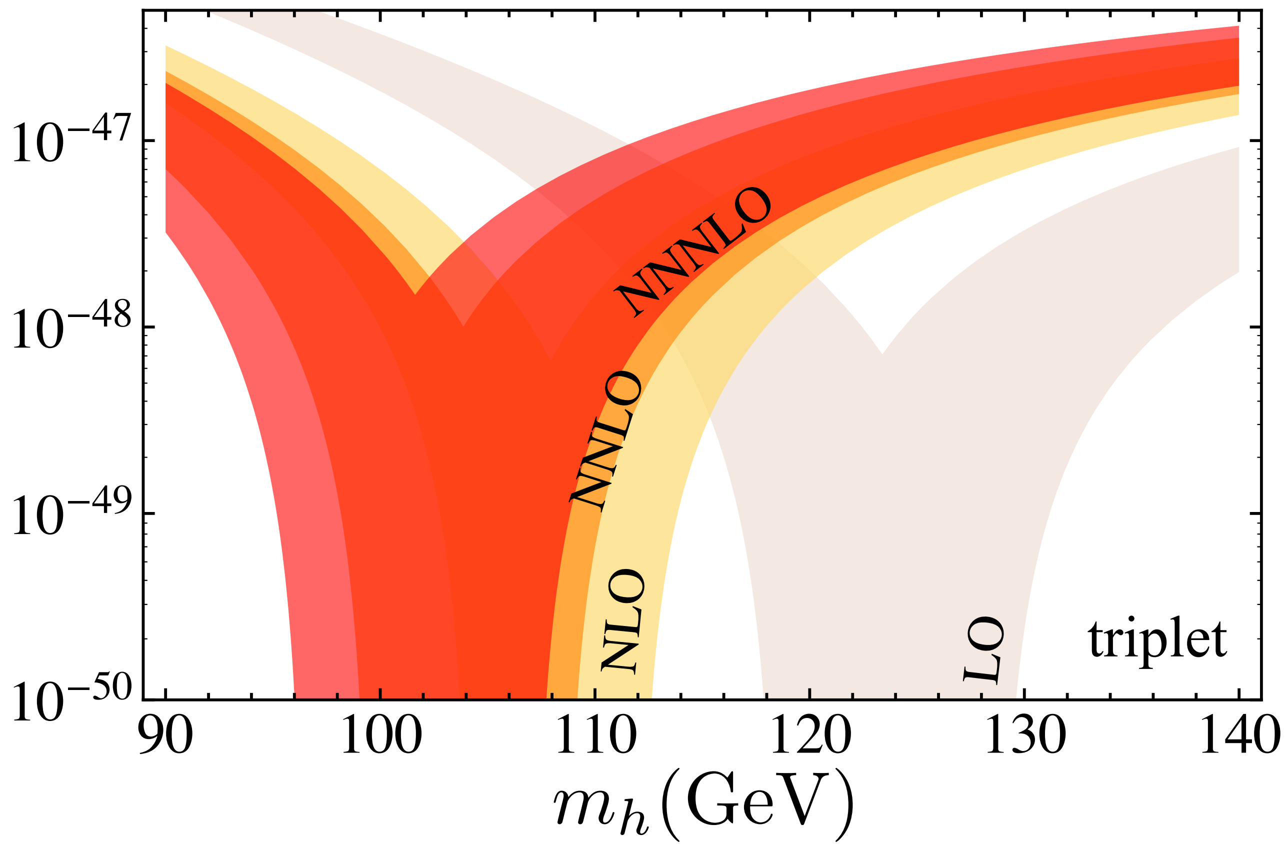
$$\boxed{\sigma_{\rm SI} = 1.3^{+1.2+0.4}_{-0.5-0.3} \times 10^{-47}\,{\rm cm}^2}$$

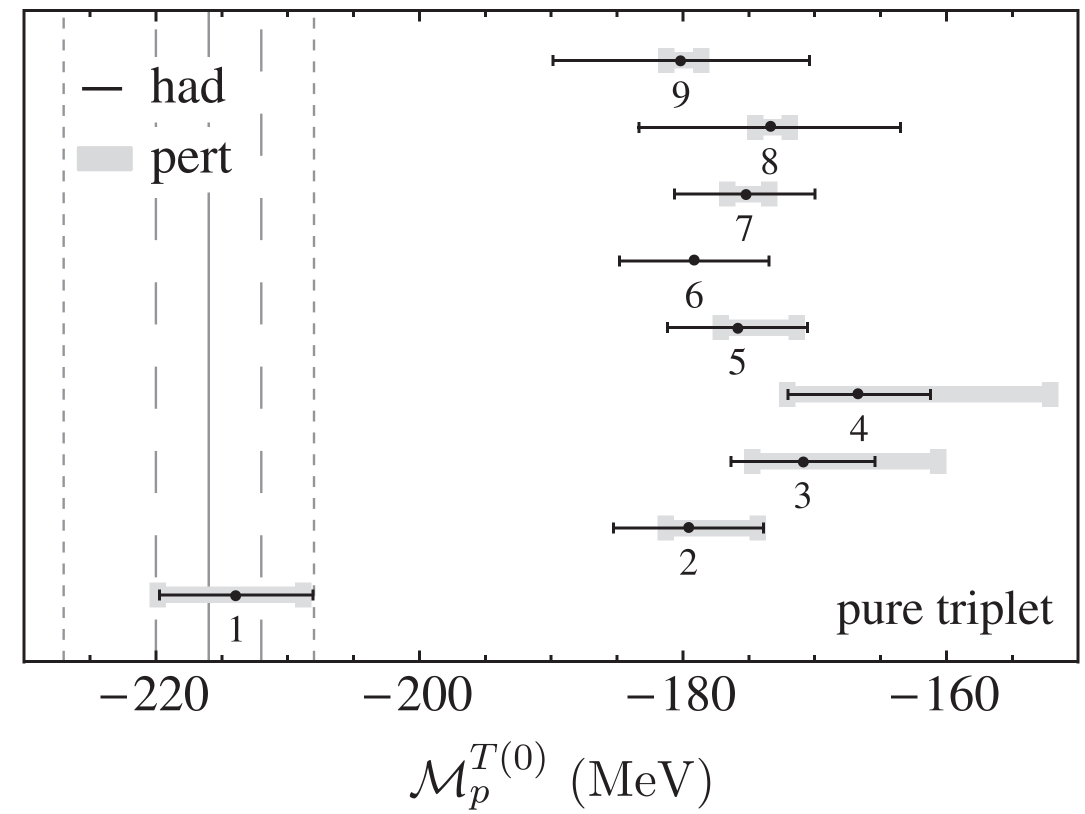
$$\boxed{\sigma_{\rm SI} \lesssim 10^{-48}\,{\rm cm}^2 \quad (95\%\,{\rm C.L.})}$$

$$\sigma_{\rm SI} \sim \frac{\alpha_2^4 m_N^4}{m_W^2}\left(\frac{1}{m_W^2}, \frac{1}{m_h^2}\right)^2 \sim 10^{-45}\,{\rm cm}^2$$

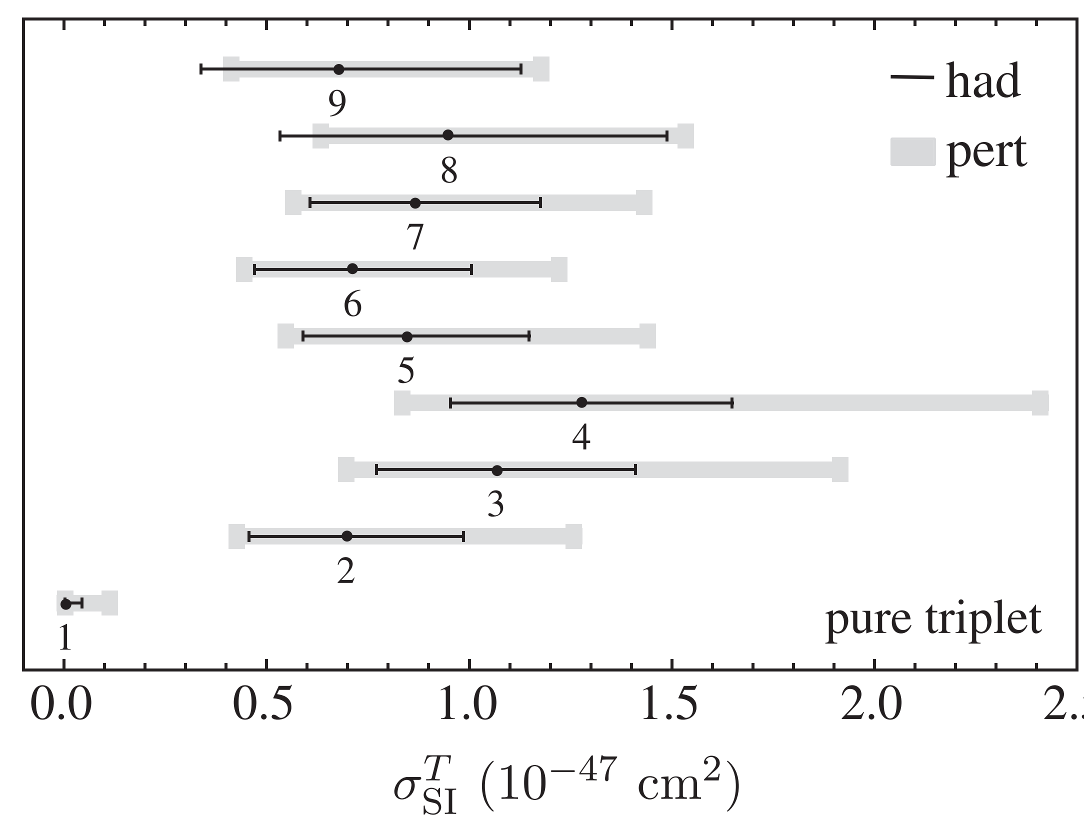
$$\sigma \approx 3\times 10^{-47}\left[1-\left(104\,{\rm GeV}/m_h\right)^2\right]^2\left[J(J+1)-\left[\frac{1+\left(104\,{\rm GeV}/m_h\right)^2}{1-\left(104\,{\rm GeV}/m_h\right)^2}\right]\frac{Y^2}{2}\right]^2$$

Model-independent uncertainties

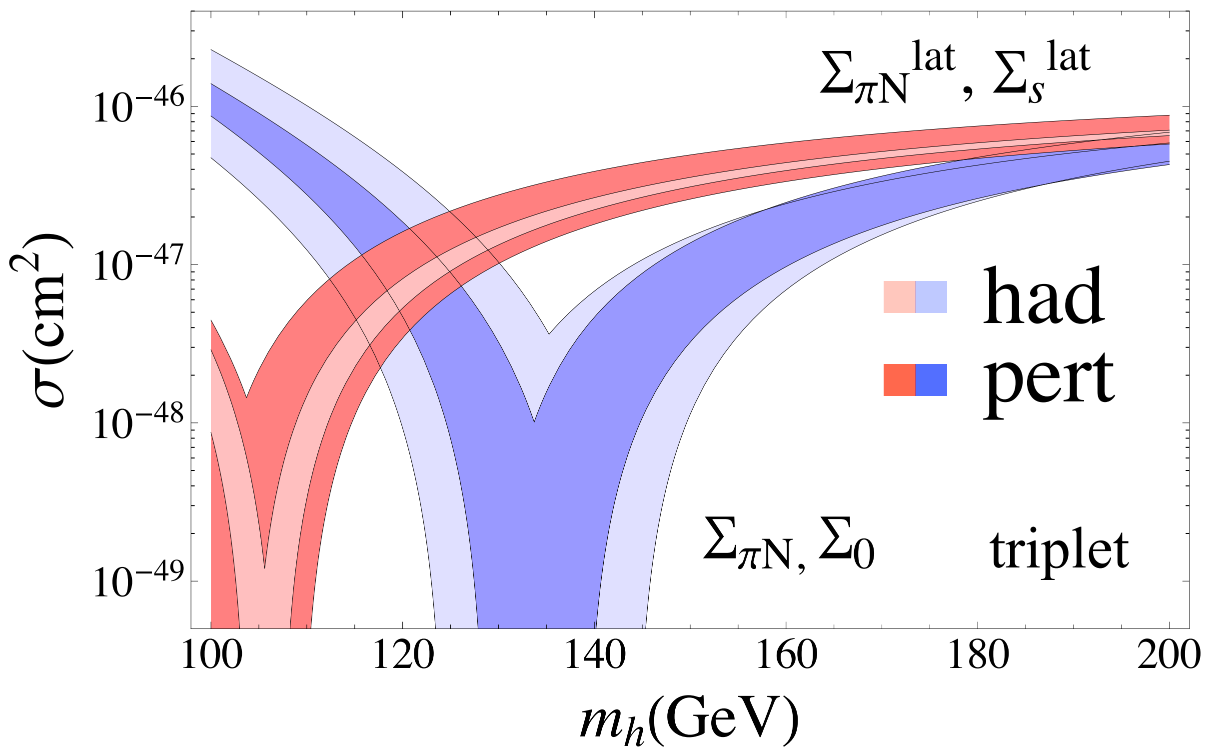


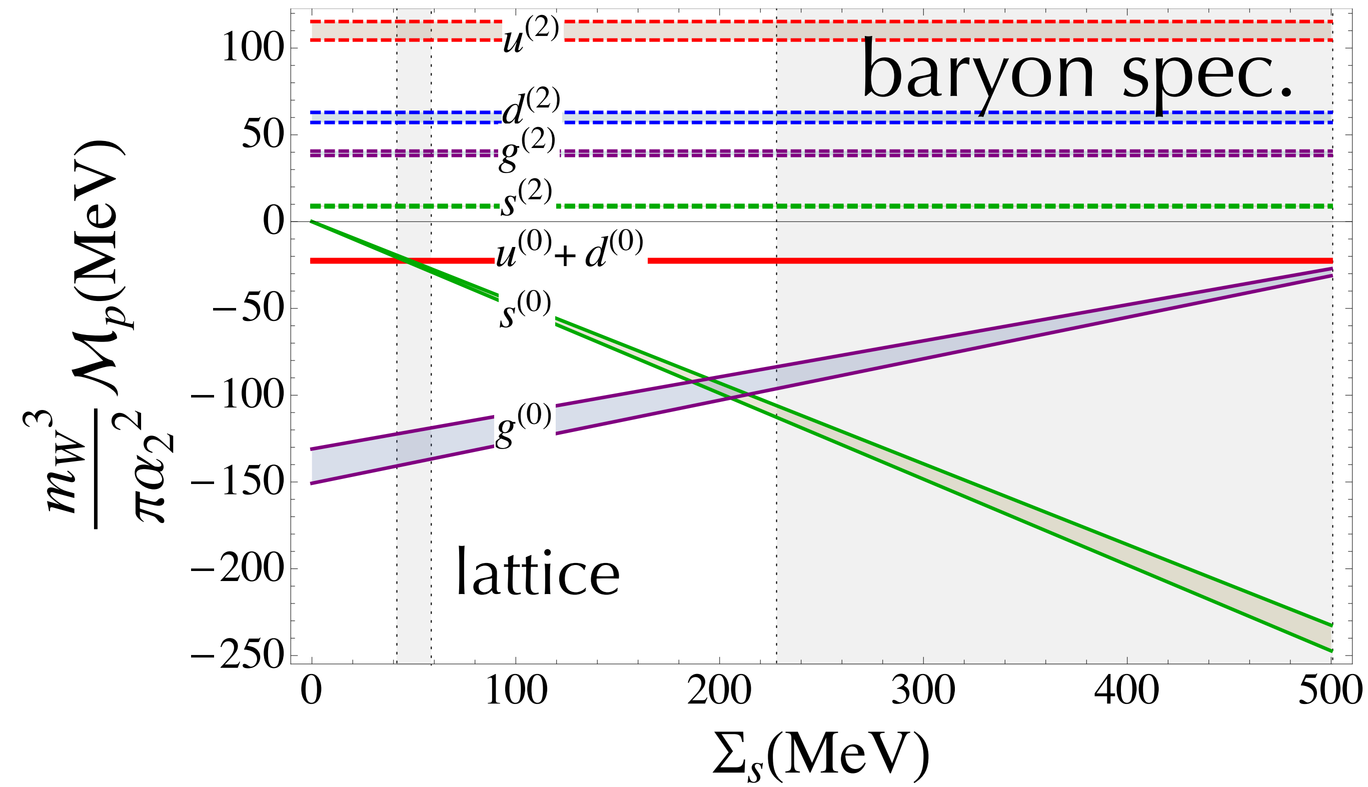


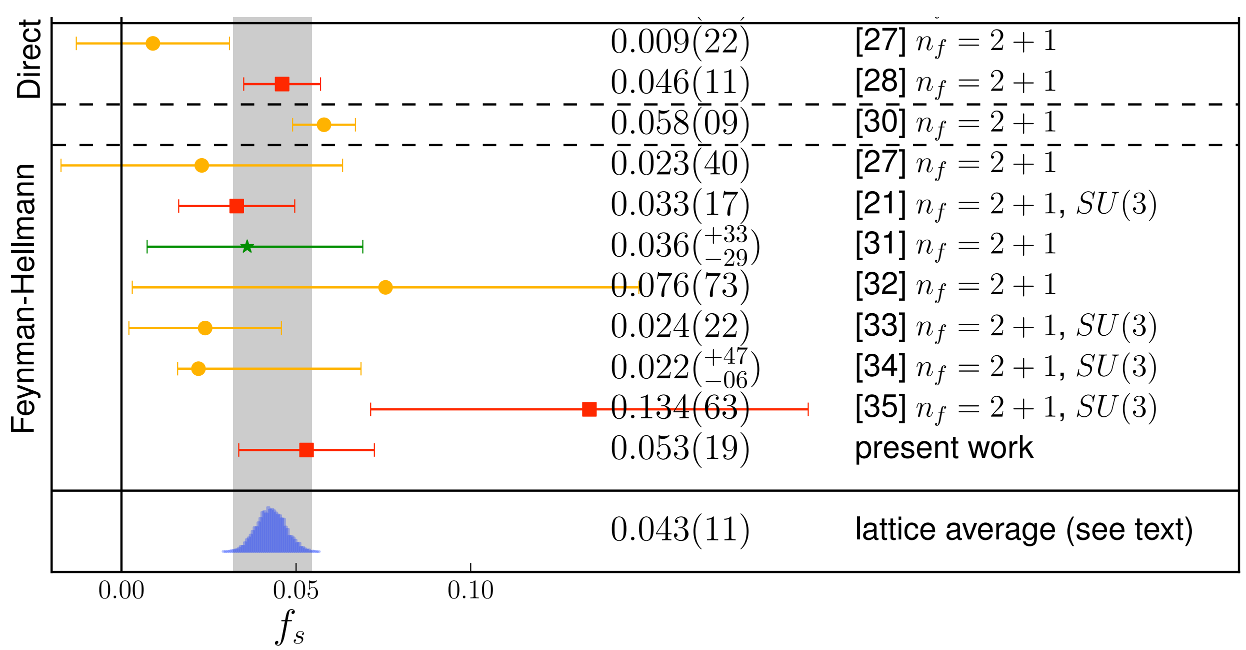
pQCD corrections in the RG running from μ_c to μ_0 and in the spin-0 gluon matrix element for triplet



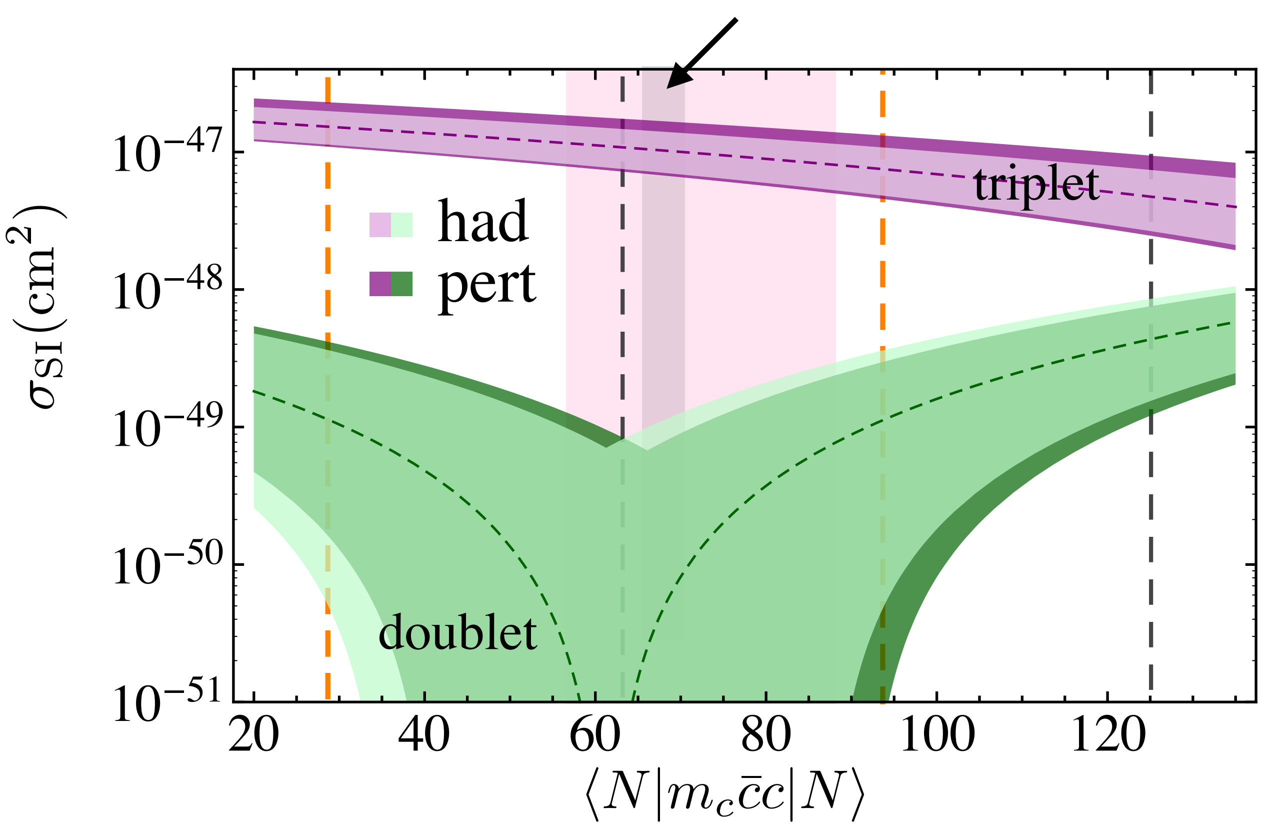
Sensitivity to model-independent inputs





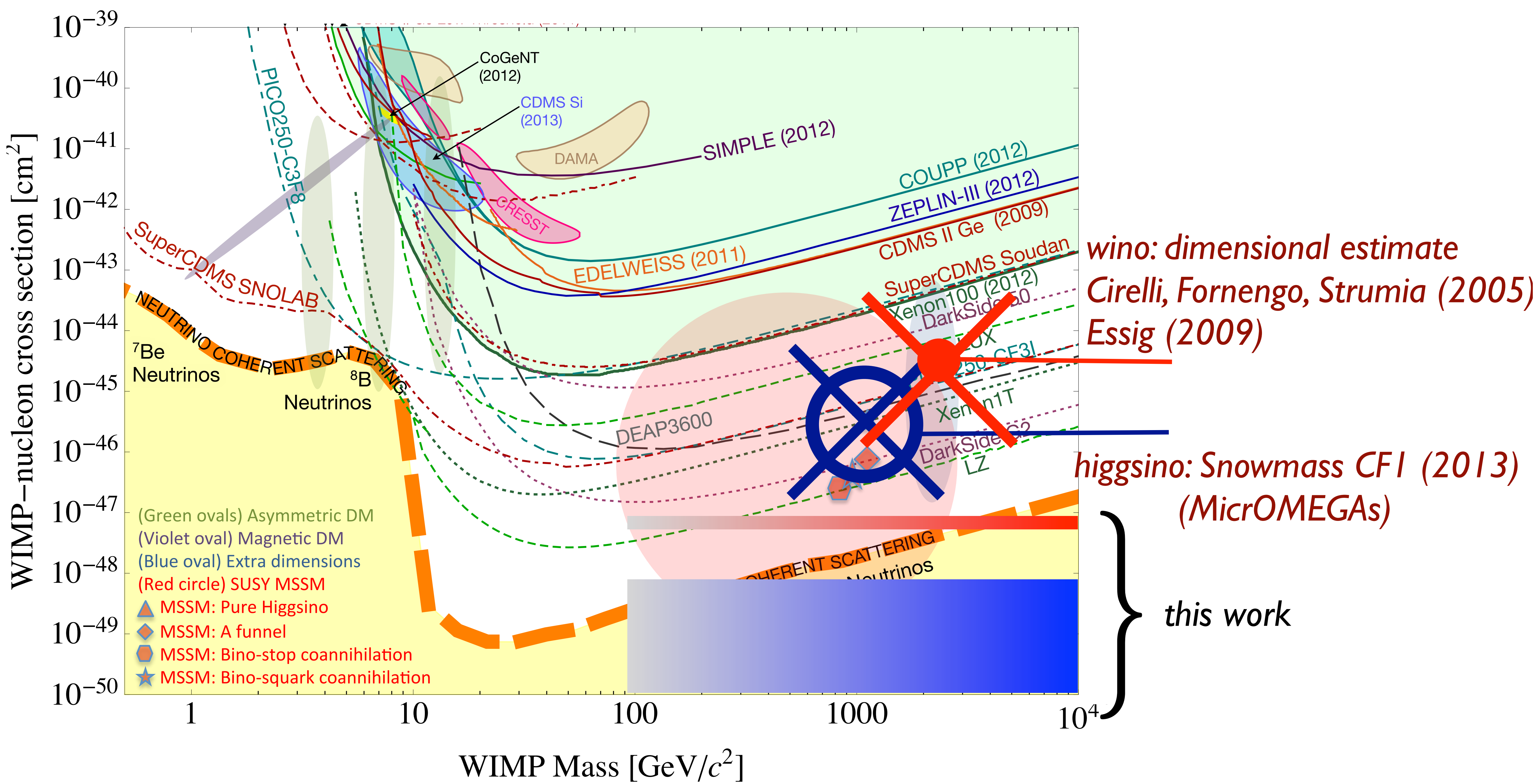


Junnarkar, Walker-Loud [1301.1114]



$$\text{J=1,Y=0:} \quad \sigma_{\rm SI} = 1.3^{+1.2+0.4}_{-0.5-0.3} \times 10^{-47}\,\mathrm{cm}^2$$

$$\text{J=1/2,Y=1/2:} \quad \sigma_{\rm SI} \lesssim 10^{-48}\,\mathrm{cm}^2 \quad (95\%\,\mathrm{C.L.})$$

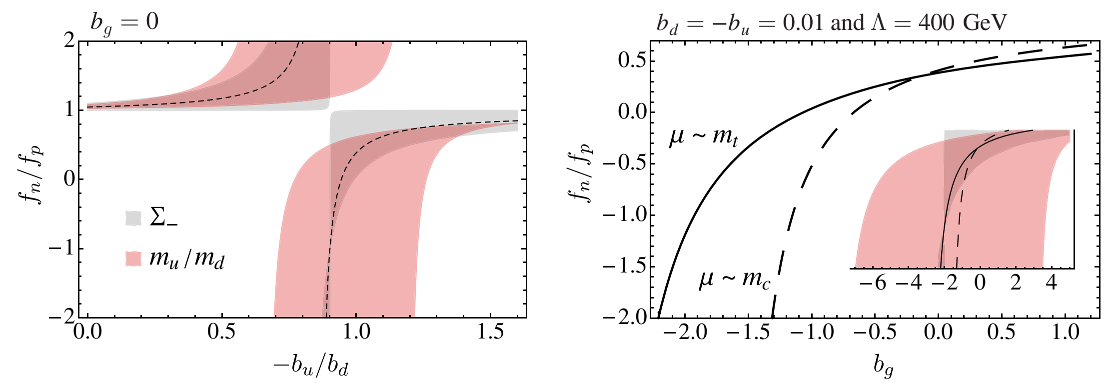


WIMP observables are interesting, multiple-scale field theory problems

M

annihilation: sommerfeld enhancement, bound states, thermal bath effects, Sudakov logs

production: complementarity



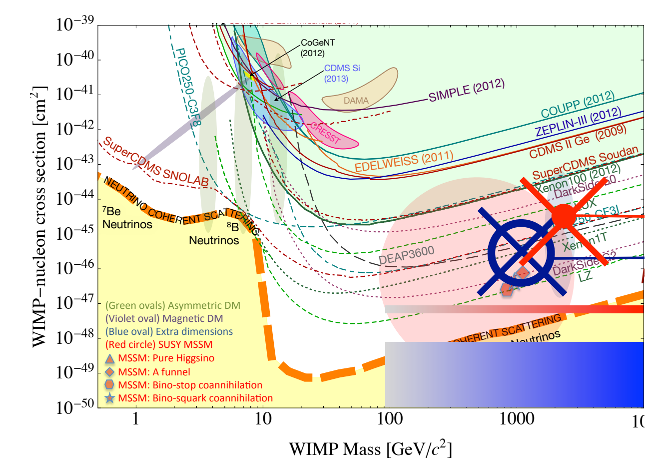
m_W

weak scale matching

power corrections, other UV completions

incorporate in other scenarios, or into something like micromegas

QCD and EW running

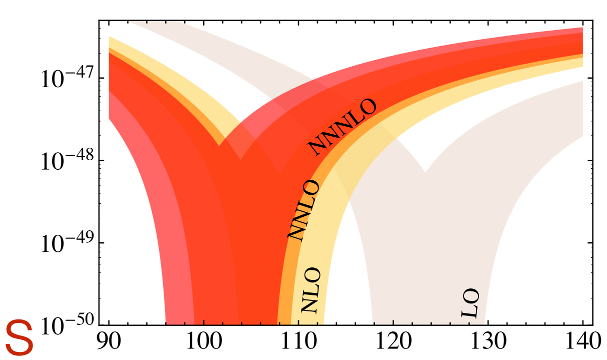


$m_b\,, m_c$

lattice: charm scalar matrix element

m_N

scattering: nucleon matrix elements, DM-nucleon EFT, multinucleon effects



Galilean vs Lorentz invariance

include other currents in analysis of multi-nucleon effects